ANALYSIS OF SEDIMENT TRANSPORT EQUATIONS FOR RAINFALL EROSION

by

Pierre Y. Julien and Daryl B. Simons

June, 1984

Civil Engineering Department
Engineering Research Center
Colorado State University
Fort Collins, Colorado

CER83-84PYJ-DBS52

ANALYSIS OF SEDIMENT TRANSPORT EQUATIONS

FOR RAINFALL EROSION

by

Pierre Y. Julien and Daryl B. Simons

June, 1984

Civil Engineering Department
Engineering Research Center
Colorado State University
Fort Collins, Colorado

CER83-84PYJ-DBS52

ANALYSIS OF SEDIMENT TRANSPORT EQUATIONS

FOR RAINFALL EROSION

Prepared by

Civil Engineering Department
Engineering Research Center
Colorado State University
Fort Collins, Colorado

P. Y. Julien
D. B. Simons

June, 1984
Revised, February, 1985

CER83-84PYJ-DBS52

ACKNOWLEDGMENTS

This study has been completed at the Engineering Research Center at Colorado State University during the post-doctoral studies of the first author. The support of a NATO post-doctoral fellowship provided by the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged. The writers wish to extend their appreciation to all those who reviewed this report, and express special thanks to D. M. Hartley, G. O. Brown, and J. S. O'Brien. Their suggestions and comments were very helpful in the final preparation of this document.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

a, c, d	coefficients
A,b	parameters for rainfall
C	Chézy coefficient
d_s	size of sediment
f	Darcy-Weisbach friction factor
g	gravitational acceleration
h	flow depth
i	rainfall intensity
I,J	coefficients
K	friction parameter with rainfall
k_o	friction parameter without rainfall
L	length of overland flow
m, N	exponents of stream power sediment transport equations
n	Manning coefficient
q	unit water discharge
q_c	critical unit water discharge
q_s	unit sediment discharge
r, r_1 r_2	constants
Re	Reynolds number
R^2	coefficient of determination
S	bed slope
S_f	slope of the energy line
u	velocity at a distance y from the free surface
$\bar{u}$	mean velocity
$\bar{u}_c$	critical mean velocity

U_*	shear velocity
w	fall velocity
x	longitudinal distance
y	vertical distance from the free surface
$\alpha,\beta,\gamma,\delta,\varepsilon$	coefficients of the sediment transport equation
δ'	thickness of the laminar sublayer
μ	dynamic viscosity
ν	kinematic viscosity
ρ	specific mass of water
ρ_s	specific mass of sediments
τ_o	bed shear stress
τ_c	critical bed shear stress
Φ	rate of energy dissipation

I. INTRODUCTION

Soil erosion by rainfall is one of the major sources of sediments transported into streams. The physical processes governing rainfall erosion are very complex and no generally accepted sediment transport equation has been developed so far. On upland areas, the flow usually begins in a very thin film of water called laminar sheet flow. Further downstream, flow concentrates and initiates the formation of rills. When the Reynolds number exceeds the critical value ($Re \cong 2000$), the flow becomes turbulent.

Various approaches have been used in the past decades to analyze sediment transport by overland flow. These can be generally classified into three main categories based on: (1) application of mechanics principles to describe equilibrium conditions; (2) regression analysis of experimental and field data; and (3) applications of probabilistic and stochastic principles. Studies in the third category are limited while the first two categories have been used extensively. In the first category, the fundamentals of fluid mechanics based on force equilibrium have been applied more extensively than the concepts of energy and power.

In streams, however, the first investigations to determine the rate of sediment transport as bed-load date from the end of the nineteenth century. Since then, several formulas were derived from both theoretical analysis and experimental investigations. These equations are essentially valid for turbulent flow with various bed form profiles. Some are limited to bed load in flumes and streams, while others are also applicable to total load.

Some of these equations valid for turbulent streams have been used to predict soil erosion from overland flow. For example, Komura (1976) used the Kalinske-Brown relationship and obtained fair agreement with observed data though his data set was relatively limited. The Meyer-Peter and Müller equation has also been used by Li (1979) in computer models for routing sediments on small watersheds. Several sediment transport equations have been examined by Alonso, Neibling and Foster (1980, 1981) to determine how well they fit observed erosion data collected on concave slopes. They recommended the Yalin equation to compute the sediment transport capacities for overland flow.

There is no doubt that the fundamental relationships for sediment transport in turbulent stream flows can provide guidance to the analysis of the sediment transport capacity by overland flow. However, since sheet flows are generally classified as laminar, a theoretical analysis is required to determine which sediment transport formulas derived for turbulent streamflows are applicable to laminar sheet flows.

The objective of this investigation on various sediment transport equations applicable to overland flow conditions are: 1) to analyze and transform several empirical equations into a rational relationship based on dimensional analysis; 2) to examine the applicability of bed-load equations under various hydraulic conditions including laminar sheet flows; and 3) to apply the concepts of energy dissipation and stream power to derive sediment transport equations for sheet flows.

In the first part of this report, the hydraulic characteristics relevant to sediment transport will be reinstated to clearly point out the differences between turbulent and laminar flows. Several sediment transport formulas valid for turbulent streamflows are then transformed for the laminar sheet flow conditions. Finally, the sediment transport equations for the transport capacity of laminar sheet flows are derived from energy dissipation and stream power concepts. These relationships are compared with regression equations obtained from experimental studies of soil erosion and overland flow.

II. OVERLAND FLOW CHARACTERISTICS

This chapter points at the detailed description of the overland flow characteristics. The principal variables and the fundamental equations describing the flow processes are investigated for three types of flow conditions: laminar sheet flows, turbulent smooth and turbulent rough flows respectively.

2.1 Variables

Overland flow refers to the thin sheet layer of surface runoff toward the stream channel system. Sheet runoff over a smooth surface is usually classified as an unsteady, nonuniform flow affected by raindrop impact.

Overland flows are open channel flows in which rills are small-scale channels and sheet flow occur in a very wide and shallow cross section. One of the major differences between stream flow and overland flow result from the relative magnitude of inertia and viscous forces. Stream flows are largely turbulent because inertia forces overcome friction forces due to the fluid viscosity. In sheet flows, raindrop impact and surface roughness disturb the flow pattern but due to the reduced flow depth the Reynolds numbers remain small. The velocity fluctuations are damped by the viscous forces and the flows behave as laminar. The classification of overland flows depends upon the Reynolds number and the relative roughness. Overland flow can be either laminar or turbulent and the surface can be either rough or smooth. The hydraulic characteristics of flow in a wide channel are shown in Figure 1. The main geometric variables are the plot length L and the slope S. The hydraulic variables are the rainfall intensity i, the flow depth h, the mean velocity $\bar{u}$, the unit water discharge q, and the thickness of the laminar sublayer δ'. The parameter generally associated with the sediment discharge q_s is the bed shear stress τ_o. While the other properties are the gravitational acceleration g, the kinematic viscosity ν and the specific mass of water ρ and of sediments ρ_s.

2.2 Fundamental Equations

The two nonlinear partial differential equations derived by de Saint-Venant are basically used to solve the problem of gradually varied unsteady flows. The continuity equation is

$$\frac{\partial h}{\partial t} + \bar{u} \frac{\partial \bar{h}}{\partial x} + h \frac{\partial \bar{u}}{\partial x} = q_o \tag{1}$$

and the momentum equation including a lateral inflow component q_o is:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} = g(S - S_f) - g \frac{\partial h}{\partial x} - \frac{q_o}{h} (\bar{u} - v) \tag{2}$$

in which:

q_o = inflow rate

S_f = friction slope

v = the velocity component along x of the lateral inflow.

Considering the principal terms of the momentum equation, the kinematic wave approximation has been most widely recommended (Wooding, 1965; Woolhiser, 1975). This approximation states that the friction slope is equal to the soil surface slope, or

$$S = S_f \tag{3}$$

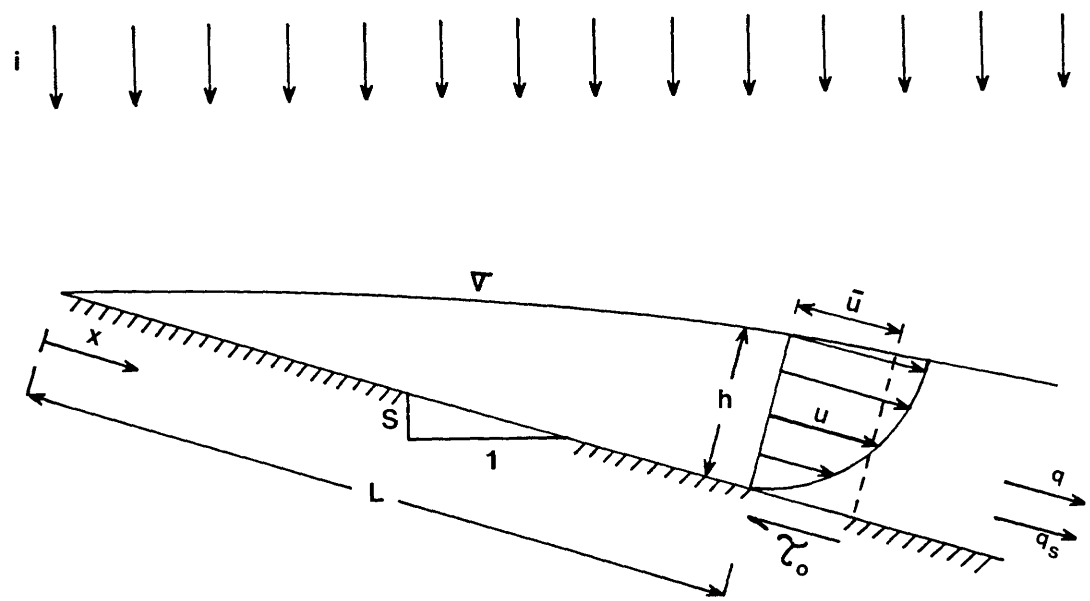

Figure 1. Overland flow variables.

For the case of steady uniform flow conditions over an impervious surface, the continuity equation can be written:

$$q = \bar{u}h = iL \tag{4}$$

The Reynolds number is defined as follows:

$$Re = \frac{\bar{u}h}{\nu} \tag{5}$$

The energy loss equation given by Darcy-Weisbach is written as a function of the Darcy-Weisbach friction factor f:

$$S_f = \frac{f}{8}\frac{\bar{u}^2}{gh} \tag{6}$$

The value of the friction factor f is a function of the Reynolds number and the relative roughness.

