ABSTRACT OF THESIS

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A CRITICAL STUDY OF FACTORS INVOLVED IN THE ECONOMICAL DESIGN OF PIPE SYSTEMS FOR PUMPING PLANTS

Submitted by

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I. INTRODUCTION

The purpose of this thesis is to make a comprehensive study of all the individual factors involved in the economical design of pipe systems for pumping plants, to analysis them in the light of hydraulics and known physical laws, and to correlate and combine them in order to obtain the most satisfactory pipe system for a given set of conditions. The factors involved are the study of the maximum pressure rise or fall due to water hammer, the computation of hydraulic losses along a pipe line, and the determination of the most economic size of pipe.

II. STUDY OF WATER HAMMER

Water hammer is the phenomenon arising in the pipes when the gate or valve is suddenly closed or opened. Due to the rapid change of velocity, a dynamic pressure will develop as a result of the sudden transformation of energies. Because this rise or fall of pressures often shows itself by making a noise similar to that produced by striking the pipe with a hammer, this phenomenon is usually referred to as water hammer. The intensity of this dynamic pressure depends upon the length of time elasping during closure of the gate, the initial velocity of flow in the pipe and the speed of propagation of the wave. By applying the momentum principle and the equation of continuity and by assuming that both the pipe walls and water are elastic, the following two fundamental differential equations can be obtained.

$$\frac{\partial H}{\partial x_1} = -\frac{1}{g}\frac{\partial V}{\partial t} \quad \text{.........................(1)}$$

$$\frac{\partial H}{\partial t} = -\frac{a^2}{g}\frac{\partial V}{\partial x_1} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

where "a" is the velocity of pressure wave in feet per second.

The solution of these two equations is

$$H - H_o = f(t + \frac{x}{a}) + F(t - \frac{x}{a}) \quad \dots\dots\dots\dots (3)$$

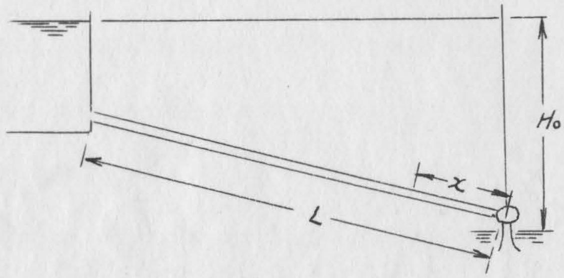
$$V - V_o = -\frac{g}{a}\left[- f(t + \frac{x}{a}) + F(t - \frac{x}{a})\right] \quad \dots\dots (4)$$

where F and f represent arbitrary functions of the quantities $t - \frac{x}{a}$ and $t + \frac{x}{a}$ respectively; $F(t - \frac{x}{a})$ is the sum of all the direct pressure waves at $(t - \frac{x}{a})$ seconds after closure begins and $f(t + \frac{x}{a})$ is the sum of all the reflected pressure waves at $(t + \frac{x}{a})$ seconds after closure begins.

In applying the theory of water hammer to pump lines, however, slight change is necessary. The serious conditions often occur after failure of electric power to the motors. Immediately after power failure, the pump continues to rotate in the positive direction pumping water up the pipe line as it slows down. A time is reached when the head in the discharge line is greater than that produced by the pump and the water flows back through the pump, with the pump continuing to rotate in the positive direction. Finally the pump slows to a stop and reverses its direction of rotation as a runaway turbine. It can be seen that after power failure the flow of water in the discharge pipe is controlled by the characteristics of the pump impeller and the inertia of the rotating elements of the pump. Since the analytical solution of water hammer problems is often too tedious and complicated, a graphical method is common in use.

The basic equations used for the graphical solution are the conjugate equations derived from the fundamental equations of water hammer (3) and (4), and the pump inertia equation deduced from the pump characteristics.

(1) Conjugate equations.



$$h_{At} - h_{Ct_1} = -2\rho(v_{At} - v_{Ct_1}) \qquad \text{..........(5)a}$$

$$h_{Ct_1} - h_{At_2} = +2\rho(v_{Ct_1} - v_{At_2}) \qquad \text{.......(5)b}$$

where $h = \frac{H}{H_o}$, $v = \frac{V}{V_o}$, and $\rho = \frac{a\,V_o}{2g\,H_o}$ which represents the pipe line characteristics.

(2) Pump inertia equation.

$$\alpha_1 - \alpha_2 = \frac{91,758\,Q_o\,H_o}{(WR^2)(N_o^2)(\eta_o)}(\beta_1 + \beta_2)\Delta t \qquad \text{.........(6)}$$

where $\alpha = \frac{N}{N_o}$, a ratio between the pump speed at any time to the normal speed for steady pumping conditions. $\beta = \frac{M}{M_o}$, a ratio between the pump input torque at any given speed and head to that corresponding to normal speed and rated pumping head. η_o= pump efficiency. WR^2 = flywheel effect of rotating parts in lb.-ft.^{2}

The procedure of the graphical solution may be outlined briefly as follows:

(1). Convert the pump performance curves to pump characteristics on the h - v diagram.

Since the pump characteristics are different for each pump, it is impossible to set up general rules to cover the solution of surge problems for all plants. The only satisfactory way is to study each pumping plant individually from the pump performance curves supplied by the manufacturer. These performance curves generally give the relations between the pumping head, the pump input horse-power, and the pump efficiency to the pump discharge. Using h and v as axes, two families of curves, α and β, are thus plotted from the pump performance curve and the known characteristics of the α and β curves.

(2) Determination of the maximum and minimum pressure due to water hammer for different length of time elapsing after the power failure and different locations along the pipe line.

From the conjugate equations, it can be seen that they may be represented graphically by straight lines with slopes equal to $\pm 2\rho$, the sign of which indicated the direction. The points A_o, $B_{\frac{1}{2}\frac{L}{a}}$, and $C_{\frac{L}{a}}$ are located at $h = 1$, and $v = 1$ on the h - v diagram, because at moments indicated by the subscripts (the pressure wave will not reach C until the end of $t = \frac{L}{a}$) these points are all in the normal operation condition. The point $A_{\frac{1}{4}\frac{L}{a}}$ is then located on the line with slope $+ 2\rho$ passing through the point $h = 1$, $v = 1$, on the h - v diagram. The exact position of this point, however, should be determined by means of the pump inertia equation by trial. The wave then reflects back until it reaches the discharge end C, the head acting on which is always a constant as unaffected by the wave surge. This can be done by drawing a line through point $A_{\frac{1}{4}\frac{L}{a}}$ with slope $= - 2\rho$ until it meets the line $h = 1$ on the h - v diagram at $C_{\frac{5}{4}\frac{L}{a}}$. By the same procedures, the points for $A_{\frac{9}{4}\frac{L}{a}}$, $C_{\frac{13}{4}\frac{L}{a}}$, etc.

can be located. Actually the wave is created one after one continuously as the pump slows down due to power failure. For simplicity, it might just trace the waves at instants $A_{\frac{1}{2}\frac{L}{a}}$, $A_{\frac{3}{4}\frac{L}{a}}$, and $A_{\frac{L}{a}}$, etc. followed the power failure. The connecting line of these points thus plotted on the h - v diagram will give the maximum and minimum pressures.

III. HYDRAULIC STUDIES

As the quantity of flow which will pass through a pipe depends upon the total head, the size of pipe, and the pipe losses, any loss of head in the pipe will tend to reduce the flow. Careful studies of hydraulic losses are, therefore, necessary to determine the economic size of pipes. The hydraulic losses in a pipe consist of the entrance loss in the trashrack, the friction loss in the pipe, the bend losses, and the losses due to contraction and expansion. The pipe friction loss may be computed by the well-known Scobey's formula.

$$H_f = K_s \frac{V^{1.9}}{D^{1.1}} \quad \text{..................(7)}$$

where H_f = head loss due to pipe friction per thousand feet, feet.

K_s = a constant which varies with the class and age of pipe.

V = velocity in pipe, feet per second.

D = inside diameter, feet.

The bend loss is computed by the formula suggested by J. Hinds.

$$H_1 = C \sqrt{\frac{\Delta}{90}} \frac{V^2}{2g} \quad \text{..................(8)}$$

where Δ = deflection angle, in degrees.

C = coefficient equal to 0.25 when $\frac{R}{D}$ is greater than 2.0

The other minor losses due to expansion and contraction of pipe and the entrance losses may be easily found from the ordinary texts on hydraulics.

IV. DETERMINATION OF THE MOST ECONOMIC SIZE OF PIPE

The most economical diameter is the one in which the total annual cost is a minimum and will be obtained when a proper balance between power loss and initial cost of pipe exists.

By expressing the total annual cost in a mathematical form, setting its first derivative with respect to D equal to zero, and solving for D_e, the following formula is obtained:

$$D_e = 0.50218 \sqrt[6.9]{\frac{K_s Q^{2.9}\ f b S_g e_j}{aHre\ (1 + i\ )}} \quad \text{............}(9)$$

where D_e = the economical diameter of pipe, feet.

Q = the rated discharge expressed in cu. ft. per sec.

K_s = a general coefficient in the Scobey formula.

f = load factor.

b = the value of the power loss in dollars per Kilowatt-hour.

S_g = the gross allowable tension in the steel pipe, pounds per square inch.

e_j = the joint efficiency of pipe.

a = the unit cost of steel in the pipe, dollars per pounds.

H = the weighted average head including water hammer, ft.

r = the ratio of the annual fixed, operating and maintenance charge to the construction cost of the pipe.

e = the over-all efficiency.

i = the percentage of overweight of steel in the pipe.

For practical design, the thickness of pipe along the whole line is not uniform for the purpose of economy. Equation (9), which is based on the weighted average water pressure and assumes uniform thickness all through the line, is therefore not exactly correct. The procedures for carrying out the detailed study are outlined as follows:

(1) Calculation of thickness of pipe.

Thickness of pipe is computed from the ordinary hoop tension formula. For different diameters, tabulate the relation between the thickness of pipe and the corresponding maximum allowable water head which can safely sustain.

(2) Graphically method for determining the length of pipe for certain pipe thickness.

Plot the profile of the pipe line from the pumping unit to the discharge end. The total head acting on the pumping unit is the summation of the lifting head, the hydraulic losses, and the maximum pressure rise due to water hammer. Determine the length of each portion of pipe line for different pipe thickness.

(3) Calculate the weight of pipe and the annual cost including interest, depreciation and maintenance for different sizes of pipes.

(4) Compute the power loss due to friction per year for different sizes of pipes.

The total annual cost for different sizes of pipes is then the summation of (3) and (4). The most economical size is the one which gives the minimum total annual cost.

V. SUMMARY

The factors involved in the economical design of pipe systems for pumping plants are (1) the determination of pressure rise and fall due to water hammer following a power failure, (2) the study of hydraulic losses in a pipe line and (3) the computation of the most economic size of pipe. The former two factors are the prerequisites in designing the pipe and the latter is the final procedure required to obtain the most economical design of pipe systems.

Based upon the principle of continuity, Newton's second law of motion and Hook's law, the fundamental equations of water hammer are developed. A graphical solution in determining the maximum and minimum pressure due to water hammer is introduced by combining the fundamental equations with the pump characteristics for a particular pumping plant. The maximum pressure thus obtained is used to determine the thickness of pipe shells, while the minimum pressure is of value to check the profile of pipe line if the hydraulic gradient falls below the pipe line.

The hydraulic losses in a pipe line consist of the entrance loss in the trashrack, the friction loss in the pipe, the bend losses, and the losses due to contraction and expansion. Their computations are based upon the principles of hydraulics. In order to make the results of study readily avaliable in convenient form to designers, tables and charts are prepared.

The most economical diameter is the one in which the total annual cost is a minimum and will be obtained when a proper balance between power loss and initial cost of pipe exists. A mathematical equation is derived in determining the most economical diameter of steel pipes.

However, for practical design, the thickness of steel pipe along the whole line is not uniform for the purpose of economy. Detailed study is, therefore, necessary. All items which have been considered as contributing to the rational selection of the most economical size of pipe have been inserted in a tabular form as shown in the table attached. By proceeding in order across the table from left to right all sixteen factors involved in the selection will be taken into consideration and properly evaluated in their effect upon the final selection. Only by this approach or a similar systematic attack can the designer be assured of a safe economical selection of pipe for each individual pumping plant.

