# RESISTANCE TO SHEET FLOW

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## ACKNOWLEDGEMENTS

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# RESISTANCE TO SHEET FLOW

## ABSTRACT

The results of a literature review on resistance to sheet flow are presented. The effects of surface roughness, rainfall, and vegetation are considered. At least in the case of laminar flow, it is found that the total flow resistance is the sum of the contributions of individual effects. The friction factor for the surface roughness effect in laminar flow is directly proportional to the relative roughness and varies inversely with the Reynolds number. A power function of rainfall intensity in laminar flow can represent the effect of rainfall on the product of friction factor and Reynolds number. For turbulent flow, however, the friction factor depends on the surface conditions which are partitioned into smooth, transition, and fully rough. The analysis of flow through vegetation is more complex and calls for further studies. For densely vegetated surfaces, the Darcy-Weisbach friction factor is shown to decrease signifcantly at Reynolds number well beyond the critical value of $R_e = 2000$ for smooth surfaces. In some cases, the flow behaved as laminar flow at $R_e = 100,000$.

# 1. INTRODUCTION

Overland flow on natural watersheds and urban drainages due to excess rainfall is commonly referred to as thin sheet flow. When the rainfall intensity exceeds the infiltration rate of the surface, sheet flow begins; sheet flow is generally unsteady and non uniform. The discharge increases in the downstream direction during the rainstorm and surface runoff rushes down the slope of watersheds, paved roads, side walks, or parking lots in urban areas. After cessation of rainfall, runoff continues during the time in which base flow sources exist; thereafter the recession phase starts. Sheet flows can be dealt with as wide open channel flows except that if the flow is generated by rainfall, excess resistance will be induced by raindrop impact. Shallow flows are more sensitive to raindrop impact because of the reduced flow depth.

The mechanics of sheet flow is of interest for several practical purposes including evaluation of: (1) surface runoff from natural watersheds; (2) soil erosion from watersheds and farmlands; (3) design discharge for urban drainage systems; (4) hydraulic characteristics of shallow flows in border irrigation system; (5) the modeling of overland flow.

In one flow classification, the ratio of the inertia to viscous forces defines the Reynolds number, $R_e$. When viscous forces dominate the Reynolds number, $R_e$ is small and usually thin flow depth exists. This kind of flow is called laminar sheet flow which classifies most of the cases of thin overland runoff. With large Reynolds numbers, the inertia forces dominate the viscous forces and the flow is turbulent which corresponds to relatively large depths.

The primary parameter in mechanics of sheet flow is resistance to flow which determines other hydraulic variables such as velocity and shear stress. The focus of this paper is confined to the evaluation of the Darcy-Weisbach friction factor for steady laminar and turbulent sheet flows in wide channels under different surface roughness conditions, and with or without rainfall effect. The surface roughness conditions include smooth and rough boundaries in addition to roughness due to vegetation.

## 2. DIMENSIONAL ANALYSIS

The following analysis pertains to the general case of steady sheet flow in a wide channel over a rough boundary through vegetation with rainfall effect. The resistance coefficient, Darcy-Weisbach f, is then a function of all the relevant variables which describe the channel geometry, roughness, rainfall, flow and fluid characteristics. The variables fall into six categories: (1) channel variables such as bed slope $S_o$; (2) roughness parameters such as boundary roughness height k, and roughness concentration C, defined as the ratio of the plan area of roughness elements to the total plane area of the base; (3) rainfall parameters such as rainfall size d, rainfall pattern $\alpha$, raindrop shape coefficient $\lambda$, rainfall intensity i, raindrop velocity entering main flow U; (4) flow parameters such as average flow velocity V, average flow depth Y, head loss gradient $S_f$; (5) fluid parameters such as fluid density $\rho$, specific weight of fluid $\gamma$, and dynamic viscosity $\mu$ ; and (6) vegetation parameters classified into two categories: geometric and physical. Among the geometric characteristics are $S_y$= the average vegetation spacing at depth y, $d_y$ = the average diameter or width of the vegetation elements at y, $G_y$ = the average gap size at y, the pattern dimensionless quantity $\psi$, and the cross-sectional shape dimensionless quantity $\theta$. The physical characteristic of plants, as adopted by Kouwen and Unny (1973), is the flexural rigidity of the plants shown by EI. The deflected height of the vegetation, K, may be regarded as a parameter of the combination of geometric and physical characteristics.

The general form of functional relationship may be shown as follows:

$$\text{Func}\ (V, Y, S_f, S_o, k, C, d, \alpha, \lambda, i, U, S_y, d_y, G_y, K, \psi, \theta, EI, \rho, \gamma, \mu) = 0 \qquad (1)$$

For flows over a rough surface without any effect of rainfall and vegetation, Eq. 1 takes the form:

$$f = \frac{8gYS_f}{V^2} = \text{func}(V, Y, S_o, k, C, \rho, g, \mu\ ) \qquad (2)$$

where f, instead of $S_f$, is the dependent variable. By selecting V,Y, and $\rho$ as the independent variables and applying the $\pi$ theorem for constant C (the maximum value similar to Nikuradse's experiments), one obtains:

$$f = \text{func}\ (S_0,\ k/Y,\ F,\ R_e) \qquad (3)$$

in which F= Froude number and $R_e$ = Reynolds number. The effect of Froude number can be dropped for laminar flow.

For boundary shear stress due to flow over a smooth surface with rainfall effect, Eq. 1 reduces to:

$$\tau = \text{func}\ (V, Y, S_o, d, \alpha, \lambda, U, i, \rho, g, \upsilon) \qquad (4)$$

where $\tau$ is the boundary shear stress equal to $\gamma YS_f$. Yoon (1970) performed a dimensional analysis to present:

$$\frac{f}{8} = \frac{\tau}{\rho V^2} = \text{func}\ \left( \frac{VY}{\upsilon},\ \frac{V}{\sqrt{gY}},\ S_o,\ \frac{id}{\upsilon},\ \alpha,\ \lambda, \frac{iY}{\upsilon},\ \frac{U}{\sqrt{gY}}\ \right) \qquad (5)$$

where $V.Y/\upsilon$ and $V/\sqrt{gY}$ are the conventional Reynolds number and Froude number respectively. Yoon experimentally found that: (1) $iY/\upsilon$ and $U/\sqrt{gY}$ showed a poor correlation with f; (2) the effect of $\alpha$ or rainfall spacing was negligible; (3) $\lambda$ was kept constant and therefore dropped from the analysis; (4) Froude number appeared to be of secondary importance; and (5) $id/\upsilon$ is proportional to i for constant $\upsilon$. Therefore, Eq. 5 becomes:

$$f = \text{func}\ (\ R_e,\ S_o,\ i) \tag{6}$$

By applying the $\pi$ theorem on Eq. 1 for the sheet flow through vegetation with rainfall effect and dropping unimportant terms of rainfall parameters based on the previous discussion, the following form is obtained:

$$\text{func}\ (\ S_f, S_o, \frac{k}{Y}, \frac{id}{\upsilon}, \frac{S_y}{Y}, \frac{d_y}{Y}, \frac{G_y}{Y}, \psi, \frac{K}{Y}, \theta, \frac{EI}{\rho V^2 y^4}, \frac{\gamma Y}{\rho V^2}, \frac{\nu}{VY}\ ) = 0 \tag{7}$$

Chen (1976) used the experimental results of Yoon (1970) and argues that the effect of rainfall would be maximum for flow on the horizontal smooth surface but would decrease with increasing k and $S_o$. He continues that since the roughness of turf surface is very high, the effect of rainfall intensity is believed to be insignificant. Also, the data by Chen (1976), Phelps (1970), and Hartley (1980) show that the flow resistance for flow through vegetation is much higher than that of flow only with rainfall.

After some modifications in Eq. 7 and using the relation $V_{max}.G = V.S$, Hartley (1980) comes up with the following equation:

$$f = \text{func}\ (S_o, \frac{S_y}{Y}, \frac{d_y}{Y}, \frac{G_y}{Y}, \psi, \theta, \frac{K}{[EI/\rho V_*^2]^{\frac{1}{4}}}, \frac{V_{max}.d}{\upsilon}, \frac{V}{\sqrt{gY}}) \qquad (8)$$

in which $V_* = \sqrt{gYS_f}$. The term k/y in Eq. 7 was dropped by assuming flow through vegetation having smooth boundary. However, the effect of roughness, if considerable compared to vegetation resistance, can be added to the vegetation resistance to yield total resistance.

In case of relatively sparse vegetation all of the terms in Eq. 8 should be considered. For grass with maximum density, however, the flow resistance is mainly due to drag on the roughness elements and concentration, shape, and pattern effects could be dropped from the analysis, as in Chen's study. In case of experiments with artificial cylinders, the restrictions and simplifications made by Hartley include: (1) the density of the system doesn't change with depth, so subscripts of the first three terms after $S_o$ may be dropped; (2) the effect of pattern and shape will be represented by a constant in the final equations; and (3) flexibility effects can be dropped for the experiments with rigid cylinders. Also for rigid system, K = Y. Therefore:

$$f = \text{func}\ ( S_o, \frac{S}{Y}, \frac{D}{Y}, \frac{G}{Y}, \frac{V_{max}.G}{\upsilon}, \frac{V}{\sqrt{gY}} ) \qquad (9)$$

In case of laminar sheet flow, usually with very shallow depth, the deflected height and flexural rigidity of the vegetation are not important and Eq. 9 still applies. The Froude number contribution in laminar flow resistance equations has not been included so far. The experiments such as Chen's have been conducted with the attempt to eliminate surface instabilities. However, Hartley reported only small free surface effect even in turbulent flow. Hence, Eq. 9 takes the form of:

$$f = \text{func}\,(S_o,\ S/Y,\ D/Y,\ G/Y,\ V_{max}.G/\upsilon) \tag{10}$$

in which $R_e = V_{max}.G/\upsilon = V.S/\upsilon$ is the Reynolds number based on vegetation spacing.

3. GOVERNING EQUATIONS

One of the most common resistance factors is the Darcy-Weisbach friction factor, f. The Darcy-Weisbach formula was first developed for flow in pipes in the following form :

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{11}$$

where $h_f$= friction loss along length L of the pipe, given the pipe diameter, D, and the mean flow velocity, V. For open channel flow, $h_f/L$ and D are substituted by $S_f$ and 4Y respectively :

$$f = \frac{8gYS_f}{V^2} \tag{12}$$

where $S_f$ = friction gradient, V = velocity, and Y = flow depth equal to hydraulic radius in a wide channel. Eq. 12 may be applied to steady uniform flow in wide channels by substituting $S_o$ for $S_f$. Other friction factors, such as Manning n and Chezy C, are mostly used for turbulent flow. The relationship between f, n, and C in English units is as follows :

$$C = \frac{1.486Y^{1/6}}{n} = \left(\frac{8g}{f}\right)^{1/2} \tag{13}$$

The sheet flow with rainfall as lateral inflow is considered to be a shallow spatially varied flow which with constant rainfall intensity and constant base flow would be steady. The derivation of governing equations for steady spatially varied flow with rainfall has been studied by many investigators; among them, Chow (1959), Woo and Brater

(1962), and Yen and Wenzel (1970). Probably Yen and Wenzel (1970) derived the most comprehensive dynamic equation for this case by both momentum and energy approaches.

The continuity equation for the flow with rainfall in a wide channel can be written as :

$$q = q_0 + ix \tag{14}$$

where q, and $q_0$ = total and base flow rates per unit width of the channel at x = 0 . Under the following basic assumptions: (1) one dimensional steady flow; (2) hydrostatic pressure distribution; (3) constant channel slope; (4) constant momentum correction factor along the channel; (5) negligible air entrainment effect; and (6) impervious boundary, Yen and Wenzel (1970) using momentum approach came up with the equation of water surface profile for steady spatially varied flow as follows :

$$\frac{dY}{dx}\left(\text{Cos}\ \theta - \frac{\beta V^2}{gD}\right) = S_o - S_f + \frac{i}{gA}\left(U\ \text{Cos}\phi - 2\beta V\right) \tag{15}$$

where x=distance in the flow direction, D= A/T= hydraulic depth at x, A= cross section area at x, T= top width at the free surface, $\theta$ = angle between x direction and horizontal direction, $\beta$ = the momentum correction factor, $S_f$ = friction slope defined as $\tau/\gamma R$, R = hydraulic radius, $\phi$ = angle between velocity U and x direction, and other variables have been already defined. For a wide channel, D and R are simply replaced by flow depth, Y.

## 4. SURFACE ROUGHNESS EFFECT

### 4.1. Laminar Flow

The study of laminar sheet flow over bare surface is the most simplified situation of interest in order to identify the variation of flow resistance coefficient due to surface roughness and Reynolds number. The following general formulation has been adopted by early investigators, such as Izzard (1944), and Woo and Brater (1961):

$$f = \frac{K}{R_e} \tag{16}$$

K value varies with the flow regime, surface roughness, rainfall effect, vegetation and probably slope. Theoretically speaking, K is equal to 24 for laminar flow over a smooth wide channel. This can be found by either applying Boussinesq equation, primarily developed for rectangular pipes having a width b and depth of 2Y, to a wide open channel with infinite width and depth of Y, or imposing equilibrium between the component of weight in the direction of flow and the shear resistance of the channel bottom. Horton, Leach, and Van Vliet (1934) experimentally confirmed the K value being 24 for laminar flow in a rectangular channel with a smooth surface, covered by white pine. Allen (1934) found the upper limit of $R_e$ for true laminar flow regime being about 300 for smooth surfaces. The University of Illinois' data given by Landsford and Robertson (1958) and Chow (1959) determined the same K value as 24 for laminar flow when $R_e<500$.

Woo and Brater (1961) tried to determine friction factor for different boundary surfaces. They partitioned the surfaces into smooth, rough, and very rough. Woo and Brater evaluated the width effect for the flow in rectangular channels, estimating an error of less than 5 percent in K when the width-depth ratio was 25. Woo and Brater's data for flow over masonite surface representing a typical rough surface showed a value of 30.8 for K. The U.S. Waterways Experiment Station (1935) had already reported K being 31.6 for laminar flow over cement surface. The upper limit of $R_e$ for laminar flow varied from 400 for a slope of 0.060 to 900 for a slope of 0.001.

Glued-sand with an average diameter of 1 mm on the masonite surface used by Woo and Brater (1961) as a very rough surface on which flow experiments were conducted. It was found that K increased with the slope (except for slopes less than 0.003), having a value of 39.2 for $S_o = 0.001$ up to 100 for $S_o = 0.060$, Fig. 1. The upper limit of laminar flow range was confined between 400 to 800, varying inversely with the slope. Generally, the data in the laminar range seems inadequate to warrant the results.

If the f variation with slope is computed based on Woo and Brater's (1961) data, it will be found that for sand surface (k=1 mm) when $S_o > 0.003$:

$$f = \frac{155.85 + 46 \log S_o}{R_e} \tag{17}$$

The application of the above equation is limited to slopes less than 0.020 after which the number of data points for each slope is lacking.

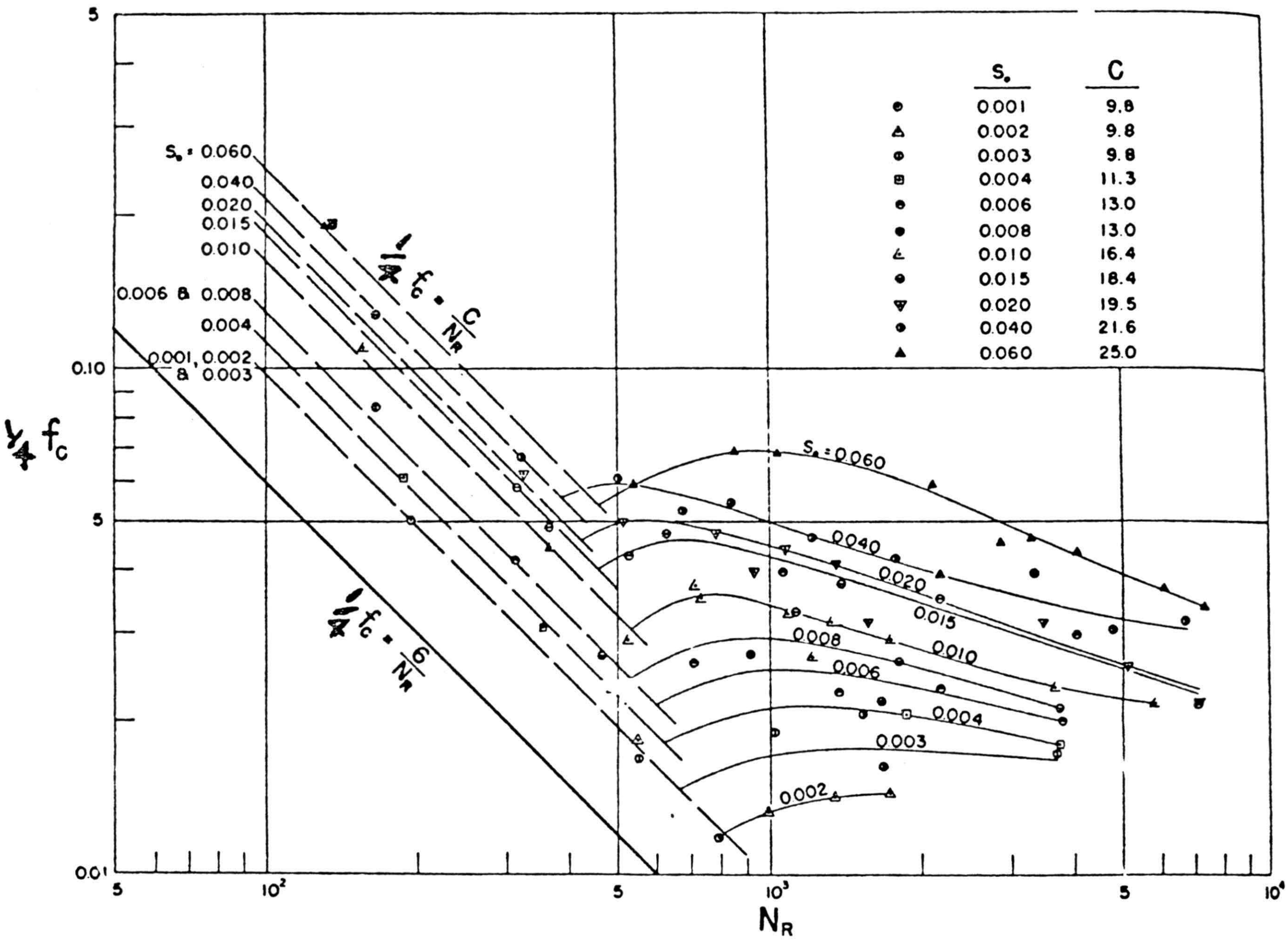


Fig. 1. The f-$R_e$ relationship for sand surface, after Woo and Brater (1961).