Three other important variables regarding resistance to flow and soil erosion are the bed shear stress τ_o, the shear velocity U_* and the thickness of the laminar sublayer δ' in turbulent flows. These variables are defined as:

$$\tau_o = \rho g h S \tag{7}$$

$$U_* = \sqrt{\frac{\tau_o}{\rho}} \tag{8}$$

$$\delta' = \frac{11.6\ \nu}{U_*} \tag{9}$$

This last variable has a physical significance since the ratio of δ'/d_s delineates the type of turbulent flow as whether the boundary is smooth or rough (see Figure 2). With varying Reynolds number and relative roughness, the friction coefficient f will follow different laws. Four types of flow ranging from laminar flow to turbulent flow will be

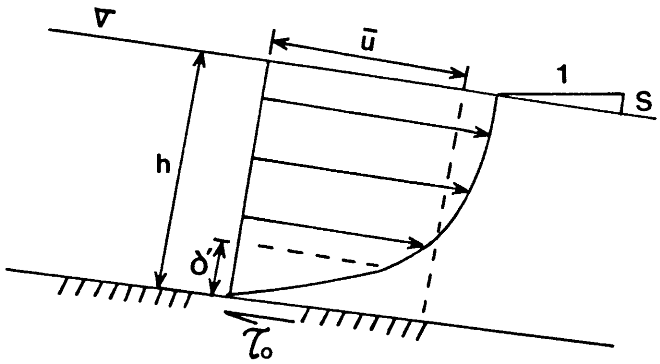

Figure 2a. Turbulent flow characteristics: smooth boundary.

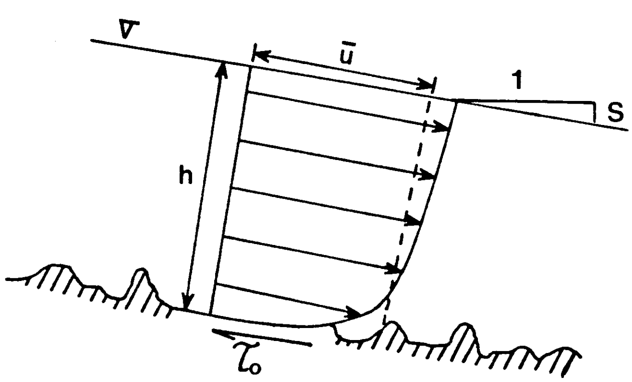

Figure 2b. Turbulent flow characteristics: rough boundary.

examined: 1) laminar sheet flow; 2) turbulent flow over a smooth surface as given by the Blasius equation; 3) turbulent flow over a rough surface given by the Manning equation; and 4) turbulent flow over a rough surface with very small relative roughness as given by the Chézy equation. This last flow type is not very likely to occur in overland flow. It has been considered as a limiting case for which the Darcy-Weisbach friction factor (or Chézy C) remains constant. For each of these flow conditions, the principal variables related to soil erosion ($\bar{u}$, h and τ_o) are defined as a function of slope and water discharge for steady flow conditions.

2.3 Laminar Flow

Laminar flows with raindrop impact can be described by the Darcy-Weisbach equation (Eq. 6) in which the friction factor f is related to: (1) the Reynolds number Re, (2) the surface friction coefficient k_o without raindrop impact, and (3) two empirical coefficients A and b for raindrop impact. The following relationship is generally used:

$$f = \frac{K}{Re} = \frac{k_o + Ai^b}{Re} \tag{10}$$

As shown in Table I, the values of k_o have been tabulated by Woolhiser for various surface types and the value $k_o = 24$ is representative of the smooth surface condition. Several sets of coefficients A and b have been obtained from experimental investigations and these values are indicated in Table II. The experimental data reported by Shen and Li are shown in Figure 3. These indicate that for a bare smooth surface, the flow is laminar for $Re < 900$ and the Blasius law is valid for turbulent flows over a smooth surface when $Re > 2000$. Chen's data in Figure 4 shows that laminar flows are observed for Reynolds numbers as large as 10^5 for vegetated surfaces.

Table I. Resistance parameters for overland flow (after Woolhiser, 1975).

Surface	Laminar Flow k_o	Turbulent Flow Manning n	Turbulent Flow Chézy C ($ft^{1/2}$/sec)
Concrete or Asphalt	24 - 108	.01 - .013	73 - 38
Bare Sand	30 - 120	.01 - .016	65 - 33
Graveled Surface	90 - 400	.012 - .03	38 - 18
Bare Clay-Loam Soil (eroded)	100 - 500	.012 - .033	36 - 16
Sparse Vegetation	1000 - 4000	.053 - .13	11 - 5
Short Grass Prairie	3000 - 10,000	.10 - .20	6.5 - 3.6
Bluegrass Sod	7000 - 40,000	.17 - .48	4.2 - 1.8

Table II. Resistance coefficients A and b for rainfall.

Reference	A*	b
Izzard (1944)	5.67	1.33
Li (1972)	27.2	0.4
Fawkes (1972)	10.0	1.0

*For i in inches per hour.

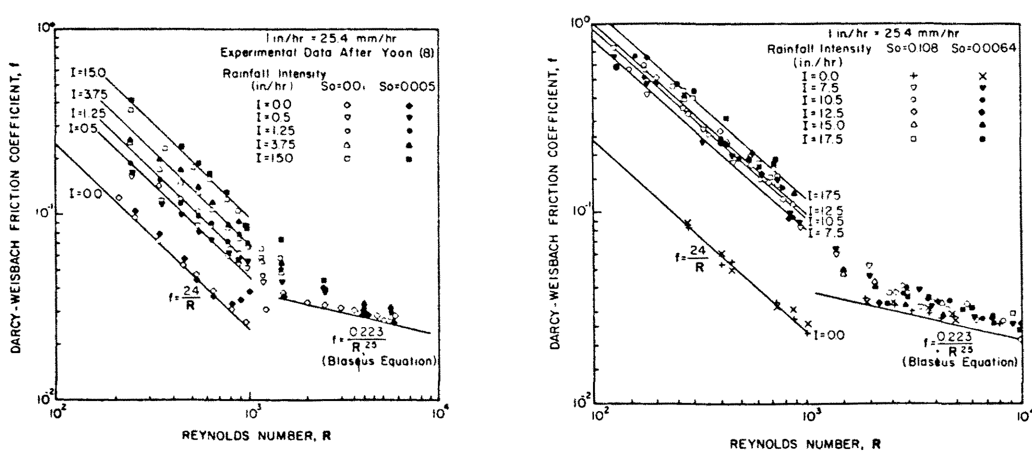

Figure 3. Influence of rainfall intensity on the friction coefficient (after Shen and Li, 1973).

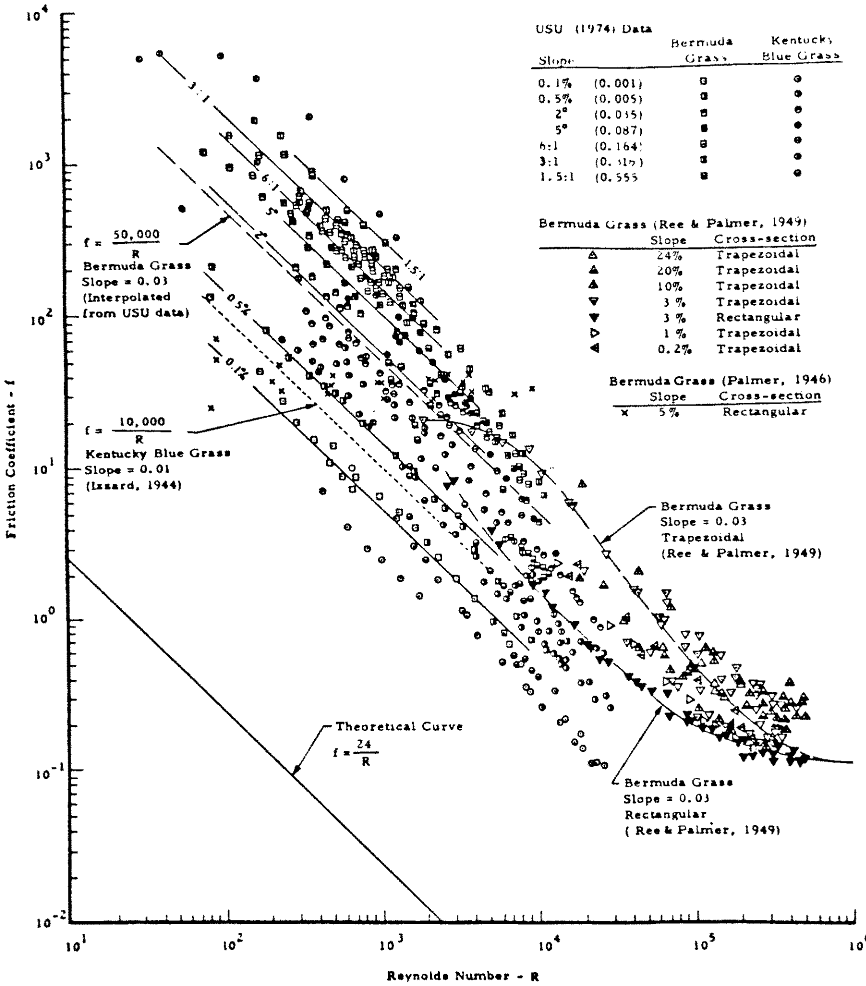

Figure 4. Influence of vegetation on the friction coefficient (after Chen, 1976).

For very low Reynolds numbers, the flow is laminar and the Darcy-Weisbach equation for that type of flow is given from Eqs. 6 and 10

$$S_f = \left(\frac{K\nu}{\bar{u}h}\right)\frac{\bar{u}^2}{8gh} \tag{11}$$

where K is the friction parameter for sheet flows.

Along a vertical profile, the shear stress and velocity distributions are described by the following equations

$$\tau = \frac{\tau_o y}{h} \tag{12}$$

and

$$u = \frac{12\ gS}{K\nu}\ (h^2 - y^2) \tag{13}$$

in which, τ is the shear stress and u is the velocity at the distance y from the free surface. The main flow velocity determined from the integration of Eq. 13 gives:

$$\bar{u} = \left(\frac{8g}{K\nu}\right)\ S\ h^2 \tag{14}$$

The general equation for energy dissipation in three dimensions with no limitations on the boundary conditions has been reported by Lamb (1932). In the simplified case under consideration, the rate of energy dissipation Φ reduces to:

$$\Phi = \mu \left(\frac{du}{dy}\right)^2 = -\tau\ \frac{du}{dy} \tag{15}$$

The profiles of shear stress (Eq. 12), velocity (Eq. 13), and rate of energy dissipation (Eq. 15) in laminar sheet flows are plotted in Figure 5.