TABLE 4

DETAILED STUDY OF ECONOMICAL DIAMETER

		Material							Hydraulic Losses					Capitalization			
Pipe dia.	Pipe dia.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
In Feet	in Inches	Plate thickness	Max. head	Weight per ft. of pipe net + 20% over weight and supports	Total length for each thickness	Weight for each plate thickness	Cost per foot in place	Total cost for each plate thickness	Velocity	Loss Per 1000 ft $K_s = 34$	Loss for each length	Friction loss per year	Friction loss per year	Friction loss per year per foot	Interest depr'c. & maint. per foot per year	Total cost of Pipe per foot	Weighted Average of Col. 15
(ft)	(in)	(in)	(ft.)	(lbs.)	(ft)	(lbs)	($)	($)	(ft/sec)	(ft)	(ft)	(KWH)	($)	($)	($)	($)	($)

T H E S I S

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1947

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Fang Yu Yang

ENTITLED A Critical Study of Factors Involved in The Economical Design of Pipe Systems for Pumping Plants

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF Science

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Permission to publish this thesis or any part of it must be obtained from the Dean of the Graduate School.

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CHAPTER I. INTRODUCTION

The design of steel pipe is not only a matter of hydraulics, but also a problem in structural analysis. Design of pipe systems, therefore, must be based upon the principles of hydraulics, the theory of structures, and at the same time, upon practicality in construction and operation. A review of the literature reveals that although each of these factors has been given considerable attention by previous investigators, no comprehensive study has been made which enables the designer to combine all of these factors to obtain the most satisfactory pipe system for a given set of conditions. The purpose of this thesis is to bring together these individual items and correlate them so that they can be used to obtain the best solution to practical design problems.

The computation of hydraulic losses along a pipe line, the determination of the maximum pressure rise due to water hammer and the economic size of pipe are the main features of design from the point of view of hydraulics, while the thickness required for the pipe shell and the design of stiffener rings are the important items for the structural design. These items are the important factors involved in the design of steel pipes. The methods of design of pipe thickness and stiffener rings have already been developed. The thickness of pipe shell is ordinarily determined by the hoop tension formula which can be found from texts on hydraulics and strength of materials. The stiffener rings are used to resist the shear force and moment developed at supports, to maintain the circularity of the pipe shell, and to carry the load either to concrete piers or to rockers. The principles of design involved have also been presented in the "Design of Large Pipe Lines" by Schorer, H.[1] and the "Penstock Analysis and Stiffener Design" by the U. S. Bureau of Reclamation[2]. Therefore these two factors will not be considered in this thesis.

In order to make the results of this study readily available in convenient form to designers, tables and charts will be prepared.

The largest sizes of penstocks that have been built are those installed at the Boulder Dam with a diameter of thirty feet and a shell thickness of $2\frac{3}{4}$"[3]. For pumping plants, however, the maximum diameter of pipe used is twelve feet.[4] It is the manufacturers' opinion that pipe thickness greater than four inches will prove too expensive for economical use. Theoretically, no limitation has yet been set for the maximum lifting head. Several pumps can be used in series when one pump is not enough to serve the purpose. The length of pipe is limited by the maximum water pressure including the water hammer, the hoop-tension stress, and the allowable pipe thickness. In all practical cases the greater the length of pipe the more severe is the effect of water hammer.

In general, pipes for pumping plants are exposed and are

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1 Schorer, Herman. Design of large pipe lines. Trans., American society of civil engineers, Vol. 98, pp. 101, 1933.

2 U. S. Bureau of reclamation. Penstocks analysis and stiffener design. Final report of Boulder Canyon Project, part V, bulletin no. 5. Washington, 1940.

3 U. S. Bureau of reclamation. Construction of Boulder Dam, power plant and appurtenant. Specification no. 519. Washington, 1931.

4 U. S. Bureau of reclamation. Plate steel discharge pipe for Grand Coulee Pumping Plant, Columbia Basin Project, Washington. Specification no. 1338. Washington, 1946.

supported by rocker supports and anchorages. The spacing between these supports usually ranges from forty feet to sixty feet. It can be shown that spans greater than sixty feet are not economical because of the rapidly increasing stress due to beam bending, although it depends also upon the diameter of pipe.

The scope of this thesis is, therefore, confined to those values which are most frequently encountered in the design of pipe systems for pumping plants.

CHAPTER II. STUDY OF WATER HAMMER

Water hammer is the phenomenon arising in pipes when the gate or valve is suddenly closed or opened. Due to the rapid change of velocity, a dynamic pressure will develop as a result of the sudden transformation of energies. Because this rise or fall of pressures often shows itself by making a noise similar to that produced by striking the pipe with a hammer, this phenomenon is usually referred to as water hammer. The intensity of this dynamic pressure depends upon the length of time elapsing during closure of the gate, the initial velocity of flow in the pipe and the speed of propagation of the wave. The longer the time taken to destroy the velocity and the less the initial velocity in the pipe, the smaller will be the magnitude of the dynamic pressure, and the less is the danger from the ensuing shock.

The same phenomenon will also occur in the case of pumping pipe lines. With low velocities of flow and the pumps driven by the engines so much in use in earlier days, the problem did not assume its present proportions; now that centrifugal pumps driven by electric motors are in common use failure of the electrical supply during operation often results in shocks on the pipes and pumps sufficient to shake the stations and often to damage the pumping system. In such cases it is important to find the magnitude of the change in pressure due to water hammer, and to design the pipe to resist these dangerous pressures.

In attacking the problem of water hammer, two broad methods of attack have been used. In the first approach both water and pipe walls are considered to be rigid, and in the second both pipe walls and water are treated as elastic. The second case is much the more general and more truly representative of the conditions that exist. It has, therefore, been accepted as the basis of theoretical analysis in this thesis.

A. Fundamental Equations of Water Hammer.

1. General Equations for Pressure Rise or Fall.

In the elastic column theory, it is generally assumed that the pipe line friction, entrance loss and the velocity head are negligible as compared with the rise in head due to water hammer.

Two principles will be used. One is the Newton's second law of motion - the resulting force acting on an element is equal to its time rate of change of momentum, i. e. $F = M\frac{dV}{dt}$ and the other is the principle of continuity.

First consider the Newton's second law of motion. Fig. 1 shows the forces acting on an element of water with length Δx_1.

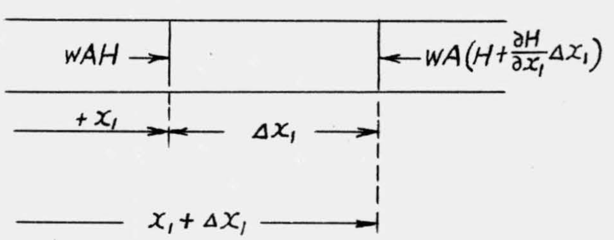

Fig. 1

Let H = total pressure head including pressure rise due to water hammer, ft.

V = velocity of flow in pipe, ft. per second.

x_1 = the distance measured positive from the reservoir (as the origin) toward the gate in the direction of flow, ft.

The unbalanced force is $wA \frac{\partial H}{\partial x_1} \Delta x_1$, the mass to be moved is $\frac{wA \Delta x_1}{g}$ and acceleration is $- \frac{dV}{dt}$.

Then,

$$wA \frac{\partial H}{\partial x_1} \Delta x_1 = - \frac{wA \Delta x_1}{g} \frac{dV}{dt}$$

or

$$\frac{\partial H}{\partial x_1} = - \frac{1}{g} \frac{dV}{dt} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where w = unit weight of water, lb. per cu. ft.

A = pipe cross-sectional area, sq. ft.

g = gravitational acceleration, ft. per sec. per sec.

Water hammer is actually a series of pressure waves created one after another as the gate is being closed, interfered and mixed, and reflected back and forth in the pipe until finally dying out by pipe friction. The velocity of flow in the pipe, affected by these pressure waves, varies for different locations and different instants of time. H and V are, therefore, both a function of x_1 and t. Then it can be written

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_1} \frac{dx_1}{dt} = \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x_1}$$

The term $V \frac{\partial V}{\partial x}$ may be negligible when compared to $\frac{\partial V}{\partial t}$, then Equation (1) becomes

$$\frac{\partial H}{\partial x_1} = - \frac{1}{g} \frac{\partial V}{\partial t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From the principle of continuity, it is apparent that if space is made available by the expansion of the pipe shell and the compression of the water under the influence of an excess pressure, water will flow into and fill the space thus provided.

Fig. 2 shows the conditions of flow at two successive instants T_1 and $T_1 + \Delta t$ during which the element AB has moved to CD. The pressure and velocity at A, B, C, and D are as given in Fig. 2.

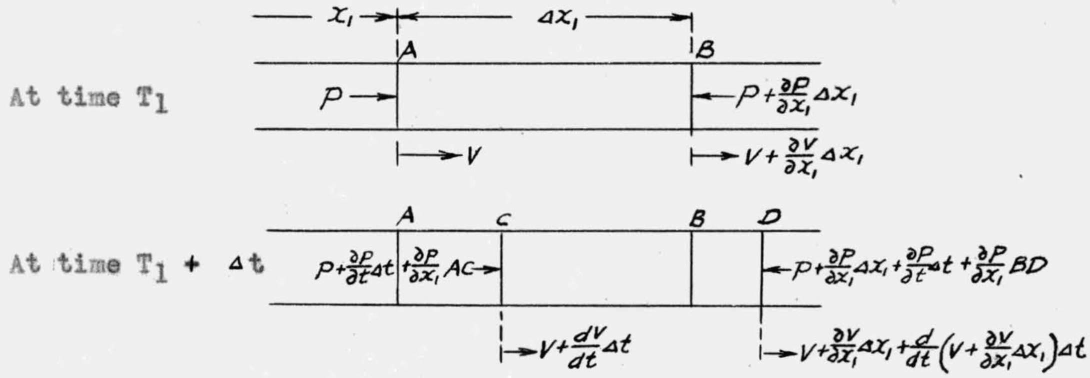

Fig. 2

To satisfy the principle of continuity, the element of water AB must compress into the element CD. The reduction in length of the element of water Δx_1 due to an increase in pressure is given by (AC - BD). Furthermore it is also equal to the effect of stretching the pipe shell and of compressing the water. First the quantity (AC - BD) should be determined. If Δt is taken sufficiently small then

AC = average velocity in the interval AC $\times \Delta t$

$$= \left[V + \frac{1}{2}\frac{dV}{dt}\Delta t\right]\Delta t = V\Delta t\left[1 + \frac{\Delta t}{2}\left(\frac{1}{V}\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_1}\right)\right]$$

and BD = average velocity in the interval BD $\times \Delta t$

$$= \left[\left(V + \frac{\partial V}{\partial x_1}\Delta x_1\right) + \frac{1}{2}\frac{d}{dt}\left(V + \frac{\partial V}{\partial x_1}\Delta x_1\right)\Delta t\right]\Delta t$$

Neglecting infinitesimals of order higher than the first, then

$$AC - BD = -\frac{\partial V}{\partial x_1}\Delta x_1\,\Delta t \qquad \text{.................}(3)$$

The reduction in length of an element of water Δx_1 due to an increase in pressure ΔP is made up of two parts, namely, that resulting from the elasticity of the conduit wall and that due to the compressibility of the fluid. The radial deflection of a shell under the action of an increased internal pressure ΔP is[1]

$$\delta = \frac{\Delta PR^2}{Ee} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

where δ = radial deflection of shell, ft.

ΔP = an increased internal pressure, lb. per sq. ft.

R = inside radius of pipe, ft.

E = modulus of elasticity, lb. per sq. ft.

e = thickness of pipe shell, ft.

Then the change in volume due to the stretching of the shell is

$$\left[\pi(R + \delta)^2 - \pi R^2\right]\Delta x_1 = 2\pi R\,\delta\,\Delta x_1 \qquad \ldots\ldots\ldots(5)$$

since $\pi\delta^2$ is very small when compared to $2\pi R\delta$ and may be neglected.

Substitution of Equation (4) in (5), gives the change in volume due to stretching of the pipe shell,

$$2\pi R\delta\,\Delta x_1 = \frac{2\pi R\Delta PR^2}{Ee}\Delta x_1 \qquad \ldots\ldots\ldots\ldots\ldots(6)$$

Equation (6) divided by $\pi(R + \delta)^2$ gives the reduction in length,

$$\frac{2\pi R\,\Delta PR^2\Delta x_1}{Ee\,\pi(R + \delta)^2} = \frac{2R\Delta P\,\Delta x_1}{Ee} \qquad \ldots\ldots\ldots\ldots\ldots(7)$$

The change in volume due to the compressibility of the water under the internal pressure ΔP is

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1 Timoshenko, S. Theory of plated and shells. pp. 404. N. Y. McGraw-Hill book co., 1940.

$\frac{\Delta P}{K}$ x volume of water in the length Δx_1

$$= \frac{\pi R^2 \Delta x_1}{K} \Delta P \quad \ldots\ldots\ldots\ldots (8)$$

where K = bulk modulus of water which is defined as the ratio of unit compressive stress to the volumetric strain.

