FINAL REPORT

LAMINAR/TURBULENT TRANSITION IN TRANSPIRED BOUNDARY LAYERS

NSF Grant
ENG-70-01360
(GK 23989)

Prepared by
Robert N. Meroney
Principal Investigator

Fluid Dynamics & Diffusion Laboratory
College of Engineering
Colorado State University
Fort Collins, Colorado

U18401 0074124

December 1974

CER74-75RNM27

LAMINAR/TURBULENT TRANSITION IN TRANSPIRED BOUNDARY LAYERS FINAL REPORT

I. INTRODUCTION

The protection of a solid surface under conditions of intense thermal potential has become important since the advent of hypersonic rocket and satellite vehicles and since the use of turbine blades and rocket nozzles which must withstand temperatures near existing metallurgical limits. It has been evident for some time that conventional internal cooling techniques are inadequate in such applications. Mass transfer in the forms of transpiration, film cooling, or ablation has received particular attention recently as the best method to reduce high heat transfer rates. On a pound per pound basis the effectiveness of injected materials to reduce heat transfer rates greatly exceeds the heat absorbing ability of solid materials.

The problem of the boundary layer has received quite a lot of attention since the early work of Prandtl, at the beginning of this century. But much of the work so far done has been devoted to consideration of boundary layer flows over a solid surface. More recently, however, interest has been shown in the boundary layer with suction or injection, the former to maintain laminar flow or to prevent separation, the latter to provide surface cooling on the wings of high speed aircraft and turbine blades. Although the sucked laminar boundary layer can be said to have been thoroughly investigated both experimentally and theoretically, such a statement could not be made for either the transpired transitional or turbulent boundary layer. No satisfactory analytical or semi-analytical methods is, in fact, well developed for the case of laminar boundary layer instability in the presence of mass injection.

Literature Review: Transition with Transpiration

Much of the original work on the laminar boundary layer stability was done in Germany by Orr, Sommerfeld and fairly recently, Tollmein and Schlichting, (although the basic formulations may be attributed to Reynolds). It is clear that since all known laminar solutions of the hydrodynamic differential equation are valid for arbitrarily high Reynolds numbers, the origin of turbulence could only be due to an instability of the laminar flow. Reynolds and his earlier successors, notably Hamel[1], sought the limit of this laminar flow stability by energy considerations but met with little success. Later investigators, principally Rayleigh, adopting small disturbance methods were able to predict laminar flow instability fairly successfully. The work of Orr[2] and Sommerfeld[3] culminating in the Orr-Sommerfeld disturbance equation are the best known originals utilizing the small perturbation method. Although both authors did not give any solutions to their disturbance equations, considerable progress has, to date, been made in this difficult field. On this basis of small perturbations a powerful body of theory concerning the boundary layer stability has been built especially for flows over solid surfaces, although Ulrich[4], Iglisch[5] and

Pretsch[6] have considered the case with suction. But the fact cannot be overlooked that a great many other factors can also cause, modify, or control transition processes. The indication one obtains in a review of transition research seems to be that every case of transition ought to be treated as a unique case. Generalized formalisms can at best give satisfactory results only for a few cases of observed transition while probably only approximating other cases.

What seems obvious in view of these facts is an escape from the conventional classical stability theory. C. duP Donaldson[7] apparently shares this view too, for in a recent report, referred to in this report, he presents a new analytical model of transition which combines both flow energy considerations and large and small mean flow perturbation. Donaldson's method is plausible; however, being yet at a developing stage it still has many limitations. It represents, nevertheless, the desired break from convention.

An approach very similar to Donaldson's but incorporating transpiration and laying greater emphasis on a flow energy consideration was followed, and was studied concurrently with a parametric solution of the Orr-Sommerfeld equation in the presence of transpiration. The latter study provided a convenient basis on which to check the new model, in view of the near-complete lack of experimental data for transpiration in transpired flows.[8,9]

Literature Review: Laminar Transpired Boundary Layers

The success of mass transfer to reduce heat transfer from a high temperature boundary layer has been amply demonstrated by extensive experimental and analytical studies. Gas injection into laminar boundary layers has received the greater portion of all previous investigative efforts because it is amenable to more or less rigorous analytical solution. Extensive bibliographies covering this work may be found in References 10, 11, 12, 13, and 52.

Experimental mass transfer data sufficiently extensive and reliable for correlation or comparison purposes is very difficult to obtain. Although results of several experimental programs for mass transfer into a laminar boundary layer are available in the open literature,[13, 34-38, 50-53] the state of the analytical understanding of injection into a laminar boundary layer is such that correlation of the various exact solutions available is just as meaningful. A summary of the most zero pressure gradient solutions and how they are obtained is given in Reference 10; exact calculations are available for the injection of air, helium, hydrogen, carbon monoxide, water vapor, and iodine into air.[10,12,13,22,39-41,55]

Literature Review: Turbulent Transpired Boundary Layers

The greater portion of all boundary layer flows, however, occur when the turbulent boundary layer typifies the flow regime. Turbulent boundary layers are more likely to prevail even at low Reynolds numbers because: first, the gas streams in propulsive devices usually involve high levels of vorticity and turbulence; second, permeable walls are inherently rough; and third, the influence of continuous injection is destabilizing with respect to a laminar boundary layer. Turbulent boundary layers characteristically do not allow rigorous solution of the governing differential equations due to lack of knowledge concerning the turbulent exchange mechanisms.

