DESIGN HYDROGRAPHS FOR VERY SMALL WATERSHEDS FROM RAINFALL

By

B. M. Reich

CIVIL ENGINEERING SECTION
COLORADO STATE UNIVERSITY
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Colorado State University
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[1/] This material is copied with the permission of the Dean of the Graduate School from a dissertation accepted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy at Colorado State University.

[2/] Presently stationed as: Hydrologist, Drakensberg Conservation Area, Department of Agricultural Technical Services, Estcourt, Natal, Republic of South Africa.

CONTENTS - continued

FIGURES

TABLES

ABSTRACT

Individual flood hydrographs observed on agricultural and range land throughout the United States of America were described by a three-parameter mathematical model. The values of the three hydrograph parameters were related to topography, land use and rainfall associated with each flood event.

Three separate multiple regressions were developed, each for predicting one of the hydrograph parameters from three independent variables. Thirty-six independent variables were tested including many which have been suggested by previous workers and which are incorporated in current design methods.

Multiple linear regression, with three independent variables, accounts for 80 percent and 61 percent of the variance in the volume and peak rate of runoff respectively. The third parameter a time variable, which also defines the hydrograph recession, can be predicted by a logarithmic relationship involving only soils, land use and topography so that about 60 percent of its variance is explained.

Due to the sample size being limited to forty-seven flood events, the developed regressions do not provide a rigorous formula for design. They are presented rather to show that this approach provides a simple method by which hydrographs could be synthesized. The discussion suggests that a large-scale study could provide the desired accuracy, and indicates ways of achieving this.

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Chapter 1

INTRODUCTION

Many efforts (19)* have been made towards rationalizing flood flow estimation for very small watersheds within the past five years. A large number of peak-flow estimates are still being made by inadequate means. Firstly, there is a paucity of hydrographs observed on watersheds within the range of one-third to four square miles. Secondly, the subject is confused by the many design methods cited in handbooks without reference to their regional restriction and limited basis.

The time seems appropriate to associate the flood hydrograph with the causative rainstorm. The intensity-frequency-duration regime for rainfall in the United States of America has been thoroughly investigated by the Weather Bureau (11). Designers should be provided with methods for estimating flood-hydrographs with commensurate accuracy.

Objectives

The purpose of this study was to describe flood hydrographs observed on small agricultural and range land watersheds by a mathematical model with three parameters. These hydrograph parameters were then correlated to other numerical variables which describe: the rainstorm causing each particular flood, the topographic characteristics of the watershed, and the soil and current land use. If this could be successfully achieved it would provide a means of synthesizing flood hydrographs for ungaged watersheds. The present work should be viewed as a pilot study to test the validity of the approach on readily available data. Its success should signal the analysis of a large quantity of unpublished hydrologic data. This pilot study could greatly reduce the expense of such a large-scale follow up by establishing techniques and by showing which variables do not warrant further study.

The multiple regressions obtained at the conclusion of this study only represent tentative relations. Their reliability can best be judged by seeing how well they predict results in a new set of observed data. Truer regressions with narrower confidence intervals will doubtless arise as more and more data become available.

Delineations

The forty-seven hydrographs and hyetographs used in this study were observed by the U. S. Agricultural Research Service (23) on fourteen watersheds. Arranged according to the code numbers given them by the Agricultural Research Service, all watersheds are listed in Table 1. They were located in the following eleven widely spaced states: Virginia, Illinois, Iowa, Ohio, Wisconsin, Texas, Nebraska, Arizona, New Mexico, Washington and Mississippi.

The largest watershed studied had an area of 2086 acres (3. 26 square miles). The smallest was 290 acres (0. 45 square miles). Beyond this arbitrary size range, the next-smaller and next-larger watersheds for which the A. R. S. had published data were 187 acres (. 292 square miles) and 4430 acres (6. 93 square miles). Thus one may generalize by saying that the present study aimed at presenting results applicable to watersheds within the general size range from one-third to four square miles.

Only floods caused by rainstorms were considered. This is justified on very small watersheds, since the flood series (21) from these stations shows the annual maxima to occur almost exclusively in the summer months, and not to be due to snow melt.

Certain watersheds seem to produce a characteristic double-peaked hydrograph. Although the mathematical model fitted was single peaked, not all the observed double peaks were excluded from the study. The approach was rather to approximate them with a broad-crested theoretical hydrograph whose parameters could hopefully be related to the particular basin characteristics responsible for this unusual watershed response (28, 29, 30). Only one hydrograph with clearly separated peaks was omitted from the study. It was the result of a storm moving transversely across a small belt of long twin tributaries. The peak rate was considerably less than for the three hydrographs actually used for this watershed. From the point of view of predicting design floods its omission seems desirable. Furthermore, uneven areal distribution is an undesirable feature for this study of very small watersheds.

The basic approach was to use the total observed runoff. That is to say, groundwater and interflow were not subtracted as in the unit hydrograph attempt to isolate "surface runoff." A simple adjustment was made to the observed hydrographs to account for antecedent flow when this was present.

It should be stressed that the forty-seven hydrographs originally selected from the available fifty were retained for the remainder of the study. The degree to which the mathematical model could be fitted to the corrected "observed" hydrographs, in no way affected their retention in the study. Likewise no further adjustments were made to the corrected "observed" hydrographs in an attempt to influence the regressions. Whatever deviations

*Numbers in parenthesis refer to items in the "Bibliography."

Table 1. Names, locations and areal extent of experimental watersheds together with the dates of storms used in this study.

Watershed Identification: ARS Code Number	Watershed Identification: Locality	Watershed Area, Acres	Years of Record	Date of Flood Event
15.1	W-I, Staunton, Virginia, Bell Creek	390	8	8 Sept. '48 13 Apr. '49 7 June '55
17.4	W-4, Edwardsville, Illinois	290	18	27 May '38 21 June '42 31 March '52 2 July '52
21.1	Ralston Creek, Iowa City, Iowa	1,926	34	1 June '43 21 July '48 1 July '50 18 July '56
26.30	Watershed 196, Coshocton, Ohio	303	24	16 June '46 16 Aug. '47 1 Sept. '50 12 June '57
29.1	W-I, Colby, Wisconsin	345	13	28 July '49 13 May '56 4 June '58
31.1	W-I, Fennimore, Wisconsin	330	24	12 Aug. '43 11 July '44 28 June '45 24 June '49
42.3	Watershed D, Riesel (Waco), Texas	1,110	18	10 June '41 15 June '42 15 July '50 24 April '57
44.1	W-3, Hastings, Nebraska	481	23	20 June '39 10 July '51 7 June '53 15 June '57
44.3	W-8, Hastings, Nebraska	2,086	23	10 July '51 7 June '53 29 Aug. '57
45.2	W-II, Safford, Arizona	682	21	26 July '40 28 Sept. '41 7 Aug. '42 9 Aug. '43
48.2	W-2, Mexican Springs, New Mexico	610	6	28 July '39 24 Aug. '39 26 Aug. '39 5 Sept. '40
60.6	Watershed G.S. 8 Pullman, Washington	762	8	13 April '37 25 Jan. '41 3 March '41 6 June '41
62.1	W-4, Oxford, Mississippi	2,000	-	22 May '57
62.2	W-5, Oxford, Mississippi	1,130	-	22 Jan. '57

remained in the hydrograph fitting and unexplained variance of the regressions were assumed to be mainly sampling error and random fluctuations. It is recognized that spatial variations in rainfall, cover, and other factors constitute the physical reasons for some of the aberrations.

Need for the Study

The design of an ever increasing number of hydraulic structures involves the estimation of flood runoff from small watersheds. As road building and headwater flood control programs gain momentum, more and more capital will be involved in culverts, valley fills and small spillways. Most of these structures are not costly enough to warrant detailed individual hydrologic investigations. Yet the cost of perfecting a universal method which could be applied quickly and easily to each field situation would bring about considerable savings. Currently much public and private capital is wasted on hydraulic structures, many of which are either overdesigned or fail due to the underestimation of floods.

By virtue of temporary storage behind road fills and above the spillway level of dams, flood outlets may sometimes be designed to discharge less than the inflow hydrograph's peak. With a view to such future refinements in design, an estimation procedure which predicts the shape of the hydrograph would be more valuable than one merely giving the peak rate.

A need also exists to move away from the regional flood frequency approach. Both it and envelope curves for flood peaks assume adequate spatial sampling and a certain degree of homogeneity throughout the region. Anomolous results will always be presented in the adjacent belts of two such regional syntheses. Relationships involving the causal elements such as rainfall intensity, land slope and soil properties would be far more realistic. In fact such regressions could reasonably be used beyond the region where the hydrographs were obtained. In the era of American technical aid to underdeveloped countries, such rational methods could be used overseas, since the topographic, soil and rainfall information can be obtained far more readily than can flood observations.

Recent Related Studies

The most closely related work to the proposed study was reported by Gray (9, 10) in 1960. In that study forty-six watersheds ranging in size from 0.27 to 32.64 square miles were studied from the unit hydrograph approach. Their range in size far exceeded that in the present study. On the other hand Gray's hydrographs came exclusively from Illinois, Iowa, Missouri, Nebraska, Ohio, Wisconsin and North Carolina where the climatic range is far narrower than that encompassed by the present study's watersheds, which represent eleven states. In contrast to Gray's approach, this investigation does not involve a representative distribution graph for each watershed. Individual differences between hydrographs observed on the same watershed have been retained here and accounted for by rainfall peculiarities. Another difference between the two studies is that Gray employed a two-parameter gamma distribution whereas the present investigation fits a three-parameter equation, which should be far more flexible, to the observed hydrographs.

Benson (2) showed, as has been suggested by Nash (17) and others, that after three or four independent meteorologic and physiographic variables have been used, further variables do not appreciably decrease the standard error in estimating floods. Benson's analysis eliminates the effect of variation of individual storms since flood peaks, of specified return periods, obtained from a frequency analysis of annual maxima were used as his dependent variable. The main-channel slope was found next in importance to drainage-area size. Benson's study has little application to very small watersheds, since only three of the 170 New England stations which he studied possessed areas of less than ten square miles.

Hickok, Keppel and Rafferty (12) made a significant contribution to hydrograph synthesis. They studied about 130 hydrographs and hyetographs from fourteen watersheds ranging in size from 11 to 790 acres in the arid Southwest. Lag time was related to watershed area, average land slope and drainage density. The estimated lag time was used to predict the hydrograph peak rate for an assumed total volume of runoff. Finally the entire synthesized hydrograph could be obtained from a generalized hydrograph, expressed dimensionlessly in terms of lag time and peak rate. Their dimensionless hydrograph appeared to be independent of rainfall pattern and of soil and cover condition. This simplification probably resulted from their four research localities possessing similar climatic and cover conditions.

A recent article by Chow (3) presents a method for determining peak discharges from rural watersheds which are smaller than 6000 acres. By trial and error the method enables one to ascertain which duration of rainfall excess gives the maximum rate of runoff and to estimate the latter by applying four charts. The method involves runoff curve numbers and relationships presented by the U. S. Soil Conservation Service (24). Combined with this is the concept of a peak-reduction factor which is defined as the ratio of the peak discharge to the equilibrium direct discharge. Although the charts presented are exclusively for Illinois, these first two phases of the method are entirely general and could be applied universally with rainfall data (11). To complete the procedure it is necessary to express the peak reduction factor as a function of the ratio of the duration of rainfall excess to lag time. The lag time must therefore also be estimated from watershed characteristics. Chow obtained these two relationships from fifty-three storms covering twenty small watersheds in the Midwest. Until similar relationships are available for other climatic and topographic areas the method will be regionally restricted.

Potter (18) has recently consolidated much of his earlier work into a very clear method of estimating floods from areas ranging in size between 0.16 and 25 square miles. Peak rates of runoff with return periods from ten to fifty years can be obtained from a topographic index and the 10-year 60-minute rainfall. Four graphical correlations are presented, each applicable to a zone of characteristic underlying rock formation. Based, as it is, on ninety-five gaged watersheds the method represents considerable unification over most of the United States east of the 105^0 meridian, to which it applies.

A recent discussion of unit hydrograph shapes for watersheds of 27.2, 50 and 290 acres has been presented by Minshall (14). He shows that the "unit hydrograph" is not a constant for one small watershed but that its peak rate and the time from beginning of excess rainfall to peak rate of runoff may be related to rainfall intensity and storm (time) pattern.

Nash (16) has discussed various methods of determining the relation between effective rainfall and storm runoff as particular cases of the general unit-hydrograph theory. His remarks are valuable to anyone relating physical characteristics of a catchment to its indical response.

No review of literature would be complete without mentioning some of the attempts to arrive at hydrographs purely from mathematical considerations. Although apparently overlooked by many later writers, the three parts which Zoch (28, 29, 30) published of an incomplete series, twenty-five years ago, showed great promise. Commencing with simplifying assumptions of rectangular or triangular watersheds and uniform rainfall rates he progressed through to the development of complex mathematical expressions for runoff under various complex conditions. More recent examples of this approach are papers by Dooge (5) and Edson (6).

Chapter 2

MATHEMATICAL TECHNIQUES

In this chapter some of the mathematical equations basic to this analysis of the hydrograph will be developed. The application of certain common statistics to the description of a rainstorm will be discussed. Finally a brief description will be presented of the regression analysis technique, which will be applied to the data in the next two chapters.

Hydrograph Model

Equation and its integration - It was decided to pursue the assumption of Yevdjevich (27) and attempt to fit the three parameter Pearson type III function to the discharge hydrographs. For the purpose of the present study this is expressed by the equation

$$q = q_o \, e^{-\frac{t}{G}} \left[1 + \frac{t}{m}\right]^{\frac{m}{G}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (1)$$

where the symbols are explained in Fig. 1.* Integrating gives the total volume of runoff,

$$W = \int_{-m}^{\infty} q_o \, e^{-\frac{t}{G}} \left[1 + \frac{t}{m}\right]^{\frac{m}{G}} dt \, .$$

Let $t + m = u$, then $dt = du$

and $W = q_o \, e^{\frac{m}{G}} \int_{o}^{\infty} e^{-\frac{u}{G}} \left[\frac{u}{m}\right]^{\frac{m}{G}} du \, .$

Let $y = \frac{u}{G}$. $\therefore$ $G\,y = u$ and $G\,dy = du$,

and

$$W = q_o \, e^{\frac{m}{G}} \left[\frac{G}{m}\right]^{\frac{m}{G}} G \int_{o}^{\infty} e^{-y} \, y^{\frac{m}{G}} \, dy \, .$$

Whence $W = q_o \, e^{\frac{m}{G}} \left[\frac{G}{m}\right]^{\frac{m}{G}} G\,\Gamma \left[1 + \frac{m}{G}\right] \quad . \; . \; . \; . \; . \; (2)$

because by definition $\Gamma(1 + x) = \int_{o}^{\infty} e^{-y} \, y^{x} \, dy$.

*Besides their definition in the text, frequently used symbols are listed along with their units and dimensions in Table 14 of the Appendix.

Apart from the ordinate q and abscissa t , equation (1) contains parameters q_o , m and G. Three parameters must therefore be specified to determine the particular shape of this model. It therefore possesses more flexibility with which it may be fitted to observed hydrographs than do the two parameter models employed by earlier workers (6, 9).

Interchangeability of time parameters, G and m - Equation (2) shows that if the volume of runoff, W , is given as well as q_o , then G and m are uniquely interrelated. Consequently m may be omitted and G used as the final definitive parameter, along with W and q_o . This change is desirable in practice, since m is difficult to ascertain and G has a physical significance.

Graphical determination of hydrograph parameters - Equations (1) and (2) uniquely determine q as a function of t , for each set of values of q_o , G and W . Complexity of the mathematical relations precludes an explicit solution, but the following graphical solution was found satisfactory:

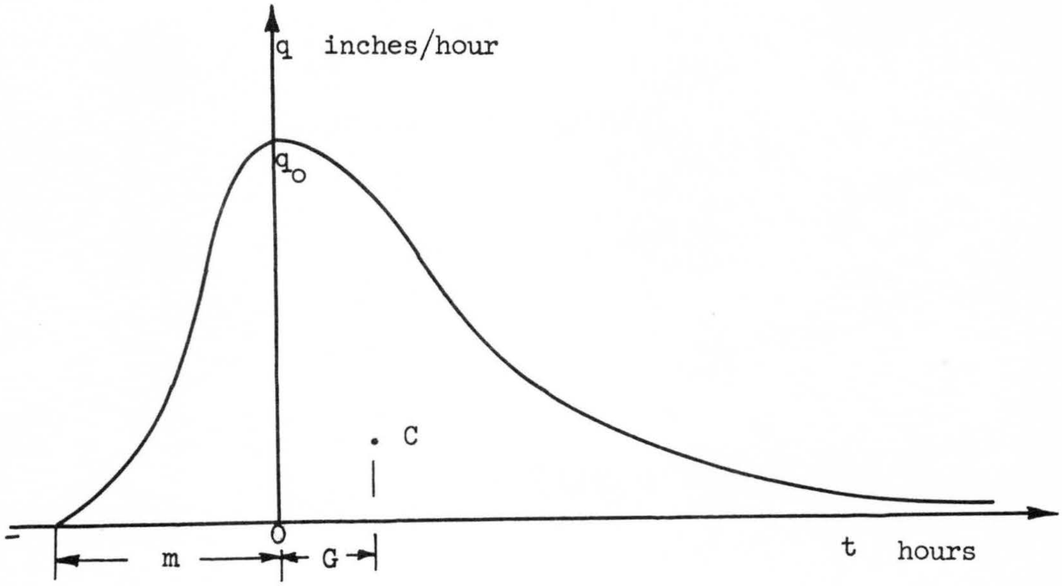
Equation (2) may be rewritten as

$$\alpha = \frac{\left[\frac{W}{q_o}\right]}{G} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (3)$$

where $\alpha = e^{\frac{m}{G}} \left[\frac{G}{m}\right]^{\frac{m}{G}} \Gamma \left[1 + \frac{m}{G}\right] \quad . \; . \; . \; . \; . \; . \; (4)$

W over q_o is grouped within parentheses in equation (3) since it represents a fixed quotient for each hydrograph of known volume and peak. Various values of G were tried and each will specify a different hydrograph for the particular $\frac{W}{q_o}$ ratio.

The selection of each G fixes the value of α from equation (3). This, in turn, assigns a unique value to $\frac{m}{G}$ according to equation (4). To obviate the repeated trial and error solution for the latter from the tables, the relationship was expressed graphically as illustrated in Fig. 2. Thus for each G selected only one ratio of $\frac{m}{G}$ satisfied equations (3) and (4) simultaneously. In summary, many paired values of G and m are available which satisfied equation (2) for each hydrograph of fixed W and q_o .



m = time from commencement of runoff to peak discharge rate, measured on the negative side of the origin.

C = center of gravity of hydrograph.

G = time between center of mass of runoff and peak discharge rate.

q = discharge rate from watershed in inches per hour; discharge rates at the outlet have been divided by the area of the watershed and converted to depth per unit time.

q_o = peak rate of discharge; converted to inches per hour as described above.

$W = \int_{-m}^{\infty} q \, .dt$ = total runoff volume in inches: consistent with the above treatment of q and q_o, division by watershed area has reduced the units to $\frac{\text{volume}}{\text{area}}$ = length. Nevertheless, the popular usage of the term "hydrograph volume" has been retained.

Fig. 1 Mathematical model of flood hydrograph.

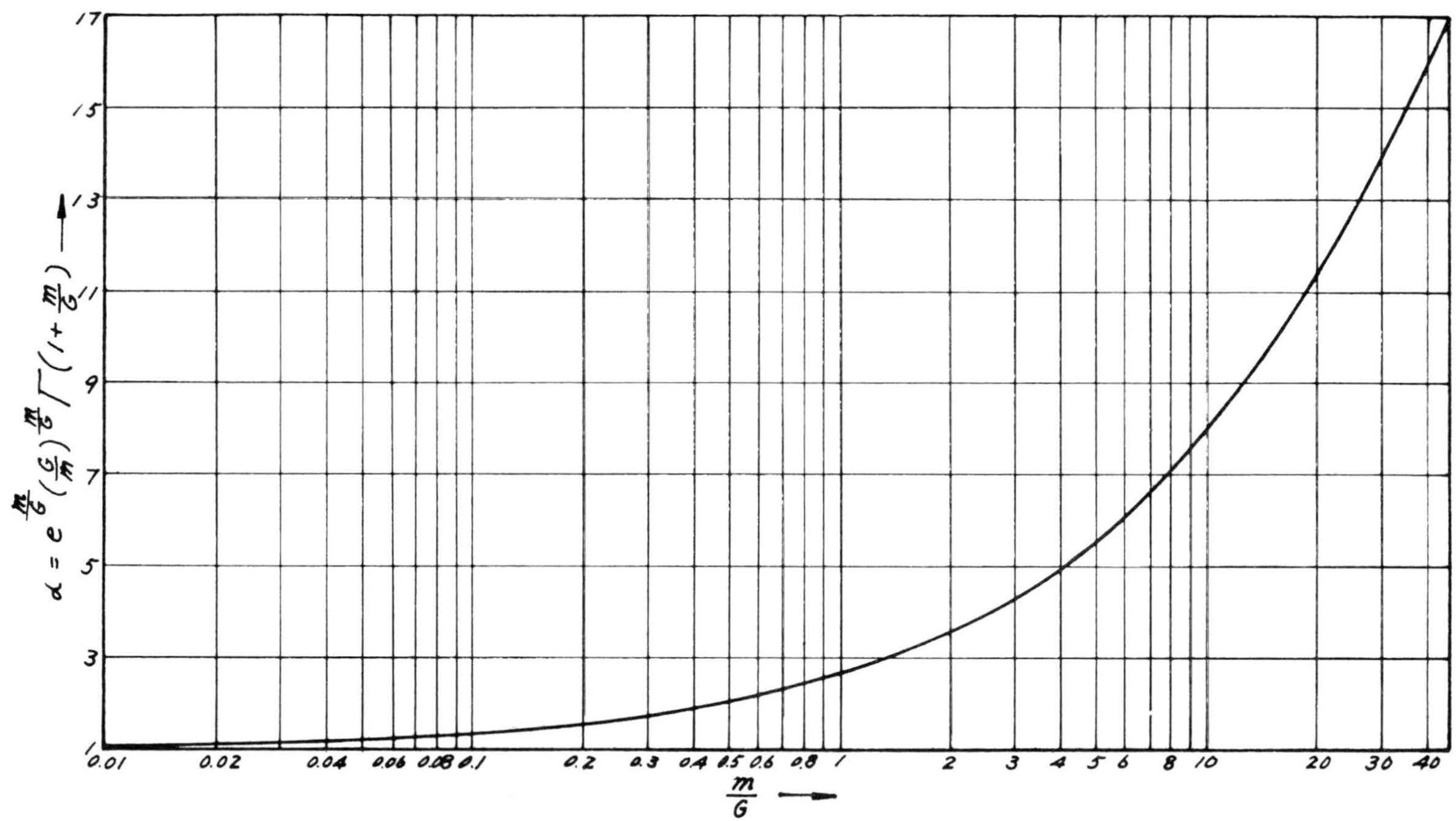


Fig. 2 Functional relationship between $\alpha = e^{\frac{m}{G}} \left(\frac{G}{m}\right)^{\frac{m}{G}} \Gamma\left(1 + \frac{m}{G}\right)$ and $\frac{m}{G}$.

Preparing each trial mathematical hydrograph involved substituting pairs of m and G values into equation (1) to determine a series of q values at various times for plotting.