The variables $\bar{u}$, h and τ_o for laminar sheet flows derived from Eqs. 4, 7 and 11 are summarized in Table III for comparison with similar relationships valid under turbulent conditions.

2.4 Turbulent Smooth Flow

Turbulent flows (Re > 2000) for bare soil surfaces behave as hydraulically smooth when the thickness of the laminar sublayer given by $\delta' = 11.6\ \nu/U_*$ is much in excess of the size of soil particles d_s ($\delta' > 3\ d_s$). In this case, Keulegan (1938) derived an equation similar to the von Karman-Prandtl logarithmic equation. When the Reynolds

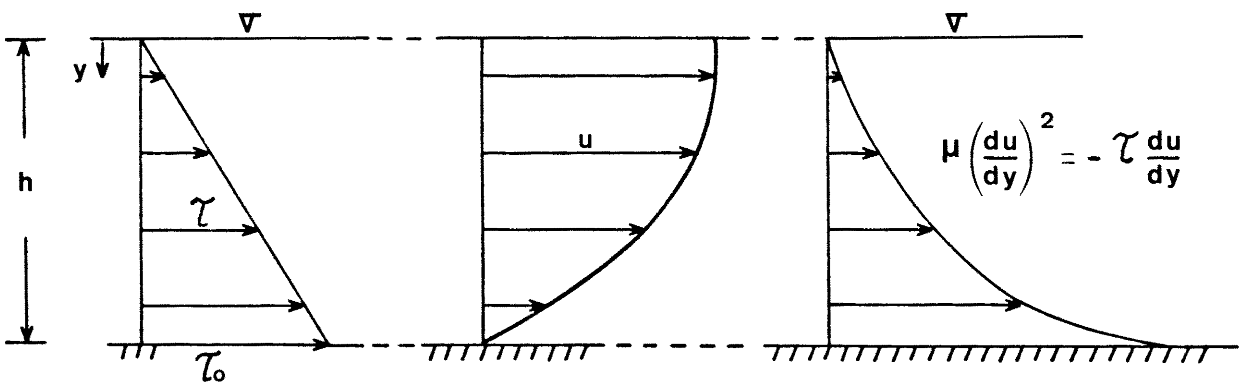

Figure 5. Shear stress, velocity, and energy dissipation profiles in laminar sheet flow.

Table III. Summary of flow characteristics (velocity, depth, and shear stress).

VELOCITY		$\bar{u} = c\ S^a\ q^d$		
Type of flow	Boundary	c	a	d
Laminar		$\left(\frac{8g}{K\nu}\right)^{1/3}$	1/3	2/3
Turbulent	smooth	$\left(\frac{8g}{0.316}\right)^{1/3}\ \nu^{-1/12}$	1/3	5/12
Turbulent (Manning)	rough	$\left(\frac{1}{n}\right)^{0.6}$	0.3	0.4
Turbulent (Chézy)	rough	$\left(\frac{8g}{f}\right)^{1/3}$	1/3	1/3
DEPTH		$h = c\ S^a\ q^d$		
Type of flow	Boundary	c	a	d
Laminar		$\left(\frac{K\nu}{8g}\right)^{1/3}$	-1/3	1/3
Turbulent	smooth	$\left(\frac{0.316}{8g}\right)^{1/3}\ \nu^{1/12}$	-1/3	7/12
Turbulent (Manning)	rough	$n^{0.6}$	-0.3	0.6
Turbulent (Chézy)	rough	$\left(\frac{f}{8g}\right)^{1/3}$	-1/3	2/3
SHEAR STRESS		$\tau_o = c\ S^a\ q^d$		
Type of flow	Boundary	c	a	d
Laminar		$\rho g\ \left(\frac{K\nu}{8g}\right)^{1/3}$	2/3	1/3
Turbulent	smooth	$\rho g \left(\frac{0.316}{8g}\right)^{1/3}\ \nu^{1/12}$	2/3	7/12
Turbulent (Manning)	rough	$\rho g\ n^{0.6}$	0.7	0.6
Turbulent (Chézy)	rough	$\rho g\ \left(\frac{f}{8g}\right)^{1/3}$	2/3	2/3

number is not too large, this equation can be approximated by the Blasius equation:

$$S_f = 0.316 \left(\frac{\nu}{\bar{u}h}\right)^{\frac{1}{4}} \frac{\bar{u}^2}{8gh} \qquad (16)$$

For turbulent smooth flows, the variables $\bar{u}$, h and τ_o given in Table III are derived from Eqs. 4, 7 and 16. The exponents of S for these variables are identical to those obtained for laminar sheet flows.

2.5 Turbulent Rough Flow

In turbulent rough channels ($Re > 2000$ for bare soil surface, $5\delta' < d_s$) without bed forms, an increase of the Reynolds number (or water discharge) raises the water level, and decreases the relative roughness and the friction factor. The logarithmic equation given by Keulegan (1938) is:

$$C = \sqrt{\frac{8g}{f}} = c_1 \log c_2 \frac{h}{d_s} \qquad (17)$$

This equation is a theoretically sound resistance relationship for turbulent rough flows. Approximate power relationships such as the Manning equation, however, remain more useful to hydraulic engineers. Since both equations are in good agreement for open channel flows, the Manning equation (SI units) is used in this report:

$$\bar{u} = \frac{1}{n} h^{2/3} S_f^{1/2} \qquad (18)$$

in which n is the Manning roughness coefficient. Strickler proposed the following formula to relate Manning n value to the median size, in feet, of the boundary roughness:

$$n = 0.0342\, d_s^{1/6} \qquad (19)$$

The combination of Eqs. 4, 7, and 18 gives the relationships for $\bar{u}$, h, and τ_o shown in Table III for turbulent rough conditions. One notices that the relationship $\tau_o \propto \bar{u}^2$ does not hold true for the Manning relationship and the values of the exponents are slightly different than those derived from the Chézy relationship. The Manning n coefficient should be constant for uniform sediment roughness without bed forms.

When the relative roughness is small, the Darcy-Weisbach equation with constant friction factor is equivalent to the Chézy equation ($f = 8g/C^2$) and after combining Eqs. 4, 6, and 7, the variables $\bar{u}$, h, and τ_o are written as a function of S and q. The resulting expressions are listed in Table III. It is shown that $\tau_o \propto \bar{u}^2$ while for the velocity $\bar{u}$ the exponents of q and S are identical and equal to 1/3.

2.6 Discussion

This analysis of the hydraulic characteristics is very instructive and the results summarized in Table III indicate clearly that when the velocity, the flow depth, and the bed shear stress are written in terms of discharge and slope, the exponent of the slope remains nearly the same for each variable under different flow conditions ranging from laminar to turbulent. The exponents of the slope for velocity, flow depth, and bed shear stress are respectively 1/3, -1/3, and 2/3. On the other hand, the exponent of the water discharge varies gradually under different conditions and for extreme conditions the exponent values differ by a factor 2. Moreover, the variation of the exponents of discharge for velocity and shear stress are in opposite directions for varying flow conditions. Indeed, for flow conditions changing from laminar to turbulent flows, the exponent of velocity varies from 2/3 to

1/3 while the exponent of shear stress varies from 1/3 to 2/3. This effect is extremely important if we consider the rate of sediment transport.

From this analysis it can be concluded that the transformation of bed-load equations from turbulent flow to laminar sheet flow will lead to completely different results whether the relationships are based on velocity or on bed shear stress.

III. SEDIMENT TRANSPORT EQUATIONS

This chapter deals with the sediment transport equations for rainfall erosion. Several approaches are investigated to obtain a theoretically sound relationship supported by experimental data. The method of dimensional analysis is first applied to the principal variables related to soil erosion. Then, several empirical relationships are transformed into the general equation obtained by dimensional analysis. In the following section several sediment transport formulas are applied to turbulent smooth and laminar sheet flow conditions. Energy dissipation and stream power concepts are applied to sheet flows to derive theoretical sediment transport equations. The last section of this chapter summarizes the results obtained in this chapter. The range of the exponents of a sound relationship is defined and the results of various approaches are discussed.

3.1 Variables and Dimensional Analysis

Sheet erosion is the result of soil particles detachment and transport from raindrop impact and overland flow. Most of the eroded soil particles are transported downstream by runoff and the unit sediment discharge is a function of several variables. A relationship for sediment transport by overland flow will be obtained from the analysis of the following variables:

$$q_s = f(L, S, i, \bar{u}, h, q, \tau_o, \tau_c, d_s, \rho_s, \rho, \nu, g) \qquad (20)$$

in which τ_c is the critical shear stress and d_s is the size of soil particles, and the other variables were defined previously. Among these variables, the first two (L, S) describe the geometry and the next five $(i, \bar{u}, h, q, \tau_o)$ are flow characteristics including rainfall intensity. The last six $(\tau_c, d_s, \rho_s, \rho, \nu, g)$ are associated with soil and water properties and the gravitational acceleration. The shear stress is difficult to measure in the field and is usually computed from other variables. In a river, the variables S, $\bar{u}$, h and g are used to describe stream flows because the velocity and depth are generally more easily measured than the rainfall intensity and the length L. For this reason, Laursen (1956) suggested to reduce some sediment transport equations to a function of the variables $\bar{u}$ and h. On the other hand, in soil erosion problems, the variables i and L have a great physical significance. The slope and unit water discharge can be more easily measured than the velocity and depth. Therefore, the variables S and q are more relevant than $\bar{u}$ and h to define a sediment transport relationship for overland flow. Elimination of the variables $\bar{u}$ and h is possible from the Darcy-Weisbach equation (Eq. 6) and the continuity equation $q = \bar{u}h$.

The critical shear stress value τ_c corresponds to the beginning of motion of the sediment particles. Its evaluation remains a complex problem requiring further investigation, but the basic relationship gives the critical shear stress as a function of the particle size and the specific masses of water and sediment. The sediment size d_s can be eliminated from a relationship between τ_c and d_s similar to critical Shields number for laminar flow. In other words, the sediment size can be replaced by the critical shear stress in a sediment transport equation. In practice, the specific masses of water and sediment are nearly constant for particle sizes ranging from clays to gravels. In the case of aggregates, equivalent conditions of shear stress can be defined while keeping the same specific mass of sediment in the analysis. Therefore, to avoid redundancy of the variables, the constant value of ρ_s and the relationship between τ_c and d_s enable us to delete the variables ρ_s and d_s from Eq. (20), while keeping the variable τ_c.