Through a different approach, Kruse et al (1965) attempted to define the friction factor for flow over rough surface in terms of roughness characteristics and channel slope. They came up with the following formula :

$$f = \frac{6000(\sigma/\lambda)S_0^{0.5}}{R_e} \tag{18}$$

where $\sigma$ = soil roughness height, and $\lambda$ = soil roughness spacing. The formula shows the correlation of friction factor with the ratio of roughness height to spacing and apparently the bed slope.

The idea of correlation of f with the relative roughness was investigated by Phelps (1975). Phelps tested the flow over spherical roughness elements with diameter of 1.17 mm (.046 in) and grain concentration of 0.1 in the slope range being 0.00048-0.0451. The data confirmed the variation of f with relative roughness not slope.

Having Phelps' data in Fig.2, the following power equation may be developed to confirm Eq.16 for constant k/Y: $f = aR_e^b$. Table 1 can be filled by using Fig.2 as the reference.

TABLE 1 - Values of a and b Based on Phelps' Data

| Relative Roughness (1) | # of Data (2) | a (3) | b (4) | K (5) |
|---|---|---|---|---|
| .23 | 4 | 35.889 | -1.00195 | 35.498 |
| .27-.28 | 5 | 43.584 | -1.02503 | 38.161 |
| .35 | 7 | 42.392 | -1.00191 | 42.040 |
| .52-.55 | 7 | 31.179 | -0.88777 | 50.61 |

As it is seen, the exponent b is very close to -1.0 except for the last series when k/Y=.52 - .55. As a result, the resistance equation may be written in this form: $f=K/R_e$, where K = func(k/y). If a regression is to be performed, the result for K will yield:

$$K = 24 + 72.1 \left(\frac{k}{Y}\right)^{1.31}, \quad \frac{k}{Y} < .50 \tag{19}$$

The application of resistance equation in the form of $f=K/R_e$ would be probably limited to k/Y values less than .50, according to Phelps' data. The result of the power model for k/y=.52 - .55 is not satisfactory to verify the equation for that specific k/Y. It is possible that free surface instability effect for high k/Y cause the discrepancies such that the correlation of f with $R_e$ decreases indicating the change in flow regime from laminar to transition and turbulent.

Phelps (1975) reported that Woo and Brater's (1961) data also validated Eq.16 as they were grouped based on relative roughness. Assuming so, K values deduced from Woo and Brater's data are higher than those of Phelps' as much as two times for a constant k/Y. One may reason that the roughness concentration used by Woo and Brater was the maximum possible similar to Nikurase's work, where Phelps' selected a concentration equal to 0.1 in his experiments.

Now, as it is clear, two different independent variables have been used for the evaluation of flow resistance, i.e. slope and relative roughness. Although Kruse et al. (1965) presented an equation in which slope was the independent variable besides the roughness size, they

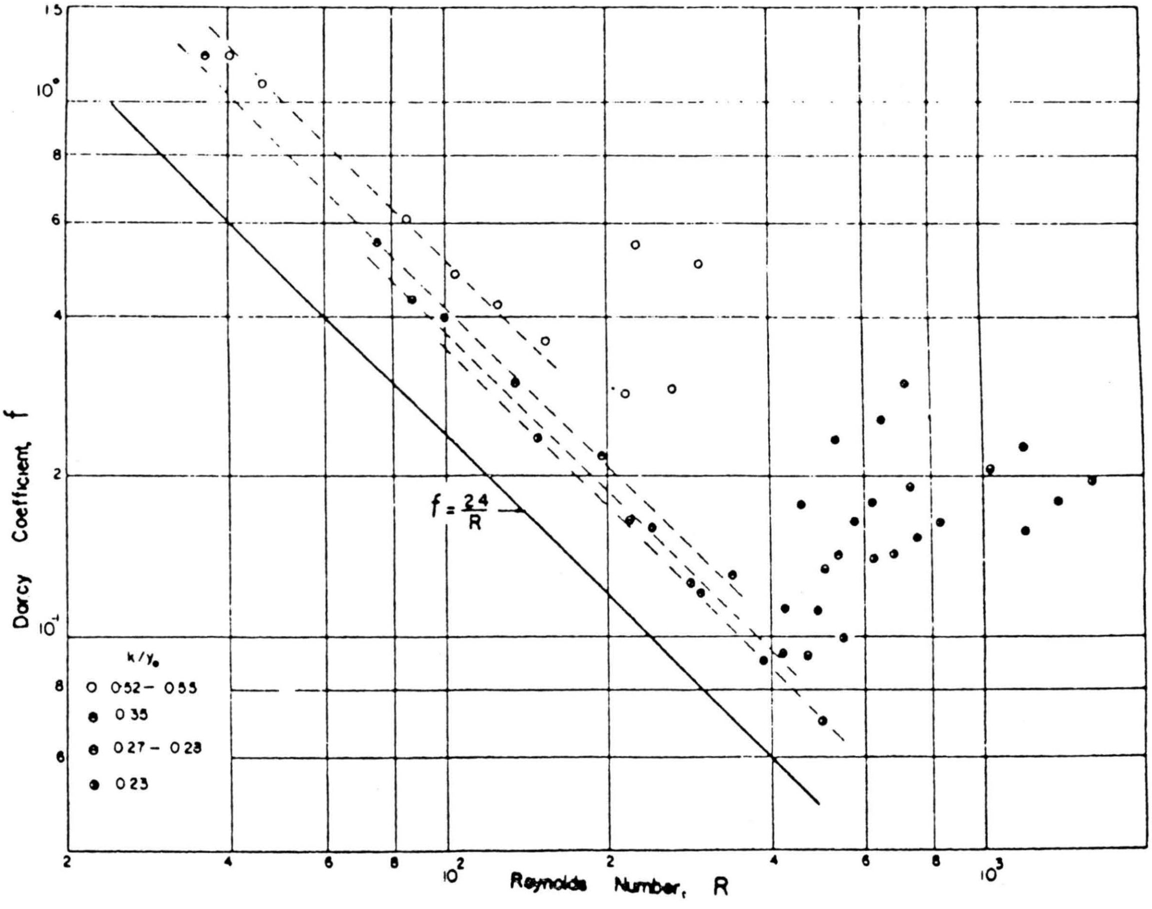


Fig. 2. The f-$R_e$ relationship for rough surfaces, after Phelps (1975).

k/Y = Relative roughness

speculated that the apparent correlation between resistance and slope could be due to relative roughness and local turbulence at the tips of the roughness elements. When slope increased while discharge and hence Reynolds number were kept constant, depth would then decrease and more resistance would be induced due to larger portion of the flow being into contact with the roughness at a higher velocity. Therefore, the basic cause of resistance variation can be relative roughness rather than slope, which in turn is responsible for changes in relative roughness. In addition, working with slope as the primary variable requires a series of experiments for each roughness size whereas the k/Y ratio reflects both roughness size and depth which varies with bed slope in the case of constant discharge. Phelps' work successfully demonstrates the effectiveness of k/Y being independent variable and the validity of equation $f = K/R_e$.

Yet, some considerations must be taken into account when working with relative roughness. First of all, the roughness concentration has to be held constant for each diagram of f vs $R_e$ and k/Y. Second, the k value, the height of the roughness, needs an accurate measurement. Third, for high k/Y, free surface instabilities may bring about additional energy dissipation whose effect on f in laminar flow region has not been quantitatively determined.

## 4.2. Turbulent Flow

The flow over a bare surface becomes turbulent when $R_e > 2000$. There are three types of turbulent flow depending on size of the boundary roughness compared to laminar sublayer thickness. Smooth conditions occur when the boundaries are hydraulically smooth such that the roughness elements are well covered under the laminar sublayer. On the contrary, turbulent flow over fully rough surface exists when the projections break through the laminar sublayer and dominate the flow behavior. Finally, transition region of turbulent flow is the region between smooth and fully rough conditions. It is noticeable that change from smooth to fully rough flow corresponds to increase in $R_e$ and therefore in discharge, which shrinks the laminar sublayer thickness. The limits of these three kinds of turbulent flows are as follows :

1. Smooth condition : $\delta > 3k$ or $V_*k/\upsilon < 4$
2. Transition : $k/5 < \delta < 3k$ or $4 < V_*k/\upsilon < 70$
3. Fully rough : $\delta < k/5$ or $V_*k/\upsilon > 70$

where k = the median size of the boundary particles and $\delta$ = the laminar sublayer thickness equal to $11.6\upsilon/V_*$.

The resistance equations were primarily developed for flow in pipes. The f-$R_e$ relationship for smooth pipes was derived by Blasius as the following :

$$f = \frac{0.223}{R_e^{0.25}} \tag{20}$$

in which hydraulic radius is used as the characteristic length in definition of $R_e$. The Blasius equation may be applied for turbulent flows over smooth boundary when $R_e < 25000$. Beyond that limit, the

Prandtl-von Karman equation based on logarithmic velocity profile is believed to hold :

$$\frac{1}{\sqrt{f}} = 2 \log (R_e \sqrt{f}) + 0.4 \tag{21}$$

The use of Eq. 20 and Eq. 21 for open channel flow has been investigated based on the data developed at the Univ. of Illinois given by Lansford and Robinson (1958) and also data of Univ. of Minnesota given by Straub et al. (1958). Fig.3 indicates that the equations for turbulent flows in smooth pipes may be representative of all smooth channels. In addition, the cross section shape of the channel in turbulent flow has little effect on friction factor whereas it is important in laminar flow. This means that for sheet flow assumed in a wide channel, Eq.20 and Eq.21 can approximate the friction factor when the boundary is smooth such as that of urban drainage systems.

Another alternative is to integrate the turbulent velocity profile over smooth boundary and then calculate the friction factor from average velocity. The final formula would be :

$$\frac{1}{\sqrt{f}} = a \log(R_e \frac{\sqrt{f}}{b}) \tag{22}$$

Basically, a is related to the von Karman's universal constant as 0.4, and b depends on the value of a as well as shape of the cross section of the channel. Keulegan's (1938) formula, which probably is the closest in result to Prandtl-von Karman equation, for a very wide, smooth

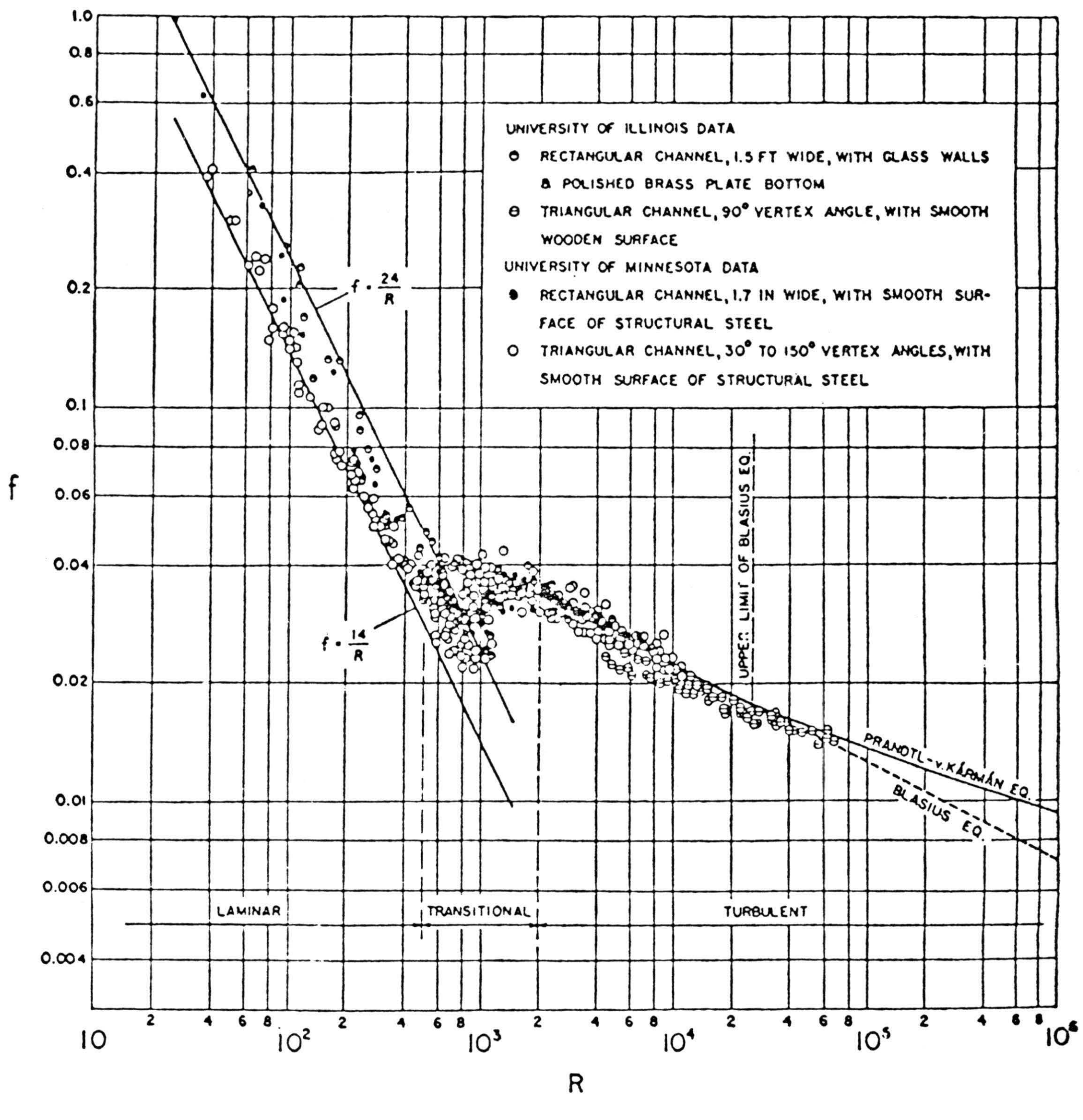


Fig. 3. The f-$R_e$ relationship for flow in smooth channels, after Chow (1959).

channel reduces to a=2.03, and b=0.853. In overland areas, however, the surface is mostly rough with fairly large relative roughness.

The flow resistance of turbulent flow in fully rough condition is entirely due to the ratio of hydraulic radius over the roughness size, R/k, and can be expressed as follows :

$$\frac{1}{\sqrt{f}} = \frac{C}{\sqrt{8g}} = a \log(b' \frac{R}{k}) \tag{23}$$

where R=hydraulic radius, and $b'$ is a constant to be determined by experiments. The value of $b'$ depends not only on the shape of the channel cross section but also on the spacing (roughness concentration) and form of the roughness elements. As a result, different investigators present different values based on the data they use. Keulegan (1938) found that a=2.03 and $b'$=11.09 for a very wide channel with sand-grain roughness in the fully rough regime. For a trapezoidal channel, however, Keulegan's formula gives similar a but $b'$=12.27. At the meeting of IAHR, Thijsee (1949) proposed a similar equation which after modifications results in a=2.03 and $b'$=12.2 for a very wide channel. In case of flow over commercial surfaces, such as concrete and wood, the k values have been presented by Ackers (1959).

If the variation of Chezy coefficient C, instead of Darcy f, is to be plotted versus $R_e$ using Eq.20 and Eq.21 for smooth condition and Eq.23 for fully rough condition, a modified Moody diagram for open channel flow will show up. Fig.4, taken from Henderson's (1966) book, indicates that in case of turbulent flow over fully rough surfaces, C only depends on R/k ratio and independent of $R_e$ effect. The R/k ratio

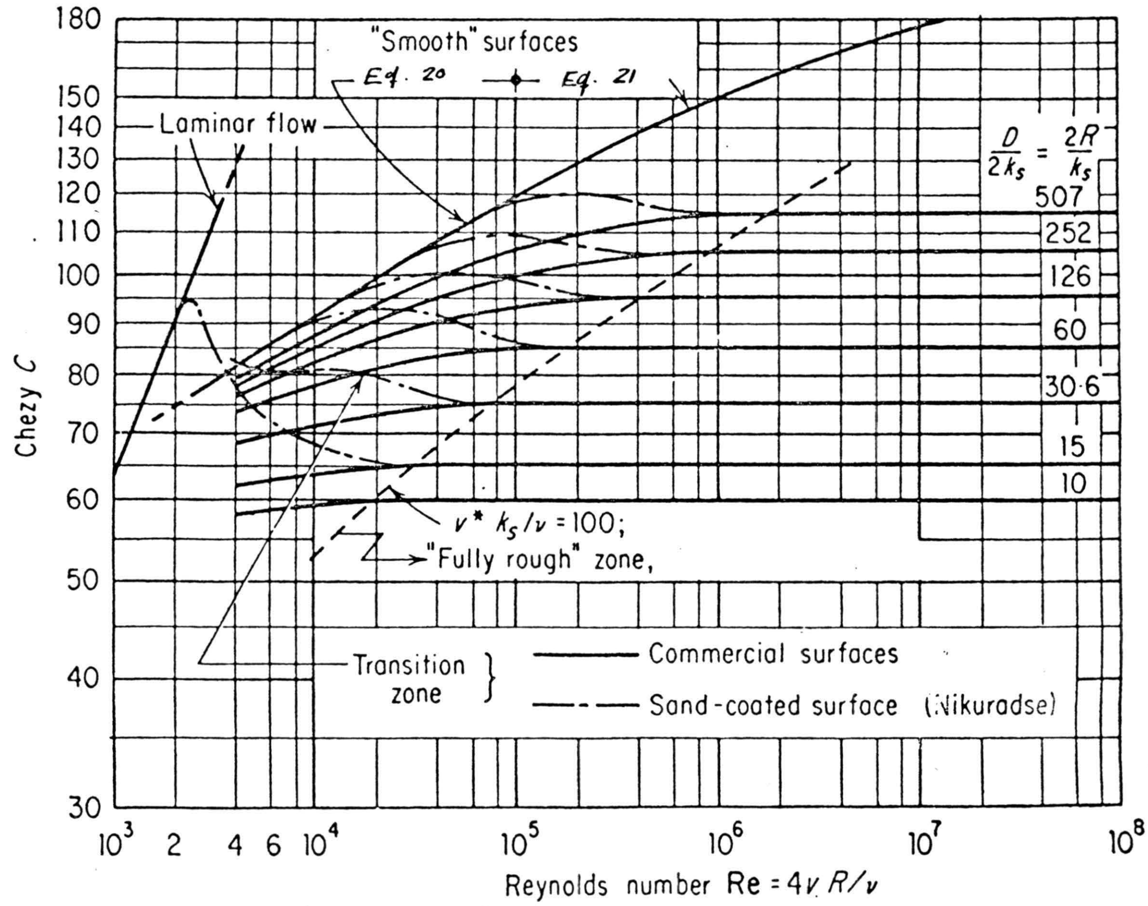


Fig. 4. Modified Moody diagram showing C-$R_e$ relationship, after Henderson (1966).

covers from 5 to 235.5, probably based on range of available data. Although turbulent flow in fully rough condition usually occurs in relatively high R/k ratios, in overland regions with steep slope one may expect turbulent sheet flow with high relative roughness, or low ratios of R/k. In that case, the applicability of Eq. 23 needs more investigations in order to complete Fig. 4 for smaller R/k ratios.

