Dissertation

Three Essays on Gender Inequality, Dynamic Bargaining, and Technology Adoption in Subsistence Agriculture

Submitted by

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In partial fulfillment of the requirements

For the Degree of Doctor of Philosophy

Colorado State University

Fort Collins, Colorado

Summer 2014

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Abstract

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The dissertation presents three essays that build on each other to highlight the consequences of gender inequality in subsistence farming. Motivated by findings that unequal access to productive resources has indisputably serious implications for relative welfare between genders and relatedly hinders economic development of households, the project provides empirical and theoretical contributions to understanding these consequences.

Together, the essays confront evidence that households engaged in agriculture do not behave as unitary decision makers but are instead sites of conflict and hierarchy. As dynamics of intrahousehold differences in capabilities, constraints, and control, bound individual agency, they define gender specific strategies. Such strategies necessarily determine individual well being and social equality between genders, as well as prospects for growth at the household level. These outcomes demand consideration from development policy by necessitating an understanding of power dynamics within the household and the role of social institutions in defining them.

In unpacking the household to explore the dynamic links between men and women and the strategic behaviors they accordingly adopt, the dissertation is ultimately concerned with the power of social institutions to affect both equity and growth in the agricultural context. While feminist and bargaining literatures have developed thorough arguments in consideration of such institutions and their effects on relative welfare, theoretical models exploring the responses of agricultural households to development policies in a world of significant risk have not. The contribution of the project is therefore to examine the effects of intrahousehold

inequality on agricultural production and develop relevant models of household behavior to consider such problems.

The first essay leverages unique data from four villages in southern Ghana to test the hypothesis that resources are allocated efficiently between genders within households. Related work has rejected this hypothesis, concluding that gender inequality in this way is inefficient and offers significant gains from reallocating productive resources to women. We confirm these findings and extend this line by applying empirical techniques from efficiency analyses of firms. In doing so, we estimate the direct cost of households failing to making such reallocations. Estimated cost increases due to gender inequality of between 35% and 51% on subsistence plots strongly support arguments for redistribution and highlight the importance of asset inequality regarding land, which is over allocated to men by 63% on average.

The significant misallocation of land supports arguments in the development literature that improving women's access to and security over land is an important means to both equity and growth. Additionally, it suggests that households in this context do not act as benevolent planners but are instead political structures with conflict and power imbalances.

Essays two and three build on the conclusion that unequal access to productive resources is a result of a gendered allocation process, and that this process generates Pareto-inferior outcomes at the household level. Each present two-stage bargaining models consistent with the inter temporal nature of the agricultural investment problem and the separate spheres system of subsistence production common to much of the developing world. By leveraging theoretical contributions from feminist and household bargaining literatures, each of these essays contribute to understanding the role of gender bias in agricultural development. By modeling the key features of gender dynamics, these papers consider the interactions between development policies, inequality, and behavioral strategies.

By accommodating the relevant aspects of cooperation and conflict through bargaining, the essays contribute to the literature on separate spheres bargaining by developing dynamic frameworks where individual strategies are necessarily linked over time. While the models assume a relatively benign power structure, the decentralized setting in which investments are made results in potential Pareto-improvements described above. Extensions to the bargaining mechanism to include additional realities of power and sanctions are discussed throughout.

While the approach to the household problem is similar in the theoretical models, the focus is importantly different. Essay two highlights the role of expectations over spousal behavior on gender specific strategies. Specifically, as men and women look forward to a bargaining process determining transfers of harvest revenues, they accordingly adjust crop choices on their own farms. This dynamic leads to divergent outcomes for individuals and households as inequality increases.

In exploring the strategies men and women adopt as a result of their access to resources and expectations over returns, this paper uncovers many sources of additional constraints to women's welfare and prospects for change. The conversation often returns to a discussion of unequal access to land which drives behavioral strategies in the model and has significant implications for future investments in soil quality. As this process is intertwined with one's position in the local political hierarchy, it encompasses additional links between social institutions and economic behavior.

The third essay takes a similar approach to the dynamic savings problem but considers the interactions between intrahousehold bargaining and the adoption of new agricultural technologies. As crop choice in this context also represents a risk management strategy, the investment decisions women and men adopt take on additional importance regarding relative welfare and responses to socially determined capabilities.

In this paper, we extend literature on bargaining models applied to financial decision making to include the necessary elements of noncooperation. In doing so, we highlight additional considerations for policy regarding how individuals optimize savings in the face of significant risk. Specifically, the recipient of policy support matters as strategic behavior of other household members may have unintended consequences. We consider the application of input subsidies and technology improvements as well as reductions in risk of the new technology.

Numerical simulation of the model suggests that egalitarian households are better able to realize gains from such policies through both higher rates of innovation uptake and improvements in food security with resulting increases in food production. Intrahousehold dynamics respond to policies in a way that household portfolio risk mirrors that of the individual with more control over household resources. In addition to simulating these strategies and considering the effects of heterogeneous risk preferences, we return to the Ghana data to test the core hypotheses of the model.

We find that households where women control fewer resources make net transfers from men to women, supporting the primary mechanism of strategic links between agents. Additionally, we find empirically that households which share productive assets more equally are better able to invest in the new technology on average, supporting the main prediction of the theoretical models. Furthermore, we conclude that gender inequality is an overarching constraint, beyond that from uncertainty from subsistence risk and unless approached proactively may continue to deter innovation, economic growth, and social change.

Acknowledgements

Thank you to Jennalyn Slootmaker for her endless love and support, and Elissa Braunstein and Daniele Tavani for their constant guidance and tireless consideration.

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Essay #1: The Cost of Gender Inequality in Subsistence Agriculture: A Stochastic Frontier Analysis of Four Villages in Ghana

1.1. Introduction

Gender plays an overriding role in determining the intrahousehold allocation of resources in Sub-Saharan Africa. Regardless of women's socioeconomic status, their rights to household resources are generally inferior to men's [1].[1] While women in developing countries contribute approximately 43 percent of agricultural labor, they typically receive lower output per unit of land and are less likely to engage in commercial farming [3]. These productivity differences are primarily due to gender differences in access to inputs and resources.

Gender inequalities in access to household resources are founded in both formal and informal institutions and have been shown to directly affect agricultural production possibilities (e.g. [4]) and investments in agricultural innovations [1, 5, 6] and land fertility [7, 2]. Resulting outcomes of these effects at both the individual and household levels determine relative and total welfare through intrahousehold risk pooling [8]), nutritional smoothing [9], and budget shares of specific expenditure types [10].

While many models of agricultural behavior continue to treat the household as a unitary decision maker, the arguments cited above and additional econometric analyses have routinely rejected the unitary framework.[2] From various angles, these studies have uncovered sources of conflict and power, and resulting inefficiencies which reduce both means and ends

[1]Additionally though, social status in the local political hierarchy has its own (gendered) effect on investments in land fertility [2], which we return to later.

[2][11] give a review of both theoretical foundations extending the unitary model and empirical evidence exploring assumptions between such extensions.

in improving livelihoods of individuals and families. As suggested, these have direct implications in terms of individual livelihoods but also indirect effects as outcomes feed back onto future opportunities and constraints.

In this paper, we estimate the direct costs imposed on the household production of subsistence crops by a gendered distribution of resources. Specifically, we estimate the cost of unequal sharing of land and labor on maize and cassava farms in Southern Ghana. To do so, we start by confirming findings that unequal access to inputs does not reflect productive efficiency, and that a reallocation of resources could make the household as a whole better off. We then offer a novel contribution to estimating the cost of such inefficiency by leveraging stochastic production frontiers, allowing us to isolate a cardinal measure of cost increases due specifically to unequal input use by gender. In doing so, we complement existing literature on household agricultural production and support models of relevant household behavior that consider non-cooperative decision making. Along the way we uncover particular aspects of household production which highlight specific institutional characteristics referenced above.

Our analysis exploits the atomistic structure of subsistence farming in Ghana, where men and women from the same household manage separate plots while often farming the same crop in a given period. This largely separate production process is common in other parts of Sub-Saharan Africa as well as in South Asia [2]. While farms are individually managed, women, men, and other household members routinely apply labor to each others plots and contribute yields to meet household subsistence demands.

Unique and detailed data on inputs, outputs, prices, and soil characteristics, collected in the Agricultural Innovation and Resource Management in Ghana survey [6], allow us to compare the relative allocation of inputs to male and female farms and isolate the cost of production possibilities lost from inequality. In exploring this somewhat natural experiment, we find that while female plots are smaller and receive significantly fewer inputs than their spouse's plots, females are no less effective as plot managers. Specifically, they are no less technically efficient at turning inputs into outputs on similar plots and in some cases are more so. As production technologies exhibit decreasing returns to scale, lower access to resources makes inputs marginally more productive on female farms so that a reallocation from male plots would increase total production of the household. We estimate that relative inequality from such misallocations increases cost by between 35% and 51% for the average household.

The rest of the paper is organized as follows. We start by describing the data and characterizing subsistence production in southern Ghana, briefly comparing it to similar contexts. Next we explore gender differences in access to productive inputs and find that females have significantly smaller farms and, controlling for relevant plot characteristics, utilize significantly less labor per acre. Section 3 tests for diminishing returns to scale and estimates production functions to compare managerial effectiveness by gender. Results here suggest that females are no less productive at turning inputs into revenue and since technologies exhibit diminishing returns to scale, a redistribution of inputs could increase female production more than would be lost on the male's farm. Section 4 compares technical efficiency between genders across and within households to support the previous finding in a more objective manner. Finally, using relative input prices at the household level, we estimate production frontiers and household-specific deviations from the cost frontier due specifically to allocative inefficiency. Section 5 concludes and discusses implications of our findings for the modeling of household agricultural production.

1.2. Household Agriculture in Southern Ghana

In the late 1990's, Ghana was in the middle of a transition from production of primarily subsistence crops to farming Pineapple for export. From Table 1.1, it is clear that men are the primary farmers of the new cash crop. However, both men and women continued to farm the traditional subsistence crops, maize and cassava, during this transition. This pattern is common to other developing regions, where women contribute significantly to agricultural production but are less likely to be active in commercial farming [3].

The data we analyze were collected over 15 rounds (during 1997/1998) from 200 households in four villages in rural Ghana [6]. As an intrahousehold panel, this data provides a unique opportunity to explore the allocation of inputs between individuals within households. Since females and males often farm maize and cassava in the same period, we can compare the allocation of household resources between genders and estimate the effect of this distribution on total household production.

Table 1.1. Land Allocation by Gender

Crop	Male		Female	
Bean	0.03	%	0.00	%
Cassava	32.36		72.61	
Cocoyam	1.32		7.46	
Garden Egg	0.11		0.00	
Maize	16.16		17.36	
Oil Palm	6.58		0.70	
Okro	0.04		0.00	
Oranges	0.81		0.23	
Pepper	0.39		0.05	
Pineapple	38.54		0.10	
Plantain	1.87		0.92	
Sugar Cane	0.53		0.00	
Tomato	0.15		0.01	
Yam	1.11		0.58	
Total	100	%	100	%

Table 1.2 presents average plot size in acres and per acre values of revenue and inputs by gender, largely reflecting the general differences by gender described in [3]. Female plots are significantly smaller for both maize and cassava plots on average. Furthermore, female revenue and wages paid per acre are significantly smaller for both crops. While total labor per acre is not statistically different on cassava plots, their is meaningful variation in individual types of labor used by males and females. Specifically, female plots receive more female and child (other) labor per acre but much less male and hired labor on average. Lower rates of male and hired labor are similar on female maize plots, although they also see less other labor and total labor per acre.

TABLE 1.2. Comparison of Annual Means by Gender

	Cassava			Maize		
Amount per Acre	Male	Female	t-statistic	Male	Female	t-statistic
Area (Acres)	10.22	6.90	2.08	5.73	2.67	4.76
	(1.39)	(0.77)	[.0385]	(0.53)	(0.36)	[.0000]
Revenue	37,500	26,132	2.27	45,115	35,634	1.22
	(3,597)	(3,482)	[.0238]	(5,833)	(5,159)	[.2246]
Wages Paid	3143.84	1561.40	1.98	14,936	1,281	1.29
	(760.84)	(340.16)	[.0587]	(10,616)	(405)	[.2004]
Labor (All)	66.92	75.41	-0.51	63.70	40.26	2.28
	(12.02)	(11.44)	[.6094]	(8.85)	(5.19)	[.0232]
Male Labor	30.60	7.56	3.71	24.01	2.42	8.29
	(5.25)	(3.31)	[.0002]	(2.30)	(1.22)	[.0000]
Female Labor	15.27	47.15	-4.81	9.66	26.80	-4.60
	(3.08)	(5.87)	[.0000]	(1.27)	(3.50)	[.0000]
Hired Labor	8.02	3.17	1.46	13.36	1.64	1.73
	(3.23)	(0.76)	[.1458]	(6.74)	(0.59)	[.0850]
Other Labor	13.02	17.52	-0.72	16.67	9.40	2.30
	(3.27)	(5.34)	[.4731]	(2.42)	(2.03)	[.0224]
Observations	191	139		170	86	

Standard errors in parentheses

$H_0 : \mu_m = \mu_f$

p-values in brackets

Revenue and wages in GH (1$ $\approx$ 2000 GH in 1997)

Labor values in hours

While the differences in plot size and input use highlight average heterogeneity across genders, they do not consider the likelihood that individual labor types are partial substitutes for one another or that they may be applied endogenously as functions of land quality. For this reason, we estimate labor inputs per plot as a function of plot size and soil quality.[3] Since in many cases individuals don't use specific types of labor on a given plot, we estimate a censored Tobit model. For labor type m applied to plot i we estimate:

$$L_{mi} = \begin{cases} X\beta & \text{if } L_{mi} > 0 \\ 0 & \text{if } L_{mi} = 0, \end{cases} \tag{1.1}$$

where β is the vector of coefficients to be estimated and X is the vector of explanatory variables including plot size, gender of the plot manager, and an index of soil quality.

To account for the possibility that labor types are substitutes (or complements in the sense that an individual applying more labor to a plot may have more access to labor in general and thus apply more of other types simultaneously), we estimate a multivariate Tobit model assuming error terms may be correlated across labor types by plot. The results of this estimation are presented in Table 1.3. While error terms between labor types are significantly (positively) correlated in most cases (as seen in Table A.1 of Appendix A), we also present results of individually estimated input demands in Table A.2 of the appendix.

[3] To capture differences in plot quality here we use an index derived from estimated partial effects of soil characteristics, topographic type, and length of the last fallow on output for each plot. Soil type and topographic sequence consist of four types each and capture a wide range of fertility differences, while length of fallow is found to be one of the primary means for improving land productivity [2].

Table 1.3. Multivariate Tobit Estimates of Labor Use per Acre

Dep. Var: Lab/Acre	Cassava			Maize		
Labor Type →	Male	Female	Hired	Male	Female	Hired
Gender	-72.462***	38.629***	-11.714*	-54.021***	19.799***	-79.816***
(Female = 1)	(6.982)	(6.132)	(6.902)	(6.331)	(3.273)	(24.440)
ln(Area)	-17.011***	-18.176***	0.086	-6.928***	-4.483***	-24.070**
	(2.865)	(2.723)	(3.155)	(2.221)	(1.531)	(10.260)
Quality	-27.140*	-23.460	30.658*	22.446*	-2.670	-18.233
	(15.924)	(15.400)	(17.063)	(13.521)	(8.477)	(55.187)
Other Labor	0.205***	0.505***	0.201***	0.187**	0.155***	-0.037
	(0.057)	(0.059)	(0.056)	(0.082)	(0.051)	(0.321)
Constant	54.767***	25.544***	-44.221***	16.549**	7.036	-32.809
	(6.796)	(6.700)	(8.248)	(8.337)	(5.501)	(35.726)
Observations	465	465	465	274	274	274
Wald χ^2	397.54			181.42		
$Pr(> \chi^2)$	0.0000			0.0000		

Standard errors in parentheses

*** p< 0.01, ** p< 0.05, * p< 0.1

Controlling for plot size and quality, female plots receive significantly less male and hired labor per acre. While female cultivators utilize more of their own labor per acre, this fails to offset the larger losses in male and hired labor on average. The significant (negative) effect of land quality on male labor and (positive) effect of on hired labor application to cassava plots suggest that male and hired labor is substituted on higher quality plots. This is additionally meaningful along gender lines as female plots are of significantly lower quality. This point is addressed more thoroughly in [2], which shows that fallowing is tied to one's position in the local political hierarchy, which is stronger for men on average. Here as well, fallow length plays a large determinant in plot productivity.

We also estimate a similar system with other labor as a dependent variable and find that female plots receive less per acre on Maize plots. This, somewhat counter intuitive result, is also found in [4] regarding sorghum production in Burkina Faso. These findings further reflect the observations in [1], that while women commonly have obligations to provide labor for male controlled farms (to the extent that such obligations take precedence over women's rights to engage in own-account farming) and that the most significant area of gender conflict in intrahousehold resource allocation revolves around the control of household (and wage) labor.

Although the residuals between crop specific regressions in Table 1.3 are correlated to a significant degree, and thus estimation of individual equations as in Table A.2 give biased estimates of labor application determinants, the bias isn't very strong (parameter estimates don't change significantly in magnitude). Still, standard errors of coefficients in the individual regressions are larger in most cases.

More importantly, the magnitudes of correlation provide insight as to the relative complementarity of labor application between genders. Residuals between male and female demand equations are positively correlated for both crops, suggesting that a plot using above average male labor typically uses above average female labor. The positive correlation between male and hired labor suggests a similar complement between these labor types. The magnitude of correlation between female and hired labor, however is either much smaller, in the case of cassava, or zero, in the case of maize. Together, these results show that while male labor (mostly applied to male plots) is complemented by hired labor and female labor, female labor (which is the primary type applied to female plots) is not complemented by hired labor to a large extent. While this was suggested earlier in comparing mean labor use by gender, we can confirm and further highlight this pattern since it holds when controlling for plot size and soil quality.

1.3. Subsistence Production

While we confidently conclude that men and women in the sample utilize significantly different amounts of land and labor on similar plots, and realize different returns on average, we have not addressed whether this is due to differences in productivity between male and female managers and therefore an optimal allocation. To compare productivities by gender we estimate production functions mapping inputs and soil characteristics into output revenue. If men and women are similar in regards to managerial productivity, and production technologies exhibit diminishing returns to scale, then the conclusion of gendered access to inputs warrants a Pareto-improving reallocation of productive resources to female plots.

Ideally we would model this production process with a multiple input Cobb-Douglas specification, as the hypothesis of production inefficiency rests on the presence of diminishing returns, and our later consideration of cost minimization is made straightforward by the duality of the Cobb-Douglas framework. To test the appropriateness of this functional form, we start by estimating the more general constant elasticity of substitution (CES) production function for both maize and cassava plots:

$$lnY_i = \beta_0 - \frac{\tau}{\rho} ln\left\{ \delta Land_i^{-\rho} + (1-\delta) Labor_i^{-\rho} \right\} + \epsilon_i. \tag{1.2}$$

Yield (revenue) on plot i is Y_i, *Land* is plot size in acres, *Labor* is the number of labor hours applied, δ is land's share of output relative to labor, elasticity of substitution is given by

$\sigma = 1/(1-\rho)$, τ is a returns to scale parameter, and e_i is an iid plot specific disturbance. Estimates of equation (1.2) for both maize and cassava plots are given in Table 1.4.

TABLE 1.4. CES Estimates

Parameter	Cassava	Maize
β_0	8.913***	8.957***
	(0.203)	(0.236)
τ	0.630***	0.470***
	(0.049)	(0.066)
ρ	0.145	0.089
	(0.189)	(0.697)
δ	0.335**	0.137
	(0.142)	(0.282)
Observations	465	305
R^2	0.272	0.212

Standard errors in parentheses
*** p< 0.01, ** p< 0.05, * p< 0.1

Here we are primarily interested in two hypotheses.[4] First, we test the null of constant return to scale $H_0 : \tau = 1$. We reject this null at the .1% level for both crop types, concluding there are diminishing returns to scale for these technologies. Second, we test if the elasticity of substitution parameter, ρ is significantly different than zero, or if $\sigma = \frac{1}{(1+\rho)}$ is different than one. We fail to reject this null for both crops, supporting the use of a Cobb-Douglas specification.

[4]It is also interesting that on maize plots, land's share of output is not statistically significant. On female plots however, where land is much more scarce, land's share approaches one.

Our initial comparison of management effectiveness by gender (across households) is done by estimating Cobb-Douglas production functions mapping land and labor into output revenue according to:

$$ln(Y)_i = \beta_0 + \beta_1 Gender_i + \beta_2 ln(Land_i) + \beta_3 ln(Labor_i) + \gamma X_i + \chi Z_i + e_i, \tag{1.3}$$

where $Gender_i = 1$ if the plot's manager is female and zero otherwise, X_i is a vector of four soil characteristics, four toposequence types, and length in years of the plot's last fallow, and Z_i is a vector of village, year, and household fixed-effects.[5] We estimate equation (1.3) for both maize and cassava plots by gender, and for all plots with interaction terms combining the gender variable with each input. The results of these estimations are below in Table 1.5.[6]

[5]We include soil and toposequence characteristics to avoid endogeneity of input choices, which would otherwise bias our estimates of the effects of inputs on yield [12]. Jointly, soil and toposequence qualities of plots are significant in explaining variation in both land and labor inputs at the 5% level. Controlling for plot characteristics and soil fertility is additionally important in comparing management effectiveness by gender as men and women farm significantly different quality plots on average.

[6]We also present estimates of output revenue for both subsistence crops using specific labor types in Table A.3 of Appendix A. Since many plots don't use each type of labor, we employ the technique in [13] to maintain observations upon taking logarithms without biasing estimates of technology parameters by replacing zeros with an arbitrarily small number.

Table 1.5. Cobb-Douglas Production Estimates

Dep. Var: $ln(Rev)$	Cassava			Maize		
	All	Male	Female	All	Male	Female
Gender	-0.196			0.664		
(Female = 1)	(0.489)			(1.061)		
ln(Area)	0.283***	0.203*	0.440***	0.210	0.282	0.933
	(0.075)	(0.104)	(0.117)	(0.140)	(0.179)	(0.646)
Female×ln(Area)	0.119			-0.039		
	(0.122)			(0.418)		
ln(Labor)	0.315***	0.322***	0.255***	0.370***	0.381***	-0.110
	(0.071)	(0.087)	(0.081)	(0.123)	(0.126)	(0.448)
Female×ln(Lab)	-0.076			-0.177		
	(0.101)			(0.288)		
Length Fallow	0.037	0.035	0.075*	-0.009	-0.003	0.104
	(0.025)	(0.028)	(0.038)	(0.062)	(0.073)	(0.150)
Female×Length Fallow	-0.015			-0.021		
	(0.029)			(0.119)		
Constant	9.604***	10.633***	9.423***	9.685***	9.425***	2.788**
	(0.890)	(0.756)	(0.940)	(1.534)	(1.162)	(0.934)
Observations	417	231	186	274	192	82
R-squared	0.648	0.717	0.726	0.685	0.761	0.889

Robust standard errors in parentheses

*** $p< 0.01$, ** $p< 0.05$, * $p< 0.1$

All coefficients on gender variables (both intercept shifters and interaction terms) are insignificant for both crops, suggesting that males and females are no different in their ability to turn inputs into revenue on plots with similar characteristics. Labor being marginally less productive on female plots on average is likely due to the significant under allocation of land to female farms and differences in labor composition (e.g. much less hired labor being applied to female plots). While land on female maize plots is only statistically significant at the 17% level, it is economically significant in that an additional unit of land is much more productive on female plots relative to male plots (for both crop types). These results are also clear in Table 1.14, where gender terms are insignificant, and controlling for individual labor types and soil characteristics, land is on average marginally (much) more productive on female plots.[7].