Assuming γ_s constant, Eq. 20 thus reduces to

$$f(q_s, q, i, L, \rho, \nu, \frac{\tau_c}{\tau_o}, S) = 0 \tag{21}$$

The following dimensionless groups are obtained from dimensional analysis after L, ρ, and ν are selected as repeated variables

$$f\left(\frac{q_s}{\rho\nu}, \frac{q}{\nu}, \frac{iL}{\nu}, \frac{\tau_c}{\tau_o}, S\right) = 0 \tag{22}$$

The expected general solution gives the sediment transport term as a function of the product of the other variables in the form

$$\left(\frac{q_s}{\rho\nu}\right) = \bar{\alpha}\, S^{\beta} \left(\frac{q}{\nu}\right)^{\gamma} \left(\frac{iL}{\nu}\right)^{\delta} \left(1 - \frac{\tau_c}{\tau_o}\right)^{\varepsilon} \text{ ; for } \tau_o > \tau_c \tag{23}$$

In this equation, $\bar{\alpha}$, β, γ, δ, and ε are experimental coefficients and the sediment equations based on tractive force and stream power concepts are best represented by the term $1 - (\tau_c/\tau_o)$.

Under dimensional form, this equation is transformed to

$$q_s = \alpha\ S^{\beta}\ q^{\gamma}\ i^{\delta}\ \left(1 - \frac{\tau_c}{\tau_o}\right)^{\varepsilon} \tag{24}$$

in which,

$$\alpha = \frac{\bar{\alpha}\ \rho\ L^{\delta}}{\nu\gamma + \delta - 1} \tag{25}$$

Equation 24 was obtained by Julien (1982) to describe the general relationship between sediment discharge and the principal flow variables. The first three factors (S, q, i) represent the potential erosion or transport capacity by overland flow, which is reduced by the last factor essentially representative of the soil resistance to erosion. It is also seen that when τ_c remains small compared to τ_o, the equation for sediment transport capacity is

$$q_s = \alpha\ S^{\beta}\ q^{\gamma}\ i^{\delta} \tag{26}$$

For stream flows, the sediment transport equation is not a function of the rainfall intensity, and therefore, $\delta = 0$ in this case.

3.2 Empirical Equations

Quantitative evaluation of the coefficients α, β, γ, δ and ε can be obtained from several types of equations based on different variables. The equations analyzed are those proposed by Musgrave (1947); Li, Shen and Simons (1973); and several regression equations obtained by Kilinc (1972) and others, including tractive force, stream power, velocity, and discharge equations. When these equations are a function of variables different than those of Eq. 24, the relationships in Table III for laminar flow are used for the variables $\bar{u}$, h, and τ_o, and the Reynolds number is replaced by $Re = \bar{u}h/\nu$. The results are shown in Table IV, and it is found that none of the actual equations is complete

Table IV. Transformation of several erosion equations.

Eq. No.	Reference	Equation	α[a]	β	γ	δ	ε
27	Musgrave (1947)	$q_s = \alpha' S^m L^n i^P$	α'	m	n	p-n	0
28	Zingg (1940)	$q_s \alpha L^{1.66} S^{1.37}$	--	1.37	1.66	-1.66	--
29	Wischmeier and Smith (1965)	$q_s \alpha L^{1.5} (.00076S^2 + .0053S + .0076)$	--	≅1.7	1.5	-1.5	--
30	Meyer and Monke (1965)	$q_s \alpha L^{1.9} S_o^{3.5}$	--	3.5	1.9	-1.9	--
31	Young and Mutchler (1969)	$q_s \alpha L^{2.24} S^{0.74}$	--	0.74	2.24	-2.24	--
32	Li et al. (1973)	$q_s = \alpha' \int_o^L \tau_o^2 \, dx$	$3 \alpha' \frac{\gamma_e^2}{5}\left(\frac{K \nu_e}{8g}\right)^{2/3}$	1.33	1.67	-1	0
33	Komura (1983)	$q_s \alpha q^{11/8} i^{1/2} S^{1.5}$	--	1.5	1.38	0.5	0
34	Kilinc (1972)	$q_s = e^{2.05}(\tau_o - \tau_c)^{2.78}$	$e^{2.05} \frac{\gamma^{0.78}}{\nu_e} \left(\frac{K\nu_e^2}{8g}\right)^{0.93}$	1.86	0.93	0	2.78
35	Kilinc (1972)	$q_s = e^{0.122} ((\tau_o - \tau_c)\bar{u})^{1.67}$	$e^{0.122} \gamma_e^{1.67}$	1.67	1.67	0	1.67
36	Kilinc (1972)	$q_s = e^{-3.17} \bar{u}^{3.625}$	$e^{-3.17} (\frac{8g}{K\nu_e})^{1.21}$	1.21	2.42	0	0
37	Kilinc (1972)	$q_s = e^{1.24} \bar{u}^{4.67} Re^{-0.878}$	$e^{1.24} {}_e^{0.878} (\frac{8g}{K\nu_e})^{1.56}$	1.56	2.24	0	0
38	Kilinc (1972)	$q_s = e^{-11.6} Re^{2.05} S^{1.46}$	$e^{-11.6} \nu_e^{-2.05}$	1.46	2.05	0	0
39	Kilinc (1972)	$q_s = e^{11.7} q^{2.035} S^{1.66}$	$e^{11.7}$	1.66	2.03	0	0

[a]Sediment discharge in pounds per ft-sec.

since some coefficients are still zero. Consequently, for each particular equation, the number of variables is reduced owing to these zero values. From this analysis the main parameters are the slope S and the discharge q. The numerical values of the coefficient β vary from 1.2 to 1.9, and γ varies from 1.4 to 2.4. These range of values will be referred to as the range of the empirical coefficients β and γ for erosion equation by overland flow.

The well-known Universal Soil Loss Equation cannot be transformed directly into the general equation since the slope factor is written in a quadratic form. The equivalent exponent, however, is expected to vary between 1 and 2. Julien (1982) suggested an equivalent exponent value near 1.7. The Kilinc and Richardson equations cannot define the parameters δ and ε since the overland flow rate is almost the same as the rainfall rate and also the bed tractive force is generally much in excess of the critical shear stress value. This analysis also shows that the number of independent parameters after transformation is the same as before transformation. For example, equations based on slope and length (Eqs. 28, 30 and 31) have two independent parameters, both before and after transformation, which imposes the condition $\delta = -\gamma$. Considering equations having one independent parameter, for Eq. 34, $\varepsilon = 3\gamma$ and $\beta = 2\gamma$; for Eq. 35, $\beta = \gamma = \varepsilon$; and for Eq. 36, $\beta = \gamma$.

Further fundamental research is therefore needed to obtain a more complete description of the soil erosion rate. The coefficients of the general equation obtained by dimensional analysis are kept variable for the purpose of this study. Accordingly, the prediction from each equation will be possible, provided the proper set of coefficients is selected from Table IV. Fair estimates can be obtained from a regression equation such as given by Kilinc (1972). Excellent results were obtained by Julien (1982) with the use of the discharge and slope formula (Eq. 39).

Soil erosion by overland flow does not remain absolutely uniform as assumed theoretically. The formation of rills locally increases the unit water discharge q such that on the whole area, the resulting erosion rate may be larger than for uniform flow conditions. The rill erosion data collected by Kilinc were analyzed by the writers and once the volume of rill erosion was subtracted from the total erosion, the following regression equation was obtained

$$q_s \propto S^{1.31} q^{1.93} \quad ; \quad (R^2 = 0.96) \tag{40}$$

This equation will be used for comparison with sediment transport equations for laminar sheet flow.

3.3 Sediment Transport Equations for Streams

In this section, we propose to transform several of the well-known sediment transport equations originally derived for turbulent stream flows in order to determine whether they are applicable or not to laminar and turbulent conditions in overland flows. So many sediment transport equations in turbulent streams have been suggested by various investigators that it is almost impossible to consider all of them.

This analysis includes the transformation of the equation suggested by Du Boys (1879), O'Brien-Rindlaub (1934), or WES (1935), Shields (1936), Schoklitsch (1934), Kalinske-Brown (1949), Meyer-Peter and Müller (1948), Bagnold (1956), Engelund-Hansen (1967), Inglis-Lacey (1968), Yalin (1977), Chang et al. (1967), Barekyan (1962), and Pedroli (1963). The Einstein bedload equation has not been treated separately since it agrees very well with the Yalin and the Meyer-Peter and Müller equations.

A constant sediment grain size is assumed and the analysis is focused on the sediment transport capacity. Other constants such as the fluid properties or the gravitational acceleration are also deleted from the investigation. Particular attention is pointed at the values of β and γ which are the exponents of the slope and water discharge in Eq. 24. For each of the types of flow described in Chapter 2, the results have been summarized in four corresponding tables: (a) laminar sheet flow (Table V); (b) turbulent flow over smooth surface as given by Blasius equation (Table VI); (c) turbulent rough flow described by Manning equation (Table VII); and (d) constant Darcy-Weisbach or Chézy coefficient (Table VIII).

The last column of these four tables represents an index of fitness of these basic equations with the observed value of exponents. This index is equal to the number of parameters (β, γ) enclosed within the ranges of empirical coefficients as determined in the previous section ($1.2 < \beta < 1.9$ and $1.4 < \gamma < 2.4$). The higher the index is, the best this equation should compare with observed data. Conversely, when the index is equal to zero, the given equation is expected to be a poor predictor for overland flow.

The sediment transport equations transformed give reasonable values of the parameter β. The value of γ, however, are generally is usually too small to fall within the range of empirical coefficients. The parameter ε is highly variable for these equations and further investigation of incipient conditions are required to better define the critical shear stress and the parameter ε.