A report by ASCE (1963) supports the use of Colebrook equation with slightly modified coefficients for flow in transition region to open channels. The equation is :

$$\frac{1}{\sqrt{f}} = \frac{C}{\sqrt{8g}} = -2\log\left(\frac{k}{12R} + \frac{0.625}{R_e\sqrt{f}}\right) \tag{24}$$

However, the above equation is applicable to commercial surfaces. Therefore, for natural rough surfaces with k being the median particle size, Eq.24 has to be tested. In Fig.4 , the difference between the curves for pipe flow and open channel flow in transition region is shown.

Manning equation, as a flow resistance equation, is the most well known power relationship which has been developed for open channel turbulent flow over rough surfaces. For R/k ratios ranging from 10 to 10000, the Manning-Strickler relationship approximately gives equivalent resistance coefficients as the logarithmic equation by Keulegan:

$$n = \frac{1.486R^{1/6}}{\sqrt{8g/f}} = 0.0342\ k^{1/6} \tag{25}$$

where k=median size of the roughness particles in feet. It should be noticed that Manning equation is suitable for all fully rough flows in which Manning's n is constant for a given particle size. For transition flows, however, f is the better resistance coefficient given by Eq. 24.

boundary shear stress, $\tau$, assuming $\beta = 1$. He found that the measured boundary shear stress, even with the difficulties in measuring flow depths with rainfall effect, was in excellent agreement with boundary shear stress computed using Eq.26. Therefore, the application of one dimensional dynamic equation of spatially varied flow appeared to be accurate enough for determination of water surface profile, provided a reasonable resistance law; i.e. an equation for f. It was also found that $S_o$ overcame the other terms in magnitude while evaluating $S_f$. Each of $S_1$ and $S_2$ contributed nearly one tenth of $S_o$ whereas $S_3$ was negligible in magnitude.

## 5.1. Laminar Flow

Izzard (1944) first studied the resistance to laminar sheet flow with rainfall effect. He considered that the K value in general formula, Eq.16, could be the sum of a constant and a function of rainfall intensity. Therefore the following function was developed and then used by many other investigators:

$$f = \frac{K}{R_e} = \frac{K_o + \phi(i)}{R_e} \tag{28}$$

where $K_o$ is a function of surface roughness. Izzard used a paved rough surface in his experiments. As a result, he determined $K_o$ being 27 for rough surface. The power function of rainfall intensity turned out to be 5.67 $i^{1.33}$, where i(in/h). In addition, Izzard observed increase in f with increasing bottom slope. However, no slope parameter was included in friction factor equation.

Li (1972) conducted his tests to determine the independent variables of friction factor for laminar flow over smooth surface with rainfall through a dimensional analysis. He assumed the following power equation:

$$f = \beta_o R_e^{\beta 1} i^{\beta 2} S_o^{\beta 3} \epsilon \tag{29}$$

where $\beta_o, \beta_1, \beta_2, \beta_3$ are constants and $\epsilon$ is the error in the regression equation. The data covered a range of $R_e$ from 126 to 900 for laminar regime, 0 to 17.5 in/h for rainfall intensity, and slopes being .0108 and .0064. The result of multiple regression showed that :

$$f = 13.517\ R_e^{-.958}\ i^{.413}\ S_0^{-.088}\ \epsilon \tag{30}$$

According to statistical tests made by Li (1972), bottom slope had an insignificant effect on the product of $f.R_e$. Furthermore, the exponent of $R_e$ was approximated to -1.

Before Li (1972), Yoon (1970) had carried out several tests to identify the independent variables affecting friction factor. Yoon (1970) found that the effect of raindrop spacing and raindrop impact velocity were almost negligible on friction factor under his test conditions. However, friction factor increased with increasing rainfall intensity and relatively bottom slope.

Li (1972) performed a regression analysis using his data and Yoon's data to derive the following power function for $\phi(i)$:

$$\phi(i) = 27.162\ i^{.407}\ , \qquad \text{for } R_e < 900 \tag{31}$$

i is in in/h. The agreement of the above equation with Yoon's data is shown in Fig.5 and with Li's data in Fig.6.

Fawkes (1972) approximated the flow with rainfall as a steady flow with a very flat water surface profile. As a result, $S_f$ would be almost equal to $S_0$. Fawkes then presented $\phi(i) = 9.982i$.

Other data based on experiments on sheet flow over smooth and rough surfaces with rainfall given by Kisisel et al.(1973) indicated no significant change in f due to slope. The data seemed to obey the same general formulation for f, though no attempt was made to deduce a certain equation for f.

In order to define friction factor experimentally for sheet flow with rainfall, most of the investigators used the kinematic wave approximation as suggested by Woolhiser (1969). The approximation assumes that all the terms in the momentum equation are negligible except $S_o$ and $S_f$, resulting in $S_f = S_o$. Then, depth and velocity in Eq.12 are measured for a cross section and the variation of f due to rainfall versus $R_e$ will be defined. Izzard (1944), Kisisel et al.(1973), and Fawkes (1972) used the kinematic wave approximation to determine the f variation.

According to Yoon's study on Eq.26, the kinematic wave approximation may involve up to 20 percent error in $S_f$ determination. Yoon (1970), and then Li (1972), directly measured the boundary shear stress by hot film sensors, in order to avoid any approximation in their analysis. Having shear stress and flow velocity, they computed friction factor, $f = 8\tau/\rho V^2$, for specific rainfall intensity and Reynolds number.

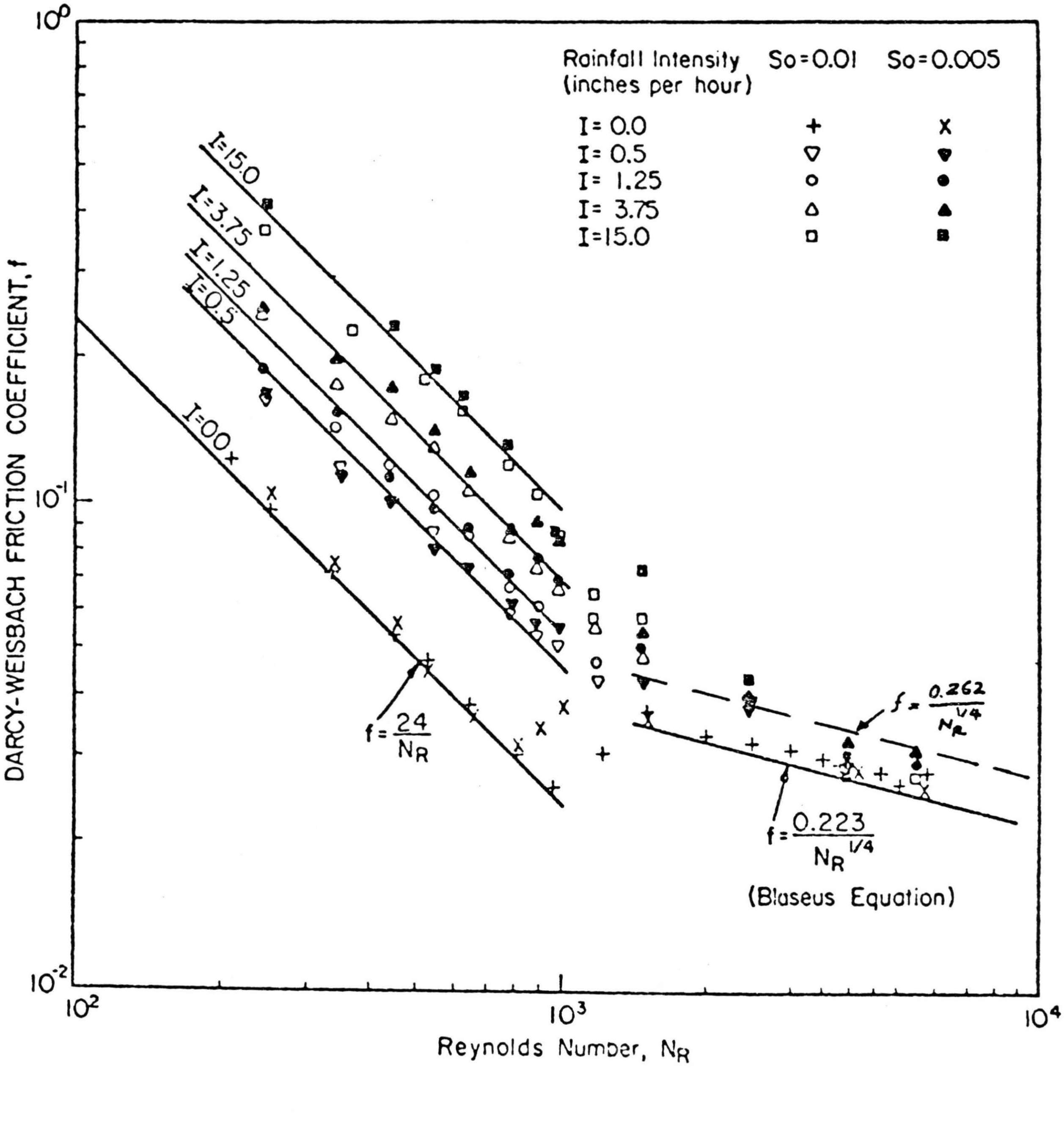


Fig. 5. The f-$R_e$ relationship for flow with rainfall, after Yoon (1970).

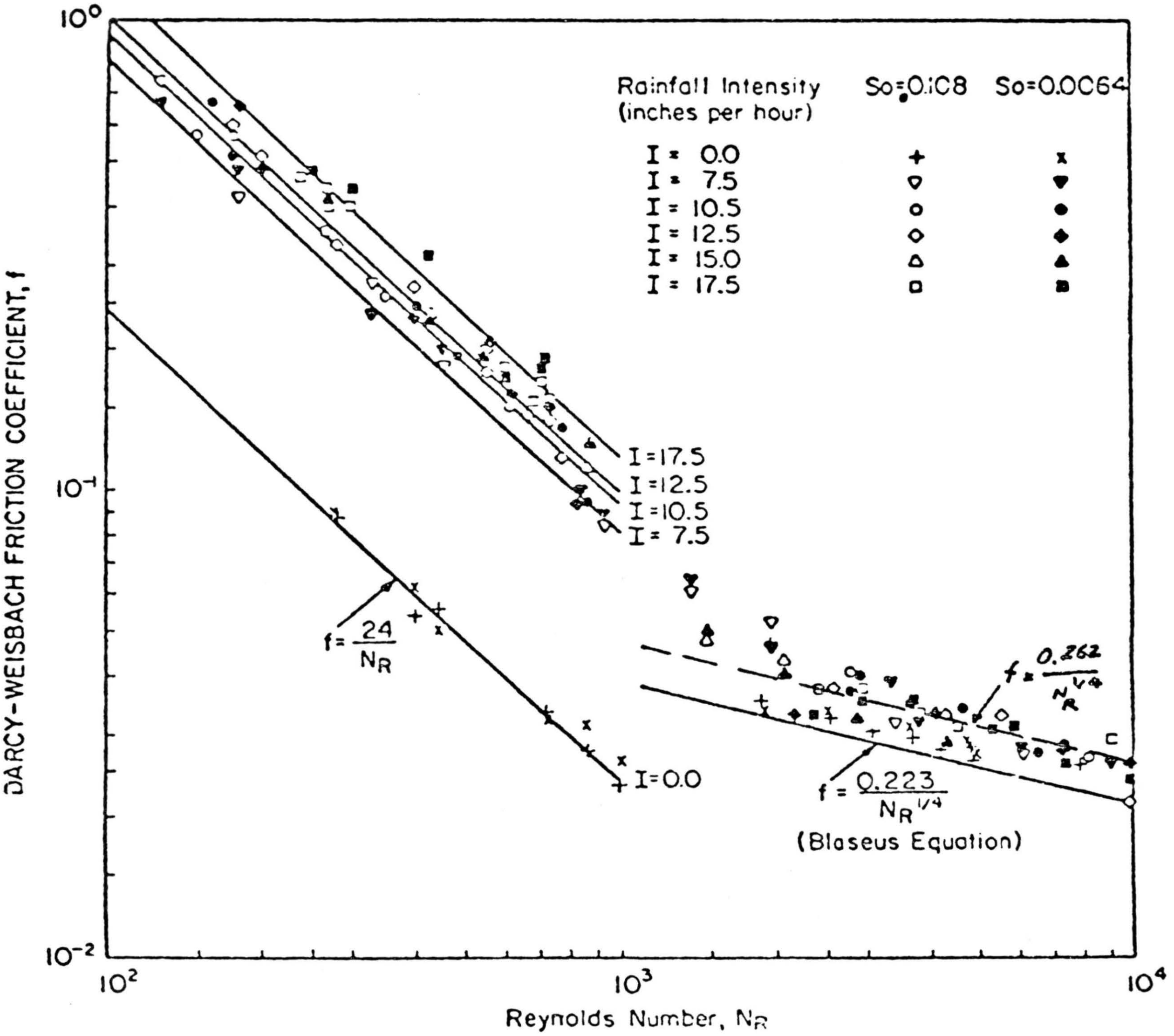


Fig. 6. The f-$R_e$ relationship for flow with rainfall, after Li (1972).

Consequently, Eq. 28 substituted by $\phi(i)$ from Eq. 31 is the most accurate equation for solving dynamic equation of spatially varied flow.

As already discussed, K in Eq.16 may be a function of slope, $S_o$, or relative roughness, k/Y. Using a function of $S_o$ would bring about an approximation by assuming steady uniform flow, which is obviously not true when rainfall exists. On the other hand, K being a function of k/Y, as used by Phelps (1975) specifically for steady uniform flow over rough boundary, reflects the effect of non-uniformity of the flow with rainfall effect. As spatially varied flow moves on, the depth changes and the boundary resistance has to change accordingly to yield the relative roughness effect. Therefore, both friction factors due to boundary roughness and rainfall will be functions of distance, simply because depth and Reynolds number are not constant for sheet flow with rainfall :

$$f = \frac{\text{func}(k/Y) + 27.162 i^{.407}}{(q_0 + ix)/\nu} \tag{32}$$

5.2 Turbulent Flow

Similar to the discussion for laminar flow with rainfall, the data provided by Yoon (1970) and Li (1972) are the most applicable and accurate compared to the other's data. Li first assumed that Blasius equation could be modified to accommodate the rainfall effect for turbulent flow over a smooth boundary :

$$f = \frac{\phi'(i)}{R_e^{0.25}} \tag{33}$$

which is valid for $R_e > 2000$ where the turbulent flow begins. The regression analysis between Yoon's and Li's data showed that for available data $\phi'$ was not a function of rainfall intensity but rather a constant. The results indicate that :

$$\phi' = 0.262 \quad \text{for} \quad 0.5 < i < 17.5 \text{ in/h}$$
$$\phi' = 0.25 \quad \text{for} \quad i = 0 \tag{34}$$

The above results mean that the flow resistance begins to increase with rainfall intensity somewhat below 0.5 in/h. Once the flow resistance is increased, any further increase of rainfall intensity doesn't change the flow resistance at least for i < 17.5 in/h. Since the major cause of increase in flow resistance due to rainfall is the creation of turbulence by rainfall impact, one should expect a little change in flow resistance when the flow is already turbulent.

As seen in Figs. 5 and 6, the f values decrease from that for the laminar range ending at $R_e = 900$ to its value for the turbulent range starting at $R_e = 2000$. Li (1972) approximated the relation between ln f and ln $R_e$ in transition range with a line and gave the following equation:

$$f = 0.0392\ (R_e/2000)^a \tag{35}$$

in which $a = -1.252 \ln(0.68 + 0.77i^{0.407})$. The equation applies only for flow in the transition range, $900 < R_e < 2000$, over a smooth boundary.

# 6. VEGETATION EFFECT

Evaluation of vegetation resistance in sheet flow involves the most complicated experiments particularly for natural vegetation. So many interrelated variables contribute in flow resistance through vegetation that no test is able to separate the effect of each variable. The problem becomes more complex when the combined effects of vegetation, bottom roughness, and rainfall are present and yet no confirmed method of separation among those effects has been developed. Nevertheless, at least in case of laminar flow, it is believed that total resistance can be represented by the linear superposition of vegetation drag, bottom roughness, and rainfall effect. The last one is minor compared to vegetation drag and the natural bottom roughness of natural vegetated areas. The bottom effects due to roughness has been already discussed.

Although no unique equation in a general form has been derived to calculate the vegetation resistance, the following literature review and discussions will clarify, to some extent, the results of past studies.

