DESIGN APPLICATION OF THE HOTTEL-WHILLIER-BLISS EQUATION

by

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A method is presented for experimentally determining the three factors that determine collector efficiency in the Hottel-Whillier-Bliss Equation. \[ Q_u = F_R A_c \left( \tau \alpha \right)_e I - U_L (T_f - T_a) \]. These factors are: the collector heat removal factor, \( F_R \); the effective transmittance-absorptance product, \((\tau \alpha)\_e\); and the overall heat loss coefficient, \( U_L \).

The method of testing requires: computation of \((\tau \alpha)\_e\) from measurements of cover transmittance and collector reflectance; computation of \( F_R \) from a test in which the heat loss term equals zero; and computation of \( U_L \) from a test in which insolation equals zero. This method was applied to collectors used on Solar House I at Colorado State University, with experimental and theoretical results being in close agreement. The method can be used to experimentally evaluate collector performance and for optimization of collector design.
INTRODUCTION

The application of the theoretically derived Hottel-Whillier-Bliss Equation as a guide to the design, developing, and testing of flat-plate solar collectors is the subject of this paper. This equation expresses heat delivery from a collector in terms of two operating variables:

1. the component of insolation normal to the absorber plate, $I$; and
2. the temperature difference between the entering heat removal fluid, $T_f,i$, and the air surrounding the collector, $T_a$. The equation is written:

$$Q_u = F_R A_c \left[ (\tau_\alpha)_e I - U_L (T_f,i - T_a) \right]$$

The equation states that useful heat collected per unit area equals solar energy absorbed by the absorber, $(\tau_\alpha)_e I$, less the heat loss, $U_L (T_f,i - T_a)$, multiplied by a factor related to the effectiveness of transferring the heat absorbed by the absorber plate to the heat removal fluid, $F_R$. The analytical determination of $F_R$, $(\tau_\alpha)_e$, and $U_L$ for design purposes are described by other authors. [1,2]

The three design factors, $F_R$, $(\tau_\alpha)_e$, and $U_L$, are measures of thermal performance and combine to yield overall collector efficiency in terms of the operating variables of temperature and insolation. The three factors can be used to identify features which would enhance performance with the highest cost-benefit. A corollary objective is to identify factors that are not economically justifiable in improving performance and that may be eliminated to reduce costs. This process is commonly referred to as value engineering.
SIGNIFICANCE OF THE DESIGN FACTORS

The effective transmittance-absorptance product, \((\tau\alpha)_e\), represents the complex interaction of optical properties in the solar radiation wavelengths. It is somewhat larger than the direct product of the cover transmittance and absorber absorptance because some of the radiation reflected from the absorber is returned to the absorber due to cover reflectance. The increase is typically about 5%. The effective transmittance-absorptance product is influenced by cover transmittance, number of covers, and absorptance of the absorber plate.

The heat removal factor, \(F_R\), is influenced by the heat transfer resistance between the heated absorber surface and the collector fluid. This would be affected by the design of the absorber plate and by the properties of the collector fluid. The heat removal factor is also affected by the flow rate of the collector fluid.

The heat loss coefficient, \(U_L\), is influenced by the number and spacing of covers and by conditions within the spaces, such as honeycomb cells or evacuation. The heat loss coefficient is also influenced by the longwave radiative properties of the absorber and covers and by wind speed.

Thus it is seen how these three factors influence total collector efficiency and how they may be dealt with independently. Physical characteristics of a collector may be judged in relation to their cost and effect upon these factors. For example, one might consider an antireflective coating on the glass covers or an increment of additional insulation to the back of the collector at the same cost per unit of area. The selection between these two may be made by comparing their
respectively effects upon \((\tau \alpha)_e\) and \(U_L\) for a representative condition of \(T_{f,i}, T_a,\) and \(I\). By such a process the optimal design may be reached within the limitations, such as the physical dimensions, set forth in the initial design.

**PROTOTYPE TESTING**

When the design is completed a prototype is constructed for testing purposes. The prototype must be large enough to exhibit as nearly as possible the behavior of a complete system of collector units. It is typical for collector systems to consist of several smaller units which are supplied in parallel by the heat removal fluid. Under this arrangement, it is logical to use one of these units as the prototype for testing thermal performance, because that is the smallest unit representing the total collector system. If the primary components (such as transparent covers, absorber, and back insulation) of the test unit can be replaced easily, the effects of the three factors, \(F_R\), \((\tau \alpha)_e\), and \(U_L\), can be readily determined. Such versatility, provided it does not influence the thermal performance, can save a great deal of effort throughout the development process.

