DEVELOPMENT OF A SUBSURFACE HYDROLOGIC MODEL AND USE FOR INTEGRATED MANAGEMENT OF SURFACE AND SUBSURFACE WATER RESOURCES

by

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December 1977

Colorado Water Resources Research Institute
Completion Report No. 82

DEVELOPMENT OF A SUBSURFACE HYDROLOGIC MODEL AND USE FOR INTEGRATED MANAGEMENT OF SURFACE AND SUBSURFACE WATER RESOURCES

Completion Report

OWRT Project No. B-144-Colorado

by

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submitted to

Office of Water Research and Technology
U.S. Department of Interior
Washington, D.C. 20240

December 1977

The work upon which this report is based was supported in part by funds provided by the U.S. Department of Interior, Office of Water Research and Technology, as authorized under the Water Resources Research Act of 1964, and pursuant to Grant Agreement No. 14-31-0001-6064

COLORADO WATER RESOURCES RESEARCH INSTITUTE

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ABSTRACT

Two new concepts useful in studies of conjunctive management of surface and ground waters are introduced.

In order to model both accurately and cost-effectively the effect of initial aquifer drawdowns on the future state of the system, *artificial* pumping rates for one period prior to the initial time are introduced. With this approach it is possible to simulate the effect of initial conditions in the same way pumping effects are predicted. In addition for long-term simulation the concept of *sequential reinitialization* is introduced. In this manner daily simulation over many years can be carried with pumping and relaxation discrete kernels calculated for only a few periods.

The concept of *reach transmissivity* introduced in earlier studies is verified by comparison with observations of return flows to a reach of the South Platte River. The study indicates that the approach is sound.

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RESEARCH OBJECTIVES

The first objective of this project was the development of a subsurface hydrologic model that portrays accurately and *efficiently* fluid movement below the ground. The second objective was to design the model so that it can be used for integrated management of a surface-ground water system.

ACHIEVEMENTS OF CONTRACT

It is not desirable to repeat in this completion report all the results obtained over the past two years and the detailed procedures by which they were obtained. These results and procedures can (or will) be found in one thesis (Peters, 1978), two reports (Morel-Seytoux, 1977; Peters and Morel-Seytoux, 1978), and several papers (Morel-Seytoux and Khanji, 1975; Morel-Seytoux, 1976 abcd; Sonu and Morel-Seytoux, 1976; Morel-Seytoux et al., 1977).

Rather a brief review of the methods of attacks and a sample of results will be given. The emphasis is on the latest developments which have not been reported elsewhere and which hold great promise for the future. Generally speaking the thrust of the research has been in the direction of development of new and imaginative methods that will greatly reduce the cost of management studies of conjunctive use of surface and ground waters without significant reduction in accuracy. In this regard the project was highly successful.

NEED FOR STUDY

On September 1, 1977, a $90,000 4-months contract funded by the Bureau of Reclamation was awarded by the State *Drought* Coordinator, Office of the Governor, to Colorado State University under the title "South Platte River Basin Water System Operations Study." The purpose of the study is "to provide the STATE with information on hydrologic and economic impacts of applying efficiency criteria to the irrigation conveyance, distribution and application system of the South Platte River Basin." The scope of the work includes the following tasks:
"a. Adapt the current CSU basin model for surface-groundwater supply to the assessment of hydrologic and economic impacts described above.
b. Operate the conjunctive model with a range of assumed "efficiency" improvements in delivery, distribution and application systems and irrigation water management practices to assess the hydrologic and economic impacts described above."

The model referred to in the contract as "the current CSU basin model for conjunctive management of surface-groundwater supply" is *precisely* the model developed for OWRT under contract B-144-Colorado and the earlier contract B-109. It is a clear indication that the OWRT funded research was addressed to an acute and real problem in the drought-stricken West.

PART I

THE CONCEPT OF SEQUENTIAL REINITIALIZATION

A. INTRODUCTION

In earlier studies (Morel-Seytoux et al., 1973; Morel-Seytoux and Daly, 1975) it was shown for an aquifer, governed by the linearized Boussinesq equation:

$$\phi \frac{\partial s}{\partial t} - \frac{\partial}{\partial x} (T \frac{\partial s}{\partial x}) - \frac{\partial}{\partial y} (T \frac{\partial s}{\partial y}) = Q_p \tag{1}$$

where ϕ is effective porosity (specific yield), s is aquifer drawdown, T is transmissivity, t is time, x and y are the horizontal cartesian coordinates, and Q_p is the withdrawal rate from a well, that the solution of Eq. (1) is always of the form:

$$s_w(t) = \int_0^t Q_p(\tau)\, k_{wp}(t-\tau) d\tau \tag{2}$$

In Eq. (2) $s_w(t)$ is the drawdown at any point w in the aquifer at time t, $Q_p(\tau)$ is the pumping rate at well p and $k_{wp}(\ )$ is the *unit impulse* kernel of drawdown due to pumping. For a homogeneous aquifer of infinite extent the kernel is known analytically, namely:

$$k_{wp}(t) = \frac{e^{\frac{-\phi R_{wp}^2}{4Tt}}}{4\pi Tt} \tag{3}$$

where R_{wp} is the distance between point w and well p. For a heterogeneous aquifer of finite extent and odd boundary shape no analytic solution is available. In this case it is preferable to use a discrete form of Eq. (2), namely:

$$s_w(n) = \sum_{\nu=1}^{n} \delta_{wp}(n-\nu+1)Q_p(\nu) \tag{4}$$

where $s_w(n)$ is aquifer drawdown at point w at discrete time n, $Q_p(\nu)$ is the average pumping rate during the time-interval $(\nu-1,\nu)$ or ν^{th} period and $\delta_{wp}(\ )$ is the so-called *discrete kernel* (or *unit pulse* kernel). The discrete kernel values can be calculated using numerical techniques (e.g., finite differences or finite elements approximations) and several such *discrete kernel generators* have been developed and adapted (Morel-Seytoux and Daly, 1975; Morel-Seytoux et al., 1975; Peters, 1978). When P wells are present in the system, Eq. (4) generalizes to the form:

$$s_w(n) = \sum_{p=1}^{P} \sum_{\nu=1}^{n} \delta_{wp}(n-\nu+1)Q_p(\nu) \tag{5}$$

Eq. (5) suffers particularly from one limitation. It does not provide the actual drawdown, only the contribution to drawdown due to pumping. To this component of drawdown must be added the *relaxation* component, that is, the drawdown that results from the *natural* tendency of the aquifer to return to equilibrium if initially it was not in equilibrium. If this *natural redistribution* component is denoted $v_w(n)$, the drawdown is given by the expression:

$$s_w(n) = v_w(n) + \sum_{p=1}^{P} \sum_{\nu=1}^{n} \delta_{wp}(n-\nu+1)Q_p(\nu) \tag{6}$$

If the evolution of the *initial condition* (or *relaxation* or *natural redistribution*) component must be calculated by numerical simulation, then much of the usefulness of the discrete kernel approach is lost. It was imperative to develop an efficient procedure to predict the natural (*internal*) evolution of the aquifer when it is not subjected to external excitations such as pumping.

B. TREATMENT OF INITIAL CONDITIONS

Broadly speaking how does a mathematician solve a new problem? He (or she) reduces it to another problem for which the solution is already known. Reasoning similarly (Morel-Seytoux, 1975) since the solution for pumping excitations is known, one tries to reduce the natural redistribution problem to an *equivalent* one of pumping.