Table V. Transformed equations for laminar sheet flow.

$$q_s \propto S^{\beta} q^{\gamma} \left(1 - \frac{\tau_c}{\tau_o}\right)^{\varepsilon}$$

Eq. No.	Investigator	Equation	β	γ	ε	Index*
41	Du Boys	$q_s \propto \tau_o(\tau_o - \tau_c)$	1.33	0.66	1	1
42	WES	$q_s \propto (\tau_o - \tau_c)^{m=1.5}$	1	0.5	1.5	0
43	Shields	$q_s \propto S^q (\tau_o - \tau_c)$	1.67	1.33	1	1
44	Schoklitsch	$q_s \propto S^{1.5}(q - q_c)$	1.5	1	-	1
45	Kalinske-Brown	$q_s \propto \tau_o^{2.5}$	1.67	0.83	0	1
46	Meyer-Peter et al.	$q_s \propto (\tau_o - \tau_c)^{1.5}$	1	0.5	1.5	0
47	Bagnold	$q_s \propto \tau_o^{0.5} (\tau_o - \tau_c)$	1	0.5	1	0
48	Engelund-Hansen	$q_s \propto \tau_o^{1.5} \bar{u}^2$	1.67	1.83	0	2
49	Inglis-Lacey	$q_s \propto \bar{u}^5 h^{-1}$	2	3	0	0
50	Yalin ($\tau_o \cong \tau_c$)	$q_s \propto \tau_o^{0.5} (\tau_o - \tau_c)^2$	1.67	0.83	2	1
51	Yalin ($\tau_o >> \tau_c$)	$q_s \propto \tau_o^{0.5} (\tau_o - \tau_c)$	1	0.5	1	0
52	Chang et al.	$q_s \propto \tau_o \bar{u}$	1	1	0	0
53	Barekyan	$q_s \propto Sq\bar{u}$	1.33	1.67	0	2
54	Pedroli	$q_s \propto \tau_o^{1.6} h^{0.2}$	1	0.6	0	0

*The index represents the number of exponents within the ranges: $1.2 < \beta < 1.9$; and $1.4 < \gamma < 2.4$.

Table VI. Transformed equations for turbulent flow over a smooth boundary.

$$q_s \propto S^{\beta} q^{\gamma} \left(1 - \frac{\tau_c}{\tau_o}\right)^{\varepsilon}$$

Eq. No.	Investigator	Equation	β	γ	ε	Index*
41	Du Boys	$q_s \propto \tau_o(\tau_o - \tau_c)$	1.33	1.17	1	1
42	WES	$q_s \propto (\tau_o - \tau_c)^{m=1.5}$	1	0.88	1.5	0
43	Shields	$q_s \propto S^q (\tau_o - \tau_c)$	1.67	1.58	1	2
44	Schoklitsch	$q_s \propto S^{3/2}(q - q_c)$	1.5	1	-	1
45	Kalinske-Brown	$q_s \propto \tau_o^{2.5}$	1.67	1.46	-	2
46	Meyer-Peter et al.	$q_s \propto (\tau_o - \tau_c)^{1.5}$	1	0.88	1.5	0
47	Bagnold	$q_s \propto \tau_o^{0.5} (\tau_o - \tau_c)$	1	0.88	1	0
48	Engelund-Hansen	$q_s \propto \tau_o^{1.5} \bar{u}^{2}$	1.67	1.71	0	2
49	Inglis-Lacey	$q_s \propto \bar{u}^5 h^{-1}$	2	2.5	0	0
50	Yalin ($\tau_o \cong \tau_c$)	$q_s \propto \tau_o^{0.5} (\tau_o - \tau_c)^2$	1.67	1.46	2	2
51	Yalin ($\tau_o >> \tau_c$)	$q_s \propto \tau_o^{0.5} (\tau_o - \tau_c)$	1	0.88	1	0
52	Chang et al.	$q_s \propto \tau_o \bar{u}$	1	1	0	0
53	Barekyan	$q_s \propto Sq\bar{u}$	1.33	1.42	0	2
54	Pedroli	$q_s \propto \tau_o^{1.6} h^{0.2}$	1	1.05	0	0

*The index represents the number of exponents within the ranges: $1.2 < \beta < 1.9$; and $1.4 < \gamma < 2.4$.

Table VII. Transformed equations for turbulent flow over a rough boundary (Manning equation).

$$q_s \alpha S^{\beta} q^{\gamma} \left(1 - \frac{\tau_c}{\tau_o}\right)^{\varepsilon}$$

Eq. No.	Investigator	Equation	β	γ	ε	Index*
41	Du Boys	$q_s \alpha \tau_o(\tau_o - \tau_c)$	1.4	1.2	1	1
42	WES	$q_s \alpha (\tau_o - \tau_c)^{m=1.5}$	1.05	0.90	1.5	0
43	Shields	$q_s \alpha S^q (\tau_o - \tau_c)$	1.7	1.6	1	2
44	Schoklitsch	$q_s \alpha S^{1.5}(q - q_c)$	1.5	1	-	1
45	Kalinske-Brown	$q_s \alpha \tau_o^{2.5}$	1.75	1.5	0	2
46	Meyer-Peter et al.	$q_s \alpha (\tau_o - \tau_c)^{1.5}$	1.05	0.9	1.5	0
47	Bagnold	$q_s \alpha \tau_o^{1/2}(\tau_o - \tau_c)$	1.05	0.9	1	0
48	Engelund-Hansen	$q_s \alpha \tau_o^{1.5} \bar{u}^{-2}$	1.65	1.7	0	2
49	Inglis-Lacey	$q_s \alpha \bar{u}^5 h^{-1}$	1.8	1.4	0	2
50	Yalin ($\tau_o \cong \tau_c$)	$q_s \alpha \tau_o^{1/2} (\tau_o - \tau_c)^2$	1.75	1.5	2	2
51	Yalin ($\tau_o >> \tau_c$)	$q_s \alpha \tau_o^{1/2} (\tau_o - \tau_c)$	1.05	0.9	1	0
52	Chang et al.	$q_s \alpha \tau_o \bar{u}$	1	1	0	0
53	Barekyan	$q_s \alpha Sq\bar{u}$	1.3	1.4	0	2
54	Pedroli	$q_s \alpha \tau_o^{1.6} h^{0.2}$	1.06	1.08	0	0

*The index represents the number of exponents within the ranges: $1.2 < \beta < 1.9$; and $1.4 < \gamma < 2.4$.

Table VIII. Transformed equations for turbulent flow over a rough boundary (Chézy equation).

$$q_s \,\alpha\, S^{\beta} q^{\gamma} \left(1 - \frac{\tau_c}{\tau_o}\right)^{\varepsilon}$$

Eq. No.	Investigator	Equation	β	γ	ε	Index*
41	Du Boys	$q_s \,\alpha\, \tau_o(\tau_o - \tau_c)$	1.33	1.33	1	1
42	WES	$q_s \,\alpha\, (\tau_o - \tau_c)^{m=1.5}$	1	1	1.5	0
43	Shields	$q_s \,\alpha\, S^q\, (\tau_o - \tau_c)$	1.67	1.67	1	2
44	Schoklitsch	$q_s \,\alpha\, S^{3/2}(q - q_c)$	1.5	1	-	1
45	Kalinske-Brown	$q_s \,\alpha\, \tau_o^{2.5}$	1.67	1.67	0	2
46	Meyer-Peter et al.	$q_s \,\alpha\, (\tau_o - \tau_c)^{1.5}$	1	1	1.5	0
47	Bagnold	$q_s \,\alpha\, \tau_o^{1/2}\, (\tau_o - \tau_c)$	1	1	1	0
48	Engelund-Hansen	$q_s \,\alpha\, \tau_o^{1.5}\, \bar{u}^{-2}$	1.67	1.67	0	2
49	Inglis-Lacey	$q_s \,\alpha\, \bar{u}^{5}\, h^{-1}$	2	1	0	0
50	Yalin ($\tau_o \cong \tau_c$)	$q_s \,\alpha\, \tau_o^{0.5}\, (\tau_o - \tau_c)^2$	1.67	1.67	2	2
51	Yalin ($\tau_o >> \tau_c$)	$q_s \,\alpha\, \tau_o^{0.5}\, (\tau_o - \tau_c)$	1	1	1	0
52	Chang et al.	$q_s \,\alpha\, \tau_o\, \bar{u}$	1	1	0	0
53	Barekyan	$q_s \,\alpha\, Sq\bar{u}$	1.33	1.33	0	1
54	Pedroli	$q_s \,\alpha\, \tau_o^{1.6}\, h^{0.2}$	1	1.2	0	0

*The index represents the number of exponents within the ranges: $1.2 < \beta < 1.9$; and $1.4 < \gamma < 2.4$.

It can be concluded that most of these equations are not applicable to sediment transport by laminar sheet flow. Among the equations examined, the formulas proposed by Engelund-Hansen and Barekyan (Eqs. 48, 53) seem relevant for predicting soil erosion by overland flow. The formulas suggested by Shields, Kalinske-Brown and Yalin (Eq. 43, 45, 50) might also be considered but the parameter γ is too small in the case of laminar sheet flow. The Inglis-Lacey equation (Eq. 49) seems fairly good for turbulent flow over rough boundaries, but clearly overestimates both parameters β and γ under different flow conditions. The other equations generally underestimate the parameters β and γ and are regarded as irrelevant for soil erosion.

3.4 Application of Energy, Work and Power Concepts to Sheet Flows

In the mid-eighteenth century the concepts of energy and work done were successfully applied to the motion of fluids with the significant contributions of Euler and Bernoulli. At that time, it was considered that no work was done by shear stress, no mechanical work was added to the fluid system and there was no heat transfer. One century later, Lord Kelvin (1845) discovered the concept of minimum kinetic energy for irrotational flow. The rate of dissipation of energy due to viscosity was then derived by Stokes (1851) and further developments were also reported by Lamb (1932) and Rouse (1959). As a result, when the shear stress components are included in the analysis, two sets of terms are added to the energy equation: (1) the total work done by shear stress; and (2) the dissipative work. The same analysis can also be extended to the work done per unit time called rate of work done, which correspond to the concept of power. At the beginning of the 20th century Gilbert (1914) formulated the major principles of work done by a stream, which were later applied by Rubey (1933) to debris-laden streams under equilibrium conditions. In his paper, Rubey essentially derived and supported with experiments the following relationship:

$$\frac{Cw}{\bar{u}S_f} = \text{constant} \tag{55}$$

in which C is the sediment concentration; w is the fall velocity of particles, $\bar{u}$ is the mean velocity and S_f is the energy gradient. Another detailed analysis of sediment transport from the energy balance of solid and fluid particles emerged from Velikanov's investigations between 1944 and 1956. His so-called gravitational theory, which has been summarized by Bogardi (1974) and Kondrat'ev (1959), is derived from the equilibrium of work done by gravity, settling of particles and friction of both fluid and solid phases. Bagnold (1960, 1966) studied the transport of sediment based on stream power per unit area $\tau_o \bar{u}$ given by the product of the bed shear stress τ_o and the mean velocity $\bar{u}$. His bed load equation is quite similar to Velikanov's (in Simons and Senturk, 1977) since they are both derived from similar principles. More recently Yang's papers (1967, 1972, 1973) emphasize on the relationship existing between sediment transport and stream power per unit weight, given by the product of velocity and slope $\bar{u}S$ also called unit stream power. In his effort to obtain a dimensionless equation, he suggests the following relationship:

$$\log C = I + J \log \left(\frac{\bar{u}S}{w} - \frac{\bar{u}_c S}{w} \right) \tag{56}$$

in which $\bar{u}_c$ is the critical velocity; I and J are coefficients. It is worth noting that Yang's Eq. 56 reduces to Rubey's Eq. 55 when

$\bar{u}_c/\bar{u}$ is small and J = 1. In the case of overland flow, Rooseboom and Mülke (1982) show interesting results for turbulent flow with rough and smooth boundaries.