Statistical Parameters Describing Rainstorms

The manner in which rainfall intensity varied with time was published (23) as a hyetograph along with each hydrograph studied. Two such diagrams are reproduced in Fig. 3. Also noted on the figure are the three statistics, mean time, standard deviation, and skewness, which summarize the major features of the time distribution of the storm. They comprise a new attempt at describing important characteristics of a rainstorm with a few values. One may visualize a skew bell-shaped curve with the same statistical parameters of skewness, standard deviation, mean, etc. as presenting a smoothed version of the hyetograph. This elimination of very short angular peaks and breaks in intensity is desirable since these steps occur at different times at different points throughout the watershed. The overall input to the watershed is more likely to approach a smooth curve than it would the steplike pattern published for a single gage. Furthermore the natural watershed immediately begins to destroy these angular peaks by storage.

Thus the computation of moments, and thence the standard deviation of time and skewness were undertaken. It was considered that the statistical parameters, S_1, S_2, and S_3, adequately describe for the watershed generally whether the storm was: extremely peaked or relatively uniform in time, early-peaking or late-peaking. To reflect the amount of rain, the storm total and the average rainfall intensity were added to the set. Three antecedent rainfall amounts complete the statistical rainfall parameters as listed under section IV of Table 2.

Mean time, S_1 - This may be regarded as fixing the center of gravity of the hyetograph along the time axis. Moments for each block of rainfall about the starting time were summed and divided by storm total to obtain this statistic S_1. It established the axes MM in Fig. 3 about which the first moments summed to zero.

Standard deviation about mean time and skewness - These Statistical Rainfall Parameters, S_2 and S_3, were computed from the hyetograph in a similar way that they would be derived from a frequency histogram. If ν_k, the k th moment about A (an arbitrary reference value), is $\frac{\Sigma f_i(x_i - A)^k}{F}$; then $\nu_o = 1$, $\nu_1 = \bar{x} - A$, and $\nu_2 = f(A)$ with a minimum at $A = \bar{x}$. If μ_k, the k th moment about $\bar{x}$, is $\frac{\Sigma f_i(x_i - \bar{x})^k}{F}$;

then: the variance, $\mu_2 = \nu_2 - \nu_1^2$

$$\mu_3 = \nu_3 - 3\,\nu_1\,\nu_3 + 2\,\nu_1^3$$

the standard deviation, $S_2 = \sigma = \sqrt{\mu_2}$

the momental skewness, $S_3 = \frac{\alpha_3}{2} = \frac{1}{2}\sqrt{\frac{\mu_3}{\sigma^3}}$.

Table 3 outlines a typical calculation. $\bar{x}$ is the mean time described in the preceding section. The computations were performed on a digital computer, as rainfall data had been punched on cards for other purposes. The program transformed the data so that A became zero; whence $\nu_1 = \bar{x} = S_1$.

Multiple Linear Regression Analysis

In broad perspective the entire study was composed of three facets.

1. It was necessary to describe each observed hydrograph by ascribing numerical values to the three parameters W, q_o, and G.
2. Another set of variables had to be evaluated which described the characteristics of the watershed topography, of the soil and current land use, of the rainstorm causing each particular flood, and of the antecedent conditions measured by preceding rainfall.
3. The final facet was to relate variables obtained in 1 above to those obtained in 2.

The basic approach to be used in this third facet will be discussed in the remainder of this chapter.

More specifically, regression equations had to be developed from which each of W, q_o, and G could be predicted (as a dependent variable) from a few of the many so-called independent variables which described the storm, the environment, and the antecedent moisture conditions. The technique chosen to establish the relationships between variables from the two sets was the stepwise multiple regression analysis. A description of this statistical technique, given before proceeding to the next chapter on data analysis, should provide an understanding of why many of the parameters were included as independent variables.

Generalized procedure - The technique of multiple regression analysis establishes a functional relationship by which the dependent variable may be approximately predicted from a number of independent variables. An anticipated relationship is set up and the least squares criteria is applied to empirical observations of both dependent and independent variables. This results in a system of equations which have to be solved simultaneously for the coefficients of each term. Since there is one equation for each variable, the computations become so cumbersome as to require a digital computer. No attempt will be

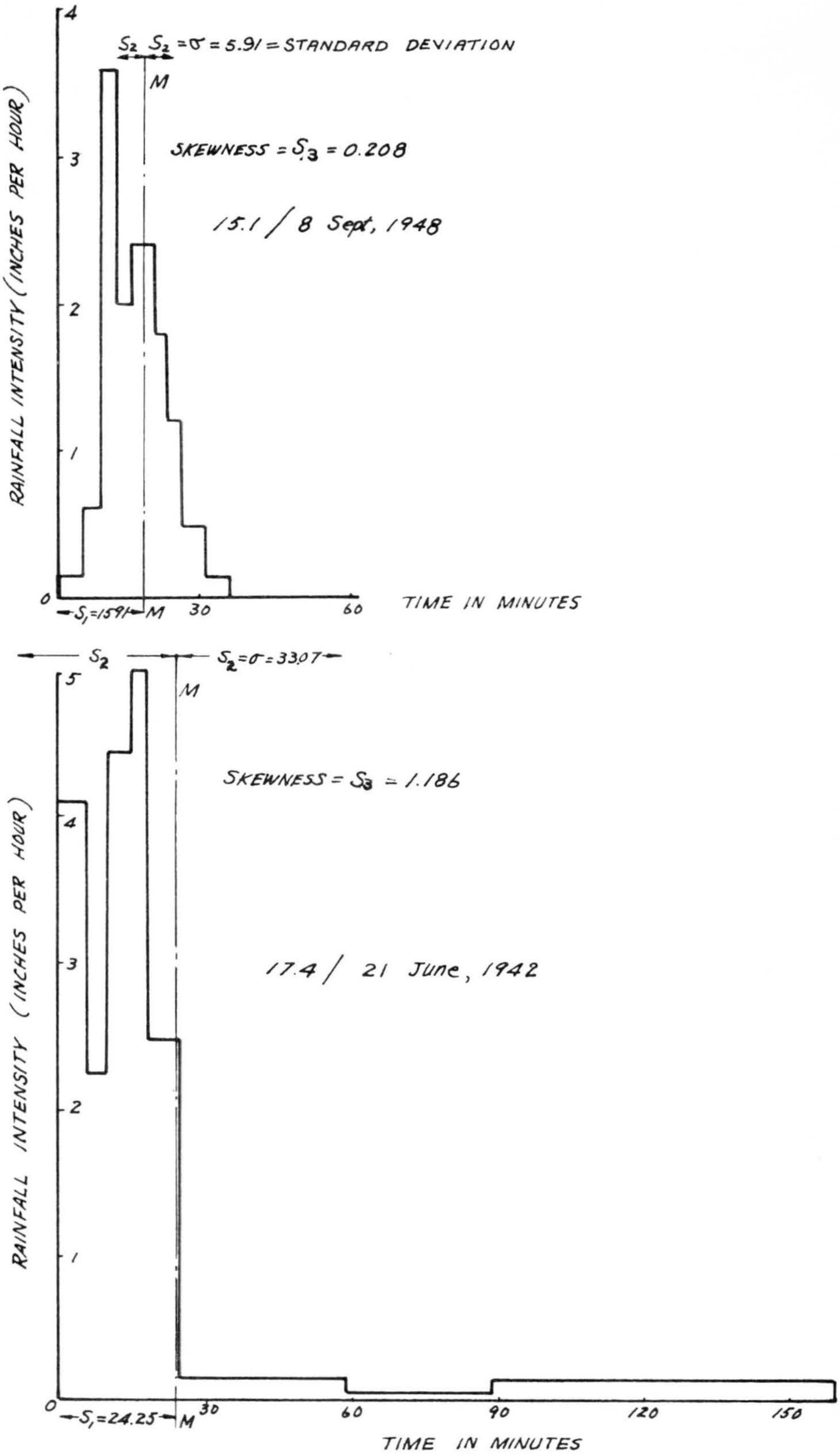


Fig. 3 Typical hyetographs with superposition of statistical rainfall parameters for: mean time, S_1; standard deviation, S_2; and skewness, S_3.

Table 2. Symbols and definitions of independent variables for stepwise multiple regression analyses.

	I. Topographic Parameters
T_1	Watershed area, acres.
*T_2	Length of the longest collector from the gaging station carried out to the watershed perimeter, L feet.
*T_3	Length along the main stream from the gaging station to the point nearest the mass center of area, L_c feet.
T_4	Fall over watershed, H feet.
*T_5	Average main-channel slope, S_c feet/foot, is the slope of a straight line drawn through the gaging point on the longitudinal section of the longest collector such that the area between it and the horizontal axis equals the area below the longitudinal section.
*T_6	Average land slope, percentage.
T_7	$L/\sqrt{S_c}$.
T_8	Drainage density, in feet of mapped channel per acre.
*T_9	Time of concentration, computed from the Soil Conservation Service nomograph involving H and L, in hours.
	II. Design Indices Depicting Land Use and Soils
*D_1	Infiltration capacity after one hour of continuous rainfall according to A.S.C.E. (21) tables of soil and cover, inches/hour.
D_6	S.C.S.'s intermediate runoff curve number (14).
D_7	S.C.S.'s curve number after correction for antecedent rainfall (14).
D_2	Runoff volume expected from S.C.S. relationship with rainfall and corrected curve number, inches.
D_4	Watershed lag from Mockus' nomogram (22) which includes S.C.S. curve number, hours.
*D_5	Cook's ΣW from runoff producing characteristics (23) such as soils, land use and topography.
D_8	Turner's C (24) for use in $Q = CIA$.

* Denotes parameters which were used in the final regression equations.

Table 2. (Continued) Symbols and definitions of independent variables for stepwise multiple regression analyses.

III. Conventional Rainfall Parameters		
*R_1	Storm total, inches.	
R_2	$P_{0/10}$	Rainfall amounts received during sequential periods of storm, inches.
R_3	$P_{10/20}$	
R_4	$P_{20/30}$	
R_5	$P_{0/30}$	
R_6	$P_{30/60}$	
R_7	Initial intensity, inches/hour.	
R_8	I_5	Maximum rainfall intensities during consecutive numbers of minutes, inches/hour.
R_9	I_{10}	
R_{10}	I_{15}	
*R_{11}	I_{30}	
R_{12}	I_{60}	
R_{13}	Minutes after which intensity equals or exceeds 2 inches/hour.	
R_{14}	$= S_5$ 5-day	Antecedent rainfalls, inches: (Used with III and IV).
R_{15}	$= S_6$ 3-day	
R_{16}	$= S_7$ 1-day	

IV. Statistical Rainfall Parameters	
S_1	Mean time in minutes after commencement of rain.
S_2	Standard deviation about the mean time, minutes.
S_3	Momental skewness of time $= \frac{\alpha_3}{2} = \frac{\mu_3}{2\alpha^3}$.
S_4	Average rainfall intensity, inches/hour.
S_5, S_6, S_7	Antecedent rainfalls, defined above.
S_8	Storm total $= R_1$ inches.

* Denotes parameters which were used in the final regression equations.

Table 3 - Specimen calculation of statistical rainfall parameters

Rainfall intensity, inches/hour i ? min	Incremental duration, minutes d	Time from increment center to start of rainstorm, minutes x_i	Amount of rain, inches $f_i = id$	$f_i x_i$	$f_i x_i^2$	$f_i x_i^3$
0.12	5	2.5	0.60	1.50	3.75	9.38
0.60	4	7.0	2.40	16.80	117.60	823.20
3.60	3	10.5	10.80	113.04	1,186.92	12,462.66
2.00	3	13.5	6.00	81.00	1,093.50	14,762.25
2.40	5	17.5	12.00	210.00	3,675.00	64,312.50
1.80	2	21.0	3.60	75.60	1,587.60	33,339.60
1.20	3	23.5	3.60	84.60	1,988.10	46,720.35
0.48	5	27.5	2.40	66.00	1,815.00	49,912.50
0.12	5	32.5	0.60	19.50	633.75	20,596.88
	Summations $\Sigma f_i = F =$		42.00	668.04	12,101.22	242,939.32

$\nu_1 = \bar{x} = S_1 = \dfrac{\Sigma f_i x_i}{F} = 15.91$ $\qquad$ $\nu_1^2 = 253.13$

$\nu_2 = \dfrac{\Sigma f_i x_i^2}{F} = 288.12$ $\qquad$ $\mu_2 = \nu_2 - \nu_1^2 = 34.99$

$\nu_3 = \dfrac{\Sigma f_i x_i^3}{F} = 5,784.27$ $\qquad$ $S_2 = \sigma = \sqrt{\mu_2} = 5.9152$

$\sigma^3 = 206.97$

$3\,\nu_1\,\nu_2 = 13,751.97$ $\qquad$ $\nu_3 - 3\,\nu_1\,\nu_2 = -7,967.70$

$2\,\nu_1^3 = 8,054.54$ $\qquad$ $\mu_3 = 3\,\nu_1\,\nu_2^2 + 2\,\nu_1^3 = 86.84$

Momental skewness $= S_3 = \dfrac{\alpha_3}{2} = \dfrac{1}{2}\sqrt{\dfrac{\mu_3}{\sigma^3}} = +\ 0.210$

Average rainfall intensity $= S_4 = \dfrac{F}{\text{Total time}} = \dfrac{42.0}{35} = 1.2$ inches/hour made here to list the formulae involved in the computation. Attention will rather be focused on how the technique can be applied to draw maximum information from a prescribed set of data.

Stepwise analysis - The stepwise multiple regression procedure adds one independent variable at a time into the regression equation. The program automatically selects the next variable which will, in combination with those variables previously included in the regression, reduce the unexplained variance the most in a single step. Output from the computer contains the regression equation applicable at each step. For the sake of discussion, this would be given by equation (5).

$$Y = a_3 + b_3 x_1 + c_3 x_2 + d_3 x_3 \quad \ldots\ldots\ldots \quad (5)$$

$$Y = a_4 + b_4 x_1 + c_4 x_2 + d_4 x_3 + e_4 x_4 \quad \ldots\ldots \quad (6)$$

All the coefficients and the constants will change when the next step gives equation (6), with one more variable.

Transformations for non-linear regression - It is entirely possible that a dependent variable is closely correlated with four independent variables, but could nevertheless not be predicted by a linear relationship such as equation (6). The true relationship may rather be multiplicative involving different exponents as in equation (7).

$$Y = a\, x_1^{\,b}\, x_2^{\,c}\, x_3^{\,d}\, x_4^{\,e} \quad \ldots\ldots\ldots\ldots \quad (7)$$

Equation (8) shows how this type of relationship can be readily evaluated by the linear regression analysis if all variables are transformed into logarithms before feeding them into the same linear regression program.

$$\log Y = \log A + b \log x_1 + c \log x_2 + d \log x_3 + e \log x_4 \quad \ldots\ldots\ldots\ldots \quad (8)$$

This and many other transformations are easy to incorporate into the data processing. The different types and combinations of transformations are virtually limited only by the cost of computing. For the purposes of the present study the log transformation was the only one run besides the straight data.

Comparing the efficiency of different equations - Besides evaluating the constants and coefficients in such equations as (5), (6) and (8), it is necessary to know the relative precision with which the alternate equations predict the hydrograph parameters. The computer prints the unbiased standard error of estimate, $\hat{S}ey$, after adding each variable in the stepwise procedure. As more variables are included, $\hat{S}ey$ decreases rapidly at first, then more slowly to a minimum from which it rises slightly in an oscillatory fashion. The standard error of any parameter y of hydrograph data before any regression has been introduced, $\hat{S}y$, is also given by the program. The rate at which $\hat{S}ey$ decreases with respect to $\hat{S}y$ provides an indication as to whether the addition of an extra variable to the prediction equation is warranted or not. Since $\hat{S}y$ is different for W, q_o and G, as well as for any of their logarithms the above approach will, however, not provide a universal standard of comparison. fig. 8

In comparing the efficiency of a linear multiple regression equation such as (6) to that of one involving a log transformation like (8) the unbiased coefficient of determination, $\hat{R}^2$, is most useful. It specifies the ratio of the variance explained by the prediction equation to the total variance of the dependent variable. Thus an $\hat{R}^2$ of 0.80 signifies that only 20 percent of the variance in the sample is unexplained by that particular regression equation. At every step in the regression analyses values of $\hat{R}^2$ were obtained by applying:

$$\hat{R}^2 = 1 - \left[\frac{\hat{S}ey}{\hat{S}y}\right]^2 \quad \ldots\ldots\ldots\ldots \quad (9)$$

This statistic compensates for different sample sizes and for different numbers of variables in the regression equations. Hence a universal standard of comparison is provided.

Chapter 3

ANALYSIS OF DATA

This chapter describes the procedural details involved in analyzing the data obtained from the Agricultural Research Service (22, 23) publications. Here again the three major divisions present themselves.

1. It was necessary to study the observed hydrographs with a view to fitting the mathematical model to them.

2. The numerous parameters which describe the topographic, storm and other possible flood influences required presentation.

3. The way in which the regression analysis was applied to relating factors from 1 and 2 needs discussion.

Hydrograph Fitting

The object here was to find what value of the recession parameter G produced a mathematical model which best approximated the shape of the observed hydrograph. Criteria for acceptance will be discussed in another section of this chapter before proceeding to evaluate how well the model simulates the prototype. Prior to this, however, it is necessary to describe how antecedent flow was separated from some observed hydrographs so that every runoff event could be associated with the corresponding rainstorm.

Total runoff considered rather than surface component - There was no attempt made in this study to separate surface runoff from ground water or interflow. The analysis merely sought empirical relationships and was not subject to any of the unit hydrograph assumptions which dictate separation. The assumption that ground-water flow is negligibly small on watersheds of these sizes was borne out by the fact that the flow was zero prior to twenty-seven of the forty-seven storms studied. For the remaining cases antecedent flow was usually less than one percent of the peak rate. The ratio of antecedent discharge to peak discharge of a particular return period increases with an increase in drainage area. Hence the simple separation permissible here could become invalid on larger watersheds.

Once a flood is in progress it is immaterial for the present purpose what proportions of the water travel as either surface runoff, interflow or groundwater flow. The objective is to provide the designer with a method of predicting the total flood hydrograph. The contention that "surface" runoff alone would produce more uniform hydrographs does not appear to offset the disadvantage of each designer having to add back an unknown quantity of base flow after estimation.

The aim was therefore to consider total runoff wherever possible and relate it to the soil, storm, and other factors which may affect its distribution with time. Thus it was felt that if a certain type of land use or soil gave rise to considerable interflow this would be reflected in a lengthening of G , which in turn would be related through the regression to the causative elements. The ideal of describing the total hydrograph provides a unifying criterion which all investigators could strive for. In contrast, separation of surface runoff is executed by different methods and to varying degrees by those who have to subjectively apply it.

Separation of antecedent flow - The only separation employed in the present study was the elimination of antecedent runoff. The object of this was to correct the "observed" hydrographs to give a discharge rising from zero after the inception of rainfall. Thereby the direct effect of rain which fell prior to the storm under study was removed. This separation consisted of subtracting a constant runoff rate throughout the entire hydrograph. This adjustment was made in twenty cases, for which it averaged only 0.7 percent of the peak rate. Its magnitude only exceeded one percent for five cases, while its maximum was four percent.

A more severe correction was applied to the record of 31 March 1952 for watershed 17.4, which involved a slightly different condition. This event comprised two bursts of rainfall separated by about one and a half hours. Rather than discard the resulting double-peaked hydrograph completely, it was decided to consider only the second runoff peak and its relation to the second burst of rainfall. Runoff separation was executed according to the previous paragraph, which led to the subtraction of a constant discharge of eight and a half percent of the peak rate. The earlier rainfall was added to the antecedent rainfall. As was done in the other twenty cases of separation, the corrected hydrograph was simply terminated when the final depletion fell below the subtractive correction. Thus a single-peaked curve was included in the analysis, which hopefully would bring into the regressions the effect of high antecedent moisture.

One other hydrograph (19 July 1953 for watershed 15.1) was completely omitted from the study because its rising limb was badly distorted by an incipient peak of magnitude of about one-third that of the final peak. The above correction was unable to handle this unusual situation.

The hydrograph from watershed 62.2 on 6 December 1957 was omitted partly because the above method could not separate the intense discharge which was superimposed on a broader flood due to protracted lighter rainfall. Furthermore, the rainfall intensity data in this case had only been averaged over one hour durations. So it was felt that too much information was missing from this case to warrant its inclusion.

Hydrograph peak, q_o - In all but six cases the maximum ordinate of the mathematical hydrograph model was made equal to the peak of the corrected observed hydrograph. This was done to satisfy the mathematics described at the beginning of the previous chapter, which required q_o to be one of the three fixed parameters. Moreover this is physically desirable as it forces a close coincidence of the peak portion which is most important for flood applications.

Figure 4 presents an example of those few cases in which it was beneficial to use a smaller q_o for the mathematical model than the actual observed peak rate of runoff. In this example the aberrant little peak which preceded the main mass of flood water is inconsequential. It is of more importance to simulate the hydrograph shape in the broader region of the flood crest. The narrow high observed surge only lasted for about two minutes. The small volume of water which it represented could be expected to be absorbed in temporary storage behind whatever structure is being designed. Moreover in routing applications, which would involve its progression through storage this almost instantaneous surge would be rapidly attenuated.

Most reductions were less than five percent of the peak rate. The most extreme reduction was twenty percent and was involved in the attempt to approximate the double-peaked hydrograph of 28 July 1939 from 48.2. The other hydrographs concerned with this type of peak reduction were: 26 July 1940 from 45.2; 10 July 1951 from 44.3; 4 June 1958 and 13 May 1956 from 29.1.

Runoff volume, W - A far more arbitrary decision had to be made regarding the volume of runoff than was required for the preceding parameter. As a result of the exponential depletion it was impossible to read one absolute value of W from the hydrograph as could be done with q_o. For the purposes of flood hydrology the long tail of low flows is unimportant. The exact time at which to terminate the integration is arbitrary, however.

In this study two or three observed values of W were tried in the process of fitting the mathematical model, each with a number of G-values. W was simply read from the table of progressive runoff volumes at times when the hydrograph tail had become unimportant to flood considerations. In cases where the aforementioned separation had been applied the appropriate rectangle at the base of the hydrograph, was subtracted so as to remove the runoff not due to the storm under study. A number of mathematical models were computed for each of these arbitrary W's. The best fitting models from each subgroup were compared. The W of the very best fitting model was used as the parameter for that flood even throughout the rest of the study. Generally the smaller values of W gave better fitting hydrographs.

It is conceded that there is considerable scope for refining this selection. The basic criteria, of using a W-value which could generate a mathematical model which closely approaches the observed hydrograph, seems sound. The expense and effort devoted to each subphase of the study had to be proportioned according to the overall scope and accuracy of the approach.

Criteria of acceptance - It has already been seen in Fig. 4 that the selected mathematical model does not follow all the serrations of the observed hydrograph exactly. The inflow-outflow computation and routing in which these estimates will be used requires rather a reliable representation of the rapidly rising section and of the early recession of the hydrograph. The exact form of the early rise, while discharge rates are relatively low, and that of late depletion when discharges have dropped below dangerous rates are relatively unimportant.

No statistical test is available for evaluating how well the mathematical model fits the observed hydrograph in terms of hydrological acceptance. Gray (9) has pointed out the need for developing such methods. If such numerical tests become available then digital computers will be able to make exacting selections of the best-fitting model with great speed. For the present study a visual comparison between the observed and numerous computed hydrographs had to suffice. By way of example Fig. 5 shows the best mathematical models which could be fitted to three of the observed hydrographs. According to the arbitrary classification depicted in Fig. 5 one may rank the goodness with which all hydrographs were fitted as follows: very good in eleven instances, good in twenty-seven instances and poor in nine instances.