Let s_π° denote the initial drawdown at any point π in the aquifer and let $\bar{s}$ denote the average initial drawdown in the entire aquifer. Eq. (5) is not applicable since the aquifer is not initially in static equilibrium. Let us assume (even though contrary to evidence) that one period prior to the initial time the aquifer was in static equilibrium with uniform drawdown of value $\bar{s}$. Let Q_λ° be the *artificial* pumping rate during this period preceding the initial time at any point λ in the aquifer. Then the future drawdowns in the aquifer can be predicted with Eq. (5) provided the time period prior to time zero be included in the summation, namely:

$$s_\pi(n) = \bar{s} + \sum_{\lambda=1}^{\Pi} \delta_{\pi\lambda}(n+1)Q_\lambda^\circ + \sum_{p=1}^{P} \sum_{\nu=1}^{N} \delta_{\pi p}(n-\nu+1)Q_p(\nu) \qquad (7)$$

where Π is the total number of aquifer (grid) points. The first two terms on the right-hand side constitute the initial condition component denoted v in Eq. (6). Naturally at time zero the predicted drawdowns by Eq. (7) must be the initial drawdowns. Thus the unknown and as yet arbitrary pumping rates Q_λ° must satisfy the system of equations:

$$\sum_{\lambda=1}^{\Pi} \delta_{\pi\lambda}(1)Q_\lambda^\circ = s_\pi^\circ \qquad \pi=1,2,\ldots\Pi \tag{8}$$

Eq. (8) could be solved numerically for a particular set of initial drawdown values since the coefficients $\delta_{\pi\lambda}(1)$ are known. However, if different initial conditions are to be studied it is preferable to solve Eq. (8) explicitly. The solution of Eq. (8) is of the form:

$$Q_\lambda^\circ = \sum_{\pi=1}^{\Pi} \theta_{\lambda\pi}^*(s_\pi^\circ - \bar{s}) \tag{9}$$

where the matrix $[\theta_{\lambda\pi}^*]$ is the inverse of the matrix $[\delta_{\pi\lambda}(1)]$. The coefficients $\theta_{\lambda\pi}^*$ are deduced from the coefficients $\delta_{\pi\lambda}(1)$ by the Gauss-Jordan elimination procedure. Substitution of Eq. (9) in Eq. (7) yields:

$$s_\pi(n) = \bar{s} + \sum_{\lambda=1}^{\Pi}\sum_{\pi=1}^{\Pi} \delta_{\pi\lambda}(n+1)\theta_{\lambda\pi}^*(s_\pi^\circ-\bar{s}) + \sum_{p=1}^{P}\sum_{\nu=1}^{n}\delta_{\pi p}(n-\nu+1)Q_p(\nu) \tag{10}$$

For simplicity in writing it is convenient to define new coefficients, namely:

$$\theta_{\pi\lambda}(n) = \sum_{\alpha=1}^{\Pi} \delta_{\pi\alpha}(n+1)\theta_{\alpha\lambda}^* \tag{11}$$

Finally the prediction of future drawdowns is obtained by the expression:

$$s_w(n) = \bar{s} + \sum_{\pi=1}^{\Pi} \theta_{w\pi}(n)(s_\pi^\circ-\bar{s}) + \sum_{p=1}^{P}\sum_{\nu=1}^{n}\delta_{wp}(n-\nu+1)Q_p(\nu) \tag{12}$$

Again the first two terms on the right-hand side represent the evolution of drawdown that would take place even if the aquifer was not excited by pumping because it was not initially at rest. Note that in Eq. (12) the initial condition term depends linearly on the difference $(s_\pi^\circ-\bar{s})$ as expected and not on the artificial pumping rates. The concept of artificial pumping rates was useful for the derivation but it is ultimately eliminated.

The discrete kernels $\delta_{wp}(\ )$ are calculated by a numerical model (the discrete kernel generator; Morel-Seytoux and Daly, 1975). This is the costly part particularly if the $\delta_{wp}(\ )$ are to be calculated for many periods. Once they have been obtained the $\theta^*_{w\pi}$ can be calculated by inversion of the matrix $[\delta_{\pi\lambda}(1)]$. This is done only once. Thus the use of Eq. (12) is particularly effective for repetitive interrupted short-term simulation, for example over a 20 weeks irrigation season starting in early spring. Every spring water-table observations are made and one wants to predict the state of the aquifer during the irrigation season as a function of various pumping practices, given the initial state of the system.

On the other hand for long-term simulation the direct application of Eq. (12) for a weekly period over a 20 years horizon would be very costly, because 1040 values of $\delta_{wp}(\ )$ would have to be calculated (and stored) for each grid point and each pumping node.

C. METHODOLOGY FOR LONG-TERM SIMULATION

The combined use of hydrologic and economic models to develop long-term planning strategies has introduced the need for methodologies to be developed that are capable of simulating inexpensively aquifer behavior far into the future.

The ability to model the natural redistribution due to non-equilibrium initial conditions makes it possible to stop a study at some point and, having calculated the drawdowns at every point, use the calculated drawdown values as initial conditions. By reinitializing the

the aquifer in this manner, it is possible to use the influence coefficients that were generated for earlier time periods to calculate drawdown values for the second time sequence. This idea can, of course, be used for any number of time sequences. The implication of using this reinitialization approach is that aquifer drawdown values can be predicted any time into the future using drawdown and natural redistribution influence coefficients, $\delta_{wp}(\ )$ and $\theta_{w\pi}(\ )$, calculated only for a few periods.

In order to test this approach the $\theta_{w\pi}(\ )$ coefficients were calculated from the $\delta_{wp}(\ )$ coefficients for a simple situation when the natural redistribution of drawdown can be calculated analytically. The hypothetical aquifer is linear, homogeneous, with no flow boundary conditions at both ends and consists of 5 grid points. Two tests were performed with different initial conditions as shown on Table 1.

Grid Point	1	2	3	4	5
Test 1	-6	-3	0	3	6
Test 2	-1.5	1	1	1	-1.5

Table 1. Initial drawdowns (meters)

Figure 1 displays the time evolution of drawdown at grid points 2 and 4 for Test 1. With or without reinitialization the results compare very well with the analytical solution. The results for Test 2 are similar (Figure 2). Another test (Test 3) was performed for a square 9 grid homogeneous aquifer. Initially the aquifer is in static equilibrium, the lower left corner well is pumped intermittently at the rate of 10,000 m^3/week. Figures 3, 4, 5 and 6 show the effect of the frequency