## 6.1. Rigid Sparse Vegetation

The relationship between resistance to flow and hydraulic parameters of sheet flow through rigid sparse vegetation can be derived by applying momentum equation to a finite increment $\Delta x$ along flow direction. For a steady flow in a wide channel one obtains :

$$F_g = F_b + F_D \tag{36}$$

where $F_g$ = fluid weight component in flow direction per unit width approximately equal to $\gamma YS_0$ in case of sparse vegetation, $F_b$ = boundary shear force per unit width, and $F_D$ = total vegetation drag per unit width. The boundary shear force is equal to $\gamma YS_f$ or $\rho f_b V^2/8$, in which $S_f$ = the friction slope due to boundary resistance, and drag force is equal to $^{Y}$ $0.5 C_D V_e^2 dA_e$ in which $C_D$ = local drag coefficient, and $dA_e$ = local area of vegetation projected normal to flow direction. If the vegetation system is composed of rigid uniform cylinders and local velocity can be approximated by mean velocity of the flow, then Eq.36 becomes :

$$\gamma YS_0 = f_b \rho V^2/8 + 0.5\ NC_D dY\ V^2 \ , \text{ for } h > Y \tag{37}$$

where N = the number of cylinders per unit area of bed, d = cylinder diameter, and h = cylinder height. When h < Y, then h should be substituted for Y in last term. In a more simplified form :

$$f_t = f_b + f_v = \frac{8gYS_0}{V^2} \tag{38}$$

where $f_v$ = friction factor due to vegetation equal to $4NC_D dY$. Hence, the contribution of vegetation effect, $f_v$, to total friction factor is dependent on flow depth as the hydraulic parameter, vegetation characteristics including number of single stems per unit area in a sparse pattern, stem diameter, and drag coefficient.

Li and Shen (1973) studied the drag coefficient for idealized vegetation, represented by rigid cylinders. As Fig.7 shows, the variation of mean drag coefficient in turbulent flow for second row cylinders in a staggered pattern is relatively small down to at least longitudinal spacing to diameter ratio of 5 at which $C_D$ is only 8% higher than that of a single cylinder or that of first row cylinders. In case of a parallel pattern, however, $C_D$ keeps continuously decreasing as the spacing is reduced for a given d, such that $C_D$ equals only 60% of $C_D$ for a single cylinder. Of course when the transverse spacing is changed, these ratios may change. Now, as long as $C_D$ remains unchanged with the spacing, roughly down to 10d in staggered pattern and 50d in parallel pattern, the vegetation is considered sparse and $C_D$ would be only function of element shape and Reynolds number, as has been classified by Hoerner (1965). Li and Shen recommend an average $C_D$ being 1.2 for sparse cylinders. This value also has been reported in standard texts such as Schlichting (1968) for drag coefficient of a single cylinder in an idealized two-dimensional flow in cylinder Reynolds number, $R_d = Vd/\upsilon$, ranging from about $8*10^3$ to $2*10^5$.

## 6.2. Dense Rigid Vegetation

Neglecting the free surface and flexibility effects, Kirsch and Fuchs (1967) studied the drag coefficient for pressure flow through parallel and staggered arrangements of dense rigid cylinders. They introduced a dimensionless coefficient of drag enhancement, $F^*$, which relates to $C_D$ as the average drag coefficient for each cylinder in an array such that :

$$C_D = \frac{2F^*}{R_d} \tag{39}$$

where $R_d$ = cylinder Reynolds number equal to $Vd/\upsilon$. If $S_1$ and $S_2$ represent the center to center spacing in the cross stream direction and in streamwise direction, $F^*$ can be empirically evaluated as :

$$F^* = 4\pi\left[ -\ln\left( \frac{d}{2S_1} - 1.33\right) + \frac{\pi^2}{3}\left( \frac{d}{2S_1} \right)^2 \right], \quad \text{for } d/S_1 < 0.7$$

$$F^* = \frac{9\pi}{2\sqrt{2}} \left( 1 - \frac{d}{S_1} \right)^{-2.5}, \quad \text{for } d/S_1 > 0.7 \tag{40}$$

Both above equations hold when $S_2 > S_1$. For $S_2 < S_1$, $F^*$ ratio decreased below unity with decrease in spacing between rows in a parallel arrangement. On the contrary, opposite relation was verified for staggered pattern in the case $S_2 < S_1$, depending on $d/S_1$ and $S_1/S_2$. Kirsh and Fuchs also found that for nonuniform pattern of cylinders and for rotating rows of cylinders relative to one another, $F^*$ showed less value than those of parallel and staggered patterns of equal density.

Chilton and Genereaux (1933) experimented pressure drop for the pressurized flow through staggered arrangement of cylinders presenting :

$$\frac{\Delta P}{L} = \frac{53 V_{max}\, \mu}{{d_e}^2} \quad \text{for laminar flow} \tag{41}$$

$$\frac{\Delta P}{L} = \frac{1.5\rho^{.8} V_{max} \mu^{.2} \sqrt{N}}{G^{.2}} \quad \text{for turbulent flow} \tag{42}$$

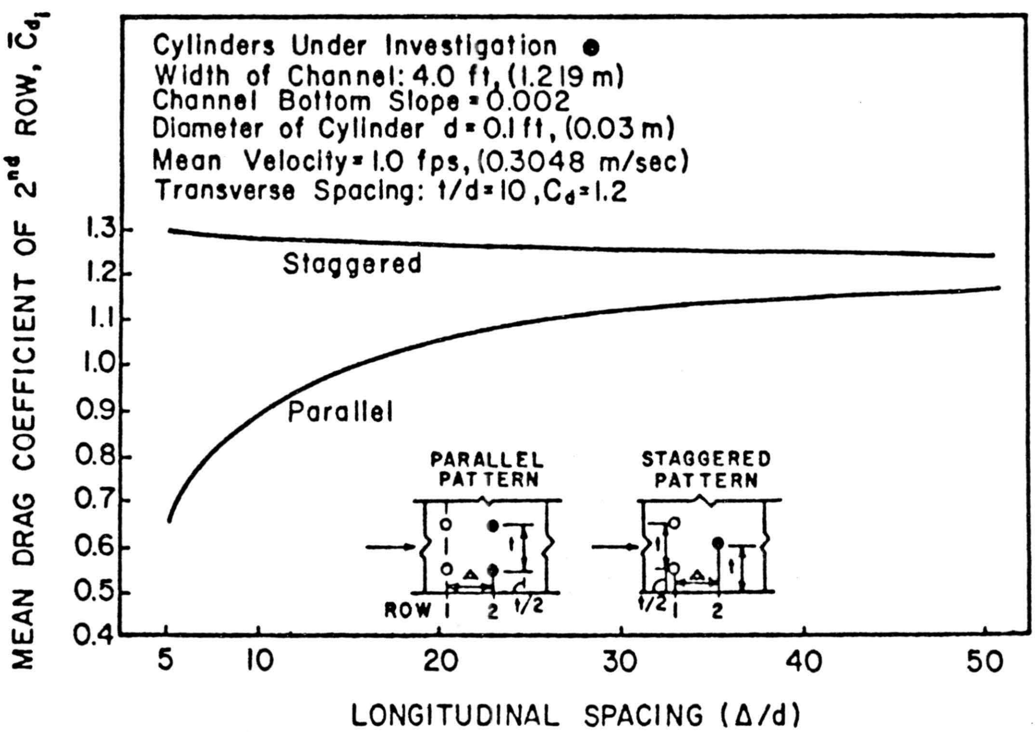


Fig. 7. The mean drag coefficient variation for staggered and parallel patterns, after Li and Shen (1973).

in which $\Delta P$ = pressure drop over length L, $V_{max}$ = maximum velocity through the gap or narrowest space between two adjacent cylinder elements, $d_e$ = equivalent diameter equal to $(4/\pi dN - d)$, d = cylinder diameter, N = number of elements per unit area of the bed, G = gap size.

Eq. 41 may be changed for the use in open channel with the aid of similarity between friction factor in open channel and pressure drop in pipes :

$$S_f = \frac{fV^2}{8gY} = \frac{53V_{max}\mu}{\gamma d_e^{\,2}} = \frac{\Delta P}{\gamma L} \tag{43}$$

or by substituting $V = GV_{max}/S$ :

$$f = 424 \, \frac{Y \; S}{d_e G} \, (R_e)_e^{-1} \tag{44}$$

where $(R_e)_e = V_{max} d_e / \upsilon$. This equation has not been verified experimentally for open channel flow. It confirms, however, the proportionality of friction factor directly with flow depth , and inversely with Reynolds number.

Similar modifications for turbulent flow relationship with recalling that $N = 1/S^2$ yield :

$$f = \frac{12YS}{G^2} \, (R_e)_G^{-.2} \tag{45}$$

where $(R_e)_G = V_{max}G/\upsilon$. Although the equation was primarily developed for pressure flow, it can confirm the linear dependence of f on flow depth, Y, in case of turbulent flow through rigid dense vegetation, similar to the relation for rigid sparse system. The small negative power of Reynolds number also satisfies the expectation for a turbulent flow.

Hartley (1980) tested the sheet flow on a smooth surface through 1/4 inch diameter cylinders representing ideal vegetation. He then measured the flow depths and velocities and used the following energy equation to evaluate friction slope :

$$S_f = (Y_1-Y_2)/\Delta x + (V_1^2-V_2^2)/2g\Delta x + S_0 \tag{46}$$

where subscripts 1 and 2 stand for upstream and downstream locations with the distance $\Delta x$ apart. He reported that since the flow was close to a uniform flow, in most cases $S_f$ showed values quite near $S_o$. Then, the total friction factor f could be calculated having $S_f$, Y, V and using Eq.12. Assuming linear superposition of drag, Hartley removed the sidewall effect applying the method by Vanoni and Brooks (1957) and then bottom resistance using $f = 24/R_e$ for laminar flow and Blasius equation for turbulent flow. In case of smooth boundaries, the sidewall effect and bottom resistance showed quite minor values compared with the vegetation resistance.

Hartley assumed the following simple power model for laminar flow: $f = A\,(Y/d)^B R_d^{C}$, where A depends on density and pattern, Y/d is the depth diameter ratio to account for form drag effects, and $R_d$ is diameter

Reynolds number equal to $V_{max}.d/\upsilon$ . By performing regression, Hartley confirmed the general form $f=K/R_d$ as:

$$f = A\ (Y/D)\ R_d^{-1.0} \tag{47}$$

Generally, having depth, instead of bed slope, as independent variable is advantageous because in case of non-uniform flow with rainfall the effect of change in depth would be included in flow resistance due to vegetation.

For turbulent flow, Hartley dropped the effect of Reynolds number, assuming negligible effect, and he allowed Froude number to enter the equation. Therefore, the power equation for turbulent flow became :

$$f = A\ (Y/d)^B\ F^E \tag{48}$$

where F = Froude number. By performing data regression, Hartley found the influence of Froude number to be marginal in its effect on resistance coefficient, even though the free surface effects were physically evident in some slopes. Also the exponent of Y/d turned out to be 1.

To account for density variation, Hartley introduced a correction factor being $(d/S)^2$. Therefore his resistance equation now becomes:

$$f = C\ \frac{dY}{S^2}\ R_d^P \tag{49}$$

in which p equals -1 for laminar flow and zero for turbulent flow. Constant C is dependent on the vegetation pattern as in the following table :

Table 2. Pattern Coefficient (C)

| Pattern | Laminar Flow | Turbulent Flow | Relative C |
|---|---|---|---|
| Staggered | 2995 | 11.4 | 1.0 |
| Parallel | 1366 | 5.2 | 0.46 |
| Random | 1576 | 6.0 | 0.53 |

Table 2 shows that the highest resistance is produced by staggered patterns for a given element density, whereas a random pattern yields somewhat more than half of that for staggered pattern. For the laminar flow, Hartley assumed that the relative pattern effect determined for turbulent flow was valid in the laminar range in order to avoid the lack of data in that range. However, no evidence has been provided to justify that assumption.

The conditions and restrictions on using Hartley's equations are as follows: (1) flow is laminar when $R_d<150$ and is turbulent otherwise -- $R_d$ may be replaced by $(V_{max}.d)/\upsilon = (S/S-d).(V.d)/\upsilon$ in which (S-d) equals the gap size; (2) the vegetation surface is smooth and either no flexibility effect occurs or the flow is very shallow; (3) the vegetation pattern can be identified as one of staggered, parallel, or random; (4) the vegetation density is approximately constant along the height of stems; and (5) the equations only give the vegetation resistance.

## 6.3. Flexible Artificial Vegetation

The effect of flexibility of vegetation simulated by artificial turf on resistance to sheet flow was noticed by Fenzel (1964). He introduced a dimensionless deflection parameter, $V^2Y^4/J$, in which J= EI, E = module of elasticity of the vegetation material, and I = moment of inertia of the turf cross section. For his particular studies on irrigation systems, Fenzel dropped this parameter from dimensional analysis because of no bending effect or other deflection of the vegetation in his experiments.

Hoerner (1965) modified the drag coefficient for a prismatic element by a factor equal to the cube of the cosine of the angle between the element and normal to the flow direction. This factor takes the degree of flexibility into account and implies that the drag coefficient for a flexible element is less than that of a rigid one. Obviously, the method can not be applied when the elements are semi-rigid which may be bent with varying angle and also the method holds for sparse vegetations.

More experiments on dense synthetic flexible turf were carried out by Phelps (1970). He did his experiments with artificial turf of raffia sewn to a jute fabric base. His procedure was to test the variation of f with $R_e$ for different constant depths. This was accomplished for a series of depths by adjusting discharge to achieve these depths on a given slope. The reason for choosing constant depths with varying Reynolds number was to reflect the effect of decreasing vegetation density with the distance from the boundary, similar to natural grass. Phelps then found that the product of $f.R_e$ was not a constant for

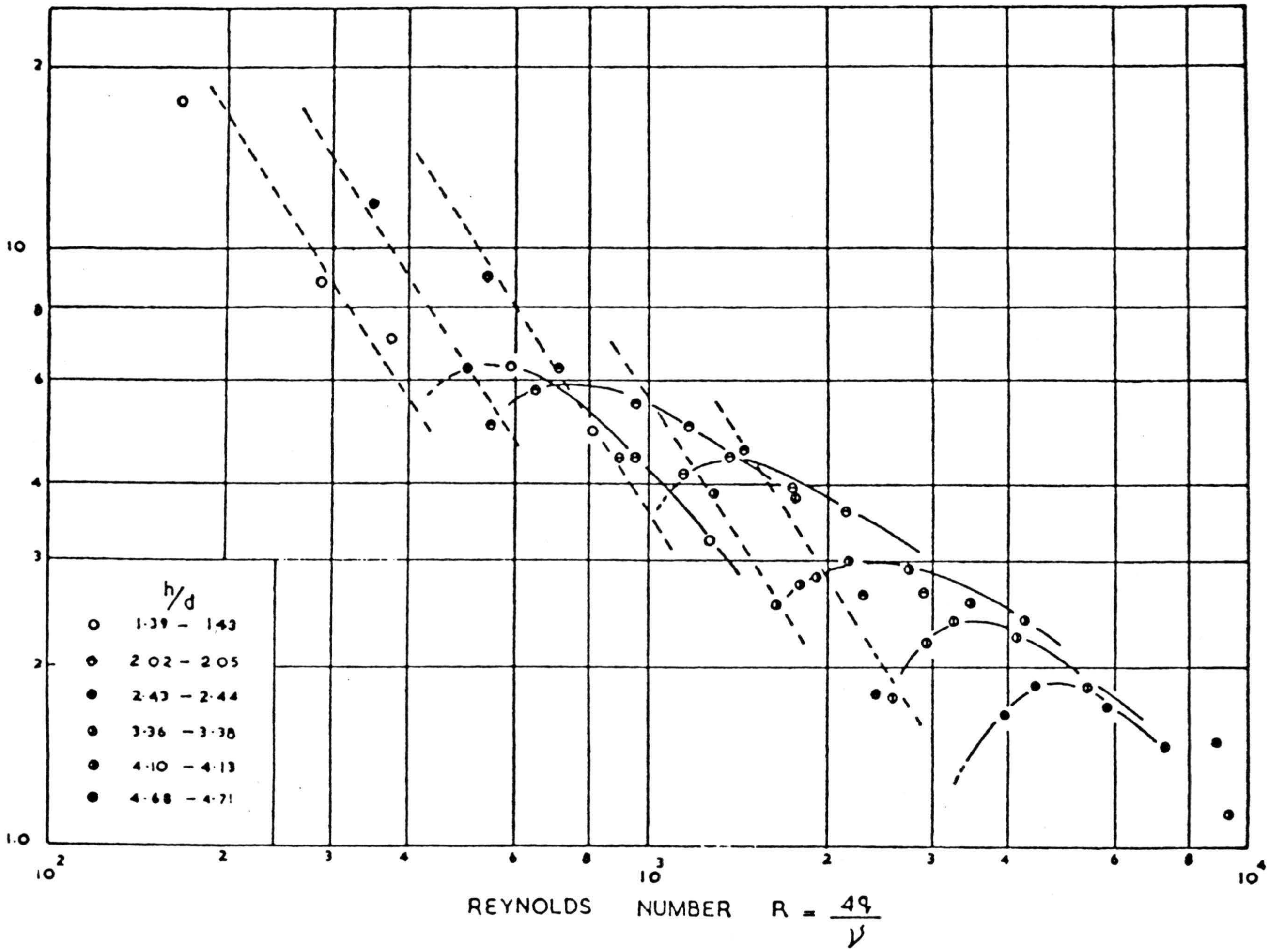


Fig. 8. The f-$R_e$ relationship for flexible artificial turf, after Phelps (1970).

laminar flow but rather decreasing with increase in $R_e$ for every constant depth. This means a steeper slope than -1 on log-log paper which is the theoretical slope. Phelps (1970) explained this departure in terms of the flexibility of the synthetic turf in response to the flow condition. As the Reynolds number and velocity increased, the expansion of average pore size caused steeper decrease in resistance.

The data are depicted in Fig.8 illustrating f vs $R_e$ for constant values of h/d, where h is flow depth and d is flow passage dimension which was set to .01 feet due to assumed similarity of flow through turf with groundwater flow through porous media, with convection d being .01. Therefore, constant lines of h/d represent constant depths. If one traces constant depth line in the direction of increasing $R_e$ or discharge, he will find that the slope is increasing in that direction. As a result, the values of constant slope lines should decrease from the bottom to the top in direction of increasing f. Now, if for constant $R_e$ or discharge the bed slope is reduced, the flow depth will increase and so will resistance. However, as will be indicated later, the same change in slope in Chen's data for natural vegetation causes less resistance. One may reason the difference in terms of the ability of contraction of pores due to lower velocity over the ability of the flow to find larger pores at higher depths in Phelps' tests. This is probably one difference between behavior of artificial turf and the natural one.

