

## Estimation of Raindrop Size Distribution Parameters from Polarimetric Radar Measurements

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### ABSTRACT

Estimation of raindrop size distribution over large spatial and temporal scales has been a long-standing goal of polarimetric radar. Algorithms to estimate the parameters of a gamma raindrop size distribution model from polarimetric radar observations of reflectivity, differential reflectivity, and specific differential phase are developed. Differential reflectivity is the most closely related measurement to a parameter of the drop size distribution, namely, the drop median diameter ( $D_0$ ). The estimator for  $D_0$  as well as other parameters are evaluated in the presence of radar measurement errors. It is shown that the drop median diameter can be estimated to an accuracy of 10%, whereas the equivalent intercept parameter can be estimated to an accuracy of 6% in the logarithmic scale. The estimators for the raindrop size distribution parameters are also evaluated using disdrometer data based simulations. The disdrometer based evaluations confirm the accuracy of the algorithms developed herein.

### 1. Introduction

Ever since the introduction of differential reflectivity ( $Z_{dr}$ ) measurement, one of the long standing goals of polarimetric radar has been the estimation of the raindrop size distribution (DSD). Seliga and Bringi (1976) showed that  $Z_{dr}$ , for an exponential DSD, is directly related to the median volume diameter ( $D_0$ ). Careful intercomparisons between radar measurements of  $Z_{dr}$  and  $D_0$  derived from surface disdrometers and airborne imaging probes have shown that  $D_0$  can be estimated to an accuracy of about 10%–15% (see, for example, Aydin et al. 1987; Bringi et al. 1998). A general gamma distribution model was suggested by Ulbrich (1983) to characterize the natural variation of the DSD. The non-spherical shape of raindrops results in anisotropic propagation of electromagnetic waves, with a difference in the propagation constant at horizontal and vertical polarization states. The specific differential propagation phase ( $K_{dp}$ ) is a forward scatter measurement (Seliga and Bringi 1978; Sachidananda and Znić 1987) where-

as the radar reflectivity ( $Z_h$ ) and  $Z_{dr}$  are backscatter measurements. The weighting of the DSD by  $Z_{dr}$  and  $K_{dp}$  is controlled by the variation of mean raindrop shape with size. A combination of the three radar measurements ( $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$ ) can be utilized to estimate the DSD, specifically the parameters of a parametric form of the DSD such as the gamma DSD. This paper presents algorithms for the estimation of parameters of a gamma DSD from polarimetric radar measurements. The paper is organized as follows. Section 2 describes the raindrop size distribution and its parameters, whereas section 3 describes the shape of raindrops and its implication for polarimetric radar measurements. Estimators of the DSD parameters are presented in section 4, and the impact of measurement errors on the estimates are discussed in section 5. Validation of the algorithms using disdrometer data are presented in section 6. Important results of this paper are summarized in section 7.

### 2. Raindrop size distribution

The raindrop size distribution describes the probability density distribution function of raindrop sizes. In practice, the normalized histogram of raindrop sizes (normalized with respect to the total number of observed

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raindrops) converges to the probability density function of raindrop sizes. A gamma distribution model can adequately describe many of the natural variations in the shape of the raindrop size distribution (Ulbrich 1983). The gamma raindrop size distribution can be expressed as,

$$N(D) = n_c f_D(D) \text{ (m}^{-3} \text{ mm}^{-1}\text{)}, \quad (1)$$

where  $N(D)$  is the number of raindrops per unit volume per unit size interval ( $D$  to  $D + \Delta D$ ),  $n_c$  is the number concentration, and  $f_D(D)$  is the probability density function (pdf). When  $f_D(D)$  is of the gamma form it is given by

$$f_D(D) = \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} e^{-\Lambda D} D^\mu, \quad \mu > -1, \quad (2)$$

where  $\Lambda$  and  $\mu$  are the parameters of the gamma pdf. Any other gamma form such as the one introduced by Ulbrich (1983),

$$N(D) = N_0 D^\mu e^{-\Lambda D}, \quad (3)$$

can be derived from this fundamental notion of raindrop size distribution. It must be noted that any function used to describe  $N(D)$  when integrated over  $D$  must yield the total number concentration, to qualify as a DSD function. This property is a direct consequence of the fundamental result that any probability density function must integrate to unity. When  $\mu = 0$ , the gamma DSD reduces to the exponential form as  $N(D) = n_c \Lambda e^{-\Lambda D}$ . The relation between  $D_0$ ,  $\mu$ , and  $\Lambda$  is given by (Ulbrich 1983)

$$\Lambda D_0 \cong 3.67 + \mu, \quad (4)$$

where  $D_0$  is the drop median diameter defined as

$$\int_0^{D_0} D^3 N(D) dD = \frac{1}{2} \int_0^\infty D^3 N(D) dD. \quad (5a)$$

Similarly, a mass-weighted mean diameter  $D_m$  can be defined as

$$D_m = \frac{E(D^4)}{E(D^3)}, \quad (5b)$$

where  $E$  stands for the expected value. Using (4),  $f_D(D)$ , the gamma pdf described by (2), can be written in terms of  $D_0$  and  $\mu$  as

$$f_D(D) = \frac{(3.67 + \mu)^{\mu+1}}{\Gamma(\mu+1) D_0} \left(\frac{D}{D_0}\right)^\mu e^{-[(3.67 + \mu)(D/D_0)]}. \quad (6)$$

The above form makes the normalized diameter ( $D/D_0$ ) as the variable rather than  $D$ . Several measurables such as water content ( $W$ ) and rainfall rate ( $R$ ) can be expressed in terms of the DSD as

$$W = \frac{\pi}{6} \rho_w n_c E(D^3), \quad \text{and} \quad (7)$$

$$R = \frac{\pi}{6} n_c E[v(D) D^3], \quad (8a)$$

where  $R$  is the still-air rainfall rate and  $v(D)$  is the terminal velocity of raindrops (Gunn and Kinzer 1949). The conventional unit of rainfall rate is millimeters per hour. Converting to this unit, rainfall rate is expressed as

$$R = 0.6\pi \times 10^{-3} n_c E[v(D) D^3] \text{ (mm h}^{-1}\text{)}, \quad (8b)$$

where  $n_c$  is in cubic millimeters,  $v(D)$  in millimeters per second, and  $D$  in millimeters.

In order to compare the pdf of  $D$  [or,  $f_D(D)$ ] in the presence of varying water contents, the concept of scaling the DSD has been used by several authors (Sekhon and Srivastava 1971; Willis 1984; Testud et al. 2000). The corresponding form of  $N(D)$  can be expressed as

$$N(D) = N_w f(\mu) \left(\frac{D}{D_0}\right)^\mu \exp\left[-(3.67 + \mu)\frac{D}{D_0}\right], \quad (9)$$

where  $N_w$  is the scaled version of  $N_0$  defined in (3):

$$N_w = \frac{N_0}{f(\mu)} D_0^\mu, \quad \text{and} \quad (10a)$$

$$f(\mu) = \frac{6}{(3.67)^4} \cdot \frac{(3.67 + \mu)^{\mu+4}}{\Gamma(\mu+4)}, \quad (10b)$$

with  $f(0) = 1$  and  $f(\mu)$  is a unitless function of  $\mu$ . One interpretation of  $N_w$  is that it is the intercept of an equivalent exponential distribution with the same water content and  $D_0$  as the gamma DSD (Bringi and Chandrasekar 2001). Thus  $N_w$ ,  $D_0$ , and  $\mu$  form the three parameters of the gamma DSD.

### 3. Raindrop shape and implication for polarimetric radar measurements

The equilibrium shape of raindrops is determined by a balance of hydrostatic, surface tension, and aerodynamic forces. The commonly used model for raindrops assumes oblate spheroidal shapes, with the axis ratio  $b/a$ , where  $b$  and  $a$  are the semiminor and the semimajor axis lengths, respectively. Pruppacher and Beard (1970) give a simple model for the axis ratio ( $r$ ) based on a linear fit to wind tunnel data as

$$r = 1.03 - 0.062D; \quad 1 \leq D \leq 9 \text{ mm}. \quad (11)$$

Rotating linear polarization data in heavy rain (Hendry et al. 1987) has indicated that raindrops fall with the mean orientation of their symmetry axis in the vertical direction. The large swing in the crosspolar power in their data implies a high degree of orientation of drops with the standard deviation of canting angles estimated to be around  $6^\circ$  assuming a Gaussian model. It is reasonable to assume that the standard deviation of canting angles is in the range  $5^\circ$ – $10^\circ$  (Bringi and Chandrasekar 2001).