While the above analysis indicates that males and females effectively face the same technology when controlling for soil quality, we have not yet compared relative allocations of inputs between genders *within* households. For this reason, we estimate production frontiers, which allow an objective comparison of productivity by comparing realized output to that at an efficient frontier. Still, the result that females are no less productive as plot managers on average, combined with the presence of diminishing returns and significantly lower levels of inputs, suggests that a reallocation of resources to female farms would increase total production overall.

[7]While female labor is marginally more productive on female plots, male labor is not. The insignificance of male labor on female plots can be attributed to two potential causes. First, female plots are land constrained. While there are diminishing returns to labor, and so we'd expect an increase in male labor to have a larger effect on female plots (which use less labor), we do not find this. Second, this may reflect the argument in [1] that men and women care more about output on their own plots, which could manifest in less productive work on one's spouse's plot.

1.4. Intrahousehold Efficiency

To further uncover the effect of unequal input use on total household production, we now estimate stochastic production frontiers. As opposed to ordinal rankings of farmer productivities (e.g. comparing predicted revenue given input use), we can now provide cardinal measures of productive efficiency by comparing actual revenue achieved by each cultivator to that which could be earned with complete efficiency. While our question is ultimately one of allocative efficiency between individuals, we start by comparing technical efficiency in section 1.4.1 to extend the analysis of the previous section. In section 1.4.2, we allow for both technical and allocative inefficiency to isolate the potential production lost by households due specifically to inferior allocations of inputs between genders.

1.4.1. Technical Inefficiency Only.

To compare relative technical efficiency of male and female farmers, we estimate stochastic frontiers (according to and following the notation of [14]). First, we assume there is no inefficiency resulting from relative misallocations of land and labor within or across plots. By separating producer specific error terms, ϵ_i into random and technical inefficiency components, $v_i \sim$ iid $N(0, \sigma_v^2)$ and $u_i \sim$ iid $N^+(0, \sigma_u^2)$, we can estimate cultivator specific deviations from the production frontier from technical inefficiency. The Cobb-Douglas production function analogous to equation (1.3) is now:

$$ln(Y)_i = \beta_0 + \beta_1 Gender_i + \beta_2 ln(Land_i) + \beta_3 ln(Labor_i) + \gamma X_i + \chi Z_i + (v_i - u_i). \tag{1.4}$$

The non-negative residual component, u_i, represents deviation from the production frontier due to technical inefficiency. Since the random productivity shock, v_i, is mean zero and $u_i \geq 0$, each producer's output $ln(Y_i)$ is bounded above by the deterministic portion of (1.4). Graphically, as in [15], technical efficiency for cultivator i is the ratio OQ/OP in Figure 1.1, where a mix of inputs x_1 and x_2 produce at most a yield of Y^*.

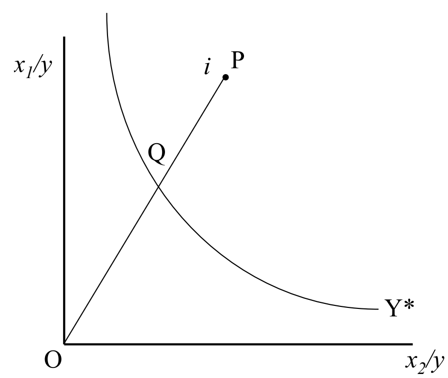

FIGURE 1.1. Technical Efficiency

A producer is on their production frontier if $u_i = 0$ (i.e. $P = Q$, so $OQ/OP = 1$) or inside the frontier as technical inefficiency is present (as $OP > OQ$, so $OQ/OP < 1$). We estimate (1.4) using maximum likelihood techniques, which provide estimates of the technology parameters and producer specific deviations from the frontier. Estimates of the technology parameters are given below in Table 1.6.

Table 1.6. Stochastic Frontier Estimates (Technical Inefficiency Only)

Dep. Var: $ln(Rev)$	Cassava			Maize		
	All	Male	Female	All	Male	Female
ln(Area)	0.281***	0.134**	0.444***	0.124*	0.044	-0.073
	(0.041)	(0.055)	(0.048)	(0.069)	(0.060)	(0.144)
ln(Labor)	0.363***	0.417***	0.241***	0.415***	0.499***	0.187*
	(0.038)	(0.047)	(0.076)	(0.054)	(0.059)	(0.101)
Length Fallow	0.032**	0.040**	0.013	0.024	-0.003	0.091
	(0.014)	(0.019)	(0.021)	(0.027)	(0.029)	(0.066)
Constant	9.809***	9.578***	8.812***	9.251***	9.146***	7.794***
	(0.450)	(0.731)	(0.638)	(0.713)	(0.610)	(0.834)
Observations	417	231	186	274	192	82
Wald χ^2	220.93	163.46	199.30	98.47	272.40	19.12
$Pr(> \chi^2)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0856

Standard errors in parentheses
*** p< 0.01, ** p< 0.05, * p< 0.1

These results largely reflect those of the Cobb-Douglas estimates in terms of relative productivities between males and females. Female cassava plots achieve higher marginal yields with land on average while labor is marginally more productive on male plots, again reflecting the combination of diminishing returns to labor, its complementarity with land, and the fact that female plots are much smaller on average. In addition to the technology parameters above, we can also compare technical efficiency (TE) between genders by calculating:

$$TE_i = exp\{u_i\} \tag{1.5}$$

for each cultivator. First, we calculate (1.5) for each producer and compare averages across households by gender. From Table 1.7, females achieve significantly higher levels of technical efficiency on cassava plots (of similar quality) and effectively the same technical efficiency on maize plots on average.

Table 1.7. Technical Efficiency by Gender (Across Households)

	Cassava			Maize		
	Male	Female	t-statistic	Male	Female	t-statistic
Technical Efficiency	0.448	0.490	-1.93	0.460	0.426	1.36
	(0.015)	(0.016)	[.0546]	(0.014)	(0.020)	[.1745]
Observations	231	186		192	82	

Standard errors in parentheses

$H_0 : \mu_m = \mu_f$

p-values in brackets

Since often times women and men farm the same crop in the same year, we can also compare technical efficiency by gender *within* households. As some individuals farm multiple plots of the same crop in a year, we weight plot specific inefficiencies to get an individual level measure for both the female and male in each household. A comparison of the 48 households where both genders farm cassava and 37 households where both farm maize in the same year are below in Table 1.8.

Table 1.8. Technical Efficiency by Gender (Within Households)

	Cassava Plots			Maize Plots		
	Male	Female	t-statistic	Male	Female	t-statistic
Technical Efficiency	0.463	0.473	-0.21	0.502	0.508	-0.16
	(0.037)	(0.033)	[.8364]	(0.028)	(0.026)	[.8764]
Observations	48	48		37	37	

Standard errors in parentheses

$H_0 : \mu_m = \mu_f$

p-values in brackets

Although men and women use vastly different amounts of land and labor in producing subsistence crops, we find no evidence that men are more productive in turning their inputs into revenue. This finding further supports the argument that under decreasing returns to scale and similar production technologies (i.e. effectiveness as plot manager), the significant over allocation of productive resources to male farms reduces total household production. Next, we estimate the cost of this over allocation.

1.4.2. Technical and Allocative Inefficiency.

Female cultivators in the sample households are no farther from the efficient production frontier than men in the same household. From the duality of the Cobb-Douglas specification, we can think of this similarity as a common distance from the minimum cost frontier. Leveraging data on input prices, we can extend our analysis of technical efficiency to estimate the distance from the cost frontier due specifically to sub-optimal allocations of resources between individuals in the household.

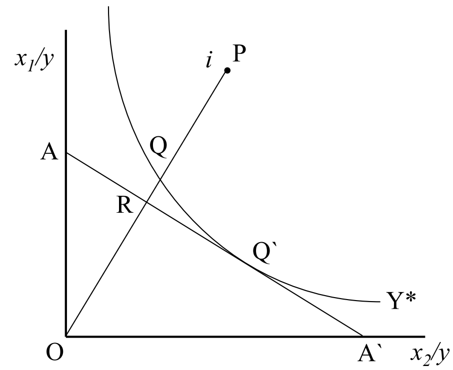

FIGURE 1.2. Technical and Allocative Efficiency

Building on the frontier presented in Figure 1.1, we are now interested in the distance QR in Figure 1.2. With relative prices A and A', we can now consider production shortfalls (or cost increases) due to a sub-optimal input mix. Where technical efficiency is as before, allocative inefficiency is represented by the deviation from the production (or cost) frontier due to a relative misallocation between x_1 and x_2. For cultivator i, the point of *both* technical and allocative efficiency given relative prices A/A', is point Q'. Allocative inefficiency is then the distance RQ, which represents the reduction in production costs that would occur if production were at the allocatively (and technically) efficient point Q' instead of the technically efficient but allocatively *inefficient* point Q [15]. Allocative efficiency between x_1 and x_2 is then the ratio OR/OQ. A producer choosing an optimal mix of x_1 and x_2 would produce such that $OR = OQ$, or $RQ = 0$.

To estimate allocative inefficiency between genders in the same household, we now treat the household as a single production unit which allocates land and labor to two individual farms (f and m for female and male plots respectively). Household i's production function analogous to (1.4) is now:

(1.6) $ln(Y_i) = \beta_0 + \beta_1 ln(Lab_{mi}) + \beta_2 ln(Lab_{fi}) + \beta_3 ln(Land_{mi}) + \beta_4 ln(Land_{fi}) + (v_i - u_i),$

where labor applied to female and male plots is: Lab_f and Lab_m and land: $Land_f$ and $Land_m$.

To separate total inefficiency into its technical and allocative components, we combine the stochastic frontier in equation (1.6) with the $(N-1)$ cost-minimizing conditions equating each pair of inputs marginal productivities with their relative prices. For a cultivator to produce at minimum cost, they would allocate inputs x_1 and x_j with marginal productivities β_1 and β_j and prices w_1 and w_j according to:

$$\frac{x_1\beta_j}{x_j\beta_1} = \frac{w_j}{w_1}, \forall j = 2, \ldots, N. \tag{1.7}$$

Taking logs of these conditions and rearranging gives the $N-1$ equations:

$$ln\left(\frac{x_1}{x_j}\right) + ln\left(\frac{\beta_j}{\beta_1}\right) - ln\left(\frac{w_j}{w_1}\right) = 0. \tag{1.8}$$

If a household is allocating resources optimally across (and within) plots, then these conditions will hold for each pair of inputs. If it is allocatively inefficient in the sense that it over or under allocates inputs relative to one another, then these conditions will not hold. To allow for the possibility that households do deviate from their cost minimizing allocations, we can add a vector of two-sided error terms specific to each household which includes each pair of inputs, $\eta_i = (\eta_{2i}, \ldots, \eta_{Ni})' \sim$ iid $N(\mu, \Sigma)$, so that the cost-minimizing conditions are now:

$$ln\left(\frac{x_1}{x_j}\right) + ln\left(\frac{\beta_j}{\beta_1}\right) - ln\left(\frac{w_j}{w_1}\right) = \eta_{ji}. \tag{1.9}$$

An over allocation of the numeraire, x_1 relative to input j is represented by a positive disturbance, $\eta_{j1} > 0$ and an under allocation by a negative one.[8] While a simpler model for estimating cost inefficiency in this way is to assume each input-pair disturbance has a mean of zero, our analysis thus far indicates there may be systematic over allocations of inputs towards male plots. For this reason, we make the distributional assumption, $\overline{\eta}_j = \mu_j$ as in [16], to allow for a non-zero average deviation from the cost-minimizing conditions across households. The system to be estimated is then:

[8]For example, in equation (1.9) if inputs j and 1 are applied equally and have similar prices, so that $x_1 = x_j$ and $w_1 = w_j$, the first and third terms on the left hand side would be zero. If the marginal productivity of j where higher than that of the numeraire, the second term would be positive, representing an over allocation of x_1 relative to x_j, or $\eta_j > 0$.

(1.10)

$$ln(Y_i) = \beta_0 + \beta_1 ln(Lab_{mi}) + \beta_2 ln(Lab_{fi}) + \beta_3 ln(Land_{mi}) + \beta_4 ln(Land_{fi}) + (v_i - u_i)$$

$$\eta_{2i} = ln\left(\frac{Lab_{mi}}{Lab_{fi}}\right) + ln\left(\frac{\beta_2}{\beta_1}\right) - ln\left(\frac{w_2}{w_1}\right)$$

$$\eta_{3i} = ln\left(\frac{Lab_{mi}}{Land_{mi}}\right) + ln\left(\frac{\beta_3}{\beta_1}\right) - ln\left(\frac{w_3}{w_1}\right)$$

$$\eta_{4i} = ln\left(\frac{Lab_{mi}}{Land_{fi}}\right) + ln\left(\frac{\beta_4}{\beta_1}\right) - ln\left(\frac{w_4}{w_1}\right)$$

Table 1.9 compares mean allocations of inputs to male and female farms for those households where both individuals farm the same subsistence crop in the same year. As before, male plots are significantly larger and receive more labor on average. While labor use and average wages are higher on male plots, these differences are significant only at the 33% and 36% levels. Here and in the estimation of the system (1.10), average wage is calculated by weighting wages of individual labor types by how much of each type was applied on an individual's farm. Higher wages on male farms are an expected result of the much greater use of hired and male labor, which demand higher wages on average, as opposed to female and other household labor which are paid lower wages on average, and applied to female plots more often.

Table 1.9. Household Averages by Gender

	Male Plot(s)	Female Plot(s)	t-statistic
Area	9.42	5.74	1.74
	(1.89)	(0.95)	[.0834]
Labor	211.52	174.02	0.98
	(20.16)	(0.03)	[.3283]
Average Wage	900.43	836.36	0.91
	(55.40)	(43.08)	[.3625]
Households	89	89	
Standard errors in parentheses			
$H_0 : \mu_m = \mu_f$			
p-values in brackets			

Estimation of (1.10) consists of minimizing the joint distribution of the error vector for each household, $f(\epsilon, \eta) = f_\epsilon(\epsilon) f_\eta(\eta)$, where $\epsilon = v_i - u_i$, according to the following distributional assumptions as in [14]:

(1) $v_i \sim$ iid $N(0, \sigma_v^2)$

(2) $u_i \sim$ iid $N^+(0, \sigma_u^2)$

(3) $\eta_i = (\eta_{2i}, \eta_{3i}, \eta_{4i})' \sim$ iid $N(\mu, \Sigma)$, and

(4) v_i is distributed independently of u_i, and each are independent of the elements of η_i.

Assumptions (1) and (2) represent the decomposition of productivity shocks into their random and technical inefficiency components as before. Assumption (3) states that household deviations from their cost-minimizing conditions are independent across households and are distributed with mean $\mu = (\mu_2, \mu_3, \mu_4)'$ and variance covariance matrix Σ.

The last assumption is known to be problematic, since allocative inefficiency necessarily raises cost and is therefore related to the inefficiency term u_i. This "Greene Problem" is not dealt with here, but does enter our estimation of system (1.10). Replacement of assumptions (3) and (4) with:

$$\begin{bmatrix} |u| \\ \\ \eta \end{bmatrix} \sim N \left[\begin{pmatrix} 0 \\ \\ \mu \end{pmatrix}, \quad \Sigma \right], \tag{1.11}$$

where

$$\Sigma = \begin{bmatrix} \sigma_u^2 & \Sigma_{\mu\eta} \\ \\ \Sigma'_{\mu\eta} & \Sigma_{\eta\eta}, \end{bmatrix} \tag{1.12}$$

would allow technical and allocative inefficiency to be correlated as in [17]. While this specification would simultaneously consider any bias on technical inefficiency imposed by allocative inefficiency and allow us to test whether households which are more technically efficient also allocate between spouses more efficiently, data requirements do not allow us to do so for these households.[9] Information on more couples farming the same crop in the same period would certainly support this line of study. Still, we don't believe the assumption of

[9]This would certainly be an interesting extension by exploring if productivity improvements (increases in technical efficiency via extension programs, training, etc.) lead to greater (or lesser) gender equality regarding access to resources.

uncorrelated inefficiencies alters our main conclusion as to the significant costs of allocative inefficiency.

In addition to including the potential for households to deviate from their cost-minimizing choice of inputs, we have fundamentally altered the behavioral assumption underlying the model itself. Whereas before our production function estimation implicitly assumed that inputs where exogenously given (after controlling for soil characteristics and land quality) so that individuals maximized yield, our current model assumes that households choose inputs endogenously to minimize the cost of producing a given amount of output. We return to the appropriateness of this assumption after obtaining estimates of the technology parameters under this assumption below.

As in [14], minimization of the joint distribution $f(\epsilon, \eta)$ is accomplished by maximizing the log likelihood function:

$$(1.13) \quad lnL = \text{constant} - Iln\sigma - \frac{I}{2}ln|\Sigma| + Ilnr - \frac{1}{2}\sum_i \left[(\eta-\mu)_i'\Sigma^{-1}(\eta-\mu) + \left(\frac{1}{\sigma^2}\right)\varepsilon_i^2\right] + \sum_i \left[1 - \Phi\left(\frac{\varepsilon_i \lambda}{\sigma}\right)\right],$$

where I is the number of households, $r = \sum \beta$ is returns to scale, $\sigma^2 = \sigma_v^2 + \sigma_u^2$, $\lambda = \sigma_u/\sigma_v$, and $\Phi(\cdot)$ is the cumulative distribution function for a standard normal variable. Following [16], we can concentrate the likelihood function (1.13) with respect to Σ and μ. Specifically, when the likelihood function is maximized, systematic misallocations μ_i will equal the average misallocation across producers, $\overline{\eta}_i$. Substituting each such average misallocation into the variance covariance matrix, Σ, we simplify each element, $\sigma_{jk} = \frac{1}{I}\sum_I(\eta_{ji} - \overline{\eta}_j)(\eta_{ki} - \overline{\eta}_k)$, which means the components of η and μ, and therefore Σ depend entirely on the vector of technology parameters, β, relative prices, and data on input use. The parameters to be estimated in maximizing the likelihood function (1.13) are then the technology parameters, σ^2, and λ. The results of this procedure are below in Table 1.10.

Table 1.10. Stochastic Frontier Estimates (With Technical and Allocative Inefficiency)

Dep. Var: $\ln(\text{Revenue}_{HH})$	Cassava and Maize
$\ln(\text{Lab}_m)$	0.536***
	(0.025)
$\ln(\text{Lab}_f)$	0.474***
	(0.132)
$\ln(\text{Area}_m)$	0.862***
	(0.007)
$\ln(\text{Area}_f)$	0.909***
	(0.032)
Constant	5.440***
	(0.415)
σ^2	1.401***
	(0.249)
λ	2,656.037
	(10,835.614)
Observations	89
Wald χ^2	130916
$Pr(> \chi^2)$	0.0000
Standard errors in parentheses	
*** p< 0.01, ** p< 0.05, * p< 0.1	

Before calculating misallocations of inputs and the cost of allocative inefficiency, we must address the immediate observation that the marginal productivities of inputs are much higher than in our previous results. Reasons for this significant difference can be attributed to two primary reasons: the assumption underlying the econometric model, in which households are now taking output as given and choosing inputs to minimize cost, combined with systematic over allocation of both land and labor to male plots, and the way we have aggregated individual production up to the household level.

If the behavioral assumption of cost minimization with exogenous output is appropriate, then direct estimation of the production function (as we have done here) is inappropriate [16]. As Schmidt and Lovell show, estimation of the cost function would provide more appropriate estimates of technology parameters. Additionally, the presence of allocative inefficiency tends to overestimate returns to scale relative to cases where it is assumed away (i.e. in models where $\eta_{ji} = 0, \forall j, i$, as in section 4.2). [18] show this using Monte-Carlo simulations under different behavioral assumptions. As we show below, there is significant (systematic) allocative inefficiency between pairs of inputs, increasing costs and thus demand for each input to produce a given output.

The second issue arises from our aggregation of two farms into one, where by adding yields as a single output but leaving inputs separate, will put upward pressure on marginal productivities since a partial increase in a given input will have a larger effect on the log of two summed outputs relative to the effect of a similar change on the sum of two logged outputs.[10] In this way, we are overstating the marginal effects of inputs on output, which also contributes to the larger estimates here.

[10] For example, consider two plots using inputs x_f and x_m to produce outputs of x_f^δ and x_m^δ. A marginal increase in x_f will have to increase output more in our specification ($ln(x_f^\delta + x_m^\delta)$) than if we were to sum the logarithms of individual outputs ($ln(x_f^\delta) + ln(x_m^\delta)$), since $\frac{\partial ln(f_f + f_m)}{\partial x_f} = \frac{\delta}{x_f + x_m} < \frac{\partial(ln(f_f) + ln(f_m))}{\partial x_f} = \frac{\delta}{x_f}$. We continue with this specification as our choice model assumes the "household producer" is producing a single output. A multiple output system would be more appropriate in this regard, although the calculation of misallocations is not affected in a meaningful way.

While these two concerns are important in explaining the larger estimates of marginal productivities, they do not alter our main conclusion. The argument for a gendered reallocation of inputs based on the combination of decreasing returns to scale, similar productivities, and different levels of inputs remains unchanged. What appears to be significant *increasing* returns to scale in Table 1.10 is a result of our aggregation to a "household plot," which is necessary to allow for different productivities and prices of inputs on male and female farms, and the effects of our behavioral assumption in the presence of (significant) allocative inefficiency.

Furthermore, cost increases due to deviations from cost-minimizing allocations depend on *relative* productivities between pairs of inputs in relation to prices (not their absolute magnitudes). Relative input elasticities are very similar to those obtained in our production estimates in section 1.3 and frontier estimates without allocative inefficiency in 1.4.1. We expect some change here as labor composition (and therefore average wages) now vary by household.

From the results in Table 1.10, we can calculate household specific deviations from the cost minimizing conditions (1.9) for each input pair. These are reported below in Table 1.11. Relative misallocations of male labor in relation to female labor and male land are given by η_2 and η_3, respectively. Relative deviations of female land relative to female labor and male land relative to female land, are the differences between the relevant ratios of other deviations (which are individually compared to the same numeraire, male labor):

$$(1.14) \qquad \eta_2 - \eta_4 = ln\left(\frac{Land_{fi}}{Lab_{fi}}\right) + ln\left(\frac{\beta_2}{\beta_4}\right) - ln\left(\frac{w_4}{w_2}\right) \text{ and}$$

$$(1.15) \qquad \eta_4 - \eta_3 = ln\left(\frac{Land_{mi}}{Land_{fi}}\right) + ln\left(\frac{\beta_4}{\beta_3}\right) - ln\left(\frac{w_3}{w_4}\right), \text{ respectively.}$$

On average, households over apply labor to male plots relative to female plots. Although marginal labor is again relatively more productive on male plots, the combination of: (i) lower average wages paid on female farms (which utilize more low wage female and other household labor compared to male plots where men and hired labor earn higher wages), (ii) the fact that less labor is applied to female plots on average, and (iii) the presence of diminishing returns to labor, results in an average over allocation of labor to male plots.

