A Coefficient to Characterize Mixing in Solar Water Storage Tanks

A dimensionless coefficient is developed to characterize the level of mixing in solar water storage tanks. The MIX number, based on the height weighted energy, or moment of energy, in the tank, ranges from 0 to 1, with 0 representing a perfectly stratified (unmixed) tank and 1 representing a fully mixed tank. Limiting values are based on theoretical determinations of the maximum and minimum values of the moment of energy in a tank without mixing and a tank with complete mixing, respectively. Use of the new MIX number is illustrated by experimental data obtained in a 372-liter storage tank operated with both a conventional drop-tube inlet and a rigid, porous stratification manifold. The initial tank temperature profile, the temperature of the water entering the tank, and test duration are varied in three testing schemes. Fluid mixing is quantified by measured vertical temperature profiles and the dimensionless MIX number.

Introduction

Experimental (Davis and Bartera, 1975; Lavan and Thompson, 1977; Fanney and Klein, 1988) and analytical (Sharp and Loehrke, 1979; Wuestling, 1983; Jesch and Braun, 1984; Wuestling et al., 1985) investigations show that thermal stratification of the solar storage tank enhances the performance of solar domestic hot water (SDHW) systems. Maximum possible thermal performance is obtained when no mixing occurs in the solar storage tank. In this case, the coldest stored fluid is circulated through the collector(s). Heated water is then returned to the tank without inducing mixing of fluid layers with different temperatures. During water draws to the load, the hottest water in the tank is withdrawn from the tank and replaced (again without mixing) with cold water from the mains water supply. Minimum performance is obtained when fluid streams entering the storage tank mix completely (and instantaneously) with water in the tank. In this case, water circulated to the collector is at the same temperature as that supplied to the load.

Loss of stratification of the thermal storage tank results from convective mixing, both forced and natural, and, to a lesser extent, from conduction between hot and cold fluid layers. Maintaining thermal stratification requires inhibition of mixing in the tank. Mixing depends on the design of the tank and the operating conditions (flow rate and temperature of incoming fluid streams and the temperature distribution in the tank). Forced convection mixing is due to the momentum of the fluid streams entering the storage tank and depends on the flow rate of the entering stream and the design of the inlet. Momentum diffusers have been employed to reduce mixing due to jet entrainment.

Mixing due to natural convection occurs when the fluid entering the storage tank is colder (or hotter) than the surrounding fluid. Differing fluid densities give rise to a natural convection current which mixes the tank. The extent of natural convection is a function of the temperature of the incoming fluid, the temperature distribution within the tank, and the design and location of the inlet. Porous distribution manifolds have been tested which allow the fluid to enter the tank to an appropriate level with reduced mixing (Sharp and Loehrke, 1979; Loehrke et al., 1979; Davidson et al., 1992).

Vertical conduction within the storage tank wall can also give rise to natural convection currents. Fluid near the wall is cooled or heated to the mean tank temperature faster than is the bulk of the fluid in the tank. The resulting horizontal temperature difference drives a natural convection circulation. Mixing due to wall conduction is reduced when the thermal conductivity of the wall is nearly the same as that of the fluid (Jaluria and Gupta, 1982) or when the inside surface of the tank is insulated and the outside surface is conductive (Shyu and Hsieh, 1987).

Effective design and optimization of SDHW systems require accurate characterization of mixing in the solar storage tanks under all operating conditions. Several parameters have been proposed to characterize mixing; however, none adequately considers both energy level and temperature distribution. The most commonly used parameters are those proposed by Phillips and Dave (1982), Satish (see McCarthy and Wood, 1990), and Wu and Bannerot (1987).