The scope of our investigation is to determine the values of the exponents β and γ, describing the sediment transport capacity in laminar sheet flows using energy and stream power concepts. The relationships will be derived from three different ways: (1) rate of energy dissipation; (2) Bagnold stream power; and (3) unit stream power. The Bagnold stream power per unit area $\tau_o\bar{u}$ is obtained from Eqs. 4 and 7. For laminar flow, the unit stream power $\bar{u}S$ is obtained from Table III. These expressions are:

$$\tau_o\bar{u} = \rho g q S \tag{57}$$

$$\bar{u}S = \left(\frac{8g}{K\nu}\right)^{2/3} S^{4/3} q^{2/3} \tag{58}$$

Two different approaches referred to as global and local are used in this section. The former determines the sediment discharge from the mean characteristics of the flow. The latter describes the process at every point along the vertical profile and the sediment discharge equation is thereafter obtained by integration along the flow depth.

3.4.1 Rate of Energy Dissipation

A theoretical sediment transport equation for overland flow, is derived by assuming that the sediment concentration is proportional to the rate of energy dissipation. After substituting Eq. 13 into Eq. 15, one obtains

$$C \propto \Phi = 576 \frac{\rho g^2 S^2 y^2}{K^2 \nu} \tag{59}$$

The local approach assumes that Eq. 59 is valid at every point along the vertical profile and the unit sediment discharge q_s is obtained by the following integral

$$q_s = \int_o^h Cu \, dy \tag{60}$$

After substituting the velocity equation for laminar flow Eq. 13 and the sediment concentration given by Eq. 59 into Eq. 60, the unit sediment discharge relationship is

$$q_s \; \alpha \; \frac{\rho g}{K\nu} \left(\frac{8g}{K\nu}\right)^{1/3} S^{4/3} \; q^{5/3} \tag{61}$$

Then assuming that in general, ρ, g, K and ν are nearly constant gives

$$q_s \; \alpha \; S^{4/3} \; q^{5/3} \tag{62}$$

This is the main equation obtained from the rate of energy dissipation. One may apply the global approach as well, which can be written in terms of the product of the mean velocity and the total energy dissipation rate:

$$q_s \; \alpha \; \bar{u} \int_o^h \Phi \, dy \tag{63}$$

After substituting Eq. 59 and the velocity and depth for laminar flow (Table III), the integration of Eq. 63 leads to the same result as Eq. 62.

3.4.2 Stream Power Approach

Bagnold pointed out that the maximum transport efficiency is larger for laminar flow than for turbulent flow. If one assumes that his sediment transport relationship is valid for laminar overland flow, the global approach gives:

$$q_s \;\alpha\; \tau_o \;\bar{u}\; \frac{\bar{u}}{w} \tag{64}$$

For a given particle size, the fall velocity in clear water remains constant, and from the equations for velocity and shear stress for laminar flow (Table III), Eq. 64 transforms to

$$q_s \;\alpha\; S^{4/3}\; q^{5/3} \tag{65}$$

This equation is the same as Eq. 62 derived from energy dissipation concept.

Another sediment transport equation based on stream power has been used in the past. This equation includes the critical shear stress τ_c and an exponent to the stream power term m

$$q_s \;\alpha\; ((\tau_o - \tau_c)\; \bar{u})^m \tag{66}$$

It is seen that when the critical shear stress τ_c is small compared to the bed shear stress τ_o the corresponding equation for the sediment transport capacity is written as follows

$$q_s \;\alpha\; (\tau_o \bar{u})^m \;\alpha\; S^m\; q^m \tag{67}$$

This means that the exponents of q and S are not independent but linked to each other because the original equation (Eq. 66) has only one degree of freedom. From the analysis of experimental data of rainfall erosion Kilinc (1972) obtained the value $m = 1.67$ by regression analysis (Eq. 35).

3.4.3 Unit Stream Power

As mentioned previously, Yang's stream power equation can reduce to Rubey's equation. Let us assume that the sediment concentration is proportional to the product $\bar{u}S/w$. Using the global approach, the sediment discharge is

$$q_s \propto Cq \propto \frac{\bar{u}S}{w} q \tag{68}$$

This relationship can only be derived by using the global approach since in this case the local approach erroneously means that the concentration is maximum near the surface. For a given size fraction (constant fall velocity), after substituting the velocity equation (Table III) in Eq. 68 gives

$$q_s \propto S^{4/3} q^{5/3} \tag{69}$$

It is then concluded that for laminar sheet flows the three different approaches used to derive a sediment transport relationship (rate of energy dissipation, stream power and unit stream power) lead to the same power function of slope and discharge.

The following sediment transport equation based on unit stream power has also been suggested:

$$C \propto ((\bar{u} - \bar{u}_c)S)^N \tag{70}$$

in which N is an exponent equivalent to J in Eq. 56 when w is constant. Here again, when the critical velocity $\bar{u}_c$ is small compared to $\bar{u}$, the sediment transport capacity is:

$$q_s \propto S^{4N/3} q^{1+2N/3} \tag{71}$$

Since Eq. 70 has only one degree of freedom, the exponents of q and S are not independent though they might have different values.

Unless there is a physical reason or theoretical evidence to support equations using one degree of freedom, a general regression analysis aiming to determine the influence of the variables q and S separately should not be made with such restrictive equations. Otherwise, the exponents obtained from regression analysis represent a compromise value between two independent exponents. Fortunately, for soil erosion, Eqs. 66 and 70 can give fair approximation since the exponents of q and S do not differ considerably. Also, Eq. 39 in Table IV can approximately reduce to Eq. 71 when $N \cong 1.3$, which is within the range of previous observations by Yang (1972) on several streams ($1.0 < N < 2.1$).

3.4.4 Other Theoretical Equations

Another theoretical equation was suggested by Li, Shen and Simons (1973). This equation assumes that the pickup rate of particles is proportional to the square of the bed shear stress:

$$q_s \propto \int_0^L \tau_o^2 \, dx \qquad (32)$$

in which x is the longitudinal distance. As pointed out by Shen (1979), this equation can be reduced to Eq. 62, for laminar sheet flows. This soil erosion equation based on force equilibrium is therefore in agreement with those based on stream power and energy dissipation approaches.

3.5 Summary of Results and Discussion

The principal sediment transport capacity relationships for laminar flows are summarized in Table IX. The discussion of the results of this study is based on the comparison between several sediment transport capacity relationships for rainfall erosion given by $q_s \propto S^\beta q^\gamma$. The values of the exponents β and γ are compared for the different approaches used in this chapter. The equations are classified between

Table IX. Summary of sediment transport capacity equations in laminar sheet flow.

Eq. No.	Relationship	$q_s \alpha S^{\beta} q^{\gamma}$	β	γ
Theoretical				
62	Energy dissipation		1.33	1.67
65	Stream power		1.33	1.67
69	Unit stream power		1.33	1.67
32	Li, Shen and Simons		1.33	1.67
Empirical				
28	Zingg (1940)		1.37	1.66
29	Universal soil-loss equation		$\cong$1.7	1.5
30	Meyer and Monke (1965)		3.5	1.9
31	Young and Mutchler (1969)		0.74	2.24
33	Komura (1983)		1.5	1.38
34	Kilinc (1972)	$q_s = f(\tau_o, \tau_c)$	1.86	0.93
35	Kilinc (1972)	$q_s = f(\tau_o, \tau_c, \bar{u})$	1.67	1.67
36	Kilinc (1972)	$q_s = f(\bar{u})$	1.21	2.42
37	Kilinc (1972)	$q_s = f(\bar{u}, Re)$	1.56	2.24
38	Kilinc (1972)	$q_s = f(Re, S)$	1.46	2.05
39	Kilinc (1972)	$q_s = f(q, S)$	1.66	2.03
40	Kilinc data (total erosion minus rill erosion)		1.31	1.93
Transformed from Turbulent Flow Equations				
41	Du Boys		1.33	0.66
42	WES		1.0	1.0
43	Shields		1.67	1.33
44	Schoklitsch		1.5	1.0
45	Kalinske-Brown		1.67	0.83
46	Meyer-Peter Müller		1.0	0.5
47	Bagnold		1.0	0.3
48	Engelund-Hansen		1.67	1.83
49	Inglis-Lacey		2.0	3.0
50	Yalin ($\tau_o \cong \tau_c$)		1.67	0.83
51	Yalin ($\tau_o >> \tau_c$)		1.0	0.5
52	Chang et al.		1.0	1.0
53	Barekyan		1.33	1.67
54	Pedroli		1.00	0.6

theoretical and empirical relationships for rainfall erosion, and also the transformation of turbulent flow relationships to laminar sheet flow conditions.

The theoretical equations give similar results ($\beta = 1.33$ and $\gamma = 1.67$) and are recommended for laminar sheet flows without rills. This relationship compares very well with Zingg relationship and with Kilinc data when the rill erosion is subtracted from the total erosion (Eq. 40).

The exponents of the empirical relationships are shown in Figure 6 to vary within the following ranges $1.2 < \beta < 1.9$ and $1.4 < \gamma < 2.4$. The increase in these exponents is attributable to rill erosion which varies for different soil types. As a first approximation, Eq. 39 ($\beta = 1.66$, $\gamma = 2.03$) should be used when rill erosion is expected to occur. This equation has been used by Julien (1982) to predict the sediment transport capacity for both rainfall and snowmelt. This equation gave excellent results and is suggested unless a site specific relationship is available. The use of the universal soil-loss equation (Eq. 29) is also advocated though the exponent β is an approximation of the quadratic function and the exponent δ is negative (from Table IV). Its wide use and calibration for various soil and climate conditions enhance the applicability of this equation for predicting rainfall erosion.