The over allocation of land to male plots is even more apparent. In part this is due to the assumption that land prices are constant, as there is not enough information on plot rental to estimate farm specific prices, which may bias our estimate of land misallocation upwards since male plots are of higher quality on average and might accordingly demand higher rents. The estimated over allocation is therefore a result of: (i) decreasing returns to land, (ii) higher marginal productivity of land on female plots, and (iii) the significantly smaller size of female farms relative to males.

While the relative price of labor to land is less meaningful due to the homogeneous (and relatively large) price of land (and meaningless comparison of units), high marginal returns to land, and the fact that much fewer acres of land are used relative to hours of labor, we can still see the over allocation of inputs to male plots by comparing the over allocation of male labor relative to male land (given by η_3) to the relative under allocation of female land relative to female labor (given by $\eta_2 - \eta_4$). Given relative prices and productivities, males are over-allocating labor relative to land by 109%. At the same time, female plots are utilizing on average 134% too much labor relative to land. Again, we see the significant constraint of smaller plots to production on female farms.

Table 1.11. Estimated Efficiency Components

	Mean	Minimum	Maximum	t-statistic
CAI	0.655	0.05	1.88	15.51
	(0.04)			[.0000]
η_2	0.387	-6.16	4.12	2.01
$\left(\frac{Lab_{mi}}{Lab_{fi}}\right)$	(0.19)			[.0479]
η_3	1.09	-3.47	5.44	6.44
$\left(\frac{Lab_{mi}}{Land_{mi}}\right)$	(0.17)			[.0000]
$\eta_2 - \eta_4$	-1.335	-4.81	2.99	-9.05
$\left(\frac{Land_{fi}}{Lab_{fi}}\right)$	(0.15)			[.0000]
$\eta_4 - \eta_3$	0.631	-3.09	5.13	3.19
$\left(\frac{Land_{mi}}{Land_{fi}}\right)$	(0.20)			[.0019]
Households	89			

Standard errors in parentheses

$H_0 : \overline{X} = 0$

p-values in brackets

To estimate the cost of allocative inefficiency for each household, we calculate $CAI_i = A - ln(r)$ for each household as in [14], where r is the returns to scale parameter as before and:

$$(1.16) \qquad A = \sum_{n>1} \left(\frac{\beta_n}{r} \right) \eta_n + ln \left[\beta_1 + \sum_{n>1} \beta_n exp\{-\eta_n\} \right].$$

While the (alarmingly large) returns to scale parameter enters our calculation of cost increases due to input misallocations, we can see here that it is only the relative productivities that matter. A scaling of each productivity parameter by an equal amount (so that the ratios between βs are maintained) increases r by the same amount, so that the first term in (16) is unaffected, and the increase in the second term is exactly offset by the decrease upon subtracting $ln(r)$.

From Table 1.11, the average cost increase due specifically to allocative inefficiency is 65%. It is important to recall that this increase in costs includes misallocations by individuals on their own farms (i.e. between land and labor). As described above, the significant over allocations of labor relative to land are at least partly a result of relative prices and meaningless unit comparisons. To consider the cost increase due specifically to intrahousehold misallocations between genders, we must isolate the portion of (1.16) that is due specifically to cost increases from allocating land and labor sub-optimally across individuals.

While equation (1.15) consists of terms that can be either directly attributed to misallocations between individuals (e.g. η_2) or indirectly (through $\eta_4 - \eta_3$), we cannot separate it analytically to isolate inter-individual misallocation costs from intra-individual ones. Instead, we estimate the marginal effects of the individual components of (1.15) on the total cost of allocative inefficiency for each household. Specifically, we start by estimating:

$$CAI_i = \beta\Gamma + \delta Z + \gamma\Omega + e_i, \tag{1.17}$$

where Γ is a vector of components representing misallocations between individuals (e.g. relative input applications across genders, relative wages, η_2 and $\eta_4 - \eta_3$, and combinations of interaction and logged terms of these components), Z is a similar vector of elements representing misallocations by individuals on their own farms, Ω is a vector of interactions between individual and gender components which cannot be attributed to one or the other, and β, δ and γ are vectors of coefficients to be estimated.[11] Estimated coefficients from (1.16) are then used to calculate household specific estimates of intrahousehold and intraindividual shifters of total CAI: $CAI_{gender,i} = \hat{\beta}\Gamma_i$ and $CAI_{individual,i} = \hat{\delta}Z_i$, respectively. Finally, to estimate the portion of the total cost of allocative inefficiency due specifically to misallocations across genders, we estimate:

$$CAI_i = \alpha CAI_{gender,i} + (1-\alpha)CAI_{individual,i} + e_i, \tag{1.18}$$

where $\alpha \in [0, 1]$ is the portion of total allocative inefficiency cost due to that across genders within households. We estimate this portion to be .85.[12] Multiplying the estimated value of

[11]Ordinary Least Squares estimation of (1.17) consists of 27 explanatory variables and results in an $R^2 = .9903$ and t-statistics which are all significant at the 10% level (many at 1%).

[12]Specifically, $\hat{\alpha} = .85$, $se(\hat{\alpha}) = .0365$.

α by each households total cost increase CAI_i, we find that on average, the misallocation of inputs across genders is 55.7%, with a standard deviation of .34.

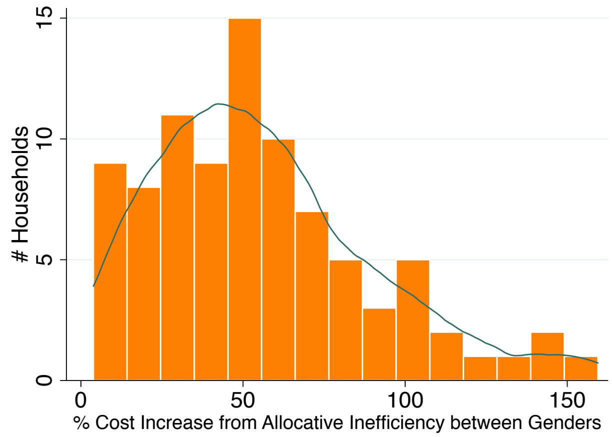

FIGURE 1.3. Cost Increases from Gender Inequality

After removing cost increases due to individual allocative inefficiencies, average production cost increases significantly due to misallocations across genders.[13] The distribution of household cost increases form gender inequality are above in Figure 1.3.[14] Many households are near or below the mean in terms of cost increases. The median cost increase from gender misallocations, of 51% may give a more realistic estimate of the average household's deviation, by removing the effect of the few households with extremely large misallocations.[15]

Still, median cost increases are significantly higher than averages calculated for agricultural households in Burkina Faso in [4]. To reconcile this significant difference, we offer three primary differences between our analyses. First, our estimates include the effect of relative

[13]Still, the greater over allocation of labor relative to land on female plots (removed here) may reflect women's unequal access to land.

[14]Additional plots of individual misallocations and cost increases from specific misallocations can be found in Appendix B.

[15]Still, we shouldn't disregard these outliers as the relative land and labor allocations between genders for the households with the highest CAI_{gender} are extremely high. In many of these cases, the land allocation is counter to the land over allocation, so that the land-to-labor ratios are even more divergent.

prices, which exacerbate the cost of misallocations as females pay less for labor on their plots. Second, as our treatment of the household applying each input to female and male farms results in gender-specific marginal productivities, we will overestimate the effect of over application to male plots (with lower input elasticities) upon reallocation. [4] on the other hand calculates gains from redistributions based on production function estimates over all plots, which in our case would use elasticities between the range of male and female specific ones. Third, land is much more productive marginally here than in the crops considered in the Burkina Faso case. Although there are also significantly large differences in land use in Burkina Faso, it is not as much of a contributing factor to output as it is here. Furthermore the higher marginal returns to land on female plots drive up the costs of misallocating land.

To consider the effect of the first and second differences, we use estimates of marginal productivities for the household as a whole (giving technology parameters between those of individual genders and therefore reducing the overestimation of reallocation gains due to divergent elasticities under diminishing returns), remove the effect of prices, and recalculate household cost increases. Doing so results in an average cost increase of 42% and a median increase of 35.9% due specifically to sub-optimal allocations between genders.

Our estimate of cost increases from gender inequality may also be temporarily high for the study region due to the rapidly changing environment which the villages in the survey were undergoing. Specifically, the (partial but quick) transition towards pineapple production may have put additional pressures on input choices as households learned to allocate land and labor to a new crop. While our findings may not therefore be representative of agricultural settings not in transition, the possibility that transition drives inequality in certain realms is still important.

Even during such a transitional period, a reallocation of resources on male cassava and maize plots to female cassava and maize plots can be done without affecting men's pineapple plots, unless there are complementary effects from males working on their own plots, generating inexpensive opportunities for monitoring hired labor, etc. on pineapple farms, or by reducing time spent moving from their spouse's farm to their own. Still, the gains generated from reallocating land appear to be significant and would not imply a loss of complementarity to pineapple management assuming labor allocations remained unchanged. Additionally, reallocations of female, hired, and other household labor to female plots would support gains from land reallocation while not taking away potential monitoring effects of males working on their own farms. If anything, this would reduce management burden on subsistence crops and support any gains from specialization in pineapple.[16]

1.5. Discussion

In our analysis of Ghanaian subsistence farming, we have confirmed empirical evidence of production inefficiency resulting from gender differences in access to inputs. While women utilize much smaller plots of land, they also receive much less labor per acre. Furthermore, the composition of labor differs significantly by gender and specific types (e.g. hired labor) are applied much less frequently with one's own on female plots.

Following the argument in [4] we similarly find that while women utilize significantly fewer inputs on their farms, they are no less productive as plot managers controlling for important soil characteristics. As production technologies exhibit decreasing returns to scale, total household production could be increased with a redistribution of resources from men

[16]Although women rarely farm pineapple, estimates of pineapple production suggest a similar combination of diminishing returns to scale and homogeneous managerial capabilities between genders.

to women. This result has important implications for the way theoretical models of agricultural households treat the production process. It highlights not only that the allocation of resources within the household matters, but that the household unit does not act as a benevolent planner would. Instead, inferior norms of gendered access to resources and atomistic production drive the household away from its production frontier.

We extend the previous analysis by developing objective comparisons of inefficiency at both the cultivator and household levels. By comparing technical efficiency between women and men across and within households, we further support the previous argument. With data on relative input prices, we also estimate the cost of allocative inefficiency due specifically to sub-optimal allocations between genders in households where both farm the same crop in the same period. Specifically, we find an average over allocation of labor to male farms of 38.7% and over allocation of land of 63.1%. After isolating total cost increases due to intrahoushold inefficiency, we conclude that production costs for these crops increase in the range of $35\% - 51\%$ due to gender inequality.

This cost is largely due to the severe under allocation of land to women cultivators. This reflects findings that the lower status of women in local political hierarchies reduces their ability to invest in soil fertility through fallowing. This results in poorer quality and smaller plots in general. This finding offers clear support for arguments that improvements in women's land rights offer a significant means for improving both gender equality and agricultural output.

While the sample households were amidst a significant transition to cash crop production, we have also offered reasons why the gains from reallocation may not be lost in a lack of representativeness. Any additional pressures of agricultural transition only add additional importance to combating gender inequality in access to inputs as policies seek agricultural innovations and the incorporation of new production techniques.

Essay #2: Dynamic Bargaining in Household Agriculture: Inefficient Norms and Gendered Strategies

2.1. Introduction

An analysis of poverty that ignores gender can be misleading in terms of both causation and consequences [19]. In the context of subsistence agriculture, gendered institutions determining control of household resources define constraints and incentives which necessarily determine individual strategies and household outcomes. These mechanisms encompass the dynamic nature of the investment problem facing agricultural households and have important implications for economic growth, food security, and social equality.

Women and men engaged in subsistence agriculture confront different opportunities, constraints, and outcomes, each of which are important for both gender equality and economic growth. Specifically, women and men have unequal access to productive inputs [4], adopt new technologies and crop types differently [5, 20], enjoy different off-farm opportunities [8], and cope with economic shocks differently [21]. While gender and bargaining literatures have explored these and similar issues, theoretical models intended to explain the importance of crop choice and policy efficacy for growth in developing regions have not.

The purpose of this paper is therefore to extend the model of the agricultural household to consider the influence of gender-bias on household strategies and outcomes over time. To do this, we leverage insights from separate spheres and intrahousehold bargaining literatures which highlight the role of gender norms as institutional conventions applicable to the agricultural context. By placing these mechanisms in a dynamic framework, we contribute to the separate spheres literature by exploring how an institution of gender bias regarding control over household resources affects the inter temporal strategies of men and women and results in divergent outcomes both between genders and across households.

As inequality in access to resources increases, individual strategies diverge from that of a unitary planner and household investment falls. This reflects findings that the allocation of productive resources in subsistence agriculture often leave potential Pareto improvements available. Beyond reducing total investment at the household level, this dynamic also has significant implications for gender specific welfare and related differences in terms of future opportunities, budget shares of household expenditures, and bargaining power within the household.

As households in this context encounter non-trivial subsistence constraints, divergent outcomes regarding budget shares and bargaining power take on especially detrimental consequences for women. In addition to incomplete vesting of land across individuals within households [8], risk pooling, nutritional smoothing, and reinvestment opportunities are also defined by one's gender ([8], [9], and [1], respectively).

With aspects of both cooperation and competition, our model highlights observations of related empirical and theoretical work and offers various other testable hypotheses. After developing a model where individuals make investment choices based on expectations over spousal behavior, we consider the effects of bargained transfers and the dynamic links between agents. We then solve a sub-game perfect Nash equilibrium and discuss how individual strategies and household outcomes depend on institutional norms. Then we discuss generalizations to the model and briefly describe how they relate to and further inform our main conclusions.

2.2. Gendered Constraints in Subsistence Agriculture

While women in developing regions contribute a large portion of agricultural labor, they typically achieve lower rates of productivity on their farms [3]. This is primarily due to significantly less access to productive inputs relative to men. We accordingly operationalize gender as an institution determining relative control over household resources. In this way, gender is a social characteristic underlying the problem itself [22]. As will be shown, the social norm's influence on an individual's access to productive resources determines relative shares of returns from production, and feeds back on their savings choice to determine optimal investments. The institution of gender becomes a constraint that shapes human interaction, meeting North's more formal definition of an institution.

The analytical mechanism of the (potential) gender-bias is an exogenously determined level of relative control over the household asset base. Specifically, it is a measure of female say relative to male say, given by the parameter $\pi \in [0, 1]$. As we'll show, this treatment of gendered access to resources, while different from the usual mechanism of bargaining power, has a similar effect in terms of altering gains from bargaining and the resulting strategies men and women adopt.

Our treatment of gender thus entails a parallel notion of power. While agents' strategies may represent mutual best-responses, one spouse may effectively have power in that their adherence to an institutional convention provides them with additional gains at the expense of their partner. What appears to be a passive coherence to the existing convention may be the same result had the agent also chosen their preferred institutional rule. However this will not be so for one's partner, as they would clearly prefer a more equal share of resources.

The specific context is one where households farm both subsistence and cash-crops and spouses farm separate plots.[17] This pattern of individual production sites is common to many parts of Sub-Saharan Africa and South Asia [2]. The atomistic nature of farming outlines the basic savings problem, where we focus on levels of investment in each crop by individuals in a two-person household, but also reflects a degree of individuality present in marriages in this context. Regarding West African marriage, specifically, [23] summarizes:

> Spouses usually enjoy little everyday companionship except, perhaps, when they grow old: they rarely sit and converse; they eat separately; they tend to have separate ceremonial and recreational activities. Considering that they are rarely seen walking down a path together, it is no wonder that they seldom work jointly to produce crops which either party may sell, or toil alongside each other on the fields. (p.124)

While in many cases, spouses do apply labor to each others plots and share portions of individual returns, both women and men care more about output on their own farms [1]. While this may be a reflection of the cultural norms described above, it also has direct implications for individual strategies as budget shares of particular goods are significantly related to the shares of income accruing to women in the household [10].

Socially determined norms regarding what individual household members can do, consume, and make decisions about, can be considered responses to a combination of cooperation and conflict [19]. While the model we develop allows for both cooperation, through both shared cash crop revenues and contributions to a household public good, and conflict,

[17]We are not necessarily concerned with the differences in these crops, although as subsistence crops may offer a less fungible savings instrument, we might consider the cash crop more of an investment in the traditional sense and therefore have different implications for economic growth. The key is that there may be unequal control over one crop (in this case the cash crop).

through individual preferences and division of the household asset base, we also consider those aspects of primarily selfish objectives mentioned above.

The primary empirical observations we consider are that: (i) males and females often farm separate crops, (ii) female spouses have less access to productive inputs, (iii) females have less say over financial assets and returns from cash crop production and therefore hold safer/buffer-asset portfolios ([21]), and (iv) fewer household resources are applied to female plots. While these observations are common in various contexts of subsistence agriculture, our model largely treats men and women as similar (through similar preferences, technologies, and a potential for a wide range of asset allocations between them).[18] The observations referenced above result from our model when the institutional norm is (realistically) biased against the female (i.e. $\pi < .5$).

In addition to not controlling inputs specific to cash crop production, women may also control much less of their returns. Beyond the inability to draw down certain assets for consumption smoothing in the case of shocks (e.g. [21]), women may also have less rights to reinvest their incomes in productivity-enhancing technologies or labor-saving equipment [1]. This puts additional pressure on the utilization of agricultural labor, the control of which, [1] argues is the focus of the most striking areas of gender asymmetry and conflict.

Conflict over household (and hired) labor has its parallel in access to land. In addition to striking differences in the size of farms, gender also has a hugely significant effect on land security. Specifically, incomplete property rights make fallowing sections of one's farm, a primary means to improving land productivity, dependent on one's perception of tenure rights. This has clear gender dimensions, as it is often a function of one's position in the local social-political network, which is systematically lower for women [2].

[18]We consider differences in preferences as an extension in section 4.1.

It has been shown that given outcomes at the household level, the allocation of productive resources between genders does not reflect choices of a unitary decision maker (or benevolent dictator) [4, 10]. Instead, production decisions must be a result of some bargained decision process. Furthermore, as resulting outcomes offer potential Pareto improvements at the household level, this bargaining process reflects some noncooperative element [11].

While in the context we consider here there are certainly aspects of cooperation, we also include relevant aspects of conflict, in the form of decentralized investments and competition over inputs. These reflect the realities of the choice problem and capture the central differences in resource access by gender, but also allow the household to operate inside of its production frontier, reflecting the theoretical motivations cited above. To do so, we follow [24] and place intrahousehold bargaining at the center of the decision framework to accommodate the aspects of conflict that are central to this setting and fundamentally alter the dynamic savings problem.

2.3. The Model

We develop a model of the separate spheres agricultural household by first defining the core aspects of gender and the dynamic links between agents. We then consider the effects of agent expectations over their spouse's behavior on optimal investments. Finally, we solve a Cournot-Nash equilibrium in which each agent is best responding to their spouse given the state of technology and the institutional sharing rule.

2.3.1. The Dynamic Savings Problem.

To include elements of both cooperation and conflict in a way consistent with the agricultural problem, we model the savings process in two stages, separated by the time between investments in crops (planting) and bargaining over transfers (harvest):

(1) Investment: Agents choose individual investments at time t to maximize their own objectives taking the behavior of their spouse as given.

(2) Bargaining: After returns are realized in period $t+1$, the household chooses wealth transfers between spouses to maximize the gains from cooperation.

Although agricultural production obviously uses multiple inputs (e.g. land, labor, fertilizer) as well as ranges within each (e.g. hired vs. household labor), we aggregate productive resources of the household to a general asset, W. While simplifying our analysis, this does abstract from gender differences in control over specific resources. For example, male plots often use higher quality land which have more secure property rights, and higher rates of hired labor, while females utilize land of lower quality and more household labor. However, as inputs in this context are generally complementary, we continue with a single asset.

From the definition of the institutional rule above, household wealth is divided between female and male agents according to:

$$\begin{aligned} W_{ft} &= \pi W_t \\ W_{mt} &= (1-\pi) W_t \end{aligned} \tag{2.1}$$

Individuals allocate their portion of household wealth between consumption, c, and a mix of investments in a food crop, Z, and cash crop, x to maximize the sum of current period utility from consumption and subsistence production, and discounted utility from cash crop revenues. Utility from subsistence is weighted by $\gamma > 0$ to consider the effects of subsistence requirements (e.g. demands of dependents, etc.) on relative preferences. Future utility from cash crop production is weighted by a common discount factor $\beta \in (0, 1]$. Where the price of the cash crop is p and prices of the consumption good and food crop are both normalized to one, female and male objectives are:

$$f: \max_{c_{ft}\in(0,W_{ft})} u(c_{ft}) + \gamma_f u(Z_{ft+1}) + \beta u(W_{ft+1}) \qquad m: \max_{c_{mt}\in(0,W_{mt})} u(c_{mt}) + \gamma_m u(Z_{mt+1}) + \beta u(W_{mt+1})$$

$$\text{s.t.} \qquad\qquad\qquad\qquad \text{s.t.}$$

$$W_{ft} = \pi W_t \geq c_{ft} + px_{ft} + z_{ft} \qquad W_{mt} = (1-\pi)W_t \geq c_{mt} + px_{mt} + z_{mt}$$

$$Z_{ft+1} = f(z_{ft}, z_{mt}) \qquad Z_{mt+1} = f(z_{ft}, z_{mt})$$

$$W_{ft+1} = f(x_{ft}) + \Theta_{t+1} \qquad W_{mt+1} = f(x_{mt}) - \Theta_{t+1} \tag{2.2}$$

To consider the possibility that food crops may be a mix of both private and public goods, we add a parameter $\alpha \in [0,1]$ which represents the degree of public access to one's investment in the food crop.[19] Where z_f and z_m denote female and male contributions to food crop production, utilities derived from subsistence are given by:

$$u_f(Z) = u\big[(1-\alpha)z_f + \alpha(z_f + z_m)\big] \text{ and}$$

$$u_m(Z) = u\big[(1-\alpha)z_m + \alpha(z_f + z_m)\big]. \tag{2.3}$$

[19]Contribution to a public good is the mechanism that generates Pareto-inferior outcomes in [25]. While there is a similar public good problem here (when $\alpha > 0$), we focus on inferior outcomes in relation to investment in the cash crop/savings instrument.

These are gender neutral in the sense that a non-zero value of α allows an individual to receive utility from their spouse's contribution while opening their own contribution to household access to the same extent. While we continue to reference "crops," the public good can also be thought of more generally as production of a household public good (e.g. caring for children, maintaining the household, etc.) which necessarily requires productive resources and similarly benefits the household overall. In the case where $\alpha = 0$ food production is fully private, while in the case where $\alpha = 1$, the food crop is a pure public good, and when $\alpha \in (0, 1)$, it is a mix.[20]

Agents maximize their own utility but are also linked through: (i) subsistence production, which provides utility to each agent when $\alpha > 0$, (ii) transfers of cash crop revenues post-harvest, given above by Θ_{t+1}, and (iii) the sharing of household wealth.[21]

Next period transfers, Θ, consist of cash-crop revenues and are defined to represent a net male-to-female transfer, so that a positive (negative) transfer adds to (subtracts from) female wealth and subtracts from (adds to) male wealth. This allows for transfers in both directions, although the common occurrence is regular transfers from the male to the female as we describe below.