#### a. Differential reflectivity

The differential reflectivity can be written as (Seliga and Bringi 1976)

$$10 \log_{10} \frac{E[\sigma_{hh}(D)]}{E[\sigma_{vv}(D)]} = 10 \log_{10}(\xi_{dr}), \quad (12)$$

where the symbol  $E$  represents expectation and  $\sigma_{hh}$  and  $\sigma_{vv}$  are the cross sections at horizontal and vertical polarizations, respectively.

Seliga and Bringi (1976) showed that for an exponential distribution and axis ratio given by (11),  $Z_{dr}$  can be expressed as a function of the median volume diameter  $D_0$ . This microphysical link between a radar measurement and a parameter of the DSD is important. More fundamentally,  $\xi_{dr}^{-1}$  may be related to the reflectivity factor-weighted mean of  $r^{7/3}$  (Jameson 1985). For a more general gamma form, an approximate power-law fit can be derived assuming  $-1 \leq \mu \leq 5$ ,  $0.5 < D_0 < 2.5$  mm, and  $N_w$  chosen to be consistent with thunderstorm rain rates. Using the fit recommended by Andsager et al. (1999) for the Beard and Chuang (1987) equilibrium shapes, power-law fits to  $D_0$  and  $D_m$  can be derived as

$$D_0 = 1.619(Z_{dr})^{0.485} \quad (\text{mm}), \quad (13a)$$

$$D_m = 1.529(Z_{dr})^{0.467} \quad (\text{mm}), \quad (13b)$$

where  $Z_{dr}$  is in decibels and the fits are valid at S-band frequency (near 3 GHz; Bringi and Chandrasekar 2001).

### b. Specific differential phase

The relation between specific differential phase ( $K_{dp}$ ) and the water content and raindrop axis ratio was described by Jameson (1985). Following Bringi and Chandrasekar (2001) a simple approach based on Rayleigh-Gans scattering is described here to derive this relation.

The specific differential phase can be expressed as

$$K_{dp} = \frac{2\pi}{k_0} \text{Re}[E(f_h - f_v)] \quad (\text{deg km}^{-1}), \quad (14)$$

where  $k_0$  is the free space propagation constant, and  $f_h$ ,  $f_v$  are the forward scatter amplitudes at horizontal and vertical polarization, respectively. For Rayleigh-Gans scattering, (14) reduces to

$$K_{dp} = \frac{\pi k_0}{12} \text{Re} \left\{ E \left[ D^3 \left( \frac{\epsilon_r - 1}{1 + \frac{1}{2}(1 - \lambda_z)(\epsilon_r - 1)} - \frac{\epsilon_r - 1}{1 + \lambda_z(\epsilon_r - 1)} \right) \right] dD \right\}, \quad (15a)$$

where  $\epsilon_r$  is the dielectric constant and the depolarizing factor  $\lambda_z$  is given by

$$\lambda_z = \frac{1 + f^2}{f^2} \left( 1 - \frac{1}{f} \tan^{-1} f \right), \quad \text{and} \quad (15b)$$

$$f^2 = \frac{1}{r^2} - 1. \quad (15c)$$

The above expectation can be substantially simplified by recognizing that

$$\text{Re} \left[ \frac{\epsilon_r - 1}{1 + \frac{1}{2}(1 - \lambda_z)(\epsilon_r - 1)} - \frac{\epsilon_r - 1}{1 + \lambda_z(\epsilon_r - 1)} \right] \cong c(1 - r), \quad (16)$$

where  $c$  is approximately constant varying between 3.3 and 4.2 with  $r$  from 1 to 0.5. This range of  $c$  is valid for  $\epsilon_r$  of water at microwave frequencies in the range 3–30 GHz. Substituting (16) in (15a) results in

$$K_{dp} = \frac{\pi k_0 c}{12} \int D^3 (1 - r) N(D) dD \quad (17a)$$

$$= \frac{\pi}{\lambda} \times \frac{c}{\rho_w} \int \frac{\pi}{6} \rho_w D^3 (1 - r) N(D) dD \quad (17b)$$

$$= \frac{\pi c W}{\lambda \rho_w} \left[ 1 - \frac{E(rD^3)}{E(D^3)} \right], \quad (17c)$$

where  $W$  is the rainwater content,  $\rho_w$  is the water density,  $E$  stands for expectation over the DSD, and  $\lambda$  is the wavelength. The ratio of expectations in (17c) can be defined as the mass-weighted mean axis ratio  $\bar{r}_m$ . In terms of conventional units for  $W$  in grams per cubic meters,  $\rho_w = 1 \text{ g cm}^{-3}$ , and  $\lambda$  in meters, (17c) can be reduced to

$$K_{dp} = \left( \frac{180}{\lambda} \right) \times 10^{-3} c W (1 - \bar{r}_m) \quad (\text{deg km}^{-1}), \quad (18)$$

where  $c \cong 3.75$  is both dimensionless and independent of wavelength. This result links the specific differential phase with parameters of the DSD (Jameson 1985). If the equilibrium axis ratio model given in (11) is used in (18) then  $K_{dp}$  is given by

$$K_{dp} = \left( \frac{180}{\lambda} \right) \times 10^{-3} c W (0.062) D_m \quad (\text{deg km}^{-1}). \quad (19)$$

Thus,  $K_{dp}$  is related to the product of  $D_m$  and water content. Though the above result was obtained using the Rayleigh-Gans approximation, it is valid up to 13 GHz (Bringi and Chandrasekar 2001).

### c. Mean raindrop shape derived from polarimetric radar measurements

Field studies of Tokay and Beard (1996) indicate that raindrops from 1 to 4 mm oscillate. Andsager et al. (1999) show that oscillations result in an upward shift of the mean axis ratio versus diameter curve, specially in the 1- to 4-mm range. Gorgucci et al. (2000) assumed a simple linear model for axis ratio versus size of the form

$$r = 1 - \beta D \quad (20)$$

and derived radar-based estimators of  $\beta$ . They also showed that  $\beta$  decreases slightly with increasing reflectivity (or, on the average, the axis ratio is smaller than the equilibrium axis ratio) perhaps indicating raindrop oscillations.

It was shown in section 3a that  $\xi_{dr}^{-1}$  is related to the reflectivity weighted axis ratio. Similar dependence on  $K_{dp}$  can be derived from (18). Let  $p(r)$  be the probability density function of the axis ratio for a given diameter. The expression for  $K_{dp}$  can be generalized as (Bringi and Chandrasekar 2001)

$$K_{dp} = \frac{2\pi c^*}{k_0} \int D^3 N(D) \int (1-r)p(r) dr dD \quad (21)$$

$$= \frac{2\pi c^*}{k_0} \int D^3 N(D) [1 - E(r)] dD, \quad (22)$$

where  $E(r)$  is the mean value of  $r$ , and  $c^*$  is a constant. The functional dependence of  $E(r)$  versus  $D$  may be modeled as in (20). Using the linear model in (20), Gorgucci et al. (2000) showed the variations of  $Z_{dr}$  and  $K_{dp}$  with respect to  $\beta$ , and in turn derived an estimator for  $\beta$  based on polarimetric radar measurements. This can be used subsequently in algorithms relating  $Z_{dr}$  and  $K_{dp}$  to the parameters of the DSD, which gives rise to a methodology for estimating the gamma DSD parameters based on radar measurements.