Another criterion which was applied, was that the theoretical line approximately crossed the observed rising limb in such a way as to keep the areas lost and gained almost equal. At the same time, it was attempted to equalize the slopes over all but the lower values of the rising limb. The way in which these requirements were met can be seen for all three cases in Fig. 5. Unfortunately cases occasionally arose where the satisfactory fitting of the rising limb produced a model which was too wide in the general region of the peak. Case C of Fig. 5 illustrates such an instance. The use of such a model would afford extra safety in spillway capacity.

Attempts to get good fits - It has been shown in the first section of the preceding chapter why it was necessary to hold q_o and W constant while substituting matched pairs of G and m, which simultaneously satisfied equations (3) and (4), into the hydrograph definition equation (1). Figure 6 shows four of the resulting models with q_o and W constant

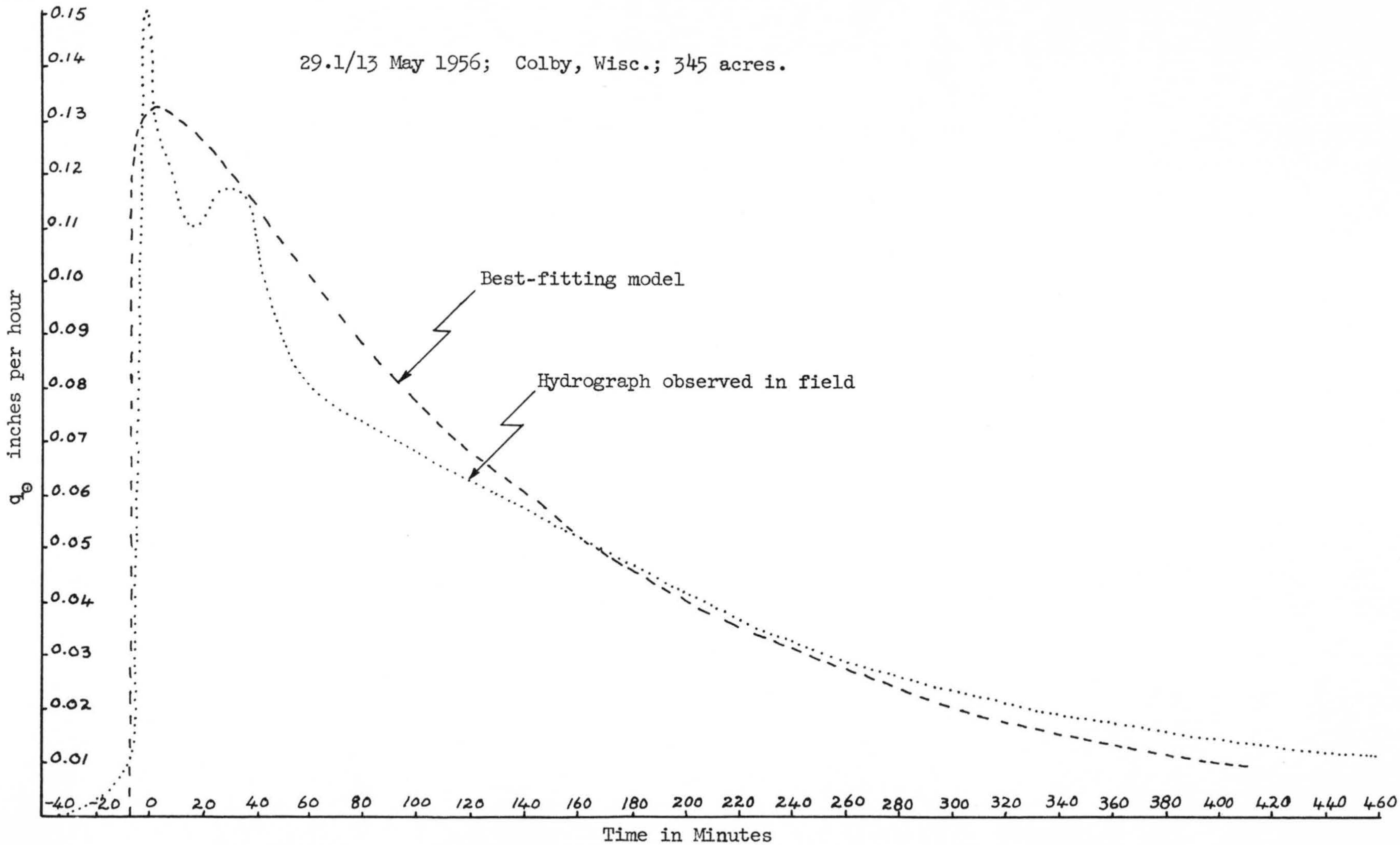


Fig. 4 Illustrative case of situation in which the peak rate of runoff (q_o) for fitted model was reduced below observed peak.

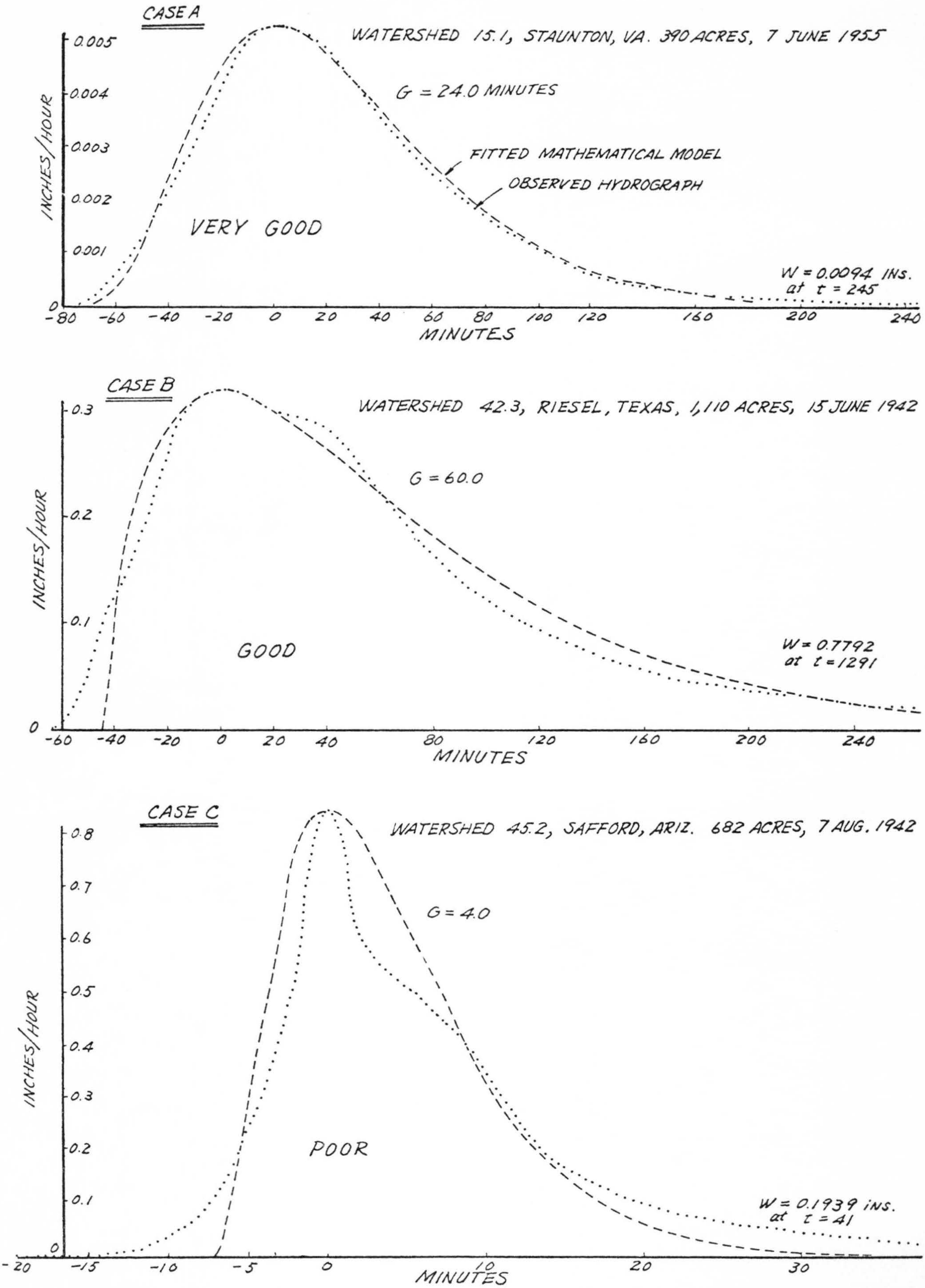


Fig. 5 Typical examples of the arbitrary classification of goodness of fit.

and with G varied in the above fashion. It has been mentioned earlier in this section that sets of models were produced for various W's and in a few cases even q_o was varied. The IBM 1620 electronic computer was employed to calculate q and t coordinates for the four hundred and twenty curves tried in the fitting process.

Although the peak of the model happens to correspond in time to that of the observed hydrograph for all cases shown in Figs. 5 and 6, this was not a rigid requirement. The general shape of the peak section and its width down to about two-thirds of peak rate were given precedence. This point is illustrated by Fig. 7, in which the model's peak was positioned twelve and a half minutes to the right of the observed peak. Thereby a better representation could also be obtained of the rising limb and early recession. Such shifting is permissible as neither the research method nor the application of the results involve the lag time between rainfall and runoff. All that is required is a reproduction of the hydrograph shape itself. Such lateral shifting was necessary in fitting twelve of the forty-seven hydrographs.

Table 9 in the appendix lists the variables W, q_o and G for the best fitting model hydrographs for the forty-seven flood events studied.

Parameters Considered as Independent Variables

For the purpose of regression analysis one refers to the variables from some of which one hopes to predict the three hydrograph parameters as "independent variables". They are actually not independent of each other. For example geomorphic processes have coupled the "length of the longest collector" to the "watershed area" and have set the "average channel slope" according to the "fall over the watershed". Likewise the rainfall intensities for various short durations are related to each other. All members of such groups of parameters have been included in the study so that the step-wise multiple regression analysis can indicate exactly which members correlate most strongly with each of the hydrograph parameters. In the final regressions only the more important "independent variables" were used in the prediction equations.

The thirty-six independent variables are listed and defined in Table 2 according to four groups. Input to the regression analysis never involved more than thirty-two of these variables simultaneously. Before proceeding to discuss these so-called independent variables another point will be discussed which alleviates possible problems of interrelated "independent variables". The final regression equations each involved only one or two independent variables from each group. As an example the prediction of W requires values of (1) infiltration capacity, (2) time of concentration, and (3) storm total. According to the way in which these terms were defined in this study they are physically independent of each other, or practically so. Hence the coefficient of determination for this equation need not be further reduced to account for interactions.

I. Topographic parameters - This group contained lengths, heights and other statistics, singly or in combination, which could readily be obtained from topographic maps. In the absence of suitable small-scale maps these features could rapidly be evaluated from a field inspection. Numerical values for each of these parameters for the forty-seven flood events studied are listed in Table 10 of the appendix.

Watershed area in acres, was tried as a parameter although much of its effect had been eliminated by the selection of the units for q_o and W. Rather than express the peak rate of discharge in cubic feet per second, q_o has been expressed in inches/hour by dividing by the watershed area and changing units. This reduced the peaks to a comparable basis regardless of the size of the various watersheds. It was felt that area itself would not be a significant variable over the restricted range of area studied. This assumption was later verified by the analyses, in which area only entered the regressions after many more important variables. In the cases of W and q_o area entered as the fifteenth and tenth variables respectively.

The thinking behind this idea of dividing the gross discharge rates and runoff amounts by the size of each watershed is equivalent to the assumption that the peak cubic feet per second from various watersheds is proportional to the watershed sizes. Had this division not been performed strong linear correlation would have been found between area and the peak rate in cubic feet per second, and the total runoff in acre feet.

Length of longest collector, is the distance L feet from the gaging station along the stream and from its end in a straight line to the nearest point on the divide.

Length to point nearest mass center of area was the distance L_c feet along the main channel from the gaging site. The center of area was determined with a plumb bob by suspending cardboard cutouts from three points. An interesting digression was the high accuracy with which an inexperienced operator could visually predict the position of the center of mass.

Fall over the watershed, was simply the difference in elevation, H feet, between the gaging site and the divide at the head of the longest collector.

Average main channel slope, S_c, was obtained by plotting the longitudinal section of the longest collector produced to the divide. S_c was calculated as the slope in feet per foot of the straight line drawn through the gaging point in such a manner that

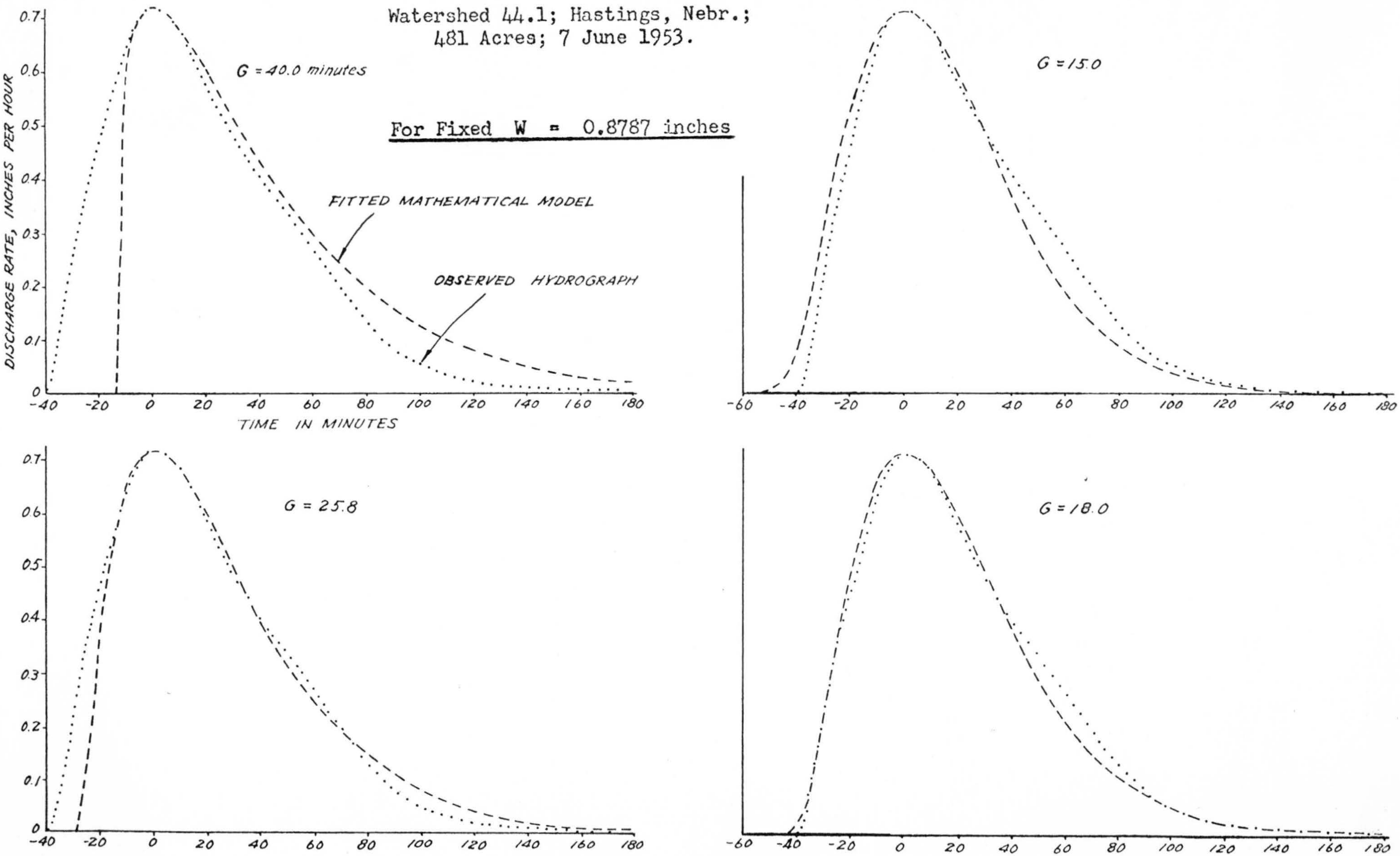


Fig. 6 Typical attempt to fit model to observed hydrograph.

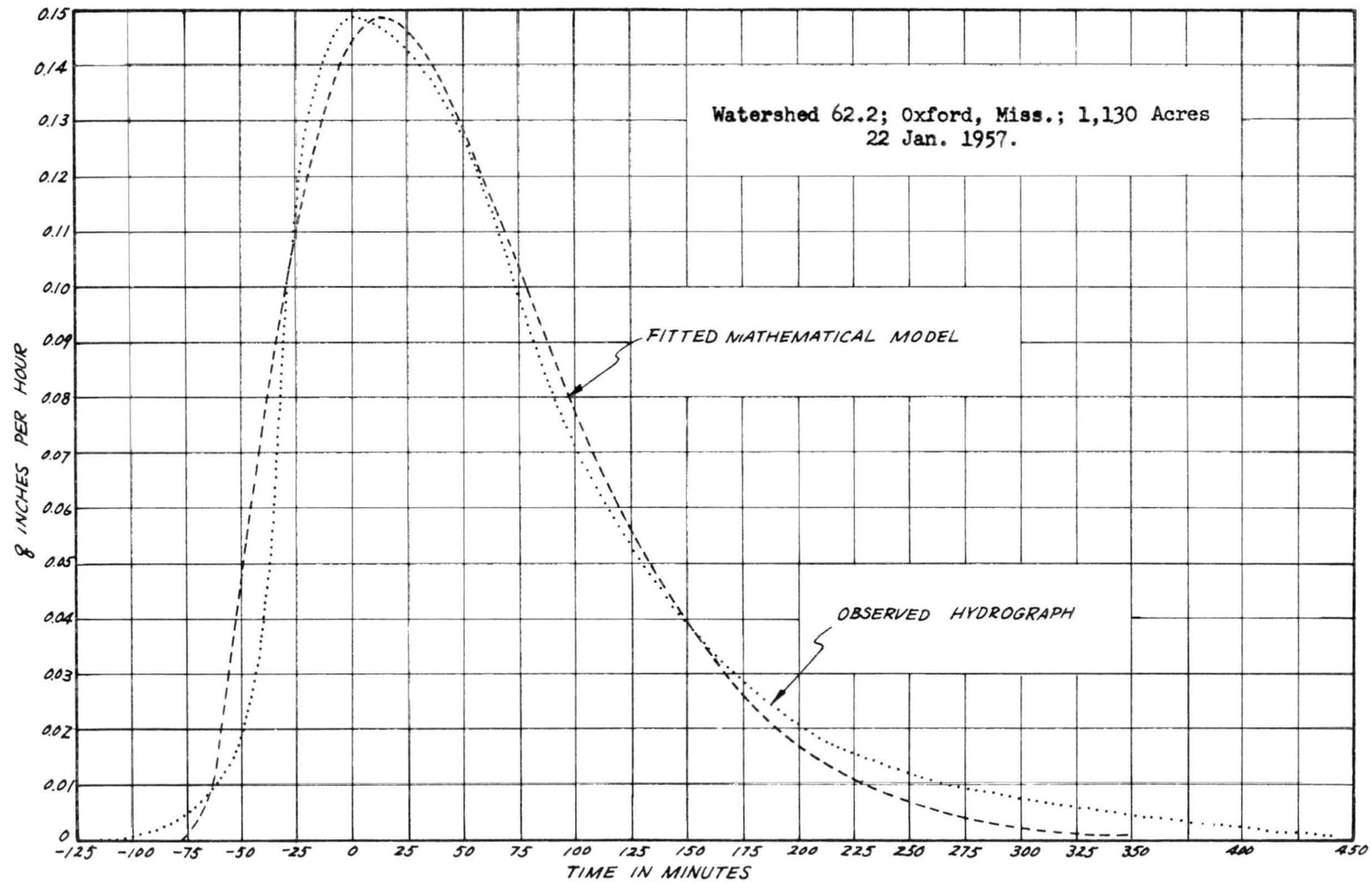


Fig. 7 Shifting of mathematical model in time to obtain better overall fit.

the area between the line and a horizontal line drawn through the channel outlet was equal to the area between the channel grade line and the same horizontal line (10). In field applications where topographic maps are not available a rapid survey along the channel could provide the information.

Average land slope, is the percentage S_a obtained by weighting the proportions of each watershed occurring in various slope-classes (22). In a few cases where this information had not been previously prepared, it was obtained from topographic maps.

A parameter coupling length and channel slope, $L/\sqrt{S_c}$, has been tentatively suggested by Clark (4) and others as related to hydrograph shape.

Drainage density, T_8, is the length of mapped channel in feet per acre of drainage area. It is difficult to determine from maps because of inconsistent map standards.

Time of concentration, T_9, is included as a topographic parameter because the Soil Conservation Service (24) nomograph from which it is obtained only employs the two parameters H and L described above. Whether or not this truly estimates the time in hours required for water to reach the outlet, from the hydraulically most remote point, is immaterial. For the purposes of this study it was simply a combination of topographic features which would hopefully be significant in the regressions for either q_o, W or G.

II. Design indices depicting land use and soils - Various numerical scales which have been proposed from time to time to signify runoff potential of watersheds according to their soils, slope and land use will be considered under this group of parameters. Many of these are obtained by combining arbitrary class marks for runoff producing characteristics of the watershed. The many possible combinations of these elements--each taken at either an extreme, high, normal or low rating--give a wide range from a small index number to one almost a hundred times as great. The way in which each experimental watershed is composed of various proportions of areas having particular soil and cover conditions further broadens the range of these numbers. They may therefore be considered, from the statistical viewpoints, as continuous variables rather than as ranks from which they were derived. Table 11 in the appendix lists the numerical value of these variables for the forty-seven cases studied. Significant correlations will be sought between these empirical indices and the three hydrograph parameters. It may be well to remember that these are not continuous variates in the sense as are measurable characteristics.

The infiltration capacity, D_1, was evaluated from Tables 9 and 10 of the A.S.C.E.'s Hydrology Handbook (1). First a value, ranging from 0.01 to 1.0 inches per hour was selected from one table. These are supposedly infiltration capacities, shown by standard curves after one hour, for bare soil. Each value selected was multiplied by the cover factor which varies from 1.0 to 7.5. Each of the cover types permanent (forest and grass), close growing crops, and row crops were ascribed a range of values for either good, medium and poor conditions.

S.C.S.'s intermediate runoff curve number, D_6. This was evaluated from Tables 3.9-1 and 3.9-2 of the Soil Conservation Service Handbook (24) according to the prevailing land use annotated on each flood event. As a preliminary to this process the hydrologic soil groups had to be determined according to definitions in their (24) Chapter 7 and the soils' descriptions given for the research watersheds (21). By definition such runoff curve numbers were considered valid for intermediate conditions of antecedent moisture.

S.C.S.'s curve number after correction for antecedent rainfall, D_7. According to the amount of rain which had actually occurred on the five days preceding each storm studied, the antecedent condition was classed as either unusually dry, wet or intermediate. If the condition was different from intermediate the runoff curve number was altered to form this new variable. The standard Soil Conservation Service procedures (24) involving their Tables 3.4-2 and 3.10-1 were employed.

The runoff volume expected from S.C.S. relationship, D_2, was obtained for each of the corrected curve numbers and the observed storm totals. This was performed by using tables (26) which are equivalent to Fig. 3-10-1 of the handbook (24).

Watershed lag from Mockus' nomograph (15) D_4, was based on the above runoff curve number as well as the average watershed slope and the main stream length (to the furthest divide). It was estimated from a nomograph (8) of the U.S. Soil Conservation Service.