The experimental mass transfer data for flow with a turbulent boundary layer is even less reliable and more limited than that for laminar boundary layers. In addition, there are no reliable exact solutions to fall back upon at this time. Several investigators have studied the injection of air into air in a turbulent boundary layer.[11,29,42-44] The reliability of certain of this data has been questioned. Mickley and Davis[30] noted that the results of Mickley, et al.,[11] are dubious due to a defect in model construction. Bartle and Leadon[45] were overcorrected for radiation by as much as 25 percent at the higher blowing rates, and they also noted that the data of Rubesin, et al.[43] scattered seriously.

Various investigators have performed analyses and experiments to determine the effect of distributed air transpiration through a flat surface over which air flows in a turbulent boundary layer.[11,14-16] These investigations revealed that transpiration reduces both skin friction and heat transfer to the porous surface, and that these effects are qualitatively the same as the behavior of injection on the laminar boundary layer. It is also known that injection of a light molecular gas into the boundary layer is much more effective than air in reducing the intensity of the heat or momentum exchange with the wall surface.[17-22] This beneficial effect of low molecular weight gases in providing mass transfer cooling is attributable to one or more of the following physical properties: their unusually high specific heat, low viscosity, high thermal conductivity, and high diffusivity, or also to their extended ability to occupy volume otherwise filled by the ambient fluid.[13,23] Various investigators have proposed approximate theories for predicting heat transfer and skin friction from a turbulent boundary layer in the presence of wall injection.[11,14,15,18,20,21,23] Most proposals are based on complex mixing length theories which require empirical data to accomplish a solution. The authors realized that the mixing length concept is only an approximate model and that the simplifications inherent in it gave their analyses mainly a heuristic value; however, they took the pragmatic viewpoint that such efforts had provided useful information in the past and might continue to do so.[24] Unfortunately, the results of such efforts appear to remain qualitative in nature, and a good interpretation sufficient for engineering purposes is still required.[25,26]

Extensive effort has been made to correlate the sparse experimental data available on mass transfer into turbulent boundary layers by semi-empirical blowing parameters.[21,26-29] Most authors attempt to compare the heat transfer or skin friction measured in the presence of mass transfer with the same quantity expected without mass transfer under otherwise identical flow conditions. This is done in the hope of finding a universal blowing parameter which isolates the effect of mass transfer on the boundary layer over a wide range of Reynolds numbers, Mach numbers, and property variations. Currently there appear to be as many correlation parameters suggested as there are sets of experimental data. Most trustworthy data is limited to air into air injection at low ambient temperatures in the absence of chemical reaction or dissociation. A limited amount of data is available for light molecular weight gas injection into high Mach number streams.[28,29,61]

Frequently, it is assumed that transfer mechanisms within a turbulent boundary layer are such that it may be fruitfully investigated by dividing the boundary into two separate regions--a wall region (inner 15%) and an outer or wake region (outer 90%). Recent experimental data[31,49,66] seem to be equally satisfied by semi-empirical relations suggested by Stevenson[49], Smith[31], or Meroney[74] in the outer 90% of the transpired boundary. None of these theories, however, enlighten the understanding of transfer mechanisms in such a flow.

Results of studies of the near wall region are less satisfactory. Because of inaccuracies in some of the data, which have been taken with inadequate experimental techniques, and the presence of significant three-dimensional tunnel effects in a large portion of the other available data[32,66,30,31,72] results generally have not been conclusive enough to establish even semi-empirical universal laws for the near wall region. Hence, because of inadequate verification, most of the previous theories are little more than data correlations.

II. RESEARCH OBJECTIVES

1. To develop a new analytical method based on flow energy consideration, as a basis on which to formulate a sound transpired boundary layer stability theory. This method will be very similar to C. duP Donaldson's analytical method with the included effects of mass injection. A satisfactory laminary boundary layer model has been adopted and satisfactorily modified to include the effects of blowing on major flow parameters.

2. Experimental verification of the resulting boundary layer models especially for the lower Reynolds number up to , say 10^6 (Reynolds numbers based on displacement or momentum boundary layer thicknesses). Most analytical models, or even regression equations, for the transpired turbulent boundary layer have been formulated to describe phenomena at moderate to high Reynolds number flows. A separate eddy diffusivity formulation expressly designed to describe near transition flowing effects would be appropriate. It is expected that a reformulation of the universal eddy diffusivity model by Mellor at low Reynolds numbers will be fruitful.

3. Because of the scarcity of comparative experimental data anticipated especially at the lower Reynolds numbers, a concurrent development of a parametric solution of the Orr-Sommerfeld disturbance equation incorporating blowing effects, is desirable.

4. A turbulent boundary layer model based on an effective viscosity hypothesis has already been successfully examined by the present investigators for flows over both solid and porous surfades. This model is, in fact, an improved extension of C. Mellor's[8] differential mean field model for the turbulent boundary layer. Further improvement to remove the limitations on this model due to wall curvature is essential.

III. RESEARCH RESULTS AND REPORT ABSTRACTS

A COMPUTER STUDY OF TRANSITION OF WALL BOUNDARY LAYERS, Anyiwo, J.C. & Meroney, R.N., CSU CER71-72JCA-RNM21, 188 p., 1972

Abstract

A direct solution by computer of the governing equations for the mean and fluctuating motions in incompressible, two dimensional fluid flow has been obtained using an invariant modelling technique for the triple and higher order velocity correlation terms. This provides a reasonably satisfactory data-generating procedure for predicting general two dimensional wall boundary layer data (laminar through transition to turbulent flows) for a wide variety of wall and free stream conditions, often impossible to model in wind tunnels.

It has also been shown that differential methods of wall boundary layer prediction utilizing the eddy viscosity concept, and integral relation methods, can be adapted through an implicit transition or intermittency function to provide a continuous prediction of general two dimensional wall boundary layers from laminar through transition to the turbulent regime. Such a transition function has been evaluated and tested in this work.