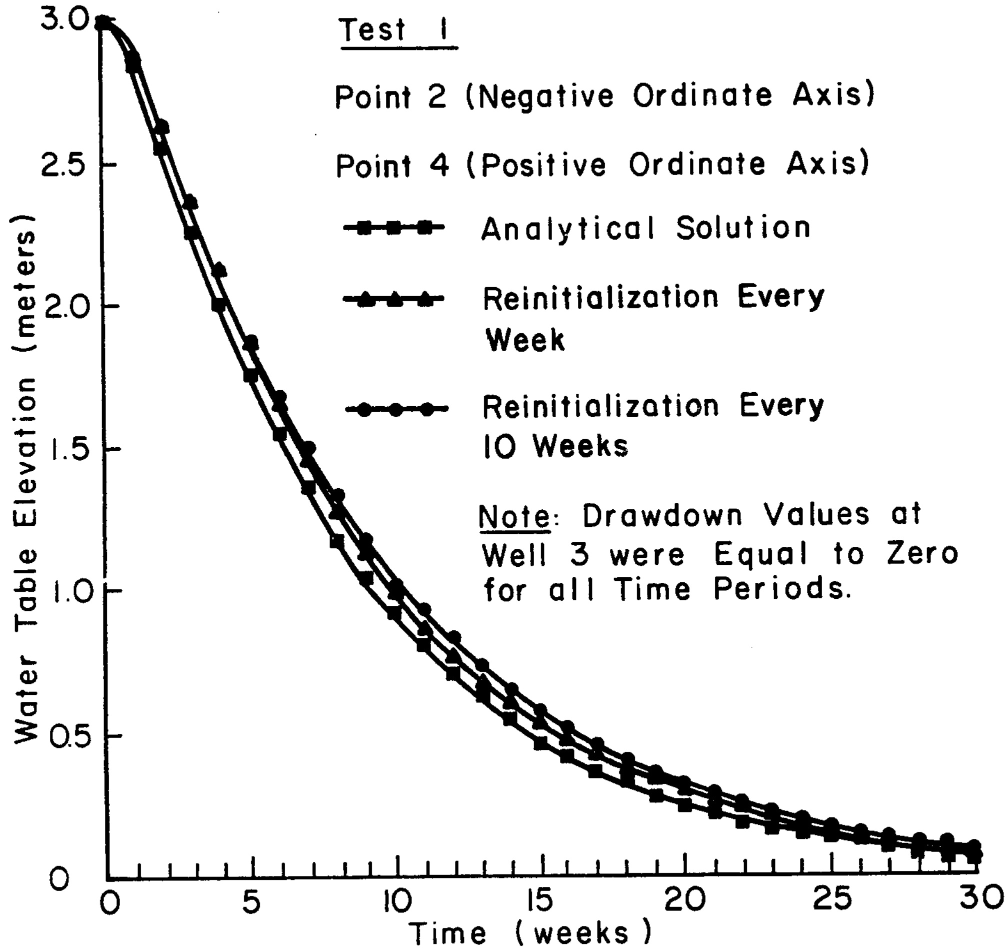

Figure 1 Comparison of predictions of aquifer redistribution at Points 2 and 4 for Test 1

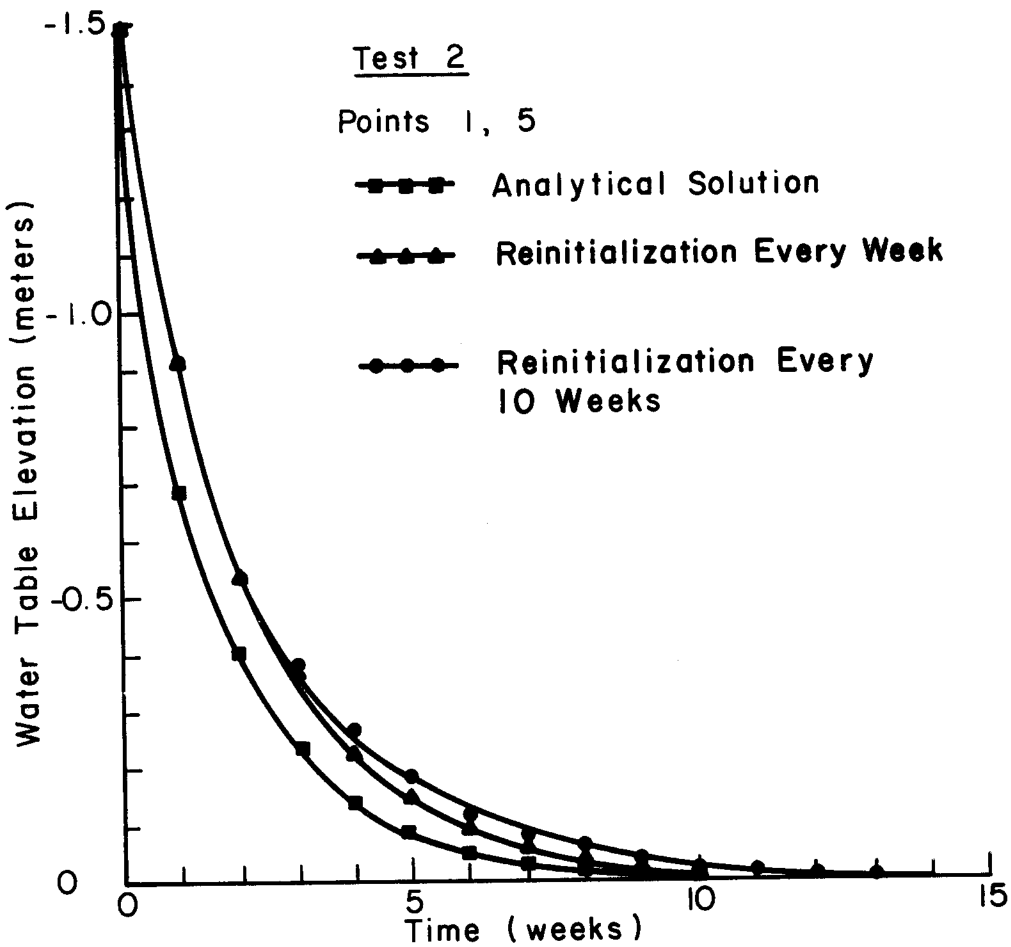

Figure 2 Comparison of predictions of aquifer redistribution at Points 1 and 5 for Test 2