Although the adequacy of Phelps' data is in doubt particularly for higher depths, Phelps made three important conclusions for sheet flow through dense flexible artificial vegetation : (1) the varying density of vegetation with depth has to be accounted for; (2) for constant depth, pore or flow tube size can expand as the velocity increases due to vegetation flexibility; and (3) the critical Reynolds number marking the limit of laminar flow decreases with the decrease in depth.

## 6.4. Natural Vegetation

The early investigations of the flow resistance in a laminar flow through natural vegetation dates back to attempts to determine K value in Eq.16. As the first investigator, Izzard (1944) conducted a series of experiments on the laminar flow with the rainfall over a turf surface covered with Kentucky Blue grass. He found K to be as high as 10,000 for bed slope being .01 and with any rainfall intensity.

An extensive study on effect of specific natural vegetation on resistance to sheet flow was carried out by Chen (1976). Bermuda grass and Kentucky Blue grass were used as the typical vegetation in overland areas. Through a dimensional analysis with considering test results, Chen assumed Reynolds number, slope, relative roughness k/Y, and rainfall intensity as the independent variables in dimensional analysis. Chen concluded that the effect of the rainfall would decrease with increase in roughness size, k, and bottom slope and therefore it may be neglected for high roughness boundary of grassed area. Later, he dropped k from the analysis for sake of simplicity and difficulties involved in k measurement. Finally, the remaining variables became $R_e$ and slope, i.e. f=func $(R_e, S_o)$. The regression analysis showed that K value for laminar flow through Bermuda grass began from 5000 up to 500,000 for slopes being .001 to .555 respectively. It was also found that the upper limit of $R_e$ for laminar flow decreased from $10^4$ for $S_o$=.001 to $10^3$ for $S_o$=.555. The equation suggested by Chen to be applied for Bermuda grass and Kentucky Blue grass surfaces in the laminar range is:

$$f = \frac{510{,}000\ S_o^{.662}}{R_e} \tag{50}$$

The increase in slope, if considered as an independent variable, would increase the friction factor of flow on a rough surface when discharge and other parameters held constant. The case of natural vegetation with higher density near the bed yields the same effect for bed slope. To reason such an effect, Kruse et al. (1965) explained the phenomena by considering the correspondence of increase in slope and decrease in depth for constant discharge and therefore higher average density opposing the flow. This trend resulted from Chen's tests on Bermuda grass.

Hartley (1980) superimposed the constant depth lines on Chen's data, as shown in Fig.9. Hartley confirmed the reason stated by Kruse et al. (1965) that for constant slope, resistance decreases as depth increases indicating lower average density of vegetation with increasing depth. Another trend in Fig.9 may be observed along constant depth lines. Generally, the friction factor grows along the path such that the tangent slope to the path starts from zero and increases toward infinity. This implies that constant depth at higher slope ranging from .001 to .164 and higher $R_e$ up to some extent, corresponds to a higher friction factor. Obviously, the preceding conclusion is in contradiction with the case of flow over a rough boundary in which friction factor decreases with slope and $R_e$ with depth held constant. Hartley explains that the increase in resistance along constant depth lines in Chen's data could be due to either instability in free surface as velocity increases or flexibility effects. The former effect requires additional energy dissipation and the latter causes an increase in biomass brought down into the flow due to bending. Kouwen and Unny (1973) state that this effect of flexibility increases resistance as long as the vegetation is not totally overtopped or channelized by the flow.

In the second part of constant depth line in Chen's data, f tends to grow very rapidly with constant $R_e$ and consequently discharge. The trend is true for depths being larger than 0.1 feet and when $S_o>0.164$. This indicates that for steep slope with constant depth, the flow resistance becomes independent of $R_e$ when $R_e>700$ and apparently flow enters the transition regime. Therefore, the upper limit for $R_e$ for laminar regime in Chen's data would be probably close to 700 for slopes steeper than 0.164, whereas Chen extends it to 1100. One may reason the phenomenon for steep slope in terms of high free surface instability causing turbulence and making the flow exit from laminar regime. For practical purposes, however, a steeper slope ($S_o>.164$) rarely occurs and the Chen's data on resistance to flow through Bermuda grass can be used for mild slope when $R_e$ is as large as $10^4$.

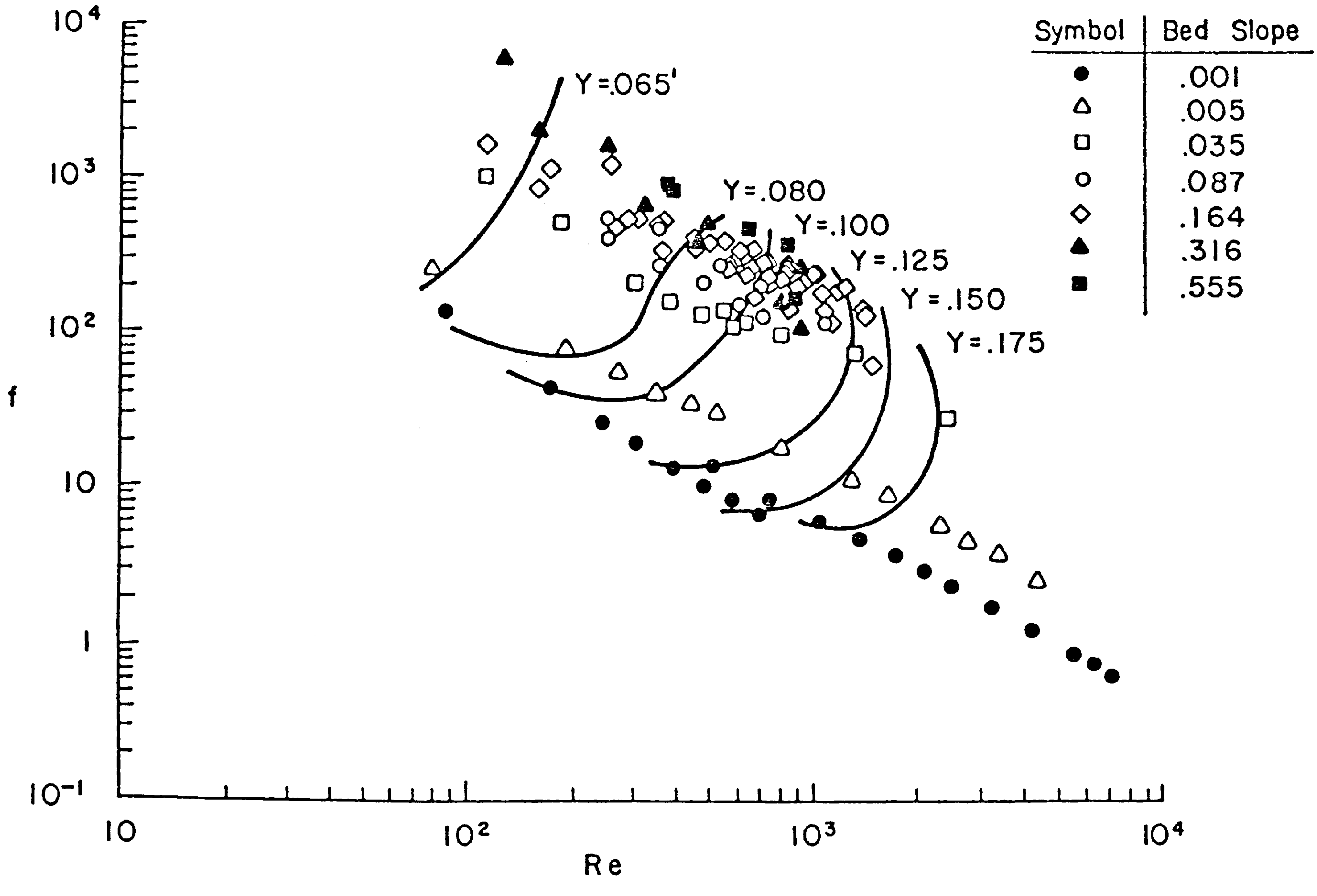


Fig. 9. The f-$R_e$ restricted data for flow through Bermuda grass, after Chen (1976).

Even though there exist a debate concerning whether the bed slope can be an independent variable, Chen's data confirms a good agreement in laminar region with the equation $f=K/R_e$. Since Chen's equation directly computes the total resistance, there is no need to separate the boundary resistance and deal with it. Also, the equation comes from the experiments in which more similarity with natural situation occurs, particularly density variation with depth in addition to flexibility effect. The comparison of the data and the equation is shown in Fig.10.

Similar data on flow through Bermuda grass has been presented by Palmer (1945). Palmer data along with Chen data are plotted in Fig.10. Although most of the Palmer data fall within laminar range as indicated by Chen, it shows an almost constant f through the laminar range rather than decreasing f with $R_e$. Chen reasons the discrepancy in the results between his and Palmer's study in terms of high difficulties involved in depth measurements with such thin flows. Whatever the reason, the Palmer data in laminar range can not be trusted because showing nearly constant f in that range means the relative independency of resistance from Reynolds number that might be true for turbulent flows.

Ree and Palmer (1949) performed extensive experiments on resistance to turbulent flow through various grasses, particularly Bermuda grass, in two different channel cross sections, trapezoidal and rectangular, with channel slope ranging from 0.002 to 0.24. They plotted curves of Manning's n versus the product of velocity and hydraulic radius. Also the results of experiments identified three conditions of vegetal roughness system in terms of flexibility: (1) erect condition corresponding to low flows with high resistance, constant n until partial submergence occurs; (2) deflected condition at intermediate flow, decreasing resistance with discharge, beyond complete submergence; and (3) prone condition at high flows and low resistance above the flattened vegetation, fully turbulent flow with constant n. Having Ree and Palmer data including the variation of n vs VR and the temperature at the time of experiments, Chen derived f vs $R_e$ using the relation between f and n and then plotted the results along with his own data in Fig.10. Three interesting conclusions are revealed from Fig.10. First, the Ree and Palmer data falls mostly into transition and turbulent ranges, having a steep drop in resistance in transition range and terminating to, as Chen puts it, a fixed f when entering fully turbulent flow. The fixed f value is claimed to be 0.11 for $R_e$ larger than $10^6$. However, almost all of the curves of n vs VR provided by Ree and Palmer terminates to a constant value for n indicating a fully turbulent flow independent of Reynolds number. Since n is proportional to $f^{1/2}R^{1/6}$, then constant n doesn't mean constant f while $R_e$, or discharge, is increasing. Therefore, referring to fixed f in f-$R_e$ diagrams, without having data in apparently constant f region, cannot be true and connection of two broken curves in Fig. 10 only indicates the independency of f from channel cross section for fully turbulent flow. In order to derive f-n relationship and use it for fully turbulent region, one can use the Manning equation in addition to Eq.12 and then eliminate the depth parameter by introducing $R_e$ into Eq.12. It yields:

$$f = 8(1.49)^{-1.8}n^{1.8}gS^{.1}R_e^{-.2}\upsilon^{-.2} \tag{52}$$

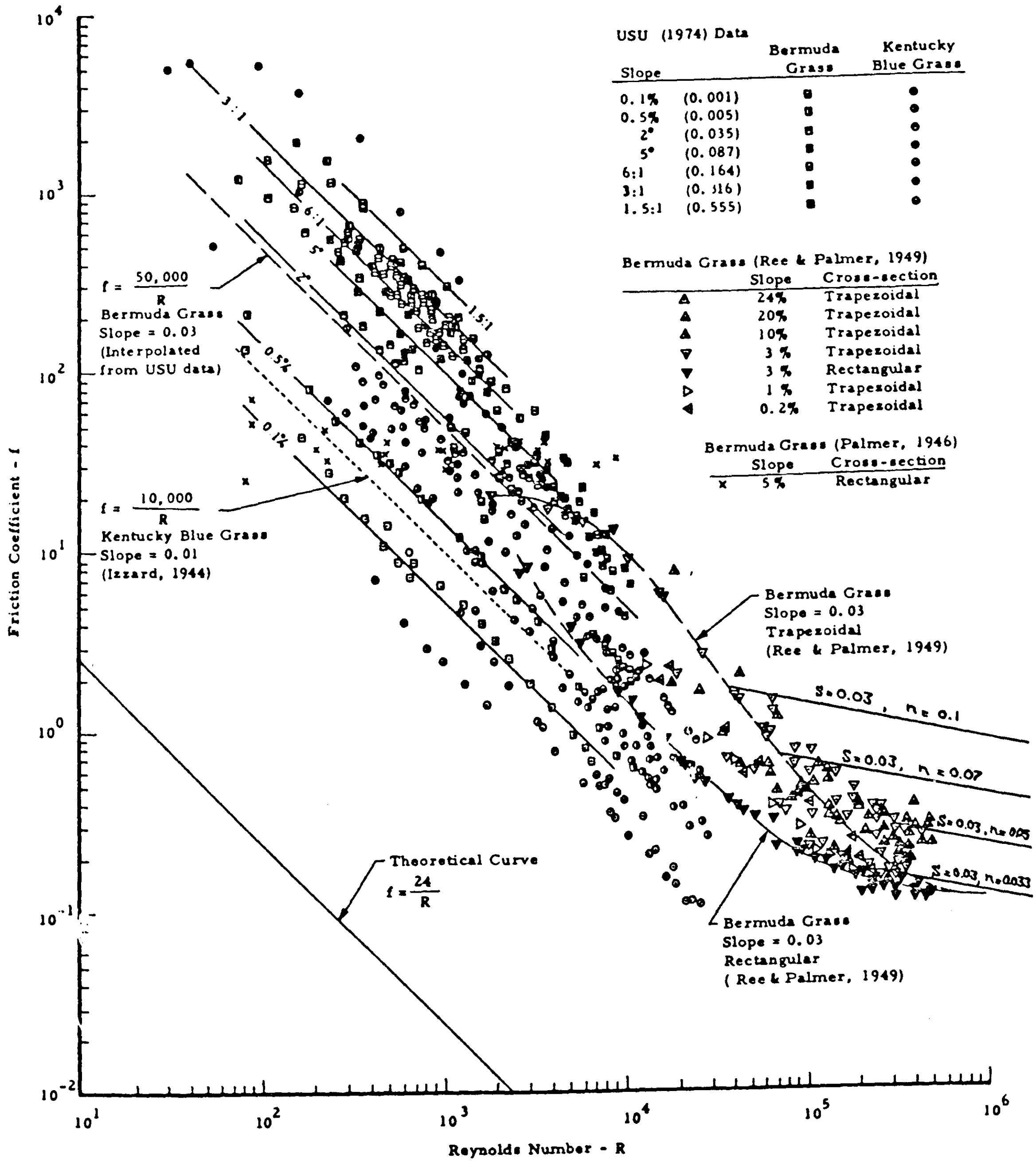


Fig. 10. The f-$R_e$ relationship for flow through Bermuda and Kentucky grasses, after Chen (1976).

For $\upsilon = 1.5*10^{-5}$ ft$^2$/s, g = 32.2 ft/s$^2$, and specific slope being 0.03, the equation simplifies to :

$$f = 819.98 n^{1.8} R_e^{-.2} \tag{53}$$

The Ree and Palmer's n-VR curves indicates a constant n being 0.033, corresponding to the line shown in Fig. 10, for fully turbulent flow when S=0.03. As it is seen that the f-n line extends the broken curves of f-$R_e$ from transition region into fully turbulent flow.

Second, the variation of f in the transition range may differ with the cross section shape for the same slope. Two broken curves in Fig.10 connect the data for trapezoidal and rectangular cross sections for 3% channel slope. The trapezoidal resistance curve represents larger f compared to that of a rectangular one for similar $R_e$, or discharge. Equal discharge in rectangular and trapezoidal cross sections requires larger depth in rectangular channel, corresponding to less resistance. This trend was also derived from Chen's data in laminar region and was explained in terms of less vegetation density at higher depths in addition to lower resistance due to flexibility effects.

Third, both broken curves seem to meet at approximately $R_e$ = 2000 at a point that flow on the 3% slope starts to deviate from the laminar region to the transition. Interestingly, the point of intersection between two broken curves almost lies on the line representing f-$R_e$ relationship in Chen's equation for laminar flow on 3% channel slope. This indicates a good agreement between Chen's and Ree and Palmer's data.

## 6.5. Deep Flow over Flexible Vegetation

The importance of vegetation flexibility on relative roughness and flow resistance was suggested by Fenzel and Davis (1964) through a series of experiments on artificial turf. Element stiffness, spacing, and shape as well as fluid properties and flow parameters were realized to affect the flow resistance. Fenzel and Davis showed that the vegetation resistance was dominant over soil resistance, even though they couldn't evaluate the significance of flexibility parameters in their analysis due to lack of data. They also noticed the importance of soil resistance only at small depths in sparsely vegetated channels whereas it could be ignored for most deep flows in densely vegetated channels.

Probably, the most comprehensive analysis, which will be explained in details, of velocity profile and flow resistance in presence of flexible vegetation in deep flows was accomplished by Kouwen and his colleagues. Kouwen et al.(1969) and then Kouwen and Unny (1973) developed a semilogarithmic velocity profile equation by introducing a new relative roughness, Y/K, to account for the deflection effect of flexible vegetation. Y is simply flow depth and K stands for the deflected height of the vegetation. The equation is :

$$\frac{V}{V_*} = C_1 + C_2 \ln\left(\frac{Y}{K}\right) \tag{53}$$

where $C_1$ and $C_2$ are constants for a given vegetation type and density. $C_1$ depends mainly on the flow through the vegetation and hence will be a function of its density. For small depths when $Y < K$, the equation reduces to $V/V_* = C_1$ or by substituting for $V_*$, it is obtained that $\tau_0 = \rho V^2/C_1^2$ which looks like the familiar drag equation where $C_D = 2/C_1^2$. Since $C_D$ is directly proportional to the number of stems per unit area, it becomes clear that $C_1$ is dependent on the density of the vegetation. $C_2$, on the other hand, is related to vegetation stiffness.