From a force-field theory developed in this work, the phenomena of laminar instability and laminar to turbulent transition have been reexamined with satisfactory predictions of incipient laminar instability, transition and non-linear disturbance amplification characteristics. The force-field theory emphasizes a dynamic fluid property which defines the fluid cohesiveness or ability to resist perturbations. This property, it appears, solely determines the flow characteristics. The force-field theory derives, in addition, a simple non-linear flow transfer function which provides a very simple procedure for the continuous prediction of simple two dimensional wall boundary layers.

Finally, by assuming that conceptual fluid particles in a boundary layer fluid would be arranged in continuous mean energy levels, a

statistical collision theory has been initiated to describe and predict turbulence characteristics in wall boundary layers. All preliminary results are satisfactory and justify continued pursuance of the methods demonstrated in this work.

Conclusions:

This work consisted of three main parts. In the first part, a numerical experimental technique has been developed to satisfactorily generate boundary layer experimental results. The main advantages of this numerical data generating process lie in its capability to provide simulated physical experimental data at a fast rate and for a wide variety of combinations of desirable boundary conditions which may be impossible to attain in physical experimentation. Data generated by the numerical technique show no appreciable inferiority to equivalent physical experimental data. Moreover, there seems to be greater hope of improving mathematical models of physical phenomena than of improving measurement hardware and technique particularly for fluctuating flow variables. Also, a linking mechanism in the form of an intermittency factor (or transition function) has been developed to extend the power of differential and integral relation methods to the complete prediction of boundary layers from the laminar through the transitional to the turbulent regimes.

The second part of this work was based on a force-field hypothesis, which emphasizes a dynamic fluid property called the "cohesiveness" or fluid "tenacity". This fluid property defines the ability of the fluid to resist perturbations, and appears to be the primary factor in the determination of flow characteristics. All the preliminary results derived on the basis of the force-field hypothesis show no significant deviation from reality. In fact, the incompleteness of some previous boundary layer theories become very obvious in view of the force-field idea. Boundary layer phenomena and, indeed, general natural phenomena seem to follow exactly according to this force-field concept. A general force-field theory is therefore stated as follows:

> "Particles in any system in nature will tend to execute independent behavior or motions in accordance with their separate internal force fields except as constrained by the prevalent external force-field. It requires a steady force field above a certain critical magnitude to establish 'order' among the particles. The magnitude of this critical force field is determined by the average internal force field of the particles."

For the special case of wall boundary layers, the following conclusions are drawn on the basis of the force-field theory:

(i) The laminar, transitional and turbulent boundary layers represent different phases of the basic boundary layer fluid at least with respect to the "cohesive" fluid property.

(ii) The general characteristics of the wall boundary layer may be described, completely, solely by the local "cohesive" fluid property which is a function of position and section stability number, only. The section stability number is given by the following relation:

$$SN \doteq [Re_\Delta^2 + a_1(Ra + Ta + A_t)]^{1/2} .$$

(iii) The position of incipient laminar instability is given approximately by:

$$SN_{ci} \approx 1000.$$

for a length scale $\Delta \equiv \delta^* \, 10^{-2.34 \tanh(\frac{10}{H} - 4)}$.

This corresponds to:

$$Re_{\delta^* ci} \doteq \frac{\{1 - 0.012 \ (Ra + Ta + A_t\}^{1/2}}{\{1 + 280 \ I_1^2\} \ \{1 + v_o/U_1\}} \quad 10^{\{3 + 2.34 \tanh(\frac{10}{H} - 4)\}}$$

(iv) The frequency of the most favored local disturbance is given approximately by:

$$fp \approx 0.016 \frac{U_1}{\nu_1 \eta^*} \cdot \frac{1}{Re_\Delta}$$

where $\Delta \equiv \delta^* \, 10^{-2.34 \tanh \, (\frac{10}{H} - 4)}$

is a modified boundary layer thickness.

(v) The position of incipient natural laminar to turbulent transition is given approximately by:

$$SN_{ct} \approx SN_{ci} - 1.3 \times 10^3 \ \ln(33I_1); \quad 0 < I_1 < 0.03$$

$$= SN_{ci}; \text{ otherwise}$$

i.e., $Re_{\delta^* ct} = Re_{\delta^* ci} - 674.2 \ \ln(33I_1)$.

(vi) The amplitude, K, for supercritical and critical disturbances, of the total disturbance kinetic energy at the critical layer is given approximately by:

$$K(x)_{max} \propto \frac{30\omega I_1^{1/2} \exp[0.003(SN-SN_{ci})]}{1 + 0.33 \ I_1 \ (1-\omega) \ \exp[0.003(SN-SN_{ci})]}$$

and the maximum amplification factor is given by

$$a^{*2}_{max} = \frac{\omega^{1/2} \exp[0.003(SN-SN_{ci})]}{1 + 0.33\ I_1(1-\omega) \exp[0.003(SN-SN_{ci})]}$$

(vii) The rate of production of turbulence (spots) may be given approximately as:

$$g \approx 1.53 \times 10^{-10} \left(\frac{U_1}{\nu_1}\right)^2 \{1 - \exp[-5.34 \times 10^{-5} (x-x_{ct})^2/\nu_1]\}.$$

An intermittency factor (or transition function) may then be defined to indicate what proportion, f, of the transitional boundary layer is turbulent at any instant.

$$f = 1 - \exp[-0.04\ g\ x^3/U_1]$$

(viii) A flow transfer function may be defined for a boundary layer scalar variable, q, such that:

$$q(x+_{\Delta}x) \approx G_o(\eta^*,SN_D) + G_1(\eta^*,SN_D)q(x) + G_2(\eta^*,SN_D)q^2(x) + G_3(\eta^*,SN_D)q^3(x)$$

$$0 \leq q \leq 1 .$$

The boundary layer does indeed behave closely to a linear system except in the developing regions such as the entrance region and the transitional boundary layer. Even in these developing regions, the non-linearity is not very strong except when the external and wall influences such as pressure gradient, heat and mass transfer and wall curvature, are appreciable.