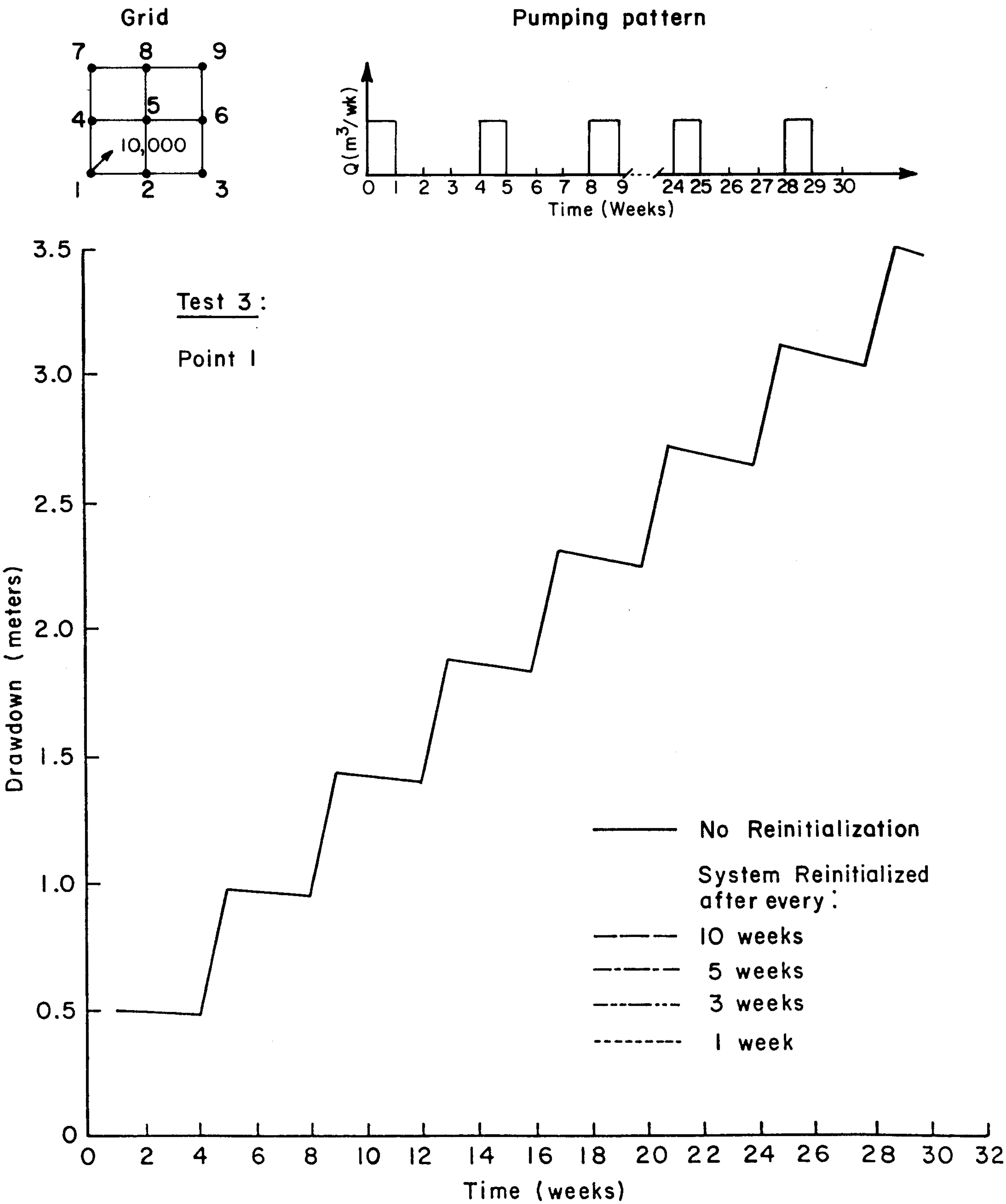

Figure 3 Grid description, pumping pattern and results of Test 3 at Point 1

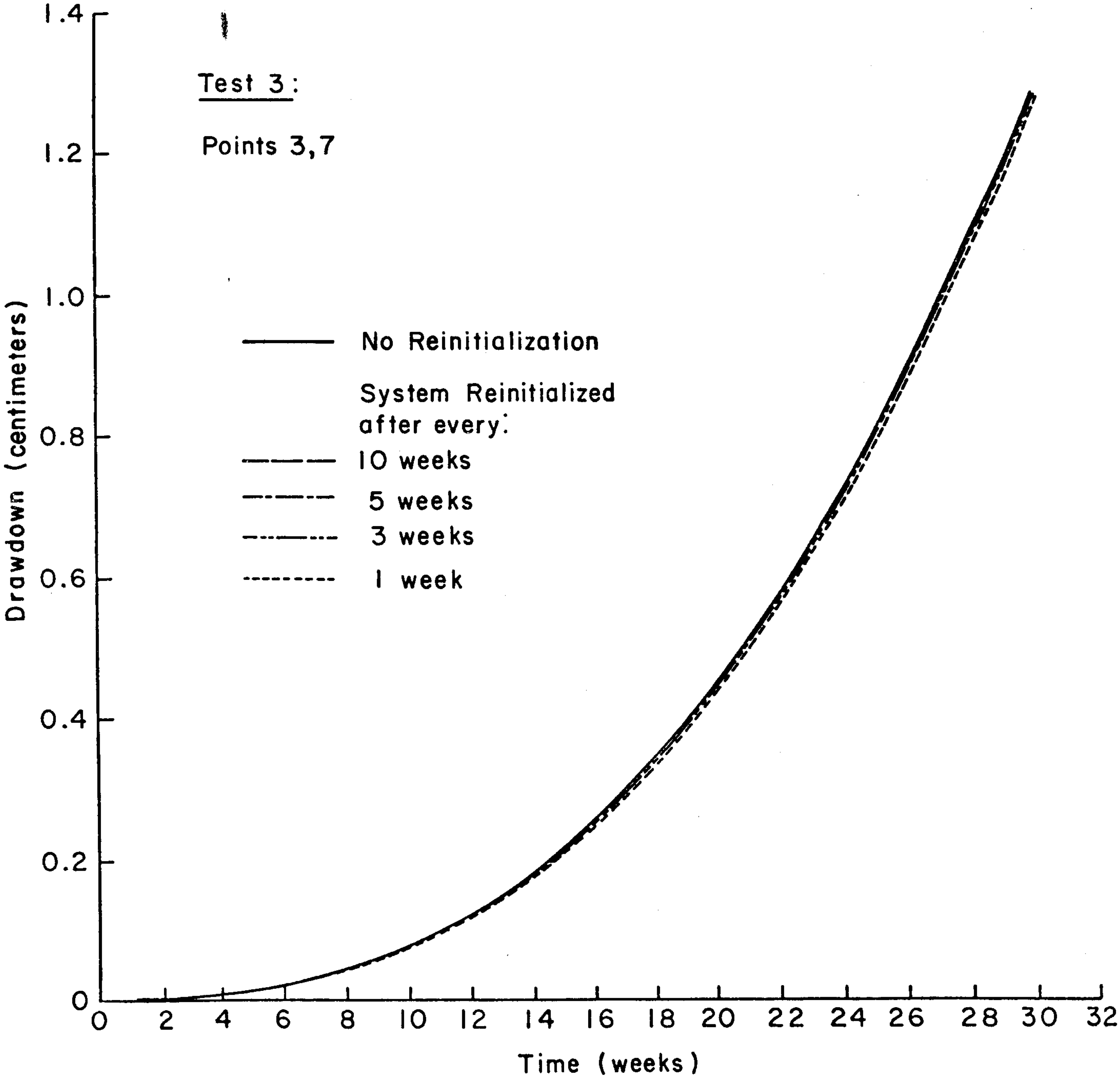

Figure 4 Results of Test 3 at Points 3 and 7

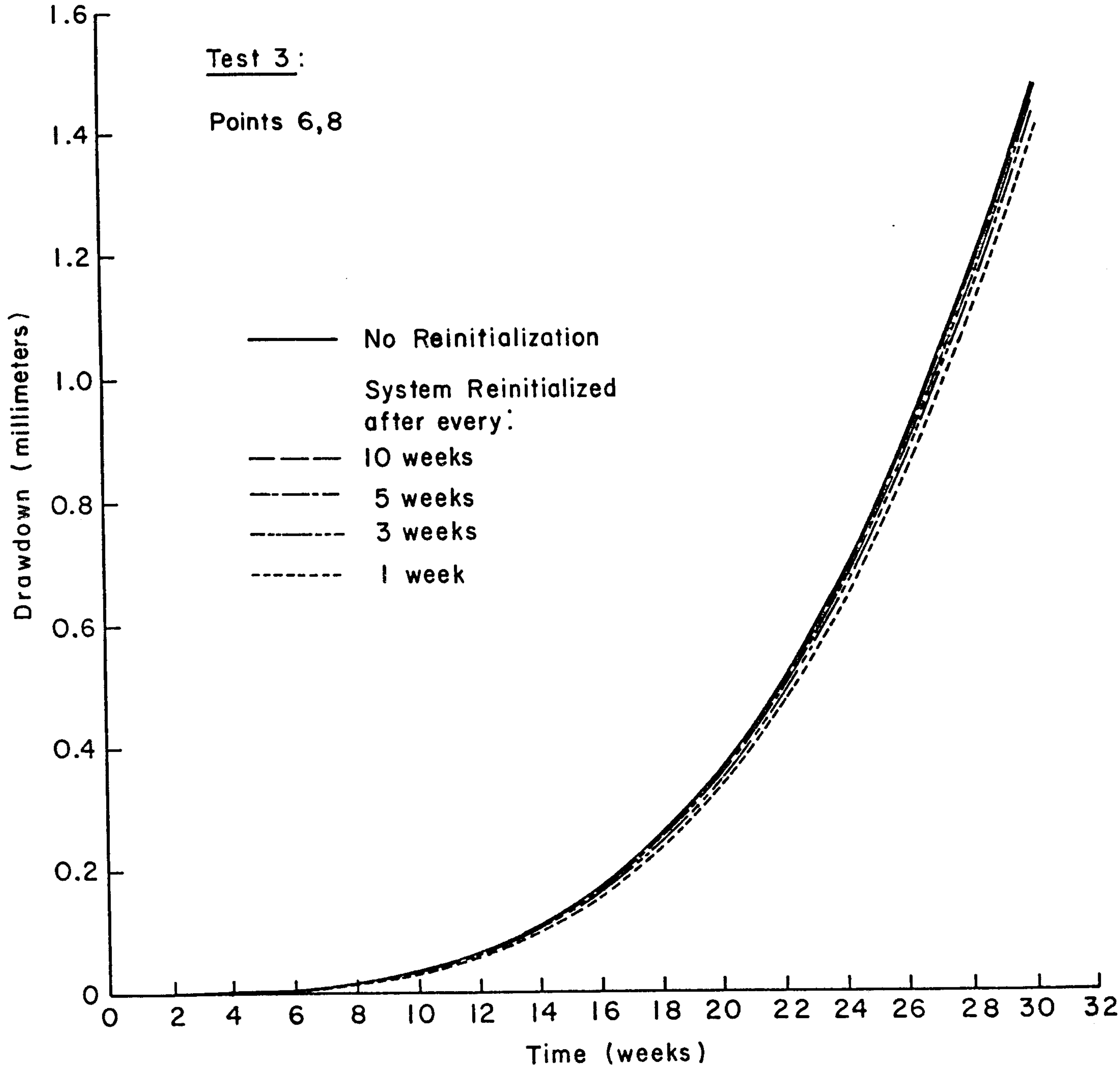

Figure 5 Results of Test 3 at Points 6 and 8

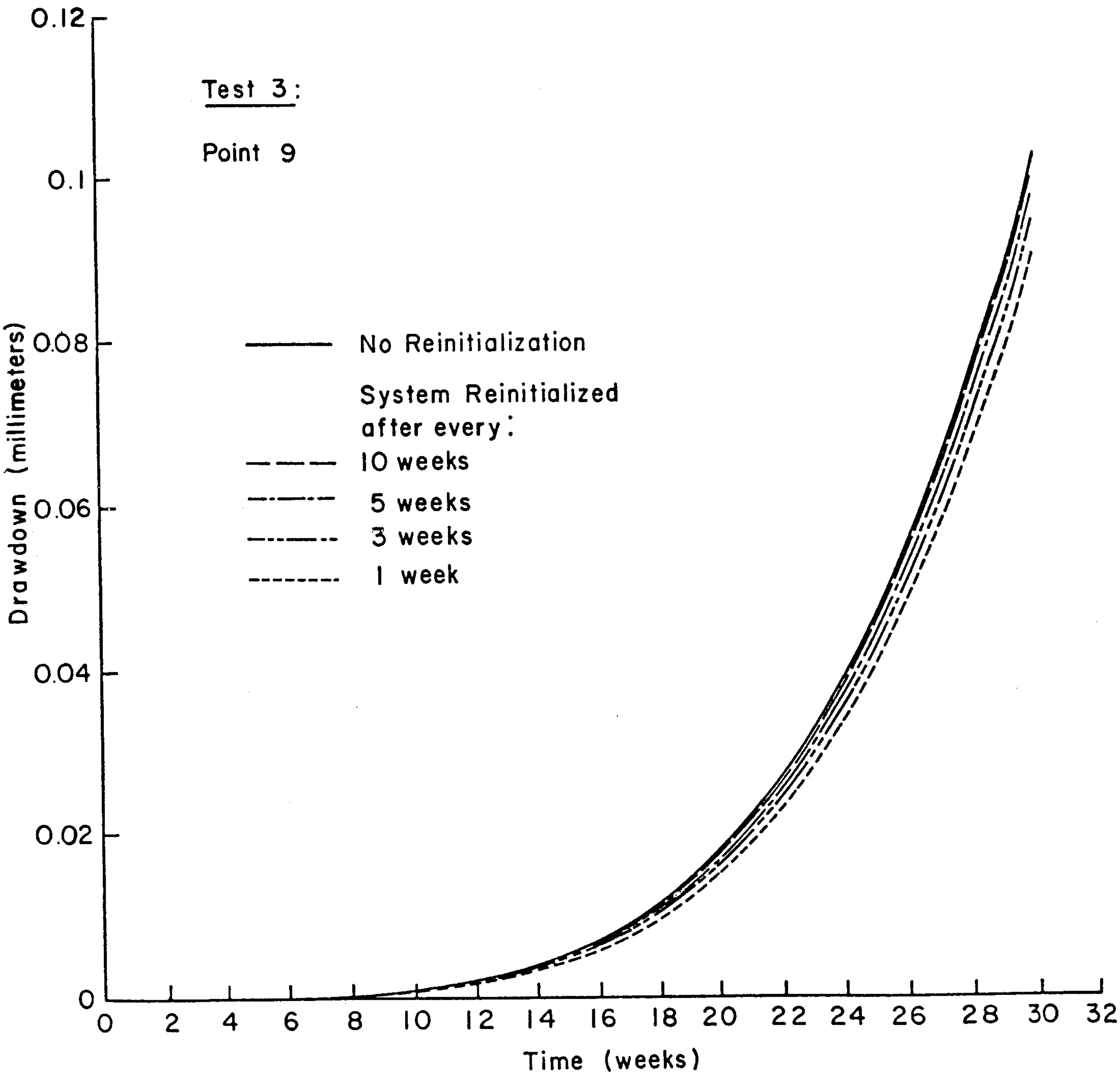

Figure 6 Results of Test 3 at Point 9

of reinitialization on the predictions of drawdowns at various points in the system. The effect is minor and if anything as evidenced on Figures 1 and 2 by comparison with an analytical solution the frequent reinitialization leads to more accurate results.

D. CONCLUSIONS

The concept of sequential reinitialization holds great promises to finally permit predictions of the state of an aquifer on a frequent basis (week or even day) at low cost, with an excellent accuracy, and for long periods of simulation (20-50 years). Frequent reinitialization eliminates the need for the large amount of multiplication involved in Eq. (12) when n is large (say 500 weeks) and of calculation and storage of $\delta_{wp}(\ )$ and $\theta_{w\pi}(\ )$ coefficients.