With logarithmic preferences, a linear cash crop production technology $f(x) = Rx$, and expectations over transfers to or from one's spouse, $(\hat{\Theta})$, and substituting agent specific budget constraints given in (2.2), individual objectives are:

[20] Utilities in (2.3) collapse to: $u_f = u(z_f + \alpha z_m)$ and $u_m = u(z_m + \alpha z_f)$ respectively.

[21] While we are primarily focused on the investment decision, and therefore current period strategies, agents are also linked over time through their vested interest in household growth. A general effect on wealth accumulation follows from additional returns from increased investments. Additionally though, gendered strategies often have individual specific effects regarding future investments. For example, while women generate own-account economic activity, they have relatively little freedom to reinvest their returns as desired [1]. While we don't consider the continued effect of gender over time in this way, expectations of future returns certainly influence savings choices in our model and highlight a similar dynamic.

$$\max_{x_f, z_f} V_f = ln(\pi W - px_f - z_f) + \gamma ln(z_f + \alpha z_m) + \beta ln(Rx_f + \hat{\Theta})$$

(2.4)

$$\max_{x_m, z_m} V_m = ln\Big((1-\pi)W - px_m - z_m\Big) + \gamma ln(z_m + \alpha z_f) + \beta ln(Rx_m - \hat{\Theta}).$$

Equations (2.4) show the individual nature of objectives to maximize the sum of current and discounted future utility, but also the links described above. Before exploring the determination of post-harvest transfers, we consider the effect of *expected* transfers ($\hat{\Theta}$) on male and female investments and begin to uncover the behavioral responses between agent strategies.

2.3.2. Effects of Expected Transfers on Investments.

Maximization of objectives (2.4) by each agent gives four first order conditions, which together define optimal pairs of investments as functions of $\hat{\Theta}$. These investments are functions of time and subsistence preferences, technology parameters, own prices, household wealth, the institutional sharing rule, and expected transfers $\hat{\Theta}$:[22]

$$x_f^*(\hat{\Theta}), x_m^*(\hat{\Theta}) = f(\alpha, \beta, \gamma, p, R, W, \pi)$$

(2.5)

$$z_f^*(\hat{\Theta}), z_m^*(\hat{\Theta}) = f(\alpha, \beta, \gamma, R, W, \pi)$$

As individuals expect transfers in their favor, they invest less in the cash crop and more in the subsistence crop:[23]

[22]Full (closed form) solutions are given in Appendix C.

[23]In most cases we describe effects on female investments, as agents are symmetric except for the opposite effects of π and Θ on males relative to females.

$$\frac{\partial x_f^*}{\partial \hat{\Theta}} = \frac{1-\alpha+\gamma}{R((\alpha-1)(1+\beta)-\gamma)} < 0$$

$$\frac{\partial z_f^*}{\partial \hat{\Theta}} = \frac{\gamma p}{R(1+\beta-\alpha(1+\beta)+\gamma)} > 0.$$

The first condition is an anticipated response to expecting more wealth next period. In this case, as the female expects more in transfers from her spouse, she can increase her own sum of two period utilities by consuming more and increasing her contribution to subsistence production. This is the traditional effect of "chop-money" which is regularly transferred from males to females for providing domestic duties and purchasing household items. Here, it manifests as more investment in the food crop.

The effects of expected transfers on investments depend on the extent at which the food crop is a public good. Specifically, as the food crop becomes more public, female responses to expected transfers get stronger:

$$\frac{\partial^2 x_f^*}{\partial \hat{\Theta} \partial \alpha} = -\frac{\beta\gamma}{R(1+\beta-\alpha(1+\beta)+\gamma)^2} < 0$$

$$\frac{\partial^2 z_f^*}{\partial \hat{\Theta} \partial \alpha} = \frac{\gamma p(1+\beta)}{(R(1+\beta-\alpha(1+\beta)+\gamma)^2} > 0. \tag{2.6}$$

As the subsistence crop is more public in nature, expected transfers from the male have a larger effect on female increases in food/household production. As throughout, this is a rational behavioral response by females in that she is adjusting investments to maximize her own utility over time. More importantly though, it describes a primary means by which men can simultaneously maintain female contributions to the household (in this case through food production) and may selfishly justify a male's continued control over cash crop production and revenue by way of specialization.

Although at this point we have simplified the analysis by not considering the effect of investments on transfers, the role of expectations is relevant on its own. Each agent is making investment choices given expectations over future outcomes. From the timing of the problem, investments (and therefore consumption patterns, contributions to the public good, and prospects for growth given by total household investments) are made now, while transfers are made in the future. While we don't include risk in this model, we can surmise the possibility that actual (future) transfers may not necessarily match (current) expected transfers. Examples of incomplete risk pooling and nutritional smoothing [26] might result from the case where they don't match.

The effect of expected transfers on investments also depends on preferences towards the subsistence good:

$$\frac{\partial^2 x_f^*}{\partial\hat{\Theta}\partial\gamma} = \frac{\beta(\alpha-1)}{R(1+\beta-\alpha(1+\beta)+\gamma)^2} < 0 \;\forall\; \alpha < 1$$

$$\frac{\partial^2 z_f^*}{\partial\hat{\Theta}\partial\gamma} = \frac{(p(1-\alpha)(1+\beta)}{R(1+\beta-\alpha(1+\beta)+\gamma)^2} < 0 \;\forall\; \alpha < 1. \tag{2.7}$$

As preferences towards the food crop become relatively larger (through greater subsistence requirements or a larger dependency ratio), the effect of transfers on private investments increases, while that on subsistence production decreases. Again, expected transfers increase the female incentive to reduce her investment in the private good as she expects more in transfers and puts more weight on subsistence production.

In addition to (potentially joint) production of the household public good, post-harvest transfers link agent strategies and clearly affect investment choices. Transfers specifically, are the link between future cooperation and current investment, and will ultimately be the mechanism driving gendered strategies in the model. Next, we determine the choice of transfers as a bargained process.

2.3.3. Bargained Wealth Transfers.

Although we have made a case for a significant sense of individuality in this context, there are also fundamental sources of shared interests. In addition to empirical findings in favor of a collective approach to resource allocations in this context, marriage also involves a degree of cooperation that may be absent in other bargains [27].[24]

Beyond production of the public good, we accommodate intrahousehold cooperation through bargaining over returns from cash crop production. After harvests are realized, the household chooses the level of net male-to-female wealth transfers, $\Theta \in (-W_{ft+1}, W_{mt+1})$ to maximize the product of individual gains from cooperation. Generally, the household problem in the bargaining stage is:

$$\max_{\Theta \in (-W_{ft+1}, W_{mt+1})} N = [V_f(W_{ft+1} + \Theta) - V_f^e][V_m(W_{mt+1} - \Theta) - V_m^e] \tag{2.8}$$

where $V_f(W_{ft+1} + \Theta)$ and $V_m(W_{mt+1} - \Theta)$ are female and male indirect utilities from next period wealth net of transfers, assuming cooperation ensues, and V_f^e and V_m^e are the amounts

[24] Examples specific to household agriculture include [28] and [4]. [11] give a thorough review of theoretical developments and empirical evidence.

of indirect utility obtained from "outside options," or threat points.[25] The choice of threat point is an important element in the bargaining mechanism as it not only determines the source of power an agent has but also says something specific about an individual's exit opportunities. For example, [27] and [29] both use indirect utility from divorce as an individual's threat point. This implicitly assumes that individuals could sell their labor at a similar wage upon divorce (their models are static ones of income and consumption). An analogous assumption in our model would be problematic for two reasons. First, the assumption of well-functioning asset markets may be even more unrealistic in the development context of subsistence agriculture. Second, as [25] argues, female perceptions of exit opportunities may be misjudged due to female seclusion (perhaps by male intention) or biased reference groups, and carry additional social stigmas if chosen. Such a stigma is interacts with the gender dimensions of land tenure, where women's rights are not as strong as men's and additionally experience additional tenure insecurity upon being widowed or divorce [1].

Katz and others have argued that utility received in the event of non-cooperation within marriage is a more justifiable threat point. We follow this strategy in order to avoid the use of potentially misestimated divorce utility as a bargaining chip in regular interactions between spouses. Instead, we assume individual threat points are equal to the value of one's investment in the cash crop.[26] be to include In this way, the "outside option" is the wealth actually generated on an individual's farm in a given period as opposed to what they might earn upon divorce. Cooperative outcomes, $V_f(W_{ft+1}(\pi) + \Theta)$ and $V_m(W_{mt+1}(\pi) - \Theta)$, are

[25]Cooperation in the Nash model here will always ensue as the threat points will never be greater than the gain from bargaining.

[26]While the institutional norm as we have specified affects agent investments and therefore relative threat points, a straightforward extension to consider additional institutional shifters of outside options, as in [29], would be to apply an additional weight reducing the amount of one's realized return in the case of noncooperation. This could account for the referenced (additional) sanction on divorced women for example from losing future rights to land if exiting the household.

an individual's utility from next period wealth if the bargain succeeds. From the cash crop technology defined above, utilities in the event of noncooperation are $V_f^e = V(Rx_f)$ and $V_m^e = V(Rx_m)$. The Nash product to be maximized is then:

(2.9)

$$\max_{\Theta \in (-W_f', W_m')} N = \Big[V_f\big(\pi R[x_f + x_m] + \Theta\big) - V_f\big(Rx_f\big)\Big]\Big[V_m\big((1-\pi)R[x_f + x_m] - \Theta\big) - V_m\big(Rx_m\big)\Big],$$

where the chosen transfer lies in the range of female returns ($-W_{ft} = -Rx_f$) and male returns ($W_{mt} = Rx_m$).[27]

The chosen transfer is most importantly a function of individual threat points and wealth levels, both of which are a function of the given institutional sharing rule which as we will show directly affects individual investments and therefore alters the elements of an individual's gain from cooperation. As the institutional rule affects investment decisions, relative say takes on the more pragmatic meaning of a gendered constraint as in [30], where it represents both a shared identity and a cultural norm that shapes the power of different groups. [7] also discusses this dual nature of gendered constraints in agriculture, as women in Africa are constrained by crop type (women are often limited to farming subsistence crops) but also consider farming for food to be part of their group identity. In the context of strategic bargaining, it follows that the resulting (lower) revenue females realize from subsistence crops gives them less say over liquid assets and cash-crop decisions.

[27] With this specification of the cooperative bargain, we have implicitly assumed equal bargaining power (as opposed to the generalized Nash bargain: $\max_{\Theta \in (-W_f, W_m)} N = [Gain_f)]^{\psi}[Gain_m]^{1-\psi}$, where $\psi \in [0,1]$ would represent female bargaining power). While our implicit assumption of $\psi = .5$ allows analytical tractability and avoids assumptions about relative skill at the bargaining table, it is likely that the general specification would be more realistic. However, with an additional weighting on the gains from cooperation, the primary effect of gender bias on bargained outcomes would not be changed.

With logarithmic preferences as before, the optimal transfer is a function of individual investments, the institutional sharing rule and the linear technology parameter:

$$\Theta^* = R\big[(1-\pi)x_f - \pi x_m\big]. \tag{2.10}$$

The optimal transfer is increasing in one's investment but decreasing in their control over household resources:

$$\frac{\partial \Theta^*}{\partial \pi} = -R(x_f + x_m) < 0$$

$$\frac{\partial \Theta^*}{\partial x_f} = (1-\pi)R > 0$$

$$\frac{\partial \Theta^*}{\partial x_m} = -\pi R < 0 \tag{2.11}$$

Transfers shift in one's favor with their investment in the cash crop. This is due to the increase in one's bargaining position with a higher threat point. However, this effect diminishes in control over resources, as $\frac{\partial^2 \Theta^*}{\partial x_f \partial \pi} = -R < 0$. This is due to the combined effects of higher resource control resulting in higher transfers to one's partner in cooperative bargaining ($\frac{\partial \Theta^*}{\partial \pi} < 0$) and the diminishing marginal returns to next period wealth relative to that derived from subsistence and consumption.

The effect of the sharing rule on the transfer again mirrors the common observation of chop money flowing from males to females, where as the individual with less control over household wealth receives side payments when the household maximizes the gains from cooperation (i.e. they are compensated for a smaller share of initial wealth, which in the usual case of $\pi << .5$ results in $\Theta > 0$, or a transfer to the female). We can also see a more detailed version of the dynamic described in section 2.3.2, where the female invests less in cash crop production when she expects more in transfers. Here, as she controls less of the household asset base, the optimal transfer will shift in her favor, causing the female to invest less in the cash crop. This incentive is somewhat counteracted by the effect of lower female investment reducing transfers in her direction (by lowering her threat point).

2.3.4. Decentralized Investments.

From the cooperative bargain above, agents can expect post harvest wealth transfers depending on each spouse's investment in the cash crop and the institutional sharing rule. Given individual preferences from section 3.1, female and male objectives are now:

$$\max_{x_f,z_f} V_f = ln(\pi W - px_f - z_f) + \gamma ln(z_f + \alpha z_m) + \beta ln(Rx_f + R\big[(1-\pi)x_f - \pi x_m\big]) \tag{2.12}$$

$$\max_{x_m,z_m} V_m = ln\Big((1-\pi)W - px_m - z_m\Big) + \gamma ln(z_m + \alpha z_f) + \beta ln(Rx_m - R\big[(1-\pi)x_f - \pi x_m\big]).$$

Individual maximization of (2.12) gives four first order conditions, which from concavity of utilities in their arguments give maximum investments.

Taking behavior of one's spouse as given (denoted by hatted variables), individual investments in each of the crop technologies are an optimal allocation of savings on one's own farm and simultaneously best responses to their partner's choices:

$$x_f^*(z_f, \hat{x}_m) = \frac{\beta(\pi W - z_f)}{(1+\beta)p} - \frac{\pi \hat{x}_m}{(1+\beta)(\pi - 2)}$$

$$z_f^*(x_f, \hat{z}_m) = \frac{\gamma \pi W}{1+\gamma} - \frac{\gamma p x_f}{1+\gamma} - \frac{\alpha \hat{z}_m}{1+\gamma}$$

$$x_m^*(z_m, \hat{x}_f) = \frac{\beta(W(1-\pi) - z_m)}{(1+\beta)p} - \frac{(\pi - 1)\hat{x}_f}{(1+\beta)(1+\pi)}$$

$$z_m^*(x_m, \hat{z}_f) = \frac{\gamma(1-\pi)W}{1+\gamma} - \frac{\gamma p x_m}{1+\gamma} - \frac{\alpha \hat{z}_f}{1+\gamma} \tag{2.13}$$

While emerging from a pair of rational choices, these outcomes are reminiscent of agency, we will show that in the cases where $\pi \neq .5$, they more accurately represent socially bounded agency. Before allowing agents to calculate the effect of each others investments on their own payoffs, we can continue to uncover the dynamic link between agents by considering expectations. For a given sharing rule, an agent invests more in the cash crop as their partner does:

$$\frac{\partial x_f^*}{\partial \hat{x}_m} = \frac{\pi}{(1+\beta)(2-\pi)} > 0$$

$$\frac{\partial x_m^*}{\partial \hat{x}_f} = \frac{1-\pi}{1+\beta+\pi+\pi\beta} > 0. \tag{2.14}$$

This is a result of expecting less transfers as one's spouse invests more (which from the partial effect of individual investments on transfers in (2.11) causes transfers to increase with one's investment), or in other words, the effect of losing ground at the bargaining table when one's spouse increases their investment. This response get's stronger in one's control over resources, as both the threat point effect and gains from one's own investment get larger:

$$\frac{\partial^2 x_f^*}{\partial \hat{x}_m \partial \pi} = \frac{2}{(1+\beta)(\pi-2)^2} > 0 \tag{2.15}$$

Holding their spouse's investment fixed, an agent invests more in the cash crop as their control over the resource base increases:

$$\frac{\partial x_f^*(\hat{x}_m)}{\partial \pi} = \frac{\frac{2\hat{x}_m}{(\pi-2)^2} + \frac{\beta W}{p}}{1+\beta} > 0$$

$$\frac{\partial x_m^*(\hat{x}_f)}{\partial \pi} = -\frac{2\hat{x}_f p + \beta(1+\pi)^2 W}{(1+\beta)p(1+\pi)^2} < 0. \tag{2.16}$$

While this is expected as an increase in wealth allows more savings, we will see below that the strategic response in one's spouse's investment causes own investment to increase in control over household resources at a diminishing rate. Individuals respond similarly to their spouse's investment in subsistence production, by investing less as their spouse invests more:

$$\frac{\partial z_f}{\partial \hat{z}_m} = \frac{\partial z_m}{\partial \hat{z}_f} = -\frac{\alpha}{1+\gamma} < 0. \tag{2.17}$$

Both decrease subsistence contributions as they expect their spouse to contribute more, which is the expected outcome of a public good game. This effect increases as the food crop becomes more public (i.e. as $\alpha \to 1$) and decreases as preferences towards subsistence increase.

Assuming agents can, in addition to performing backward induction from the cooperative game, exact the effects of their behavior on their spouse's (and vice versa), best responses in (13) give a decentralized Nash equilibrium. Pairs of investments for each agent are mutual best responses given preferences, household wealth, and the institutional sharing rule: x_f^{NE}, z_f^{NE}, x_m^{NE}, $z_m^{NE} = f(\alpha, \beta, \gamma, W, \pi)$

Definition 1. *The pair of each agent's best response strategies* $\left\{x_f^*(x_m^*), z_f^*(z_m^*)\right\}$ *and* $\left\{x_m^*(x_f^*), z_m^*(z_f^*)\right\}$ *are a sub-game perfect Nash equilibrium* $\left(x_f^*(x_m(x_f)\right)$ *and* $\left(z_f^*(z_m(z_f))\right)$.

Proposition 1. *If the food crop is private to any extent* ($\alpha < 1$)*, total household investment in the cash crop is maximized in the case of equality:* $\frac{\partial x_{hh}^{NE}}{\partial \pi} = 0 \iff \pi = .5$.

In the case of equal control over the household asset base, each agent invests the same in cash crop production and there are no transfers made post harvest. This effectively mirrors the unitary model, where each agent has similar levels of wealth and given homogeneous preferences and productivities, invests equal amounts in each crop. In this optimistic case, there would be no need for transfers.

As the institutional rule diverges from equality ($\pi \neq .5$), the expectation of transfers lowers total household investment in sum. This is a result of the optimizing adjustments made by each agent as described above. Although each agent is still making an optimal allocation of their wealth (looking forward to their spouse doing the same and the resulting transfers) total household investment is lower than in the case of equality. Furthermore, this is a result of individual investments increasing in one's control but at a diminishing rate. Specifically, as one continues to transfer more to their partner, they allocate towards private consumption to maximize their own two-period utility.

From the objectives (2.12) it is clear that indirect utility is increasing in one's control over household wealth, so an autonomous change in the sharing rule in favor of one's partner runs counter to the selfish objective of maximizing own utility.[28] As total production would be increased with shifts to equality, both could be made better off *if* side payments were made to compensate the individual giving up wealth initially. While our model shows the *potential* gains from reallocating wealth, the nonbinding nature of the investment stage means that the aforementioned compensation is not guaranteed. For this reason, agents continue to do what's in their best interest given the institutional norm. Unequal access to the household asset base, in conjunction with private incentives, means the household outcome generates potential Pareto improvements.

[28]In relation to [31] this persistence would remain as there would be no reason to for mutual innovation to a more efficient norm.

Proposition 2. *Subsidies to cash crop production increase individual and equilibrium household investment in the cash crop. When the food crop is private to any extent, the effect of a subsidy is strongest in the case of equality.*

A reduction in the input price of the cash crop technology increases total investment, as expected. Additionally, this effect is strongest in the case of equality. This is a result of each individual investing more with price decreases but less so as their control over the asset base grows. This follows from the effect of control on transfers, as in (2.11), where transfers shift in one's spouse's favor as control increases. As the cash crop input price falls, an individual invests more while gains from marginal investments fall as control increases, and so the subsidy effect on individual investments diminishes.

While this result highlights an important consideration for policies (e.g. in estimating the gains from subsidies relative to other pursuits), it also reflects the argument in [11], that an analytical approach that does not take into account the multiplicity of decision makers in the household cannot be entirely satisfactory. While we have clearly followed this notion, we highlight it here, as subsidy performance clearly depends on the state of relative say within the household.

Beyond the effect of a subsidy on total household investment, that on individual investments matters even more, especially in the context of expectations developed in sections 2.3.2 and 2.3.4. As females have little control over specific resources and the returns from their use, they may respond opposite that of policy intentions. If input subsidies (in this case) were intended to target certain expenditures within the household, then the effectiveness of such a payment would clearly depend on which individual in the household received it. While in the unitary framework this wouldn't matter, in this case the response of the non-recipient matters [32]. Additionally, as budget shares of particular goods are significantly related to shares of income accruing to women, strategic responses to such policies matter even more [10].

2.4. Generalizations

Although we have alluded to extensions above, we also suggest additional generalizations and limitations to the model presented so far.

2.4.1. Heterogeneous Preferences.

Gendered strategies in the our model diverge as a consequence of unequal access to resources within the household and the resulting behavioral responses of each agent. However, we might also consider the possibility that individuals have different preferences for subsistence production.

Acknowledging the point in [7] that women are often constrained by crop type but also consider subsistence farming part of their identity, it may be hard to separate the result from the constraint. Are females farming subsistence crops out of choice, or because they are constrained in a way that it becomes their optimal strategy? In either case, the outcomes of shared identity and the result of constrained action may be similar in appearance. So far, we haven't supposed this is a result of divergent preferences, but instead that of socially bounded agency.

To some extent, heterogeneous preferences may simply reflect other gender norms embedded in the context of subsistence agriculture. For example, social sanctions to deviating from expected behavior (e.g. failure to contribute to household production while receiving chop-money) may cause women to place their own bias on relative crop choices. While in this example, the behavior is a result of a different social phenomenon, resulting investments may reflect the same behavior as if preferences were truly different. Again, an individual's revealed preference may not be the result of pure agency.

In a model with $\gamma_f \neq \gamma_m$, some interesting results emerge.[29] From above, when preferences for the food crop are similar and it is at least somewhat private, household investment in the cash crop is maximized in the case of equality. As subsistence preferences diverge however, maximum cash crop production depends on the combination of γ_f/γ_m and α. Specifically, if the female has stronger preferences towards household production, maximum household production is achieved in the case where she controls more wealth if the food crop is more than half public ($\pi > .5$ and $\alpha > .5$).

This is a result of more utility being gained from the public good allowing greater investment in the cash crop, which is increasing in control over household wealth, in this case in the hands of the female. The opposite is true if the food crop is significantly more private ($\alpha < .5$). As far as social norms put additional pressure on women to fulfill household responsibilities, this result may add additional benefits to increasing female control over household resources.

2.4.2. Bargaining Power and Related Considerations.

While we don't model power explicit (e.g by generalizing the cooperative Nash bargain to include gender specific weights on gains from cooperation), the dynamic responses to expectations of transfers and behavior in sections 2.3.2 and 2.3.4 reflect the notion of power in [33], as an ability of an individual to deliberately generate economic results (potentially, but not necessarily) against the will of others. While the institutional norm is exogenous in our model, it is clear that one individual can benefit from it's persistence as well as their spouse's actions resulting from it.