The third portion of this work assumes that conceptual fluid particles in a boundary layer fluid are arranged in continuous mean energy levels according to their mean energy content. The subsequent application of a statistical "collision theory" with these conceptual fluid particles as the basic particles has yielded qualitatively very good predictions of the characteristics of turbulence in the boundary layer, such as the rates of production and dissipation of turbulence energy, and the amplification characteristics of disturbances in shear flows.

Published in the open literature as:

Anyiwo, J.C. "A Force Field Theory, Part I -- Laminar Flow Instability," AIAA J., Vol. 11, No. 1, Jan. 1973, pp. 43-49.

Anyiwo, J.C. and Meroney, R.N., "A Semi-Empirical Transition Criterion for Laminar Wall Boundary Layers," Accepted by J. of Fluid Engineering, 17 pp. (1974).

PENETRATIVE CONVECTIVE INSTABILITIES IN PARALLEL FLOW, Kahawita, R.A. and Meroney, R.N., CSU CER71-72 RK-RNM-24, 29 pp., 1972.

Abstract:

An analysis has been performed of penetrative convective instabilities arising from the combined action of thermal and centrifugal buoyancy forces. The theory allows for the fact that in the atmosphere, convection arising in an unstable layer may penetrate into a neighboring stable region. The objective has been to examine the effect of various mean temperature and velocity profiles on the critical limit and convective penetration of the disturbances. The linearized perturbation equations have been solved employing an approximate technique. The results obtained indicate that nonlinear profiles are more unstable and penetrative than linear ones. The close analogy between streamline curvature and thermal stratification effects has been demonstrated. It is found that for parallel layers of fluid along curved heated walls, a unique stability curve for neutral disturbances may be obtained if the quantity plotted along the abscissa is $Ra + \kappa N_G^2$ where Ra is the Rayleigh Number, N_G is the Goertler Number and κ a constant which expresses the relative importance of the mean temperature and velocity profiles.

Conclusions:

The object of this analysis was to explore the stability of parallel layers of fluid under the simultaneous influence of curvature and heating. The simple linear theory indicates not surprisingly, that the two effects are additive, demonstrating the close analogy between streamline curvature and buoyancy. A similar result has been obtained for the case of Thermohaline Convection by Lindberg who arrives at the conclusion that the thermal Rayleigh Number and an analogously defined "Concentration Rayleigh Number" add linearly to form a stability parameter. The stability of a few nonlinear temperature profiles was investigated and their strongly penetrative nature demonstrated. Since such profiles are not uncommon in the atmosphere, it is reasonable to expect that convective instabilities could be generated in the lower layers, causing considerable modification of the vertical transport of heat, moisture and momentum, as is stated by Estoque.

Future work should include the interaction of the Tollmein Schlichting wave instabilities with truly three dimensional convective disturbances. The disturbances analyzed here have been quasi-two-dimensional in that no variations in the streamwise direction have been assumed. The behavior of penetrative instabilities in an Ekman layer flow should also prove interesting since this approximates more closely the true atmospheric situation. Finally, the need for some simple nonlinear analyses to establish the interaction mechanism between the disturbances and the mean flow is now becoming of vital necessity.

Published in open literature as:

Kahawita, J.C. and Meroney, R.N., "Longitudinal Vortex Instabilities in Laminar Boundary Layers of Curved Heated Surfaces," Physics of Fluids, 17(9), pp. 1661-1666, 1974.

THE STABILITY OF PARALLEL, QUASI-PARALLEL AND STATIONARY FLOWS, Kahawita, R.A. and Meroney, R.N., CSU CER73-74RK-RNM12, 162 pp., 1973.

Abstract:

The methods of linear perturbation theory have been used to study the stability of various flows, among them being

(i) The stability of boundary layers along concave heated walls;

(ii) The stability of boundary layers along concave walls with suction;

(iii) The stability of wall jets along concave and convex walls;

(iv) The spin up of a two-dimensional cylinder in an infinite medium;

(v) The stability of stationary layers of fluid with arbitrary temperature stratification;

(vi) The stability of natural convection flow along inclined plates.

During the course of this work, three different solution techniques were employed; one of them was an approximate analytic technique, the remaining two were numerical. Three-dimensional spatially and temporally amplifying disturbances were considered in this study.

The results indicated that the normal velocity component of the mean flow in a boundary layer, although much smaller than the stream-wise component had a profound effect in reducing the stability of the flow. On the other hand, suction at the wall improved the stability characteristics. For the flow of parallel layers of fluid along heated walls with small curvature, it was found that a unique stability curve for neutral disturbances may be obtained if the quantity plotted along the abscissa is $Ra + K_s N_G^2$ where Ra is the Rayleigh Number, N_G is the Goertler number and K_s is a constant which expresses the relative importance of the mean temperature and velocity profiles.

It was demonstrated also that wall jets are unstable on concave as well as convex walls.

The results obtained for the stability of the spin up of a cylinder in an infinite medium are in qualitative agreement with experiment.

The dependence of the onset of convective overturning in an unstable layer of fluid with a nonlinear basic temperature profile and bounded above by fluid of varying stability on Rayleigh number was established.