PART II

THE CONCEPT OF REACH TRANSMISSIVITY

A. INTRODUCTION

In earlier studies (Morel-Seytoux et al., 1973; Morel-Seytoux and Daly, 1975; Morel-Seytoux, 1975) it was postulated that return flow from an aquifer to a river reach could be expressed in the form:

$$Q_r = \Gamma_r \, [\sigma_r - s_r] \tag{13}$$

where Γ_r is the *reach transmissivity* (dimension area per time), σ_r is the drawdown to the water level in the stream (or canal) and s_r is the drawdown in the aquifer in the vicinity of the reach. Studies of the literature on seepage from canals indicated that the reach transmissivity could be expressed in terms of stream and aquifer parameters (Peters, 1978) by the equation:

$$\Gamma_r = \frac{TL}{e} \left[\frac{\frac{W_p}{2} + e}{5 \, W_p + \frac{e}{2}} \right] \tag{14}$$

where T is the aquifer transmissivity, e its saturated thickness, L the length of the reach, and W_p its wetted perimeter. It remained to verify the validity of both Eqs. (13) and (14) on an actual river.

B. CASE STUDY

The study area consists of an 81-mile reach of the South Platte River between Balzac and Julesburg, Colorado, shown on Figure 7. The aquifer is an alluvial-filled valley ranging from two to nine miles in width. The alluvium, which is comprised mainly of sand and gravel, is underlain by sandstone and shale of Cretaceous geologic age.

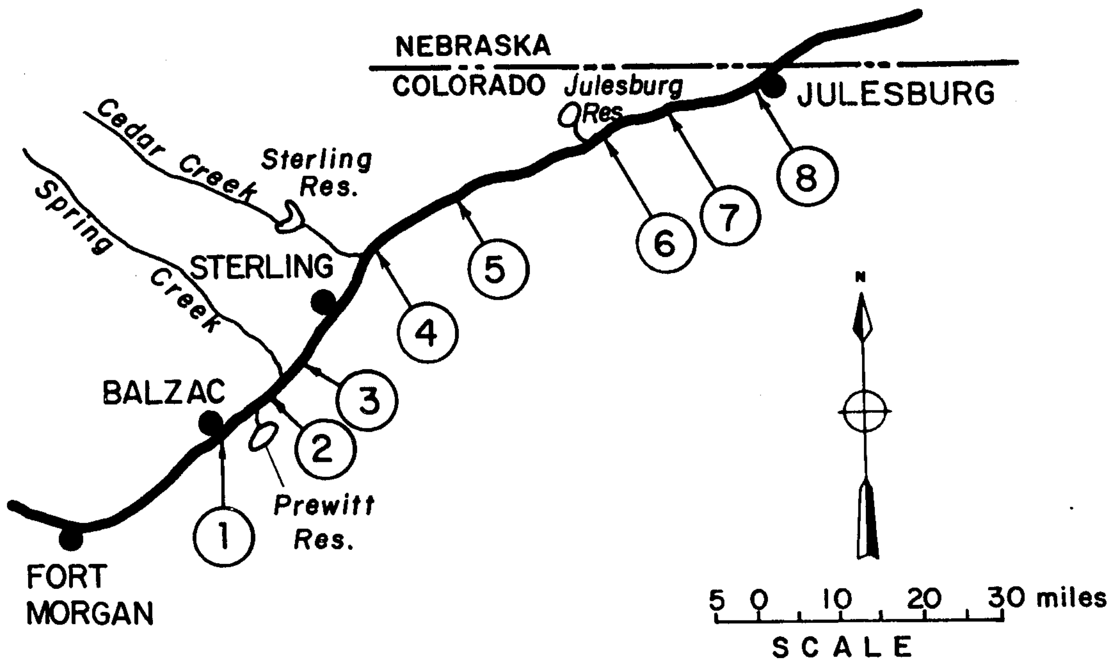

Figure 7 Location of cross sections used to determine $(\sigma_r - s_r)$ values.

As the South Platte River flows from Balzac to Julesburg, ground water and surface water return flows maintain it as a perennial stream. Extensive agricultural development of both surface water and ground water exist. The ground water aquifer is recharged largely from canal seepage and deep percolation of irrigation water.

Return flows were calculated in two independent ways. The net return flows between the two gauged stations at Balzac and Julesburg were calculated by a material balance from the observations of streamflows at both stations for the seasonal period December through April inclusive for the period 1921-1975. The results are displayed graphically on Figure 8. No trend is apparent from the data.

For the second estimation of return flows the river was subdivided into 89 reaches. For each reach the reach transmissivity was estimated from Eq. (14) using data of transmissivities and saturated thickness from the U.S. Geological Survey, lengths read from maps and estimates of wetted perimeters from field inspection along the river. Drawdown observations in wells were available along 8 cross-sections perpendicular to the river, unfortunately only for 5 years (1971-1975). To determine the aquifer drawdowns at 5 surface widths away from the river center, water table profiles were drawn between the observations as shown on Figure 9 for section 6. The head drop difference $(\sigma_r - s_r)$ was thus estimated for each year for each section. These values were then interpolated linearly between sections to get an estimate of the head drop for every reach as shown on Figure 10. Because the precise date of the water table observations were not known, 5-year average estimates were also obtained by drawing a single water table profile through the individual year observations as shown on Figure 11.