Equation (8) divided by the area gives the change in length due to compressibility of water,

$$\frac{\pi R^2 \Delta x_1 \Delta P}{K \pi R^2} = \frac{\Delta x_1 \Delta P}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Then the reduction in length of an element of water Δx_1 due to an increased pressure ΔP is the sum of (7) and (9),

$$\frac{2R}{Ee} \Delta P \Delta x_1 + \frac{\Delta x_1 \Delta P}{K} = \Delta x_1 \left[\frac{D}{Ee} + \frac{1}{K}\right] \Delta P \quad \ldots\ldots (10)$$

The total pressure change, ΔP, during Δt is

$$\Delta P = \frac{dP}{dt} \Delta t \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

But P is a function of x_1 and t, and expressed as

$$\frac{dP}{dt} = \frac{\partial P}{\partial t} + \frac{\partial P}{\partial x_1}\frac{dx_1}{dt} = \frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x_1} \quad \ldots\ldots\ldots\ldots\ldots (12)$$

Substitution of Equations (11) and (12) in (10), gives

$$\text{the reduction in length} = \Delta x_1 \left[\frac{D}{Ee} + \frac{1}{K}\right]\left[\frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x_1}\right] \Delta t \quad \ldots\ldots (13)$$

Now we may equate Equation (3) to (13),

$$-\frac{\partial V}{\partial x_1} \Delta x_1 \Delta t = \Delta x_1 \left[\frac{D}{Ee} + \frac{1}{K}\right]\left[\frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x_1}\right] \Delta t$$

$$\frac{\partial V}{\partial x_1} = -\left[\frac{D}{Ee} + \frac{1}{K}\right]\left[\frac{\partial P}{\partial t} + V \frac{\partial P}{\partial x_1}\right]$$

The term $V \frac{\partial P}{\partial x_1}$ is very small when compared to $\frac{\partial P}{\partial t}$ and may be neglected, then

$$\frac{\partial V}{\partial x_1} = -\left[\frac{D}{Ee} + \frac{1}{K}\right] \frac{\partial P}{\partial t} \quad \ldots\ldots\ldots\ldots\ldots (14)$$

Since $P = wH$ and $\frac{\partial P}{\partial t} = w \frac{\partial H}{\partial t}$, Equation (14) becomes

$$\frac{\partial V}{\partial x_1} = -\left[\frac{D}{Ee} + \frac{1}{K}\right] w \frac{\partial H}{\partial t}$$

or $$\frac{\partial H}{\partial t} = -\frac{a^2}{g}\frac{\partial V}{\partial x_1} \qquad \text{.....................(15)}$$

where $$a = \sqrt{\frac{1}{\frac{w}{g}\left[\frac{D}{Ee} + \frac{1}{K}\right]}} \qquad \text{.....................(15)a}$$

and represents the velocity of pressure wave.

From the two principles of motion, Equations (2) and (15) may now be solved simultaneously. Two differential equations are obtained:

$$\frac{\partial^2 V}{\partial t^2} = a^2 \frac{\partial^2 V}{\partial x_1^2} \qquad \text{.....................(16)}$$

and,

$$\frac{\partial^2 H}{\partial t^2} = a^2 \frac{\partial^2 H}{\partial x_1^2} \qquad \text{.....................(17)}$$

By solving these two partial differential equation, we get

$$H - H_o = f(t - \frac{x_1}{a}) + F(t + \frac{x_1}{a}) \qquad \text{...............(18)a}$$

$$V - V_o = \frac{g}{a}\left[f(t - \frac{x_1}{a}) - F(t + \frac{x_1}{a})\right] \qquad \text{...............(18)b}$$

Equations (18) are the fundamental water hammer equations. But for convenience, the distance from the gate to the reservoir will be considered as positive since the disturbance occurs initially at the gate as shown in fig. 3 and moves toward the reservoir. By substituting $-x$ for $+x_1$, Equations (18) becomes[1]

$$H - H_o = F(t - \frac{x}{a}) + f(t + \frac{x}{a}) \qquad \text{...........(19)a}$$

$$V - V_o = -\frac{g}{a}\left[F(t - \frac{x}{a}) - f(t + \frac{x}{a})\right] \qquad \text{...........(19)b}$$

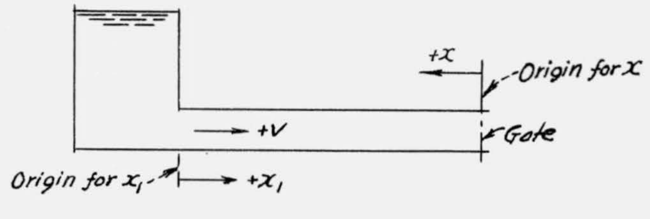


Fig. 3

[1] Angus, Robert W. Water hammer in pipes, including those supplied by centrifugal pumps. University of Toronto, Toronto, Canada. Bulletin no. 152, 1938.

In these expressions H is the pressure head, and V, the velocity, at a point on the pipe x feet from the gate at t seconds after closure begins; $F(t - \frac{x}{a})$ is the sum of all the direct pressure waves at $(t - \frac{x}{a})$ seconds after closure begins; $f(t + \frac{x}{a})$ is the sum of all the reflected pressure waves at $(t + \frac{x}{a})$ seconds after closure begins; and H_o and V_o are the initial pressure head and velocity before the closure of gate.

2. Velocity of Pressure Wave, "a".

The velocity of pressure wave, "a", in a pipe is defined in Equation (15)a,

$$a = \sqrt{\frac{1}{\frac{w}{g}\left[\frac{D}{Ee} + \frac{1}{K}\right]}} \quad \text{..............(15)a}$$

where w = unit weight of water = 62.5 lb. per cu. ft.

g = gravitational acceleration = 32.2 ft. per sec. per sec.

K = bulk modulus of water = 294,000 lb. per sq. in.

= 294000 x 144 lb. per sq. ft.

E = Young's modulus for steel pipe wall approximately

= 29,400,000 lb. per sq. in.

= 29400000 x 144 lb. per sq. ft.

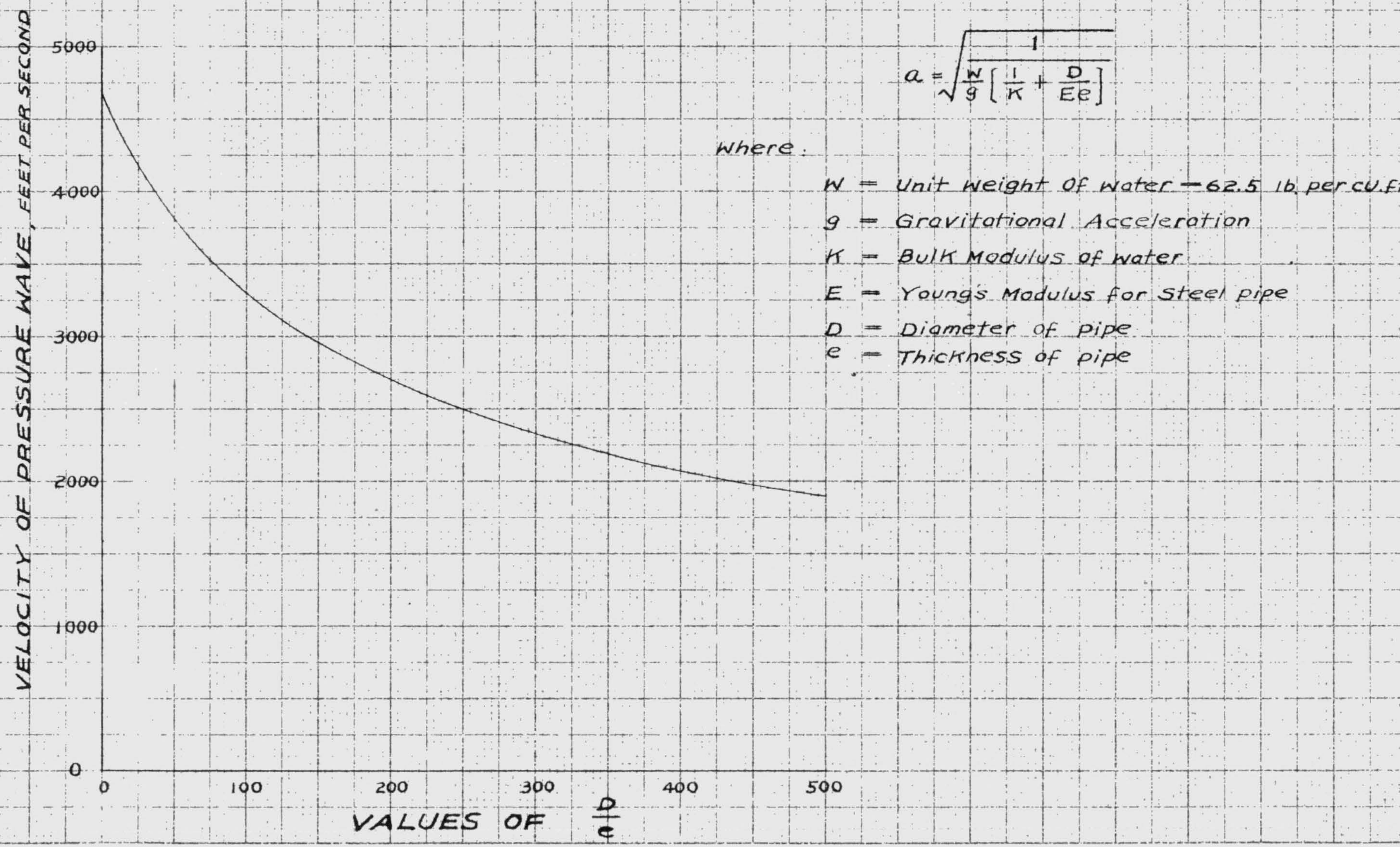
Substituting these values in Equation (15)a,

$$a = \frac{4,660}{\sqrt{1 + \frac{D}{100e}}} \quad \text{...........(15)b}$$

In order to simplify the calculation, the curve of "a" is plotted by using the ratio of $\frac{D}{e}$ against "a" as shown in Fig. 4, where D is the inside diameter of pipe, and e is the thickness of pipe wall of the same unit.

FIG. 4

VELOCITY OF PRESSURE WAVE IN PIPES



B. Graphical Solution of Water Hammer for Pumping Plants.

When a pumping plant and its pipe line are being designed it is necessary to know what water hammer conditions are likely to be encountered. Ordinarily, the worst conditions occur after failure of electric power to the pump motors. For a system without a check valve at the pump, where the water in the discharge line can return through the pump, a certain series of events occurs after power failure. Immediately after power failure, the pump continues to rotate in the positive direction pumping water up the pipe line as it slows down. A time is reached when the head in the discharge line is greater than that produced by the pump and the water flows back through the pump, with the pump continuing to rotate in the positive direction. Later the pump slows to a stop and reverses its direction of rotation, then it operates in reverse as a runaway turbine.

It can be seen that after power failure the flow of water in the discharge pipe is controlled by the characteristics of the pump impeller and the inertia of the rotating elements of the pump. Since the pump characteristics are different for each pump, it is not possible to set up general rules to cover the solution of surge problems for all plants. The only satisfactory procedure is to study each pump plant individually.

The analytical solution of water hammer problems is often very tedious and complicated. The process of tracing the effects of the different waves (direct and reflected waves) produced, so as to make a proper summation, is very lengthy. Therefore the following graphical method is presented for use instead of the analytical method previously outlined.

1. Basic Equations for Graphical Solution.

Two sets of basic equations are necessary in solving the water hammer problems. They are the conjugate equations and the pump inertia equation as developed in the following two articles.

The symbols used in this article are defined as follows:

H = pumping head at any time, ft.

H_o = rated pumping head, ft.

$h = \frac{H}{H_o}$

N = pump speed at any time in revolutions per minute.

N_o = normal pump speed for steady pumping conditions.

$\alpha = \frac{N}{N_o}$

M = pump input torque corresponding to a given speed and head.

M_o = pump input torque corresponding to normal speed and rated pumping head.

$\beta = \frac{M}{M_o}$

η = pump efficiency.

η_o = pump efficiency at the rated pumping head.

ω = angular velocity in radians per second = $\frac{2\pi N}{60}$

Q = pump discharge at any time in cu. ft. per second.

Q_o = pump discharge at rated pumping head in cu. ft. per second.

$q = \frac{Q}{Q_o} = v = \frac{V}{V_o}$

WR^2 = flywheel effect of rotating parts in pounds ft.^{2}

I = moment of inertia of rotating parts = $\frac{W R^2}{g}$

(a). Conjugate Equations.

In using graphical methods, two conjugate equations should be first derived from the fundamental equations.