The angle at which the two-dimensional wave instability passes into the three-dimensional mode in natural convection along an inclined plate was calculated. The result was found to be in good agreement

with experiment. Other results obtained for this flow were in good quantitative agreement with experiment.

Finally, some simple wind tunnel experiments with boundary layers along curved heated walls were performed. Photographic evidence of longitudinal vortices was obtained together with some qualitative data.

Conclusions:

This report presented some results of a theoretical investigation into the stability of Parallel, Quasi-Parallel and Stationary Flows. The analyses were confined to three dimensional disturbances that evolve either with distance or with time, as a result of the action of unstable body forces. The body forces considered herein were due to thermal stratification and centrifugal action. Similar analyses may be performed when the body forces are due to say Coriolis or magnetic effects.

Linearized Perturbation Theory was used to obtain the stability equations for the following flows:

(i) Parallel as well as Quasi-Parallel flows along Curved Heated Walls;

(ii) Stationary (Stagnant) Layers of Fluid with arbitrary temperature stratification;

(iii) Natural Convection along Inclined Plates.

The resulting system of coupled ordinary differential equations was solved mainly numerically using two different computing schemes. For a particular case, approximate analytical methods were also used. The computer program developed was able to handle the wide variety of flows examined. The results obtained were that:

(a) For Quasi Parallel Flows along Curved, Unheated Walls the critical curve of Goertler number versus Wavenumber did not display a minimum but extraploated to a critical Goertler number of zero at a finite value of Wavenumber. At higher wall temperatures, or at finite amplification rates, the critical curve displayed a minimum.

(b) For stagnant layers of fluid with arbitrary temperature stratification, the onset of convective activity was influenced by the nonlinearity of the profile and imposition of the semi-infinite boundary condition which tends to reduce the critical limit.

(c) For Natural Convection Flow along an Inclined Plate, the vortex instability becomes the dominant mode at angles of inclination from the vertical, greater than about 20°. At lower angles of inclination, the two dimensional wave-type oscillation dominates the transition process.

The results of the analyses presented provided lower bounds on the stability of the various flows to infinitesimal disturbances. This has been confirmed by experiment in most cases, where linear theory has been found to consistently underestimate experimental results. In the light of this experience, the theoretical results obtained are in as good agreement with experimental data (where available) as may be expected. The computer programs developed during this study have been successful in solving the large number of "stiff" differential equations describing the stability of the various flows investigated. A further step in the analysis, would be the incorporation of variable property effects which should provide results approaching the limit possible with linearized perturbation theory. Beyond this lies the domain of nonlinear analyses and numerical simulation, -- an expensive and time-consuming process.

Published in literature as:

Kahawita, R.A. and Meroney, R.N., "Vortex Mode of Instability in Natural Convection Flow Along Inclined Plates, IJHMT, Vol. 17, #5, pp. 541-548, 1974.

Kahawita, R.A. and Meroney, R.N., "The Influence of Heating on the Stability of Laminar Boundary Layers Along Concave Heated Walls," Accepted by Canadian J. of Mechanical Engineering, Fall, 1973.

Kahawita, R.A. and Meroney, R.N., "Taylor Goertner Vortices in Laminar Wall Jets Along Curved Surfaces," Canadian Symposium on Theoretical Fluid Dynamics, Proceedings of, U. of Western Ontario, London, Ontario, June 17-21, 1974.

MEASUREMENTS OF TURBULENT BOUNDARY LAYER GROWTH OVER A LONGITUDINALLY CURVED SURFACE, Meroney, R.N., CER73-74 RNM26, 69 pp., 1974.

Abstract:

The result of an "additional rate of strain" on a turbulent parcel of fluid as it undergoes even mild streamline curvature can be very large. Yet until recently skin friction and heat transfer calculations have ignored this effect. Recent measurements over turbine cascades suggest curvature influences heat transfer by an order of magnitude. In addition there exists a strong analogy between the effects of centrifugal body forces and the buoyancy body force arising in density stratified flow in a gravity field.

This note reports the results of a set of measurements of boundary layer development over convex and concave surfaces and compares the results with various turbulence models utilized in computational programs. A moderate curvature wind tunnel test section was constructed ($\delta/R \approx .01$ to $.02$) to examine the influence of curvature on boundary layer structure.

The boundary layer rate of growth, compared to that of a boundary layer in the same pressure gradient on a flat surface, was decreased on

the convex surface and increased on the convex surface by ten to twelve percent as a result of only an apparent one to two percent perturbation on the size of the source terms in the Reynolds stress equations. Measurements are available of longitudinal static wall pressure, vertical stagnation pressure and single and cross-wire anemometer voltages at a sequence of five downstream stations. Lateral transverses at six heights for two downstream stations were completed over the concave side. Analog and digital interpretation of anemometer signals provided data of $\bar{u}$, $\bar{v}$, $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$, $\overline{u'v'^2}$, $\overline{u'^2v'}$, $\overline{u'^3}$, and $\overline{v'^3}$.

Conclusions:

As a result of this investigation a body of data was developed on boundary layer flow over moderately curved convex and concave surfaces. The following conclusions may be drawn from examination of the results and comparison with recent analytical models.

Turbulent Boundary Layers along Convex Surfaces:

i) The Law of the Wall holds in a modified form along convex surfaces.

ii) Initial and subsequent decreases in the intensities of turbulence are due partly to favorable pressure gradient and partly to curvature. The curved streamlines interact with the boundary layer to inhibit vertical mixing.

iii) The shear stress decreases steeply outside the near wall region and approaches zero well inside the typical boundary layer (about 0.88 for $\delta/R \simeq .01$).

iv) A length scale correction of the sort proposed by Bradshaw suffices to predict the effect of moderate convex curvature in skin friction, shape factor, and momentum thickness.

v) A small change in curvature ($\delta/R \simeq .01$) arouses a large (10%) change in integral properties of the flow field.