#### 4. Estimators of the gamma DSD parameters

Seliga and Bringi (1976) showed that for an exponential distribution, the two parameters of the DSD, namely  $N_w$  and  $D_0$ , can be estimated using  $Z_{dr}$  and  $Z_h$ . They used a two-step procedure where they estimated  $D_0$  using an equilibrium raindrop shape model and subsequently used that in the expression for  $Z_h$  to estimate  $N_w$ . This procedure can essentially be applied for a gamma DSD, and generalized to account for raindrop oscillations using the linear model in (20). The procedure for estimating the gamma DSD parameters is as follows: first, estimate  $\beta$  using the algorithm described by Gorgucci et al. (2000) and, subsequently, estimate  $D_0$ ,  $N_w$ , and  $\mu$ , recognizing the right  $\beta$  value.

##### a. Estimation of $D_0$

It was noted in section 3a that  $D_0$  can be estimated from  $Z_{dr}$  as a simple power-law expression (13a). This parameterization was based on the Beard and Chuang (1987) equilibrium axis ratios and essentially corresponds to a fixed equivalent  $\beta$ . Gorgucci et al. (1994) obtained approximate parameterizations for  $Z_h$  and  $Z_v$  of the form

$$Z_h = c_h g_h(\mu) N_0 \left( \frac{D_0}{3.67 + \mu} \right)^{\alpha_h}, \quad (23a)$$

$$Z_v = c_v g_v(\mu) N_0 \left( \frac{D_0}{3.67 + \mu} \right)^{\alpha_v}, \quad (23b)$$

where  $c_h$ ,  $c_v$ ,  $g_h(\mu)$ ,  $g_v(\mu)$ ,  $\alpha_h$ , and  $\alpha_v$  are constants that depend on  $h$  and  $v$  polarizations. From the above and with some modest algebra, it can be shown that a parameterization for  $D_0$  can be pursued of the form

$$\hat{D}_0 = a_1 Z_h^{b_1} (\xi_{dr})^{c_1}, \quad (24)$$

where  $\xi_{dr} = 10^{0.1Z_{dr}}$  is the differential reflectivity in linear scale, and  $Z_h$  is the reflectivity factor at horizontal polarization (in  $\text{mm}^6 \text{m}^{-3}$ ). Though the above parameterization form was obtained from the approximation in (23), the coefficients in (24) can be derived from the simulation of gamma DSDs directly as follows. Once the gamma DSD is given in the form in (9), it is straightforward to compute radar parameters such as  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$ . The mean axis ratio versus  $D$  relation is modeled by (20). Under these conditions and at a temperature of  $20^\circ$ ,  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  are computed for widely varying DSD by randomly varying  $N_w$ ,  $D_0$ , and  $\mu$  over the following ranges:

$$10^3 \leq N_w \leq 10^5 \quad (\text{mm}^{-1} \text{m}^{-3}), \quad (25a)$$

$$0.5 \leq D_0 \leq 3.5 \quad (\text{mm}), \quad (25b)$$

$$-1 \leq \mu \leq 5, \quad (25c)$$

with the constraint  $R < 300 \text{ mm h}^{-1}$ . While  $D_0$  and  $\mu$  are varied randomly over their respective ranges,  $\log_{10} N_w$  is randomly varied over its range. This range falls within the range of parameters suggested by Ulbrich (1983). Once  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  values are simulated, a nonlinear regression analysis is performed to estimate the coefficients  $a_1$ ,  $b_1$ ,  $c_1$ . Though these coefficients are accurate for a single  $\beta$ ,  $c_1$  changes significantly with  $\beta$ . Figure 1 shows the plot of the coefficients  $a_1$ ,  $b_1$ ,  $c_1$  as a function of  $\beta$  to demonstrate the sensitivity. This variation of  $c_1$  with  $\beta$  can be further parameterized by fitting power-law expressions. These coefficients are (valid for S band)

$$a_1 = 0.56, \quad (26a)$$

$$b_1 = 0.064, \quad (26b)$$

$$c_1 = 0.024\beta^{-1.42}. \quad (26c)$$

In summary,  $D_0$  can be estimated by first estimating  $\beta$  using the approach of Gorgucci et al. (2000) as

$$\beta = 2.08 Z_h^{-0.365} K_{dp}^{0.380} \xi_{dr}^{0.965} \quad (\text{mm}^{-1}), \quad (27)$$

and then using coefficients (26) in (24). For the equilibrium axis ratios (24) reduces to

$$\hat{D}_0 = 0.56 Z_h^{0.064} \xi_{dr}^{1.245} \quad (\text{mm}), \quad (28a)$$

whereas, when  $\beta = 0.0475$  (typical for tropical rain, discussed in section 6),

$$\hat{D}_0 = 0.56 Z_h^{0.064} \xi_{dr}^{1.817} \quad (\text{mm}). \quad (28b)$$

Simulations can also be utilized to evaluate the performance of the estimator of  $D_0$  in (24). Figure 2a shows a scatterplot of  $\hat{D}_0$  versus true  $D_0$ , for widely varying

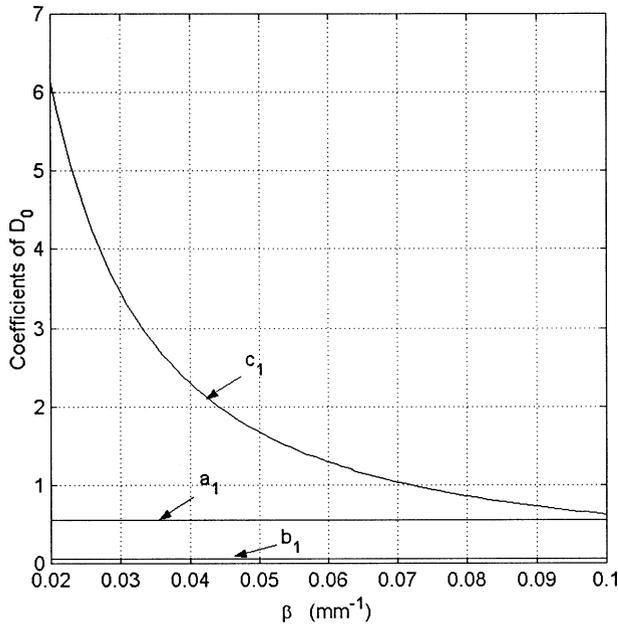


FIG. 1. The coefficients  $a_1$ ,  $b_1$ ,  $c_1$  of  $D_0(Z_h, Z_{dr})$  algorithm given by (24) as a function of  $\beta$ .

$\beta$ , and gamma DSD parameters as given by (25). Quantitative analysis of the scatter gives a correlation coefficient of 0.963. It can be seen from Fig. 2a that  $D_0$  is estimated fairly well with negligible bias over a wide range. Figure 2b shows the normalized standard deviation (NSD) of  $\hat{D}_0$  as a function of  $D_0$ , where NSD is defined as

$$NSD = \frac{[\text{var}(\hat{D}_0)]^{1/2}}{D_0} = \frac{SD(\hat{D}_0)}{D_0}, \quad (29)$$

where SD indicates standard deviation. Figure 2b shows that  $D_0$  can be estimated to an accuracy of about 10% when  $D_0 > 1$  mm. A similar estimate of  $D_0$  can be derived using  $K_{dp}$  and  $Z_{dr}$  as

$$\hat{D}_0 = a_2 K_{dp}^{b_2} (\xi_{dr})^{c_2} \text{ (mm)}, \quad (30)$$

where

$$a_2 = 0.41\beta^{-0.34}, \quad (31a)$$

$$b_2 = 0.076, \quad (31b)$$

$$c_2 = 0.097\beta^{-0.97}. \quad (31c)$$

This estimator is similar to the estimator in (24) except that  $K_{dp}$  estimates are difficult to obtain at low rain rates. On the other hand, this estimator is immune to variations in absolute calibration of the radar system. Error analysis of the estimator given by (30) yields a correlation coefficient of 0.963. The normalized standard deviation of the estimate of  $D_0$  given (30) is also shown in Fig. 2b. It can be seen that in the absence of any measurement errors these two estimates are comparable. For equilibrium axis ratios this estimator for  $D_0$  reduces to