The Phillips (1982) coefficient

\[ K_\mu = \frac{F_B F_D \alpha A_{co}}{(UA)(T_m - T_a)} \]  \[ F_B F_D \alpha A_{co} \]  \[ (T_m - T_a) \]

(1)

focuses on the effect of mixing in the tank on daily thermal performance of the entire system. The disadvantages of the
The coefficient proposed by Satish,

$$\xi = \frac{\Delta T_{\text{actual}}}{\Delta T_{\text{ideal}}} \left( \frac{T_{\text{avg}}}{T_{\text{avg}, \text{ideal}}} \right)_{\text{actual}},$$  \hspace{1cm} (2)

is the product of the ratio of the actual temperature difference between water at the top and bottom of the storage tank to an "ideal" temperature difference and the ratio of the actual average temperature of the tank to the average temperature calculated assuming a linear vertical temperature profile. The "ideal" temperature difference is defined as the temperature of boiling water minus that of the cold supply water. Stratification is defined in terms of the thermocline in the tank. This focus ignores the possibility that a tank of uniform (hot) temperature could result from an ideal tank in which hot water displaces colder water without mixing. A tank in which mixing is complete would also produce uniform (although lower) temperature water for the same operating conditions. In either case, $\xi$ is zero. Energy level is not characterized. Theoretically, $\xi$ can equal 1 in an "ideally" stratified tank containing boiling water at the top of the tank and mains temperature water at the bottom of the tank.

The Wu and Bannerot (Wu and Bannerot, 1987) stratification factor,

$$ST = \frac{\sum m_i (T_i - T_{\text{avg}})^2}{M \cdot T_{\text{avg}}},$$  \hspace{1cm} (3)

also focuses on the temperature gradient and not the degree of mixing. Equation (3) is the mass weighted mean square temperature divided by total mass of the tank. In this expression, $T_i$ is the temperature of the water in the $i$th vertical tank element. This factor is a clear indicator of the instantaneous deviation of the vertical temperature profile from that of an isothermal tank at $T_{\text{avg}}$, but does not characterize energy stored in the tank. The factor increases as the temperature difference between the top and bottom of the tank increases. However, like the Satish coefficient, if at any time the tank becomes isothermal, $ST$ is zero, regardless of energy level. The coefficient could be modified by using a reference temperature $T_r$ rather than $T_{\text{avg}}$. However, the number would not be single valued. For example, let $T_r$ be a temperature between the lowest and highest temperatures in the tank. Then a tank having a uniform temperature 2 units greater than $T_r$, would have the same $ST$ value as one with a uniform temperature 2 units greater than $T_{\text{avg}}$. In an SDHW system, mixing would have to be less in order for the final temperature to be greater for the same conditions.

This paper presents a dimensionless mix number which characterizes storage tank performance in terms of both total energy stored in the tank and vertical temperature profile. Incorporation of analytical values of the height weighted energy of a fully mixed tank and of an unmixed (fully stratified) tank into the mix number allows a relative comparison of tank performance. The value of the mix number ranges from 0 to 1, with 0 representing an ideal unmixed tank and 1 representing a fully mixed tank. Use of the new mix number is illustrated by a comparison of the performance of a storage tank operated with a conventional drop-tube inlet to performance of the same tank operated with a stratification enhancing manifold.

### Analytical Development

The mix number is based on the energy level in the storage tank weighted by vertical location. The first moment of energy, similar to the first moment of mass, $M$, used in solid mechanics, is introduced to account for energy location. The first moment of mass,

$$M = \int_0^H x \, dm,$$  \hspace{1cm} (4)

is a measure of the tendency of the distributed mass to cause an horizontal beam of length $L$ to rotate about point "0." By replacing mass with stored energy and the horizontal coordinate $x$ with the vertical coordinate $y$, the vertical moment of energy in a solar storage tank is defined as

$$M_E = \int_0^H y \, dE,$$  \hspace{1cm} (5)

for a storage tank of height $H$. For example, for a tank divided into four regions of uniform temperature (energy), the moment of energy is approximated as

$$M_E = \sum_{i=1}^4 y_i E_i,$$  \hspace{1cm} (6)

where $y_i$ is the distance measured from the bottom of the tank to the center of node $i$, and the energy of node $i$ is $E_i = \rho \cdot C_p \cdot V \cdot T_i$. Of course, when considering mixing as a function of time of day, the moment of energy increases as water heated in the solar collector is added to the tank and decreases when hot water is withdrawn from the tank to meet the load. A larger value of the moment of energy is obtained if no mixing occurs.