Kouwen and Unny (1973) used flexible plastic strips to model and determine $C_1$ and $C_2$ for different conditions : prone and otherwise. The prone condition was found when the shear velocity exceeded a critical shear velocity as follows :

$$V_* > V_{*c} = 0.028 + 6.33\ (MEI)^2 \qquad (54)$$

where MEI = a bulk stiffness parameter. The above relationship was primarily developed for elastic roughness which returns to its initial position after cessation of the flow. An analysis of Eastgate's (1966) data revealed that for tall natural grasses the critical shear velocity given by Eq.54 was too high. For natural long stiff grasses, which acts plastically under the flow, Eastgate's data indicated that :

$$V_{*c} = 0.23\ (MEI)^{.106} \qquad (55)$$

Thus Eq.54 represents the shear velocity required to elastically bend the roughness to a prone condition and Eq.55 represents the plastic case. Both equations, which are not dimensionless, are in SI units. In practice, the smaller value between Eq.54 and Eq.55 is recommended to be used.

Assuming the validity of semilogarithmic velocity profile, the resistance coefficients, f and n, can be written in SI units as :

$$\frac{1}{\sqrt{f}} = a + b \log\left(\frac{Y}{K}\right) \tag{56}$$

and

$$n = \frac{Y^{1/6}}{\sqrt{8g}\ [\ a + b \log(Y/K)]} \tag{57}$$

Using the data on synthetic plastic roughness, Kouwen and Unny determined a and b as 0.15 and 1.85 for $V_*/V_{*c} < 1.0$; 0.20 and 2.70 for $1.0 < V_*/V_{*c} < 1.5$; 0.28 and 3.08 for $1.5 < V_*/V_{*c} < 2.5$; and 0.29 and 3.50 for $V_*/V_{*c} > 2.5$.

Kouwen and Li (1980) established an equation to evaluate the deflected height of the vegetation, in SI units, as :

$$K = 0.14\ h\ \left[\left(\frac{MEI}{\rho V_*^2}\right)^{.25}/h\right]^{1.59} \tag{58}$$

The remaining difficulty is the value of MEI (in $N.m^2$) for each grass type. Because there were no reported measurements of the deflected vegetation heights, K, for the experiments modeling the flow over natural vegetations, Kouwen and Li used a backward method to calibrate MEI values. They collected the experimental data of Chen (1975), Cox and Palmer (1948), Eastgate (1966), and Ree and Palmer (1949) including measurement of vegetation height, h, flow velocity, V, and effective slope, $S_f$. In their method, Kouwen and Li assumed an initial value for MEI for each grass. Then they calculated K, n, $V_{cal}$, and $Q_{cal}$ for each individual experiment. That assumed value of MEI, which gave the smallest summation among the differences between calculated discharges and corresponding measured discharges for all experiments with one grass, was tabulated as the value of MEI for that specific grass. The table was confirmed by computing retardance curves, n vs VR, and comparing with the measured retardance curves presented by Chen, Cox and Palmer, and the others. The good fit between the retardance curves was assumed to be an indication to justify the use of flexible plastic strips to model the flow over natural vegetation. Finally, Kouwen and Li proposed an iterative procedure for the design of a channel with vegetative lining. Kouwen (1969) classified flow through and over vegetation according to whether vegetation was erect and stationary, bent and waving, or prone. Shen and Li (1973) cited element waving as a possible mechanism increasing flow resistance. However the method by Kouwen and Li (1980) doesn't consider the element waving as a middle condition between erect and prone and only deflection effect contributes in the equations. Even though no report of applying Kouwen and Li' s method is available, the method can be considered as a collection of existing data on turbulent flow resistance through various natural vegetations.

7. CONCLUSIONS

The following conclusions emerged from the discussion of the literature on resistance to sheet flows:

(1) total resistance in sheet flow can be represented by the sum of resistances due to rainfall, roughness, and vegetation;

(2) the relative roughness may represent a more general variable compared to bed slope, in flow resistance equation for laminar flow over a rough boundary. According to Phelps' paper, the friction factor equation in the form $f = K/R_e$ has been verified. K is constant for a given relative roughness;

(3) the friction factor for turbulent flow depends on the condition of roughness related to the flow. Flow resistance under hydraulically smooth conditions is a function of Reynolds number whereas under fully rough condition the primary variable becomes the relative roughness;

(4) the friction factor, here defined as $8\tau/\rho V^2$, depends on Reynolds number and rainfall intensity for laminar flow over a smooth boundary and only on Reynolds number for turbulent flow. The resistance equation given by Li (1972) is recommended for the computation of flow resistance with rainfall;

(5) flow through vegetation is very complicated. Nevertheless, in limited number of cases several methods can be applied. Chen's equation is suggested for total friction factor due to laminar flow through Bermuda and Kentucky Blue grasses. For either flow through rigid vegetation with constant density along depth of flow, or very shallow flow through grass, Hartley's equations may be used to compute friction

factor for different vegetation patterns in both laminar and turbulent flow;

(6) in case of deep turbulent flow through natural vegetation, Ree and Palmer's resistance curves can provide Manning's n. Also in this case Kouwen and Unny's method is suitable for channel design with vegetative lining; and

(7) the relative magnitude of resistance to flow due to rainfall, roughness, and vegetation (represented by Bermuda grass) shows that rainfall resistance and roughness resistance for laminar flow are generally comparable whereas vegetation resistance drastically overcomes that of both rainfall and roughness combined.

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## APPENDIX II - LIST OF SYMBOLS

The following symbols are used in this paper:

$A$ = cross sectional area;

$C$ = concentration of roughness elements; also Chezy C;

$C_D$ = drag coefficient of vegetation elements;

$D$ = average diameter;

$d$ = rainfall size; also diameter of vegetation elements;

$D$ = pipe diameter; also depth;

$EI$ = stiffness of vegetation;

$F$ = Froude number = $V/\sqrt{gy}$;

$f$ = Darcy-Weisbach friction factor;

$g$ = gravitational acceleration;

$G$ = average gap size;

$h$ = vegetation height;

$h_f$ = head loss in pipes;

$i$ = rainfall intensity;

$K$ = deflected height of the vegetation; also constant for description of $f$-$R_e$ relationship;

$k$ = mean boundary roughness height;

$N$ = number of cylinders per unit area;

$n$ = Manning's n;

$q$ = unit discharge;

$q_0$ = unit base flow rate in case of rainfall;

$R$ = hydraulic radius;

$R_e$ = Reynolds number = $q/\nu$;

$R_d$ = diameter Reynolds number = $VD/\upsilon$;

S = average vegetation spacing;

$S_o$ = bed slope;

$S_f$ = friction or energy gradient;

T = free surface width of the channel;

U = velocity of raindrop entering main flow;

V = mean flow velocity;

Y = average flow depth;

x = distance in the main flow direction;

$\beta$ = velocity distribution factor in momentum equation;

$\beta_i$ = regression coefficient in regression equation;

$\alpha$ = rainfall pattern dimensionless quantity;

$\gamma$ = Specific gravity of water;

$\epsilon$ = error in regression equation;

$\lambda$ = parameter describing raindrop shape; also soil roughness spacing;

$\rho$ = density of water;

$\tau$ = boundary shear stress;

$\theta$ = angle between main flow direction and horizontal; also cross sectional shape dimensionless quantity of vegetation elements;

$\mu$ = dynamic viscosity of water;

$\upsilon$ = kinematic viscosity of water;

$\psi$ = dimensionless vegetation pattern parameter;

$\phi$ = angle between the velocity U and x-direction;

$\sigma$ = soil roughness height;

$\delta$ = laminar sublayer thickness;

APPENDIX III. Tables of Data

f-Re Data Based on Relative Roughness, after Phelps (1975)

| Data number | Channel slope | Relative roughness | Unit discharge (m2/s) | Mean depth (mm) | Reynold number | Darcy f |
|---|---|---|---|---|---|---|
| 1 | 0.00083 | 0.35 | 0.000067 | 3.35 | 75 | 0.553 |
| 2 | 0.00083 | 0.28 | 0.000142 | 4.21 | 161 | 0.242 |
| 3 | 0.00083 | 0.23 | 0.000253 | 5.00 | 286 | 0.128 |
| 4 | 0.00083 | 0.52 | 0.000034 | 2.26 | 36 | 1.225 |
| 5 | 0.00154 | 0.35 | 0.000125 | 3.38 | 135 | 0.301 |
| 6 | 0.00154 | 0.28 | 0.000276 | 4.24 | 296 | 0.121 |
| 7 | 0.00154 | 0.23 | 0.000466 | 5.00 | 500 | 0.070 |
| 8 | 0.00238 | 0.54 | 0.000043 | 2.18 | 46 | 1.079 |
| 9 | 0.00238 | 0.35 | 0.000180 | 3.35 | 194 | 0.219 |
| 10 | 0.00238 | 0.28 | 0.000390 | 4.24 | 423 | 0.094 |
| 11 | 0.00238 | 0.23 | 0.000511 | 5.16 | 551 | 0.099 |
| 12 | 0.00199 | 0.52 | 0.000038 | 2.24 | 40 | 1.220 |
| 13 | 0.00199 | 0.27 | 0.000333 | 4.29 | 489 | 0.112 |
| 14 | 0.00458 | 0.53 | 0.000078 | 2.18 | 85 | 0.613 |
| 15 | 0.00458 | 0.35 | 0.000314 | 3.30 | 342 | 0.131 |
| 16 | 0.00458 | 0.27 | 0.000461 | 4.29 | 507 | 0.134 |
| 17 | 0.00458 | 0.23 | 0.000578 | 5.08 | 626 | 1.141 |
| 18 | 0.00302 | 0.35 | 0.000240 | 3.38 | 257 | 0.159 |
| 19 | 0.00302 | 0.27 | 0.000446 | 4.27 | 469 | 0.093 |
| 20 | 0.00048 | 0.27 | 0.000840 | 4.32 | 92 | 0.432 |
| 21 | 0.00048 | 0.23 | 0.000142 | 5.03 | 148 | 0.237 |
| 22 | 0.00048 | 0.35 | 0.000034 | 3.35 | 36 | 1.240 |
| 23 | 0.00120 | 0.23 | 0.000360 | 5.00 | 388 | 0.091 |
| 24 | 0.00120 | 0.28 | 0.000208 | 4.24 | 220 | 0.166 |
| 25 | 0.00120 | 0.35 | 0.000095 | 3.35 | 99 | 0.398 |
| 26 | 0.00612 | 0.54 | 0.000100 | 2.16 | 104 | 0.485 |
| 27 | 0.00612 | 0.35 | 0.000401 | 3.35 | 426 | 0.113 |
| 28 | 0.00612 | 0.28 | 0.000508 | 4.24 | 538 | 0.143 |
| 29 | 0.00612 | 0.23 | 0.000659 | 5.05 | 691 | 0.143 |
| 30 | 0.00761 | 0.55 | 0.000117 | 2.13 | 126 | 0.424 |
| 31 | 0.00761 | 0.28 | 0.000528 | 4.24 | 576 | 0.164 |
| 32 | 0.00761 | 0.23 | 0.000707 | 5.03 | 756 | 0.153 |
| 33 | 0.01000 | 0.55 | 0.000145 | 2.13 | 154 | 0.362 |
| 34 | 0.01000 | 0.28 | 0.000579 | 4.24 | 621 | 0.180 |
| 35 | 0.01000 | 0.23 | 0.000770 | 4.98 | 829 | 0.164 |
| 36 | 0.01490 | 0.53 | 0.000207 | 2.13 | 216 | 0.287 |
| 37 | 0.01490 | 0.27 | 0.000689 | 4.27 | 733 | 0.192 |
| 38 | 0.01980 | 0.53 | 0.000244 | 2.21 | 265 | 0.284 |
| 39 | 0.01980 | 0.35 | 0.000499 | 3.35 | 534 | 0 235 |
| 40 | 0.01980 | 0.23 | 0.001122 | 5.03 | 1204 | 0.158 |
| 41 | 0.02970 | 0.53 | 0.000214 | 2.21 | 229 | 0.550 |
| 42 | 0.02970 | 0.27 | 0.000953 | 4.32 | 1039 | 0.207 |
| 43 | 0.02970 | 0.23 | 0.001285 | 5.03 | 1393 | 0.180 |
| 44 | 0.04510 | 0.53 | 0.000271 | 2.18 | 297 | 0.507 |
| 45 | 0.04510 | 0.35 | 0.000659 | 3.33 | 716 | 0.301 |
| 46 | 0.04510 | 0.27 | 0.001107 | 4.29 | 1194 | 0.229 |
| 47 | 0.04510 | 0.23 | 0.001515 | 5.03 | 1600 | 0.197 |

f-Re, Data for Sand Surface, after Woo and Brater (1962)

| Data number | Bed slope | Discharge (cfs/ft) | Depth (in) | Reynolds number | Darcy f |
|---|---|---|---|---|---|
| 1 | 0.001 | 0.000786 | 0.145 | 66 | 0.7360 |
| 2 | 0.001 | 0.002345 | 0.195 | 195 | 0.2008 |
| 3 | 0.001 | 0.009700 | 0.310 | 789 | 0.0472 |
| 4 | 0.002 | 0.006140 | 0.210 | 547 | 0.0732 |
| 5 | 0.002 | 0.011980 | 0.295 | 992 | 0.0532 |
| 6 | 0.002 | 0.016600 | 0.375 | 1340 | 0.0570 |
| 7 | 0.002 | 0.021650 | 0.450 | 1730 | 0.0580 |
| 8 | 0.003 | 0.002017 | 0.145 | 166 | 0.3360 |
| 9 | 0.003 | 0.006730 | 0.190 | 553 | 0.0676 |
| 10 | 0.003 | 0.012600 | 0.300 | 1025 | 0.0762 |
| 11 | 0.003 | 0.020970 | 0.400 | 1687 | 0.0652 |
| 12 | 0.003 | 0.046550 | 0.695 | 3695 | 0.0694 |
| 13 | 0.004 | 0.002323 | 0.130 | 188 | 0.2432 |
| 14 | 0.004 | 0.004460 | 0.160 | 354 | 0.1232 |
| 15 | 0.004 | 0.006110 | 0.180 | 482 | 0.0932 |
| 16 | 0.004 | 0.023970 | 0.430 | 1870 | 0.0828 |
| 17 | 0.004 | 0.047700 | 0.650 | 3720 | 0.0720 |
| 18 | 0.006 | 0.003834 | 0.140 | 311 | 0.1672 |
| 19 | 0.006 | 0.008900 | 0.210 | 707 | 0.1048 |
| 20 | 0.006 | 0.017470 | 0.315 | 1384 | 0.0920 |
| 21 | 0.006 | 0.027560 | 0.430 | 2188 | 0.0936 |
| 22 | 0.006 | 0.048500 | 0.595 | 3795 | 0.0804 |
| 23 | 0.008 | 0.005800 | 0.145 | 469 | 0.1080 |
| 24 | 0.008 | 0.011500 | 0.230 | 919 | 0.1096 |
| 25 | 0.008 | 0.021060 | 0.320 | 1670 | 0.0880 |
| 26 | 0.008 | 0.022950 | 0.360 | 1811 | 0.1056 |
| 27 | 0.008 | 0.048100 | 0.550 | 3768 | 0.0856 |
| 28 | 0.010 | 0.002130 | 0.110 | 157 | 0.4380 |
| 29 | 0.010 | 0.005060 | 0.145 | 367 | 0.1780 |
| 30 | 0.010 | 0.007240 | 0.160 | 523 | 0.1164 |
| 31 | 0.010 | 0.009970 | 0.215 | 718 | 0.1496 |
| 32 | 0.010 | 0.009210 | 0.200 | 732 | 0.1404 |
| 33 | 0.010 | 0.014920 | 0.270 | 1080 | 0.1316 |
| 34 | 0.010 | 0.015580 | 0.260 | 1221 | 0.1080 |
| 35 | 0.010 | 0.018250 | 0.305 | 1330 | 0.1272 |
| 36 | 0.010 | 0.022400 | 0.340 | 1740 | 0.1168 |
| 37 | 0.010 | 0.047200 | 0.520 | 3664 | 0.0940 |
| 38 | 0.010 | 0.074100 | 0.685 | 5820 | 0.0872 |
| 39 | 0.015 | 0.002256 | 0.105 | 167 | 0.5100 |
| 40 | 0.015 | 0.004330 | 0.125 | 316 | 0.2328 |
| 41 | 0.015 | 0.004450 | 0.120 | 367 | 0.1952 |
| 42 | 0.015 | 0.007310 | 0.160 | 530 | 0.1716 |
| 43 | 0.015 | 0.008680 | 0.185 | 626 | 0.1880 |
| 44 | 0.015 | 0.014830 | 0.250 | 1070 | 0.1592 |
| 45 | 0.015 | 0.014370 | 0.230 | 1130 | 0.1320 |
| 46 | 0.015 | 0.019040 | 0.290 | 1393 | 0.1504 |
| 47 | 0.015 | 0.026650 | 0.355 | 2178 | 0.1408 |
| 48 | 0.015 | 0.052100 | 0.525 | 4075 | 0.1192 |
| 49 | 0.015 | 0.092100 | 0.690 | 7100 | 0.0868 |
| 50 | 0.020 | 0.001824 | 0.095 | 136 | 0.7720 |

f-Re Data for Sand Surface, after Woo and Brater (1962)