$$\hat{D}_0 = 1.055 K_{dp}^{0.076} \xi_{dr}^{1.439} \text{ (mm)}, \quad (32a)$$

whereas, for tropical rain with  $\beta \cong 0.0475$  (discussed in section 6),

$$\hat{D}_0 = 1.155 K_{dp}^{0.076} \xi_{dr}^{1.864} \text{ (mm)}. \quad (32b)$$

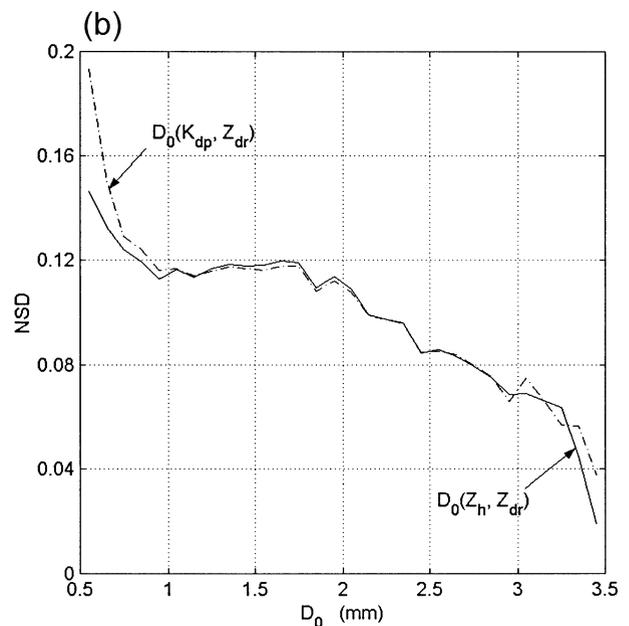
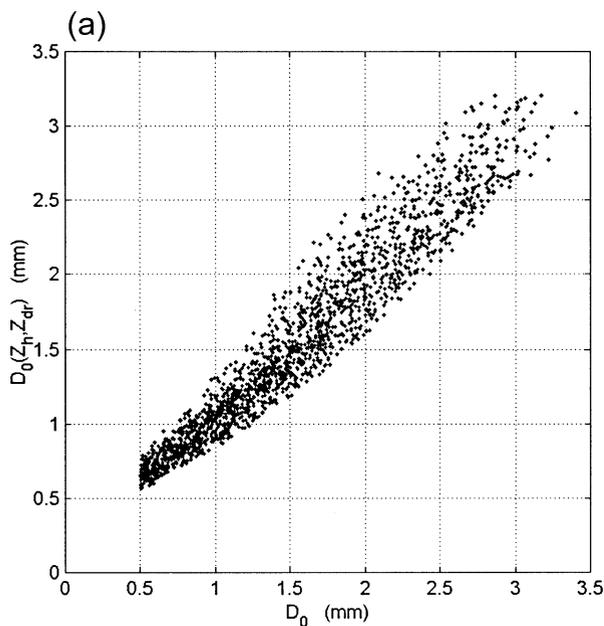


FIG. 2. (a) Scatterplot of  $D_0(Z_h, Z_{dr})$  vs the true value of  $D_0$  for widely varying DSD. (b) Normalized standard deviation (NSD) in the estimates of  $D_0$  as a function of the true value of  $D_0$ .

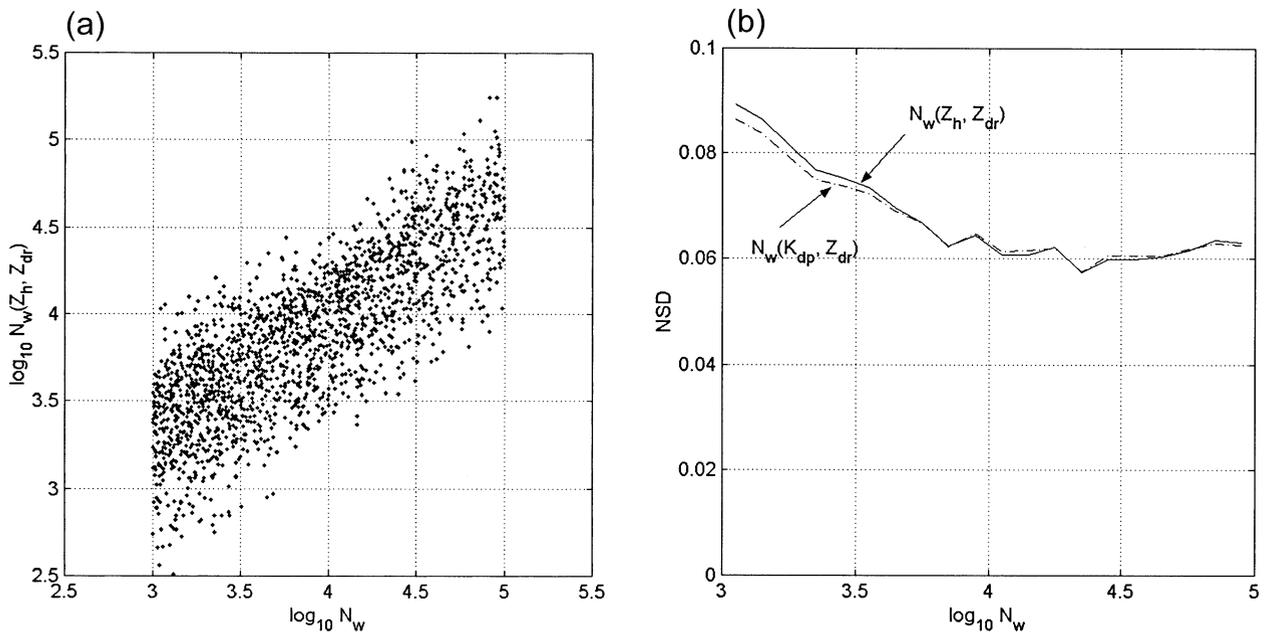


FIG. 3. (a) Scatterplot of  $\log_{10}N_w(Z_h, Z_{dr})$  vs the true value of the  $\log_{10}N_w$  for widely varying DSD. (b) Normalized standard deviation in the estimates of  $\log_{10}N_w$ , as a function of  $\log_{10}N_w$ .

*b. Estimation of  $N_w$*

Once  $D_0$  is estimated,  $N_w$  can be easily estimated using one of the moments of the DSD such as  $Z_h$  or  $K_{dp}$ . For example, it was shown in (19) that  $K_{dp}$  is proportional to the product of  $W$  and  $D_m$  (or approximately  $D_0$ );  $Z_h$  can be written in terms of the gamma DSD parameters as (see also Ulbrich and Atlas 1998)

$$\frac{Z_h}{N_w} = F_z(\mu)D_0^7, \tag{33a}$$

where

$$F_z(\mu) = \frac{f(\mu)\Gamma(7 + \mu)}{(3.67 + \mu)^{7+\mu}}. \tag{33b}$$

For an exponential distribution ( $\mu = 0$ ),

$$N_w = \frac{Z_h (3.67)^7}{D_0^7 6!} = 12.45 \frac{Z_h}{D_0^7} \text{ (mm}^{-1} \text{ m}^{-3}\text{)}. \tag{33c}$$