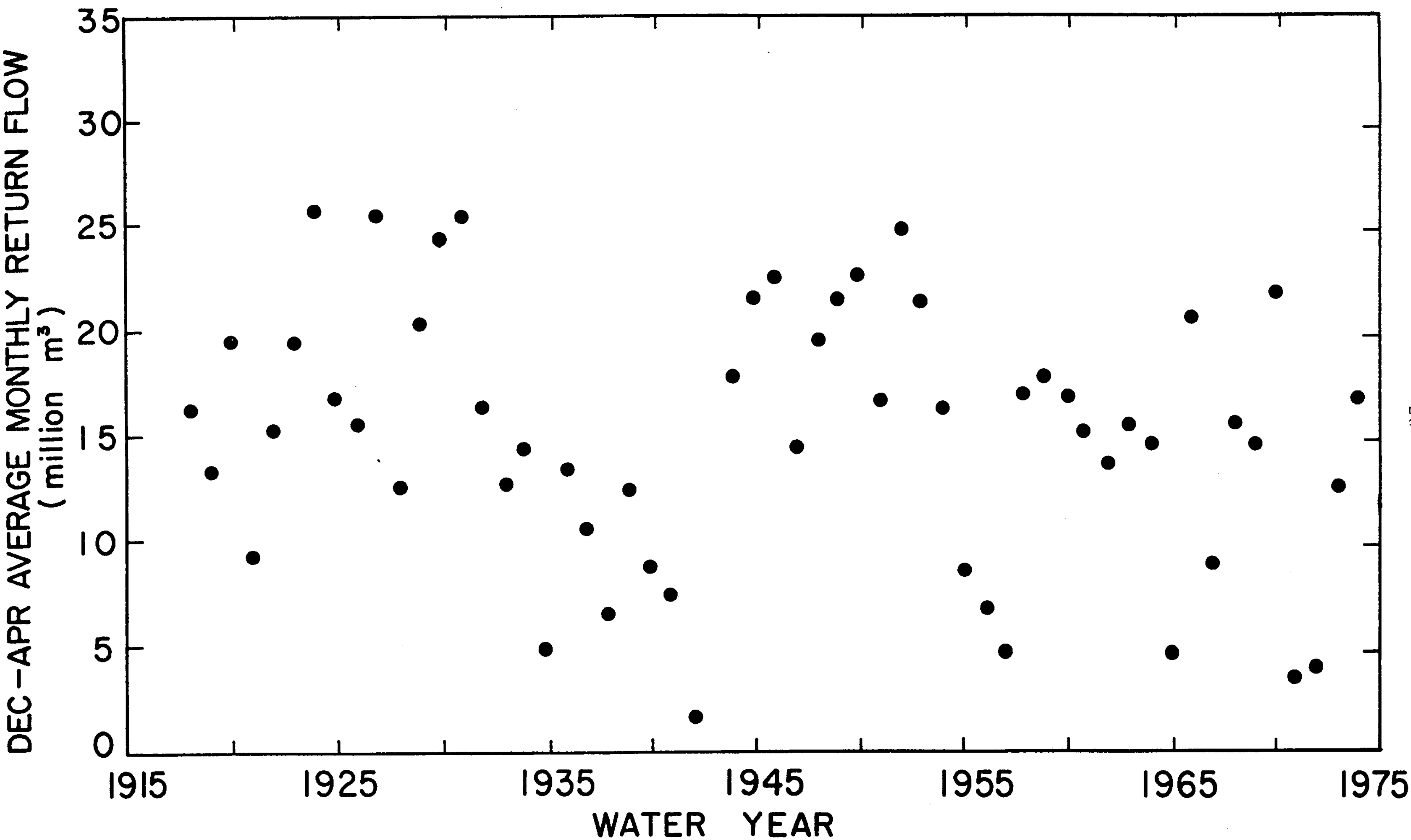

Figure 8 Average monthly return flows for winter months (December-April)

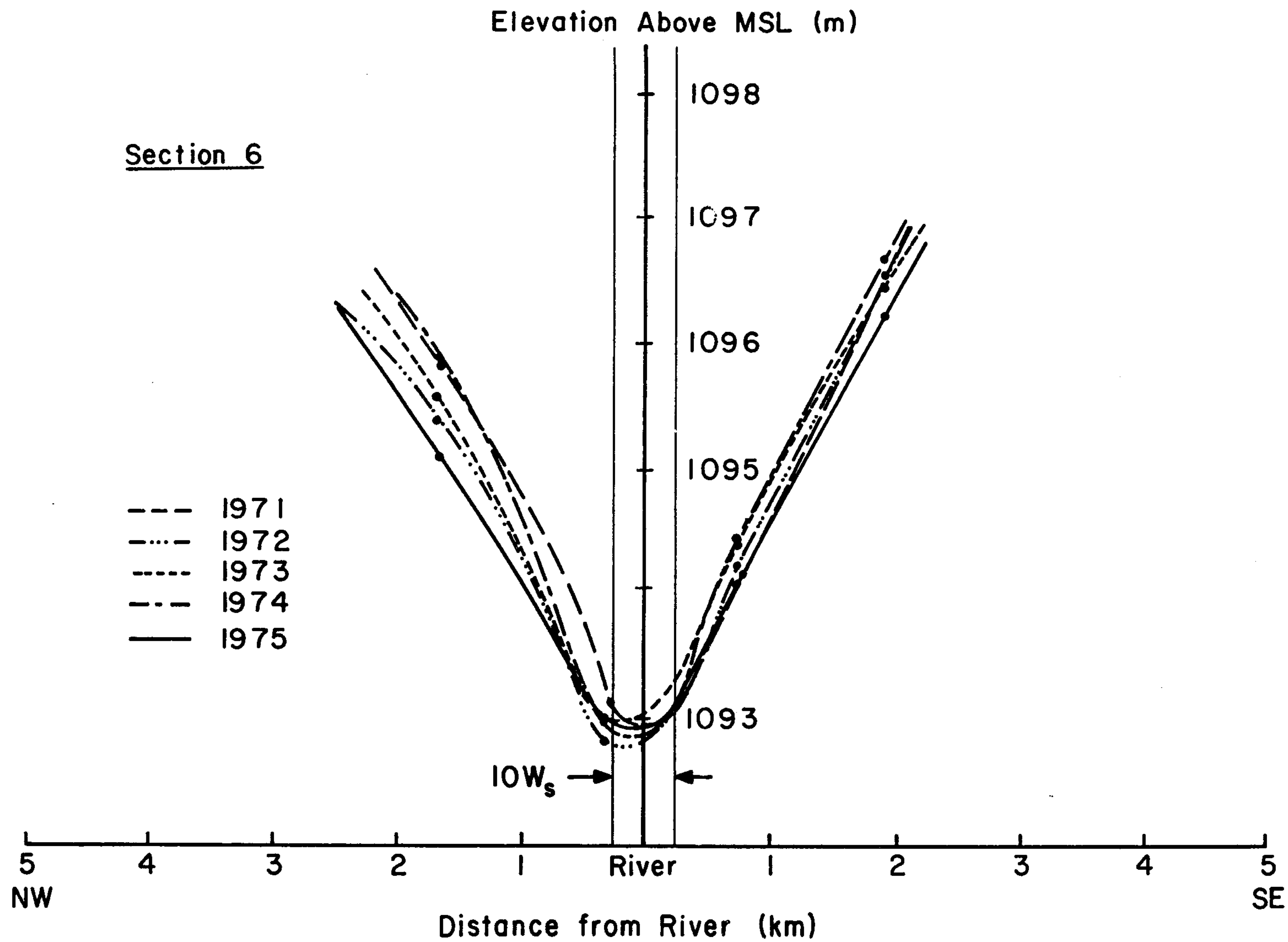

Figure 9 Water table profiles at cross section 6 used for the determination of winter month head drop (σ_r-s_r) values.