*Cook's ΣW, D_5, has been used by the Soil Conservation Service (25) in some areas. The way in which it is used along with a specific set of curves to relate runoff peak to drainage area is explained by Frevert et al (8).

Turner's C, D_8, was the last of the seven design indices studied here. This Australian (20) proposed a system of ranking by which an estimate of the runoff coefficient C in the classical Rational Formula, Q = CIA, can be obtained. He proposed one table for catchments less than one square mile and one for those bigger than this. He discusses

*The symbol ΣW has become established in the literature to express this index of runoff potential. It should not be confused in any way with the symbol W used in this study to signify the total runoff volume.

many (1, 25) existing schemes for estimating C, including Australian (13) curves which involved rainfall intensity as a factor besides soil and land use. After presenting qualitative arguments Turner arrives at his weights for rainfall intensity, topographic relief, storage, infiltration and cover.

III. Conventional rainfall parameters - Thirteen parameters describing the observed rainstorm, and three describing the antecedent rainfall were considered in this group of independent variables. The title "conventional" was given to these because equivalent amounts of storm rainfall, maximum rainfall intensities and so on can be estimated for a design situation from available publications. For example the rainfall intensity atlas (11) could be used to obtain the thirty-minute maximum rainfall intensity for a chosen return period.

The reason why a number of very similar variables were included in the analysis was to allow the statistical technique to select from many possible variables which were the more strongly correlated to each hydrograph parameter. In this way the stepwise regression analysis showed that the maximum rainfall intensity for thirty consecutive minutes was far more closely correlated with the hydrograph peak than were the maximum intensities for five, ten, fifteen or sixty minutes. In the same way the antecedent rainfall for one- and three-day periods were included besides the five-day amount currently used (24).

Values of these parameters for the observed flood events are listed in Table 12 of the appendix.

IV. Statistical rainfall parameters - The purpose of developing this new set of descriptive parameters was discussed in the previous chapter, where the necessary calculations were also outlined. These eight parameters have been defined in Table 2 and illustrated in Fig. 3. All that may be added is that from the applications point of view these statistical rainfall parameters are not yet available in published form. If the analysis had shown one or more of these statistical rainfall parameters to give the best regressions for hydrograph predictions, then they would need to be evaluated from typical storms in each area. Values obtained for the forty-seven storms studied here can be found in Table 13 of the appendix.

Stepwise Linear Regression Analysis

The final phase of the analysis was the application of stepwise multiple regression techniques to both dependent and independent variables obtained thus far. The objects of this were as follows:

1. To determine whether some conventional rainfall parameters were better than statistical rainfall parameters, both in combination with the most significant parameters from the topographic group and the indices depicting land use and soils.

2. To establish the most efficient equations for predicting W, q_o and G, each from about three variables.

Details of this part of the study will comprise the next chapter.

Chapter 4

DISCUSSION OF RESULTS

In this chapter results of the stepwise multiple regression analysis are represented. From a comparison of these results a selection was made of the final regression equations for predicting each of W, q_o and G. For the sake of convenience the groups of parameters were referred to by the Roman numerals under which they are listed in Table 2, page 10. Likewise individual parameters were designated by their numbered letters. In every analysis data were included for all forty-seven flood events.

The concluding sections of this chapter show how closely the observed hydrograph can be predicted from the tentatively proposed regression equations.

Modifying Influences upon the Coefficient of Determination

As was mentioned at the close of Chapter 2, the unbiased coefficient of determination, $\hat{R}^2$, was used in this study for comparing the efficiency of different prediction equations. Before enumerating these results it will be helpful to discuss the biased coefficient of determination, R^2. It will provide a basis from which the other factors, such as interdependence and serial correlation, which influence the interpretation can be considered.

The relationship between the biased and unbiased coefficient of determination is given by Ezekial and Fox (7) as

$$\hat{R}^2 = 1 - (1 - R^2)\left[\frac{n-1}{n-M}\right] \quad \ldots\ldots\ldots\ldots \quad (10)$$

In this equation n is the number of observations. The total number of constraints imposed in fitting the regression equation, M, is one more than the number of variables employed in the equation. It can be seen from equation (10) that $\hat{R}^2$ is approximately equal to R^2 when the number of observations is many times greater than the number of variables included in the regression equation. When $n = 500$, the reduction ratio $\frac{n-1}{n-M}$ becomes 1.008 and 1.028 in the cases where M is equal to 5 and 15 respectively. These corrections are virtually negligible. They substantiate the view that for samples containing almost five hundred observations regressions containing fifteen or less variables will give values of $\hat{R}^2$ essentially identical to those for R^2.

Examining Fig. 8 in the light of the above discussion suggests the improvement which could be made in the values of $\hat{R}^2$ if another study could be based upon 500 observations rather than the forty-seven available here. The shaded region between the two curves indicates the order of magnitude of the improvement possible by overcoming the restriction of small sample size. Whether a future sample of, say, 500 observations would actually give larger values of $\hat{R}^2$ than were obtained for the biased R^2 from this small sample remains a matter for conjecture. The amount of improvement would depend on the representativeness of the present small and future large samples.

Two other factors, besides sample size and number of constraints discussed above, may reduce the reliability of the regression equations below that suggested by the unbiased coefficient of determination. Firstly any serial correlation of the observed variables will reduce the reliability with which an event can be predicted. Fortunately, floods from a small watershed are not related to each other in sequence. They are individually caused by convective storms of random extent and intensity. When the considerable distance between these experimental watersheds is remembered, the influence of serial correlation upon this study can be seen to disappear.

The last factor which decreases the reliability of regression is the dependence which may exist between some of the so-called independent variables included in the regression equations. This fact was given serious consideration in planning the stepwise multiple linear regression analysis and in the final selection of regression equations. It was for this reason that the conventional rainfall parameters and the statistical rainfall parameters were not entered into the same program. The object behind the inclusion of a large number of somewhat interrelated "independent variables" from each group was to permit the computer to select the most significant from each of the groups depicting topography, rainfall and design indices. The hope was that one parameter would be selected from each of the virtually independent groups. Table 4 shows that this ideal was achieved with regard to the prediction of W. The prediction equation selected involves the storm total, R_1, a tabulated infiltration capacity, D_1, and a relationship involving the length and drop of the watershed, T_9. These three factors are physically independent of each other and consequently do not reduce the reliability of predicting W below that indicated by the $\hat{R}^2$ of 0.80. On the other hand the prediction of q_o involves the simultaneous substitution of T_2 and T_3. These two parameters from the same group are lengths which are morphologically related to each other. The effect of this interdependence of the "independent variables" is that predictions of q_o are less reliable than is suggested by an $\hat{R}^2$ of 0.61. Similar limitations exist on the $\hat{R}^2$ of 0.62 for G. It is hoped that even if similar interdependence occurs when a large-sample study is made that the original

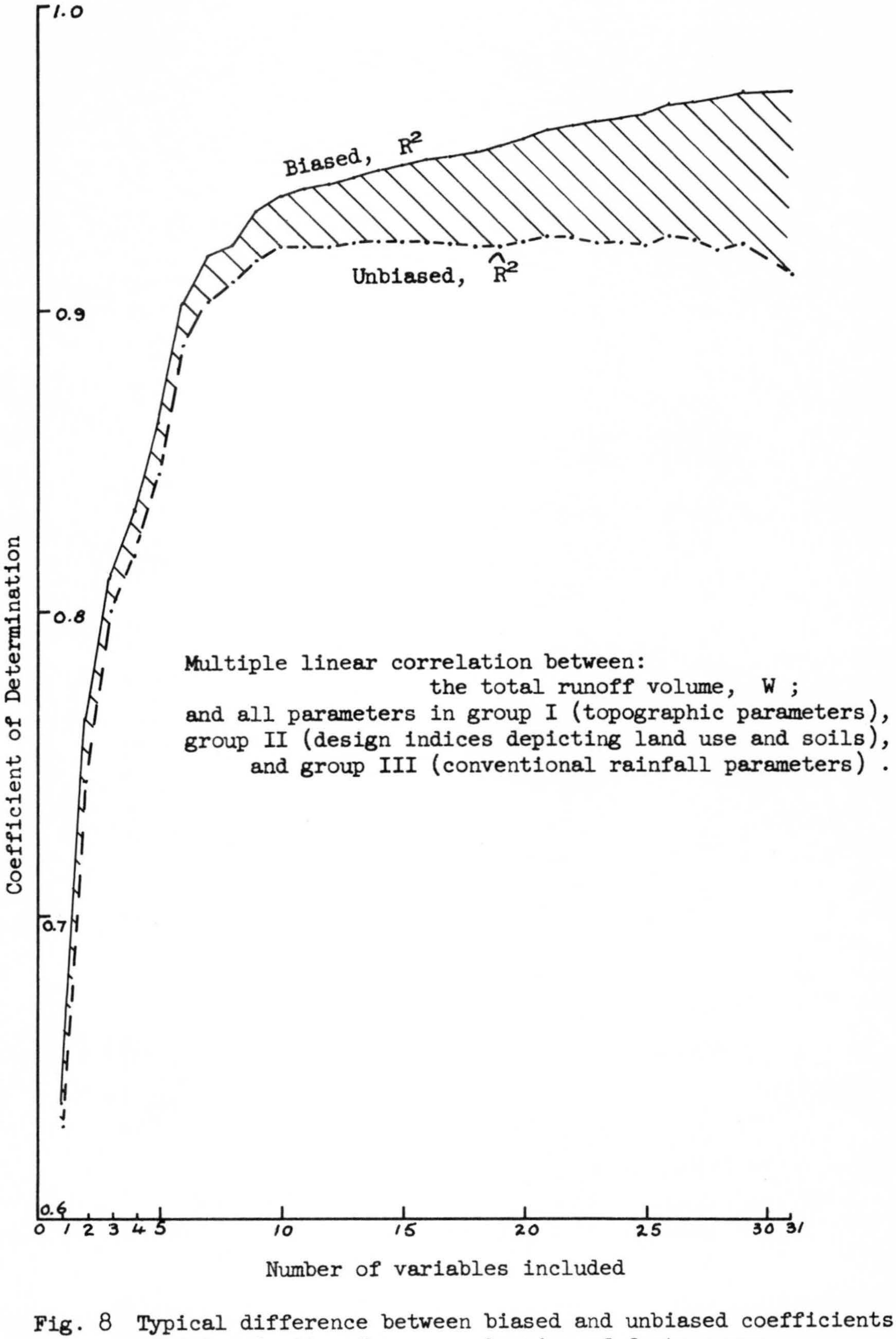


Fig. 8 Typical difference between biased and unbiased coefficients of determination for a sample size of forty-seven.

Table 4. Unbiased coefficients of determination, $\hat{R}^2$, attained with some of the better equations using various parameters.

Dependent Variable	Groups I, II, and III Topographic..., Design..., and Conventional Rainfall: linear		Groups I, II, and III: log transformation		Groups I, II, and IV Topographic..., Design..., and Statistical Rainfall: linear		Groups I, II, and IV: log transformation	
W	R_1, D_1, T_9	.80 *	R_1, T_6, D_7	.66	R_1, D_1, T_9	.80	R_1, T_6, D_1	.51
	R_1, D_1, R_{14}	.75	R_1, T_6, R_7, R_{13}	.78				
	R_1, D_1, R_{14}, T_3	.80	R_1, T_6, R_7	.61				
			R_1, T_6, D_1	.61				
q_o	R_{11}, T_3, T_2	.61	R_{12}, R_7, R_{13}	.63	R_1, T_1, D_4	.49	R_1, S_1, D_1	.58
	R_{11}, T_3, D_4	.55			R_1, T_1, D_4, T_3	.53	R_1, S_1, D_1, D_7	.65
	R_{11}, R_6, T_2	.53						
	R_{11}, T_3, R_9	.53						
	R_{11}, T_3, T_2, R_6	.69						
G	T_7, D_6, R_2	.39	T_5, D_5, T_6	.62	T_7, D_6, T_1	.37	T_5, S_4, T_8	.57
	T_7, D_6, R_2, T_1, T_8	.42	T_5, D_5, T_6, D_6	.65	T_7, D_6, T_1, T_3	.39	T_5, S_4, T_8, T_3	.58

W Volume of model hydrograph, inches.

q_o Peak rate of model discharge, inches per hour.

G Time between peak and center of mass of best-fitting model hydrograph, minutes.

R_1 Storm total, inches.

D_1 ASCE's Infiltration Capacity, inches per hour.

T_9 Time of concentration from SCS nomograph, involving length and drop from watershed perimeter to site.

R_{11} I_{30}, the maximum average intensity for thirty consecutive minutes in inches per hour.

T_3 Length along the main stream from the gaging station to the point nearest the mass center of area, L_c feet.

T_2 Length of the longest collector from the gaging station carried out to the watershed perimeter, L feet.

T_5 Average main channel slope, S_c feet per foot

D_5 Cook's ΣW, involving soils, landuse and topography

T_6 Average land slope, Sa %

* Underlining within the table indicates the variables and $\hat{R}^2$ of each regression equation finally selected. Explanation of some symbols is reviewed below the table according to the equations in which they occur. Others can be found in Table 2.

coefficients of determination will be so high as to outweigh the reduction in reliability.

Typical Results from an Entire Group

For the sake of an example let us consider the results of W as the dependent variable with groups I, II and III providing the independent variables. Thirty-two independent variables were therefore studied simultaneously. Figure 9 shows how the unbiased coefficient of variation, $\hat{R}^2$, rises rapidly with the addition of the first few independent variables into the regression equation. After about six variables, the inclusion of more bring about a negligible reduction in the unexplained variance. With fifteen variables included a maximum $\hat{R}^2$ of 0.9217 was reached, while it drops to 0.9110 for all thirty-two variables.

Pairs of "independent variables" which were drawn into the stepwise regression in the early steps may actually have been dependent among each other. If this were the case it was possible that the exclusion of the former member of such a pair could have enhanced the contribution of its counterpart when that was combined with other variables. Thus subsets were selected with the aim of eliminating interrelations suspected on physical grounds. This certainly gave many alternative subsets which could be used for prediction purposes, but they actually never brought about an improvement in the $\hat{R}^2$. In the thirty-eight subsets tried with the three dependent variables no $\hat{R}^2$ was ever obtained which was higher, for the same number of variables, than that selected originally by the stepwise analysis from all variables. A few of the better $\hat{R}^2$'s obtained from such subsets are included in Table 4. Data including all possible variables from three groups, similar to that shown in Fig. 9, were prepared for all twelve cases. An inspection of the way in which variables were ranked within these provided a basis for selecting the thirty-eight subsets.

Selection of Best Three-Variable Groups

A concise idea of the possibilities available for selecting the prediction equations can be gained from Table 4. For each of the three dependent variables, W, q_o and G, equations could be obtained in four ways. Firstly, the groups of independent variables I, II and III could be used or the groups I, II and IV could be used. That is to say conventional rainfall parameters, III, could be tested against the newly proposed statistical rainfall parameters, IV. Both these comparisons were made in the presence of the topographic parameters, I, and the design indices depicting land use and soils, II. The simultaneous inclusion of groups I and II was necessary to adjudge the relative importance of elements from III or IV in complete prediction equations.

Secondly, each of these three-group combinations were run with and without a log transformation. It should be noted that in the case involving the statistical rainfall parameters some values of skewness were negative. This one variable, therefore, had to be omitted from the analysis involving logarithms.

It can be seen from Fig. 9 that $\hat{R}^2$ could be increased from about 0.8 to almost 0.9 by using a regression equation with six variables rather than one with three. Corresponding improvements are possible in predicting the other two hydrograph parameters. One will always be faced with the dilemma of fewer independent variables in the equations resulting in less satisfactory hydrograph synthesis. Some compromise is necessary between bigger values of $\hat{R}^2$ and the use of a few variables. If three different independent variables are used to evaluate each of W, q_o and G then a total of nine parameters would need to be inserted into the regression equations. It hardly seems justified to expect field personnel to estimate more than nine such parameters from maps or watershed inspections. In fact it should be remembered that the present study was based on too small a sample to become a sophisticated design approach. The intention was rather to show that this approach has potential for the rapid synthesis of hydrographs. Thus three independent variables for each equation satisfied the present need.

A possibility existed that the same variable would recur in more than one of the three selected equations. So a fourth variable was sometimes included to see if the coincidence occurred. As will be seen from the underlined groups in Table 4, all regressions actually selected contained three different variables.

Prediction of Total Runoff Volume, W

As mentioned earlier, the volume of runoff, W, with which we are concerned is the one which allows us to obtain a mathematical model which best fits the observed hydrograph. This W has been compared to either thirty-two or twenty-four watershed and storm parameters in the stepwise multiple regression analysis. It is the purpose of the following paragraphs to evaluate the objective results yielded by the computer program. Consideration of the physical significance of the alternative parameters aided in the final selection of the three regression equations.

Physical significance of variables selected - The final selection of a regression equation for W (as shown by the underlined parameters and $\hat{R}^2$ in Table 4) contained: R_1, the total storm rainfall; D_1, the infiltration capacity of the watershed estimated from the A.S.C.E.'s tables; and T_9, the time of concentration obtained from the Soil Conservation Service's nomograph. The total storm rainfall, R_1, can logically be expected to influence the volume of the hydrograph, W. In fact it was the first variable to be selected in all of the four groups, as shown in Table 4. Figure 9 shows that R_1 accounted for 63 percent of the variation in W.

The infiltration capacity of the watershed

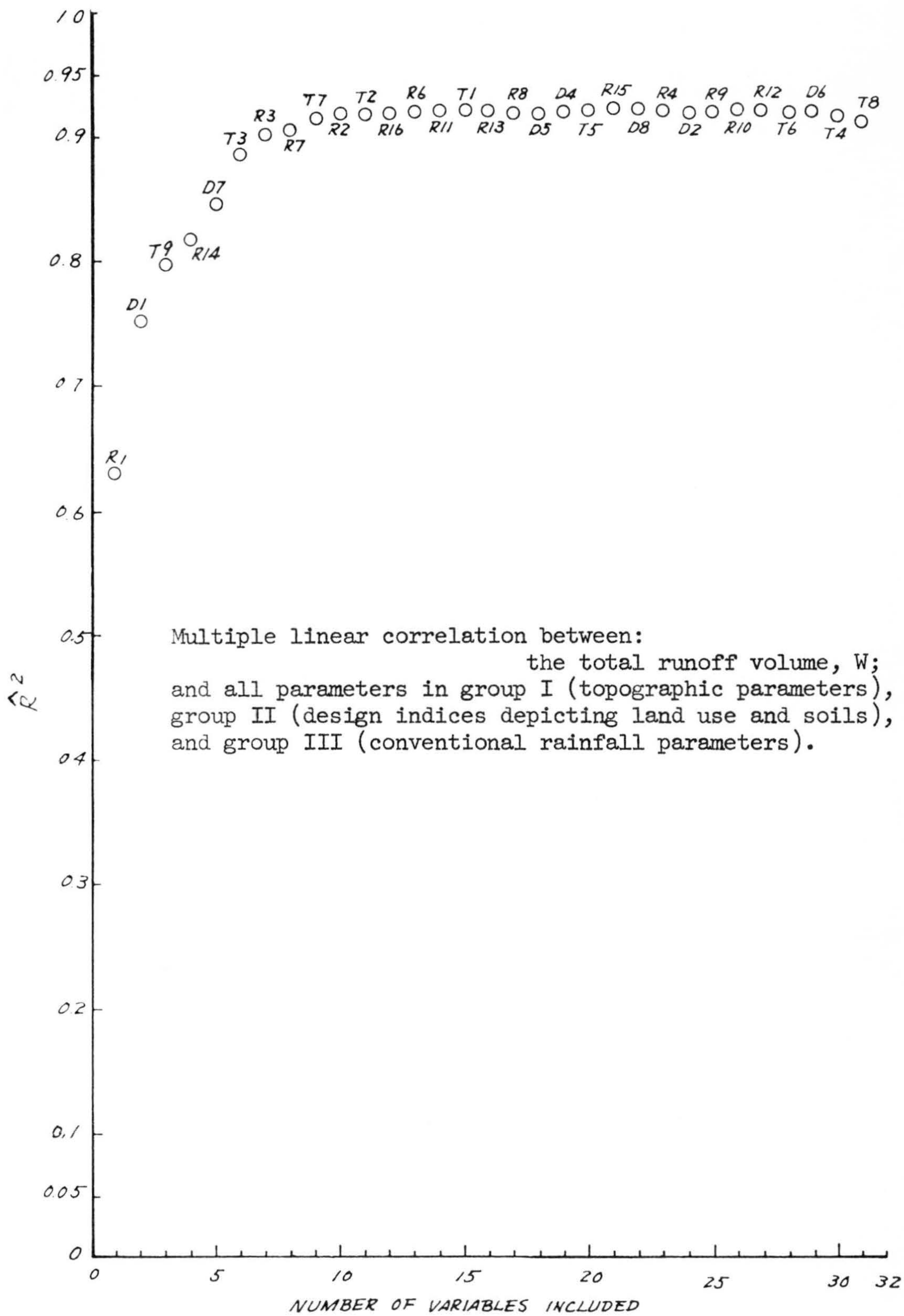


Fig. 9 **Typical variation of the** unbiased coefficient of determination, $\hat{R}^2$, as more variables are included in the regression.

would physically be expected to influence volume of runoff. It is encouraging that so simple an index of infiltration capacity as D_1 contributes to a high unbiased coefficient of determination. D_1 can be readily estimated from two tables (1). The first table simply requires an assessment of the infiltration characteristic of a soil as being: high, intermediate or low. The second table introduces the type of cover as either: permanent, close growing crops or row crops.

The third factor in the selected W-equation may appear to hold less physical significance if it is considered as a time of concentration, T_9 . More meaning attaches to it if considered simply as a combination of the fall over the watershed and the length of the longest collector. These two features were also entered into the analysis as T_4 and T_2 respectively. Neither of them, however, were in any way as significant as their combination in T_9 . Because T_9 actually involves two basic topographic features, the estimation of W requires the evaluation of four elements D_1 , R_1 , and T_2 and T_4 (for T_9). One of these, however, T_2 , is also used as the third parameter in the regression for q_o. So the estimation of W , q_o and G will still only require the evaluation of nine basic parameters.

Comparison with next best group of variables - Examination of the upper left-hand block of Table 4 shows which other sets of variables were almost as good for prediction as the group selected above. It can be seen that if the five-day antecedent rainfall index, R_{14} , replaces T_9 an $\hat{R}^2$ of 0.75 is obtained. This suggests almost as satisfactory an equation which involves only: storm rainfall, R_1 , infiltration capacity, D_1 and antecedent rainfall, R_{14} . Practical difficulties would, however, arise in the application of such a regression which involves both storm rainfall and antecedent rainfall. Additional consideration would have to be given to the joint probabilities of certain pairs of R_{14} and R_1 occurring simultaneously with a view to the return period aspect.

Final regression equation - The equation for predicting W in inches, from D_1 in inches per hour, T_9 in hours and R_1 in inches is:

$$W = 0.1315 - 0.5792\, D_1 + 0.1902\, T_9 + 0.4261\, R_1 \quad \ldots \quad (11)$$

Predicted versus observed W - Table 5 lists the observed values along with the values of all three dependent variables as predicted by the selected regression equations. The amount by which the actual exceeds the predicted is termed the residual. Residuals have been expressed as percentages of the corresponding predicted value. Restricting attention to W , it is seen that the residuals are less than thirty percent for twenty-one cases. For thirty-five of the forty-seven cases, the residuals are less than fifty percent.

It can be seen from Fig. 10 that the large differences between predicted and observed values of W , occur for small actual peak discharges, q_o . Thus if observed peaks smaller than 0.4 inches per hour are discarded, then the highest residual will be sixty percent and all others will be less than forty-six percent. This consideration lends confidence to the prediction equation, as it will be used in association with high q_o for flood estimation.