Incorporation of analytical models of the performance of

### Nomenclature

- $A$ = area, m$^2$
- $c_p$ = specific heat, J/kg K
- $E$ = energy, J
- $F_s$ = heat exchanger penalty factor
- $F_R$ = collector heat removal factor
- $H$ = height of tank, m
- $K_s$ = Phillips stratification coefficient, Eq. (1)
- $L$ = length, m
- $m$ = mass, kg
- $m$ = mass flow rate, kg/s
- $M$ = moment of energy, J m
- $MIX$ = mix number, Eq. (9)
- $q$ = solar flux, W/m$^2$
- $ST$ = Bannerot stratification factor, K$^2$, Eq. (3)
- $t$ = time, s
- $T$ = temperature, K
- $UA$ = overall heat-transfer coefficient, W/K
- $V$ = volume, m$^3$
- $x$ = horizontal distance, m
- $y$ = vertical distance, m

### Greek Letters

- $\alpha$ = effective transmittance product
- $\Delta$ = difference
- $\rho$ = density, kg/m$^3$
- $\xi$ = Satish stratification coefficient, Eq. (2)

### Subscripts

- $a$ = surroundings at ambient temperature
- $act$ = actual = refers to actual (experimental) tank
- $avg$ = average
- $c$ = cold fluid, minimum tank temperature
- $col$ = collector aperture
- $h$ = hot fluid, maximum tank temperature
- $i$ = tank node
- $ideal$ = ideal temperatures in Satish coefficient, Eq. (2)
- $in$ = inlet
- $mix$ = mixed tank model
- $r$ = refers to collector receiver area
- $str$ = stratified tank model
- $T$ = tank conditions
unmixed and fully mixed storage tanks into a parameter based on the first moment of energy provides limiting values to gauge the relative level of mixing in an actual storage tank. A plug flow model, which considers the storage tank to be made up of isothermal disks of fluid, is used to predict tank temperature profiles and thus energy content of an "ideal" storage tank in which no mixing occurs. The initial temperature profile and inlet flow conditions throughout some period of time must be specified. In this paper, the simulated tank is initially made up of isothermal disks of volume and temperature consistent with experimental conditions. Over a fixed time step, as a volume of water at temperature $T_i$ enters the tank, a new isothermal disk is placed in the tank at the vertical location which ensures that temperature inversions do not exist. At the same time, a volume of water equivalent to that entering the tank is removed from the bottom of the tank. After accounting for thermal losses to the surroundings, the temperature of the i-th element of the stratified tank is given by the analytical solution of

$$\rho c_p V_i \frac{dT_i}{dt} = - (UA) r (Ti - To) .$$

(7)

The length of the time step must be short enough to represent accurately variations in inlet conditions.

In contrast, the fully mixed temperature profile is determined by assuming that any time water enters the tank, the entire tank mixes completely. The mass weighted average temperature of the experimental tank is used as the initial condition for the mixed tank model. From an energy balance accounting for energy added to the tank and energy lost to the surroundings, a new isothermal tank temperature is determined analytically. The energy balance for each time step is expressed as

$$\rho c_p V_T \frac{dT_T}{dt} = m_{in} c_p (Ti - T_T) - (UA) r (T_T - To) .$$

(8)

Once the theoretical temperature profiles in the unmixed and fully mixed tanks are determined, the moments of energy for the unmixed (fully stratified) ($M_{str}$) and fully mixed ($M_{mix}$) simulated tanks are determined and incorporated into a MIX number given by

$$\text{MIX} = \frac{(M_{str} - M_{actual})}{(M_{str} - M_{mix})} .$$

(9)

For a given set of inlet conditions, $M_{str}$ and $M_{mix}$ represent the largest and smallest values of the moment of energy, respectively. The MIX number equals zero for an actual tank with equal amounts of energy at vertical locations identical to those predicted by the unmixed tank model. If an actual tank has an equal amount of energy at heights identical to those predicted by the fully-mixed tank model, MIX equals 1.