Turbulent Boundary Layers along Concave Surfaces:

i) The Law of the Wall appears to hold in a modified form along concave surfaces.

ii) Concave curvature may induce parallel sets of longitudinal rolls in the turbulent boundary layer. These rolls appear to extend the height of the boundary layer and characteristically show a wave length of the order of the boundary layer thickness.

iii) As a result of increased mixing promoted by the concave curvature there is a substantial increase in the turbulent energy all across the boundary layer.

iv) The various turbulence correlations and mean velocities are distributed laterally in a wave-like manner indicating the presence of a vortex system.

v) The shear correlation coefficient appears to remain large for an extended distance from the wall before it begins to diminish.

Published in open literature as:

Meroney, R.N., "Turbulent Boundary Layers Growth Over a Longitudinally Curved Surface,Accepted by AIAA J., June 1974.

IV. FINAL RECOMMENDATIONS

It is expected, of course, that new information developed by this research will be incorporated into the growing body of literature describing flow instabilities. It is hoped that prediction procedures provided will alow design decisions to be made where effects of transition are critical. Additional problems which warrant consideration remain. They are:

1. The completion of a datum set of measurements concerning the influence of transpiration on transition (this particular problem is being pursued with the completion of a suitable flow facility at CSU.)

2. The analytical/or numerical linkage of the various subprocesses of turbulent transition. how important are the three-dimensional mechanisms? Can they be suitably approximated in two dimensional numerical programs?

3. What is the joint influence of transition on the transport of heat, moisture, or scalar tracers as compared to the influence on momentum transport?

V. REFERENCES

1. Hamel, G., "Zum Turbulenz Problem." Nachr. Ges. Wiss. Gött. (1911).

2. Orr, W. M. F., "The Stability or Instability of Steady Motions of Perfect Liquid and of a Viscous Liquid." Part I: A Perfect Liquid; Part II: A Viscous Liquid. Proc. Roy. Irish Acad. 27, 9-68 and 69-138, (1907).

3. Sommerfeld, A., "Ein Beitrag zur Hydrodynamischen Erklärung der Turbulenten Flüssigkeitsbewegungen." Atti del 4. Congr. Internat. de Mat., Vol. III, 116-124, Roma 1908.

4. Ulrich, A., "Theoretische Untersuchungen über die Widerstandserparnis durch Laminarhaltung mit Absaugung." Schrift. dtsch. Akad. d. Luftfahrtf., 8B (1944), s. 427.

5. Iglisch, R., "Exakte Berechnung der Laminaren Reibungsschicht an der Längs Angeströmten ebenen Platte mit Homogener Absaugung." Schrift, dtsch. Akad. Luftfsch, 8B (1944), N.A.C.A. TM No. 1205 (1949).

6. Pretsch, J., "Umschlagbeginn und Absaugung," Jb. dtsch. Luftfahrtf., 1 (1942), s. 1-7.

7. C. duP Donaldson, "A Computer Study of an Analytical Model of Boundary Layer Transition." AIAA Journal Vol. 7, No. 2, February 1969.

8. Lin, C. C., "Theory of Hydrodynamic Stability." (C.U.P. 1955).

9. Betchov, R. and W. D. Criminale, Jr., "Stability of Parallel Flows." Applied Math. and Mechanics Series Vol. 10, (Academic Press, 1967).

10. Gross, J. F., J. P. Hartnett, E. J. Masson, and C. Gazley, Jr., "A Review of Binary Boundary Layer Characteristics," RAND Corp., Santa Monica, Calif. Report P1729 (1959). (IJHMT 3, 198-221, 1961).

11. Mickley, H. S., R. C. Ross, A. C. Squyers, and W. E. Stewart, "Heat, Mass and Momentum Transfer for Flow over a Flat Plate with Blowing or Suction," NACA TN-3208 (1955).

12. Brown, B. W. and P. L. Donoughe, "Tables of Exact Laminar Boundary Layer Solutions When the Wall is Porous and Fluid Properties are Variable," NACA TN-2479 (1951).

13. Dorrance, W. H., Viscous Hypersonic Flow, McGraw-Hill Book Company, Inc., New York (1962).

14. Dorrance, W. H. and F. J. Dore, "The Effects of Mass Transfer on the Compressible Turbulent Boundary Layer Skin Friction and Heat Transfer," J. Aeronaut. Sci. 21, 404 (1954).

15. Rubesin, M. W., "An Analytical Estimation of the Effect of Transpiration Cooling on the Heat Transfer and Turbulent Boundary Layer," NACA TN-3341 (1954).

16. Brunk, W. E., "Experimental Investigation of Transpiration Cooling for a Turbulent Boundary Layer in Subsonic Flow Using Air as a Coolant," NACA TN-4091 (1957).

17. Eckert, E. R. G., P. J. Schneider, and F. Koehler, "Mass Transfer Cooling of a Laminar Air Boundary Layer by Injection of a Light Weight Gas," Heat Transfer Lab., Univ. of Minn. Tech. Note No. 8 (1956).

18. Van Driest, E. R., "On the Mass Transfer near the Stagnation Point," RAND Symposium on Mass Transfer Cooling for Hypersonic Flight, June (1957).

19. Denison, M. R., "The Turbulent Boundary Layer on Chemically Active Ablating Surfaces," J. Aeronaut. Sci. 28, 271 (1961).