Along similar lines we may consider the two cases in which W was estimated as a negative quantity. Such negative values have, of course, no physical meaning. These are simply the result of the superposition of error elements on to a small true value of W . As in the previous paragraph this anomaly only occurred for very small values of q_o , and consequently would not affect flood estimates required in practice.

Prediction of Hydrograph Peak, q_o

The equation selected for the prediction of the peak of the hydrograph model was:

$$q_o = -0.2917 + 0.4600\, R_{11} - 0.00040\, T_3 + 0.00018\, T_2 \quad \ldots\ldots\ldots\ldots \quad (12)$$

where q_o is in inches per hour;

R_{11} is the thirty-minute rainfall intensity in inches per hour;

T_3 is the length from the gage site along the main stream to the point nearest the mass-center of the watershed, in feet;

and T_2 is the length, in feet, of the longest collector carried to the divide.

Physical significance of variables selected - As can be expected the rainfall intensity, averaged over the most intense thirty minutes, greatly influenced the flood peak. Disregarding the constant, -0.2917, for the sake of discussing large q_o values, one may in a restricted sense consider 0.46 as a coefficient of runoff. For every inch per hour by which the rainfall intensity increases there is a corresponding increase of 0.46 inches per hour in the peak rate of runoff. Of course for lighter rainfall intensities this type of approximation to the old Rational Formula Q = CIA (or q_o = CI , with C = 0.46) becomes progressively more distorted. Moreover when different watersheds with varying lengths and shapes are considered the other two variables in equation (12) effect the magnitude of the peak discharge.

Table 5. Values of dependent variables observed on best-fitting hydrographs and values predicted from selected regression equations.

Watershed	Date	W, inches: Best fitting model	W, inches: Predicted	W, inches: Residual as per-cent of predicted	q_o, inches per hour: Best fitting model	q_o, inches per hour: Predicted	q_o, inches per hour: Residual as per-cent of predicted	G, minutes: Best fitting model	G, minutes: Predicted	G, minutes: Residual as per-cent of predicted
15.1	8 Sept. '48	.0028	-.13546	-	0.0059	.19620	-96.99	9.5	12.75	-25.49
	13 Apr. '49	.3715	.28636	29.73	0.3990	.53660	-25.64	10.0	12.75	-21.56
	7 June '55	.0094	.04349	-78.39	0.0053	.23484	-97.74	24.0	12.75	88.24
17.4	27 May '38	.5869	.77646	-24.41	1.0550	.84412	24.98	10.5	9.734	7.91
	21 June '42	.6743	.89112	-24.43	0.9793	1.00788	- 2.84	14.5	9.734	4.90
	31 March '52	.7735	.73520	5.21	1.6220	.83170	95.02	6.2	9.734	-36.26
	2 July '52	1.1047	1.26782	-12.87	1.8600	1.88049	- 1.88	15.0	9.734	54.14
21.1	1 June '43	.9070	.61766	46.84	0.4881	.21658	125.36	23.5	30.90	-23.95
	21 July '48	.6827	.89462	-23.69	0.3392	.20968	61.77	12.5	30.90	-59.55
	1 July '50	1.1866	1.18197	0.39	0.6490	.64759	0.22	21.0	30.90	-32.04
	18 July '56	.9999	1.03523	- 3.41	0.8580	.93646	- 8.38	40.0	30.90	29.45
26.30	16 June '46	1.4413	1.40074	2.89	1.8990	1.43660	33.19	18.0	6.825	163.81
	16 Aug. '47	.2421	.23800	1.72	0.5830	.80364	-27.46	9.0	6.825	31.94
	1 Sept. '50	1.7711	1.61412	9.72	1.7690	1.21120	46.05	6.0	6.825	-12.01
	12 June '57	1.4668	1.10596	32.63	3.7200	2.14498	73.43	6.0	6.825	-12.01
29.1	28 July '49	.2740	.51768	-47.07	0.0808	.48031	-83.18	50.0	37.30	34.05
	13 May '56	.3838	.28759	33.45	0.1330	.26181	-49.20	138.0	37.30	269.97
	4 June '58	1.0909	1.29744	-15.92	0.5050	1.02908	-50.93	55.0	37.30	47.45
31.1	12 Aug. '43	.4263	.72192	-40.95	0.8940	1.68428	-46.92	4.0	6.640	-39.76
	11 July '44	.2252	.65561	-65.65	0.3020	1.45382	-79.23	7.5	6.640	12.95
	28 June '45	.4081	.29922	36.38	0.9800	.62905	55.79	5.0	6.640	-24.70
	24 June '49	.4015	.73198	-45.15	0.7230	1.17783	-38.62	10.0	6.640	50.60
42.3	10 June '41	1.3451	1.01414	32.63	0.7466	.85695	-12.88	30.0	49.77	-39.72
	15 June '42	.7792	.73912	5.42	0.3220	.60671	-46.93	60.0	49.77	20.55
	15 July '50	.8328	1.09882	-24.21	0.5360	.76357	-29.80	30.0	67.89	-55.81
	24 Apr. '57	1.6405	1.02560	59.95	0.8200	1.08602	-24.49	35.0	67.89	-48.45
44.1	20 June '39	.9041	.74814	20.84	1.1500	1.30626	-11.96	20.0	20.68	- 3.29
	10 July '51	1.6620	1.20406	38.03	1.7400	1.33570	30.27	12.0	20.68	-41.97
	7 June '53	.8787	.74148	18.50	0.7180	.77865	- 7.79	18.0	25.19	-28.54
	15 June '57	1.2488	.89607	39.36	1.8200	.97644	86.39	20.0	25.19	-20.60
44.3	10 July '51	1.3165	1.55974	-15.59	0.3480	.63090	-44.84	80.0	31.86	151.10
	7 June '53	.8174	1.04603	-21.86	0.2640	-.13406	-	70.0	31.86	119.71
	29 Aug. '57	1.3187	1.32572	- 0.53	0.2170	-.09726	-	100.0	35.05	185.31
45.2	26 July '40	.3284	.22668	44.87	0.9500	1.00844	- 5.80	2.2	6.219	-64.64
	28 Sept. '41	.6753	.47382	42.52	1.3800	1.61932	-14.78	6.0	6.219	- 3.54
	7 Aug. '42	.1939	.32042	-39.49	0.8480	1.05490	-19.61	4.0	6.219	-35.70
	9 Aug. '43	.3389	.23946	41.52	1.0000	1.11332	-10.18	5.0	6.219	-19.62
48.2	28 July '39	.0946	.52622	-82.02	0.2360	.69791	-66.18	8.0	9.404	-14.89
	24 Aug. '39	.0773	.19387	-60.13	0.2090	.21952	- 4.79	20.8	9.404	112.71
	26 Aug. '39	.0852	.22796	-62.63	0.1790	.31152	-42.54	15.0	9.404	59.54
	5 Sept. '40	.0337	.01491	125.96	0.1790	-.08407	-	5.0	9.404	-46.78
60.6	13 Apr. '37	.0524	.05866	-10.67	0.0166	-.08799	-	52.0	20.84	149.52
	25 Jan. '41	.0525	-.00371	-	0.0256	-.10087	-	20.0	20.84	- 4.03
	3 March '41	.0431	.00906	375.61	0.0311	-.04567	-	12.0	20.84	-42.42
	6 June '41	.0373	.02304	61.85	0.0342	.10750	-68.22	18.3	20.84	-12.19
62.1	22 May '57	.4264	.59365	-28.17	0.2445	.12688	92.69	15.0	33.35	55.02
62.2	22 Jan '57	.3870	.23230	66.59	0.14890	.11462	29.90	40.0	36.27	10.28

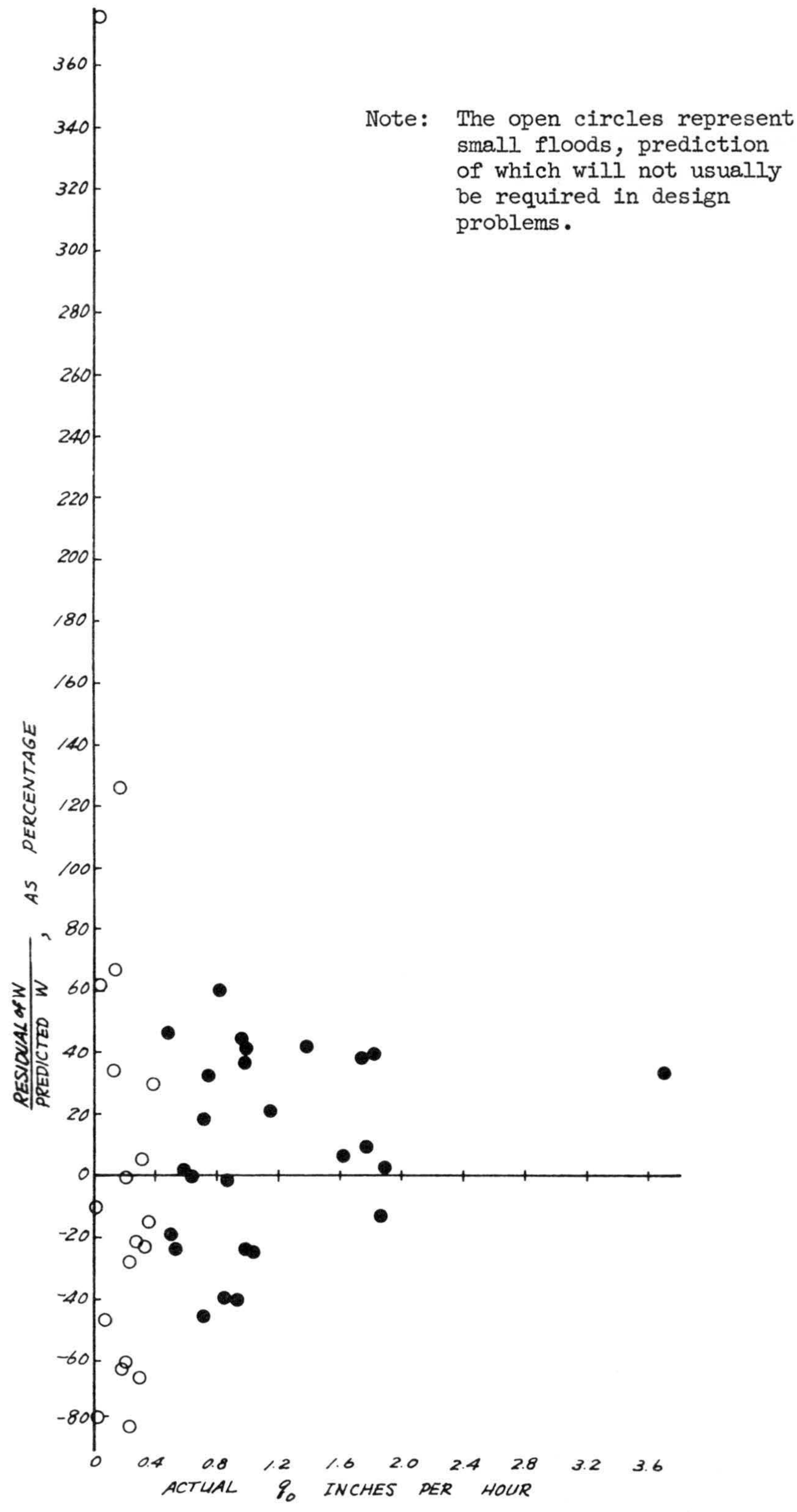


Fig. 10 Variation of the residual of W with actual peak discharge, q_o .

The constant for T_3 is negative. Thus if the mass center of area was further upstream of a site one would not expect as large a peak as if the mass of the watershed was concentrated a short distance above the site. Such a relationship has its physical explanation in peak attenuation along the longer channels.

The counteracting effect of a positive but smaller constant for T_2 accounts for the interrelationship between the length of the entire main channel and the length up to the center of area.

Comparison with next best groups of variables - Reference to Table 4 shows that equation (12), selected for predicting q_o, gave an unbiased coefficient of determination, $\hat{R}^2$, of 0.61. The log-transformation involving R_{12}, R_7, and R_{13} gave a slightly higher $\hat{R}^2$ of 0.63. The most important parameter of this group R_{12} is the maximum rainfall intensity average over sixty consecutive minutes. It can be as readily evaluated from published maps as can R_{11}, the thirty-minute intensity, required by the selected equation (12). The other two variables, R_7 and R_{13}, however, are more difficult to estimate in a design situation. Being respectively the initial intensity and the time after which the intensity reached 2 inches per hour, they would vary greatly from storm to storm even in the same region. Their use would involve the postulation of design storms for various regions. Such difficulties led to the choice of equation (12) even though its $\hat{R}^2$ was slightly smaller.

Predicted versus observed q_o - As with the previous variable the actual and predicted values were listed in Table 5 along with the residuals. To an even greater extent than with W, predictions were poor in the region of small q_o. In fact, six predicted q_o's became negative. Doubtless the negative constant, -0.2917, in equation (12) contributed towards this situation.

An examination of Fig. 11 shows that if only q_o's larger than 0.4 inches per hour were considered much variation would be removed. Moreover the remaining points on Fig. 11 tend to follow a rising pattern. This suggests that if a large enough original sample was available above a lower limit of q_o, then a new regression would be fitted with far less scatter.

In the present data, only twenty-seven of the predicted q_o's were within fifty percent of the observed peaks. Nevertheless, $\hat{R}^2$ indicates that only thirty-nine percent of the variance remains unexplained by R_{11}, T_3 and T_2.

Prediction of Recession Parameter, G

In this instance Table 4 shows how much better the logarithmic law accounts for this behavior than does the untransformed linear relationship. As was the case for W and q_o, the statistical rainfall parameters did not provide as good a prediction as did the conventional rainfall parameters. The equation selected was:

$$(\log G) = -8.13587 - 0.72653(\log T_5) - 0.93866(\log T_6) + 5.01570(\log D_5) \quad \ldots\ldots (13)$$

This transforms, after some rounding to:

$$G = \frac{7.314 \times 10^{-9} \times D_5^{\;5}}{T_5^{\;0.727} \times T_6^{\;0.939}} \quad \ldots\ldots\ldots (14)$$

where G is the time between peak and center of mass of model in minutes;

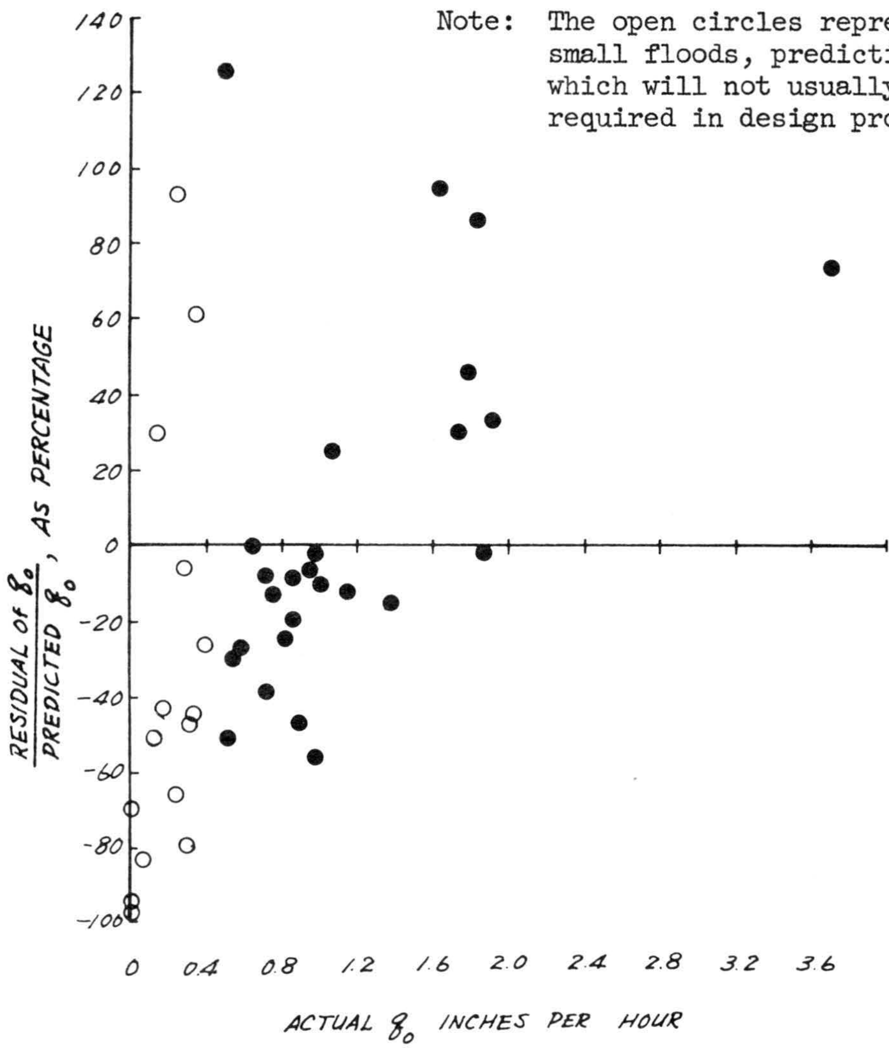
D_5 is Cook's ΣW, involving soils, land use and topography;

T_6 is the average land slope, S_a %;

T_5 is the average main channel slope, S_c feet per foot.

Physical significance of variables selected - The striking feature of equation (13) is that G is independent of rainfall parameters and can be predicted from topographic parameters alone. T_5 and T_6 have nothing to do with soils or land use and can be measured consistently from maps. D_5 again involves topographic relief, but compounded with this are the rankings for soil infiltration and vegetal cover. It is important to realize that this parameter, G, which affects the recession of the flood hydrograph should therefore be approximately constant for each watershed. Severe changes in land use may, however, be expected to influence G by altering D_5. Nevertheless, if G can be established for a particular watershed in a certain cover condition, this should provide one constant value for all of its hydrographs.

Comparison with next best groups of variables - The variables D_5, T_6 and T_5 used above gave an unbiased coefficient of determination, $\hat{R}^2$, equal to 0.62. The next highest value of $\hat{R}^2$ for a three variable group was 0.57 for the logarithmic transformation applied to T_5, T_8 and S_2, the average rainfall intensity. This last element would make G depend on a storm parameter, which is undesirable. Moreover T_8, the drainage density is a difficult parameter to evaluate consistently. The untransformed linear regressions only gave $\hat{R}^2$'s of 0.39 and 0.37 respectively. Clearly the selected equation (14) is the best currently available.



Note: The open circles represent small floods, prediction of which will not usually be required in design problems.

Fig. 11 Variation of the residual of q_o with actual peak discharge, q_o .

Predicted versus observed G - Table 5 lists the observed values along with the predicted values and residuals. Of the forty-seven cases twenty have residuals less than thirty percent. Thirty-two cases have residuals less than fifty percent of the predicted values.

In a similar manner as was seen with W and q_o, Fig. 12 shows how all but one of the large residuals of G can be eliminated if 0.4 inches per hour is set as the lower value of q_o in collecting data.

Conventional versus Statistical Rainfall Parameters

In conclusion attention should be focused on the fact that parameters from group III were seen to be better predictors of all three hydrograph parameters than were those from group IV. This meant that the introduction of new statistical parameters to describe rainstorms was not warranted. If, however, another study was to consider floods on a larger range of watershed sizes then it is conceivable that statistical rainfall parameters would become the more important.

In actual fact the only two rainfall parameters needed for the hydrograph estimate are easy to obtain. Firstly, R_{11}, the thirty-minute rainfall intensity is obtained from the rainfall frequency atlas (11) for a particular return period. R_1, the total storm precipitation, is not currently available on a frequency basis for the entire country. Design values for a particular region could readily be obtained by analyzing the series of storm totals. Maps presenting such information for continental United States on a frequency basis could readily be prepared. Treatment of the joint probability of R_1 and R_{11} attaining certain magnitudes should be included.

Some Successful Predictions of Hydrographs

The discussion of results would not be complete unless it illustrated the accuracy with which the estimated hydrographs can predict actual field events. Thus Fig. 13 presents three cases for which the regression equations had predicted values of q_o, W and G almost equal to those values of the best-fitting model. The corresponding crossed and dashed lines represent estimated hydrographs and best-fitting models, which are essentially identical.

Moreover, if one compares each of the estimated hydrographs in Fig. 13 to their observed hydrograph the representation meets the criteria of acceptance discussed in Chapter 3. Thus it is felt that this technique can satisfactorily predict the shape of a hydrograph, provided the three parameters q_o, W, and G (especially the first two) are determined with reasonable accuracy. Distortions in the hydrograph shape brought about by malestimation of one of, or both, q_o and W are illustrated by two examples in the next section. They should serve to reiterate the need for obtaining high $\hat{R}^2$'s for the regressions of q_o and W.

It should be pointed out that a truly unbiased evaluation of how adequately the estimated hydrographs simulate field conditions should depend rather on previously unused data. Thus Fig. 13 which employs observed hydrographs already used in formulating the regression equations does not provide a very powerful test. Additional hydrographs were however not available. It was also considered inadvisable to set aside some hydrographs, at the outset, for this purpose. Thereby the degrees of freedom in the regressions would have been even further reduced. If another study is undertaken then a large enough sample of flood events should be gathered so as to meet this provision.

Limitations in Accurately Predicting a Hydrograph

The Pearson type III curve has been shown to possess ample flexibility with which it can be made to fit observed hydrographs. Difficulty in predicting hydrograph shapes is still experienced, however, as a result of inaccuracies in the prediction of the three hydrograph parameters W, q_o and G from storm and watershed properties. An error in the estimation of any one of these will affect the resultant hydrograph shape. The problem is aggravated by the fact that the model requires the prediction of three parameters. An aberration of any one of these or a combination will seriously influence the shape of the model obtained from the regressions. With the $\hat{R}^2$'s of 0.80, 0.61 and 0.62 obtained for W, q_o and G respectively the chances of a significant malprediction of one or more of these variables is rather high for any case.

Figure 14 shows the effect upon the hydrograph model of one poorly predicted variable. In this case the peak, q_o, was predicted by the regression as not much more than half of the observed value. W and G were more closely predicted, so that apart from the peak section the model fits the observed hydrograph fairly well.

An instance of where predictions for both W and q_o differed considerably from those of the best fitting model is illustrated by Fig. 15. In this instance the regressions result in what may be considered a scaled-down version of the observed hydrograph or best-fitting model.

Figures 14 and 15 clearly indicate the need for further studies to increase the $\hat{R}^2$'s. By thereby reducing the magnitude of the residual errors in W, q_o and G the distortions in the hydrograph models will be held to a minimum. The sixth chapter will be devoted to suggestions for developing such improved regressions.