Still, we might expect a more direct form of power to emerge with specific outcomes. For example, women in poorer households may bear the brunt of productivity shocks in

[29]We don't present derivations for these findings but they can be drawn from the existing framework.

terms of nutritional smoothing [9]. Specifically, relative nutritional allocations after negative productivity shocks depend on relative returns of individuals. In the framework of our model, allocation of the public good might also be dependent on relative production (which we have allowed only in relation to cash crops).[30] This dynamic would only increase the motivation of females to allocate resources away from cash crop production under additional subsistence risk.

The finding in [21] that men and women draw down assets differently after negative shocks might have an additional implication here. While our model already suggests that the ability to leverage certain revenues in the future affects current investments, it might also be the case that outcome-contingent bargaining power drives additional strategic responses. As far as this further increases a shift of female controlled resources to subsistence production, there may be an additional relationship between exit opportunities/outside options and asset choice, here magnified by the threat of potential shocks.

2.4.3. Risk.

We have considered how strategies and outcomes in our model might be affected by productivity shocks, but have not allowed risk to enter the decision problems of agents here.[31] Uncertain returns (e.g. in cash crop production) would certainly be expected to alter the investment strategies of individuals. Additionally, the likelihood of negative shocks might very well have gendered consequences, especially given the additional bargaining considerations mentioned previously.

[30]The authors also find that variables related to outside options at divorce matter in terms of nutritional smoothing. In our model, this suggests that threat points, which we have treated in the cooperative bargain over cash, might also be related to food allocations in some instances.

[31]Although we do in a related paper.

Risk pooling within households has been routinely rejected in the context of subsistence farming [8, 9]. As this interacts with behavioral responses in investment strategies, risk may further exacerbate the divergence caused by institutions as described here.

For example, [34] find that social norms determine whether productivity shocks increase private or public expenditures. Specifically, intrahousehold budget shares depend on which (gendered) crop realizes a positive rainfall shock. In our case, men and women choose crops as a function of what they expect to realize in terms of their spouse's contribution to the public good and any resulting cash transfers. But, we have also considered the potential for women and men to have different preferences for public good expenditures and differences in nutritional smoothing capabilities dependent on additional power dynamics regarding relative returns. As these differences manifest in relative investment decisions, risk will magnify the divergence in strategies by gender.

2.4.4. Endogenous Norms.

Women and men play different roles in agricultural production and as a result occupy different socioeconomic positions [35]. Our treatment of gender bias as an exogenous sharing rule captures the effect of such differences on strategies and outcomes but not the suggested feedback of outcomes on the institutional rule itself. As relative returns determine one's future position in the social-political hierarchy, there may be an additional effect of rational strategies which serves to further embed the existing norm.[32]

This supposed vicious cycle is especially clear in the case of land tenure. In many instances, investment in soil fertility is a primary means to improving agricultural productivity. In an environment where fertilizer is expensive (and risky), land is relatively abundant, and

[32] The effect of relative returns would likely influence future institutional rules more directly as well, as in [36].

crop returns are low, letting one's farm lay fallow is the primary mechanism for increasing yields [2]. Women often obtain rights to use land (for household or private farming) through men [1]. At the same time, incomplete property rights mean that fallowing is risky due to potential appropriation. One's position in the local political hierarchy plays a primary role in determining this risk, and therefore whether the investment in fertility is made. As females hold significantly lower status in this hierarchy, they are less likely to fallow their land.

Additionally, [7] highlights the additional pressure on land fertility resulting from low fertilizer use due to imperfect credit markets and subsidy reductions after structural adjustment programs. As access to credit (and fertilizer itself as a cash purchase) also takes on gendered dimensions, the simultaneous constraints of credit access and property rights on females, combined with the effects of outcomes on future socioeconomic status, put persistent downward pressure on female agency and outcomes over time. The effect of social institutions on land use is additionally worrisome in [37], where the gendered allocation of land is seemingly resistant to legislation. Instead, social, administrative, and ideological factors override new legal rights.

From the brief discussion of the gendered components of land tenure alone, combined with the clear importance of soil fertility, defining the institutional requirements for endogenous change towards equality is a very important next step. Determining the conditions under which policies can incentivize agents to drive such a shift will be even more important. Exploring the conditions, both theoretically and empirically, that support men and women *jointly* deviating towards more egalitarian norms is a hugely important goal, especially regarding an asset as fundamental to both agricultural development and social equality as land.

2.5. Discussion

We have modeled the agricultural household as a site of both competition and cooperation. In doing so, we have considered the primary implications that a gendered control over household resources has for both individual strategies and household outcomes.

Through the two stage game developed here, we simultaneously reflect realities of the agricultural savings problem and highlight dynamic links between agents expectations of future (shared) outcomes and current (individual) strategies. Furthermore, in considering the isolated nature of individual production, we have allowed the household outcome to reflect Pareto inferior outcomes as shown to result from gendered access to resources in related empirical work. While this has long been accepted as a reality in feminist and bargaining literatures, it has not been treated in the context of agricultural development in this way.

Although we have assumed a very benign form of power, we have also suggested extensions from related studies and models, which might further support an understanding of the significant effects of social norms on individual behavior and group livelihoods.

Essay #3: Technology Adoption, Risk, and Intrahousehold Bargaining in Subsistence Agriculture

3.1. Introduction

As a majority of the world's poor continue to farm for food, the adoption of productivity enhancing innovations offers a relevant means of development. Growth in this way is especially important for Sub-Saharan Africa (SSA), where farmers have been slow to uptake new technologies and seen relatedly low levels of productivity growth [38]. A primary explanation for this relative stagnation is that new technologies are simply too risky to adopt [39–41]. Demand for fertilizer, for example, is limited due to high variability in crop output and incomplete information regarding the availability and cost of fertilizer as well as a general lack of knowledge on how to use it efficiently [38]. As binding subsistence constraints are both non-trivial and possible, the downside of uncertainty deters innovation and results in safer (but lower return) asset portfolios [42].

Growth and development literatures have offered various applied and theoretical explanations that feature the relationship between risk aversion and agricultural stagnation. At the same time, better access to intrahousehold panels has allowed researchers to isolate behavioral strategies resulting from living in a risky environment [43]. In doing so, they have been able to uncouple the related but separate strategies of managing risk ex ante and dealing with it ex post. By making this separation, Dercon argues that researchers have been able to supplement an understanding of how households cope with negative shocks with better insight as to how they manage risky environments through specific investment strategies, and ultimately how management and coping are interrelated. Conclusions from these models have suggested policies to support innovation uptake and improve the ability of households to both strategically manage risk in the face of potential shocks and cope with it after the fact if necessary.

By attempting to reduce risk exposure through improving credit and insurance mechanisms, providing asset and consumption stability with social protection schemes [44], and reducing aid pressure on poor households [40], resulting policy suggestions have largely targeted market imperfections as constraints to growth. These proposals have seen their corollary in applied micro and experimental literatures, where optimal subsidies, delivery mechanisms, and savings instruments attempt to plug inefficiencies (e.g. [45], which simultaneously addresses inter temporal bias and fixed cost (time) burdens). Although these conclusions have much to say about the growth implications of risk in this context, they provide little flexibility regarding institutional or political elements of the decision making process itself. As individuals *within* households pursue different strategies to manage risk and employ unique coping strategies as a result, we consider these elements here.

Insights from feminist and bargaining theories have improved our understanding of intrahousehold dynamics and related effects on household consumption, labor choice, and economic development ([11] and [32] provide theoretical and empirical surveys). In doing so, both fields have challenged the assertion that households behave as efficiency maximizing firms (or at best benevolent planners). Instead, social institutions of power, culture, and distributive conflict arise to explain the resulting choices of household members. Regarding household production in agriculture, such political processes have been shown to reduce household efficiency and imply diverging welfare outcomes [4, 10]. While these arguments convincingly portray the household as a site of both cooperation and conflict, related theoretical models have not put forward much in terms of long-run outcomes. In explaining the allocation of income and time, they have served to isolate the effects of intrahousehold processes and their interactions with common policy levers, but in a static setting. Here, we highlight a dynamic one.

In subsistence agriculture, one institution that has routinely come forward as a source of inefficiency is that of intrahousehold inequality [46, 47]. While a gendered division of labor and separate spheres of influence might be optimal under heterogeneous productivities, this optimistic result has been discredited [10]. In maintaining household welfare, those with more equal bargaining power have been more resilient [28].

These observations necessarily have their corollary in risk coping, where men and women drawn down assets differently in dealing with negative shocks [21], experience different portfolio sensitivities to price instability [48], and have different off-farm opportunities in general [8]. Gender specific strategies (and outcomes) on either side of the shock are necessarily linked as the ability to draw down assets and smooth consumption is a direct function of the portfolio chosen prior. At the same time, portfolio choice represents an expectation of individual asset control and coping requirements. This relationship is only magnified by any heterogeneous preferences women and men may have towards certain household expenditures (e.g. child welfare, dependent care), and/or acceptable levels of risk.

The contribution of this paper is to underline the effects of gender inequality on the technology adoption process. The specific setting is one where households farm both subsistence and cash-crops and spouses farm separate plots.[33] The primary empirical observations our

[33]While this context and related conclusions of productive inefficiency have been drawn in [4] and [10] regarding Burkina Faso, our model presented here builds directly on our empirical work exploring intrahousehold inequity and resulting inefficiencies in Ghana, which utilizes data from the Agricultural Innovation and Resource Management Study from Ghana [6]. Similar patterns of atomized household agriculture are common in other developing regions including much of sub-Saharan Africa and south Asia [2]. Although this setting provides a natural experiment for empirical analysis to both motivate our model and test its primary hypotheses, it does produce some bounds in terms of representativeness. Still, the general question of unequal control over household resources described above and in other investment applications (e.g. the effect

model considers are: (i) males and females often farm separate crops; (ii) female spouses have less access to productive inputs; (iii) females have less say over financial assets and returns from cash crop production and therefore invest less in the latter, and that (iv) fewer household resources are applied to female plots.

We develop a deterministic and stochastic model to explore the interaction between an institution of gender bias and household portfolio choice and the effects of both on development outcomes and uptake policies. In doing so, we show that more equal households are better able to invest in risky technologies while maintaining food security. Specifically, as egalitarian households negate strategic responses to intrahousehold transfers, they are able to invest more, maintaining consumption and other forms of savings over time.

Policies aimed at supporting technology uptake and asset accumulation have their strongest impacts on both household investment and consumption growth in the egalitarian case. In contexts where individuals within households enjoy different levels of access to productive resources and relatedly different control over their returns, policies will necessarily be bounded by the extent of inequality. This outcome is further exacerbated by heterogeneous preferences towards risk. While of course gender equality is an end in itself, we also show how it is a means to the complementary end of economic development.

The paper is organized as follows. First, we briefly discuss unequal access to productive inputs as an institutional bias a la [22] and related intrahousehold bargaining models that have treated gender in such a way. Then we develop an analytical framework centering this institution in the technology adoption process. In doing so, we momentarily set aside the realities of diminishing returns, uncertainty, and multiple crop portfolios to highlight the role of gender inequality in the investment decision. While this section uncovers the core

of intrahousehold bargaining power on retirement investment strategies in the U.S. in [49] and [50]) has its parallel here.

interaction between the institution of gender bias and uptake policy, we then extend the framework to a stochastic environment to consider the previously disregarded elements as well as heterogeneous risk preferences and welfare dynamics. Finally, we return to the data collected in [6] to test the core hypotheses of the model and discuss conclusions and next steps.

3.2. Gender as an Institution

The analytical portion of our model necessarily relies on an operationalization of gender norms, representing a social characteristic underlying the problem itself. For this we adopt North's (1990) conceptualization of institutions as "the rules of the game". Since gender is not a biological determinant of individual productivity, and enters the individual's investment problem directly by defining one's control over the household asset base, gender in our model also matches North's more formal definition of an institution: a humanly devised constraint that shapes human interaction.[34,35] Whether the unequal sharing of resources and division of wealth is a rule of thumb surviving as an artifact from earlier forms of production that *did* merit specialization, or a cultural convention unrelated to productivity differences, gender inequality in access to productive resources exists and influences household investment by altering individual strategies.

A definition of power also contributes to our analysis of the effect gender has on individual choices and household outcomes. Even as a socially determined convention, the institution of gender-bias has real effects on the distribution of wealth and the resulting strategies agents

[34]Gender of the plot manager was not found to be a statistically significant determinant of output variation in the data set from Ghana. This conclusion was also drawn in a related context but slightly different manner in [4].

[35]The suggested endogeneity of institutions as humanly devised, which might evolve with such interaction is not treated here but is certainly a relevant (and intended) extension.

adopt. Our treatment thus entails a parallel notion of power. We make this connection as the bargaining portion of our model includes gender-bias in a way that simulates power. While we distinguish the two approaches (gender-specific bargaining power versus institutional bias), our specification resembles the definition of power in [33]: the ability of one person to deliberately generate economic results even (but not necessarily) against the will of others.

The household is then characterized not as a calculating machine (as we might think of a firm) but as a social/political institution where some members routinely give commands while others are constrained by the threat of sanctions ([51]). Individuals have agency in the sense that they maximize their own objectives, however this agency is necessarily bounded by the social institution determining the distribution of the household asset base and related spheres of influence. While as we will show, the outcomes of this agency represent individually optimal best-responses, one spouse may effectively have power in that their adherence to an institutional convention provides them with additional gains at the expense of their partner. What appears to be a passive coherence to the existing convention may be the same result had the agent also chosen their preferred institutional rule.

3.2.1. Bargaining under Inefficient Institutions.

As unequal access to household resources results in productive inefficiency, and given the related observations cited above, we follow [11] by starting with the assumption that an analytical approach that does not consider the multiplicity of decision makers in the household cannot be entirely satisfactory . The referenced contradiction between self-interested firms and supposedly altruistic families is one that leads [24] towards intrahousehold bargaining as a way to accommodate conflict within the household.

In the context of household agricultural production, resource allocations resulting in production within the household frontier implies that a reallocation could make one person better off without decreasing the utility of the other. The presence of allocative efficiency across individuals within a household is therefore a necessary (but not sufficient) condition for Pareto efficient production decisions. We find significant inefficiency of this form in the Ghana sample as do [4] in a similar context in Burkina Faso. Therefore, like the models of [52] and [53], we include a non-cooperative element to allow for Pareto-inferior outcomes.

Agents are firstly linked through the distribution of the household asset base, affecting individual investments and household outcomes. While individual goals are essentially at odds in that they maximize their own objectives and compete over household resources, it is also reasonable to expect that within marriage, cooperation develops over time, ([27]). In Ghana, as in similar contexts, this involves both the application of one's labor time and other inputs to each others plots as well as regular intrahousehold transfers or "chop-money". This provides the second link between individual strategies and their spouse's outcomes.[36] While this has contextual relevance as intrahousehold risk sharing and transfers made for maintenance of the household, it also results in strategic responses to spousal behavior that policies must consider.

To simultaneously accommodate individual objectives based on competition over resources and intrahousehold cooperation, our decision framework proceeds in two stages:

(1) Investment: Agents choose individual investments at time t to maximize their own objectives taking the behavior of their spouse (and bargained transfers of returns) as given.

(2) Bargaining: After returns are realized in period $t+1$, the household chooses wealth transfers between spouses to maximize the gains from cooperation in a Nash-bargain,

[36] In [52] individuals are linked through joint production of a household good, while in [53] they are linked through both a household good and "caring," via somewhat altruistic preferences.

where individual farm revenues (assumed to be controllable gains in the event of non-cooperation) act as outside options.

This process is based on the separate spheres literature of [54] and [52], and gives the first primary characteristics of our model. First, since (even complete) information regarding spousal behavior gives best-response functions by each agent in the investment stage, optimal choices are a result of strictly competitive non-cooperative play. The allocation of resources amongst uses thus represents a decentralized equilibrium as opposed to an explicit bargaining mechanism.[37]

The existence of an intrahousehold bargaining process is clearly a fundamental departure from the unitary decision framework utilized by the existing growth models described above. As will be shown below, the non-cooperative element defining the investment stage can generate Pareto-inferior outcomes, placing our model beyond the realm of explicitly cooperative ones as well. Additionally, non-cooperative play allows strategic behavior to result in this inferiority endogenously.

3.3. A One Technology, Deterministic Model

Before simultaneously addressing the effects of gender and risk, we start with an analytical treatment to highlight the mechanism of intrahousehold inequality and its role in the investment process. Here, we show that household investment is maximized and uptake policies most effective in the case of equality.

3.3.1. Intrahousehold Cooperation.

While individuals in Ghanaian farming households make portfolio decisions largely independent from one another [46], we have also made a case for intrahousehold cooperation.

[37] The bargaining over transfers in the second stage implies an enforceable contract but the investment stage does not.

We accommodate such cooperation in the bargaining stage, where agents bargain over wealth/resource transfers once harvests are realized. Since we let wealth, W_t represent a composite asset base, these transfers are a similar amalgamation. They could contain both in-kind or cash transfers/chop-money, or represent the sharing of individual assets such as labor time or other inputs.[38] Effectively transfers are side payments made between individuals to maximize the gains from cooperation and maintain the partnership over time. In this way, transfers may amount to [19]'s idea that inequalities often survive by making allies out of the deprived. Here, post-harvest transfers can effectually maintain the marriage without fully giving up one's position of power over household wealth.

The household problem in period $t+1$ is to choose a net male-to-female transfer, Θ, from the range of individually controlled assets, $(-W_{ft+1}, W_{mt+1})$ to maximize the Nash Product:

$$\max_{\Theta \in (-W_{ft+1}, W_{mt+1})} N = [V_f(W_{ft+1} + \Theta) - V_f^e][V_m(W_{mt+1} - \Theta) - V_m^e], \tag{3.1}$$

where $V_f(W_{ft+1} + \Theta)$ and $V_m(W_{mt+1} - \Theta)$ represent female and male indirect utilities from next period wealth (net of transfers), and V_f^e and V_m^e are the amounts of indirect utility obtained from outside options.[39,40]

The choice of outside option or threat point is clearly an important element in the bargaining mechanism as it not only represents the source of power an agent has but also says

[38]Traditionally men control financial assets so that chop-money is regularly transferred to women for maintenance of the household. A more specific relationship will be developed below and confirmed in the data.

[39]This is effectively a social planner's problem where the solution to the bargain will be Pareto-Optimal. We make this assumption for two important reasons. Firstly, it considers the joint objective of household members to cooperate over time. Second, it forces any inefficiency in production choices to come from the investment stage, which allows us to focus on the role of unequal access to inputs in the technology *adoption* problem.

[40]This accommodates transfers in either direction, depending on the form of household inequality. A positive transfer $\Theta > 0$ is then a (net) payment from the male to the female, while a negative transfer, $\Theta < 0$ is a payment from the female to the male.

something specific about an individual's exit opportunities. For example, [27] and [29] both consider the indirect utility from divorce to be an individual's threat point. This implicitly assumes that individuals could sell their labor at a similar wage upon divorce. An analogous assumption in our model would be problematic for two reasons. First, the assumption of well-functioning asset markets may be even more unrealistic in the development context of subsistence agriculture. Second, as [25] argues, female perceptions of exit opportunities may be misjudged due to female seclusion (perhaps by male intention) or biased reference groups, and carry additional social stigmas if chosen.

Katz and others have argued that utility received in the event of non-cooperation within marriage is a more justifiable threat point. We follow this strategy in order to avoid the use of potentially misestimated divorce utility as a bargaining chip in regular interactions between spouses. We therefore assume individual threat points $V_f(f_f(x_f))$ and $V_m(f_m(x_m))$, utilities from female and male production on their farms, which are functions of their respective investment choices x_f and x_m. In this way the outside option cementing one's bargaining position is the wealth actually generated on one's farm in a given period and what one could expect to receive without cooperation. Where π is a socially determined rule governing the share of wealth between genders, cooperative outcomes $V_f(W_{ft+1}(\pi)+\Theta)$ and $V_m(W_{mt+1}(\pi)-\Theta)$ are next period indirect utilities if the bargain succeeds, or that from their individually controlled share of household wealth net of transfers. The Nash product to be maximized is then:

(3.2)

$$\max_{\Theta \in (-W_{ft+1}, W_{mt+1})} N = [V_f(W_{ft+1}(\pi) + \Theta) - V_f(f_f(x_f))][V_m(W_{mt+1}(\pi) - \Theta) - V_m(f_m(x_m))].$$

In this specification of the household problem, we formalize our treatment of gender bias as an institutional parameter influencing an agent's relative gain from the bargain (as opposed to the more common, generalized Nash product, $N = [\text{gain}_f]^{\pi}[\text{gain}_m]^{1-\pi}$ which weights relative gains from bargaining by skill or power). This allows us to focus on the influence of relative say as it changes the elements defining individual gains as opposed to their relative skill at the bargaining table.

Since as we will show, the level of investment in the production technology also depends on the institutional sharing rule, the points of non-cooperation in equation (3.2) are also functions of π. Although the Nash bargain as we have specified does not incorporate voice as a parameter determining the relative weights of cooperative gains as [25] describes, both terms (the gain from success and outside option) are effected by the institutional rule π which is voice in the more vague sense of decision making power over household wealth. As individual investments, and therefore outside options are also functions of the gender institution, our specification is similarly based on the dual notion of "exit".

Since Θ represents net (male-to-female) transfers, it captures both the effects of outside options but also transfers that support survival of the partnership. In the former, the institutional sharing rule acts similarly to bargaining power, as it will effect individual investments and therefore threat points. We believe this is an important result as the institutional sharing rule which enters the individual investment decision directly (and therefore investment revenues), alters one's position in the cooperative bargain without making any additional assumptions about relative bargaining skill. As a direct constraint in the investment decision though, relative say takes on the more pragmatic meaning of a gendered constraint as in [30], where it represents both a shared identity and a cultural norm that shapes the power of different groups. [7] also discusses this dual nature of gendered constraints in agriculture, as women in Africa are constrained by crop type (women are often limited to farming subsistence crops) but also consider farming for food to be part of their group identity. In the framework of our model it follows that the resulting (lower) revenue females realize from subsistence crops gives them less say over liquid assets and cash-crop decisions, which simultaneously deters their investments in cash-crops if they can't be leveraged as a coping mechanism.

We now turn to the investment stage where agents choose their agricultural portfolios given their expectations of the future bargain. As they look forward to the cooperative process just described, we will return to our specification of the Nash bargain in terms of the model specifics. Then, we can fully account for the dynamic links between individuals and develop the inseparability between risk management and coping and how gender affects throughout.