Using Equations (19)a and (19)b, multiply Equation (19)b by $\frac{a}{g}$ and subtract from Equation (19)a, then

$$H - H_o = -\frac{a}{g}(V_o - V) + 2F(t - \frac{x}{a}) \qquad \text{.............(20)a}$$

and add to Equation (19)a,

$$H - H_o = \frac{a}{g}(V_o - V) + 2f(t + \frac{x}{a}) \qquad \text{.............(20)b}$$

Equation (20)a evidently applies to the direct wave as it includes the F term only while Equation (20)b applies to the reflected wave as it includes the f term only.

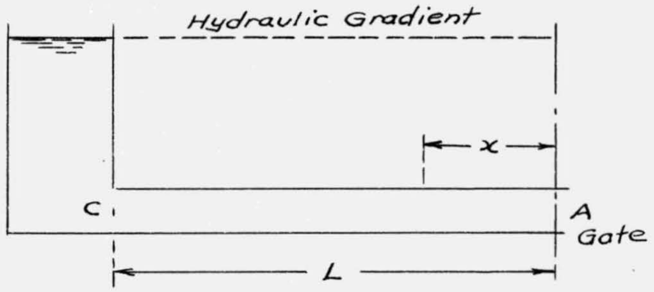


Fig. 5

Now consider a pipe line AC as shown in Fig. 5. Since the entrance and friction losses and the velocity head are neglected, the hydraulic gradient is horizontal. If the subscript o is used to denote the condition of initial steady flow, it follows that:

$$H_{Ao} = H_{Co} = H_o \qquad \dots\dots\dots\dots(21)$$

Applied to Equation (20), it gives the following series:

For the direct wave:

$$H_{At} - H_{Ao} = -\frac{a}{g}(V_{Ao} - V_{At}) + 2F(t) \quad (x = 0) \qquad \dots\dots\dots(22)$$

$$H_{Ct_1} - H_{Co} = -\frac{a}{g}(V_{Co} - V_{Ct_1}) + 2F(t_1 - \frac{L}{a}) \quad (x = L)$$

$$\text{or} \quad H_{Ct_1} - H_{Co} = -\frac{a}{g}(V_{Co} - V_{Ct_1}) + 2F(t) \quad (t_1 = t + \frac{L}{a}) \qquad \dots\dots\dots(23)$$

For the reflected wave:

$$H_{Ct_1} - H_{Co} = +\frac{a}{g}(V_{Co} - V_{Ct_1}) + 2f(t_1 + \frac{L}{a}) \quad (x = L)$$

$$\text{or} \quad H_{Ct_1} - H_{Co} = +\frac{a}{g}(V_{Co} - V_{Ct_1}) + 2f(t_2) \qquad \dots\dots\dots(24)$$

$$H_{At_2} - H_{Ao} = +\frac{a}{g}(V_{Ao} - V_{At_2}) + 2f(t_2) \qquad \dots\dots\dots(25)$$

Subtracting Equation (23) from (22), gives

$$H_{At} - H_{Ao} - H_{Ct_1} + H_{Co} = -\frac{a}{g}(V_{Ao} - V_{At} - V_{Co} + V_{Ct_1})$$

Since $H_{Ao} = H_{Co}$, $V_{Ao} = V_{Co}$

$$\text{or} \quad H_{At} - H_{Ct_1} = +\frac{a}{g}(V_{At} - V_{Ct_1}) \qquad \dots\dots\dots(26)$$

Similarly, subtracting Equation (25) from (24), gives

$$H_{Ct_1} - H_{At_2} = -\frac{a}{g}(V_{Ct_1} - V_{At_2}) \qquad \dots\dots\dots(27)$$

By dividing Equation (26) by H_o and rewriting in the form,

$$\frac{H_{At}}{H_o} - \frac{H_{Ct_1}}{H_o} = 2\frac{aV_o}{2gH_o}\left[\frac{V_{At}}{V_o} - \frac{V_{Ct_1}}{V_o}\right]$$

we obtain,

$$h_{At} - h_{Ct_1} = 2\rho(v_{At} - v_{Ct_1}) \qquad \dots\dots\dots(28)$$

where $h = \frac{H}{H_o}$, $v = \frac{V}{V_o}$, and ρ is the pipe line characteristics, and

$$\rho = \frac{aV_o}{2gH_o} \qquad \text{.......................(29)}$$

Similarly, by dividing Equation (27) by H_o and rewriting in the form

$$\frac{H_{Ct_1}}{H_o} - \frac{H_{At_2}}{H_o} = -2\,\frac{aV_o}{2gH_o}\left[\frac{V_{Ct_1}}{V_o} - \frac{V_{At_2}}{V_o}\right]$$

we obtain,

$$h_{Ct_1} - h_{At_2} = -2\rho(v_{Ct_1} - v_{At_2}) \qquad \text{................(30)}$$

Equations (28) and (30) are conjugate equations for solving the water hammer problem graphically both for penstock lines and pumping pipe lines. However, when they are applied to pumping pipe lines, the sign for the slope "2ρ" should be reversed. This is because in the case of pumping lines the pressure wave developed is in the same direction with the flow immediately following a power failure while for power penstocks, the wave is opposite in direction with the flow when the gate is being closed.

(b). Pump Inertia Equation.

In order to make a water hammer analysis of the transient conditions a relation between the pump speed and torque at all times will be needed.

In a rotating system the torque is equal to the moment of inertia of the rotating system multiplied by the angular acceleration. That is

$$M = I\frac{d\omega}{dt} = \frac{WR^2}{g}\,\frac{d\omega}{dt} \qquad \text{...............(31)}$$

Consider a small time interval Δt and let the subscripts 1 and 2 denote the values at the beginning and end of the interval. Then Equation (31) becomes

$$\frac{M_1 + M_2}{2} = \frac{WR^2}{g}\,\frac{\omega_1 - \omega_2}{\Delta t} = \frac{2\pi WR^2}{60\,g}\,\frac{N_1 - N_2}{\Delta t}$$

or
$$N_1 - N_2 = \frac{15g}{\pi(WR^2)}(M_1 + M_2)\,\Delta t$$

This equation may be then written in the following form since

$\alpha = \frac{N}{N_o}$ and $\beta = \frac{M}{M_o}$,

$$\alpha_1 - \alpha_2 = \frac{15g}{\pi(WR^2)}\,\frac{M_o}{N_o}\,(\beta_1 + \beta_2)\,\Delta t \qquad \text{............(32)}$$

But the pump input torque at the rated head is given by

$$M_o = \frac{60\,Q_o H_o w}{2\pi N_o \eta_o}$$

Then
$$\alpha_1 - \alpha_2 = \frac{91,758\,Q_o H_o}{(WR^2)(N_o^2)\eta_o}\,(\beta_1 + \beta_2)\,\Delta t$$

or
$$\alpha_1 - \alpha_2 = K_1(\beta_1 + \beta_2)\,\Delta t \qquad \text{.............(33)}$$

where
$$K_1 = \frac{91,758\,Q_o\,H_o}{(WR^2)(N_o^2)\eta_o} \qquad \text{.............(33)a}$$

Equation (33) is the necessary relation between the speed and torque at any time following power failure.

2. Graphical Solution.

(a). Conversion of Pump Performance Curves to Pump Characteristics on the h - v Diagram.

In order to use the graphical method for the solution of water hammer with the conjugate equations, Equations (28) and (30), it will be necessary to have the pump speed and torque relations plotted on the coordinates of h and v where h is the head ratio and v is the velocity ratio.

For the normal pump operation, the required pump torque and speed curves on the h - v diagram may be readily obtained from the usual pump performance data which is supplied by the manufacturer. A typical example of the pump performance data is shown in Fig. 6.

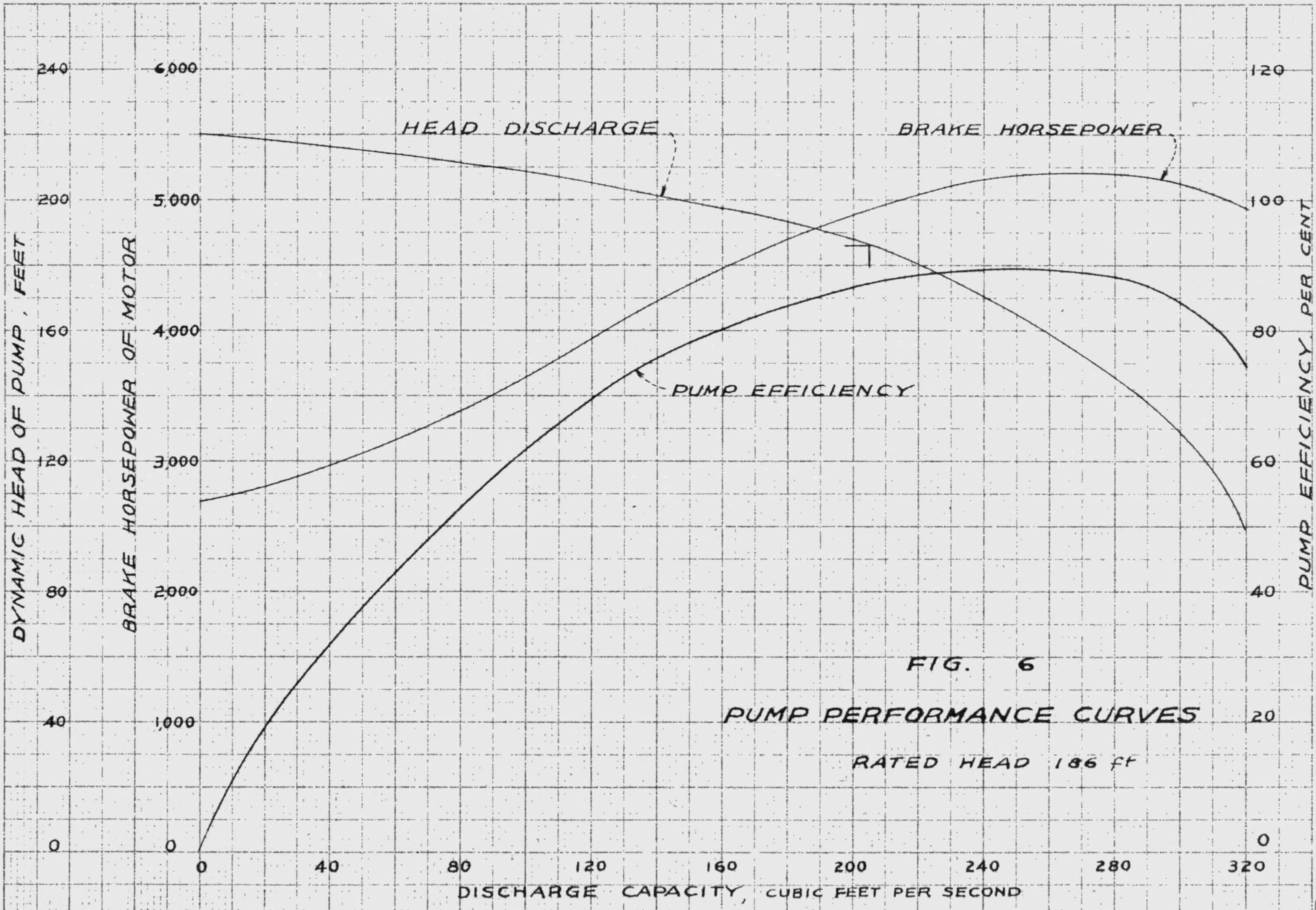


FIG. 6

PUMP PERFORMANCE CURVES

RATED HEAD 186 ft

These curves generally include the following data for the normal speed of the pump:

(1) H, the pumping head or dynamic head, plotted against Q, the pump discharge.

(2) Motor brake horsepower or pump input horsepower plotted against Q.

(3) η, the pump efficiency, plotted against Q.

Using h and v as axes, two families of curves for α and β are plotted as shown in Fig. 7. The procedure used in plotting α and β curves is briefly outlined as follows:

First step : Plot the curve for $\alpha = 1$ on the h - v diagram from the head discharge curve given in Fig. 6.

Second step : Since the manufacturers' pump performance curves are all based on the rated speed of the pump, the motor brake horsepower is directly proportional to the pump input torque. Taking the brake horsepower at the rated head as unity, the discharge corresponding to a torque of 0.9M, 0.8M, etc. may be computed from the brake horsepower curve and plotted on the $\alpha = 1$ curve as $\beta = 0.9$, 0.8 etc.

Third step : Since h and β are proportional to v^2 or q^2, the parabola curves of the type $h = k\,v^2$ can be draw through the points $\beta = 1.0$, 0.9, 0.8, etc. which have already been located on the $\alpha = 1.0$ line.