Most of the turbulent sediment transport relationships transformed to fit the laminar sheet flow conditions give a wide range of exponents β and γ, which values are usually outside the range of empirical relationships. As mentioned in section 3.3, it can be concluded that most of these equations are not applicable to rainfall erosion in laminar sheet flows. The best relationships are those of Engelund-Hansen

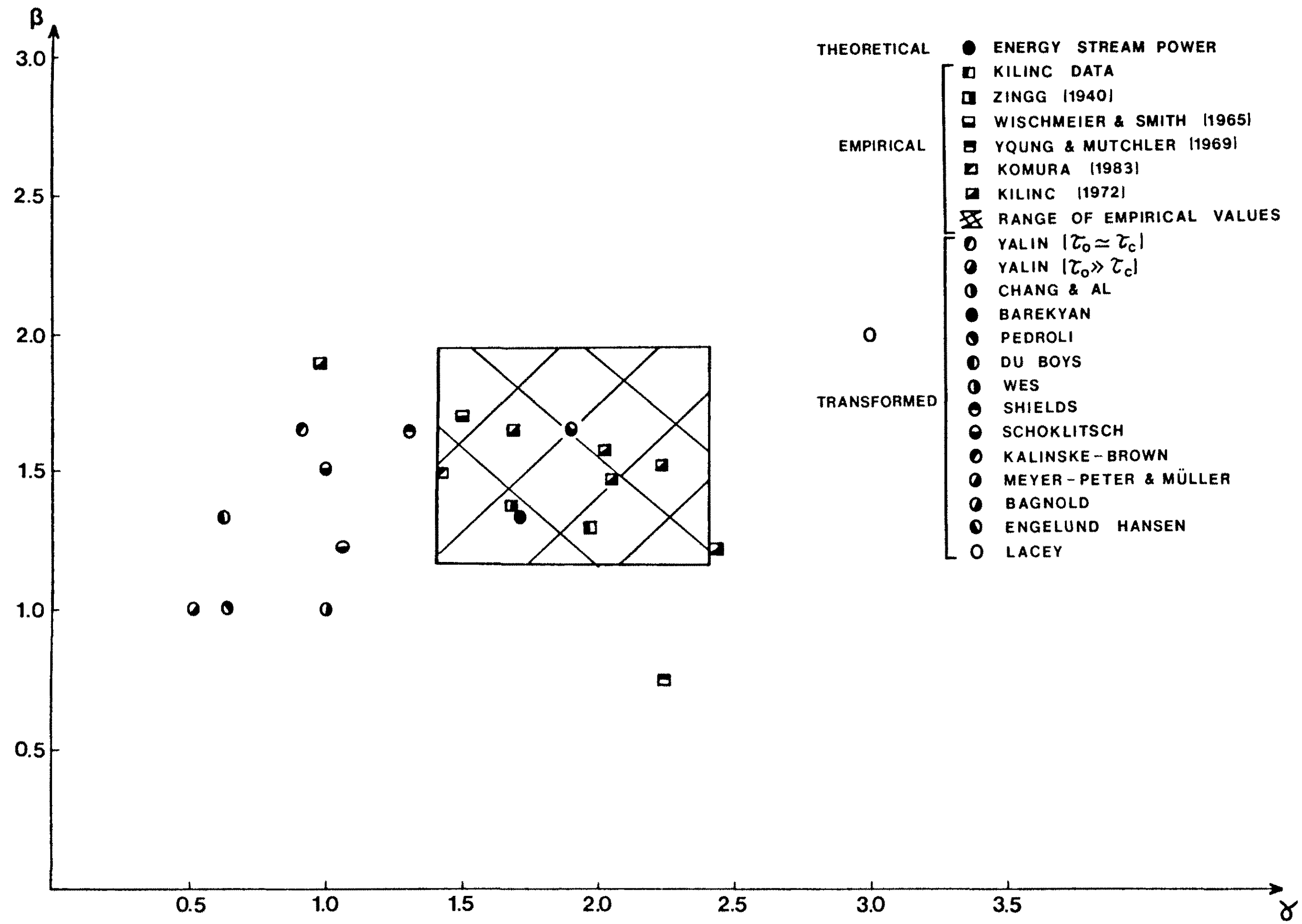

Figure 6. Sediment transport parameters β and γ for laminar sheet flows.

and Barekyan and to a certain extent, those of Shields, Kalinske-Brown and Yalin. In general, the exponent β of the stream sediment transport relationships is in agreement with the theoretical and empirical values. The exponents γ for these bed-load equations, however, are outside the range of observed values.

IV. CONCLUSION

This report deals with sediment transport capacity relationships for overland flow. The objectives are mainly to determine: 1) whether equations derived for bed-load and total load in turbulent streams can be applied to laminar sheet flows and 2) whether stream power and energy dissipation concepts can be used to define soil erosion equations.

The method used to achieve these goals was to point out the hydraulic characteristics of overland flow. Then the sediment transport variables were combined into a relationship for sediment transport capacity of rainfall erosion $q_s \alpha S^{\beta} q^{\gamma}$ derived from dimensional analysis. The exponents of this relationship were obtained from both theoretical analysis and transformation of bed-load equations. These exponents were then compared with empirical values obtained from experimental data. The principal conclusions of this investigation are summarized as follows.

Most of the sediment transport equations valid for turbulent flow in streams cannot be applied to rainfall erosion in laminar sheet flows. Among the equations examined, only those proposed by Engelund-Hansen and Barekyan (Eqs. 48 and 53) seem relevant for predicting soil erosion losses by overland runoff. The formulas suggested by Shields, Kalinske-Brown and Yalin (Eqs. 43, 45 and 50) might also be considered but the exponent of discharge is clearly too small in the case of laminar sheet flow. The other equations generally underestimate the parameters β and γ and are irrelevant to predict rainfall erosion.

The application of energy and stream power concepts to rainfall erosion by laminar overland flow is conclusive. The theoretical derivations for the case of uniform sheet flow are based on: (1) the rate of energy dissipation; (2) the total stream power; and (3) the unit stream power. These theoretical derivations lead to the same equation for the sediment transport capacity ($q_s \propto S^{1.33}\ q^{1.67}$). It is also interesting to note that only the concept of energy dissipation can be applied to every point along the vertical profile. The resulting equation is similar to an equation derived from force equilibrium concepts (Eq. 32), and show close agreement with regression equations based on experimental data (Eq. 40). Equations based on energy dissipation and stream power concepts are therefore recommended to predict the sediment transport capacity of uniform sheet flows without rills. Some other equations having one degree of freedom (Eqs. 32, 34, 35, 36, 67 and 70) can give fair approximations of the rate of soil erosion from overland flow. These equations are theoretically worthless since the exponents of the main variables are interdependent.

The range of values of the exponents of slope β and of discharge γ were well-defined from this analysis. In the case of uniform laminar sheet flows, the values $\beta = 1.33$ and $\gamma = 1.67$ are recommended to define the sediment transport capacity of overland flow. In the cases where rills develop, both exponents must be increased. The values obtained from Eq. 39 ($\beta = 1.66$ and $\gamma = 2.03$) are suggested as a first approximation unless a better empirical equation is available for the specific site and soil type under study.

Further theoretical analysis in this field should be focused on the processes of rill formation, considering the nonuniformity of flow depth and the difference between both laminar and turbulent flows.

BIBLIOGRAPHY

Alonso, C.V., W.H. Neibling and G.R. Foster. Estimating sediment transport capacity in watershed modeling. Transactions of the ASAE, Vol. 24, No. 5, 1981, pp. 1211-1220.

ASCE Task Committee. Sedimentation Engineering. Edited by V. Vanoni, Manuals and Reports on Engineering Practice, No. 54. New York, 1977, pp. 190-214.

Bagnold, R.A. The flow of cohesionless grains in fluids. Phil. Trans. of the Royal Society of London. Vol. 249, No. 964, 1956, pp. 235-297.

Bagnold, R.A. Sediment discharge and stream power, a preliminary announcement. USGS Circular 421, 1960, 23 p.

Bagnold, R.A. An approach to the sediment transport problem from general physics. USGS Prof. Paper 422I, 1966, 37 p.

Barekyan, A.S. Discharge of channel forming sediments and elements of sand waves. Soviet Hydol. Selected Papers (Transactions of AGU No. 2), 1962, pp. 128-130.

Bogardi, J. Sediment transport in alluvial streams. Akademiai Kiado, Budapest, 1974, pp. 242-285.

Brown, C.B. Sediment transportation. Chapter XII in Engineering Hydraulics, Ed. by H. Rouse, 1949, pp. 769-857.

Chang, F.M., D.B. Simons, and E.V. Richardson. Total bed-material discharge in alluvial channels. Proceedings of the Twelfth Congress of the IAHR, Fort Collins, CO, 1967, pp. 132-140.

Chen, C.L. Flow resistance in broad shallow grassed channels. Journal of the Hydraulics Division, ASCE, Vol. 102, No. HY3, 1976, pp. 307-322.

Chen, C.L. and V.E. Hansen. Theory and characteristics of overland flow. Trans. ASAE, Vol. 9, No. 1, 1966, pp. 20-26.

Chow, V.T. Open Channel Hydraulics, McGraw-Hill, 1959, pp. 205-206.

Du Boys, D. Le Rhône et les rivières à lit affouillable. Annales des Ponts et Chaussées, Série 5, Vol. 18, 1879, pp. 141-195.

Emmett, W.W. The hydraulics of overland flow on hill slopes. USGS Prof. Paper 662-A, 1970.

Engelund, F. and E. Hansen. A monograph on sediment transport in alluvial streams. Teknisk Vorlag, Copenhagen, Denmark, 1967.

Fawkes, P.E. Roughness in a model of overland flow. M.Sc. Thesis, Colorado State University, Fort Collins, 1972, 109 p.

Foster, G.R. and L.D. Meyer. Transport of soil particles by shallow flow. Trans. ASAE, Vol. 15, No. 1, 1972, pp. 99-102.

Gilbert, G.K. Transportation of debris by running water. 1914, 262 p.

Glymph, L.M., Jr. Importance of sheet erosion as a source of sediment. Trans. AGU, Vol. 38, No. 6, 1957, pp. 903-907.

Grace, R.A. and D.S. Eagleson. The modeling of overland flow. Water Resources Research, Vol. 2, No. 3, 1966, pp. 393-403.

Graf, W.H. Hydraulics of Sediment Transport. McGraw-Hill, 1971, pp. 123-160.

Henderson, F.M. and R.A. Wooding. Overland flow and groundwater flow from a steady rainfall of finite duration. Journal of Geophysical Research, Vol. 69, No. 8, 1964, pp. 1531-1540.

Inglis, C.C. Discussion of systematic evaluation of river regime. Journal of Waterways and Harbor Division, ASCE, Vol. 94, No. WW1, 1968, pp. 109-114.

Izzard, C.F. The surface profile of overland flow. Trans. AGU, Vol. 6, 1944, pp. 959-968.