3.3.2. Decentralized Investments.

The general allocation decision for the household is to distribute wealth between consumption and investment. The former provides utility in the current period while returns from the latter provide (discounted) utility next period. We approach our problem this way to account for the inseparability between consumption and production decisions facing agricultural households as in [55]. For the separate spheres household which may not allocate wealth evenly among members, we now specify two agents, $i \in \{f, m\}$, who share total wealth according to the socially determined allocation rule $\pi \in [0, 1]$. This rule determines control over household wealth in each period, W_t between agents f and m linearly so that each control a portion of total wealth, W_{ft} and W_{mt}:

$$
\begin{aligned}
& W_{ft} = \pi W_t \\
(3.3) \qquad & W_{mt} = (1-\pi) W_t \\
& W_t = W_{ft} + W_{mt}
\end{aligned}
$$

Female and male agents allocate their individually controlled portions of household wealth between consumption and investment on their own farms. Individual agents thus face potentially different budget constraints and while maximizing their own objectives may adopt heterogeneous savings decisions. With the social convention π dividing the household asset base W_t, the female and male problems are:

$$
\begin{aligned}
& f: \max_{c_{ft} \in (0, W_{ft})} u(c_{ft}) + \beta u(W_{ft+1}) && m: \max_{c_{mt} \in (0, W_{mt})} u(c_{mt}) + \beta u(W_{mt+1}), \\
& \text{s.t.} && \text{s.t.} \\
& W_{ft} = \pi W_t \geq c_{ft} + p x_{ft} && W_{mt} = (1-\pi) W_t \geq c_{mt} + p x_{mt} \\
(3.4) \qquad & W_{ft+1} = f(x_{ft}) + \Theta_{t+1} && W_{mt+1} = f(x_{mt}) - \Theta_{t+1},
\end{aligned}
$$

where individual budgets are allocated between a consumption good c_{it} with price normalized to 1, and an investment product x_{it} with price p, and next period utility is discounted by a factor $\beta \in (0, 1]$.[41] From the perspective of the investment stage in time t, the evolution of

[41] By characterizing household resources in this way, we are effectively forming a composite of potentially multiple asset types. In this light, we can consider household wealth as an amalgam of capital resources, labor power of household members, or cash used to purchase labor or other inputs. Individually controlled portions of household wealth thus represent the portion of total resources that each may leverage. In the Ghanaian villages we consider, males and females apply labor and inputs to each others plots, the key is that there are gender specific levels of command over these resources overall.

individually controlled wealth consists of returns from one's own farm, $f(x_{it})$, plus or minus any net transfer Θ made in the bargaining stage described in section 3.3.1. The sharing of household wealth in period t and intrahousehold transfers in period $t+1$ describe not only the elements of competition and cooperation, but also the inter-temporal connections between agents and the dynamic components of the bargaining framework.

3.3.3. Individual Best Responses.

From equation (3.2) in section 3.3.1, the post-harvest transfer chosen in cooperation is clearly a function of individual investments, as they determine an agent's threat point, but also the amount of household wealth to be divided upon cooperation. At the same time, the transfer determines a portion of one's utility received next period, so it also affects the level of wealth each agent is willing to allocate away from consumption towards investment on their farm in period t. We assume that agents are able to take this into account and therefore calculate the effect of their (and their spouse's) investment on the bargain, the resulting transfer that will maximize the gains from cooperation, and finally the transfer they can expect to give or receive. This is the link between *future* cooperation and *current* investment.

With logarithmic preferences $u(c) = ln(c)$, a simple linear production technology $f(x) = Rx$ with productivity parameter R, the Nash bargain that will be maximized at time $t+1$ is:

(3.5)

$$\max_{\Theta\in(-W_{ft+1},W_{mt+1})} N = \Big[ln\big(\pi R[x_f+x_m]+\Theta\big)-ln\big(Rx_f\big)\Big]\Big[ln\big((1-\pi)R[x_f+x_m]-\Theta\big)-ln\big(Rx_m\big)\Big],$$

where the range of net male-to-female transfers is bounded by individually controlled wealth levels next period, $W_{ft+1} = \pi R[x_f + x_m]$ and $W_{mt+1} = (1-\pi)R[x_f + x_m]$.[42]

The post-harvest transfer that maximizes the gains from cooperation is a function of the technology parameter, the institutional sharing rule, and individual investments:[43]

(3.6) $$\Theta^* = R\big[(1-\pi)x_f - \pi x_m\big].$$

Proposition 1. *Holding constant individual investments (which also depend on the institutional sharing rule), post-harvest transfers are inversely related to one's control over the household asset base.*[44]

An increase in one's control of household wealth causes the transfer to shift towards the other agent. From equation (3.21) the more (less) wealth the female controls, the less (more) she can expect as transfers from the male. This parallels our treatment of transfers as both chop-money and a side payment to maintain the partnership by maximizing the gains from cooperation. In many cases (as in Ghana), males control most of the resource

[42]We make this assumption in part for analytical tractability and for satisfying the Inada conditions, but primarily because they exhibit decreasing absolute risk aversion. The latter quality allows us to focus on the interaction between gender inequality and investment separate from that of wealth (level) effects. While the latter effect might also be relevant as individuals might be more amenable to risk as wealth increases, it is not our concern here.

[43]Derivations and proofs can be found in Appendix C.

[44]This relationship also holds generally when considering the additional effect of π on investments as $\Theta^*\big(x_f^*(\pi), x_m^*(\pi)\big) > 0 \ \forall \ \pi \in [0, .5]$ and $< 0 \ \forall \ \pi \in [.5, 1]$.

pool ($\approx 80-90\%$), and make regular transfers to the female for consumption purposes. This relationship is both statistically and economically significant for the Ghanaian villages.

If we first consider the individual objectives of agents taking transfers as given, we can see the dynamic link between the bargaining stage and investment strategies.[45] By solving the individual budget constraints for consumption, and using preferences, technologies, and time preferences from above, the two period problems for the male and female are:

$$\max_{x_f} V_f = ln(\pi W - px_f) + \beta ln(Rx_f + \hat{\Theta})$$

(3.7)

$$\max_{x_m} V_m = ln\Big((1-\pi)W - px_m\Big) + \beta ln(Rx_m - \hat{\Theta}).$$

Individual maximization of these objectives show that agents will invest less as they expect more transfers in their direction:[46]

$$x_f^*(\hat{\Theta}) = \frac{\pi\beta W}{p(1+\beta)} - \frac{\hat{\Theta}}{(1+\beta)R}$$

(3.8)

$$x_m^*(\hat{\Theta}) = \frac{(1-\pi)\beta W}{p(1+\beta)} + \frac{\hat{\Theta}}{(1+\beta)R}.$$

[45] Data collected from Ghanaian households shows significant discrepancies in the values men report giving relative to what females report receiving, which is at least interesting, but potentially a source of information inefficiency in the bargaining process as well. We return to this issue when testing the hypothesis of Proposition 1 below.

[46] Recall a negative transfer represents a payment from the female to the male.

$$\frac{\partial x_f^*(\hat{\Theta})}{\partial \hat{\Theta}} = -\frac{1}{(1+\beta)R} < 0$$

(3.9)

$$\frac{\partial x_m^*(\hat{\Theta})}{\partial \hat{\Theta}} = \frac{1}{(1+\beta)R} > 0.$$

This outcome is necessarily a result of the atomistic strategies we have portrayed, which beyond its contextual relevance also provides the dynamic link between agents and is crucial in determining the main result of the model, described in Proposition 2.

As individual investments also alter one's position in and gains from the cooperative bargain, we must consider that investments also change the value of the expected transfer. Assuming that agents can calculate the optimal transfer as a function of individual investments, male and female objectives in the investment stage become functions of each other's strategy. In this way, backward induction from the household bargain to the individual game allows us to combine individual objectives from equations (3.7) with the optimal transfer in equation (3.6), giving the pair of Bellman equations:

$$V_f(x_f, x_m) = \max_{x_f} \Big\{ ln(\pi W - px_f) + \beta ln\Big(Rx_f + R\big[(1-\pi)x_f - \pi x_m\big]\Big)\Big\}$$

(3.10)

$$V_m(x_m, x_f) = \max_{x_m} \Big\{ ln\Big((1-\pi)W - px_m\Big) + \beta ln\Big(Rx_m - R\big[(1-\pi)x_f - \pi x_m\big]\Big)\Big\}.$$

Next period utility is now a function of returns from one's own investment as well as from their spouse's. Taking each other's behavior as given, individuals choose the level of investment on their own farm to maximize two period indirect utility. Optimal strategies are now best responses to their partner's investment:

$$x_f^*(\hat{x}_m) = \frac{\pi\beta W}{p(1+\beta)} + \frac{\pi\hat{x}_m}{(1+\beta)(2-\pi)}$$

(3.11)

$$x_m^*(\hat{x}_f) = \frac{(1-\pi)\beta W}{p(1+\beta)} + \frac{(1-\pi)\hat{x}_f}{(1+\pi)(1+\beta)}.$$

Definition 1. *The pair of individual best response strategies, $x_f^*(x_m^*)$ and $x_m^*(x_f^*)$ are a subgame-perfect Nash equilibrium $\big(x_f^*(x_m(x_f)\big)$.*

As agents take into account the behavior of their spouse and assume their partner will do the same, best responses to each other's strategies give a decentralized Cournot-Nash equilibrium that is subgame-perfect. Solving the conditions in equations (3.11) simultaneously gives equilibrium strategies, x_f^* and x_m^* as functions of household wealth, the investment's price, the discount factor, and the institutional sharing rule:

$$x_f^* = \frac{\pi\beta W(1+\pi)\big(2\pi+\beta(\pi-2)-3\big)}{p\big[\beta(1+\pi)(\pi-2)(2+\beta)-2\big]}$$

(3.12)

$$x_m^* = \frac{\beta W(2-\pi)(\pi-1)\big(1+\beta+\pi(2+\beta)\big)}{p\big[\beta(1+\pi)(\pi-2)(2+\beta)-2\big]}.$$

Individual investments are increasing in one's control over the household asset base but at a diminishing rate. This follows from Proposition 1, where as an individual controls more of total wealth, they transfer more to their partner post-harvest.

Proposition 2. *Total household investment is maximized in the case of equal control over household wealth.*

In the case where agents have equal preferences and similar productivities, (household) investment is largest when $\pi = .5$. In this case, each agent controls half of the household asset base and makes no transfer post-harvest as they invest the same amount in equilibrium. With any inequality on the other hand ($\pi \neq .5$) the expectations of transfers lower total household investment. Assuming household wealth accumulates strictly from production returns, household growth is defined as the periodic rate of wealth accumulation:

$$g = \frac{Rx_{hh}^{NE} - W}{W}, \tag{3.13}$$

Proposition 2 implies that the household's growth rate is also highest in the case of equality. While the simplifications made in preferences and technology limit the scope of comparative statics for the model so far, we can still examine the interaction between gender bias and two common policies in this context: input subsidies and extension programs.

Proposition 3. *Input subsidies and productivity improvements have their largest impact on the household growth rate in the case of equality.*

Household investment and per period growth rate are both highest in the case of equal sharing. As equations (3.31)-(3.34) show, inputs subsidies and extension programs aimed at supporting investment also have their largest impact in the case of $\pi = .5$. This parallels the empirical observation in [28] that household welfare is better protected in households where bargaining power is spread evenly between spouses. While we have abstracted from some realities of our specific context, we can see the important interaction between intra-household resource distribution, and investment uptake policies. In ranking policy options, relative benefit comparisons must consider the effect of inequality on the expected gains from increased investment and obviously the ends which such investments hope to achieve. If the cooperative bargain as we have assumed is in reality less benign (e.g. due to unequal bargaining power, imperfect information regarding investments and transfers, less cooperation in the case of negative shocks, or even bargaining breakdowns), then the recipient of policy benefits and resulting strategic responses will be additionally important.

3.4. Stochastic Portfolio Choice

To include the realities of uncertainty and portfolios of multiple crops, we now develop a stochastic model with two investment technologies, x_i and z_i. Where the investment product x_i is still the technology of interest, we add a random (covariant) productivity shock, $\tilde{R} \sim N(\overline{R}, \sigma_R^2)$, so that the risky technology produces according to $f(x_i) = \tilde{R}x_i^d$ with diminishing returns parameter $d < 1$.[47] While this form of uncertainty might represent productivity shocks, price instability, or imperfect information on effective input use, the key mechanism is an imperfect mapping between input cost and output revenue. The risk-free alternative, z_i may also exhibit diminishing returns to scale, and generates revenue according to $f(z_i) = Gz_i^h$.

As before individuals make allocation choices, now between consumption and a two technology portfolio, in relative isolation from one another. Agents still compete over household resources encompassed by the asset base W_t, and cooperate by bargaining over wealth transfers to maximize the gains from cooperation. Since in the context of subsistence agriculture

[47] We consider only the case where each individual is equally effected as opposed to others (e.g. [42]), who consider both covariant and idiosyncratic shocks. This might be additionally relevant in our case where individual specific shocks put additional pressure on the cooperation of agents to smooth consumption after negative outcomes.

we might consider the risky technology a cash crop, we assume that the bargain entails revenue from the cash crop x_i as opposed to z_i which we now consider a subsistence food crop that can be either consumed or held as personal savings (e.g. seeds for future production).[48] This specification might also represent other pairs of technologies where one is subject to an institution of unequal control. The point is that social conventions allow separate spheres of influence between household members.

Although we also extend the choice model to include a more general form of CRRA preferences, $u_i(c_i) = (c_i)^{1-\alpha_i}/(1-\alpha_i)$, we maintain logarithmic preferences in the cooperative Nash bargain so that heterogeneous risk preferences do not alter the relative valuation of an individual's utility gain from intrahousehold transfers. We allow risk preferences to diverge only in regards to investment portfolios. The household problem in the cooperative bargaining stage is now:[49]

(3.14)

$$\max_{\Theta\in(-W_{ft+1},W_{mt+1})} N = \Big[ln\big(\pi R[x_f^d+x_m^d]+\Theta\big)-ln\big(Rx_f^d\big)\Big]\Big[ln\big((1-\pi)R[x_f^d+x_m^d]-\Theta\big)-ln\big(Rx_m^d\big)\Big],$$

The optimal transfer is again a function of individual investments, technology parameters, and the institutional sharing rule:

$$\Theta^* = R\big[(1-\pi)x_f^d - \pi x_m^d\big]. \tag{3.15}$$

[48]Similar models employ a known return equal to $1/\beta$ so that the safe asset represents a risk-free savings instrument. We are only concerned with it providing a known return which is not subject to bargained transfers.

[49]Since this bargain takes place after harvests are realized, the revenues from cash-crop production are presented here as a known quantity.

As agents continue to calculate the effect of their and their partner's investments on the bargaining stage, we can substitute the value of the optimal transfer into each agent's maximization problem. Where the price of the safe asset and consumption good are normalized to one and individuals have isoelastic preferences over consumption with constant rates of relative risk aversion, α_f and α_m, the pair of individual objectives are now:

(3.16)

$$V_f(x_f, x_m, z_f) =$$

$$\max_{x_f, z_f} \left\{ \frac{\Big(\pi W - p x_f - z_f\Big)^{1-\alpha_f}}{1-\alpha_f} + \beta E_{\tilde{R}} \left[\frac{\Big(\tilde{R} x_f^d + G z_f^h + \tilde{R}\big[(1-\pi)x_f^d - \pi x_m^d\big]\Big)^{1-\alpha_f}}{1-\alpha_f} \right] \right\}$$

$$V_m(x_f, x_m, z_m) =$$

$$\max_{x_m, z_m} \left\{ \frac{\Big((1-\pi)W - p x_m - z_m\Big)^{1-\alpha_m}}{1-\alpha_m} + \beta E_{\tilde{R}} \left[\frac{\Big(\tilde{R} x_m^d + G z_m^h - \tilde{R}\big[(1-\pi)x_f^d - \pi x_m^d\big]\Big)^{1-\alpha_m}}{1-\alpha_m} \right] \right\}.$$

Agents maximize two-period utility by allocating their individual assets between consumption and a combination of the two production technologies according to the four first order conditions in equations (3.35). The solutions to these represent best response functions to each others portfolios and again give a pair of decentralized equilibrium strategies.

Definition 2. *The pair of indirect utilities $V_f(x_f, x_m, z_f)$ and $V_m(x_f, x_m, z_f)$, and strategies $x_f^*(x_m; s_t)$ and $x_m^*(x_f; s_t)$, is a Markov perfect equilibrium such that given x_m and x_f, x_f^* and x_m^* satisfy equations (3.16), where the state of the world at time t, s_t, encompasses the household endowment W_t, institutional rule π, time and risk preferences, technology parameters, and the form of uncertainty over $\tilde{R}$.*

We utilize numerical methods to solve the four first order conditions simultaneously, maximizing the pair of equations (3.16). This is done by iterating on each agent's pair of strategies until both are playing a best response to each others portfolio given their (institutionally bounded) state of the world and expectations over the return of the risky technology.[50] This includes the following parameter values based on the Ghana data mentioned above:[51]

Parameters		
W_0	100	Initial household wealth
p	2	Price of risky technology
α_f, α_m	1.4	Relative risk aversion
$\overline{R}$	10.0	Mean return on risky technology
σ_R	8	Std. dev. of return on risky technology
d	.756	Decreasing returns parameter on risky technology
G	7.0	Productivity parameter on safe technology
d	.607	Decreasing returns parameter on safe technology
β	.95	Discount factor

When both agents share the same preferences for risk, total household investment in the risky asset is again maximized in the case of equality. Household portfolio risk, defined as the portion of total investment expenditure invested in the risky technology, is plotted against the value of the institutional sharing rule in Figure 3.1 below.

[50] Defined as updating each of their investments by at most $\epsilon = 2.2204 \times 10^{-16}$.

[51] The primary characteristics we consider are that the risky technology has higher marginal returns than the safe technology and that both exhibit decreasing returns to scale. As we have abstracted from the realities of multiple inputs per technology as well as assumed generic time and risk preferences, specific parameters are relatively unimportant beyond the primary characteristics.

FIGURE 3.1. Household Portfolio Risk with Homogeneous Preferences

While this result is expected given our analytical treatment in section 3.3, it is likely that males and females may have different preferences for risk. To explore this impact on the household portfolio, we repeat the previous simulation with heterogeneous levels of risk aversion, specifically, where the male becomes more risk averse relative to the female. The results of four simulations are below in Figure 3.2. Here, we show the result predicted (and confirmed empirically with U.S. data on retirement portfolio choice) in [49] and [50] that the risk level of the household portfolio approaches that of the individual with more bargaining power over household assets.[52]

[52]Their models use fully cooperative frameworks to develop this result.

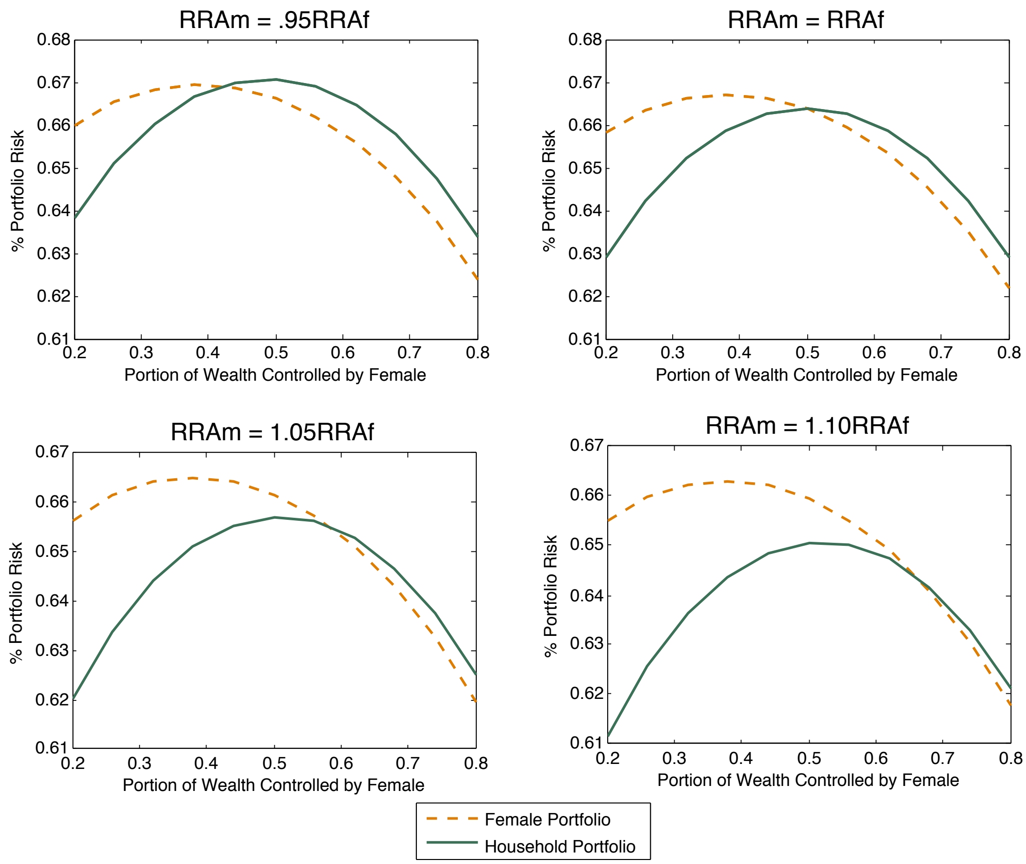

FIGURE 3.2. Female and Household Portfolio Risk with Heterogeneous Preferences

Moving from the top left to bottom right frame of Figure 3.2, the female becomes more amenable to risk. At the same time, the household portfolio converges to her preferred level of risk at higher levels of female control (as $\pi \to 1$). As this relationship results from a decentralized equilibrium in non-cooperative play, it is interesting in relation to the cooperative models cited above. However, the primary importance is in consideration of development policy, which if targeted to specific individuals within the household or for specific expenditure types (which may also vary by individual), will depend on differences in both risk preferences and relative access to resources.[53] Since household portfolio risk may

[53]We should also interpret the seemingly counter-intuitive (eventual) fall in female portfolio risk as she has more control over the household asset base. Since we have made no assumptions about bargaining power per se, higher female investment in the risky technology reduces her marginal gains from additional transfers in the bargain relative to her partner, meaning that she must continually transfer more in his direction

reflect that of one individual, policies must additionally consider separate spheres of resource control if comparing outcomes at the household level.

To consider the effect of gender inequality as it interacts with the level of uncertainty about the return on the risky asset, we solve for equilibrium portfolios across both levels of the sharing rule and variance of the risky return. The results of this procedure are presented graphically in Figure 3.3.

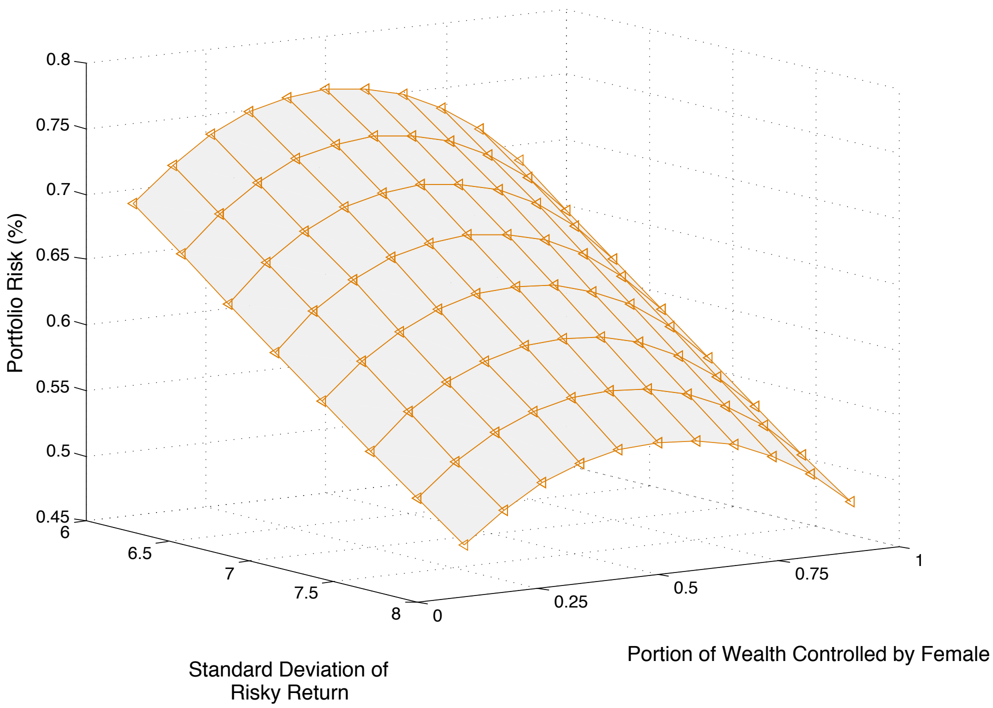

FIGURE 3.3. Household Portfolio Risk vs. Variance and Sharing Rule

As we expect, investment in the risky technology is decreasing both in inequality and variance of the productivity parameter. For given levels of uncertainty, household investment is maximized in the case of equality, as before. What runs somewhat counter to expectations

as the household maximizes the gains from cooperation. We have simulated a model including bargaining power in the traditional way and verified that the additional weighting of bargaining gains does produce the expected result that individual portfolio risk continues to increase with one's control. In doing so, the core relationship between unequal access to resources and household investment is unchanged. For this reason we continue to make no assumptions about relative skill at the bargaining table and focus on the role of the institutional parameter, π as we have.

at this point (considering the interaction between policies and inequality in the analytical model) is that the reduction in variance has a similar effect across levels of equality (see Figure B.6 in Appendix B for a side view of Figure 3.3, which holds the institutional rule constant at equal and unequal levels). This apparent contradiction is quickly resolved when we consider the indirect effect(s) of reducing uncertainty about the risky technology. As the household becomes less uncertain they both invest more in the risky technology but also allocate more wealth to the safe asset (and consumption).