Edge losses from the test collector are likely to exceed those for a complete collector system where most edges are shared with adjacent collector units. One solution is to utilize a low density and low conductivity insulation on the edges, such as urethane foam. This is not a complete solution as some heat is still lost. If edge losses are expected to exceed 10% of the useful heat gain, as with perhaps an
unusually high edge-to-area ratio, a controlled electrical heat strip to maintain the edge temperature may be advisable.

Back losses could also vary in the test unit from that of the intended ultimate application. One may want to place the test unit on an enclosure similar to an attic space if such mounting is intended. The necessity of this measure should be quite small, however, with a reasonable amount of back insulation.

Testing of the collector unit may be as described in the National Bureau of Standards method for collector performance testing. [3] It is necessary to measure the useful heat gain delivered by the heat removal fluid, the solar insolation, the wind speed, the temperature of the ambient air above the collector, and temperatures of the heat removal fluid entering and leaving the collector.

MEASUREMENT OF THE DESIGN FACTORS

Three sets of tests were conducted to determine the values of $(\tau_\alpha)_e$, $F_R$, and $U_L$. The effective transmittance-absorptance product is determined as a function of incidence angle using a series of pyranometer measurements. The product $F_R(\tau_\alpha)_e$ can be determined by a test in which the heat loss term is set equal to zero. The product $F_RU_L$ can be determined by a test during which insolation is zero. The tests were conducted with the collectors on CSU Solar House I. These collectors have aluminum structural supports, two sheets of B-quality, double strength window glass, an aluminum absorber plate with internal tubes, and insulation beneath the absorber. The heat removal fluid is a mixture of water and ethylene glycol.
Effective Transmittance-Absorptance Product Test. The effective transmittance-absorptance product was determined as a function of incidence angle based on a series of pyranometer measurements. The equation for determining \((\tau_\alpha)_e\) is:

\[
(\tau_\alpha)_e = \tau - \left[ \frac{\left( I_r/I \right) - r}{\tau} \right]
\]

The disposition of the incoming insolation is shown in Figure 1. Data collection consisted of two parts: measuring \(\tau\); and measuring \(I_r/I\). Transmittance was measured by use of a frame placed over the pyranometer and measuring insolation with and without the cover glasses inserted in the frame. (Refer to Figure 2.) The cover glass used was the same as that used in the CSU collectors. The glass was B-quality, double strength, 1/8" window glass. The pyranometer was an Eppley black and white pyranometer, model number 8-48. The results of the transmittance testing are shown in Figure 3.

Next, the collector reflectance, \(I_r/I\), was determined by alternately measuring \(I\) and \(I_r\) for the collectors. Incident insolation, \(I\), was measured with the pyranometer parallel to and facing from the collectors (Refer to Figure 4.) Reflected insolation, \(I_r\), was measured with the pyranometer parallel to and facing toward the collectors. (Refer to Figure 5.) The collector reflectance, \(I_r/I\), is also shown in Figure 3.

The effective transmittance-absorptance product was computed for several incidence angles using the curves for \(\tau\) and \(I_r/I\) presented in Figure 3. The resulting curve for \((\tau_\alpha)_e\) is also shown in Figure 3.

No Heat Loss Test. The second test is for the fluid supplying the collector be equal in temperature to the ambient air. Equation (1) becomes:

\[
Q_U = F_R A_c (\tau_\alpha)_e I.
\]
This is not precisely zero heat loss because the plate temperature is non-uniform and unequal to $T_{f,i}$. However, for good heat transfer design, this disparity is not large and is accounted for in the $F_R$ term. This test is extremely simple with an air heating collector, since the ambient air is supplied to the collector. The liquid collector requires a liquid reserve at $T_a$ or controlling of a hot/cold mixing valve to supply to unit.

The testing procedure was to run the CSU Solar House I collector pump and to run cold water from the city main through the storage side of the collector-storage heat exchanger. Collector inlet temperature was controlled by adjusting the flow rate of cold water through the heat exchanger. Useful heat gain is computed using the equation:

$$Q_u = \dot{m}c_p (T_{f,o} - T_{f,i}).$$

Insolation, $I$, was measured directly with a pyranometer mounted parallel to the collectors. A schematic of the test set up is shown in Figure 6.

The measured value of $F_R(\tau_a)_e$ was 0.623 at an incidence angle of 45°. The resulting value of $F_R$ was 0.886.