Julien, P. Prédiction d'apport solide pluvial et nival dans les cours d'eau nordiques à partir du ruissellement superficiel, Thèse de doctorat, Université Laval, Québec, 1982, 240 p.

Keulegan, G.H. Laws of turbulent flow in open channels. Research paper 1151, Journal of Research, U.S. Nat. Bureau of Standards, Vol. 21, 1938, pp. 707-741.

Kibler, D.F. and D.A. Woolhiser. The kinematic cascade as a hydrologic model. Hydrology Paper No. 35, Colorado State University, 1970.

Kilinc, M.Y. Mechanics of soil erosion from overland flow generated by simulated rainfall. Ph.D. dissertation, Colorado State University, Fort Collins, 1972, 183 p.

Kisisel, I.T. An experimental investigation of the effect of rainfall on the turbulence characteristics of shallow flow. Ph.D. Dissertation, Purdue University, West Lafayette, Indiana, 1971.

Komura, S. Hydraulics of slope erosion by overland flow. Journal of the Hydraulics Division, ASCE, Vol. 102, No. HY10, October 1976, pp. 1573-1585.

Komura, S. A method of predicting sediment yields from watersheds. Proceedings of the D.B. Simons Symposium on Erosion and Sedimentation, Section 9, 1983, pp. 23-41.

Kondrat'ev, N.E. (editor). River flow and river channel formation. Selected chapters from Ruslovoi Protsess, translated from Russian and published by NSF, 1959, 172 p.

Lamb, H. Hydrodynamics. 6th edition. Dover Publications, New York, 1932, pp. 579-581.

Laursen, E.H. The application of sediment transport mechanics to stable channel design. Journal of the Hydraulics Division, ASCE, Vol. 82, No. HY4, 1956, 11 p.

Li, R.M. Sheet flow under simulated rainfall. M.Sc. thesis, Department of Civil Engineering, Colorado State University, Fort Collins, Colorado, 111 p.

Li, R.M. Water and sediment routing from watersheds. Chapter 9 in Modeling of Rivers, Edited by H.W. Shen, J. Wiley & Sons, 1979, 88 p.

Li, R.M., H.W. Shen and D.B. Simons. Mechanics of soil erosion by overland flow. Paper presented at 15th Congress IAHR, Istanbul, Turkey. Proc. Paper A-54, 1973, pp. 437-446.

Liggett, J.A. and D.A. Woolhiser. Difference solutions of the shallow water equation. Journal of the Engineering Mechanics Division, ASCE, 93(EM2), 1967.

Liggett, J.A. and D.A. Woolhiser. Unsteady one-dimensional flow over a plane - The rising hydrograph. Water Resources Research, Vol. 3, No. 3, 1967, pp. 753-771.

Meyer, L.D. Soil erosion by water on upland areas. Chapter 27 in River Mechanics, Edited by H.W. Shen, Fort Collins, 1971, 25 p.

Meyer, L.D. and E.J. Monke. Mechanics of soil erosion by rainfall and overland flow. Transactions of the ASAE, Vol. 8, No. 4, 1965, pp. 572-580.

Meyer-Peter, E. and R. Müller. Formulas for bed-load transport. Proceedings of the Second Meeting of IAHR, Stockholm, 1948, pp. 39-64.

Morgali, J.R. Laminar and turbulent overland flow. Proceedings of the ASCE, Vol. 96, No. HY2, 1970.

Morgali, J.R. and R.K. Linsley. Computer analysis of overland flow. Journal of the Hydraulics Division, ASCE, Vol. 91, No. HY3, March 1965.

Musgrave, G.W. The qualitative evaluation of factors in water erosion: a first approximation. Journal of Soil and Water Conservation, Vol. 2, No. 3, 1947, pp. 1331-138.

Mutchler, C.K. Splash amounts from waterdrop impact on a smooth surface. Water Resources Research, Vol. 7, 1971.

Neibling, W.H. and G.R. Foster. Sediment transport capacity of overland flow. Chapter 10 in CREAMS, USDA, Conservation Research Report No. 26, 1980, pp. 463-473.

O'Brien, M.P. and B.D. Rindlaub. The transportation of bed-load in streams. Transactions of the AGU, 1934, pp. 573-603.

Partheniades, E. Erosion at deposition of cohesive soils. Journal of the Hydraulics Division, ASCE, Vol. 91, No. HY1, 1965.

Pedroli, R. Trasporto di materiale solido in canali a fondo fisso e liscio. Communicazione de'l Ufficio federale dell'economia della acque, No. 63, 1963.

Phelps, H.D. The friction coefficient for shallow flows over a simulated turf surface. Water Resources Research, Vol. 6, 1970.

Podmore, T.H. and G.E. Merua. Silt transport by thin film flow. Trans. of the ASAE, Vol. 14, No. 6, 1971.

Raudkivi, A.J. Loose boundary hydraulics. Pergamon Press, Second Ed., 1976, pp. 137-158.

Rooseboom, A. and F. J. Mülke. Erosion initiation. In: Recent developments in the explanation and prediction of erosion and sediment yield. Proceedings of the Exeter Symposium, July, 1982, IAHS Publ. No. 137, pp. 59-66.

Ross, B.B., D.N. Contractor and V.D. Shanholtz. Finite element simulation of overland and channel flow. Trans. of the ASAE, Vol. 20, No. 4, 1977.

Rouse, H. (Editor). Advanced mechanics of fluids. Wiley and Sons, 1959, pp. 205-206.

Rowlison, D.L. and G.L. Martin. Rational model describing slope erosion. Journal of the Irrigation and Drainage Division, ASCE, Vol. 97, No. IR1, 1971.

Rubey, W.W. Equilibrium conditions in débris-laden streams. AGU Trans., 14th Annual Meeting, 1933, pp. 497-505.

Schoklitsch, A. Geschiebetrieb und die geschiebefracht. Wasser Kraft und Wasser Wirtschaft. Jgg. 39, Heft 4, 1934.

Shen, H.W. Wash load and bed load. Chapter 11 in River Mechanics, Vol. 1, Edited by H.W. Shen, 1971, 30 p.

Shen, H.W. Introduction, flow resistance, sediment transport. Chapter 1 in Modeling of Rivers, Ed. by H.W. Shen, 1979, pp. 1-27.

Shen, H.W. and R.M. Li. Rainfall effect on sheet flow over smooth surface. Journal of the Hydraulics Division, ASCE, May 1973, pp. 771-792.

Shields, A. Anwendung der Ähnlichkeitsmechanik und turbulenz forschung auf die geschiebebewegung. Mitteil Preuss, Versuchsanst Wasser und Schiffsbau, Berlin No. 26, 1936. Translated to English by W.P. Ott and J.C. van Uchelen, Calif. Inst. of Tech.

Simons, D.B. and F. Sentürk. Sediment transport technology. Water Resources Publications, Fort Collins, CO, 1977, pp. 504-543.

Stokes, N. On the effect of the internal friction of fluids on the motion of pendulums. Trans. of the Cambridge Phil. Soc., Vol. 9, 1851.

Swanson, N.P. and A.R. Dedrick. Soil particles and aggregates transported in water runoff under various slope conditions using simulated rainfall. Transactions of the ASAE, Vol. 10, No. 2, 1967.

Thompson, G.T. and G.A. Robertson. A theory of flow resistance for vegetated channels. Trans. of the ASAE, Vol. 19, 1976.

Thompson, W. On the vis-viva of a liquid in motion. Cambridge and Dublin Math. Journal, 1849.

U.S. Waterways Experiment Station. Studies of river bed materials and their movement with special reference to the lower Mississippi River. USWES, Vicksburg, Paper No. 17, 1935.

Velikanov, M.A. Transport of suspended sediments in turbulent flow. Izd. An. SSR, OTN, No. 3, 1944.

Velikanov, M.A. Movement of sediment. Izd. Min. Rech flota, SSSR, Moscow, 1948.

Velikanov, M.A. Principle of the gravitational theory of the movement of sediments. Acad. of Science Bulletin, USSR, Geophys. Series, No. 4, 1954, pp. 349-359.

Velikanov, M.A. Stream flow dynamics. I-Flow Structure (1954), II-Sediment and Rivers (1955), Gostekhizdat, Moscow.

Willis, J.C. Erosion by concentrated flow. USDA-ARS 41-179, 1971.

Wischmeier, W.H. Upslope erosion analysis. Chapter 15 in Environmental Impact on Rivers, edited by H.W. Shen, Fort Collins, 1973, 26 p.

Wischmeier, W.H. and D.D. Smith. Predicting rainfall erosion losses--a guide to conservation planning. USDA Agriculture Handbook No. 537, 1978, 58 p.

Wooding, R.A. A hydraulic model for the catchment stream, problem I - Kinematic wave theory. Journal of Hydrology, Vol. 3, 1965.

Woolhiser, D.A. Simulation of unsteady overland flow, Chapter 12 in Unsteady flow in open channels. Edited by K. Mahmood and V. Yevjevich, WRP, 1975, pp. 485-508.

Yalin, M.S. Mechanics of sediment transport. Pergamon Press. Second Ed., 1977, pp. 112-165.

Yang, C.T. Minimum unit stream power and fluvial hydraulics. Journal of the Hydraulics Division, ASCE, Vol. 102, No. HY7, Proc. Paper 12238, July, 1967, pp. 919-934.

Yang, C.T. Unit stream power and sediment transport. Journal of the Hydraulics Division, ASCE, Vol. 98, No. HY10, Proc. Paper 9295, Oct. 1972, pp. 1805-1836.

Yang, C.T. Incipient motion and sediment transport. Journal of the Hydraulics Division, ASCE, Vol. 99, No. HY10, Proc. Paper 10067, Oct. 1973, pp. 1679-1704.

Yen, B.C. and H.G. Wenzel. Dynamic equations for steady spatially varied flow. Journal of the Hydraulics Division, ASCE, Vol. 96, No. HY3, Proc. Paper 7179, March 1970, pp. 801-813.

Yoon, Y.N. and H.G. Wenzel. Mechanics of sheet flow under simulated rainfall. Journal of the Hydraulics Division, ASCE, Vol. 93, No. HY9, 1971, pp. 1367-1401.

Young, R.A. and C.K. Mutchler. Soil movement on irregular slopes. Water Resources Research, Vol. 5, No. 5, 1969, pp. 1084-1085.

Young, R.A. and C.K. Mutchler. Effects of slope shape on erosion and runoff. Trans. of the ASAE, Vol. 12, No. 2, 1969.

Zingg, A.W. Degree and length of land slope as it affects soil loss by runoff. Agricultural Engineering, Vol. 21, No. 2, 1940, pp. 59-64.