The full effect of reducing uncertainty over the risky return can be seen below in figures 3.4 and 3.5. Over the first four periods of production, the effect of reducing uncertainty grows stronger over time for households with egalitarian sharing rules. Although we do not model a subsistence threshold as in [42], we can surmise a similar importance here. For households operating against subsistence constraints, we can see that an equitable household will be more able to accumulate wealth and avoid falling below a threshold level of nutrition. Zimmerman and Carter show that poorer households continue to adopt safer portfolios due to this fear. Here, we add the compounding effect of intrahousehold inequality as a significant constraint to growth in a risky environment.

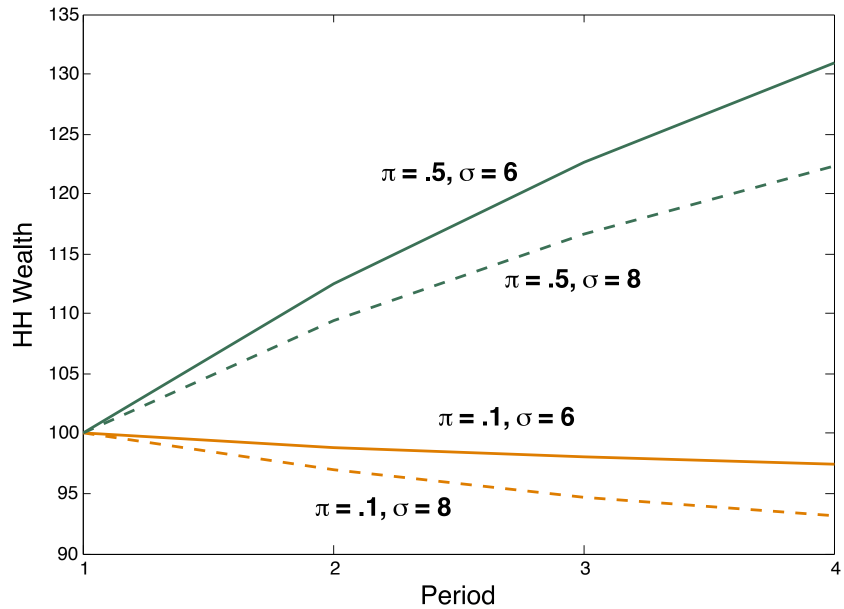

FIGURE 3.4. Household Wealth over Time

The interaction between inequality and uncertainty also has significant consequences in terms of the household consumption path. Figure 3.5 shows consumption growth over the first three periods. Although consumption mostly mirrors the growth in household assets, there are two important things to note. Firstly, initial period consumption is actually lowest in the cases of equality and low variance. This initial result is expected as a low level of risky investment is supplemented with greater consumption. However, this trade-off quickly disappears over the second and third periods, showing the potential gains from moving to a more equal sharing rule. While consumption in the first period falls, we can clearly see the potential Pareto improvement from a redistribution of assets by the second period. Secondly, the redistributive effect of lower variance on consumption is again lower for more unequal households as their consumption paths diverge less with variance changes relative to egalitarian ones. As wealth accumulation also showed less divergence for unequal households, we conclude that the redistributive gains are higher in terms of both savings in the safe asset and current period consumption under more equal sharing rules.

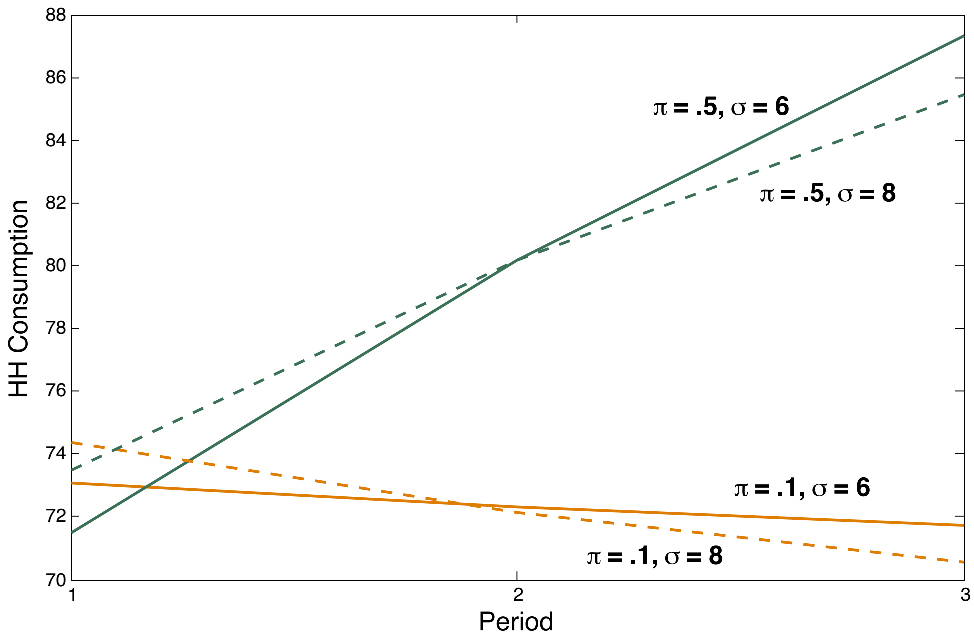

FIGURE 3.5. Household Consumption over Time

Although we have not modeled endogeneity of the institutional sharing rule, we can suggest here a potential reason why an inefficient institution might persist over time. A shift to a more equal sharing rule would necessarily reduce indirect utility of the individual giving up say (as well as the household in sum, by reducing total consumption) in period t. While our model projects that total consumption would be higher in future periods from such a shift, the promise of future repayment may not be satisfactory, especially given the lack of binding contracts to ensure such a payment. As individual objectives are strictly increasing in controllable wealth, an autonomous shift in the household sharing rule would be unlikely without an enforceable agreement.

3.5. Testing Predictions of the Model

We now return to the data that motivated our model to test its core hypotheses, namely that: (i) Post-harvest transfers are inversely related to one's control over the household asset base, and (ii) Risk of the household portfolio is decreasing in inequality. To test these hypotheses we utilize data collected over 15 rounds (during 1997/1998) from 200 households in 4 villages in rural Ghana in the Agricultural Innovation and Resource Management survey [6]. As an intrahousehold panel, this data set provides a unique opportunity to explore the allocation of inputs across individuals within households. Furthermore, since females and males often farm similar crops in the same year, we can compare household choices with individual resource allocations.[54]

We test the first hypothesis that intrahousehold transfers are negatively related to one's control over household wealth, formalized in Proposition 1 of section 3.3.3. We do this by estimating the following model using ordinary least squares:

$$\Theta_{ivt} = \beta_0 + \beta_1 \pi_{ivt} + \beta_2 X(f)_{ivt} + \beta_3 X(m)_{ivt} + u_{ivt}, \tag{3.17}$$

[54]Since individuals in surveyed households were interviewed separately (by persons of similar gender) we can also assume that reports of intrahousehold transfers and input sharing are not biased by fears of repercussion resulting from incriminating reports. However we later conclude that reports of cash transfers are biased in some way, specifically by over- and underestimations of benefactors relative to recipients.

where the actual male-to-female transfer by household i in village v and year t is a function of the household sharing rule, π, individual investments, $X(f)$ and $X(m)$, and a random shock, u_{ivt}.[55] The results of this estimation are below in Table 3.1.[56]

Table 3.1. Actual Transfers as a function of Sharing Rule

Dependent Variable: Θ (1000's GH)	
π	-141.161**
	[40.388]
$X(f)$	0.263***
	[.078]
$X(m)$	0.075***
	[.022]
Constant	225.092***
	[28.00]
Observations	231
R-squared	0.223
standard errors in brackets	
* significant at 10%; ** significant at 5%; *** significant at 1%	

Controlling for individual investments and relevant fixed-effects, we reject the null that the household sharing rule is unrelated to intrahousehold transfers, and conclude that the level of transfer is negatively related to the sharing rule. Although the division of household assets is largely male biased (see distribution of sharing rules in figure 3.7 below), the positive and significant intercept implies that at the mean of the data, the expected transfer would be towards the female even without any investment. This could reflect a combination of

[55]We sum total investments as there are not enough observations of female investment in Pineapple to accommodate a comparison of strictly risky investments.

[56]We also control for village and year fixed effects to account for variation across geographical space and period specific endogeneity.

two things: (i) The systematic over reporting of transfers, or (ii) The possibility that our theoretical model underestimates the flow of intrahousehold transfers since in the case of the Ghana survey, very few females invest in Pineapple production.[57] Given the phenomena described in footnote 31, we can at least support the former, which suggests that there is imperfect information about transfers between agents. While we could model this effect here, we continue to focus on the distribution of resources under cooperation and competition and leave information problems for later work.

Next, we test the hypothesis that inequality has no relationship to the household's level of portfolio risk. At the time of the survey, Ghana was in the middle of a major transformation to production of the cash-crop pineapple for export. Investment in the new crop entailed uncertainty about the process itself, and also regarding fertilizer commonly used in its production.[58] To test this hypothesis, we use maximum likelihood techniques to estimate a generalized linear model with a logistic distribution of portfolio risk, ρ_{hh}:

$$\ln\left\{\frac{\rho_{ivt}}{1-\rho_{ivt}}\right\} = \beta_0 + \beta_1 Ineq_{ivt} + \beta_2 X(hh)_{ivt} + u_{ivt} \tag{3.18}$$

Where $\rho \in (0,1)$ is the household's level of portfolio risk, defined as the portion of total agricultural expenditure applied to pineapple production, *Ineq* is the difference between the

[57]The sum of male reported transfers (net male to female) minus the sum of female transfers (net male to female) should equal zero, however this is not so in the data. Actually, there is an average over reporting of the transfer of 245,000 GH, which is statistically significant at the 1% level (see Figure B.5). Subtracting this average over-reporting of Θ from each household and estimating equation (3.17) again gives an estimated intercept which is not significantly different than zero.

[58]Although we have used the terms risk and uncertainty interchangeably, we have operationalized randomness as [56] would define risk. Here, decision makers know the probability distribution defining their expectation of outcomes. Alternatively, uncertainty would in Knight's terms reflect less than perfect information about the distribution itself. While we have assumed agents know the distribution of *risk*, anecdotal reports in the data reflect uncertainty as well. Specifically, many farmers avoided investment in the new crop due to perceptions of scale effects, which are not confirmed in empirical analysis [6].

actual sharing rule and equality ($|\pi - .5|$), multiplied by two so that $Ineq \in [0, 1]$ represents a range between equality and total inequality, and X is the level of total household expenditure (in $1000's$ GH) of household i in village v in round t. Linear (un-exponentiated) coefficients of this estimation are given in Table 3.2 below.[59]

TABLE 3.2. Household Portfolio Risk

Dependent Variable: Household Risk, $\rho \in (0, 1)$	
Inequality	-0.329*
	[0.187]
HH Expenditure	0.012***
	[0.002]
Constant	-1.047***
	[0.172]
Observations	1618
standard errors in brackets	
* significant at 10%; ** significant at 5%; *** significant at 1%	

As the household sharing rule deviates from equality, the average level of portfolio risk falls to a significant degree (statistically and economically). We reject the null that inequality is unrelated to household portfolio risk and conclude that there is a significantly negative relationship between the two. Specifically, as the household sharing rule deviates from equality to full inequality, the logit function, $\ln\left\{\frac{\rho_{hh}}{1-\rho_{hh}}\right\}$ decreases by .33 on average. In other words, the level of relative household portfolio risk changes on average by $\exp\{\beta_1\} - 1 = \exp\{-.33\} - 1 = -.28$, or falls by 28.0% [57].

Estimated portfolio risk (controlling for total expenditure and village and year fixed effects) falls from 37.5% in the case of equality to between 31.7% and 26.5% in the cases

[59]We also estimate a Tobit model truncated at zero and full risk and receive similar results.

where $\pi = 0$ and $\pi = 1$ respectively, reflecting the predicted drop in portfolio risk associated with a shift from equality to pure inequality in Table 3.2. Figure 3.6 presents the distribution of predicted portfolios against the household sharing rule, including a quadratic best fit line revealing this relationship and that predicted theoretically in Figure 3.1 of section 3.4.

Figure 3.6. Predicted (Household) Portfolio Risk

3.6. Discussion

For the agricultural households considered here, gender inequality and uncertainty interact crucially to affect investment and prospects for growth. While risk certainly lowers rates of investment and household wealth accumulation, we conclude that gender inequality is an overarching constraint that unless removed will continue to limit the gains from policies aimed at limiting the deterrence of risk's downside. Although we have treated the institution of gender bias as an exogenous variable, we have suggested gains from a more egalitarian rule and generated multiple testable hypotheses to apply to similar contexts.

By unpacking the household and treating it simultaneously as a site of cooperation and conflict amongst individual members, we have shown the binding effect of unequal access to resources and how it operates through its impact on optimal investment strategies. Here we offer theoretical contributions of combining intrahousehold processes with stochastic growth. While mirroring results of fully cooperative models, we also incorporate noncooperative play to allow for inferior outcomes suggested by empirical work.

Furthermore, we have addressed important considerations for development policies as they interact with a more realistic intrahousehold process supported by both theoretical and empirical studies. As females have less say over the returns to cash crop production, they accordingly adjust their portfolios, thus limiting their ability to accumulate assets and improve their future bargaining position. While we have also explored the interaction between uptake policies and the institution of gender bias in this way, we have not offered claims regarding how to reverse this cycle.

Explicit description of any endogeneity of the institutional norm will certainly allow us to consider the ability of policies to help households achieve higher rates of investment through more equal sharing rules. As far as individual outcomes drive changes in the distribution of resources through bargaining, policies may be able to simultaneously support investment, food security, and endogenous shifts towards intrahousehold equity.

As uptake policies have different impacts on individual portfolios, they must consider not only unequal improvements in individual prospects, but also the strategic responses of other household members. If males maintain control over the household asset base, the household portfolio will react to policies in a similar fashion to his own. As females expect more revenue transfers from male production, policies that increase investments on male plots may actually decrease the willingness of females to make greater investments. As far as this continues to affect relative bargaining positions over time, it may unfortunately serve to sustain a given institutional rule and deter gains in both investment and intrahousehold equality. If the pooling of risk implied by our bargaining process is in reality less concerned with cooperative gains, individual consumption possibilities targeted by policies may also be limited by the extent of inequality.

While we have highlighted the dynamic links between agents in separate spheres households and the resulting impact on development goals, we have also left important considerations for future work. Although our relatively benign form of cooperation assumes away additional inefficiencies that may affect technology adoption, we have underlined the core relationship between gender and the household's choice problem. Extension of the bargaining stage to include context specific realities such as bargaining breakdowns, imperfect information, and a stricter form of power may allow deeper insight into the strategic responses of household members to investment policy and the resulting effects on growth. Similarly, considering the evolution of the institutional rule as a result of individual strategies and outcomes will allow additional policy leverage in dealing with inequality proactively.

While this paper makes important connections between the dynamics of risk and realities of the household, there is much more to be done. In building an initial framework linking these previously isolated problems, we hope to suggest a relevant avenue for approaching the dual concerns of social equality and economic development.

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APPENDIX A

Additional Tables

Table A.1. Correlations Between Tobit Residuals

	Cassava	Maize
$\rho(e_{Male}, e_{Female})$	0.356***	0.146**
	(0.043)	(0.067)
$\rho(e_{Male}, e_{Hired})$	0.497***	0.248***
	(0.046)	(0.072)
$\rho(e_{Female}, e_{Hired})$	0.162***	0.0154
	(0.058)	(0.086)

Standard errors in parentheses

*** p< 0.01, ** p< 0.05, * p< 0.1

Table A.2. Tobit Estimates of Labor Use per Acre

	Cassava			Maize		
	Male	Female	Hired	Male	Female	Hired
Gender	-78.281***	39.066***	-10.725	-54.080***	19.809***	-83.663***
(Female = 1)	(7.465)	(6.214)	(8.195)	(6.111)	(3.263)	(25.513)
ln(Area)	-16.540***	-17.974***	-0.725	-6.600***	-4.468***	-25.890**
	(2.990)	(2.754)	(3.773)	(2.431)	(1.542)	(10.742)
Quality	-30.747*	-24.089	34.809*	21.580	-2.646	-18.400
	(16.899)	(15.683)	(20.990)	(13.503)	(8.486)	(57.057)
Other Labor	0.216***	0.509***	0.216***	0.190**	0.154***	-0.057
	(0.062)	(0.059)	(0.066)	(0.077)	(0.050)	(0.341)
Constant	54.898***	24.977***	-56.620***	16.537*	6.956	-35.215
	(7.207)	(6.930)	(10.460)	(8.710)	(5.521)	(37.357)
Observations	465	465	465	274	274	274
LR χ^2	151.85	164.89	16.01	105.47	62.61	14.64
$Pr(>\chi^2)$	0.0000	0.0000	0.0030	0.0000	0.0000	0.0120

Standard errors in parentheses

*** p< 0.01, ** p< 0.05, * p< 0.1

Table A.3. Cobb-Douglas Production Estimates (with Specific Labor Inputs)

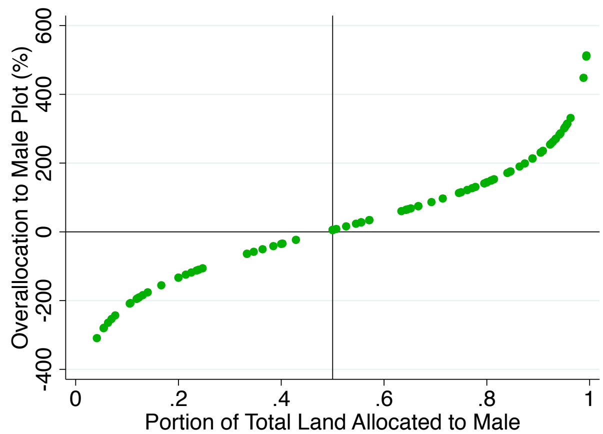
	Cassava and Maize		
	All	Male	Female
Gender	-0.219		
(Female = 1)	(0.357)		
ln(Area)	0.266***	0.255***	0.409***
	(0.058)	(0.069)	(0.109)
Female×ln(Area)	0.067		
	(0.099)		
ln(Male Labor)	0.155***	0.171**	0.122
	(0.058)	(0.072)	(0.138)
Female×ln(Male Lab)	-0.043		
	(0.090)		
ln(Female Labor)	0.174***	0.134*	0.217**
	(0.063)	(0.073)	(0.093)
Female×ln(Female Lab)	-0.034		
	(0.084)		
ln(Hired Labor)	-0.067	-0.194	-0.021
	(0.085)	(0.119)	(0.113)
Female×ln(Hired Lab)	0.000		
	(0.065)		
ln(Other Labor)	0.091	0.092	0.013
	(0.062)	(0.088)	(0.095)
Female×ln(Other Lab)	-0.019		
	(0.059)		
Length Fallow	0.028	0.022	0.051
	(0.021)	(0.025)	(0.046)
Female×Length Fallow	0.011		
	(0.034)		
Constant	9.335***	10.392***	8.829***
	(0.692)	(0.917)	(1.201)
Observations	691	423	268
R-squared	0.523	0.571	0.534