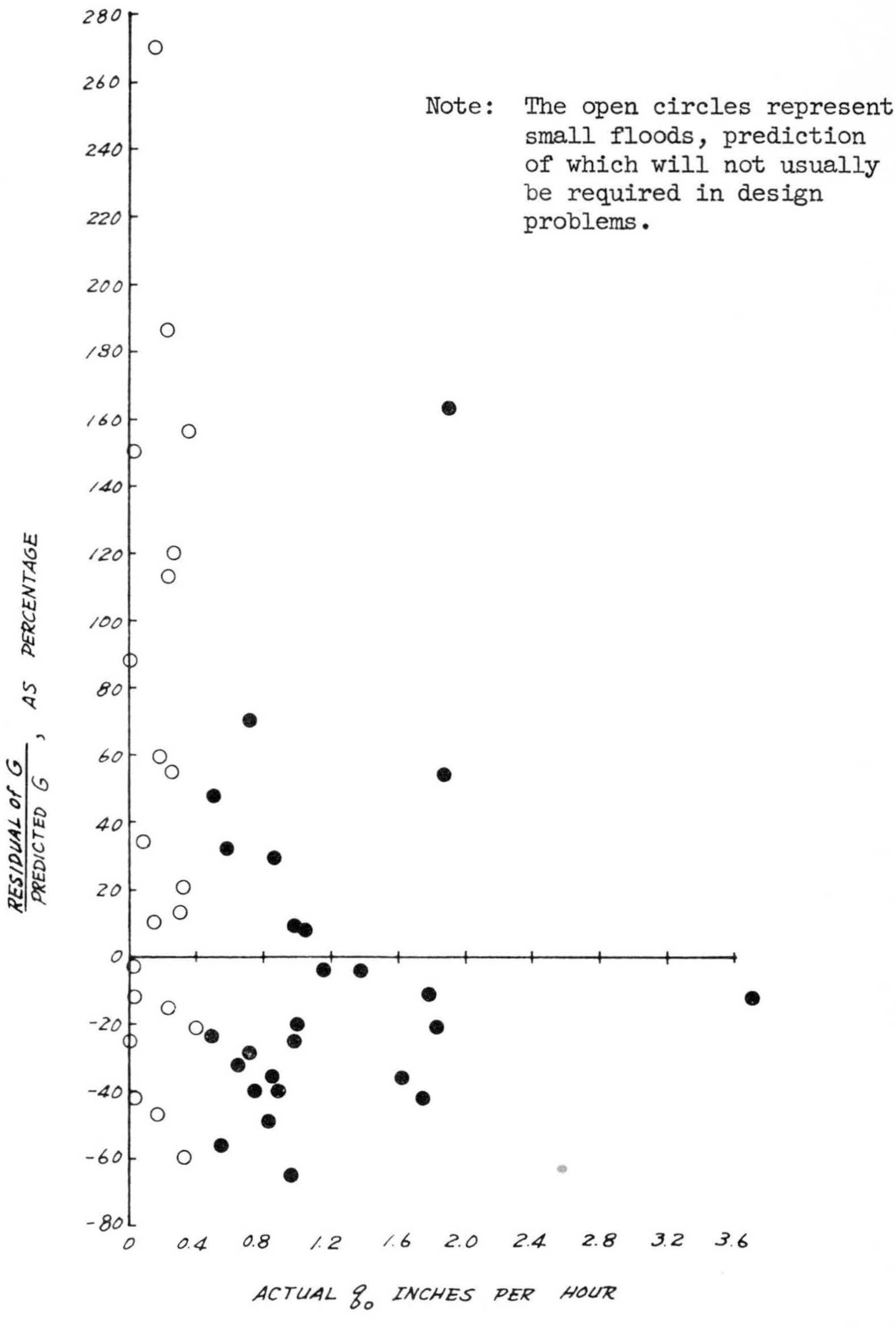


Fig. 12 **Variation of the residual of G with actual peak discharge, q_o .**

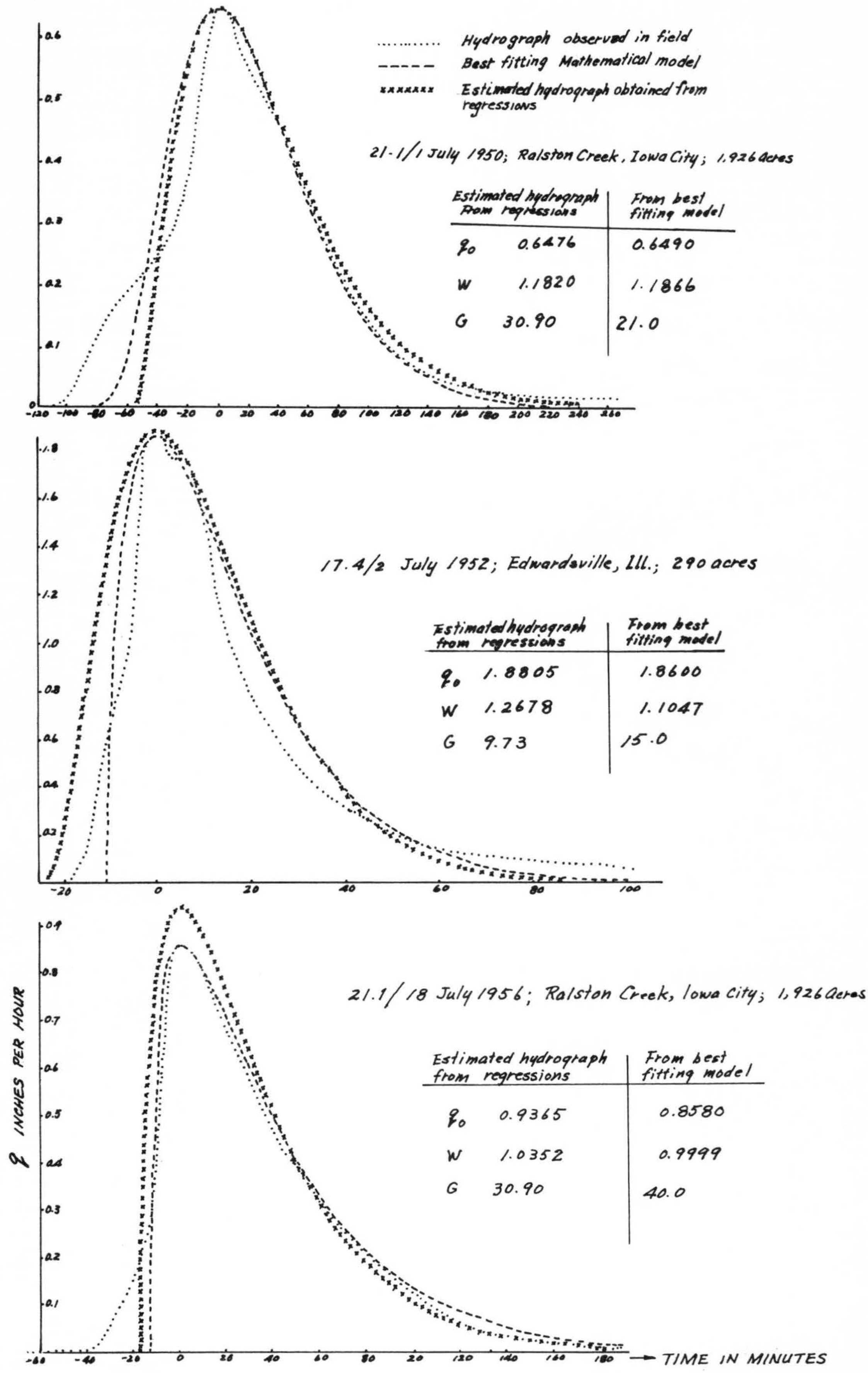


Fig. 13 Closeness with which estimated hydrographs, predicted from regression equations, can approximate the observed hydrograph and the best fitting model.

26.30 / 12 June 1957; Coshocton, Ohio; 303 Acres.

	Estimated hydrograph from regression equations	From best fitting model
q_o	2.1450	3.7200
W	1.1060	1.4668
G	6.82	6.0000

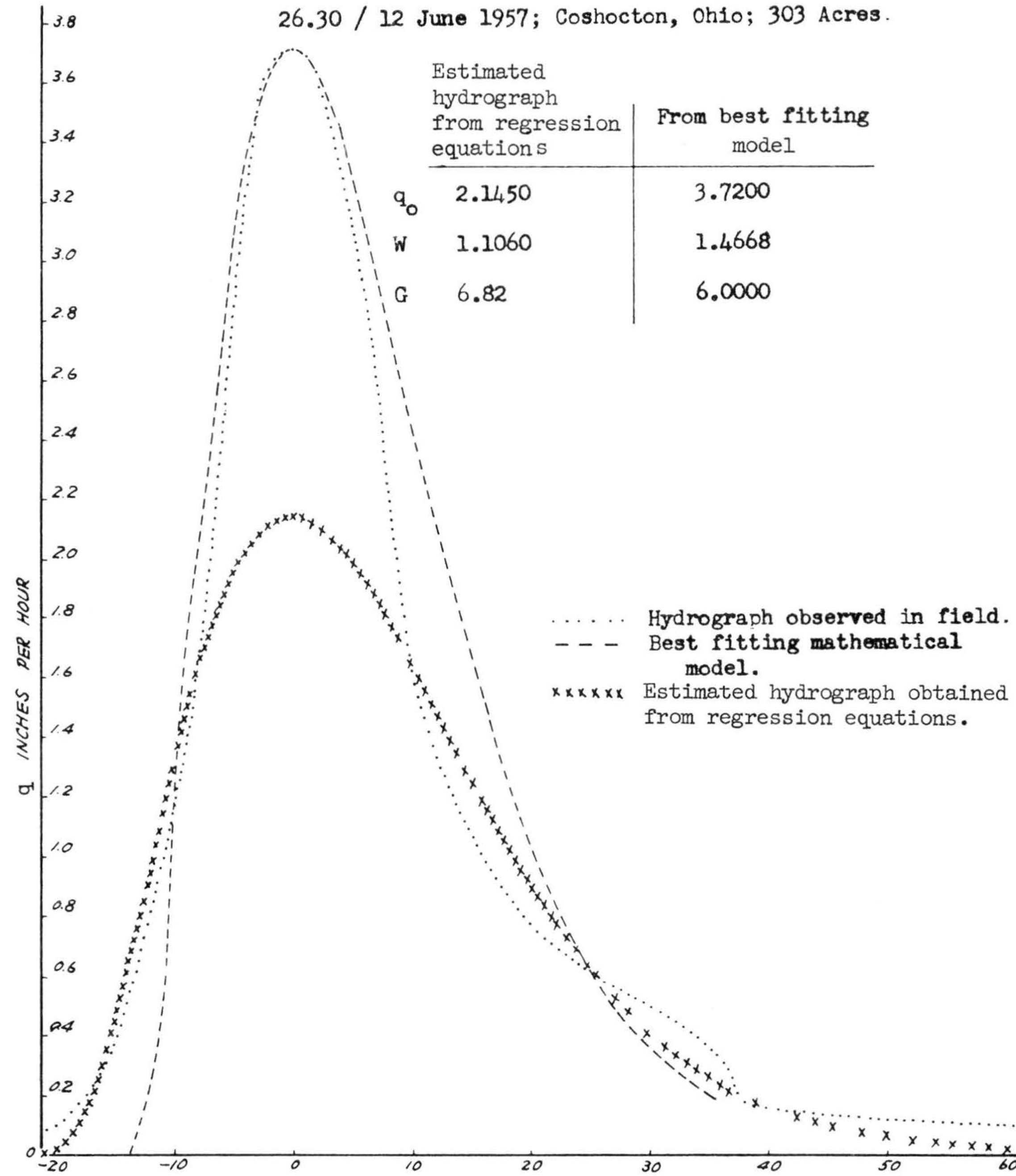


Fig. 14 Difference between estimated hydrograph obtained from regressions and observed or best-fitting hydrographs, when the peak rate has been poorly predicted.

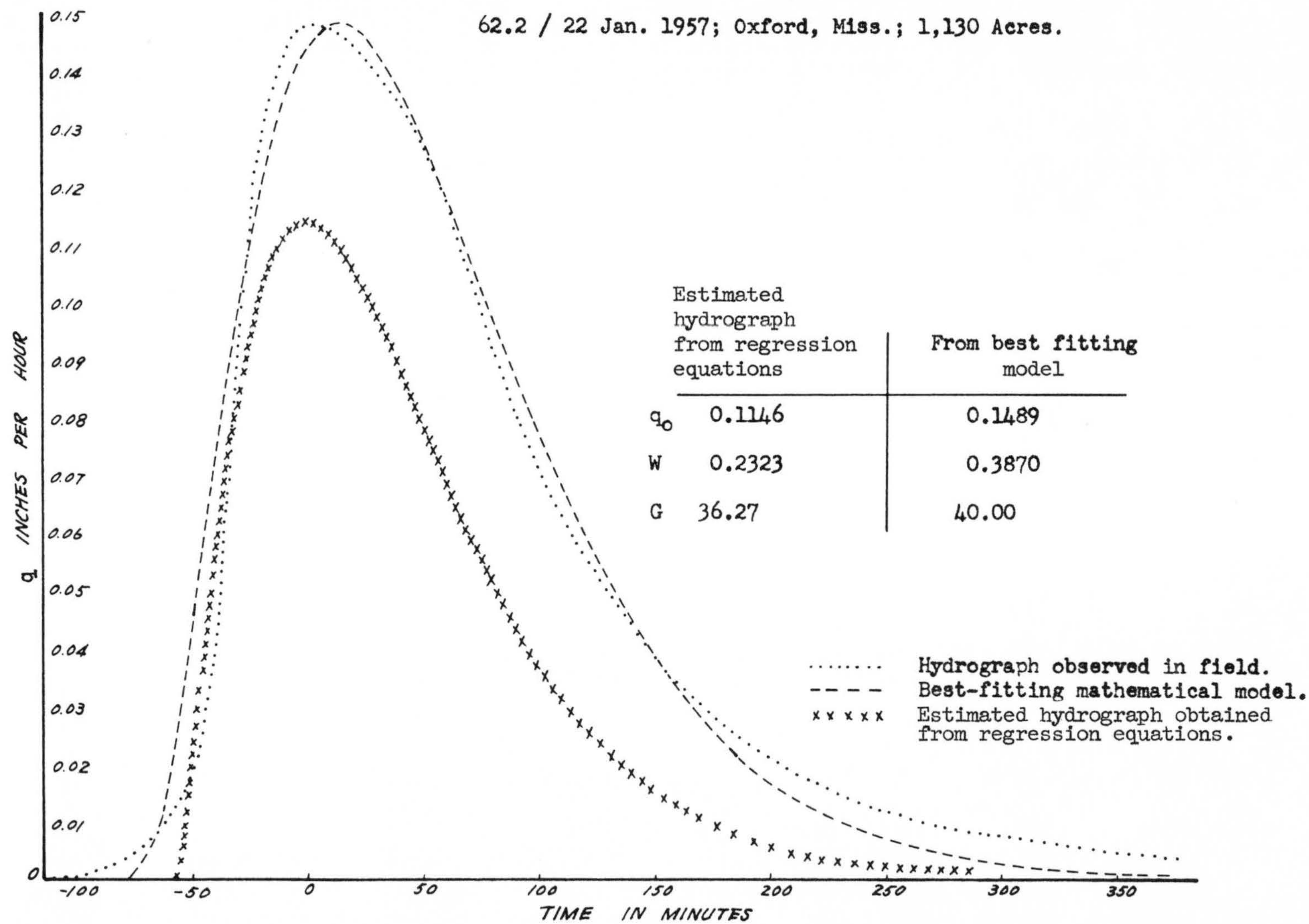


Fig. 15 Difference between estimated hydrograph obtained from regressions and observed or best-fitting hydrographs, when the peak rate and the volume have been poorly predicted.

Chapter 5

MEANS OF PREDICTING A HYDROGRAPH

By way of review it seems appropriate to outline how the regressions, developed in the previous chapter, could be used to predict a design hydrograph. No claims are made to extreme accuracy attainable by applying the tentative regressions. The intention is rather to show how such a method could be used for design considerations. Limitations of the existing study and proposals for improving the accuracy are discussed in the next chapter.

Obtaining Parameters for Hydrograph Model

First it is necessary to evaluate the parameters D_1, T_9 and R_1 for substitution into equation (11), so as to estimate W. D_1 is obtained as the product of f_1 and F which are estimated according to Tables 6 and 7 respectively. These have simply been copied from the A. S. C. E. Handbook (1) for convenient reference.

Table 6. Values of f_1 (infiltration capacity shown by standard curves at time 1 hour) for Bare Soils

Infiltration Characteristic	Range in f_1 (inches per hour)
High	0.50 to 1.00
Intermediate	0.10 to 0.50
Low	0.01 to 0.10

Table 7. Cover factor, F.

Cover		Range in value of cover factor, F.
Type	Condition	
Permanent (forest and grass)	Good	3.0 to 7.5
	Medium	2.0 to 3.0
	Poor	1.2 to 1.4
Close growing crops	Good	2.5 to 3.0
	Medium	1.6 to 2.0
	Poor	1.1 to 1.3
Row crops	Good	1.3 to 1.5
	Medium	1.1 to 1.3
	Poor	1.0 to 1.1

T_9, the time of concentration is obtained from Fig. 16, which has been reproduced here from the Soil Conservation Service Handbook (24).

The total storm precipitation, R_1, is not readily available from the rainfall frequency atlas (11) nor from any other national summary. Until

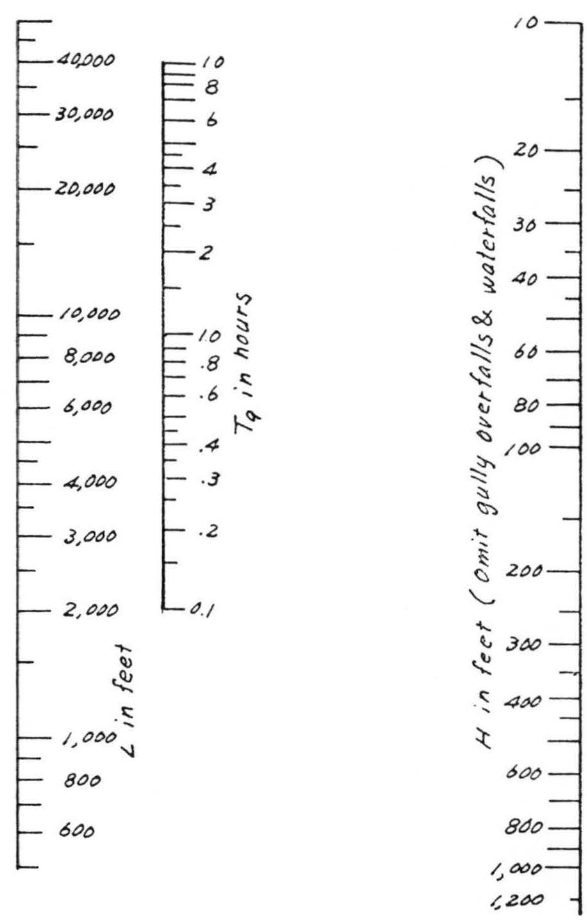


Fig. 16 Time of concentration, T_c or T_g, for head-water areas, based upon dimensions from outlet to ridge.

such information is published on a frequency basis design values will have to be judiciously estimated from local records of flood producing storms. The forty-seven cases studied involved convective storms in which most of the rain occurred in less than an hour. Thus the storm total in inches was generally of the same order of magnitude and often was numerically equal to I_{60} (the maximum rainfall intensity in inches per hour for sixty consecutive minutes). This suggests another approximation for estimating R_1, the storm total, of a particular return period from the rainfall frequency atlas in the absence of more suitable data.

Having thus evaluated D_1, T_9 and R_1 it is a simple matter to substitute them into equation (11) to obtain W, the volume of runoff in the model.

$$W = 0.1315 - 0.5792\,D_1 + 0.1902\,T_9 + 0.4261\,R_1 \quad \ldots \quad (11)$$

For field application this equation can be solved rapidly from the nomograph presented in Fig. 17.

The next requirement is to evaluate the peak of the hydrograph model q_o. The lengths T_3 and and T_2 can be readily measured from a map or on the watershed itself. The last parameter needed in equation (12) is R_{11}, the maximum rainfall intensity in inches per hour for thirty consecutive minutes. This can be obtained from the rainfall frequency atlas (11) for the desired return period.

$$q_o = -0.2917 + 0.4600\,R_{11} - 0.00040\,T_3 + 0.00018\,T_2 \quad \ldots\ldots \quad (12)$$

Rapid evaluation of this equation is likewise possible with the nomograph presented in Fig. 18.

The last of the three hydrograph parameters is G, and is predicted by equation (14).

$$G = \frac{7.314 \times 10^{-9} \times {D_5}^{5}}{{T_5}^{0.727} \times {T_6}^{0.939}} \quad \ldots\ldots \quad (14)$$

A nomograph could be prepared for its solution along the same lines as Figs. 17 and 18. The main-channel slope, T_5, and the average land slope, T_6, are easy to evaluate from maps or a field inspection of the watershed. D_5 is Cook's ΣW and can be evaluated from Table 8, which is reproduced (8) for convenient reference.

Calculation of Hydrograph Model

Having obtained values for W, q_o and G for an ungaged watershed, according to the preceding section, it remains only to sketch the corresponding hydrograph. With the three hydrograph parameters given one may easily obtain α from:

$$\alpha = \frac{\left[\frac{W}{q_o}\right]}{G} \quad \ldots\ldots \quad (3)$$

Since α is known, the relationship

$$\alpha = e^{\frac{m}{G}} \left[\frac{G}{m}\right]^{\frac{m}{G}} \Gamma\left[1 + \frac{m}{G}\right] \quad \ldots\ldots \quad (4)$$

fixes a value of $\frac{m}{G}$. This is conveniently obtained graphically as in Fig. 2. The product of this ratio and G gives a unique value of m.

Thus the three parameters q_o, G and m are available for the preparation of the hydrograph from its definition equation:

$$q = q_o\, e^{-\frac{t}{G}} \left[1 + \frac{t}{m}\right]^{\frac{m}{G}} \quad \ldots\ldots \quad (1)$$

This equation is solved for values of t from slightly larger than -m to a positive value at which the tail becomes insignificant.

A Sample Problem

Suppose we are required to sketch the flood hydrograph caused by the 100-year return period rainfall for the following 682 acre watershed located (for ease of illustration) where the four states Utah, Colorado, Arizona and New Mexico meet. Its characteristics are as follows:

a. the soil type and cover give, from Tables 6 and 7, a value of $D_1 = 0.77$ inches;
b. the fall over the watershed is $H = 1205$ feet;
c. the length of the longest collector from the outlet carried out to the watershed perimeter is $L = 17{,}880$ feet $= T_2$;
d. the length along the main stream from the outlet to the point nearest the mass center of area is $T_3 = 6940$ feet;
e. the average main channel slope was found from plotting the longitudinal section to be $T_5 = 0.0399$ feet per foot;
f. the average land slope was determined from a topographic map to be 35.0 percent;
g. an evaluation of the runoff-producing characteristics of the watershed in terms of Table 8 gave Cook's $\Sigma W = D_5 = 59$.