| Data number | Bed slope | Discharge (cfs/ft) | Depth (in) | Reynolds number | Darcy f |
|---|---|---|---|---|---|
| 51 | 0.020 | 0.004275 | 0.115 | 313 | 0.2480 |
| 52 | 0.020 | 0.007100 | 0.150 | 516 | 0.2000 |
| 53 | 0.020 | 0.010800 | 0.195 | 785 | 0.1900 |
| 54 | 0.020 | 0.011770 | 0.195 | 928 | 0.1600 |
| 55 | 0.020 | 0.014770 | 0.235 | 1076 | 0.1772 |
| 56 | 0.020 | 0.018350 | 0.265 | 1360 | 0.1648 |
| 57 | 0.020 | 0.020330 | 0.260 | 1570 | 0.1268 |
| 58 | 0.020 | 0.045600 | 0.445 | 3490 | 0.1264 |
| 59 | 0.020 | 0.066800 | 0.535 | 5110 | 0.1024 |
| 60 | 0.020 | 0.094800 | 0.640 | 7160 | 0.0868 |
| 61 | 0.040 | 0.001812 | 0.075 | 136 | 0.7660 |
| 62 | 0.040 | 0.004370 | 0.095 | 321 | 0.2684 |
| 63 | 0.040 | 0.006900 | 0.125 | 503 | 0.2444 |
| 64 | 0.040 | 0.009300 | 0.145 | 676 | 0.2108 |
| 65 | 0.040 | 0.011580 | 0.170 | 840 | 0.2188 |
| 66 | 0.040 | 0.016660 | 0.205 | 1220 | 0.1856 |
| 67 | 0.040 | 0.019700 | 0.175 | 1530 | 0.0828 |
| 68 | 0.040 | 0.023500 | 0.250 | 1780 | 0.1688 |
| 69 | 0.040 | 0.044250 | 0.360 | 3340 | 0.1424 |
| 70 | 0.040 | 0.063500 | 0.435 | 4800 | 0.1220 |
| 71 | 0.040 | 0.089500 | 0.555 | 6690 | 0.1272 |
| 72 | 0.060 | 0.001795 | 0.065 | 131 | 0.7620 |
| 73 | 0.060 | 0.007590 | 0.115 | 538 | 0.2368 |
| 74 | 0.060 | 0.012080 | 0.165 | 857 | 0.2756 |
| 75 | 0.060 | 0.016180 | 0.200 | 1048 | 0.2732 |
| 76 | 0.060 | 0.029600 | 0.285 | 2110 | 0.2368 |
| 77 | 0.060 | 0.028080 | 0.240 | 2165 | 0.1568 |
| 78 | 0.060 | 0.039240 | 0.315 | 2880 | 0.1820 |
| 79 | 0.060 | 0.043700 | 0.340 | 3300 | 0.1844 |
| 80 | 0.060 | 0.054350 | 0.385 | 4080 | 0.1728 |
| 81 | 0.060 | 0.080800 | 0.475 | 6020 | 0.1472 |
| 82 | 0.060 | 0.099100 | 0.530 | 7380 | 0.1356 |

f-Re Data with Rainfall, Laminar Flow, after Yoon (1970)

| Data number | Rainfall intensity (in/h) | Base flow rate (cfs/ft) | Combined flow rate (cfs/ft) | Flow depth (ft) | Reynolds number | Darcy f |
|---|---|---|---|---|---|---|
| 1 | 0.50 | 0.00254 | 0.00270 | 0.01000 | 249.0 | 0.16849 |
| 2 | 1.25 | 0.00237 | 0.00278 | 0.01060 | 247.8 | 0.18626 |
| 3 | 3.75 | 0.00150 | 0.00272 | 0.01167 | 245.3 | 0.25289 |
| 4 | 15.00 | 0.00141 | 0.00280 | 0.01467 | 249.9 | 0.41389 |
| 5 | 0.50 | 0.00368 | 0.00384 | 0.01092 | 354.1 | 0.11359 |
| 6 | 1.25 | 0.00339 | 0.00380 | 0.01208 | 348.8 | 0.15227 |
| 7 | 3.75 | 0.00248 | 0.00370 | 0.01308 | 345.4 | 0.19871 |
| 8 | 15.00 | 0.00182 | 0.00390 | 0.01625 | 349.4 | 0.29931 |
| 9 | 0.50 | 0.00471 | 0.00487 | 0.01250 | 449.0 | 0.10322 |
| 10 | 1.25 | 0.00463 | 0.00504 | 0.01308 | 443.2 | 0.11121 |
| 11 | 3.75 | 0.00366 | 0.00488 | 0.01517 | 444.4 | 0.17171 |
| 12 | 15.00 | 0.00153 | 0.00500 | 0.01733 | 449.4 | 0.23023 |
| 13 | 0.50 | 0.00585 | 0.00601 | 0.01300 | 548.0 | 0.07943 |
| 14 | 1.25 | 0.00554 | 0.00595 | 0.01383 | 547.9 | 0.09632 |
| 15 | 3.75 | 0.00479 | 0.00601 | 0.01625 | 544.9 | 0.14077 |
| 16 | 15.00 | 0.00182 | 0.00599 | 0.01792 | 539.3 | 0.18611 |
| 17 | 0.50 | 0.00693 | 0.00709 | 0.01433 | 648.3 | 0.07310 |
| 18 | 1.25 | 0.00666 | 0.00707 | 0.01505 | 647.6 | 0.08801 |
| 19 | 3.75 | 0.00585 | 0.00707 | 0.01683 | 644.1 | 0.11497 |
| 20 | 15.00 | 0.00218 | 0.00704 | 0.01933 | 630.4 | 0.16550 |
| 21 | 0.50 | 0.00853 | 0.00869 | 0.01542 | 798.9 | 0.06172 |
| 22 | 1.25 | 0.00829 | 0.00870 | 0.01600 | 797.0 | 0.07085 |
| 23 | 3.75 | 0.00750 | 0.00872 | 0.01758 | 798.8 | 0.08860 |
| 24 | 15.00 | 0.00385 | 0.00871 | 0.02058 | 779.9 | 0.13246 |
| 25 | 0.50 | 0.00962 | 0.00978 | 0.01667 | 899.1 | 0.05676 |
| 26 | 1.25 | 0.00938 | 0.00979 | 0.01867 | 896.9 | 0.07671 |
| 27 | 3.75 | 0.00853 | 0.00975 | 0.02000 | 893.2 | 0.09190 |
| 28 | 0.50 | 0.00260 | 0.00276 | 0.00792 | 248.4 | 0.16373 |
| 29 | 1.25 | 0.00235 | 0.00276 | 0.00825 | 248.9 | 0.18785 |
| 30 | 3.75 | 0.00150 | 0.00272 | 0.00900 | 245.3 | 0.24179 |
| 31 | 15.00 | 0.00000 | 0.00278 | 0.01075 | 249.8 | 0.36311 |
| 32 | 0.50 | 0.00371 | 0.00387 | 0.00883 | 348.8 | 0.11909 |
| 33 | 1.25 | 0.00345 | 0.00386 | 0.00942 | 347.2 | 0.14191 |
| 34 | 3.75 | 0.00262 | 0.00384 | 0.01000 | 344.9 | 0.17122 |
| 35 | 15.00 | 0.00000 | 0.00417 | 0.01183 | 373.0 | 0.22309 |
| 36 | 0.50 | 0.00481 | 0.00497 | 0.01017 | 447.9 | 0.10332 |
| 37 | 1.25 | 0.00456 | 0.00497 | 0.01067 | 447.3 | 0.11845 |
| 38 | 3.75 | 0.00372 | 0.00494 | 0.01150 | 444.6 | 0.14740 |
| 39 | 15.00 | 0.00000 | 0.00486 | 0.01317 | 434.0 | 0.22842 |
| 40 | 0.50 | 0.00592 | 0.00608 | 0.01092 | 547.9 | 0.08585 |
| 41 | 1.25 | 0.00565 | 0.00606 | 0.01158 | 547.0 | 0.10214 |
| 42 | 3.75 | 0.00481 | 0.00603 | 0.01250 | 544.3 | 0.12577 |
| 43 | 15.00 | 0.00113 | 0.00599 | 0.01433 | 531.1 | 0.18143 |
| 44 | 0.50 | 0.00701 | 0.00717 | 0.01158 | 647.9 | 0.07348 |
| 45 | 1.25 | 0.00677 | 0.00718 | 0.01208 | 646.4 | 0.08517 |
| 46 | 3.75 | 0.00594 | 0.00716 | 0.01317 | 643.3 | 0.10261 |
| 47 | 15.00 | 0.00229 | 0.00715 | 0.01508 | 634.0 | 0.15137 |
| 48 | 0.50 | 0.00869 | 0.00885 | 0.01233 | 797.5 | 0.05882 |
| 49 | 1.25 | 0.00841 | 0.00882 | 0.01267 | 796.3 | 0.06544 |
| 50 | 3.75 | 0.00759 | 0.00881 | 0.01408 | 793.3 | 0.08402 |
| 51 | 15.00 | 0.00395 | 0.00881 | 0.01608 | 781.1 | 0.11945 |

f-Re Data with Rainfall, Laminar Flow, after Yoon (1970)

| Data number | Rainfall intensity (in/h) | Base flow rate (cfs/ft) | Combined flow rate (cfs/ft) | Flow depth (ft) | Reynolds number | Darcy f |
|---|---|---|---|---|---|---|
| 52 | 0.50 | 0.00977 | 0.00993 | 0.01292 | 897.2 | 0.05313 |
| 53 | 1.25 | 0.00951 | 0.00992 | 0.01325 | 895.7 | 0.06023 |
| 54 | 3.75 | 0.00867 | 0.00989 | 0.01458 | 893.0 | 0.07404 |
| 55 | 15.00 | 0.00497 | 0.00983 | 0.01650 | 880.1 | 0.10364 |

f-Re Data with rainfall, Laminar Flow, after Li (1972)

| Data number | Rainfall intensity (in/h) | Base flow rate (cfs/ft) | Combined flow rate (cfs/ft) | Flow depth (ft) | Reynolds number | Darcy f |
|---|---|---|---|---|---|---|
| 1 | 7.5 | 0.00000 | 0.00148 | 0.00950 | 127.1 | 0.66496 |
| 2 | 7.5 | 0.00382 | 0.00530 | 0.01584 | 459.3 | 0.20148 |
| 3 | 7.5 | 0.00835 | 0.00983 | 0.01960 | 864.2 | 0.10132 |
| 4 | 7.5 | 0.00056 | 0.00204 | 0.01070 | 181.6 | 0.47095 |
| 5 | 7.5 | 0.00225 | 0.00373 | 0.01173 | 334.7 | 0.23783 |
| 6 | 15.0 | 0.00000 | 0.00295 | 0.01266 | 272.5 | 0.41859 |
| 7 | 15.0 | 0.00313 | 0.00608 | 0.01669 | 557.4 | 0.19564 |
| 8 | 15.0 | 0.00176 | 0.00471 | 0.01533 | 431.8 | 0.23227 |
| 9 | 15.0 | 0.00673 | 0.00968 | 0.01784 | 880.9 | 0.12677 |
| 10 | 17.5 | 0.00000 | 0.00334 | 0.01491 | 294.5 | 0.43650 |
| 11 | 17.5 | 0.00487 | 0.00831 | 0.01952 | 706.3 | 0.19197 |
| 12 | 10.5 | 0.00000 | 0.00207 | 0.01110 | 176.6 | 0.51200 |
| 13 | 10.5 | 0.00658 | 0.00865 | 0.01899 | 739.6 | 0.13498 |
| 14 | 17.5 | 0.00371 | 0.00715 | 0.01733 | 595.6 | 0.18171 |
| 15 | 10.5 | 0.00371 | 0.00578 | 0.01616 | 487.4 | 0.19630 |
| 16 | 17.5 | 0.00000 | 0.00304 | 0.01422 | 254.7 | 0.48261 |
| 17 | 17.5 | 0.00200 | 0.00504 | 0.01837 | 422.3 | 0.31544 |
| 18 | 17.5 | 0.00552 | 0.00856 | 0.02178 | 722.2 | 0.19090 |
| 19 | 10.5 | 0.00000 | 0.00182 | 0.01092 | 160.3 | 0.66452 |
| 20 | 10.5 | 0.00779 | 0.00961 | 0.01828 | 857.5 | 0.09497 |
| 21 | 10.5 | 0.00275 | 0.00457 | 0.01509 | 405.0 | 0.24792 |
| 22 | 12.5 | 0.00000 | 0.00217 | 0.01288 | 180.7 | 0.65689 |
| 23 | 12.5 | 0.00464 | 0.00681 | 0.01816 | 619.7 | 0.16169 |
| 24 | 12.5 | 0.00774 | 0.00991 | 0.02079 | 842.0 | 0.09710 |
| 25 | 17.5 | 0.00000 | 0.00952 | 0.02271 | 736.2 | 0.15090 |
| 26 | 17.5 | 0.00000 | 0.00344 | 0.01559 | 266.3 | 0.43960 |
| 27 | 12.5 | 0.00000 | 0.00680 | 0.01891 | 558.7 | 0.20812 |
| 28 | 12.5 | 0.00000 | 0.00246 | 0.01260 | 199.1 | 0.49223 |
| 29 | 12.5 | 0.00238 | 0.00918 | 0.02102 | 743.3 | 0.15252 |
| 30 | 12.5 | 0.00238 | 0.00484 | 0.01537 | 400.3 | 0.24377 |
| 31 | 7.5 | 0.00000 | 0.00148 | 0.00829 | 126.2 | 0.74339 |
| 32 | 7.5 | 0.00382 | 0.00530 | 0.01290 | 459.3 | 0.18719 |
| 33 | 7.5 | 0.00835 | 0.00983 | 0.01519 | 864.2 | 0.09680 |
| 34 | 7.5 | 0.00056 | 0.00204 | 0.00868 | 181.6 | 0.42328 |
| 35 | 7.5 | 0.00225 | 0.00373 | 0.00104 | 334.7 | 0.27483 |
| 36 | 15.0 | 0.00000 | 0.00295 | 0.01114 | 274.5 | 0.40388 |
| 37 | 15.0 | 0.00313 | 0.00608 | 0.01416 | 553.4 | 0.20530 |
| 38 | 15.0 | 0.00176 | 0.00471 | 0.01290 | 431.8 | 0.23961 |
| 39 | 15.0 | 0.00673 | 0.00968 | 0.01566 | 880.9 | 0.11916 |
| 40 | 17.5 | 0.00000 | 0.00344 | 0.01270 | 296.6 | 0.40490 |
| 41 | 17.5 | 0.00487 | 0.00831 | 0.01711 | 711.1 | 0.17250 |
| 42 | 10.5 | 0.00000 | 0.00207 | 0.01005 | 176.7 | 0.57594 |
| 43 | 10.5 | 0.00658 | 0.00865 | 0.01501 | 739.6 | 0.11721 |
| 44 | 17.5 | 0.00371 | 0.00715 | 0.01570 | 599.6 | 0.18398 |
| 45 | 10.5 | 0.00371 | 0.00578 | 0.01361 | 487.4 | 0.18901 |
| 46 | 12.5 | 0.00000 | 0.00217 | 0.01015 | 175.9 | 0.59854 |
| 47 | 12.5 | 0.00583 | 0.00800 | 0.01527 | 670.6 | 0.14492 |
| 48 | 12.5 | 0.00116 | 0.00333 | 0.01104 | 279.1 | 0.33646 |
| 49 | 17.5 | 0.00000 | 0.00304 | 0.01224 | 245.8 | 0.46890 |
| 50 | 17.5 | 0.00534 | 0.00838 | 0.01620 | 688.4 | 0.15486 |
| 51 | 10.5 | 0.00000 | 0.00182 | 0.00809 | 149.8 | 0.67114 |

f-Re Data with rainfall, Laminar Flow, after Li (1972)

| Data number | Rainfall intensity (in/h) | Base flow rate (cfs/ft) | Combined flow rate (cfs/ft) | Flow depth (ft) | Reynolds number | Darcy f |
|---|---|---|---|---|---|---|
| 52 | 10.5 | 0.00246 | 0.00428 | 0.01139 | 351.9 | 0.25592 |
| 53 | 10.5 | 0.00537 | 0.00719 | 0.01434 | 602.9 | 0.15105 |
| 54 | 17.5 | 0.00000 | 0.00952 | 0.01806 | 742.0 | 0.15712 |
| 55 | 17.5 | 0.00000 | 0.00344 | 0.01197 | 268.4 | 0.36170 |
| 56 | 12.5 | 0.00000 | 0.00680 | 0.01487 | 555.1 | 0.18122 |
| 57 | 12.5 | 0.00000 | 0.00246 | 0.01104 | 200.8 | 0.51461 |
| 58 | 12.5 | 0.00238 | 0.00918 | 0.01592 | 746.9 | 0.14248 |
| 59 | 12.5 | 0.00238 | 0.00484 | 0.01347 | 403.0 | 0.27640 |

f-Re Data with Rainfall, Turbulent Flow, after Yoon (1970)

| Data number | Rainfall intensity (in/h) | Base Flow rate (cfs/ft) | Combined flow rate (cfs/ft) | Flow depth (ft) | Reynolds number | Darcy f |
|---|---|---|---|---|---|---|
| 1 | 0.50 | 0.02690 | 0.02706 | 0.02833 | 2498.8 | 0.03840 |
| 2 | 1.25 | 0.02680 | 0.02721 | 0.02925 | 2493.6 | 0.04091 |
| 3 | 3.75 | 0.02596 | 0.02718 | 0.02883 | 2490.9 | 0.04024 |
| 4 | 15.00 | 0.02273 | 0.02759 | 0.03042 | 2479.0 | 0.04424 |
| 5 | 0.50 | 0.04325 | 0.04341 | 0.03583 | 4008.5 | 0.02992 |
| 6 | 1.25 | 0.04340 | 0.04381 | 0.03708 | 3993.2 | 0.03147 |
| 7 | 3.75 | 0.04280 | 0.04402 | 0.03767 | 3994.1 | 0.03247 |
| 8 | 15.00 | 0.03933 | 0.04419 | 0.03833 | 3974.0 | 0.02774 |
| 9 | 0.50 | 0.05923 | 0.05939 | 0.04333 | 5499.3 | 0.03109 |
| 10 | 1.25 | 0.05980 | 0.06021 | 0.04392 | 5488.2 | 0.02946 |
| 11 | 3.75 | 0.05863 | 0.05985 | 0.04458 | 5485.4 | 0.03128 |
| 12 | 15.00 | 0.05610 | 0.06096 | 0.04550 | 5477.2 | 0.03050 |
| 13 | 0.50 | 0.02743 | 0.02759 | 0.02300 | 2492.5 | 0.04060 |
| 14 | 1.25 | 0.02725 | 0.02766 | 0.02283 | 2191.4 | 0.03844 |
| 15 | 3.75 | 0.02643 | 0.02756 | 0.02308 | 2489.2 | 0.04101 |
| 16 | 15.00 | 0.02281 | 0.02767 | 0.02350 | 2477.3 | 0.03887 |
| 17 | 0.50 | 0.04399 | 0.04415 | 0.02892 | 3988.4 | 0.03270 |
| 18 | 1.25 | 0.04394 | 0.04435 | 0.02908 | 3987.9 | 0.03119 |
| 19 | 3.75 | 0.04290 | 0.04412 | 0.02925 | 3985.1 | 0.03296 |
| 20 | 15.00 | 0.03953 | 0.04439 | 0.02908 | 3974.1 | 0.03111 |
| 21 | 0.50 | 0.06088 | 0.06104 | 0.03525 | 5539.2 | 0.02967 |
| 22 | 1.25 | 0.06030 | 0.06071 | 0.03525 | 5483.7 | 0.03000 |
| 23 | 3.75 | 0.05956 | 0.06078 | 0.03567 | 5490.1 | 0.03089 |
| 24 | 15.00 | 0.05637 | 0.06123 | 0.03408 | 5467.1 | 0.02690 |