Robust standard errors in parentheses

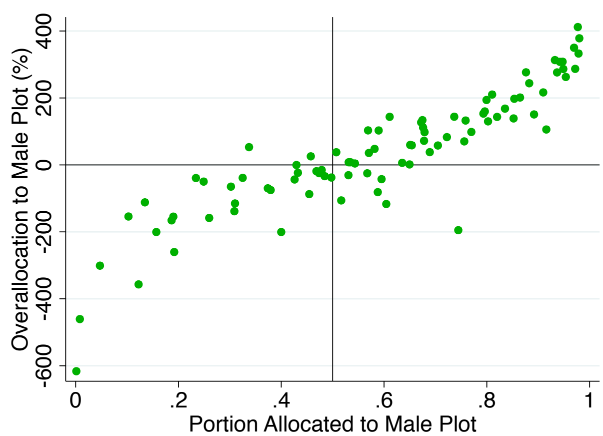
*** $p< 0.01$, ** $p< 0.05$, * $p< 0.1$

APPENDIX B

Additional Figures

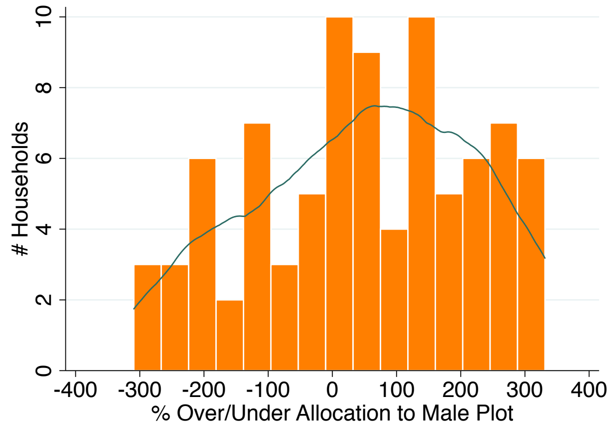


(a) Land

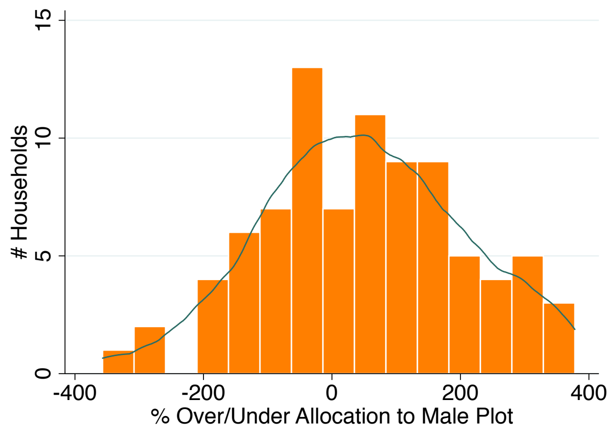


(b) Labor

Figure B.1. Relative Misallocation of Inputs



(A) Land



(B) Labor

FIGURE B.2. Distribution of Household Misallocations

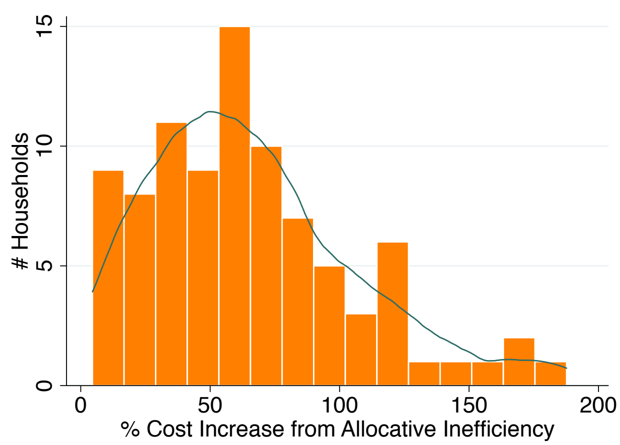


FIGURE B.3. Total Cost Increases from Allocative Inefficiency

Figure B.4. Distribution of Household Sharing Rules

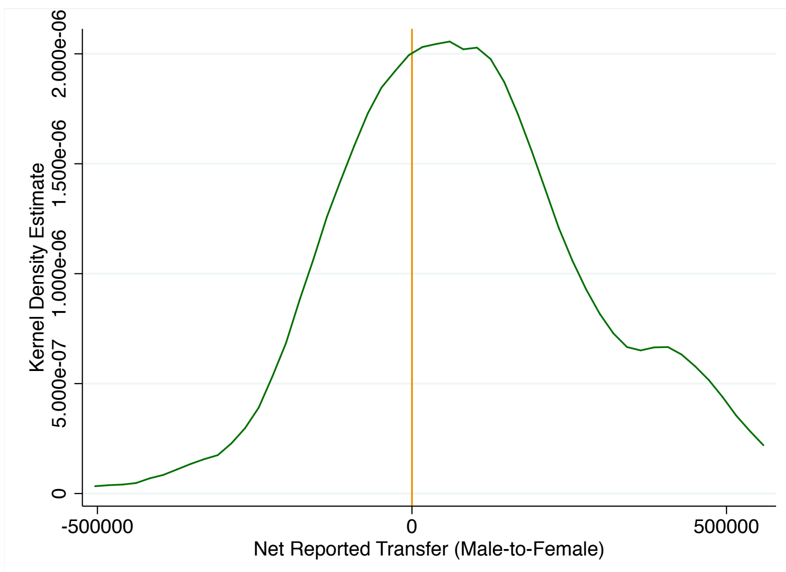


Figure B.5. Bias in Reported Transfers

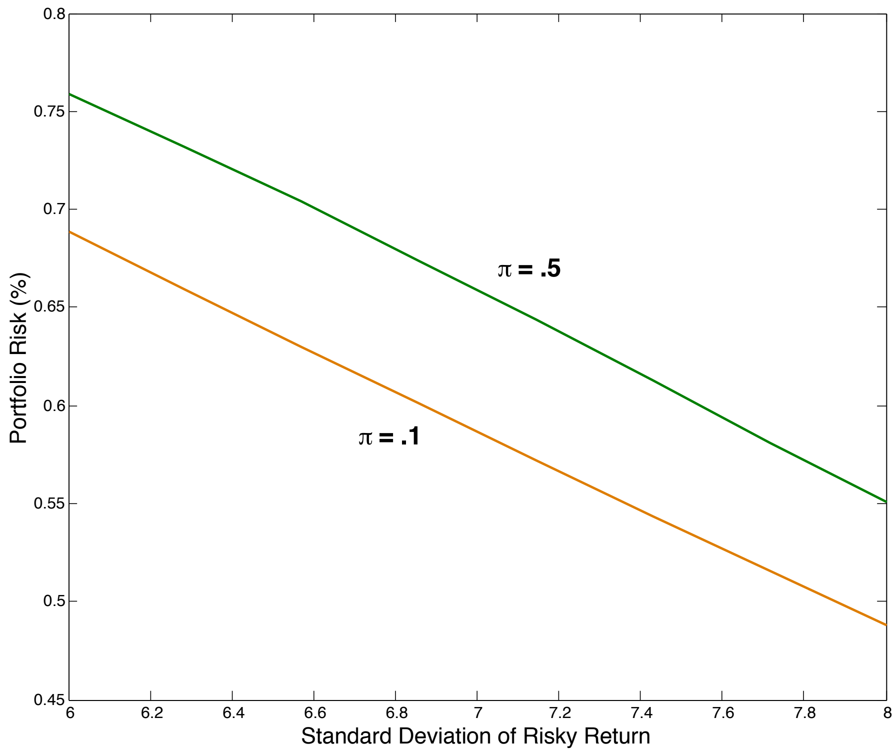


FIGURE B.6. Household Portfolio Risk as a function of Variance

APPENDIX C

Solutions, Derivations and Proofs

C.1. Derivations for Essay #2

Optimal investments as functions of expected transfers:

$$x_f^*(\hat{\Theta}) = \frac{\beta W(\pi(\alpha-1)(1+\alpha+\beta+\alpha\beta+\gamma)-\alpha\gamma)}{p(\alpha^2(1+\beta)^2-(1+\beta+\gamma)^2)} + \frac{(1-\alpha+\gamma)(1+\alpha+\beta+\alpha\beta+\gamma)\hat{\Theta}}{R(\alpha^2(1+\beta)^2-(1+\beta+\gamma)^2)}$$

$$z_f^*(\hat{\Theta}) = \frac{\gamma W(\alpha(1+\beta)(\pi-1)+\pi(1+\beta+\gamma))}{(\alpha^2(1+\beta)^2-(1+\beta+\gamma)^2} - \frac{\gamma p(1+\alpha+\beta+\alpha\beta+\gamma)\hat{\Theta}}{R(\alpha^2(1+\beta)^2-(1+\beta+\gamma)^2}$$

$$x_m^*(\hat{\Theta}) = \frac{\beta W((1+\beta+\gamma)(\pi-1)-\alpha\gamma\pi-\alpha^2(1+\beta)(\pi-1))}{p(\alpha^2(1+\beta)^2-(1+\beta+\gamma)^2)} + \frac{(1+\alpha+\beta+\alpha\beta+\gamma)(\alpha-1-\gamma)\hat{\Theta}}{R(\alpha^2(1+\beta)^2-(1+\beta+\gamma)^2)}$$

$$z_m^*(\hat{\Theta}) = \frac{\gamma W(\pi(1+\alpha+\beta+\alpha\beta+\gamma)-1-\beta-\gamma)}{(\alpha^2(1+\beta)^2-(1+\beta+\gamma)^2} + \frac{\gamma p(1+\alpha+\beta+\alpha\beta+\gamma)\hat{\Theta}}{R(\alpha^2(1+\beta)^2-(1+\beta+\gamma)^2}.$$

Nash equilibrium investments:

$$x_f^{NE} = \frac{\begin{array}{c}\beta W(1+\pi)(2\alpha\gamma - (\alpha-1)((1+\alpha)(3+2\beta)+3\gamma)\pi \\ +(\alpha-1)(2+\beta+\alpha(2+\beta)+2\gamma)\pi^2)\end{array}}{\begin{array}{c}p(2(1+\beta+\gamma)^2+\beta(2+\beta+2\gamma)\pi-\beta(2+\beta+2\gamma)\pi^2 \\ +2\alpha\beta\gamma(1+(\pi-1)\pi+\alpha^2(\beta(2+\beta)(\pi-2)(1+\pi)-2))\end{array}}$$

$$z_f^{NE} = \frac{\gamma W(\pi(1+\beta+2\gamma+\pi\beta)-\alpha(\beta(\pi-2)-2)(\pi-1)}{\begin{array}{c}2(1+\beta+\gamma)^2+\beta(2+\beta+2\gamma)\pi-\beta(2+\beta+2\gamma)\pi^2 \\ +2\alpha\beta\gamma(1+\pi(\pi-1))+\alpha^2(\beta(2+\beta)(\pi-2)(1+\pi)-2)\end{array}}$$

$$x_m^{NE} = \frac{\begin{array}{c}\beta W(\pi-2)(1+\beta-\alpha^2(1+\beta)+\gamma+\alpha\gamma \\ -(\alpha-1)(1+\alpha+\gamma)\pi+(\alpha-1)(2+\beta+\alpha(2+\beta)+2\gamma)\pi^2)\end{array}}{\begin{array}{c}p(2(1+\beta+\gamma)^2+\beta(2+\beta+2\gamma)\pi-\beta(2+\beta+2\gamma)\pi^2 \\ +2\alpha\beta\gamma(1+\pi(\pi-1))+\alpha^2(\beta(2+\beta)(\pi-2)(1+\pi)-2))\end{array}}$$

$$\text{(C.1)} \qquad z_m^{NE} = \frac{\gamma W(2(1+\gamma)-2(1+\alpha+\gamma)\pi-\beta(\pi(3+\alpha+(\alpha-1)\pi)-2))}{\begin{array}{c}2(1+\beta+\gamma)^2+\beta(2+\beta+2\gamma)\pi-\beta(2+\beta+2\gamma)\pi^2 \\ +2\alpha\beta\gamma(1+\pi(\pi-1))+\alpha^2(\beta(2+\beta)(\pi-2)(1+\pi)-2)\end{array}}$$

Proof of proposition 1 Equilibrium household investment in the cash crop, $x_{hh}^{NE} = x_f^{NE} + x_m^{NE}$ is maximized with respect to the sharing rule when:

$$\text{(C.2)} \qquad \frac{\partial x_{hh}^{NE}}{\partial \pi} = \frac{\begin{array}{c}2W(\alpha-1)\beta((\alpha-1)(4+3\beta)-4\gamma) \\ (1+\alpha+\gamma)(1+\alpha+\beta+\alpha\beta+\gamma)(2\pi-1)\end{array}}{\begin{array}{c}p(2(1+\beta+\gamma)^2+\beta(2+\beta+2\gamma)\pi-\beta(2+\beta+2\gamma)\pi^2 \\ +2\alpha\beta\gamma(1+\pi(\pi-1))+\alpha^2(\beta(2+\beta)(\pi-2)(1+\pi)-2))^2\end{array}} = 0.$$

From the numerator, this is true when $\alpha = 1$ or $\pi = .5$. In the case where $\alpha = 1$, household investment is not affected by changes in the sharing rule. The effect of the sharing rule diminishes in the case of a fully private food crop only under the assumption of similar preferences over subsistence ($\gamma_f = \gamma_m$). Relaxation of this (simplifying but potentially unrealistic) assumption is discussed in section 2.4.1. In the case of a fully public food crop, household investment increases as the individual with stronger preferences for the food crop controls more resources. Additionally, in the case where $\gamma_f \neq \gamma_m$ and $\alpha < 1$, equilibrium investment in the cash crop is highest as resource control and food crop preferences correlate positively in much of parameter space.

Proof of proposition 2 From the optimal conditions taking one's spouse's behavior as given, (2.13), partial changes in individual cash crop investments with increases in price (holding spousal investment constant) are:

$$\frac{\partial x_f^*(z_f, \hat{x}_m)}{\partial p} = -\frac{\pi \beta W}{(1+\beta)p^2} + \frac{z_f}{(1+\beta)p^2}\frac{\partial z_f}{\partial p}$$

$$\frac{\partial x_m^*(z_m, \hat{x}_f)}{\partial p} = \frac{\beta W(\pi - 1)}{(1+\beta)p^2} + \frac{z_m}{(1+\beta)p^2}\frac{\partial z_m}{\partial p}. \tag{C.3}$$

From equilibrium investments (C.1), investments in the subsistence crop are not functions of the price of the cash crop (i.e. $\frac{\partial z_f^{NE}}{\partial p} = \frac{\partial z_m^{NE}}{\partial p} = 0$), so that the second terms in (C.3) are each zero. As the first terms are both negative, each individual invests less with price increases (i.e. subsidies increase individual investments). While we held spousal cash crop investments constant, the indirect effect would only support the effect of price changes, since individual cash crop investments are positively related (as in (2.14)).

The fact that the price effect is strongest in the case of equality (with $\alpha < 1$) is more straightforward. From equilibrium household investment in the cash crop (C.1), the price effect is maximized with respect to the sharing rule when:

(C.4)

$$\frac{\partial^2 x_{hh}^{NE}}{\partial p \partial \pi} = \frac{2(\alpha-1)\beta((\alpha-1)(4+3\beta)-4\gamma)(1+\alpha+\gamma)(1+\alpha+\beta+\alpha\beta+\gamma)(2\pi-1)W}{\begin{array}{c} p^2(2(1+\beta+\gamma)^2+\beta(2+\beta+2\gamma)\pi-\beta(2+\beta+2\gamma)\pi^2 \\ +2\alpha\beta\gamma(1+(\pi-1)\pi)+\alpha^2(\beta(2+\beta)(\pi-2)(1+\pi)-2))^2 \end{array}} = 0.$$

If the food crop is at least partially private, this condition is met only when $\pi = .5$.

C.2. Derivations for Essay #3

Derivation of equation (3.6), the optimal post-harvest transfer, Θ^*:

The level of male-to-female transfer that will maximize the gains from cooperation will satisfy the following first order condition:

(C.5)

$$\frac{\partial N}{\partial \Theta} = \frac{ln\big(\pi R[x_f+x_m]+\Theta\big)-ln\big(Rx_f\big)}{\pi R[x_f+x_m]} - \frac{ln\big((1-\pi)R[x_f+x_m]-\Theta\big)-ln\big(Rx_m\big)}{(1-\pi)R[x_f+x_m]} = 0.$$

This condition is only satisfied when:

$$ln\big(\pi R[x_f+x_m]+\Theta\big)-ln\big(Rx_f\big) = ln\big((1-\pi)R[x_f+x_m]-\Theta\big)-ln\big(Rx_m\big) = 0$$

(C.6)

$$\Theta = R\big[(1-\pi)x_f - \pi x_m\big]$$

From the concavity of the Nash objective in transfers, we can conclude that the solution to the first order condition above is indeed a maximum.

Proof of Proposition 1

Holding investments fixed, the optimal transfer is decreasing in the value of π:

$$\frac{\partial \Theta^*}{\partial \pi} = -R[x_f + x_m] < 0. \tag{C.7}$$

As the transfer is a net male-to-female transfer, equation (C.7) shows the balance of the transfer is in the male's favor as his control of household wealth decreases. Likewise, a decrease in π which represents more male control would cause the optimal transfer to increase, or shift in the female's favor.

Derivation of equations (3.11), female and male best response functions:

Individuals choose optimal investments taking the behavior of their spouse as given according to the first order necessary conditions:

$$\begin{aligned} \frac{\partial V_f}{\partial x_f} &= -\frac{p}{\pi W - p x_f} + \beta \frac{(2-\pi)R}{R x_f + R\big[(1-\pi)x_f - \pi x_m\big]} = 0, \text{and} \\ \frac{\partial V_m}{\partial x_m} &= -\frac{p}{(1-\pi)W - p x_m} + \beta \frac{(1+\pi)R}{R x_m - R\big[(1-\pi)x_f - \pi x_m\big]} = 0. \end{aligned} \tag{C.8}$$

Again, with the concavity of the objective functions in the choice variables, we know that the second order conditions will hold and the first order conditions indeed give maxima. While individual investments are increasing in one's say, they do so at a diminishing rate:

$$\frac{\partial x_f^{NE}}{\partial \pi} = \frac{\beta\Big(2(1+\beta)^2(3+2\beta)+4(1+\beta)^3\pi-(2+\beta)\Big(6+\beta(8+3\beta)\Big)\pi^2 -2\beta(2+\beta)^2\pi^3+\beta(2+\beta)^2\pi^4\Big)W}{p\Big(2+\beta(2+\beta)(2+\pi-\pi^2)\Big)^2} > 0,$$

(C.9)

$$\frac{\partial x_m^{NE}}{\partial \pi} = -\frac{\beta\Big((4(5-3\pi)\pi-2+\beta^3(2+\pi-\pi^2)^2+2\beta^2(\pi-2)^2\Big(1+2\pi(2+\pi)\Big) +\beta\Big(2+2\pi\Big(20+\pi(2(\pi-2)\pi-11)\Big)\Big)\Big)W}{p\Big(2+\beta(2+\beta)(2+\pi-\pi^2)\Big)^2} < 0,$$

$$\frac{\partial^2 x_f^{NE}}{\partial \pi^2} = -\frac{4\beta(1+\beta)\Big(\beta^2+\beta^3-2-2\beta+3(1+\beta)(2+\beta)^2\pi +3\beta(1+\beta)(2+\beta)\pi^2+\beta(2+\beta)^2\pi^3\Big)W}{p\Big(2+\beta(2+\beta)(2+\pi-\pi^2)\Big)^3} < 0,$$

(C.10)

$$\frac{\partial^2 x_m^{NE}}{\partial \pi^2} = \frac{4\beta(1+\beta)\Big(2(6\pi-5)+\beta(\beta^2(\pi-2)^3-32+2\pi(24+\pi(2\pi-9)) +\beta(\pi(45+\pi(4\pi-21))-29))\Big)W}{p\Big(2+\beta(2+\beta)(2+\pi-\pi^2)\Big)^2} < 0.$$

Proof of Proposition 2 From the individual equilibrium investments in equations (3.12), total household investment is given by:

(C.11) $$x_{hh}^{NE} = x_f^{NE} + x_m^{NE} = \frac{\beta W\Big(4\pi(\pi-1)+\beta(\pi-2)(1+\pi)-2\Big)}{p\Big(\beta(2+\beta)(\pi-2)(1+\pi)-2\Big)}.$$

Supposing a social planner could choose the institutional sharing rule π with the goal of maximizing total investment, her problem would be to:

$$\max_{\pi\in[0,1]} x_{hh}^{NE} = \frac{\beta W\big(4\pi(\pi-1)+\beta(\pi-2)(1+\pi)-2\big)}{p\big(\beta(2+\beta)(\pi-2)(1+\pi)-2\big)}, \tag{C.12}$$

which would be done according to the following first order condition:

$$\frac{\partial x_{hh}^{NE}}{\partial \pi} = -\frac{2\beta W(1+\beta)(4+3\beta)(2\pi-1))}{p\big(2+\beta(2+\beta)(2+\pi-\pi^2)\big)^2} = 0, \tag{C.13}$$

which is met when

$$\begin{aligned} &2\beta W(1+\beta)(4+3\beta)(2\pi-1)) = 0 \\ &\pi = .5. \end{aligned} \tag{C.14}$$

An equal sharing rule of $\pi = .5$ is indeed a maximum as the second derivative of household investment with respect to π is negative under equality:

$$\left.\frac{\partial^2 x_{hh}^{NE}}{\partial \pi^2}\right|_{\pi=.5} = -\frac{4\beta W(1+\beta)(4+3\beta)}{p\big(2+2.25\beta(2+\beta)\big)^2} < 0. \tag{C.15}$$

Proof of Proposition 3 Household growth as defined in equation (3.13) is increasing in the productivity of the production technology and decreasing in its price:

$$\frac{\partial g}{\partial R} = \frac{\beta\big(4(\pi-1)\pi + \beta(\pi-2)(1+\pi) - 2\big)}{p\big(\beta(2+\beta)(\pi-2)(1+\pi) - 2\big)} > 0,$$

(C.16)

$$\frac{\partial g}{\partial p} = \frac{\beta R\big(2(1+\beta) + \pi(4+\beta) - \pi^2(4+\beta)\big)}{p^2\big(\beta(2+\beta)(\pi-2)(1+\pi) - 2\big)} < 0.$$

While these conditions hold over the range of sharing rules, they have their largest effect in the case of equality. This is confirmed when the comparative static results derived in equations (C.16) are compared over the range of π. In regards to the productivity parameter, the effect of a productivity increase on growth is largest in the case of equality, as the derivative of the growth effect with respect to π is 0 in the case of equality while the second derivative with respect to π is negative:

$$\frac{\partial^2 g}{\partial R \partial \pi} = -\frac{2\beta(1+\beta)(4+3\beta)(2\pi-1)}{p\big(2+\beta(2+\beta)(2+pi-\pi^2)\big)^2} = 0$$

(C.17)

$$2\beta(1+\beta)(4+3\beta)(2\pi-1) = 0$$

$$\pi = .5$$

(C.18)

$$\left.\frac{\partial^3 g}{\partial R \partial \pi^2}\right|_{\pi=.5} = -\frac{4\beta(1+\beta)(4+3\beta)}{p\big(2+2.25\beta(2+\beta)\big)^2} < 0.$$

While a price increase lowers the growth rate as in equation (C.16), and therefore a subsidy increases it, this effect is strongest in the case of equality as the growth effect from price with respect to the sharing rule is maximized (it's most minimum) when $\pi = .5$:

$$\frac{\partial^2 g}{\partial p \partial \pi} = \frac{2\beta(1+\beta)(4+3\beta)R(2\pi-1)}{p^2\Big(2+\beta(2+\beta)\big(2+\pi-\pi^2\big)\Big)^2} = 0$$

(C.19)

$$2\beta(1+\beta)(4+3\beta)R(2\pi-1) = 0$$

$$\pi = .5$$

(C.20)

$$\left.\frac{\partial^3 g}{\partial p \partial \pi^2}\right|_{\pi=.5} = \frac{4\beta(1+\beta)(4+3\beta)R}{p^2\big(2+2.25\beta(2+\beta)\big)^2} > 0.$$

First order necessary conditions for maximization of stochastic portfolios

Each agent chooses their levels of investment in the safe and risky technologies in order to maximize the objectives in equations (3.16) so that the following conditions are met (two for each agent):

(C.21)

$$\frac{\partial V_f}{\partial x_f} = -p(\pi W - px_f - z_f)^{-\alpha_f} + \beta E_{\tilde{R}}\Big((2-\pi)d\tilde{R}x_f^{d-1}\Big)\Big((2-\pi)\tilde{R}x_f^d - \pi\tilde{R}x_m^d + Gz_f^h\Big)^{-\alpha_f} = 0$$

$$\frac{\partial V_f}{\partial z_f} = -(\pi W - px_f - z_f)^{-\alpha_f} + \beta Ghz_f^{h-1}E_{\tilde{R}}\Big((2-\pi)\tilde{R}x_f^d - \tilde{R}x_m^d + Gz_f^d\Big)^{-\alpha_f} = 0$$

$$\frac{\partial V_m}{\partial x_m} = -p((1-\pi)W - px_m - z_m)^{-\alpha_m} + \dots$$

$$\beta E_{\tilde{R}}\Big((1+\pi)d\tilde{R}x_m^{d-1}\Big)\Big((\pi-1)\tilde{R}x_f^d + (1+\pi)\tilde{R}x_m^d + Gz_m^h\Big)^{-\alpha_m} = 0$$

$$\frac{\partial V_m}{\partial z_m} = -((1-\pi)W - px_m - z_m)^{-\alpha_m} + \beta Ghz_m^{h-1}E_{\tilde{R}}\Big((\pi-1)\tilde{R}x_f^d + (1+\pi)\tilde{R}x_m^d + Gz_m^h\Big)^{-\alpha_m} = 0.$$