20. Ness, N., "Foreign Gas Injection into a Compressible Turbulent Boundary Layer on a Flat Plate," J. Aeronaut. Sci. 28, 645 (1961).

21. Rubesin, M. W. and C. C. Pappas, "An Analysis of the Turbulent Boundary Layer Characteristics on a Flat Plate with Distributed Foreign Gas Injection," NACA TN-4149 (1958).

22. Libby, P. A. and M. Pierruci, "Laminar Boundary Layer with Hydrogen Injection Including Multicomponent Diffusion," AIAA J. 2, 2118-2126 (1964).

23. Tifford, A. N., "On Surface Mass Transfer Effects in a Binary Fluid," Aeronautical Res. Labs. Report ARL-62-396 (1962).

24. Townsend, A. A., "The Structure of the Turbulent Boundary Layer," Proc. Cambridge Phil. Soc. 47, Part 3, 373-395 (1951).

25. Woodruff, L. W., (Unpublished Report), Boeing Aircraft Co., Seattle, Wash., Report D2-22-02 (1962).

26. Knuth, E. L. and H. Dershin, "Use of Reference States in Predicting Transport Rates in High-Speed Turbulent Flows with Mass Transfers," Int. Jn. Heat and Mass Transfer 6, 999-1018 (1963).

27. Stewart, J. D., "Transpiration Cooling an Engineering Approach," General Electric Co., Missile and Space Dept., Document R59SD338 (1959).

28. Scott, C. J., G. E. Anderson, and D. R. Elgin, "Laminar, Transitional, and Turbulent Mass Transfer Cooling Experiments at Mach Numbers From 3 to 5," Inst. of Tech., Univ. of Minn., Research Report 162, AFOSR TN 59-1305 (1959). (ASTIA AD 231 974).

29. Bartle, E. Ray and B. M. Leadon, "Mass Transfer Cooling on a Turbulent Boundary Layer," NTFMI, Proceedings of, Stanford Univ. Press, pp. 27-41 (1962).

30. Mickley, H. S. and R. S. Davis, "Momentum Transfer for Flow Over a Flat Plate with Blowing," NACA TN-4017 (1957).

31. Smith, D. A., "The Transpired Turbulent Boundary Layer," Dr. Sci. Thesis, Chem. Eng. Dept., M.I.T. (May 1962).

32. Goodwin, B. M., "Summary of the Transpired Turbulent Boundary Layer with Zero Pressure Gradient," Dr. Sci. Thesis, Chem. Eng. Dept., M.I.T. (May 1961).

33. Fraser, M. D., "The Equilibrium Transpired Turbulent Boundary Layer on a Flat Plate," Dr. Sci. Thesis, Chem. Eng. Dept., M.I.T. (October 1964).

34. Baron, J. R., "The Binary Boundary Layer Associated with Mass Transfer Cooling at High Speed," M.I.T. Naval Supersonic Lab. Report 160 (1956).

35. Sziklas, E. A. and C. M. Banas, "Mass Transfer Cooling in Compressible Laminar Flow," In D. J. Masson's (comp.) Mass Transfer Cooling for Hypersonic Flight, Rand Corp., Santa Monica, Calif. Paper S-51 (1956).

36. Eckert, E. R. G., A. A. Hayday, and W. J. Minkowycz, "Heat Transfer, Temperature Recovery, and Skin Friction on a Flat Plate Surface with Hydrogen Release into a Laminar Boundary Layer," Univ. of Minn. Heat Transfer Lab. Tech. Report 27 (1961).

37. Gollnick, A. F., "Thermal Effects on a Transpiration Cooled Hemisphere," J. Aeronaut. Sci. 29, 538-590 (1960).

38. Craven, A. H., "The Compressible Laminar Boundary Layer with Foreign Gas Injction," College of Aeronautics, Cranfield, England Report 155 (1962). (ASTIA AD 13819).

39. Eckert, E. R. G., P. J. Schneider, A. A. Hayday, and R. M. Larson, "Mass Transfer Cooling of a Laminar Boundary Layer by Injection of a Light Weight Foreign Gas," Jet Propulsion 28, 34-39 (1958). (ASTIA AD 143 395).

40. Livingood, J. N. B., and P. L. Donoughe, "Summary of Laminar Boundary Layer Solutions for Wedge-Type Flow over Convection and Transpiration Cooled Surfaces," NACA TN-3588 (1955).

41. Sziklas, E. A., "An Analysis of the Compressible Laminar Boundary-Layer with Foreign Gas Injection," United Aircraft Corp. Report SR-0539-8 (1956).

42. Tewfik, O. E., E. R. G. Eckert, and L. S. Jurewicz, "Measurement of Heat Transfer from a Circular Cylinder to an Axial Stream with Air Injection to a Turbulent Boundary Layer," Univ. of Minn. Heat Transfer Lab. Tech. Rept. 38, AFOSR 1397 (1961).

43. Rubesin, M. W., C. C. Pappas, and A. F. Okuno, "The Effect of Fluid Injection on the Compressible Turbulent Boundary Layer--Preliminary Tests on Transpiration Cooling of a Flat Plate at M=2.7 with Air as the Injected Gas," NACA RM A55119 (1955).

44. Leadon, B. M. and C. J. Scott, "Transpiration Cooling Experiments in a Turbulent Boundary Layer at M=3," J. Aeronaut. Sci. 23 (1956).

45. Bartle, E. R. and B. M. Leadon, "Experimental Evaluation of Heat Transfer with Transpiration Cooling in a Turbulent Boundary Layer at M=3.2," J. Aeronaut. Sci. Readers Forum 27, 78-80 (1960).