Fourth step : Points on the torque ratio lines, β , which correspond to a uniform change in torque ratio's are then

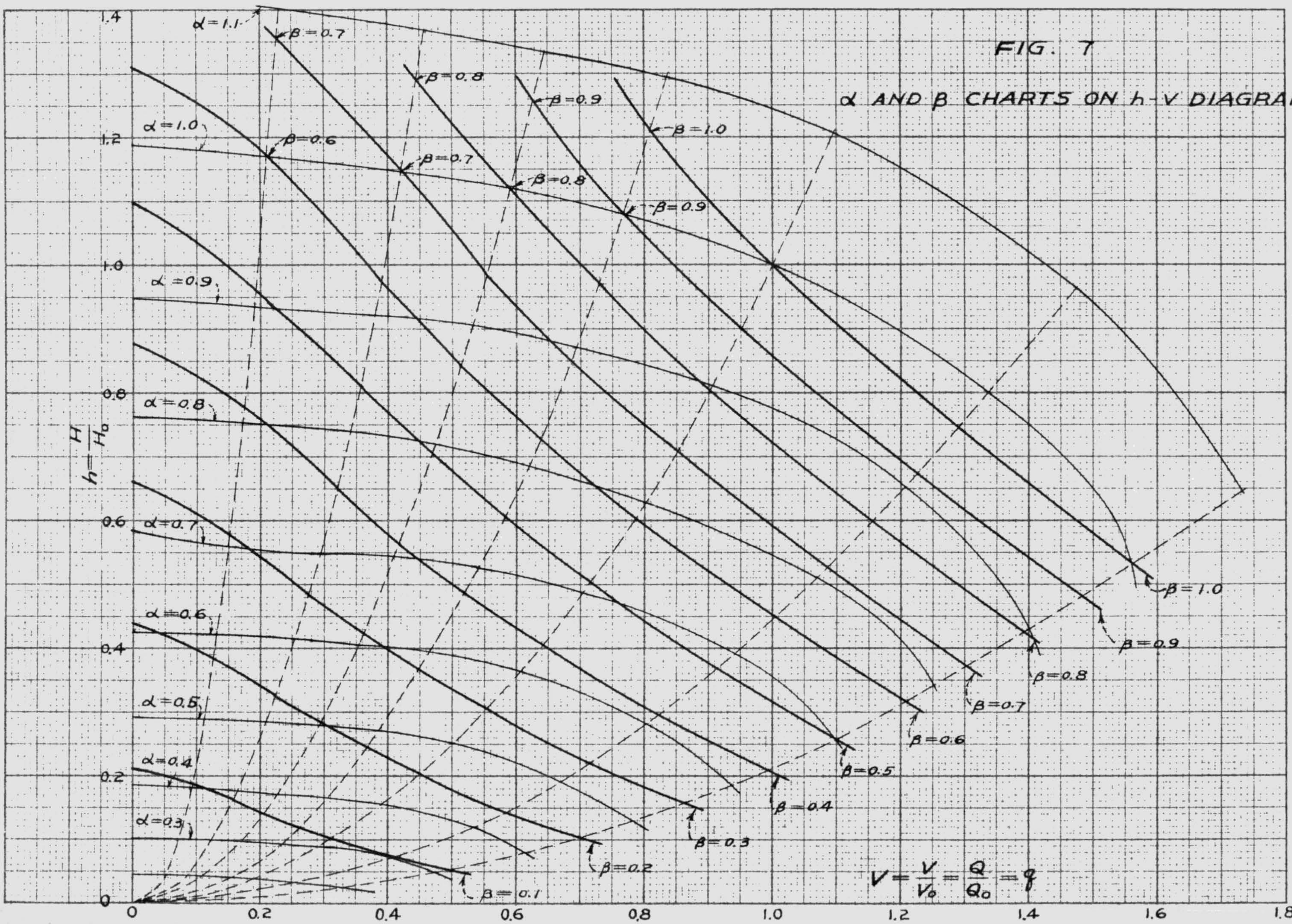


FIG. 7

α AND β CHARTS ON h-V DIAGRAM located on these parabolas at equal head spacings. For example, the parabola passing through the point $\beta = 0.8$ on the $\alpha = 1.0$ line is at $h = 1.12$. The vertical distance, $h = 1.12$, is now divided into eight equal parts and the values $\beta = 0.7$, $\beta = 0.6$ are plotted on the parabolas at $h = \frac{7}{8}(1.12) = 0.98$, $h = \frac{6}{8}(1.12) = 0.84$, etc. Then the β curves are obtained by drawing smooth curves through the points corresponding to each value of β which appears on each parabola.

Fifth step : Since the pump discharge, Q, varies with the pump speed, the speed ratio, α, is therefore proportional to the ratio of velocity, v ($\alpha \propto v \propto h^{\frac{1}{2}}$). Then α^2 is proportional to h. This relation will be used to determine the speed ratio curves for α. The values of $\alpha = 0.9$, $\alpha = 0.8$, etc. may be determined by using the parabolas drawn above and the value of h where each parabola cross the $\alpha = 1$ curve. For example, consider the parabola which passes through the point $\alpha = 1.0$ and $\beta = 0.8$. For this point, $h = 1.12$. Then the point $\alpha = 0.9$ may be plotted on this parabola at $h = 1.12(0.9)^2 = 0.902$ and $\alpha = 0.8$ at $h = 1.12(0.8)^2 = 0.717$, etc. Then the α curves are obtained by drawing smooth curves through these points.

Sixth step : After the curves for α and β are drawn the parabolas which were used to assist in the construction may be removed.

(b). Graphical Solution in A Pump Discharge Line Following A Power Failure when Reverse Flow Passes Through Pump.

In order to clarify this procedure, a typical example is shown inFig. 8. The data given are as follows:

a = 2820 ft. per sec.

V_o = 5.81 ft. per sec.

Q_o = 33.7 cu. ft. per sec.

H_o = 220 ft.

WR^2 of rotating parts = 1154.7 lb.-ft^2

pump speed = 1760 RPM

pump efficiency = 85 %

$\frac{L}{a} = 1.4$ sec.

The time interval $\Delta t = \frac{1}{4}\frac{L}{a}$ will be used.

$$\Delta t = \frac{1}{4}\frac{L}{a} = \frac{1}{4} \times 1.4 = 0.35 \text{ sec.}$$

From Equation (29),

$$\rho = \frac{aV}{2gH_o} = \frac{2820 \times 5.81}{2 \times 32.2 \times 220} = 1.155 \qquad 2\rho = 2.31$$

From Equation (33)a,

$$K_1 = \frac{91758\ Q_o H_o}{(WR^2)\ N_o^2 \eta_o} = \frac{91758 \times 33.7 \times 220}{1154.7 \times 1760^2 \times 0.85} = 0.2238$$

From Equation (33),

$$\alpha_1 - \alpha_2 = K_1(\beta_1 + \beta_2)\Delta t = 0.2238\ (\beta_1 + \beta_2) \times 0.35$$

$$= 0.0783\ (\beta_1 + \beta_2)$$

The required conjugate equations from A to the midlength point B, and from B to C are written as follows:

$$h_{Ao} - h_{B\frac{L}{2a}} = -2.31(v_{Ao} - v_{B\frac{L}{2a}})$$

$$h_{B\frac{L}{2a}} - h_{C\frac{L}{a}} = -2.31(v_{B\frac{L}{2a}} - v_{C\frac{L}{a}})$$

$$h_{C\frac{L}{a}} - h_{B\frac{3L}{2a}} = +2.31(v_{C\frac{L}{a}} - v_{B\frac{3L}{2a}})$$

$$h_{B\frac{3L}{2a}} - h_{A\frac{2L}{a}} = +2.31(v_{B\frac{3L}{a}} - v_{A\frac{2L}{a}})$$

etc.

From these equations it can be easily seen that they may be represented graphically by straight lines with slopes equal to ± 2.31. The points A_o, $B\frac{L}{2a}$ and $C\frac{L}{a}$ are located at $h = 1$ and $v = 1$ on the $h - v$ diagram, because at the beginning these points are all in the normal operation conditions. The point $A\frac{1L}{4a}$ is then located on the line of slope $+2.31$ passing through $h = 1$, $v = 1$, on the $h - v$ diagram. The exact position of this point, however, should be determined by means of the pump inertia equation, Equation (33). After several trials, this point will be found to be at a value of $\beta = 0.760$ and $\alpha = 0.863$. Other points for $A\frac{1L}{2a}$, $A\frac{3L}{4a}$, etc. are determined in a similar manner as shown in Table I.

From Fig. 8, the minimum pressure above the intake elevation at the pump due to water hammer is (0.08)(220) = 18 ft. and at the midlength of the pipe the minimum pressure head above the reservoir intake elevation is (0.31)(220) = 68 ft. The maximum head above the reservoir intake elevation at the pump due to water hammer is (1.61)(220) = 354 ft. and the maximum head above the reservoir intake elevation at the midlength of the pipe is (1.35)(220) = 297

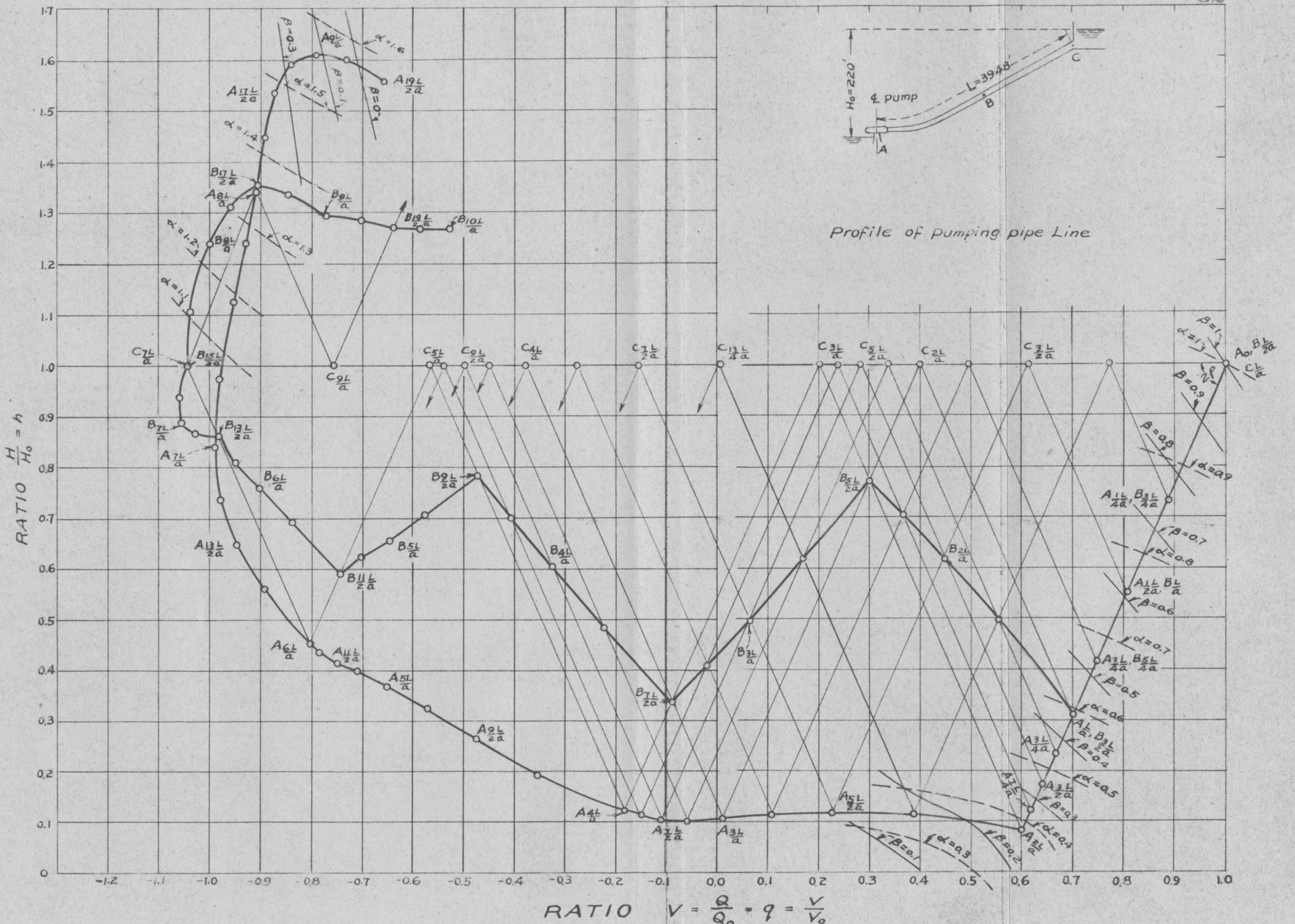
Table I

Values of α and β in determining the magnitude of pressure wave at different instants of time

Location	Pump torque ratio β	Pump speed ratio α	$\alpha_1 - \alpha_2$	$0.0783(\beta_1 + \beta_2)$
A_o	1.000	1.000	-	-
$A\frac{1L}{4a}$	0.760	0.863	0.137	0.138
$A\frac{1L}{2a}$	0.610	0.755	0.108	0.107
$A\frac{3L}{4a}$	0.520	0.668	0.087	0.088
$A\frac{L}{a}$	0.440	0.592	0.076	0.075
$A\frac{5L}{4a}$	0.380	0.528	0.064	0.064
$A\frac{3L}{2a}$	0.325	0.472	0.056	0.055
$A\frac{7L}{4a}$	0.285	0.425	0.047	0.047
$A2\frac{L}{a}$	0.250	0.383	0.042	0.042
etc.				

ft. The maximum and minimum pressure head may be plotted by using the reservoir intake elevation as h = 0 as shown in Fig. 9.