Solution -

h. Applying $H = 1205$ and $L = 17{,}880$ to

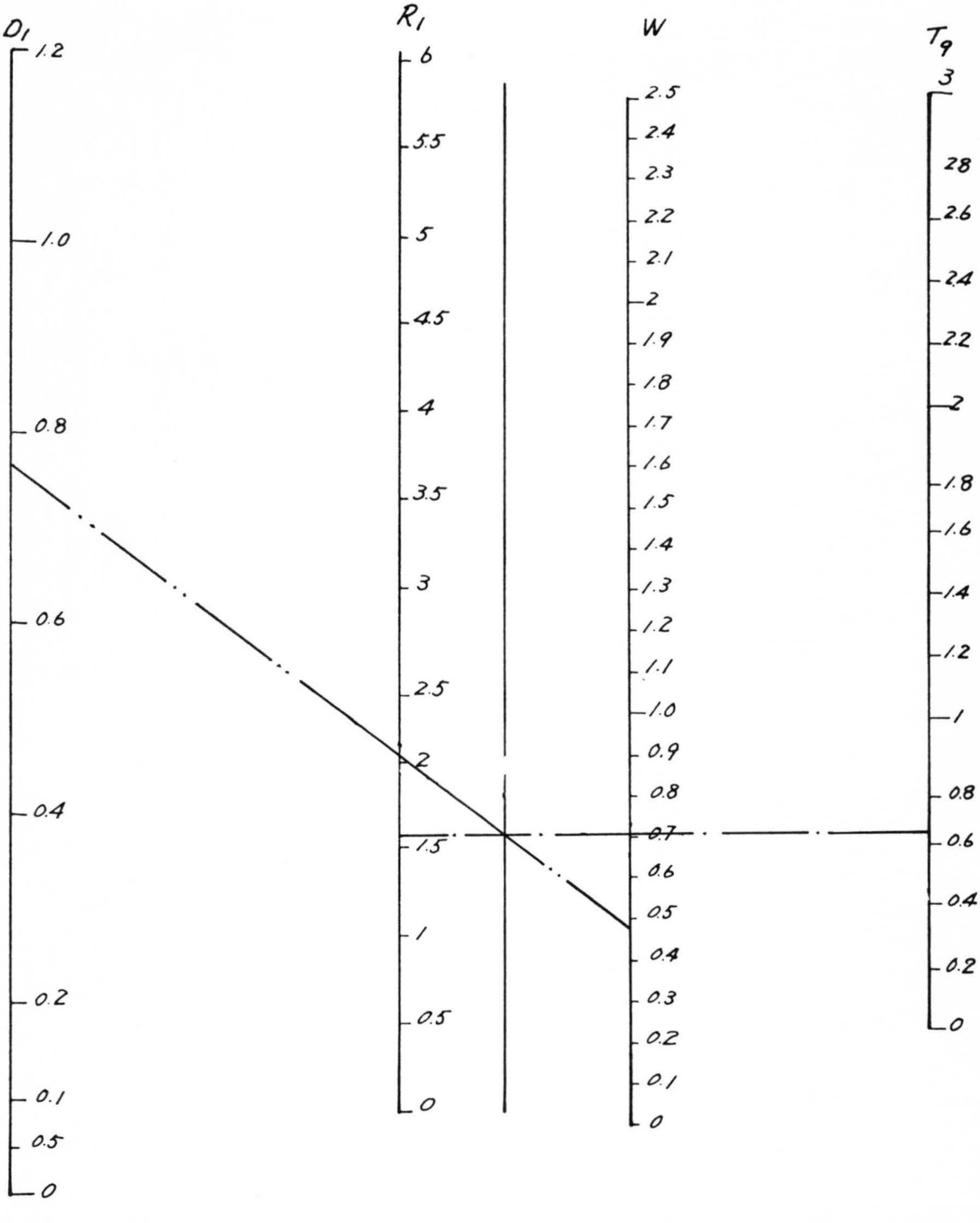


Fig. 17 **Nomograph for predicting** total runoff volume, W inches from:

a) **storm total** R_1 **inches,**

b) **time of concentration** T_9 **hours,**

c) **infiltration capacity of soil,** D_1 **inches/hour.**

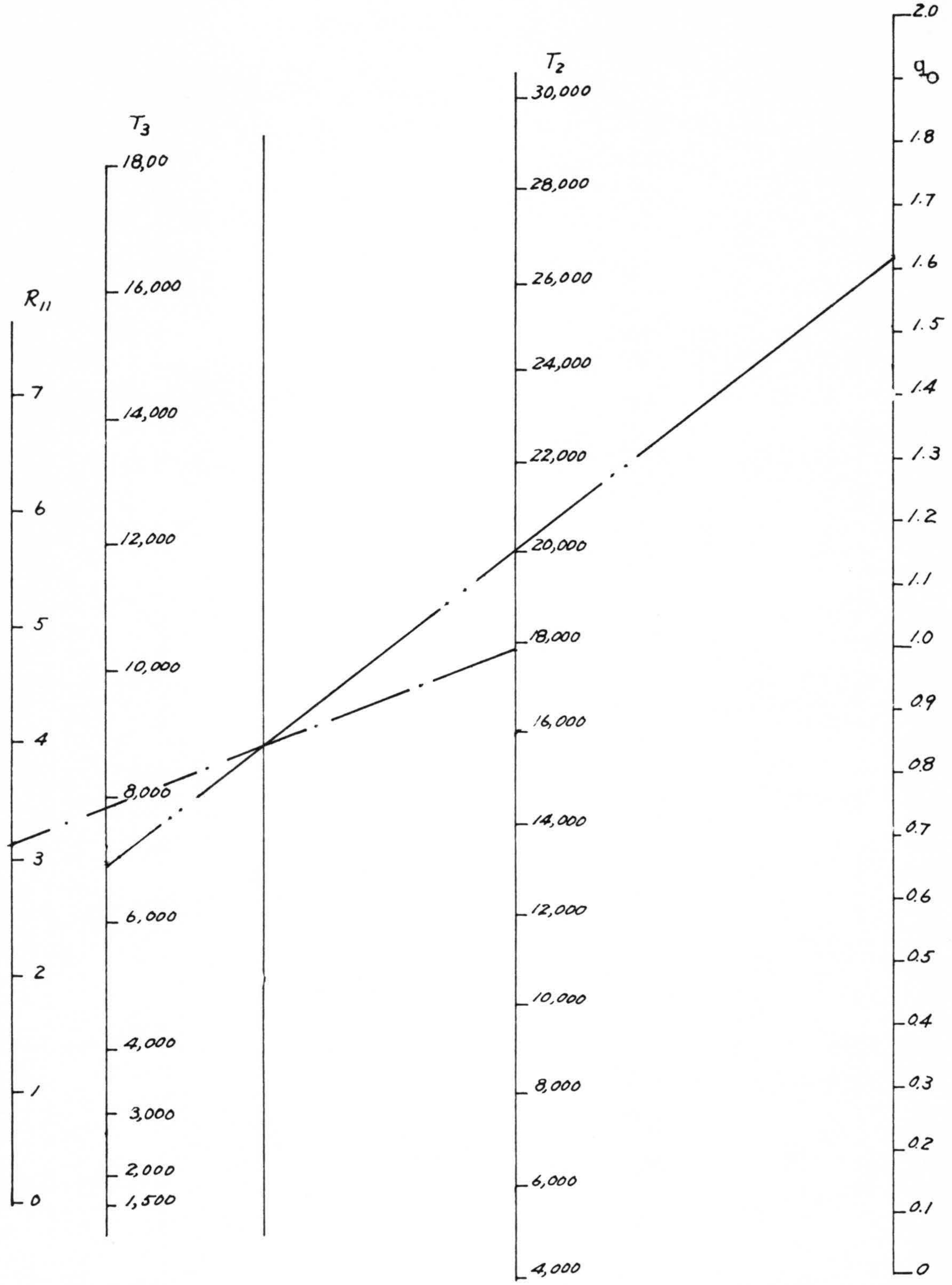


Fig. 18 **Nomograph for predicting** peak runoff rate, q_o inches/hour from:
a) 30 minute rainfall intensity, R_{11} inches/hour,
b) length up to center of area, T_3 feet,
c) length of longest collector to the divide, T_2 feet.

Table 8. Runoff-producing characteristics for the determination of Cook's ΣW.

Designation of watershed characteristics	Runoff-producing characteristics			
	100 Extreme	75 High	50 Normal	25 Low
Relief	(40) Steep, rugged terrain, with average slopes generally above 30%	(30) Hilly, with average slopes of 10 to 30%	(20) Rolling, with average slopes of 5 to 10%	(10) Relatively flat land, with average slopes of 0 to 5%
Soil infiltration	(20) No effective soil cover, either rock or thin soil mantle of negligible infiltration capacity	(15) Slow to take up water; clay or other soil of low infiltration capacity, such as gumbo	(10) Normal; deep loam with infiltration about equal to that of typical prairie soil	(5) High; deep sand or other soil that takes up water readily and rapidly
Vegetal cover	(20) No effective plant cover; bare or very sparse cover	(15) Poor to fair; clean-cultivated crops or poor natural cover; less than 10% of drainage area under good cover	(10) Fair to good; about 50% of drainage area in good grassland, woodland, or equivalent cover; not more than 50% of area in clean-cultivated crops	(5) Good to excellent; about 90% of drainage area in good grassland, woodland, or equivalent cover
Surface storage	(20) Negligible; surface depressions few and shallow; drainageways steep and small; no ponds or marshes	(15) Low; well-defined system of small drainageways; no ponds or marshes	(10) Normal; considerable surface-depression storage; lakes, ponds and marshes less than 2% of drainage area	(5) High; surface-depression storage high; drainage system not sharply defined

Fig. 16 gives the time of concentration, T_9, = 0.65 hours;

i. records of storm totals in the vicinity suggested the design value selected of R_1 = 1.56 inches;

j. the rainfall frequency atlas (11) gives the thirty-minute rainfall intensity of 100-year return period to be $R_{11} = 3.12$ inches per hour;

k. substituting the above values for D_1, T_9, R_1, R_{11}, T_3, T_2, T_5, T_6 and D_5 into equations (11), (12) and (14) gives: $W = 0.4738$ inches, $q_o = 1.6193$ inches per hour and $G = 6.22$ minutes. These evaluations may either be made by arithmetic substitution or by nomographs like Figs. 17 and 18;

l. it is necessary to obtain the value of m corresponding to this set of W, q_o and G. Using equation (3) one obtains:

$$\alpha = \frac{\left[\frac{W}{q_o}\right]}{G} = \frac{\left[\frac{0.4738}{1.6193}\right]}{\left[\frac{6.22}{60}\right]} = 2.8224 \; .$$

Entering Fig. 2 with this value of α one obtains $\frac{m}{G} = 1.08$. Multiplying by $G = 6.22$ minutes gives $m = 6.72$ minutes;

m. equation (1) becomes:

$$q = 1.619\, e^{-\frac{t}{6.22}} \left[1 + \frac{t}{6.72}\right]^{\frac{6.72}{6.22}}$$

which is solved for t ranging from -6 to 49 minutes.

n. The resultant model hydrograph is shown by the crossed curve in Fig. 19. The accompanying dotted curve represents a hydrograph actually obtained under field conditions (a) to (g), which prevailed in southern Arizona at Safford. The storm causing this event happened to have a thirty-minute rainfall intensity corresponding to the 100-year value at the Four Corners locality as listed in (j). For comparison the model best fitted to a hydrograph actually observed, for a watershed and storm assumed in this problem, has been added.

o. If the result is required in cubic feet per second all ordinates should be increased in ratio of 688 to 1. This is obtained by multiplying the area 682 acres by 1.008.

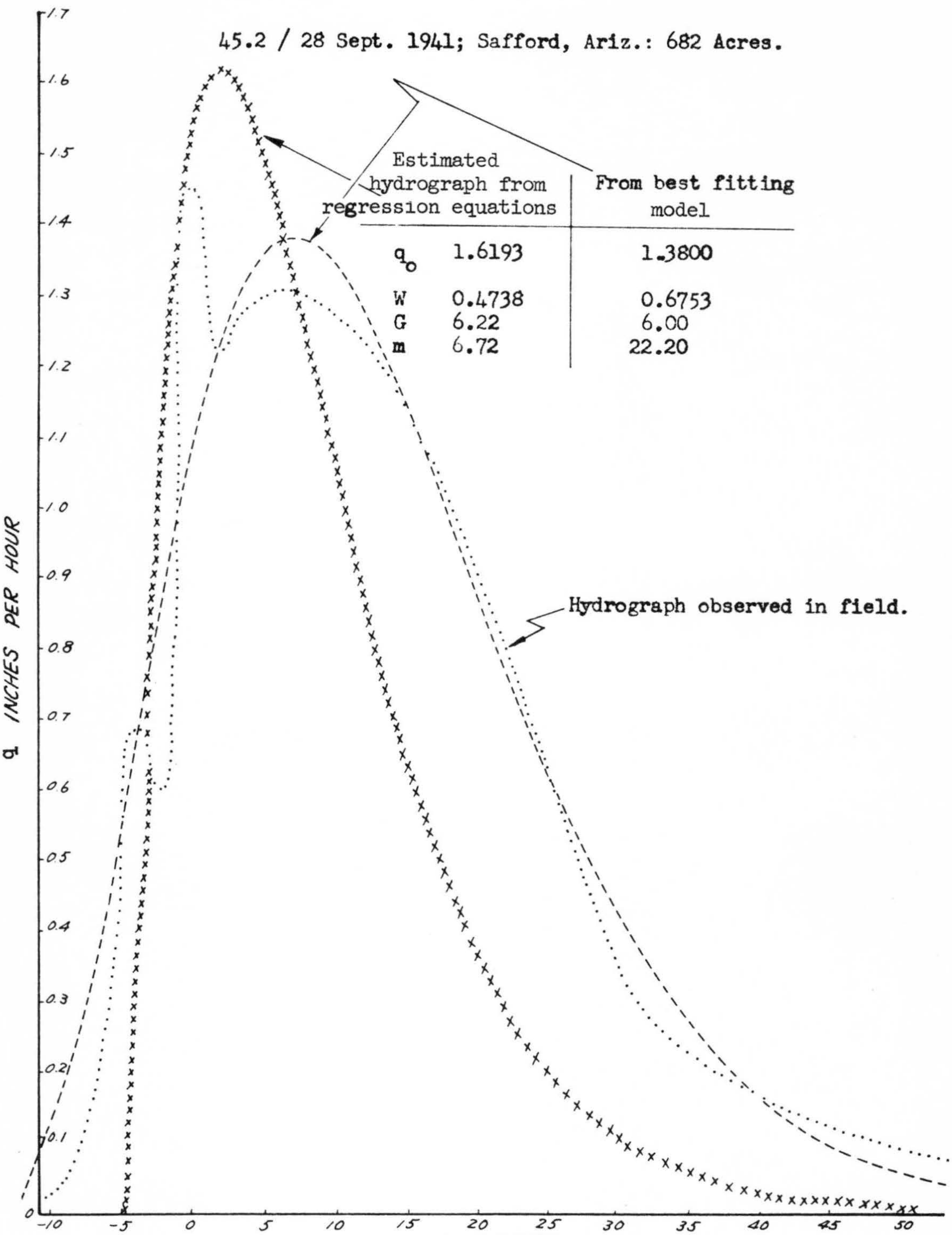


Fig. 19 Estimated hydrograph, computed from watershed and storm **parameters, compared to one actually observed.**

Chapter 6

SUGGESTIONS FOR FURTHER RESEARCH

The limited time and funds available for this study have restricted the scope of the investigation. The results obtained are not conclusive but they do establish the feasibility of the approach. In the hope that further research will be conducted, some suggestions are made for their possible inclusion.

Larger and More Selective Sample Needed

Degrees of freedom should be increased - Perhaps the greatest limitation to the present study was the fact that only forty-seven hydrographs were readily available for the study. Such a small number of cases gave very few degrees of freedom in the multiple regression analysis. For instance with the maximum number of variables ever to be considered together the degrees of freedom fell as low as fourteen. It must be pointed out, however, that development of the three-variable equations actually involved forty-three degrees of freedom. Nevertheless if a new study could be made with say five hundred hydrographs, far more reliable regressions could be obtained. Such an enlarged sample could be collected from many more localities throughout the United States. Forested watersheds should also be included because of the abundance of national and other forests in headwater areas.

Small floods should be excluded from the study - It has been shown in Figs. 10, 11 and 12 that all three regressions were adversely affected by inclusion in the data of observed floods having small q_o . Results will be used to predict large floods. So it seems desirable to set a lower limit to q_o , below which observed flood events will be disregarded. Because of the few observations available to the present study and because of its exploratory nature, peak rates as small as one two-hundredth of an inch per hour were included. These are clearly not of the order to be required for design purposes.

Larger range in watershed sizes possible - There is no reason for future studies to adhere rigidly to the arbitrary delineation set on watershed size in this study. Larger or smaller watersheds could be studied by the method developed here. Stratification of the sample according to size appears desirable so as to maintain classes within which areal effects are kept to a minimum. For instance, difficulties are envisaged if some watersheds which are so large that isohyetal patterns are important should be included with smaller watersheds on which the assumption of uniform areal distribution of rainfall is a valid approximation.

Watershed Response Characteristics

A fifth group of independent variables, basically different from those studied here, could be combined with some of these storm and watershed parameters in a future study. This fifth group could possibly relate certain indices of watershed response to other observed hydrograph features. For instance the depletion characteristic of a hydrograph is conceivably relatively constant for all floods on one watershed. Some support for this belief can be found in the fact that equation (14) describes G without any reference to rainfall parameters. Some numerical index of depletion may integrate the combined effect of many physical factors. If this is true it may be worth the designer's effort to collect a few hydrographs at a site prior to designing an important hydrologic structure. Likewise the timing of runoff features with respect to the rainfall trace, or the Φ-index may provide useful indices of watershed response. If this were found to be the case rainfall recorders could be positioned along with the flow recorder for a season or two.

Need for Statistical Test in Fitting Hydrograph Models

As described in Chapter 2, the best-fitting models were evaluated by a visual appraisal of how well each trial plot approximated the observed hydrograph. This required the expensive and time-consuming task of actually plotting each of the four hundred and twenty curves evaluated by the mathematical model. Consequently the number of values of G , W (and even q_o under special circumstances already discussed) had to be limited. If a numerical test could be developed for deciding whether an acceptable fit has been achieved by the model, then much more could be achieved. Such a test could be applied automatically by a digital computer which could continue to vary the trial parameters until one or more well-fitting models are attained. Some computer facilities could actually plot the observed and mathematical hydrographs. The added precision with which W , q_o and G could thus be determined for the best-fitting hydrograph would be reflected by more reliable regressions relating them to storm and watershed characteristics.

Simplification of Practical Application Desired

Once enough data has been studied to establish adequate reliance on the regressions, the method of producing design hydrographs should be adapted to simple field use. Figures 17 and 18 showed how nomographs could simplify the evaluation of the regression equations for W , q_o and G . Step (m) in the solution of the sample problem in Chapter 5 involves the solution of:

$$q = q_o e^{-\frac{t}{G}} \left[1 + \frac{t}{m}\right]^{\frac{m}{G}} \quad \ldots\ldots\ldots \quad (1)$$

With numerical values substituted for m and G this simplifies considerably. However, where a large number of solutions are to be made, it may be well to reduce the individual computations by the preparation of tables or graphical aids.

Physical Laws Should Provide the Form of Regression Equations

It cannot be assumed that the best regression equations will have either the linear or logarithmic form of equations (11), (12) and (13). The stepwise multiple linear regression technique lends itself to a virtually unlimited number of equational forms. Squares, powers, exponents, products and sums can be entered into the analysis in place of any of the untransformed data. The number of such empirical trials to obtain the best functional relationship is limited practically only by cost. It would be most valuable if an independent study of hydraulic laws and the other physical processes influencing runoff could develop the mathematical form in which the causative elements should be combined (28, 29, 30). In the same way that dimensional analysis forms the basis for the experimental evaluation of constants in mechanical engineering, such physical considerations could enhance the interpretation of hydrologic data. The resulting combination of physical and statistical investigations will yield the most rational understanding of flood hydrographs.

Probability Aspects Concern Designers

The question of how a flood frequency differs from that of the rainstorm with which it is associated deserves serious consideration. Brief mention was made of the concept of joint probability in connection with the simultaneous requirement for an estimate of total storm rainfall and the thirty-minute rainfall intensity each of the same return period. Particularly where two elements involved are so closely correlated as the thirty-minute intensity and the storm total a correlation table study should be made to provide a means of modifying the multiplicative law of probability. Similar joint probability considerations involve the occurrence of a particular storm total following severe antecedent rainfall. They warrant special study from the national rainfall data.

National Study of Rainstorms is Needed

It has been shown in equation (11) that the volume of the flood hydrograph is related to the total storm rainfall. Maps should be made available for readily estimating the depth of storm rainfall expected at any locality with a particular return period. This would supplement the summary which Hershfield (11) recently published for rainfall intensity. Such a study should include depth-area investigations. In fact its scope could be extended to a broad consideration of parameters for defining rainfall distributions in time and space so as to most effectively describe the runoff process.

Chapter 7

SUMMARY

One purpose of this study was to determine whether the shape of flood hydrographs observed on small watersheds throughout the entire country could be satisfactorily described by the same mathematical function. It appears that total runoff may be adequately approximated by a Pearson Type III curve. This unimodel skew curve was described by the three parameter equation:

$$q = q_o \, e^{-\frac{t}{G}} \left[1 + \frac{t}{m}\right]^{\frac{m}{G}} \quad \ldots\ldots\ldots \quad (1)$$

Mathematical properties facilitated a transformation whereby this hydrograph model could be specified by the following three parameters:

1. the peak rate of discharge per unit of watershed area, q_o ,
2. volume of flood runoff per unit of watershed area, W ,
3. G , a characteristic time depicting flood recession.

In the second phase each of these three hydrograph parameters were related to the topography, land use or rainfall associated with each flood event. The regression equations, thus produced, can be used to tentatively estimate hydrographs over vast areas of the United States of America. The hydrograph volume depends upon: the total storm rainfall, an existing tabulation of infiltration capacity, and the time of concentration as obtained by an existing nomograph from the entire length of the watercourse and its rise from site to divide. The peak rate of discharge per unit of watershed area can be predicted from: the thirty-minute rainfall intensity, and the lengths along the longest collector to the divide and nearest to the mass center of the watershed respectively. The recession characteristic, G , was related to: an index of runoff potential once proposed by the Soil Conservation Service and reproduced under the name "Cook's Σ W ," and to the average land and channel slopes.

After solving a practical problem, some suggestions are given of how further work could refine the three regressions and could otherwise improve the prediction of design hydrographs.

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APPENDIX

Table 9. Values of dependent variables for observed floods.

W. No.	M/D/Y	W inches	q_o in/hr	G min.
15.1	9/ 8/48	0.0028	0.0059	9.5
	4/13/49	0.3715	0.3990	10.0
	6/ 7/55	0.0094	0.0053	24.0
17.4	5/27/38	0.5869	1.0550	10.5
	6/21/42	0.6743	0.9793	14.5
	3/31/52	0.7735	1.6220	6.2
	7/ 2/52	1.1047	1.8600	15.0
21.1	6/ 1/53	0.9070	0.4881	23.5
	7/21/48	0.6827	0.3392	12.5
	7/ 1/50	1.1866	0.6490	21.0
	7/18/56	0.9999	0.8580	40.0
26.30	6/16/46	1.4413	1.8990	18.0
	8/16/47	0.2421	0.5830	9.0
	9/ 1/50	1.7711	1.7690	6.0
	6/12/57	1.4668	3.7200	6.0
29.1	7/28/49	0.2740	0.0808	50.0
	5/13/55	0.3838	0.1400	138.0
	6/ 4/58	1.0909	0.5050	55.0
31.1	8/12/43	0.4263	0.8940	4.0
	7/11/44	0.2252	0.3020	7.5
	6/28/45	0.4081	0.9800	5.0
	6/24/49	0.4015	0.7230	10.0
42.3	6/10/41	1.3451	0.7466	30.0
	6/ 5/42	0.7792	0.3220	60.0
	6/15/50	0.8328	0.5360	30.0
	4/24/57	1.6405	0.8200	35.0
44.1	6/20/39	0.9041	1.1500	20.0
	7/10/51	1.6620	1.7400	12.0
	6/ 7/53	0.8787	0.7180	18.0
	6/15/57	1.2488	1.8200	20.0
44.3	7/10/51	1.3165	0.3480	80.0
	6/ 7/53	0.8174	0.2640	70.0
	8/29/57	1.3187	0.2170	100.0
45.2	6/26/40	0.3284	0.9500	2.2
	9/28/41	0.6753	1.3800	6.0
	8/ 7/42	0.1939	0.8480	4.0
	8/ 9/43	0.3389	1.0000	5.0
48.2	7/28/39	0.0946	0.2360	8.0
	8/24/39	0.0773	0.2090	20.8
	8/26/39	0.0852	0.1790	15.0
	9/ 5/40	0.0337	0.1790	5.0
60.6	4/13/37	0.0524	0.0166	52.0
	1/25/41	0.0525	0.0256	20.0
	5/ 3/41	0.0431	0.0311	12.0
	6/ 6/41	0.0373	0.0342	18.3
62.1	5/22/57	0.4264	0.2445	15.0
62.2	1/22/57	0.3870	0.1489	40.0

Table 10. Values of topographic variables for watersheds studied, I.