Thus it can be seen that  $N_w$  can be estimated in terms of  $D_0$ . However, the estimate of  $D_0$  can be obtained in terms of  $Z_h$  and  $Z_{dr}$  (or  $K_{dp}$  and  $Z_{dr}$ ). Therefore, a direct estimate of  $N_w$  can be pursued of the form

$$\log_{10}(N_w) = a_3 Z_h^{b_3} \xi_{dr}^{c_3}. \tag{34}$$

The variability of  $a_3, b_3, c_3$  can be parameterized in terms of  $\beta$  as

$$a_3 = 3.29, \tag{35a}$$

$$b_3 = 0.058, \tag{35b}$$

$$c_3 = -0.023\beta^{-1.389}. \tag{35c}$$

In summary, the estimator for  $N_w$  is obtained as follows. Using  $Z_h, Z_{dr}$ , and  $K_{dp}$ , first estimate  $\beta$  as given in (27). Subsequently, calculate the coefficients in (35) and use in (34) to estimate  $N_w$ . Figure 3a shows a scatterplot of  $\log_{10}\hat{N}_w$  versus true  $\log_{10}N_w$ , where  $\log_{10}\hat{N}_w$  is estimated using (34). It can be seen from Fig. 3a that  $\log_{10}\hat{N}_w$  is estimated fairly well. Quantitative analysis of the scatter yields a correlation coefficient of 0.831. Figure 3b shows the normalized standard deviation of  $\log_{10}N_w$  as a function of  $\log_{10}N_w$ . It can be seen, from Fig. 3b, that  $\log_{10}N_w$  is estimated to a normalized standard deviation of better than 7% when  $\log_{10}N_w > 3.5$ . Note that due to the wide variability of  $N_w$ ,  $\log_{10}N_w$  is the preferred scale of comparison (similar to dB scale for reflectivity). For equilibrium axis ratios, (34) reduces to

$$\log_{10}\hat{N}_w = 3.29 Z_h^{0.058} \xi_{dr}^{-1.094}, \tag{36a}$$

whereas for tropical rain with  $\beta \cong 0.0475$  (discussed in section 6)

$$\log_{10}N_w = 3.29 Z_h^{0.058} \xi_{dr}^{-1.585}. \tag{36b}$$

Similarly, another estimate of  $N_w$  can be derived using  $K_{dp}$  and  $Z_{dr}$  as

$$\log_{10}N_w = a_4 K_{dp}^{b_4} (\xi_{dr})^{c_4}. \tag{37}$$

This variability of  $a_4, b_4, c_4$  can be parameterized in terms of  $\beta$  as

$$a_4 = 5.99, \tag{37a}$$

$$b_4 = 0.133\beta^{0.26}, \tag{37b}$$

$$c_4 = -0.042\beta^{-1.16}. \tag{37c}$$

For equilibrium axis ratios, (37) reduces to

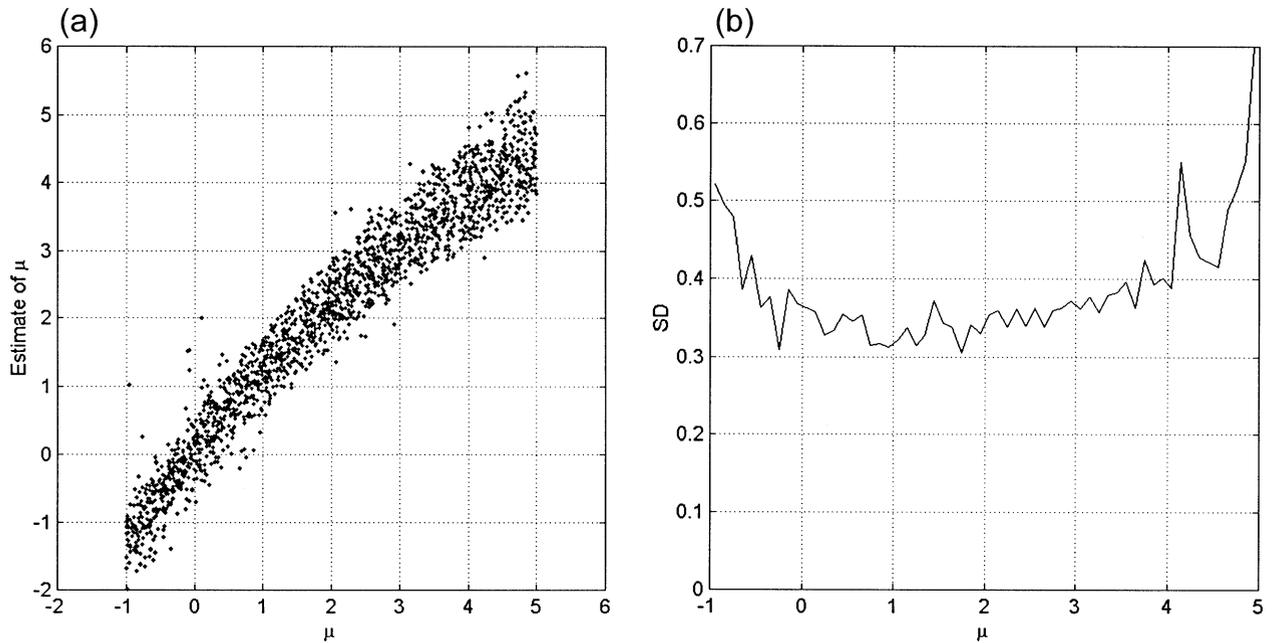


FIG. 4. (a) Scatterplot of the estimates of  $\mu$ , using Eq. (39) (under the assumption that  $D_0$  is known), vs  $\mu$ . (b) Standard deviation in the estimates of  $\mu$  (under the assumption that  $D_0$  is known), as a function of  $\mu$ .

$$\log_{10} \hat{N}_w = 5.99 K_{dp}^{0.065} \xi_{dr}^{-1.057}, \quad (38a)$$

whereas for tropical rain ( $\beta \cong 0.0475$ )

$$\log_{10} \hat{N}_w = 5.99 K_{dp}^{0.06} \xi_{dr}^{-1.44}. \quad (38b)$$

The normalized standard deviation in the estimate of  $\log_{10} N_w$  given by (37) is also shown in Fig. 3b. It can be seen from Fig. 3b that the two estimators for  $\log_{10} N_w$  are comparable in the absence of the measurement errors.

c. Parameterization of  $\mu$

The parameter  $\mu$  describes the overall shape of the distribution. Once  $D_0$  is estimated,  $\mu$  can be estimated from the following parameterization, which was constructed empirically as

$$\hat{\mu} = \frac{a_5 D_0^{b_5}}{(\xi_{dr} - 1)} - c_5 (\xi_{dr})^{d_5}. \quad (39)$$

The variability of  $a_5, b_5, c_5,$  and  $d_5$  can be parameterized in terms of  $\beta$  as

$$a_5 = 200\beta^{1.89}, \quad (40a)$$

$$b_5 = 2.23\beta^{0.039}, \quad (40b)$$

$$c_5 = 3.16\beta^{-0.046}, \quad (40c)$$

$$d_5 = 0.374\beta^{-0.355}, \quad (40d)$$

$D_0$  calculated from either (24) or (30) can be utilized in (39) to estimate  $\mu$ . Figure 4a shows the scatterplot of  $\hat{\mu}$  given by (39) versus  $\mu$  under the assumption that  $D_0$  is known. The results of Fig. 4a indicate that  $\mu$  can

be parameterized of the form given by (39) (though it appears complicated). Figure 4b shows the corresponding standard deviation in the estimate of  $\hat{\mu}$ , which is about 0.3. However, in practice  $D_0$  has to be estimated using (24) or (30), using  $Z_h, Z_{dr},$  and  $K_{dp}$ . Estimating  $\mu$  under such conditions will result in higher error than that projected by Fig. 4b. Estimating  $\mu$  accurately under practical conditions, especially in the presence of measurement errors is very difficult using the procedures discussed here.

5. Impact of measurement error on the estimates of  $D_0$  and  $N_w$

Estimators of  $D_0$  given by (24) and (30) as well as  $N_w$  given by (34) and (37) use measurements of  $Z_h, Z_{dr},$  and  $K_{dp}$ . Any error in the measurement of these three parameters will directly translate into errors in the estimates of  $D_0$  and  $N_w$ . The three measurements  $Z_h, Z_{dr},$  and  $K_{dp}$  have completely different error structures.