**No Insolation Test.** The final test is to operate the collector at night to measure the heat loss with no incoming solar radiation. Equation (1) is reduced to:

$$Q_u = -F_R A_U L (T_{f,i} - T_a).$$

The useful gain is now negative. The requirements here are for a low ambient air temperature, $T_a$, so that a useful heat gain of sufficient magnitude may be measured accurately. In addition, temperature stability must be obtained with heating from the fluid as well as constant wind conditions.
The testing procedure was to run the CSU Solar House I collectors at night when there was no insolation. Useful heat gain was determined in the same manner as for the no heat loss test. Wind speed was also measured with these tests. A schematic of the test set up is shown in Figure 7.

Results of this testing are shown in Figure 8. The points show the total heat loss coefficient as a function of fluid inlet temperature. The curve showing the total loss coefficient for the CSU collectors and a theoretical curve for the top loss of a two cover collector, both as a function of absorber plate temperature, are shown for comparison. For the CSU Solar House I collectors, the effect of wind speed on $U_L$ is small when compared with the effect of plate temperature for wind speeds from 0 to 25 km hr$^{-1}$.

The three conditions just described thus generate the necessary independent equations to determine $(\tau \alpha)_e$, $F_R$, and $U_L$.

**No Useful Gain Test.** A third independent relation may be generated by stopping flow of the heat removal fluid in the present of insolation. The collector will reach an equilibrium no-load temperature where:

$$(\tau \alpha)_e I = U_L (T_p - T_a).$$

The temperature relating to the heat loss is the absorber plate temperature, $T_p$, rather than the inlet fluid temperature, $T_{f,i}$, since the fluid is not circulating.

The difficulty with this procedure lies in obtaining an equilibrium plate temperature which is nearly equal to that in the other tests with fluid circulation. It was necessary in the CSU procedure to obtain a uniformly overcast period near solar noon. Thus the shading and optical conditions were suitable and the solar intensity was low enough to maintain lower plate temperature.
This test is not recommended because of the difficulty found in establishing steady-state conditions. Also this relation is dependent upon \( U_L \), which could differ enough from the no-heat loss \( U_L \) to impair the accuracy of this method.

The testing procedure must be carried out under typical or average values of the operating conditions anticipated. That is, with the temperatures and flow rates at or near those used for the initial design. Furthermore, it is required to operate at steady-state conditions in order to avoid heat capacity effects. Steady-state conditions of ambient temperature, wind, and solar insolation are not controllable and thus must be sought as constant as possible for the testing period. Tests with solar insolation should be performed within three hours of solar noon to minimize optical and edge shading variations. The steady-state condition is determined by the temperature stability of the absorber plate as well as other components of the collector. The dynamic or unsteady-state thermal performance is influenced by collector heat capacity and climatic transients, which are not applied in this analysis. The heat capacity does not greatly affect the total day-long performance of collectors.\([4]\)

RESULTS OF MEASUREMENT PROCEDURE

The three tests just described were performed on the CSU Solar House I collectors to evaluate \((\tau a)_e\), \( F_R \), and \( U_L \). The comparison with theoretical values of these three factors is shown in Table 1.

It is necessary to perform the no heat loss and no insolation tests, which include \( U_L \), at nearly constant wind speeds and absorber
plate temperatures. After $(\tau_0)_e$ and $F_R$ are established, with a constant $U_L$, it may be desirable to continue testing to obtain the dependence of $U_l$ upon wind speed and/or absorber plate temperature.

The $(\tau_0)_e$ test indicates that the absorptance of the absorber plate is only about 0.92. The absorber coating that was used can have an absorptance as high as 0.98 when new and clean. The difference is primarily due to the dust on the absorber plate.

It is important in the measurement procedure to exercise good accuracy. Values which depend on small differences between large values do occur.

The $(\tau_0)_e$ test should be done with clean surfaces unless the dirt effect is under consideration. The variability in cover transmission due to dirt is usually less than 5%. This was established by direct pyranometer measurement under the glass covers.

The steady-state requirement was not difficult to satisfy when near-constant weather conditions existed for 15-20 minutes.

The curve derived as a result of these tests is shown in Figure 9 as the efficiency based on net collector area (exposed glass area). Also shown in this figure are data from actual operation of the CSU Solar House I. The points represent values over a one-hour period during which the collectors operated continuously and wind speeds were less than 10 km hr$^{-1}$.