In plotting the minimum pressure line for the pipe line, it frequently happens that at some points the pressure line will fall more than 34 ft. below the pipe line profile. Such a sudden drop in pressure may cause the pipe to collapse. To avoid this difficulty it is sometimes desirable to rearrange the profile of the pipe line to keep the pressure in the pipe line above absolute zero at all times.


Profile of pumping pipe Line
$H_o = 220'$
℄ pump
L=3948'
A
B
C
RATIO $\frac{H}{H_o} = h$
RATIO $V = \frac{Q}{Q_o} = q = \frac{V}{V_o}$

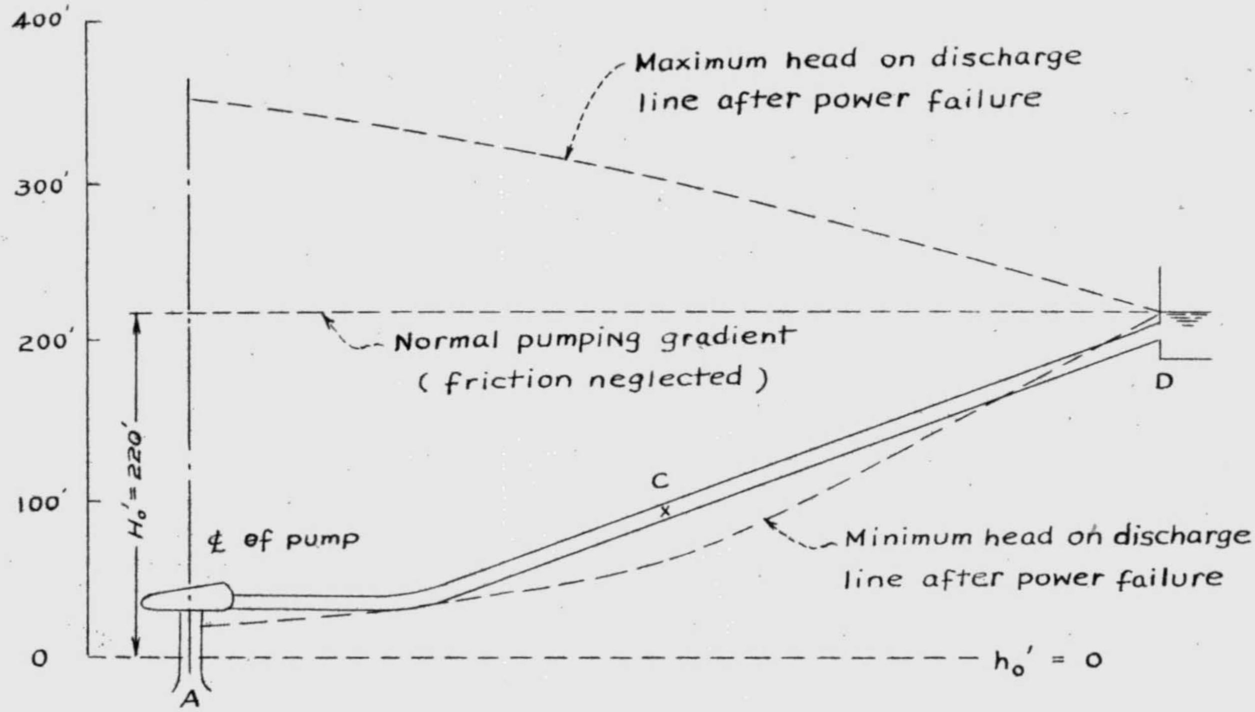

400'
300'
200'
100'
0
Maximum head on discharge line after power failure
Normal pumping gradient (friction neglected)
$H_o' = 220'$
C
D
℄ of pump
Minimum head on discharge line after power failure
$h_o' = 0$
A

FIGURE 9

CHAPTER III. HYDRAULIC STUDIES

As the quantity of flow which will pass through a pipe depends upon the total head, the size of pipe, and the pipe losses, any loss of head in the pipe will tend to reduce the flow. Careful studies of hydraulic losses, therefore, are necessary before any design is made. The hydraulic losses in a pipe consist of the entrance loss in the trashrack, the friction loss in the pipe, the bend losses, and the losses due to contraction and expansion.

If H represents the total head to be delivered in the pipe, then,

$$H = H_d + H_e + H_f + H_1 + H_2 + H_3 \qquad \text{..............(34)}$$

where H_d = difference in elevation between the level of water surface at the intake and the level of water surface at the discharge end of the pipe, or in other words, the lifting head.

H_e = loss of head at the entrance in the trashrack.

H_f = loss of head in the pipe line due to friction.

H_1 = loss of head due to bends along the pipe.

H_2 = loss of head due to enlargement of pipe.

H_3 = loss of head due to contraction in the pipe.

Usually well designed pipe lines are installed without sharp bends or sudden enlargements or contractions. In the design of pumping pipe lines, however, all minor losses should be considered in order to fulfill the requirement for delivering a required amount of water in the pipe line.

The method of computing these different losses may be outlined as follows:

A. Friction Loss.

Perhaps the greatest authority on the flow capacity of conduits is Scobey, F, C. For many years he made a special study of the various formulae developed in this field and has conducted numerous field experiments to prove or modify their results.

In his paper, "The Flow of Water in Riveted Steel and Analogous Pipes"[1], Scobey divides iron and steel pipes into three classes in accordance with the smoothness of their interior surface:

"Class 1, full riveted pipe, having both longitudinal and girth seams held by one or more lines of rivets with projecting heads.

"Class 2, girth-riveted pipe, having no retarding rivet heads in the longitudinal seams, but having the same girth seams as full-riveted pipe.

"Class 3, continuous-interior pipe, having the interior furface unmarred by plate offsets or by projecting rivet heads in either longitudinal or girth seams."

The formula which Scobey has developed as a result of his extensive experiments is

$$H_f = K_s \frac{V^{1.9}}{D^{1.1}} \qquad \text{.....................(35)}$$

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[1] Scobey, F. C. The Flow of Water in Riveted Steel and Analogous Pipes. Technical bulletin no. 150, U. S. Department of Agriculture. Washington, U. S. Govt. print. off., 1939.

where H_f = head loss due to pipe friction per thousand feet, in feet.

K_s = a constant which varies with class and age of pipe.

V = velocity, feet per sec.

D = inside diameter, feet.

For these three classes of pipes as mentioned above, Scobey gives the following coefficients:

Class 1a. $K_s' = 0.38$ for new sheet metal full riveted pipes up to 3/16" thickness.

Class 1b. $K_s' = 0.44$ for new plate metal full riveted pipes from 3/16" to 7/16" thickness with either taper or cylinder joints.

Class 1c. $K_s' = 0.48$ for new plate metal full riveted pipes from 1/2" thickness up with either taper or cylinder joints and for pipes from 1/4" to 7/16" thickness when butt jointed.

Class 1d. $K_s' = 0.52$ for new butt strap full rivet plate metal pipes from 1/2" thickness up.

Class 2. $K_s' = 0.34$ for new girth riveted pipes.

Class 3. $K_s' = 0.32$ for new continuous interior pipes. Welded steel pipe with welded field joints or connected with bolted couplers of the Dresser type belong in this class.

Scobey also found that the coefficient K_s is a function of time, that means, K_s will increase as the age of pipe increases. The formula of K_s is suggested by Scobey as:

$$K_s = K_s' \, e^{0.01t} \qquad \text{.................. (36)}$$

where K_s' = Scobey coefficient for new pipe.

e = natural logarithmic base.

t = time, years.

The values of K_s for different values of K_s' and different durations of service are tabulated as shown in Table 2.

Table 2

Values of coefficients K_s for any age pipe

Age of pipe in years	K_s' = 0.32	K_s' = 0.34	K_s' = 0.38	K_s' = 0.44	K_s' = 0.48	K_s' = 0.52
1	0.323	0.343	0.384	0.444	0.485	0.525
2	0.326	0.347	0.388	0.449	0.490	0.530
3	0.330	0.351	0.392	0.454	0.495	0.536
4	0.333	0.354	0.396	0.458	0.500	0.541
5	0.336	0.357	0.400	0.462	0.505	0.547
6	0.340	0.361	0.404	0.467	0.510	0.552
7	0.343	0.365	0.408	0.472	0.515	0.558
8	0.347	0.368	0.412	0.477	0.520	0.563
9	0.350	0.372	0.416	0.481	0.525	0.569
10	0.354	0.376	0.420	0.486	0.530	0.575
11	0.357	0.380	0.424	0.491	0.536	0.581
12	0.361	0.384	0.429	0.496	0.541	0.587
13	0.364	0.387	0.433	0.501	0.547	0.592
14	0.368	0.391	0.437	0.506	0.552	0.598
15	0.372	0.395	0.442	0.511	0.558	0.604
16	0.375	0.399	0.446	0.516	0.563	0.610
17	0.379	0.403	0.450	0.521	0.569	0.616
18	0.383	0.407	0.455	0.527	0.575	0.622
19	0.387	0.411	0.459	0.532	0.580	0.629
20	0.391	0.415	0.464	0.537	0.586	0.635

In order to simplify the calculations, a family of curves based on the Scobey formula is plotted by using K_s = 0.34 for various pipe sizes ranging from 12 to 120 inches in diameter as shown in Fig. 10.

For other values of K_s, the charts still can be used by multiplying the values found in the chart by a correction factor.

B. Entrance Loss in Trashrack.

The entrance loss in trashrack depends largely upon the intake velocity. The allowable velocity through the net rack section to avoid too great a loss of head will vary from about 1 to 2 feet per second. The following figures may be used for computing the entrance loss in trashrack:

For velocity of 1.0 ft. through the rack, $H_e = 0.10$ ft.

For velocity of 1.5 ft. through the rack, $H_e = 0.30$ ft.

For velocity of 2.0 ft. through the rack, $H_e = 0.50$ ft.

C. Bend Loss.

The loss of head due to bends in pipes is considered as the excess loss over that which would occur in a straight pipe of the same material and equal length.

The formula suggested by Hinds[1] may be recommended for use. The formula is

$$H_1 = C\sqrt{\frac{\Delta}{90}}\ \frac{V^2}{2g} \qquad \text{.................(37)}$$

where Δ = the deflection angle, in degrees.

C = coefficient.

Also Hinds suggested that the coefficient of C may be equal to 0.25

[1] Hinds, J. Loss of head in pipe line due to curvature. Technical memorandum no. 10, U. S. Bureau of reclamation, 1919.