The  $Z_h$  is based on absolute power measurement and has a typical accuracy of 1 dB. The  $Z_{dr}$  is a relative power measurement that can be estimated to an accuracy of about 0.2 dB. The slope of the range profile of the differential propagation phase  $\Phi_{dp}$  is  $K_{dp}$ , which can be estimated to an accuracy of a few degrees. The subsequent estimate of  $K_{dp}$  depends on the procedure used to compute the range derivative of  $\Phi_{dp}$  such as a simple finite-difference scheme or a least squares fit. Using a least squares estimate of the  $\Phi_{dp}$  profile, the standard deviation of  $K_{dp}$  can be expressed as (Gorgucci et al. 1999)

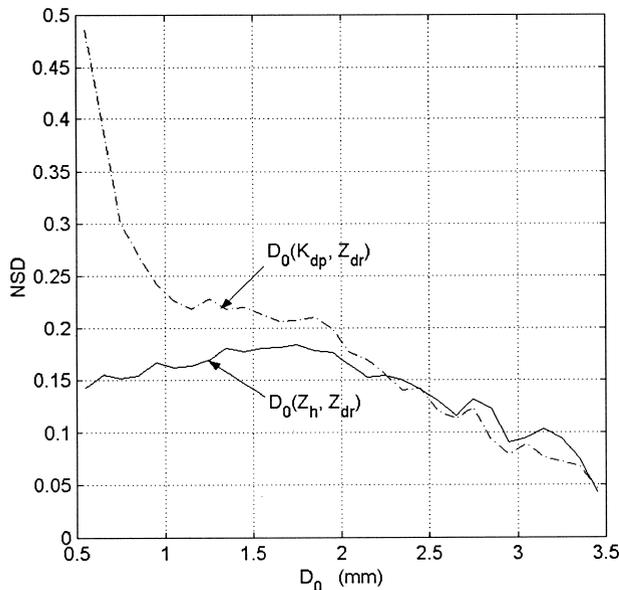


FIG. 5. Normalized standard deviation in the estimates of  $D_0$  as a function of  $D_0$  in the presence of radar measurement errors.

$$\sigma(K_{dp}) = \sqrt{3} \frac{\sigma(\Phi_{dp})}{N\Delta r} \sqrt{\frac{N}{(N-1)(N+1)}}, \quad (41)$$

where  $\Delta r$  is the range resolution of the  $\Phi_{dp}$  estimate and  $N$  is the number of range samples along the path. For a typical 150-m range spacing, and with  $2.5^\circ$  accuracy of  $\Phi_{dp}$ ,  $K_{dp}$  can be estimated over a path of 3 km, with a standard error of  $0.32^\circ \text{ km}^{-1}$ .

The measurement errors of  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  are nearly independent. In the following, simulations are used to quantify the error structure of the estimates of  $D_0$  and  $N_w$ . The simulation is done as follows. For each gamma DSD the corresponding  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  are computed. The random measurement errors are simulated using the procedure described in Chandrasekar et al. (1986). The principal parameters of the simulation are as follows: 1) wavelength  $\lambda = 11 \text{ cm}$ ; 2) pulse repetition time (PRT)  $1 = \text{ms}$ ; 3) number of sample pairs used in the estimates is 64; 4) Doppler velocity spectrum width  $\sigma_v = 2 \text{ m s}^{-1}$ ; 5) copolar correlation between horizontally and vertically polarized signals  $\rho_{hv} = 0.99$ ; 6) range sample spacing is 150 m; and 7)  $K_{dp}$  is estimated as a least squares fit over a path consisting of 50 range samples. Subsequently, estimates of  $D_0$  and  $N_w$  are obtained using the simulated measurements of  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$ . There are some practical issues associated with the estimation of  $D_0$  and  $N_w$  from radar measurements (or simulated radar measurements). At low rain rates,  $K_{dp}$  is small, and in the presence of measurement errors,  $K_{dp}$  estimates could be very small (fluctuating around zero). Under this condition (say, when  $K_{dp} < 0.2 \text{ deg km}^{-1}$ ), (27) cannot be used to estimate  $\beta$ . Therefore, when  $K_{dp}$  is small the following procedure is adopted: whenever  $\hat{K}_{dp} < 0.2^\circ \text{ km}^{-1}$  the equilibrium model for axis ratios are

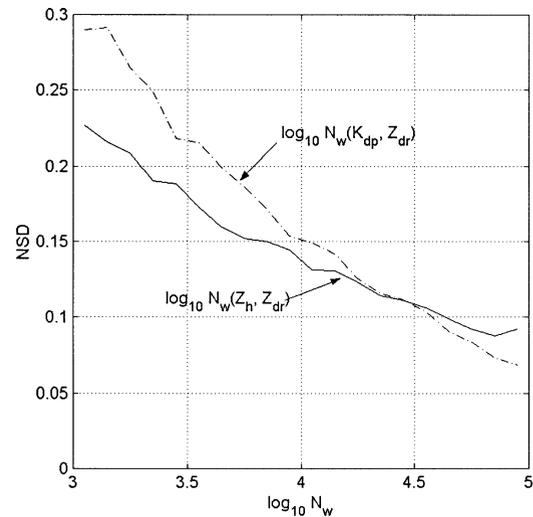


FIG. 6. Normalized standard deviation in the estimates of  $\log_{10} N_w$  as a function of  $\log_{10} N_w$  in the presence of radar measurement errors.

assumed and (28a) and (36a) are used for estimating  $D_0$  and  $N_w$ , respectively. It can be noted that (13a) also could be used for estimating  $D_0$ , followed by either (33c) or (36b) for  $N_w$ . In light rain all these algorithms provide similar results for estimates of  $N_w$  and  $D_0$ .

The normalized standard deviation in the estimates of  $D_0$  and  $N_w$  including the effect of measurement error are evaluated and shown in Figs. 5 and 6, respectively. Figure 5 shows the NSD in the estimates of  $D_0$  given by (24) and (30). Comparing Fig. 5 to Fig. 2b it can be seen that in general, there is about a 10% increase in the NSD of  $D_0$  estimate, computed from (24), due to measurement error. The NSD of  $D_0$  estimate from (30) gets worse for smaller  $D_0$  primarily due to the error in  $K_{dp}$ . The NSD of the  $N_w$  estimates, given by (34) and (37) in the presence of measurement errors, are shown in Fig. 6. Again comparing these to the NSD computations without measurement error (Fig. 3b), a 4% to 16% increase is noted depending on the value of  $N_w$ . For an  $N_w$  value of  $8000 \text{ mm}^{-1} \text{ m}^{-3}$  the NSD of  $\log_{10} N_w$  is about 12% in the presence of measurement errors. Once again the estimate of  $\log_{10} N_w$  from (37) has higher standard deviation when  $N_w < 20\,000 \text{ mm}^{-1} \text{ m}^{-3}$ . Thus,  $D_0$  and  $N_w$  can be estimated fairly well from radar measurements at least for convective rainfall with  $R \geq 5\text{--}10 \text{ mm h}^{-1}$ . These errors can be further reduced using other techniques such as spatial averaging whenever possible, which may be especially useful for stratiform rain. The following section presents evaluation of the algorithms developed here using disdrometer observations.

## 6. Evaluation of the algorithms using disdrometer data

The algorithms developed in this paper to estimate  $D_0$  and  $N_w$  are applied to data collected with a J-W