The efficiency of the plotted points is computed by using the gross (total) collector area as the area intercepting insolation. When the curve determined by the tests outlined here is adjusted to give the efficiency based on gross collector area, the results of the testing procedure compare favorably with the operating data.
CONCLUSION

The independent factors $F_R$, $(\tau \alpha)_e$, and $U_L$ from the Hottel-Whillier-Bliss Equation were found to be easily measurable. Thus the relationship serves as a useful design tool because each term can be experimentally validated. These three factors may be employed to systematically improve an existing collector design, and they can be compared with quite different designs for general evaluation.

Many developments are under way which influence one or more of these contributors to collector performance as quantified by $F_R$, $(\tau \alpha)_e$, and $U_L$. It is important as this work develops and becomes more detailed that the specific effects be presented with common understanding. The Hottel-Whillier-Bliss relationship can fulfill this need.
NOMENCLATURE

\[ A_c \quad = \quad \text{Gross collector area (m}^2\text{)} \]

\[ c_p \quad = \quad \text{Specific heat of collector fluid (J kg}^{-1}\text{°C}^{-1}\text{)} \]

\[ F_R \quad = \quad \text{Collector heat removal factor} \]

\[ I \quad = \quad \text{Normal global insolation (Wm}^{-2}\text{)} \]

\[ I_r \quad = \quad \text{Normal insolation reflected by collectors (Wm}^{-2}\text{)} \]

\[ \dot{m} \quad = \quad \text{Mass flow rate of collector fluid (kg hr}^{-1}\text{)} \]

\[ Q_u \quad = \quad \text{Heat gained in the collectors (W)} \]

\[ r \quad = \quad \text{Reflectance of collector covers} \]

\[ T_a \quad = \quad \text{Ambient air temperature (°C)} \]

\[ T_{f,i} \quad = \quad \text{Collector fluid inlet temperature (°C)} \]

\[ T_{f,o} \quad = \quad \text{Collector fluid outlet temperature (°C)} \]

\[ U_L \quad = \quad \text{Collector heat loss coefficient (W m}^{-2}\text{°C}^{-1}\text{)} \]

\[ \theta_T \quad = \quad \text{Incidence angle of beam radiation on collector (degrees)} \]

\[ \tau \quad = \quad \text{Transmittance of collector covers} \]

\[(\tau a)_e \quad = \quad \text{Effective transmittance-absorptance product}\]
REFERENCES


Figure Legends

1. Disposition of Insolation on a Flat Plate Collector
2. Apparatus for Measuring Cover Transmittance
3. Cover Transmittance, Effective Transmittance-Absorptance, and Collector Reflectance for the CSU Solar House I Collectors
4. Measuring Incident Insolation
5. Measuring Reflected Insolation
6. Equipment Layout for the No Heat Loss Test
7. Equipment Layout for the No Insolation Test
8. Heat Loss Coefficient for the CSU Solar House I Collectors
9. Performance of the CSU Solar House I Collectors
Disposition of Insolation on a Flat Plate Collector

\[ I_r = \left[ r + \tau(\tau - (\tau\alpha)_e) \right] I \]

Figure 1
Figure 2

Apparatus for Measuring Cover Transmittance
Cover Transmittance, Effective Transmittance-Absorptance and Collector Reflectance for the CSU Solar House I Collectors

Figure 3
Figure 4

Measuring Incident Insolation
Figure 5

Measuring Reflected Insolation
Equipment Layout for the No Heat Loss Test

Figure 6
Loss = U_L (T_{f,i} - T_a)

I = 0

Q_u = \dot{m} C_p (T_{f,o} - T_{f,i})

Equipment Layout for the No Insolation Test

Figure 7
Heat Loss Coefficient for CSU Solar House I Collector

Figure 8
Points are hourly averages from system operating data.
Curves are from tests outlined in this paper.

Performance of CSU Solar House I Collectors

Figure 9


<table>
<thead>
<tr>
<th>Design Factor</th>
<th>Theoretical Value</th>
<th>Experimental Value</th>
<th>Condition</th>
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<tbody>
<tr>
<td>((\tau_0)_{e})</td>
<td>(0.729 (\alpha = 0.95)) (0.693 (\alpha = 0.90))</td>
<td>0.704</td>
<td>(\theta_T = 45^\circ)</td>
</tr>
<tr>
<td>(F_R)</td>
<td>0.882</td>
<td>0.886</td>
<td>(\theta_T = 45^\circ)</td>
</tr>
<tr>
<td>(U_L (Wm^{-2}C^{-1}))</td>
<td>3.7</td>
<td>3.6</td>
<td>Flow = 0.775 ls(^{-1})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(T_{f,i} = 65^\circ C)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Wind = 18 kmhr(^{-1})</td>
</tr>
</tbody>
</table>