46. Stevenson, T. J., "Turbulent Boundary Layer with Transpiration," AIAA J. 2, 1500-1502 (1964).

47. Clarke, J. H., H. R. Menkes, and R. A. Libby, "A Provisional Analysis of Turbulent Boundary Layer with Injection," J. Aeronaut. Sci. 22, 255 (1955).

48. Black, T. J. and A. J. Sarnekei, "The Turbulent Boundary Layer with Suction or Injection," ARC (Great Britain) Report ARC 20, 501, FM 2745 (1958).

49. Mickley, H. S. and K. A. Smith, "Velocity Defect Law for a Transpired Boundary Layer," AIAA J. 1, 1685 (1963).

50. Sparrow, E. M. and J. B. Stan, "The Transpiration Cooled Flat Plate with Various Thermal and Velocity Boundary Conditions," IJHMT Vol. 9, No. 5, p. 508, May 1966.

51. Kubata, T. and F. L. Fernandez, "Boundary Layer Flows with Large Injection and Heat Transfer," AIAA J., Vol. 6, No. 1, January 1968, pp. 22-28.

52. Lachmann, G. V., Ed., "Boundary Layer Control," Vol. I and II. Pergamon Press, N.Y., 1961.

53. Dewey, C. F., Jr., et al., "The Laminar Boundary Layer with Large Pressure Gradient and Surface Mass Transfer," Rand Corporation, August 1967, AD657749.

54. Frea, W. J., and J. H. Hamelink, "Heat Transfer from the Wall of a Porous Solid Involving Gas Injection and Vaporization," 68-HT-46.

55. King, William S., "The Non-Similar Boundary Layer with Blowing," Aerospace Corp. El Segundo, Calif., AD 666 913 TR 015 (3240 10) 7, SAMSO TR 68-97 F0469567 C 0158 44P, February 1968.

56. Fannelop, T. K., "Displacement Thickness for Boundary Layer with Surface Mass Transfer," AIAA J., Vol. 4, No. 6, p. 1142, June 1966.

57. Baker, E., "Influence of Mass Transfer on Surface Friction at a Porous Surface," Imperial College, London, January 1967, TWF/R/3.

58. Rosenbaum, "Turbulent Compressible Boundary Layer on a Flat Plate with Heat Transfer and Mass Diffusion," AIAA J., Vol. 4, No. 9, September 1966, p. 1548.

59. Drake, R. L., and F. J. Molz, "A First Approximation for Flow Through a Porous Tube", ASME publication, 69-FE-44, March 1969.

60. McQuaid, J., "Aeronautical Research Council London Experiments on Incompressible Turbulent Boundary Layers with Distributed Injection," AD 847 236, FLD/GP 20/4, January 1967.

61. Wooldridge, C. E. and R. J. Muzzey, "Boundary Layer Turbulence Measurements with Mass Addition and Combustion," AIAA J., Vol. 4, No. 11, pp. 2009-20016, November 1966.

62. Kinney, R. B., "Skin Friction Drag of a Constant Property Turbulent Boundary Layer with Uniform Injection," AIAA Journal, Vol. 5, #4, April 1967, pp. 624-630.

63. Muzzy, R. J., "Surface Mass Addition into Turbulent Boundary Layer," AIAA Journal, Vol. 5, #5, pp. 1029-1032, May 1967.

64. King, W. S., "The Non-Similar Boundary Layer with Blowing," Aerospace Report TR-0158 (3240-10)-7 Air Force Report, February 1968. SAMSO-TR 68-97, AD 666913.

65. Julliand, Michel, "Ire These, Etude d'un ecoulement en conduite cylindrique poreuse a faible nombre de Reynolds, 2eme These, Mesure des flux de chaleur," Instationnaire au tube a choc: sonde a film metallique mince, 21 Fevier 1969.

66. Kendall, R. M., "Interaction of Mass and Momentum Transfer in the Turbulent Boundary Layer," Sc.D. Thesis, Chem. Engr. Dept., M.I.T. (1959).

67. Dvorak, F. A. and M. R. Head, "Effect of Uniform Injection on Heat Transfer in the Constant Property Turbulent Boundary Layer," ASME Publication, 68-WA/Ht-24 August 1968.

68. Simpson, R. L., R. J. Moffat and O. W. M. Kays, "The Turbulent Boundary Layer on a Porous Plate: Experimental Skin Friction with Variable Injection and Suction," IJHMT, Vol. 12, pp. 771-789, July 1969.

69. Dvorak, F. A., and M. R. Head, "Effect of Uniform Injection on Heat Transfer in the Constant Property Turbulent Boundary Layer," ASME Publication 68-WA/Ht-24, 1969.

70. Dershin, H., C. A. Leonard, and W. H. Gallagher, "Direct Measurement of Compressible Turbulent Boundary Layer Skin Friction on a Porous Flat Plate with Mass Injection," 5th Aerospace Sciences Mtg., Jan. 23-26, 1967 AIAA Paper 67-194.

71. Jeromin, L. O. F., "An Experimental Investigation of the Compressible Turbulent Boundary Layer with Air Injection," Aeronautical Research Council, A.R.C. 28 549, 25th, Nov. 1966 (F.M. 389) AD 813273.

72. Fraser, M. D., "A Study of the Equilibrium Transpired Turbulent Boundary Layer on a Flat Plate," Sci. D. Thesis, Chem. Engr. Dept., M.I.T. (1964).

73. Meroney, R. N., "The Effect of Mass Injection on Heat Transfer from a Partially Dissociated Gas Stream," Ph.D. Thesis, Dept. of Mech. Engr., Univ. of California, Berkeley (1965).