Experimental Validation

Use of the new MIX number is discussed in terms of experimental data obtained in a simulated direct solar hot water heating system. The experimental facility, shown in Fig. 1, includes an insulated, Lexan 372 liter water storage tank (measured $UA$ of the insulated tank = 2.7 W/K), a 310 liter electric water heater with two 4500 W heating elements used to simulate collector return water, and a cold mains water supply. Tank temperatures are measured with 19 T-type thermocouples (±0.5°C or 0.4 percent of reading, whichever is greater) mounted in a thermocouple tree located midway between the inlet and one wall of the tank. Inlet water temperature is measured with a thermocouple inserted in the pipe just upstream of the inlet. A turbine flow meter (±0.0033 l/s) is used to measure the volumetric flow rate of the water entering the storage tank.

Two inlet designs are studied. The conventional inlet is a vertical 2.54-cm diameter PVC tube which supplies water to the top of the tank. The alternate inlet, shown in Fig. 2, is a rigid porous manifold constructed of 8.9-cm diameter PVC pipe with both vertical and horizontal resistance elements. Details of the manifold design are given in Carlson (1990) and Davidson et al. (1992). The manifold includes a momentum diffuser to reduce the vertical component of momentum of water entering the tank and a distribution manifold with vertical orifice plates which reduce plume entrainment by forcing fluid to exit at the vertical height where the temperature of the fluid entering the tank equals the temperature of the fluid in the tank.

Three testing schemes are used to gain an accurate measure of how well the MIX number characterizes temperature distributions in the tank under various operating conditions. In each test, as water enters at the top of the tank, water is drained from the bottom of the tank at the same fixed flow rate (0.07 l/s based on conventional flow rates of 0.01-0.02 kg/s per m² of collector area). Tank temperature profiles, inlet water temperature, and flow rate are recorded at one-minute intervals.

In Scheme I, the upper half of the tank is filled with hot (50 ≤ $T_a$ ≤ 55°C) water and the bottom half is filled with cold (15 ≤ $T_a$ ≤ 20°C) water. Water is then delivered to the tank at a constant intermediate temperature (≈ 30°C). The length of the test is 48 minutes, the time required for the cold water to be removed from the tank (assuming no mixing occurs). In Schemes II and III, the storage tank is initially filled with 15 to 20°C water and test duration is 90 minutes, the time necessary to replace the initial volume of water in the tank. In Scheme II, water is input at approximately 50°C. In Scheme III, temperature of the inlet water is varied every ten minutes as shown in Table 1.

Moments of energy calculated for each experimental test are based on measured temperatures acquired at one minute intervals. Likewise, analytical values of the moment of energy of fully stratified and fully mixed tanks are computed in one-minute time steps.
Results

Table 2 lists the MIX numbers determined at the end of the three testing schemes for both the conventional tank and the tank operated with the rigid manifold. These numbers are interpreted in terms of measured tank temperature profiles.

Scheme I

Normalized tank temperature profiles obtained during Scheme I tests are plotted at four-minute intervals in Figs. 3 and 4, for the conventional inlet and rigid manifold, respectively. The ordinate is the normalized tank height, the distance from the bottom of the tank (Y) divided by the total tank height (H). The abscissa is a normalized tank temperature defined as the local temperature (T) minus the initial minimum tank temperature (T_c) divided by the maximum initial tank temperature difference (T_h - T_c). The thick solid line (final-str) represents the theoretical temperature profile that would exist at the end of the test if no mixing occurred. The thinner solid line (final-mix) represents the theoretical tank temperature profile that would exist if water in the tank was continuously mixed.