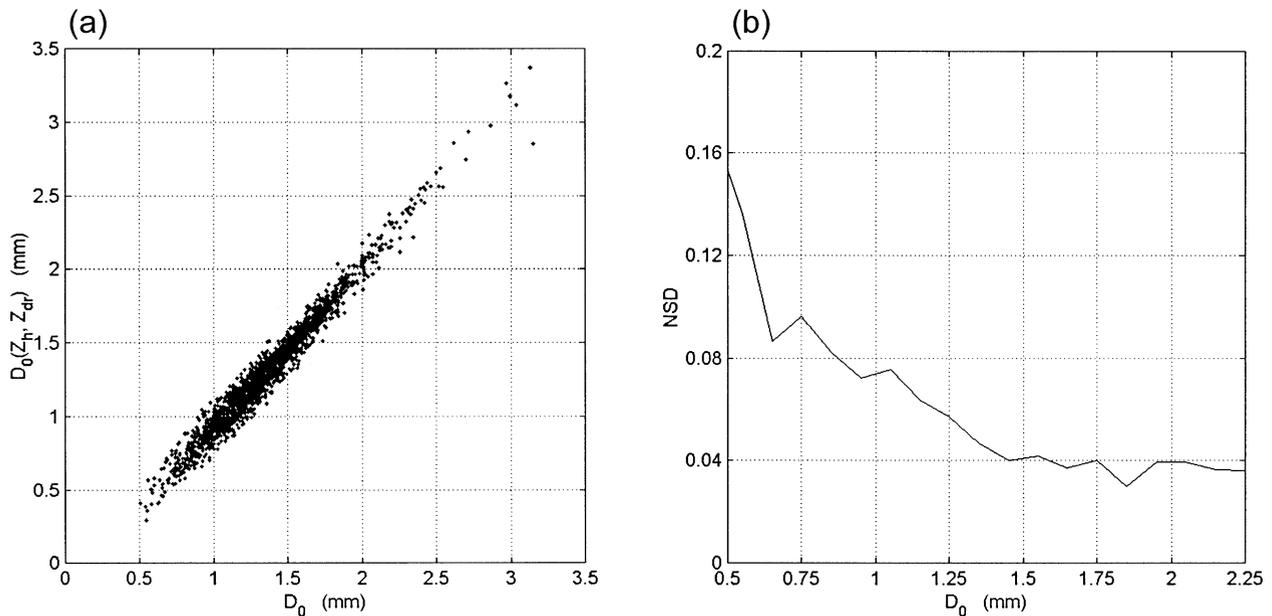


FIG. 7. (a) Scatterplot of the estimate of  $D_0$  computed from simulations of  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$ , vs the direct estimate of  $D_0$  for DSD obtained from a disdrometer located near Darwin, Australia. There are 2159 two-min DSD samples from the disdrometer. (b) Normalized standard deviation in the estimate of  $D_0$ , computed from simulations of  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$ , vs the direct estimate of  $D_0$  for DSD obtained from a disdrometer located near Darwin, Australia.

impact disdrometer (Joss and Waldvogel 1967) during a rainfall season (covering about 3 months) from Darwin, Australia. This dataset was collected by the Bureau of Meteorology Research Center (BMRC) and includes a variety of rainfall types from a tropical regime with rain rates between 1 and 150 mm h<sup>-1</sup>. The disdrometer data consists of measurements of  $N(D)$  in discrete intervals of  $\Delta D$  at 30-s intervals, which are subsequently averaged over 2 min. While several methods are available to fit the measured  $N(D)$  to a gamma form (e.g., Willis 1984), the method used here is based on Bringi and Chandrasekar (2001). First,  $D_m$  is estimated using the definition in (5b), that is, as a ratio of the fourth to third moments of the measured  $N(D)$ . Next the water content  $W$  is estimated from the definition in (7). The parameter  $N_w$  in (9) is then calculated as the intercept of the equivalent exponential DSD that has the same  $W$  and  $D_m$  as the measured  $N(D)$ , as

$$N_w = \frac{4^4}{\pi \rho_w} \left( \frac{10^3 W}{D_m^4} \right) \quad (\text{mm}^{-1} \text{m}^{-3}) \quad (42)$$

(where  $W$  is in g cm<sup>-3</sup>,  $D_m$  is in mm, and the water density  $\rho_w$  is in g cm<sup>-3</sup>). Finally, the parameter  $\mu$  is estimated by minimizing the absolute deviation between observed  $\log_{10} N(D)$  and that given by (9). Here,  $D_0$  is estimated from  $D_m$  as (Ulbrich 1983)

$$D_0 = D_m \frac{(3.67 + \mu)}{(4 + \mu)}. \quad (43)$$

Once the set of  $(N_w, D_0, \mu)$  parameters are obtained,

the radar observables  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  are simulated based on the following assumptions:

- 1) axis ratio versus  $D$  relation based on the fit proposed by Andsager et al. (1999) for  $D$  up to 4 mm; beyond 4 mm, the equilibrium axis ratios of Beard and Chuang (1987) are used;
- 2) Gaussian canting angle distribution with mean of 0° and standard deviation 10°; and
- 3) truncation of the gamma DSD at  $D_{\max} = 3.5 D_m$  [see Ulbrich and Atlas (1998) for a discussion of the drop truncation of the DSD].

The simulated set of radar observables ( $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$ ) when used in (27) gives an “effective”  $\beta$  of 0.0475 (for comparison, the equilibrium  $\beta$  is 0.062).

Note that the algorithms for  $D_0$  and  $N_w$  are constructed to be insensitive to the actual value of  $\beta$ , so that the details of the assumptions used in simulating the set of radar observables are not of particular relevance, and this fact is indeed the power of the proposed  $D_0$  and  $N_w$  algorithms in (24), (30), (34), and (37). In order to evaluate these algorithms using disdrometer measurements, the simulated values of  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  are used in (24), (34), and (39) to calculate  $\hat{D}_0$ ,  $\hat{N}_w$ , and  $\hat{\mu}$ , which are then compared against  $D_0$ ,  $N_w$ , and  $\mu$  estimated by gamma fits to the set of measured  $N(D)$ . Once again, to be consistent when  $K_{dp} < 0.2^\circ$  km<sup>-1</sup>, the estimates are computed using the practical approximation discussed in section 5. Figure 7a shows the  $D_0$  comparisons while Fig. 7b shows the NSD. Note that the  $\hat{D}_0$  algorithm can retrieve the “true”  $D_0$  quite

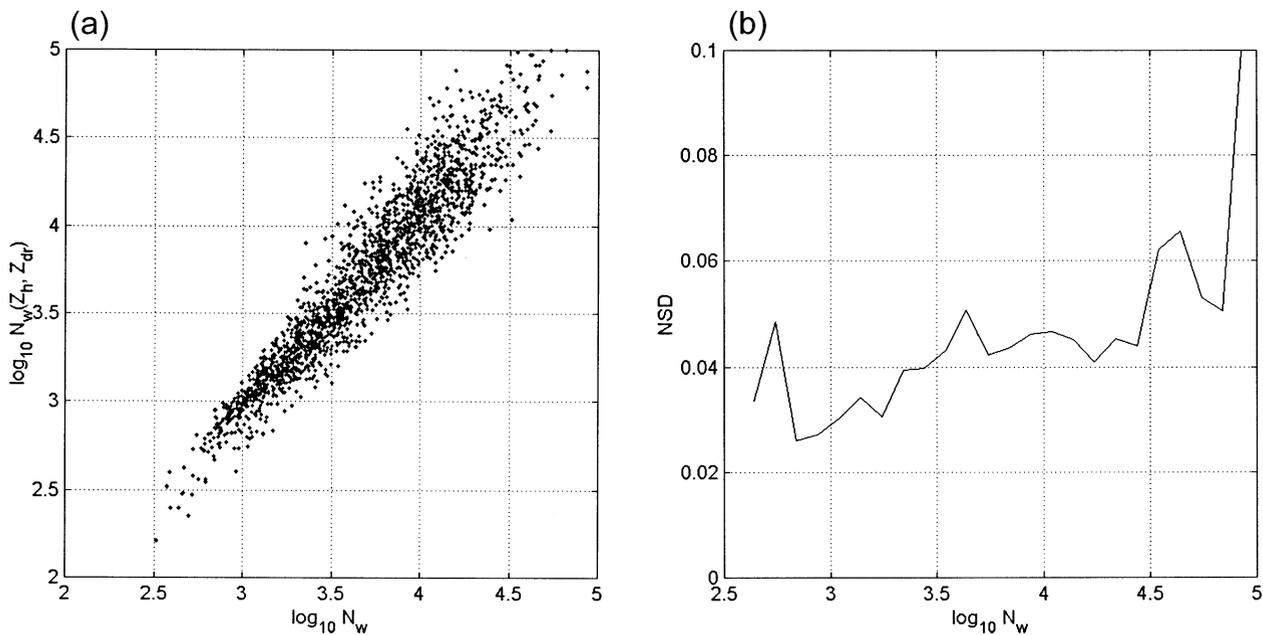


FIG. 8. (a) Scatterplot of  $\log_{10} N_w(Z_h, Z_{dr})$ , vs the direct estimate of  $\log_{10} N_w$  for DSD obtained from a disdrometer located near Darwin, Australia. (b) Normalized standard deviation of  $\log_{10} N_w(Z_h, Z_{dr})$ , vs the direct estimate of  $\log_{10} N_w$  for DSD obtained from a disdrometer located near Darwin, Australia.