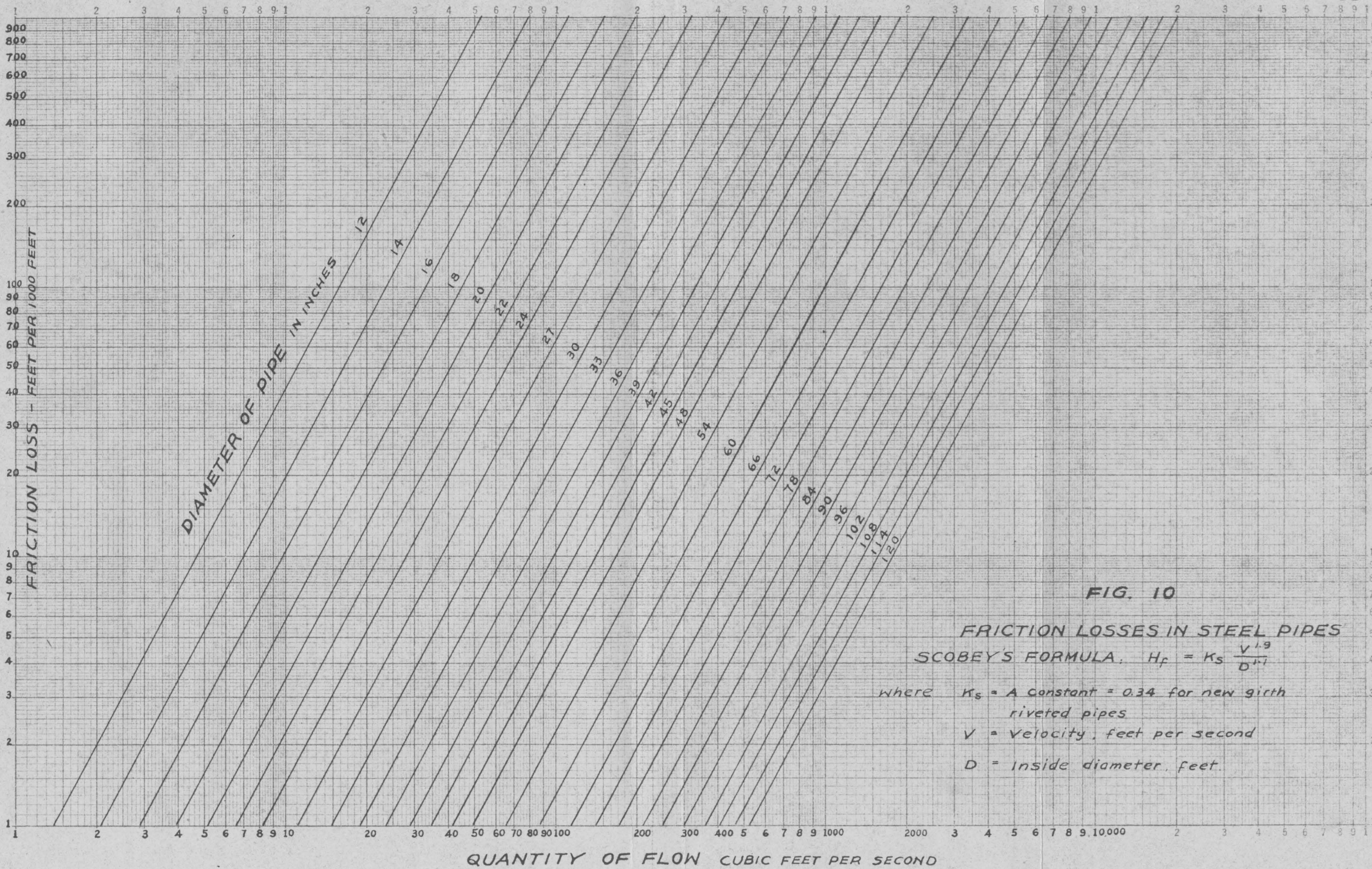

FIG. 10

FRICTION LOSSES IN STEEL PIPES

SCOBEY'S FORMULA. $H_f = K_s \frac{V^{1.9}}{D^{1.1}}$

where K_s = A constant = 0.34 for new girth riveted pipes

V = Velocity, feet per second

D = Inside diameter, feet.

when R/D is greater than 2.00, where R is the radius of bend curve and D is the diameter of pipe. Practically the R/D ratio is seldom less than 2.00. For convenience to use, Equation (37) is plotted as shown in Fig. 11.

D. Gradual Enlargement Losses.

In the design of pumping pipe lines sudden enlargement can be avoided, but gradual enlargements will still exist. The formula of loss of head due to gradual enlargement may be written as

$$H_2 = f\,\frac{V_1^2 - V_2^2}{2g} \qquad \text{..............(38)}$$

where V_1 = velocity in smaller pipe.

V_2 = velocity in large pipe.

f = an empirical coefficient depending upon the angle θ.

θ = double the angle between the axis of the pipe and its side.

King gives the values of "f" as follows:[1]

θ	2°	3°	4°	5°	6°	7°	8°	9°	10°
f	.033	.036	.039	.042	.046	.050	.055	.066	.078
θ	11°	12°	15°	20°	30°	40°	50°	60°	75°
f	.090	.100	.160	.310	.490	.600	.670	.720	.720
θ	90°								
f	.670								

E. Gradual Contraction Losses.

In accordance with the practice of the Bureau of Reclamation, the formula for loss of head due to gradual contraction may be

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[1] King, H. W. Handbook of hydraulics, pp. 191, 1939.

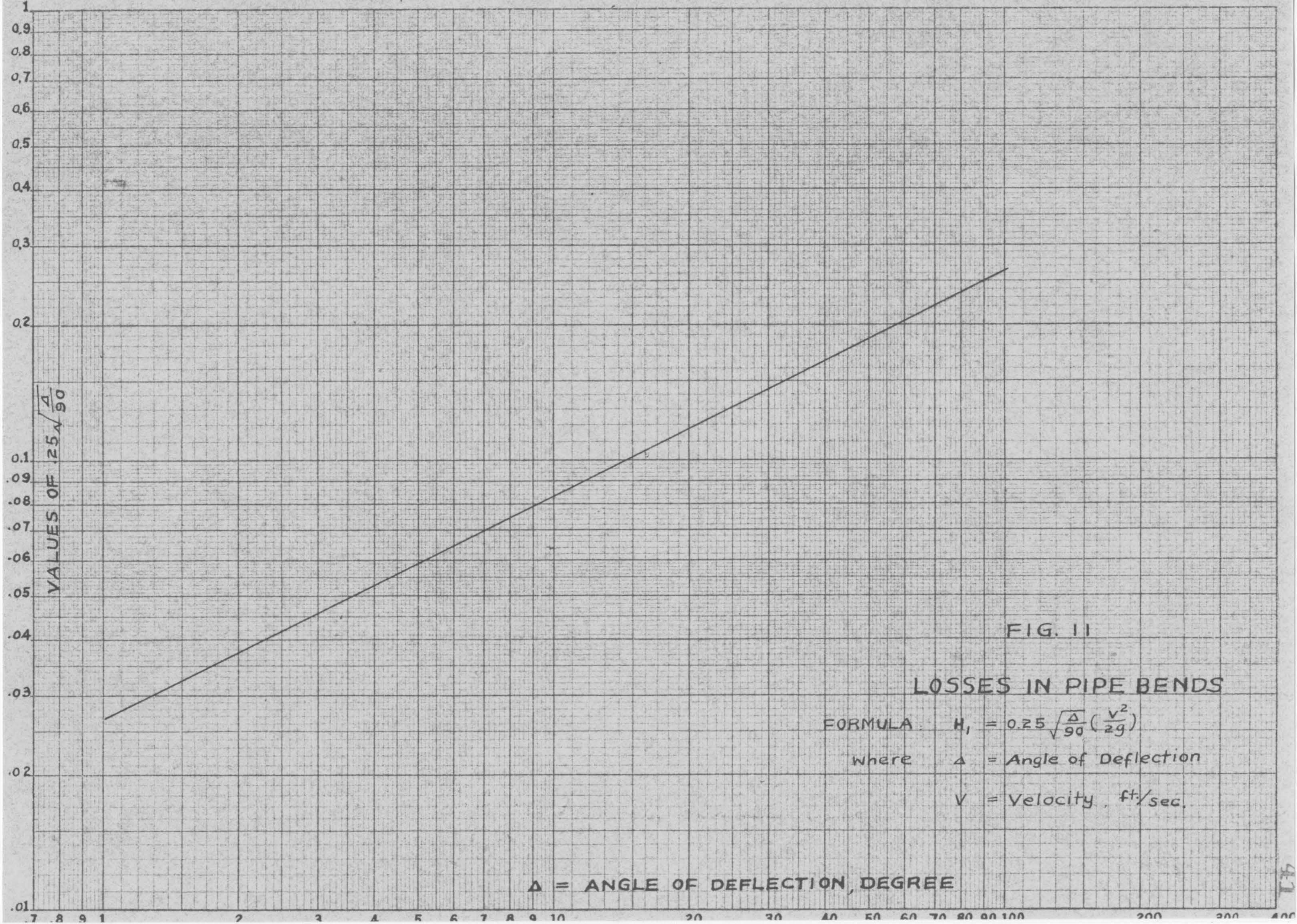

FIG. 11

LOSSES IN PIPE BENDS

FORMULA: $H_1 = 0.25\sqrt{\frac{\Delta}{90}}\left(\frac{V^2}{2g}\right)$

where Δ = Angle of Deflection

V = Velocity, ft./sec.

written as :

$$H_3 = 0.1 \frac{V_2^2 - V_1^2}{2g} \qquad \text{...................(39)}$$

where V_2 = velocity in smaller pipe, ft. per sec.

V_1 = velocity in larger pipe, ft. per sec.

CHAPTER IV. DETERMINATION OF THE MOST ECONOMIC SIZE OF PIPE

A definite determination of economic diameter of pipe is very difficult from a hydraulic point of view, since there are an indefinite number of sizes of pipes which can carry the given discharge. The smaller the diameter, the faster the water flows, and the greater will be the loss of head in the pipe lines. Consequently, a larger percentage of the total power is used in overcoming pipe friction. On the other hand, the smaller the pipe is, the cheaper the pipe will be, and the smaller the initial cost. The most economical diameter, therefore, is the one in which the annual cost is a minimum and will be obtained when a proper balance between power loss and initial cost of pipe exists. The formula for the most economical pipe for penstocks has already been derived by Voetsch and Fresen in 1938.[1] There is, however, a slight difference between penstocks and pumping pipes. Based on the same principle, the formula of the most economical size for pumping pipes will be developed.

A. Formula used for Determining Economic Diameter of Pipe.

In the analysis of economic pipe diameter, some assumptions will be presented:

(1) Uniform diameter through whole pipe line.

(2) The weighted average pumping head including water hammer will be used.

[1] Voetsch, Charles and Fresen, M. H. Economic diameter of steel penstocks. Trans., American society of civil engineers, Vol. 103, pp. 89, 1938.

(3) The entrance losses, bend losses, etc., commonly of minor effect, could be neglected, without appreciable effect upon result.

(4) The head consumed in friction loss forms a small part of the static pumping head and the size of the pumping motors and plant remains constant for a reasonable variation in the size of pipe.

(5) The pumping plant is continuously running through the whole year.

(6) The Scobey formula for flow of water in riveted steel pipes is used.

Based on these assumptions, the formula for economical diameter of steel pipe will be derived as follows:

For annual cost of power loss in friction, the equation is

$$E_f = \frac{1.176\ K_s\ Q^{2.9}\ f\ b}{e\ D^{4.9}} \qquad \text{..................(40)}$$

where E_f = the annual cost of the lost power due to friction in a pipe.

K_s = a general coefficient in the Scobey formula which determine the head loss in friction.

Q = the rated discharge expressed in cu. ft. per sec.

f = load factor.

b = the value of the power loss in dollars per kilowatt-hour.

e = the over-all efficiency of the plant to the point at which power is purchased.

D = the inside diameter of the pipe, feet.

For annual cost of pipe, the equation is

$$E_p = \frac{334\ H\ D^2\ ar\ (1 + i)}{s_g\ e_j} \qquad \text{..................(41)}$$

where H = the weighted average head including water hammer, ft.

a = the unit cost of steel in the pipe, in dollars per pound.

r = the ratio of the annual fixed, operating and maintenance charges to the construction cost of the pipe.

i = the percentage of overweight of steel in the pipe, due to laps, cover-plates, rivets, welds, etc., expressed as a decimal.

s_g = the gross allowable tension in the steel pipe, pounds per square inch.

e_j = the joint efficiency of the pipe, expressed as a decimal.

Thus, the total annual cost, E_t, of 1-ft. section of pipe is the sum of equations (40) and (41), that is

$$E_t = \frac{1.176\ K_s\ Q^{2.9}\ f\ b}{e\ D^{4.9}} + \frac{334\ H\ D^2\ a\ r\ (1 + i)}{s_g\ e_j} \qquad \text{..................(42)}$$

To determine the economic diameter, we may take the first derivative of E_t with respect to D and set it equal to zero; thus

$$\frac{dE_t}{dD} = \frac{1.176\ K_s\ Q^{2.9}\ f\ b(-\ 4.9)}{e\ D^{5.9}} + \frac{668\ H\ D\ a\ r(1 + i)}{s_g\ e_j}$$

$$= -\frac{5.7624\, K_s\, Q^{2.9}\, f\, b}{e\, D^{5.9}} + \frac{668\, H\, D\, a\, r\, (1+i)}{s_g\, e_j} = 0$$

$$\text{..............} (43)$$

Solving Equation (43) for D, we get the economic diameter of pipe, D_e,

$$D = 0.50218 \sqrt[6.9]{\frac{K_s\, Q^{2.9}\, f\, b\, s_g\, e_j}{a\, H\, r\, e\, (1+i)}} \quad \text{.......} (44)$$

For preliminary determination of economic diameters, we can make use of curves instead of Equation (44) and save time in computation. Designating

$$0.50218 \sqrt[6.9]{K} = A \quad \text{.............} (45)$$

$$\sqrt[6.9]{\frac{Q^{2.9}}{H}} = B \quad \text{.............} (46)$$

we obtain, instead of Equation (44)

$$D_e = A \times B \quad \text{.............} (47)$$

where $K = \dfrac{K_s\, f\, b\, s_g\, e_j}{a\, r\, e\, (1+i)}$

Then, using a logarithmic scale, plot two curves, one representing Equation (45) and the other representing Equation (46). From these two curves, as shown in Fig. 12, if we have A and B, D_e can be easily obtained.