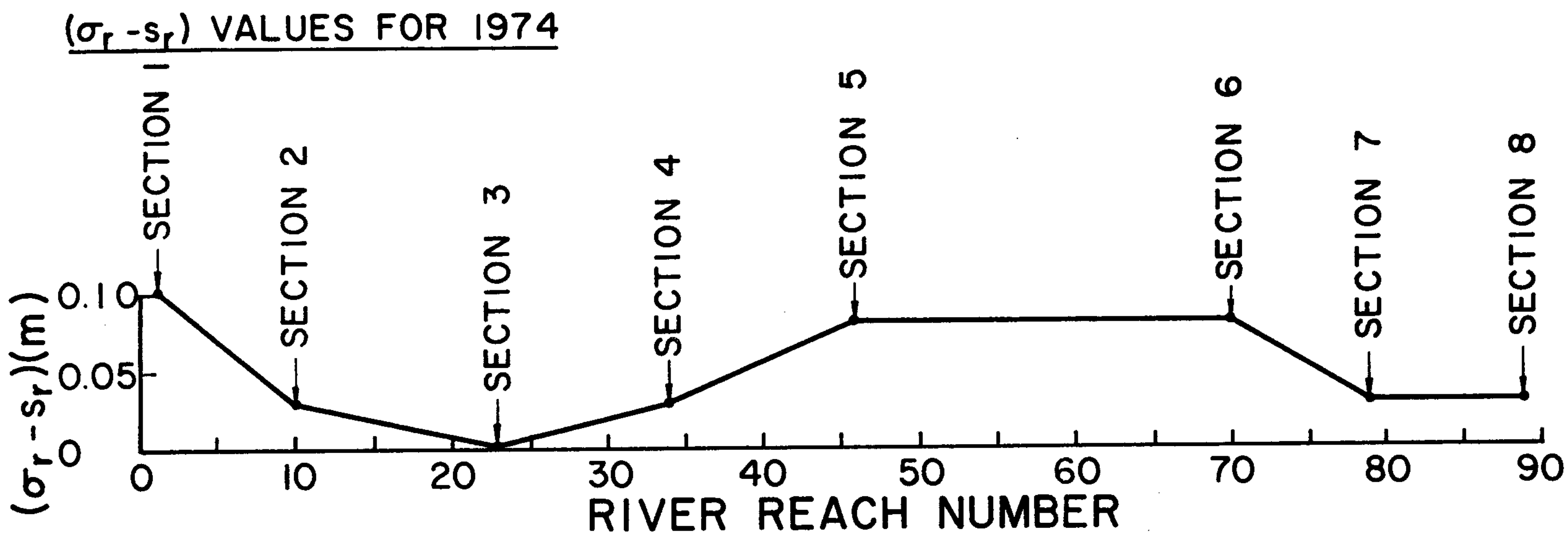

Figure 10 Linear interpolation of head drop (σ_r - s_r) values obtained from river reach cross sections--water year 1974.

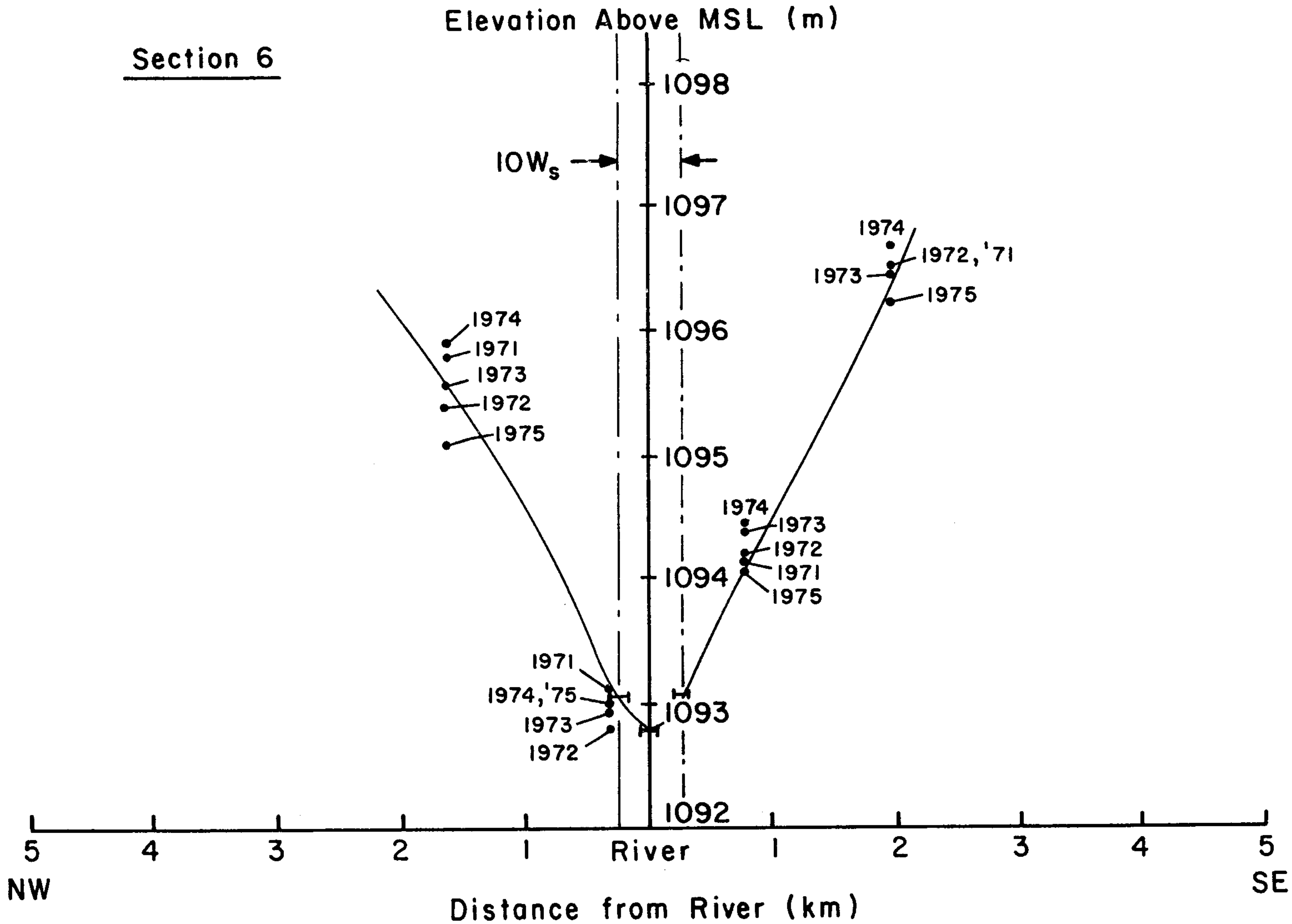

Figure 11 Water table profile at cross section 6 used for the determination of 5-year average head drop (σ_r - s_r) value.

C. RESULTS AND DISCUSSION

First of all it must be remarked that the head-drop values which drive the return flow to the river are relatively small never exceeding 0.30 meter (1 foot) and on the average of the order of 5 inches. Clearly minor errors in river stage reading or water-table reading may lead to large errors in estimation of return flows. The relatively close agreement between the predicted and observed values displayed in Table 2 may therefore be largely fortuitous but it is very encouraging. The five-year average comparison would seem to indicate that predictions exceed observations by 50%. The discrepancy would be removed by *reducing* the reach transmissivities by one third (33%) . On the other hand, the comparison with the average of the yearly values would indicate that the reach transmissivities should be increased by 15%. Thus the available

Average return flows in millions of cubic meters/month							
	Water Year					Average of Yearly Values	Five-Year Average Value
Method	1971	1972	1973	1974	1975		
Mass Balance	3.34	3.95	12.29	16.94	12.23	9.75	9.75
Integrated Darcy Equation	9.16	7.23	9.98	6.72	8.12	8.24	14.62
Percent Error*	+174	+83	-19	-60	-34	-15	+50

*Assuming the values obtained using mass balance considerations are correct.

Table 2. Summary of average monthly return flow values calculated using mass balance (observed) and integrated form of Darcy's equation

information is not sufficient to state conclusively whether the estimates of reach transmissivities using Eq. (14) are too high or too low. The results indicate that the formula yields a very close first estimate and that it would take a lot of good basic data to improve further on this first estimate by calibration.

PART III

VARIABLE RAINFALL INFILTRATION

New equations for variable rainfall infiltration, including the intermittent situation, have been derived and the validity of the equations verified by comparison with experimental data. The complete study has been documented in a separate report. For this reason the study is not repeated here. For the interested reader the complete reference for the report is the following:

Morel-Seytoux, H. J., 1977. "Derivation of Equations for Variable Rainfall Infiltration," HYDROWAR Program Report, Engineering Research Center, Colorado State University, Fort Collins, Colorado, May 1977, CEP76-77HJM47, 30 pages.

A paper based on this report was accepted in January 1978 for publication in Water Resources Research Journal and will appear there with the usual delays (hopefully before December 1978).

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