accurately (NSD < 7%) especially for  $D_0 > 1$  mm. As expected the  $D_0$  estimates get very accurate for higher values. The  $\log_{10}(N_w)$  comparison are shown in Fig. 8a, while Fig. 8b shows the NSD. The scatter in Fig. 8a shows that the accuracy in the retrieval of  $\log_{10} N_w$  is

quite high (<5%) for  $N_w > 1000 \text{ mm}^{-1} \text{ m}^{-3}$  (for reference, the Marshall–Palmer value for  $N_w$  is  $8000 \text{ mm}^{-1} \text{ m}^{-3}$ ). Figure 9a shows the  $\mu$  comparison, while Fig. 9b shows the corresponding standard deviation. The results of Fig. 9 show that it is difficult to retrieve  $\mu$  with any

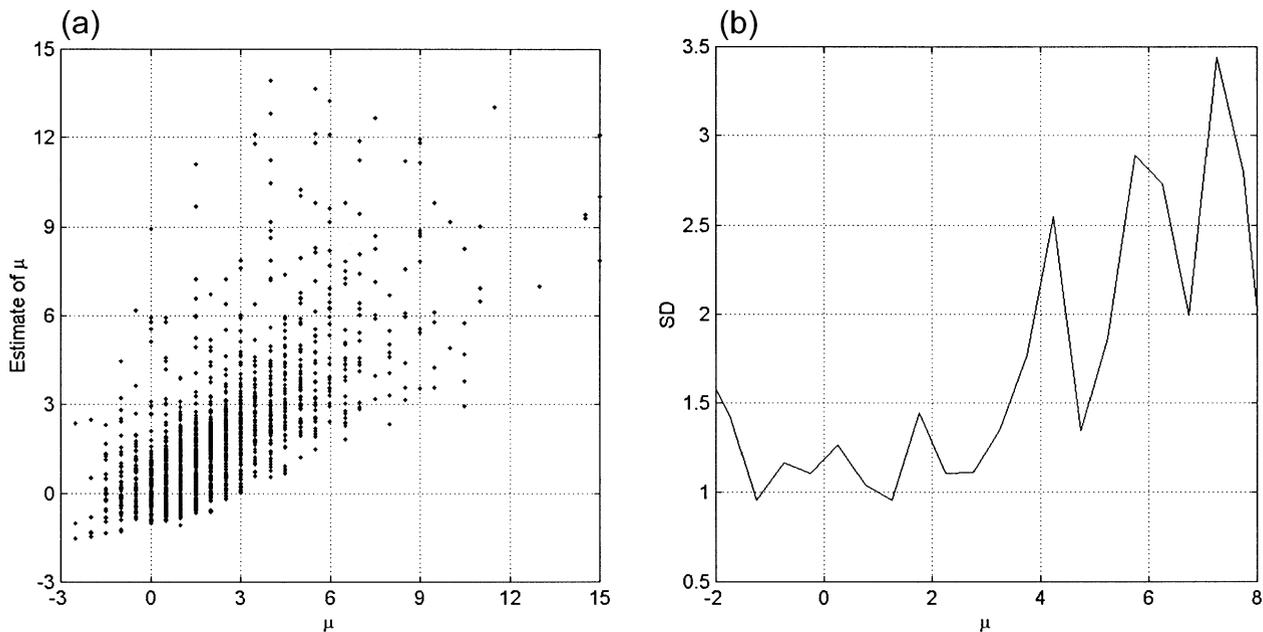


FIG. 9. (a) Scatterplot of the estimates of  $\mu$ , vs the direct estimate of  $\mu$  for DSD obtained from a disdrometer located near Darwin, Australia. (b) Standard deviation in the estimates of  $\mu$  as a function of the direct estimate of  $\mu$  for DSD obtained from a disdrometer located near Darwin, Australia.

reasonable accuracy with the current algorithms, though it may be possible to distinguish between certain ranges of  $\mu$ , for example,  $\mu = 0$  as opposed to  $\mu > 5$ , which may be sufficient in practice.

## 7. Summary and conclusions

One of the long-standing goals of polarimetric radar has been the estimation of the parameters of the raindrop size distribution. Estimators for the parameters of a three-parameter gamma model, namely  $D_0$ ,  $N_w$ , and  $\mu$ , are developed in this paper based on the radar observations  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$ . The behavior of the three radar observations  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  are influenced by the underlying DSD, and the mean shape of raindrops. Reflectivity  $Z_{dr}$  is proportional to the reflectivity-weighted axis ratio, whereas  $K_{dp}$  is proportional to the volume-weighted deviation of the axis ratio from unity. In addition, reflectivity is proportional to the sixth moment of the DSD, with corresponding variability due to polarization. Thus, the different polarimetric radar observations weight the DSD differently. It should be noted that the DSD estimates computed here correspond to radar measurements from the radar resolution volume. Among the three measurements ( $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$ ),  $Z_{dr}$  is the most closely related to a parameter of the DSD, namely  $D_0$ . Gorgucci et al. (2000) described a procedure to estimate the mean shape–size relation of raindrops based on a simple linear model. Therefore, after the prevailing shape–size relation is established,  $Z_{dr}$  can be used to estimate  $D_0$  directly. This concept is implemented in this paper as an algorithm to estimate  $D_0$  from  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$ . Statistical analysis of the estimator of  $D_0$  indicates that it can be estimated to an accuracy of 10% when  $D_0$  is 2 mm (and similar accuracies at the other  $D_0$  values). Once  $D_0$  is estimated, other measurements such as  $Z_h$  or  $K_{dp}$  can be used to estimate  $N_w$ , to a normalized standard deviation of about 6.5% when  $N_w = 8000 \text{ mm}^{-1} \text{ m}^{-3}$ , and similar order at the other values. The estimation of  $\mu$  is not easy because of the least influence of this parameter on the three measurements  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$ . Therefore, the parametric estimates of  $\mu$  derived are not as accurate. Measurement errors in  $Z_h$ ,  $Z_{dr}$ , and  $K_{dp}$  play a key role in the final accuracy of DSD estimates. Reflectivities  $Z_h$  and  $Z_{dr}$  are based on backscatter power measurement whereas  $K_{dp}$  is a forward scatter phase measurement. In addition,  $Z_{dr}$  is a differential power measurement between two correlated signals, and can be measured accurately. This high degree of accuracy in  $Z_{dr}$  translates to high accuracy in  $D_0$ . However, to estimate the prevailing mean shape–size relation,  $K_{dp}$  is needed that is relatively noisy at low rainrates. A hybrid approach is implemented in this paper such that when  $K_{dp} \leq 0.2 \text{ deg km}^{-1}$  the equilibrium shape model is used to estimate  $D_0$ . This procedure yields estimates of  $D_0$  to an accuracy of the order of 15%. Similarly,  $\log_{10} N_w$  can be estimated in the presence of measurement error to an accuracy of 15% when  $N_w$

$= 8000 \text{ mm}^{-1} \text{ m}^{-3}$ . This accuracy deteriorates to about 20% when  $N_w$  is of the order  $1000 \text{ mm}^{-1} \text{ m}^{-3}$  but improves to 10% if  $N_w$  is of the order  $40\,000 \text{ mm}^{-1} \text{ m}^{-3}$ . At low rainrates the best estimate of  $D_0$  or  $N_w$  is still the original estimates by Seliga and Bringi (1976). At low rainrates accurate estimates of  $Z_{dr}$  can be obtained by doing sufficient areal averaging, which can then be used in (13a) to estimate  $D_0$  and a subsequent exponential distribution algorithm given by (33c) to estimate  $N_w$ . In the presence of measurement error,  $\mu$  is difficult to estimate using the procedure described here in a meaningful manner. However, it may be possible to distinguish between  $\mu \approx 0$  versus  $\mu > 5$ , which may be sufficient in practice. The algorithms developed here were applied to one rainy season of disdrometer data collected in Darwin, Australia. The disdrometer analysis indicates that the algorithms work fairly well for the estimation of  $D_0$  and  $N_w$ . In summary, the algorithms presented in this paper can be used to estimate the parameters of the raindrop size distribution, from polarimetric radar data at a frequency near 3 GHz (S band).

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