W. No.	M/D/Y/	Area Acres T_1	Length Feet T_2	L_c Feet T_3	H Feet T_4	S_c Ft./Ft. T_5	S_a Percent T_6	$L/\sqrt{S_c}$ T_7	Drainage Density T_8	T_c hrs. T_9
15.1	9/ 8/48	390	8050	4020	225	0.01770	14.530	1.143	98.700	0.500
	4/13/49	390	8050	4020	225	0.01770	14.530	1.143	98.700	0.500
	6/ 7/55	390	8050	4020	225	0.01770	14.530	1.143	98.700	0.500
17.4	5/27/38	290	5160	2310	54	0.00596	5.370	1.264	68.400	0.530
	6/21/42	290	5160	2310	54	0.00596	5.370	1.264	68.400	0.530
	3/31/52	290	5160	2310	54	0.00596	5.370	1.264	68.400	0.530
	7/ 2/52	290	5160	2310	54	0.00596	5.370	1.264	68.400	0.530
21.1	6/ 1/53	1926	19750	9860	113	0.00562	11.600	4.980	21.900	1.850
	7/21/48	1926	19750	9860	113	0.00562	11.600	4.980	21.900	1.850
	7/ 1/50	1926	19750	9860	113	0.00562	11.600	4.980	21.900	1.850
	7/18/56	1926	19750	9860	113	0.00562	11.600	4.980	21.900	1.850
26.30	6/16/46	303	4610	1880	220	0.03630	16.210	0.458	26.400	0.270
	8/16/47	303	4610	1880	220	0.03630	16.210	0.458	26.400	0.270
	9/ 1/50	303	4610	1880	220	0.03630	16.210	0.458	26.400	0.270
	6/12/57	303	4610	1880	220	0.03630	16.210	0.458	26.400	0.270
29.1	7/28/49	345	6070	3460	55	0.00831	2.470	1.262	13.600	0.630
	5/13/55	345	6070	3460	55	0.00831	2.470	1.262	13.600	0.630
	6/ 4/58	345	6070	3460	55	0.00831	2.470	1.262	13.600	0.630
31.1	8/12/43	330	5910	2690	130	0.01880	5.970	0.816	25.400	0.450
	7/11/44	330	5910	2690	130	0.01880	5.970	0.816	25.400	0.450
	6/28/45	330	5910	2690	130	0.01880	5.970	0.816	25.400	0.450
	6/24/49	330	5910	2690	130	0.01880	5.970	0.816	25.400	0.450
42.3	6/10/41	1110	11360	4860	70	0.00392	2.100	3.430	27.700	1.200
	6/ 5/42	1110	11360	4860	70	0.00392	2.100	3.430	27.700	1.200
	6/15/50	1110	11360	4860	70	0.00392	2.100	3.430	27.700	1.200
	4/24/57	1110	11360	4860	70	0.00392	2.100	3.430	27.700	1.200
44.1	6/20/39	481	8710	3480	65	0.00580	5.500	2.165	59.200	0.900
	7/10/51	481	8710	3480	65	0.00580	5.500	2.165	59.200	0.900
	6/ 7/53	481	8710	3480	65	0.00580	5.500	2.165	59.200	0.900
	6/15/57	481	8710	3480	65	0.00580	5.500	2.165	59.200	0.900
44.3	7/10/51	2086	27820	14570	135	0.00408	5.620	8.250	58.900	2.500
	6/ 7/53	2086	27820	14570	135	0.00408	5.620	8.250	58.900	2.500
	8/29/57	2086	27820	14570	135	0.00408	5.620	8.250	58.900	2.500
45.2	6/26/40	682	17880	6940	1205	0.03990	10.770	1.695	63.400	0.650
	9/28/41	682	17880	6940	1205	0.03990	10.770	1.695	63.400	0.650
	8/ 7/42	682	17880	6940	1205	0.03990	10.770	1.695	63.400	0.650
	8/ 9/43	682	17880	6940	1205	0.03990	10.770	1.695	63.400	0.650
48.2	7/28/39	610	9910	4670	770	0.04780	35.000	0.793	14.300	0.390
	8/24/39	610	9910	4670	770	0.04780	35.000	0.793	14.300	0.390
	8/26/39	610	9910	4670	770	0.04780	35.000	0.793	14.300	0.390
	9/ 5/40	610	9910	4670	770	0.04780	35.000	0.793	14.300	0.390
60.6	4/13/37	762	9810	4210	300	0.02500	16.500	1.211	66.800	0.550
	1/25/41	762	9810	4210	300	0.02500	16.500	1.211	66.800	0.550
	5/ 3/41	762	9810	4210	300	0.02500	16.500	1.211	66.800	0.550
	6/ 6/41	762	9810	4210	300	0.02500	16.500	1.211	66.800	0.550
62.1	5/22/57	2000	13830	6275	145	0.00607	7.730	2.985	12.800	1.050
62.2	1/22/57	1130	10110	4040	95	0.00575	6.130	1.915	8.000	0.920

Table 11. Values of design indices depicting land use and soils, II.

W. No.	M/D/Y	D_1	D_2	D_4	D_5	D_6	D_7	D_8
15.1	9/ 8/48	1.140	0.005	0.48	64	74	74	42
	4/13/49	1.140	0.210	1.50	64	74	56	35
	6/ 7/55	1.140	0.040	0.48	64	74	74	58
17.4	5/27/38	0.072	0.410	0.22	43	86	86	46
	6/21/42	0.069	0.150	0.60	43	86	72	34
	3/31/52	0.066	0.360	0.22	43	86	86	55
	7/ 2/52	0.066	0.530	0.60	43	86	72	51
21.1	6/ 1/53	1.210	0.110	2.00	62	81	64	67
	7/21/48	1.210	0.300	2.00	62	81	64	61
	7/ 1/50	1.170	0.570	2.00	62	81	64	63
	7/18/56	1.210	0.440	2.00	62	81	64	61
26.30	6/16/46	0.820	1.710	0.26	64	76	76	43
	8/16/47	0.800	0.130	0.24	64	78	78	45
	9/ 1/50	0.790	0.870	0.81	64	76	58	52
	6/12/57	0.800	0.360	0.81	64	76	58	54
29.1	7/28/49	0.570	1.100	0.12	51	87	97	55
	5/13/55	0.570	0.001	1.05	51	87	72	59
	6/ 4/58	0.570	0.950	1.05	51	87	72	60
31.1	8/12/43	0.710	0.170	1.05	48	82	65	67
	7/11/44	0.670	0.160	1.00	48	83	67	58
	6/28/45	0.660	0.720	0.09	48	84	96	44
	6/24/49	0.700	1.840	0.09	48	84	97	41
42.3	6/10/41	0.084	0.110	2.30	47	69	69	65
	6/ 5/42	0.088	3.320	0.22	47	87	97	57
	6/15/50	0.107	2.140	0.22	50	85	97	48
	4/24/57	0.101	1.410	0.22	50	86	97	47
44.1	6/20/39	0.430	0.070	1.30	50	83	67	59
	7/10/51	0.430	0.410	1.45	50	82	66	45
	6/ 7/53	0.390	0.100	1.15	52	85	70	46
	6/15/57	0.410	0.180	1.25	52	84	68	43
44.3	7/10/51	0.540	0.630	3.00	52	84	68	50
	6/ 7/53	0.500	0.110	3.00	52	84	68	52
	8/29/57	0.510	0.390	2.70	53	85	70	43
45.2	6/26/40	0.770	0.001	7.00	59	63	43	62
	9/28/41	0.770	0.001	7.00	59	63	43	72
	8/ 7/42	0.770	0.001	7.00	59	63	43	64
	8/ 9/43	0.770	0.001	7.00	59	63	43	70
48.2	7/28/39	0.550	0.001	0.92	82	77	59	81
	8/24/39	0.550	0.001	0.92	82	77	59	69
	8/26/39	0.550	0.001	0.92	82	77	59	69
	9/ 5/40	0.550	0.001	0.92	82	77	59	86
60.6	4/13/37	0.630	0.001	0.57	76	85	70	60
	1/25/41	0.620	0.001	0.57	76	85	70	60
	5/ 3/41	0.620	0.001	0.57	76	85	70	60
	6/ 6/41	0.640	0.001	0.63	76	84	68	60
62.1	5/22/57	0.540	0.290	0.52	59	83	83	52
62.2	1/22/57	0.540	0.001	1.45	57	83	67	47

Table 12. Values of conventional rainfall parameters, III.

W. No.	M/D/Y	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	R_{13}	R_{14}	R_{15}	R_{16}
15.1	9/ 8/48	0.700	0.110	0.420	0.160	0.690	0.010	0.120	2.960	2.640	2.320	1.380	0.700	9.	1.09	1.09	0.56
	4/13/49	1.690	0.015	0.155	0.100	0.180	0.163	0.120	4.080	3.020	2.720	2.120	1.230	18.	0.24	0.22	0.20
	6/ 7/55	1.120	0.035	0.042	0.108	0.185	0.555	0.120	2.920	2.180	1.920	1.464	0.842	0.1	1.73	1.73	1.73
17.4	5/27/38	1.375	0.270	0.115	0.568	0.952	0.690	0.360	4.442	4.125	3.582	2.440	1.300	5.	1.62	0.43	0.43
	6/21/42	1.640	0.528	0.662	0.208	1.400	0.073	4.100	4.502	3.974	3.676	2.796	1.471	0.1	0.69	0.67	0.001
	3/31/52	1.270	0.495	0.473	0.240	1.208	0.010	1.970	3.795	3.197	2.781	2.413	1.217	3.	1.65	1.65	1.65
	7/ 2/52	2.520	0.526	1.033	0.787	2.346	0.119	1.950	7.827	6.371	6.593	4.693	2.465	0.1	0.70	0.70	0.70
21.1	6/ 1/53	1.960	0.125	0.675	0.143	0.943	1.017	1.500	4.200	4.200	3.212	1.886	1.960	10.	0.74	0.74	0.23
	7/21/48	2.610	0.065	0.108	0.153	0.326	0.228	0.390	3.915	3.387	2.683	1.871	1.658	100.	0.09	0.09	0.09
	7/ 1/50	3.230	0.583	0.433	0.396	1.412	1.802	3.500	3.500	3.500	3.274	2.823	1.878	0.1	0.001	0.001	0.001
	7/18/56	2.940	0.420	0.550	0.630	1.620	1.090	2.660	3.750	3.750	3.709	3.451	2.762	0.1	0.46	0.46	0.02
26.30	6/16/46	3.973	0.175	0.264	0.460	0.899	1.655	1.710	5.031	4.688	4.345	3.570	2.732	16.	1.50	0.77	0.001
	8/16/47	1.217	0.168	0.510	0.415	1.093	0.076	0.800	3.917	3.508	3.145	2.194	1.169	8.	1.85	1.85	1.12
	9/ 1/50	4.433	0.288	0.288	0.234	0.811	1.414	1.730	4.112	3.536	3.437	3.080	2.769	32.	0.04	0.001	0.001
	6/12/57	3.254	0.120	0.326	0.727	1.173	1.894	0.510	8.010	6.010	5.913	5.110	3.067	16.	1.23	0.35	0.35
29.1	7/28/49	1.400	0.326	0.387	0.417	1.130	0.270	2.250	3.440	2.860	2.504	2.295	1.400	0.1	2.57	2.47	1.55
	5/13/55	0.860	0.119	0.548	0.193	0.860	0.001	0.280	4.943	3.970	3.123	1.820	0.910	15.	0.62	0.62	0.30
	6/ 4/58	3.230	0.023	0.023	0.710	0.757	1.432	0.140	5.299	4.784	4.482	3.488	3.046	22.	0.67	0.10	0.10
31.1	8/12/43	2.150	1.252	0.912	0.220	2.383	0.001	1.500	9.350	8.395	7.650	4.301	2.150	2.	1.25	1.25	1.05
	7/11/44	1.940	0.540	1.060	0.300	1.900	0.040	3.240	6.360	6.360	5.320	3.800	1.940	0.1	0.03	0.03	0.03
	6/28/45	1.090	0.860	0.121	0.023	1.003	0.069	8.200	6.873	5.159	3.812	2.007	1.073	0.1	2.16	1.21	1.16
	6/24/49	2.160	0.545	0.697	0.358	1.600	0.305	1.650	4.702	4.445	4.241	3.200	1.905	3.	2.90	2.36	2.06
42.3	6/10/41	1.650	0.185	0.175	0.302	0.662	0.916	1.200	3.578	3.280	2.932	2.235	1.578	24.	2.07	0.58	0.54
	6/ 5/42	1.010	0.352	0.380	0.113	0.846	0.101	2.110	2.872	2.569	2.417	1.691	0.947	2.	3.68	1.36	1.36
	6/15/50	1.880	0.166	0.266	0.106	0.538	0.429	0.320	4.360	3.838	3.250	2.032	1.442	8.	2.55	2.25	0.09
	4/24/57	1.700	0.439	0.614	0.314	1.367	0.298	1.340	4.435	3.793	3.614	2.733	1.627	4.	10.31	4.38	3.67
44.1	6/20/39	1.630	0.525	0.610	0.394	1.529	0.101	2.500	4.965	4.250	3.707	3.057	1.630	0.1	0.60	0.001	0.001
	7/10/51	2.700	0.179	0.022	0.022	0.223	0.169	2.020	5.026	4.923	4.486	3.121	2.164	0.1	0.001	0.001	0.001
	6/ 7/53	1.560	0.305	0.420	0.227	0.952	0.413	1.180	3.134	2.788	2.500	1.910	1.365	4.	0.68	0.50	0.25
	6/15/57	1.950	0.268	0.248	0.248	0.762	1.130	1.600	5.520	4.060	3.320	2.340	1.891	42.	1.64	0.86	0.001
44.3	7/10/51	2.970	0.205	0.010	0.010	0.225	0.255	2.400	5.923	5.760	4.924	3.703	2.603	0.1	0.01	0.001	0.001
	6/ 7/53	1.710	0.391	0.363	0.267	1.020	0.470	1.400	3.300	2.892	2.520	2.040	1.490	3.	0.48	0.48	0.001
	8/29/57	2.380	0.250	0.410	0.375	1.035	0.626	1.500	3.564	3.120	2.590	2.120	1.661	12.	0.09	0.09	0.001
45.2	6/26/40	0.980	0.363	0.417	0.116	0.896	0.084	1.880	3.380	3.076	2.677	1.792	0.980	8.	0.001	0.001	0.001
	9/28/41	1.560	0.754	0.559	0.247	1.560	0.001	5.320	5.320	4.520	3.923	3.120	1.560	0.1	0.75	0.75	0.75
	8/ 7/42	1.200	0.029	0.051	0.460	0.540	0.503	0.180	4.800	3.780	3.087	1.893	1.140	26.	0.08	0.001	0.001
	8/ 9/43	1.010	0.430	0.513	0.067	1.010	0.001	1.000	4.200	3.960	3.307	2.020	1.010	3.	0.27	0.27	0.001
48.2	7/28/39	1.500	0.180	0.285	0.315	0.780	0.705	1.080	3.240	3.240	2.560	2.300	1.485	15.	0.001	0.001	0.001
	8/24/39	0.720	0.300	0.310	0.020	0.630	0.040	1.800	2.400	2.100	2.000	1.260	0.670	10.	0.29	0.29	0.29
	8/26/39	0.800	0.460	0.230	0.040	0.730	0.033	3.600	3.600	2.760	2.480	1.460	0.730	0.1	0.38	0.38	0.001
	9/ 5/40	0.300	0.300	0.001	0.001	0.300	0.001	3.000	3.000	1.800	1.200	0.600	0.300	0.1	0.001	0.001	0.001
60.6	4/13/37	0.440	0.005	0.005	0.015	0.025	0.025	0.030	0.240	0.240	0.240	0.228	0.221	999.9	0.52	0.26	0.10
	1/25/41	0.280	0.010	0.010	0.019	0.039	0.054	0.060	0.200	0.200	0.200	0.200	0.158	999.9	0.24	0.24	0.24
	5/ 3/41	0.310	0.015	0.015	0.060	0.090	0.123	0.090	0.360	0.360	0.360	0.320	0.230	999.9	0.39	0.36	0.02
	6/ 6/41	0.370	0.280	0.030	0.017	0.327	0.028	1.800	2.400	1.680	1.160	0.653	0.355	4.	0.03	0.01	0.01
62.1	5/22/57	1.350	0.038	0.038	0.143	0.219	0.368	0.210	2.500	1.900	1.299	0.905	0.793	999.9	1.57	0.001	0.001
62.2	1/22/57	0.560	0.040	0.045	0.050	0.135	0.210	0.240	0.900	0.700	0.600	0.400	0.300	999.9	0.40	0.40	0.40

Table 13. Values of statistical rainfall parameters, IV.

W. No.	M/D/Y	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
15.1	9/ 8/48	15.91	1.200	5.912	0.208	1.09	1.09	0.56	0.700
	4/13/49	74.45	0.596	35.851	-0.469	0.24	0.22	0.20	1.690
	6/ 7/55	91.53	0.328	33.650	0.155	1.73	1.73	1.73	1.120
17.4	5/27/38	23.68	1.560	10.343	-0.217	1.62	0.43	0.43	1.375
	6/21/42	24.25	0.657	33.072	1.186	0.69	0.67	0.00	1.640
	3/31/52	12.65	2.236	7.083	0.127	1.65	1.65	1.65	1.270
	7/2 /52	18.51	2.134	11.738	0.868	0.70	0.70	0.70	2.520
21.1	6/ 6/43	31.47	1.960	18.736	0.219	0.74	0.74	0.23	1.960
	7/21/48	106.16	0.771	46.744	-0.344	0.09	0.09	0.09	2.610
	7/ 1/50	51.89	0.906	44.941	0.705	0.00	0.00	0.00	3.230
	7/18/56	34.28	1.423	22.546	0.450	0.46	0.46	0.02	2.940
26.30	6/16/46	105.75	0.352	135.557	0.937	1.50	0.77	0.00	3.973
	8/16/47	23.79	0.427	28.987	2.329	1.85	1.85	1.12	1.217
	9/ 1/50	61.46	1.143	32.340	0.123	0.04	0.00	0.00	4.433
	6/12/57	35.75	1.455	16.750	0.796	1.23	0.35	0.35	3.254
29.1	7/28/49	20.25	1.527	11.745	0.198	2.57	2.47	1.55	1.400
	5/13/56	16.20	1.820	4.656	-0.497	0.62	0.62	0.30	0.860
	6/ 4/58	48.81	1.920	19.696	0.151	0.67	0.10	0.10	3.230
31.1	8/12/43	10.54	4.656	5.923	0.193	1.25	1.25	1.05	2.150
	7/11/44	14.28	2.328	7.686	0.360	0.03	0.03	0.03	1.940
	6/28/45	8.67	1.000	11.649	1.090	2.16	1.21	1.16	1.090
	6/24/49	25.76	0.637	29.312	1.420	2.90	2.36	2.06	2.160
42.3	6/10/41	31.96	1.172	15.954	0.152	2.07	0.58	0.54	1.650
	6/15/42	19.62	0.499	22.637	1.415	3.68	1.36	1.36	1.010
	6/15/50	49.66	0.495	31.714	0.719	2.55	2.25	0.00	1.880
	4/24/57	20.17	0.931	15.976	1.180	0.29	4.36	3.65	1.700
44.1	6/20/39	16.58	1.746	10.114	0.269	0.60	0.00	0.00	1.630
	7/10/51	76.65	0.866	29.187	-0.249	0.00	0.00	0.00	2.700
	6/ 7/53	28.58	0.883	23.315	0.558	0.68	0.50	0.25	1.560
	6/15/57	38.71	0.682	23.411	0.380	0.86	0.86	0.00	1.950
44.3	7/10/51	74.98	0.594	29.614	0.311	0.01	0.00	0.00	2.970
	6/ 7/53	29.62	1.036	22.861	0.476	0.48	0.48	0.00	1.710
	8/29/57	49.40	0.205	58.641	2.360	0.09	0.09	0.00	2.380
45.2	6/26/40	12.92	1.547	7.683	0.437	0.00	0.00	0.00	0.980
	9/28/41	11.21	3.343	7.268	0.121	0.75	0.75	0.75	1.560
	8/ 7/42	35.33	0.923	15.145	0.558	0.08	0.00	0.00	1.200
	8/ 9/43	11.35	2.755	5.000	0.034	0.27	0.27	0.00	1.010
48.2	7/28/39	28.18	1.385	13.494	-0.030	0.00	0.00	0.00	1.500
	8/24/39	16.63	0.480	18.163	1.037	0.29	0.29	0.29	0.720
	8/26/39	13.16	0.480	17.450	1.470	0.38	0.38	0.00	0.800
	9/ 5/40	3.33	1.800	1.863	0.894	0.00	0.00	0.00	0.300
60.6	4/13/37	160.66	0.080	69.921	-0.116	0.52	0.26	0.10	0.440
	1/25/41	76.27	0.108	36.029	0.033	0.24	0.24	0.24	0.280
	5/ 3/41	48.55	0.169	26.265	0.245	0.39	0.36	0.02	0.310
	6/ 6/41	11.86	0.211	18.109	1.402	0.03	0.01	0.01	0.370
62.1	5/22/57	67.51	0.577	32.624	0.025	1.57	0.00	0.00	1.350
62.2	1/22/57	71.63	0.249	35.017	0.240	0.40	0.40	0.40	0.560

Table 14 - List of frequently used symbols.

Symbol	Item	Units	Dimensions
G	Time between center of mass of runoff and peak discharge rate: see Fig. 1.	Hours, for mathematical derivation of Chapter 2; minutes, in tabulations and regression equations of later chapters	time
m	Time from commencement of runoff to peak discharge rate: see Fig. 1.	Hours, for mathematical derivation of Chapter 2; minutes in Chapter 5, when used in conjunction with predicted G.	time
q	The hydrograph ordinate at any time; or the discharge rate per unit of watershed area: see Fig. 1.	Inches per hour.	volume/time/ area = length/ time
q_o	Peak of best-fitting hydrograph model, (or the peak of the estimated hydrograph): peak rate of discharge per unit of watershed area: see Fig. 1.	Inches per hour.	volume/time/ area = length/ time
W	Total runoff volume of best-fitting mathematical model per unit of watershed area; (or the predicted value for the estimated hydrograph); see Fig. 1.	Inches	volume/area = length
α	A mathematical function of $\frac{m}{G}$, which can also be evaluated numerically from W, q_o and G.	None	dimensionless
$\hat{R}^2$	Unbiased coefficient of determination: last section of Chapter 2.	None	dimensionless
R^2	Biased coefficient of determination: first section of Chapter 4.	None	dimensionless
T_a	Symbols for individual "topographic parameters", where a represents numerical subscripts defined under Group I of Table 2.	Various	various
D_b	Symbols for individual "design indices depicting land use and soils", where b represents numerical subscripts defined under Group II of Table 2	Various	various
R_c	Symbols for individual "conventional rainfall parameters", where c represents numerical subscripts defined under Group III of Table 2.	Various	various
S_d	Symbols for individual "statistical rainfall parameters", where d represents numerical subscripts defined under Group IV of Table 2.	Various	various