Experimental data in Fig. 3 show that at the end of the 48-minute test, the tank operating with a conventional drop-tube inlet is isothermal. However, since most of the mixing occurs in the top half of the tank, there is more energy stored in the actual tank than in the simulated fully mixed tank. A MIX number of 0.62 correctly indicates that the final tank temperature is greater than the temperature in a hypothetical tank in which fluid is continuously mixed. This result points out the fallacy of basing level of mixing solely on the shape of the vertical temperature profile. As expected, at the end of the test, water in the upper half of the fully stratified tank remains at the initial temperature (minus losses to the surrounding), and water temperature in the lower half of the tank equals that of the incoming fluid.

As shown by the temperature profiles in Fig. 4 and a MIX number of 0.26, use of the rigid manifold significantly reduces mixing. At the end of the test, water temperatures in the upper half of the tank are only slightly lower than those predicted by the ideal unmixed tank model and in the lower half of the tank.

Table 2 Mix numbers

<table>
<thead>
<tr>
<th>Inlet Type</th>
<th>Scheme I</th>
<th>Scheme II</th>
<th>Scheme III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional Drop-Tube</td>
<td>0.62</td>
<td>0.56</td>
<td>0.74</td>
</tr>
<tr>
<td>Rigid Manifold</td>
<td>0.26</td>
<td>0.41</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Fig. 3 Tank temperature profiles of tank with conventional inlet operating under Scheme I

Fig. 4 Tank temperature profiles of tank with manifold operating under Scheme I
tank, measured water temperatures are only slightly greater than in an ideal tank.

Note that MIX number does change with time. If the tests were extended, the final moments of energy would be different. However, relative ordering of the numbers would remain the same. Of course, with a very long test, with a constant inlet temperature, every tank would approach the same uniform temperature.

Scheme II

Tank temperature profiles for Scheme II tests are plotted at ten-minute intervals in Figs. 5 and 6 as a function of normalized vertical position. In both these tests and scheme III tests, temperatures are not normalized since there are not constant hot and cold bounding tank temperatures. As shown in Fig. 5, after the first ten minutes, water temperatures rise throughout the tank with the conventional inlet. During this same time period, as shown in Fig. 6, mixing in the tank with the rigid manifold is restricted to the top 40 percent of the tank. This difference in fluid mixing is reflected in a difference in MIX numbers, 0.56 for the conventional tank and 0.41 for the tank with the manifold. Both inlets restrict mixing as compared to the theoretically fully mixed tank.

Scheme III

Tank temperature profiles obtained every ten minutes are plotted in Figs. 7 and 8 for the Scheme III tests. As in Scheme II, mixing occurs throughout the conventional tank after only ten minutes. In contrast, data in Fig. 8 illustrate that more than 60 minutes pass before water temperature increases in the bottom of the tank with the rigid manifold. At the end of the 90-minute test, the conventional tank is nearly isothermal. With the rigid manifold, temperatures in the lower portion of the tank are equal to those predicted for a stratified tank. In the upper part of the tank, measured temperatures are as much as 10°C less than the ideal case, but are significantly higher than in the conventional tank. The difference in mixing is quantified by a MIX number of 0.74 for the conventional tank and a value of 0.40 for the tank with manifold.

Summary

A new index to quantify mixing in water storage tanks is based on height weighted energy, calculated from the vertical temperature profile. Upper and lower bounding values of this moment of energy are determined theoretically over any thermal test history by assuming perfect stratification and complete mixing, respectively. The MIX number incorporating these theoretical values is 1 for a completely mixed tank and is 0 for an ideally stratified tank. Based on a comparison of measured temperature profiles and MIX number, this dimensionless coefficient gives a realistic appraisal of the effectiveness of tank designs in achieving thermal stratification.

References