B. Detailed Study.

For practical design, the thickness of steel pipe along the whole line is not uniform for the purpose of economy. It varies with the water pressure which certain thickness of plate can sustain. Hence Equation (44) as mentioned above, which is based on the weighted average water pressure in the whole line and assumes

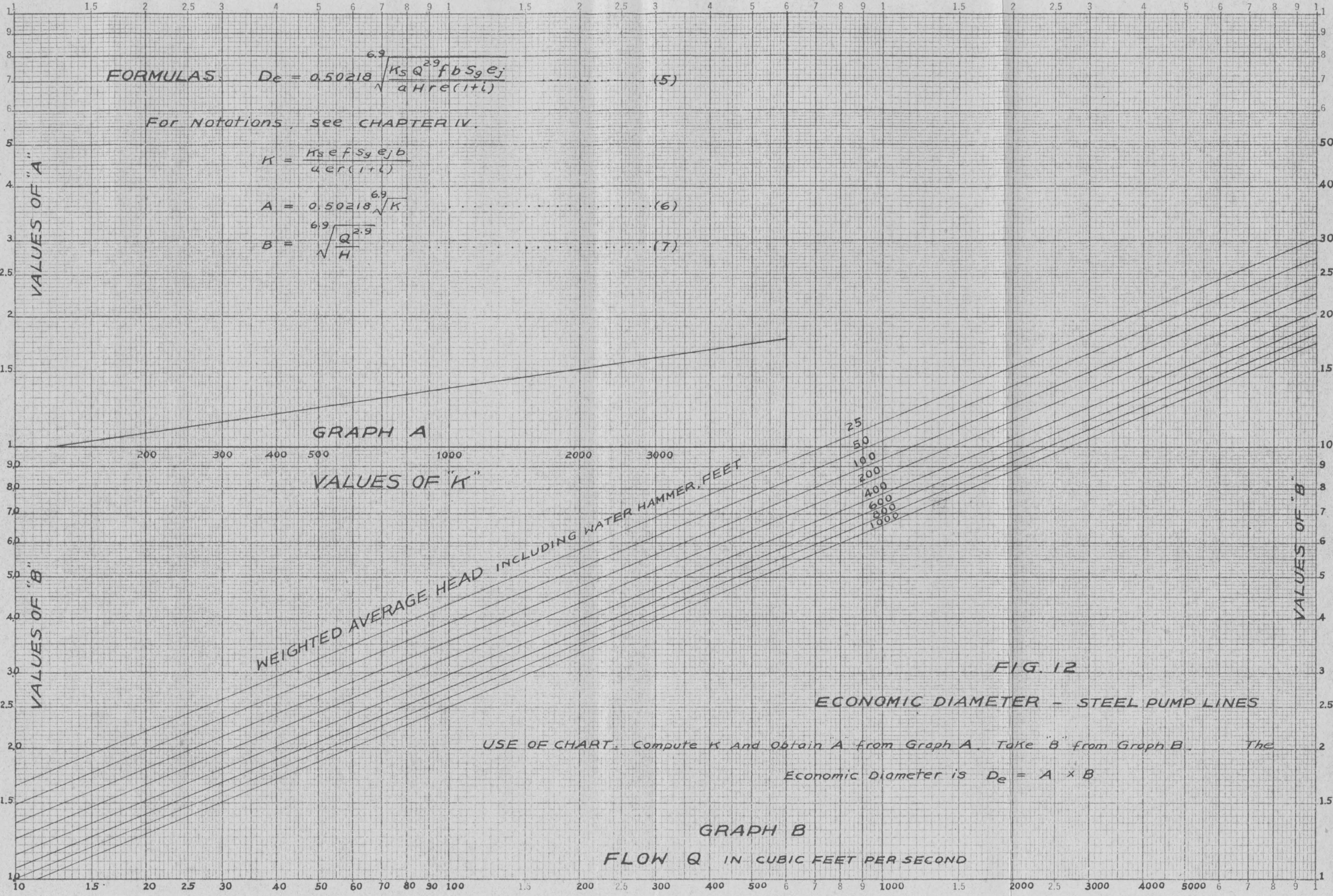

FIG. 12

ECONOMIC DIAMETER – STEEL PUMP LINES

uniform thickness all through the line, is not exactly correct. Detailed study should be carried out as follows:

1. Calculation of Thickness of Pipe.

Thickness of pipe is computed from the ordinary hoop-tension formula which may be easily found from texts on hydraulics and strength of materials.

For different diameters, we can tabulate the relation between the thickness of pipe and the corresponding maximum allowable water head which it can safely sustain.

For example: (using allowable stress 13,500 lb. per sq. in. and the joint efficiency 0.90)

D(diameter)	t(thickness)	H(pressure head)
	$\frac{1''}{4}$	195'
6'	$\frac{5''}{16}$	243'
	$\frac{3''}{8}$	293'
	---	---
	$\frac{1''}{4}$	166'
7'	$\frac{5''}{16}$	209'
	$\frac{3''}{8}$	251'
	---	---

2. Graphical Method for Determining The Length of Pipe for Certain Wall Thickness.

For certain diameters of pipe, plot the profile of the pipe line from the pumping unit to the discharge end, as shown in Fig.13.

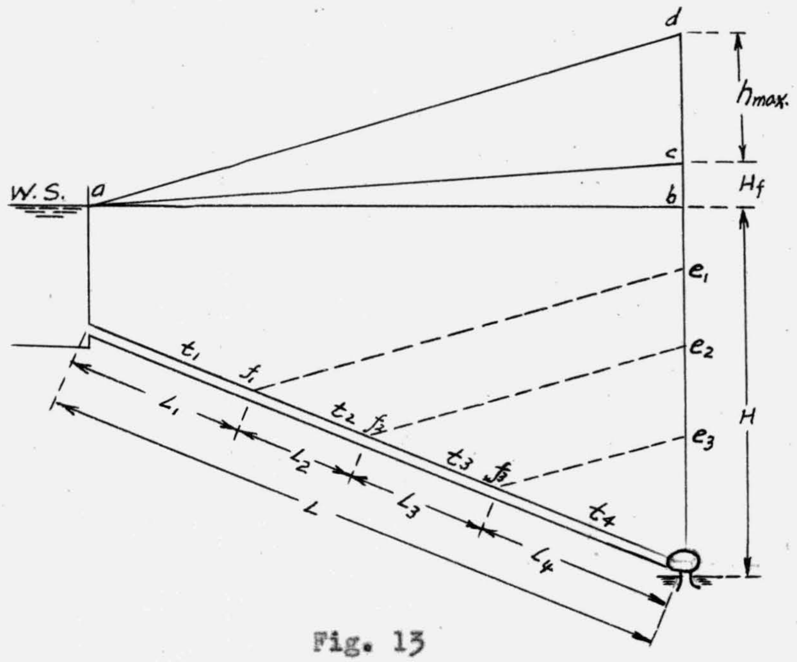

Fig. 13

For simplicity, assume the pipe is a straight line. In Fig. 13, "ab" is the static head level, "ac" is hydraulic gradient including friction head, "ad" is the maximum hydraulic gradient including the rise of head, $h_{max.}$, due to water hammer.

From point "d", measure down the distances equal to the maximum water heads that can be sustained by different thickness of pipe wall as obtained from above. Draw lines from those points e_1, e_2, etc., parallel to the maximum hydraulic gradient "ad" until they intersect the profile of the pipe at f_1, f_2, etc. The portion of steel pipe which lay above these intersecting points are those which can be installed with such pipe thickness, because they are within the allowable limit of the corresponding maximum water pressures.

Measure the length of each portion for different pipe thickness, and tabulate it as shown in Table 3. Follow the same procedure as mentioned above, find the length of pipe and the corresponding pipe thickness for other diameter of pipes, as D_2, D_3, etc.

Table 3.

Length of pipe for corresponding pipe thickness

Diameter of pipe	Thickness of pipe	Length of pipe
	t_1	L_1
D_1	t_2	L_2
	t_3	L_3
	---	---
	t_1	L_1
D_2	t_2	L_2
	t_3	L_3
	---	---
	t_1	L_1
D_3	t_2	L_2
	t_3	L_3
	---	---

3. <u>Calculate The Weight of Pipe per Foot Length for Different Size of Pipe and Different Thickness</u>.

The weight of pipe per foot length is given by

$$\frac{\pi D\ t}{144}(1 + i)\ w$$

where D and t both expressed in inches.

w = unit weight of steel = 487 lb. per cu. ft.

i = the percentage of over weight of steel in the pipe.

The values of i may vary from 0.05 to 0.25, depending on the span or type of support used. In general, using $i = 0.2$ is satisfactory.

4. Calculate The Velocity in Pipe for Different Size of Pipes.

The velocity of flow in pipe is based on the rated discharge,

$$V = \frac{Q}{A}$$

5. Calculate The Pipe Friction Loss for Different Size of Pipes.

For the detailed study, all minor losses may be neglected and the Scobey formula for flow in steel pipe may be used. The formula is

$$H_f = K_s \frac{V^{1.9}}{D^{1.1}}$$

6. Calculate The Friction Loss per Year in KWH for Different Size of Pipes.

The friction loss per year in KwH is given by

$$\frac{Q\ H_f}{10} \times 0.746 \times 8760 = 653.5\ H_f$$

7. Tabulation Form for Computing The Economic Diameter.

All items which have been considered as contributing to the rational selection of the most economical size of pipe have been inserted in a tabular form as shown in Table 4.

TABLE 4

DETAILED STUDY OF ECONOMICAL DIAMETER

Pipe dia. In feet	Pipe dia. in Inches	Material							Hydraulic Losses					Capitalization			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
		Plate thickness	Max. head	Weight per ft. of pipe net + 20% over weight and supports	Total length for each thickness	Weight for each plate thickness	Cost per foot in place	Total cost for each plate thickness	Velocity	Loss per 1000 ft K_s=34	Loss for each length	Friction loss per year	Friction loss per year	Friction loss per year per foot	Interest dep'rc. & maint. per foot per year	Total cost of pipe per foot	Weighted Average of Col. 15
(ft)	(in)	(in)	(ft.)	(lbs.)	(ft)	(lbs)	($)	($)	(ft/sec)	(ft)	(ft)	(KWH)	($)	($)	($)	($)	($)

CHAPTER V. SUMMARY

The factors involved in the economical design of pipe systems for pumping plants are (1) the determination of pressure rise and fall due to water hammer following a power failure, (2) the study of hydraulic losses in a pipe line and (3) the computation of the most economic size of pipe. The former two factors are the prerequisites in designing the pipe and the latter is the final procedure required to obtain the most economical design of pipe systems.

Based upon the principle of continuity, Newton's second law of motion and Hook's law, the fundamental equations of water hammer are developed. A graphical solution in determining the maximum and minimum pressures due to water hammer is introduced by combining the fundamental equations with the pump characteristics for a particular pumping plant. The maximum pressure thus obtained is used to determine the thickness of pipe shells, while the minimum pressure is of value to check the profile of pipe line if the hydraulic gradient falls below the pipe line.

The hydraulic losses in a pipe line consist of the entrance loss in the trashrack, the friction loss in the pipe, the bend losses, and the losses due to contraction and expansion. Their computations are based upon the principles of hydraulics. In order to make the results of study readily avaliable in convenient form to designers, tables and charts are prepared.

The most economical diameter is the one in which the total annual cost is a minimum and will be obtained when a proper balance between power loss and initial cost of pipe exists. A mathematical equation is derived in determining the most economical diameter of steel pipes.

However, for practical design, the thickness of steel pipe along the whole line is not uniform for the purpose of economy. Detailed study is, therefore, necessary. To determine the length of pipe for a certain given thickness, the graphical method here presented is very helpful.

All items which have been considered as contributing to the rational selection of the most economical size of pipe have been inserted in a tabular form as shown in table 4. By proceeding in order across the table from left to right all sixteen factors involved in the selection will taken into consideration and properly evaluated in their effect upon the final selection. Only by this approach or a similar systematic attack can the designer be assured of a safe economical selection of pipe for each individual pumping plant.

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