f-Re Data with Rainfall, Turbulent Flow, after Li ( 1972)

| Data number | Rainfall intensity (in/h) | Base Flow rate (cfs/ft) | Combined flow rate (cfs/ft) | Flow depth (ft) | Reynolds number | Darcy f |
|---|---|---|---|---|---|---|
| 1 | 7.50 | 0.03951 | 0.04099 | 0.03362 | 3435.5 | 0.03910 |
| 2 | 7.50 | 0.10471 | 0.10619 | 0.05736 | 9209.5 | 0.02556 |
| 3 | 7.50 | 0.06676 | 0.06824 | 0.04401 | 6087.0 | 0.02865 |
| 4 | 7.50 | 0.04169 | 0.04317 | 0.03352 | 3850.6 | 0.03209 |
| 5 | 15.00 | 0.04415 | 0.04710 | 0.03481 | 4317.3 | 0.02931 |
| 6 | 15.00 | 0.02912 | 0.03207 | 0.02737 | 2939.6 | 0.03265 |
| 7 | 15.00 | 0.01999 | 0.02294 | 0.02397 | 2087.5 | 0.04131 |
| 8 | 17.50 | 0.05914 | 0.06258 | 0.04098 | 5281.3 | 0.03186 |
| 9 | 167.50 | 0.12076 | 0.12420 | 0.06260 | 10698.0 | 0.02363 |
| 10 | 17.50 | 0.03016 | 0.03360 | 0.02975 | 2894.3 | 0.03633 |
| 11 | 10.50 | 0.05283 | 0.05490 | 0.03869 | 4632.6 | 0.03422 |
| 12 | 10.50 | 0.03025 | 0.03232 | 0.02911 | 2802.8 | 0.03721 |
| 13 | 17.50 | 0.04019 | 0.04363 | 0.03254 | 3732.5 | 0.03265 |
| 14 | 17.50 | 0.11512 | 0.11816 | 0.06333 | 10038.9 | 0.02407 |
| 15 | 17.50 | 0.06538 | 0.06842 | 0.04409 | 5812.9 | 0.03134 |
| 16 | 17.50 | 0.02459 | 0.02763 | 0.02447 | 2363.4 | 0.03303 |
| 17 | 10.50 | 0.06765 | 0.06947 | 0.04231 | 6275.8 | 0.02765 |
| 18 | 10.50 | 0.04058 | 0.04240 | 0.03285 | 3677.6 | 0.03578 |
| 19 | 10.50 | 0.14381 | 0.14563 | 0.06679 | 12630.8 | 0.02173 |
| 20 | 10.50 | 0.03137 | 0.03319 | 0.03070 | 2878.8 | 0.04178 |
| 21 | 12.50 | 0.04715 | 0.04932 | 0.03556 | 4052.6 | 0.03341 |
| 22 | 12.50 | 0.12327 | 0.12544 | 0.06416 | 10307.3 | 0.02604 |
| 23 | 12.50 | 0.02586 | 0.02803 | 0.02647 | 2365.4 | 0.03329 |
| 24 | 12.50 | 0.08585 | 0.08802 | 0.05020 | 7328.9 | 0.02737 |
| 25 | 17.50 | 0.08585 | 0.08889 | 0.05158 | 7450.8 | 0.02595 |
| 26 | 10.50 | 0.08585 | 0.08767 | 0.05038 | 7300.0 | 0.02855 |
| 27 | 17.50 | 0.03515 | 0.04467 | 0.03619 | 3670.5 | 0.02933 |
| 28 | 17.50 | 0.03515 | 0.03859 | 0.03214 | 3171.2 | 0.03238 |
| 29 | 12.50 | 0.03677 | 0.04357 | 0.03463 | 3527.9 | 0.03668 |
| 30 | 12.50 | 0.03677 | 0.03923 | 0.03236 | 3223.5 | 0.03347 |
| 31 | 7.50 | 0.03951 | 0.04099 | 0.02615 | 3458.7 | 0.03225 |
| 32 | 7.50 | 0.10471 | 0.10619 | 0.04850 | 9339.1 | 0.02543 |
| 33 | 7.50 | 0.06676 | 0.06824 | 0.03635 | 6087.0 | 0.02724 |
| 34 | 7.50 | 0.04169 | 0.04317 | 0.02856 | 3850.6 | 0.03354 |
| 35 | 15.00 | 0.04415 | 0.04710 | 0.03006 | 4317.3 | 0.03346 |
| 36 | 15.00 | 0.02912 | 0.03207 | 0.02508 | 2939.6 | 0.03719 |
| 37 | 15.00 | 0.01999 | 0.02294 | 0.02039 | 2087.5 | 0.04333 |
| 38 | 17.50 | 0.05914 | 0.06258 | 0.03637 | 5281.3 | 0.03050 |
| 39 | 17.50 | 0.12076 | 0.12420 | 0.05411 | 10698.0 | 0.02572 |
| 40 | 17.50 | 0.03016 | 0.03360 | 0.02534 | 2864.3 | 0.03707 |
| 41 | 20.50 | 0.05283 | 0.05490 | 0.03372 | 4632.6 | 0.03406 |
| 42 | 10.50 | 0.03025 | 0.03232 | 0.02540 | 2802.8 | 0.04102 |
| 43 | 17.50 | 0.04019 | 0.04363 | 0.02939 | 3732.5 | 0.03646 |
| 44 | 12.50 | 0.04812 | 0.05029 | 0.03196 | 4202.0 | 0.03494 |
| 45 | 12.50 | 0.12466 | 0.12683 | 0.05273 | 10345.0 | 0.02193 |
| 46 | 12.50 | 0.06584 | 0.06801 | 0.03833 | 5547.3 | 0.03316 |
| 47 | 12.50 | 0.03009 | 0.03226 | 0.02565 | 2631.3 | 0.03827 |
| 48 | 17.50 | 0.05488 | 0.05792 | 0.03341 | 4567.7 | 0.03170 |
| 49 | 17.50 | 0.02682 | 0.02986 | 0.02267 | 2435.4 | 0.03835 |

f-Re Data with Rainfall, Turbulent Flow, after Li ( 1972)

| Data number | Rainfall intensity (in/h) | Base Flow rate (cfs/ft) | Combined flow rate (cfs/ft) | Flow depth (ft) | Reynolds number | Darcy f |
|---|---|---|---|---|---|---|
| 50 | 17.50 | 0.10979 | 0.11283 | 0.05431 | 9203.0 | 0.02997 |
| 51 | 10.50 | 0.05075 | 0.05257 | 0.03190 | 4288.2 | 0.03365 |
| 52 | 10.50 | 0.03369 | 0.03551 | 0.02690 | 2976.8 | 0.03858 |
| 53 | 10.50 | 0.09474 | 0.09656 | 0.04757 | 8184.8 | 0.02687 |
| 54 | 17.50 | 0.03515 | 0.04467 | 0.03004 | 3617.0 | 0.03332 |
| 55 | 17.50 | 0.03515 | 0.03859 | 0.02729 | 3150.5 | 0.03315 |
| 56 | 12.50 | 0.03677 | 0.04357 | 0.02832 | 3527.9 | 0.03182 |
| 57 | 12.50 | 0.03677 | 0.03923 | 0.02656 | 3223.5 | 0.03170 |

f-Re Data on Bermuda grass, after Chen (1976)

| Data number | Bed slope | Discharge (cfs/ft) | Depth (in) | Mean velocity (fps) | Darcy f | Reynolds number |
|---|---|---|---|---|---|---|
| 1 | 0.001 | 0.0105 | 1.717 | 0.073 | 6.813 | 696 |
| 2 | 0.001 | 0.0088 | 1.634 | 0.064 | 8.329 | 585 |
| 3 | 0.001 | 0.0073 | 1.550 | 0.056 | 10.396 | 483 |
| 4 | 0.001 | 0.0059 | 1.469 | 0.047 | 14.258 | 391 |
| 5 | 0.001 | 0.0046 | 1.406 | 0.039 | 18.942 | 309 |
| 6 | 0.001 | 0.0037 | 1.344 | 0.033 | 26.245 | 246 |
| 7 | 0.001 | 0.0026 | 1.262 | 0.025 | 41.314 | 178 |
| 8 | 0.001 | 0.0013 | 1.157 | 0.013 | 129.365 | 88 |
| 9 | 0.001 | 0.0077 | 1.769 | 0.052 | 13.567 | 515 |
| 10 | 0.001 | 0.0112 | 1.919 | 0.070 | 8.296 | 745 |
| 11 | 0.001 | 0.0156 | 2.154 | 0.087 | 6.068 | 1036 |
| 12 | 0.001 | 0.0205 | 2.370 | 0.104 | 4.679 | 1362 |
| 13 | 0.001 | 0.0260 | 2.575 | 0.121 | 3.758 | 1721 |
| 14 | 0.001 | 0.0313 | 2.694 | 0.139 | 2.975 | 2070 |
| 15 | 0.001 | 0.0375 | 2.831 | 0.159 | 2.391 | 2486 |
| 16 | 0.001 | 0.0485 | 3.027 | 0.192 | 1.754 | 3210 |
| 17 | 0.001 | 0.0636 | 3.250 | 0.235 | 1.261 | 4210 |
| 18 | 0.001 | 0.0829 | 3.445 | 0.288 | 0.886 | 5483 |
| 19 | 0.001 | 0.0955 | 3.587 | 0.319 | 0.753 | 6319 |
| 20 | 0.001 | 0.1069 | 3.662 | 0.350 | 0.639 | 7071 |
| 21 | 0.005 | 0.0012 | 0.787 | 0.019 | 216.076 | 86 |
| 22 | 0.005 | 0.0029 | 0.957 | 0.036 | 76.655 | 193 |
| 23 | 0.005 | 0.0041 | 1.076 | 0.046 | 53.173 | 277 |
| 24 | 0.005 | 0.0053 | 1.146 | 0.056 | 39.126 | 354 |
| 25 | 0.005 | 0.0067 | 1.278 | 0.063 | 33.947 | 448 |
| 26 | 0.005 | 0.0080 | 1.374 | 0.070 | 29.382 | 533 |
| 27 | 0.005 | 0.1030 | 1.571 | 0.078 | 27.196 | 681 |
| 28 | 0.005 | 0.0121 | 1.532 | 0.094 | 18.288 | 800 |
| 29 | 0.005 | 0.0195 | 1.790 | 0.131 | 11.196 | 1294 |
| 30 | 0.005 | 0.0248 | 1.951 | 0.153 | 8.931 | 1647 |
| 31 | 0.005 | 0.0350 | 2.116 | 0.198 | 5.759 | 2316 |
| 32 | 0.005 | 0.0420 | 2.209 | 0.218 | 4.962 | 2661 |
| 33 | 0.005 | 0.0514 | 2.384 | 0.258 | 3.818 | 3402 |
| 34 | 0.005 | 0.0660 | 2.477 | 0.320 | 2.593 | 4369 |
| 35 | 0.005 | 0.0909 | 2.618 | 0.416 | 1.616 | 6015 |
| 36 | 0.005 | 0.1157 | 2.551 | 0.544 | 0.923 | 7655 |
| 37 | 0.005 | 0.1459 | 2.704 | 0.647 | 0.691 | 9655 |
| 38 | 0.005 | 0.1706 | 2.871 | 0.713 | 0.605 | 11287 |
| 39 | 0.035 | 0.0096 | 1.281 | 0.090 | 117.206 | 639 |
| 40 | 0.035 | 0.0200 | 1.799 | 0.133 | 75.142 | 1328 |
| 41 | 0.035 | 0.0369 | 1.953 | 0.226 | 28.394 | 2443 |
| 42 | 0.035 | 0.0443 | 2.129 | 0.249 | 25.571 | 2930 |
| 43 | 0.035 | 0.0492 | 2.259 | 0.261 | 24.718 | 3258 |
| 44 | 0.035 | 0.0551 | 2.346 | 0.282 | 22.075 | 3648 |
| 45 | 0.035 | 0.0652 | 2.433 | 0.321 | 17.578 | 4316 |
| 46 | 0.035 | 0.0743 | 2.482 | 0.359 | 14.401 | 4914 |
| 47 | 0.035 | 0.1352 | 2.704 | 0.600 | 5.620 | 8945 |
| 48 | 0.035 | 0.1649 | 2.788 | 0.709 | 4.144 | 10907 |
| 49 | 0.035 | 0.0088 | 1.184 | 0.089 | 110.655 | 585 |

f-Re Data on Bermuda grass, after Chen (1976)

| Data number | Bed slope | Discharge (cfs/ft) | Depth (in) | Mean velocity (fps) | Darcy f | Reynolds number |
|---|---|---|---|---|---|---|
| 50 | 0.035 | 0.0072 | 1.096 | 0.078 | 131.609 | 477 |
| 51 | 0.035 | 0.0058 | 1.010 | 0.069 | 157.528 | 386 |
| 52 | 0.035 | 0.0046 | 0.946 | 0.058 | 207.550 | 305 |
| 53 | 0.035 | 0.0028 | 0.918 | 0.036 | 511.154 | 186 |
| 54 | 0.035 | 0.0017 | 0.825 | 0.025 | 954.938 | 116 |
| 55 | 0.035 | 0.0083 | 1.229 | 0.081 | 138.873 | 552 |
| 56 | 0.035 | 0.0121 | 1.408 | 0.103 | 98.920 | 801 |
| 57 | 0.087 | 0.0161 | 1.332 | 0.145 | 117.886 | 1067 |
| 58 | 0.087 | 0.0300 | 1.740 | 0.207 | 75.659 | 1990 |
| 59 | 0.087 | 0.0408 | 1.709 | 0.287 | 38.754 | 2705 |
| 60 | 0.087 | 0.0529 | 1.914 | 0.331 | 32.523 | 3499 |
| 61 | 0.087 | 0.0612 | 1.967 | 0.373 | 26.332 | 4054 |
| 62 | 0.087 | 0.0751 | 2.014 | 0.447 | 18.808 | 4970 |
| 63 | 0.087 | 0.0891 | 2.040 | 0.524 | 13.877 | 5896 |
| 64 | 0.087 | 0.1002 | 2.053 | 0.585 | 11.188 | 6628 |
| 65 | 0.087 | 0.1108 | 2.063 | 0.644 | 9.291 | 7328 |
| 66 | 0.087 | 0.1217 | 2.079 | 0.702 | 7.868 | 8054 |
| 67 | 0.087 | 0.0038 | 0.771 | 0.059 | 404.085 | 254 |
| 68 | 0.087 | 0.0054 | 0.855 | 0.076 | 274.946 | 359 |
| 69 | 0.087 | 0.0072 | 0.945 | 0.091 | 211.224 | 477 |
| 70 | 0.087 | 0.0092 | 1.009 | 0.109 | 156.253 | 612 |
| 71 | 0.087 | 0.0109 | 1.062 | 0.123 | 129.491 | 725 |
| 72 | 0.087 | 0.0038 | 0.851 | 0.054 | 542.208 | 254 |
| 73 | 0.087 | 0.0054 | 1.023 | 0.063 | 471.334 | 359 |
| 74 | 0.087 | 0.0081 | 1.119 | 0.087 | 274.288 | 539 |
| 75 | 0.087 | 0.0106 | 1.206 | 0.105 | 201.797 | 703 |
| 76 | 0.164 | 0.0260 | 1.415 | 0.220 | 102.622 | 1721 |
| 77 | 0.164 | 0.3699 | 1.626 | 0.272 | 77.184 | 2443 |
| 78 | 0.164 | 0.0536 | 1.893 | 0.340 | 57.718 | 1549 |
| 79 | 0.164 | 0.0768 | 1.976 | 0.466 | 31.999 | 5082 |
| 80 | 0.164 | 0.0918 | 1.822 | 0.604 | 17.566 | 6076 |
| 81 | 0.164 | 0.1147 | 1.825 | 0.754 | 11.316 | 7589 |
| 82 | 0.164 | 0.1331 | 1.821 | 0.877 | 8.343 | 8806 |
| 83 | 0.164 | 0.1459 | 1.815 | 0.964 | 9.426 | 10090 |
| 84 | 0.164 | 0.0211 | 1.374 | 0.184 | 142.454 | 1397 |
| 85 | 0.164 | 0.0017 | 0.575 | 0.036 | 1520.310 | 116 |
| 86 | 0.164 | 0.0041 | 0.694 | 0.072 | 469.603 | 257 |
| 87 | 0.164 | 0.0055 | 0.743 | 0.089 | 330.415 | 365 |
| 88 | 0.164 | 0.0086 | 0.929 | 0.111 | 263.367 | 571 |
| 89 | 0.164 | 0.0103 | 0.914 | 0.135 | 176.149 | 681 |
| 90 | 0.164 | 0.0129 | 0.997 | 0.155 | 144.779 | 857 |
| 91 | 0.164 | 0.0170 | 1.117 | 0.182 | 117.873 | 1126 |
| 92 | 0.164 | 0.0090 | 0.963 | 0.112 | 267.470 | 598 |
| 93 | 0.164 | 0.0103 | 1.124 | 0.109 | 328.209 | 681 |
| 94 | 0.164 | 0.0113 | 1.039 | 0.130 | 215.328 | 748 |
| 95 | 0.164 | 0.0135 | 1.136 | 0.143 | 195.210 | 898 |
| 96 | 0.164 | 0.0151 | 1.311 | 0.138 | 241.397 | 1000 |
| 97 | 0.164 | 0.0180 | 1.361 | 0.158 | 190.204 | 1192 |
| 98 | 0.164 | 0.0055 | 0.866 | 0.076 | 524.376 | 365 |

f-Re Data on Bermuda grass, after Chen (1976)

| Data number | Bed slope | Discharge (cfs/ft) | Depth (in) | Mean velocity (fps) | Darcy f | Reynolds number |
|---|---|---|---|---|---|---|
| 99 | 0.164 | 0.0076 | 0.974 | 0.094 | 384.754 | 508 |
| 100 | 0.164 | 0.0094 | 1.071 | 0.105 | 337.078 | 625 |
| 101 | 0.164 | 0.0111 | 1.135 | 0.118 | 286.119 | 740 |
| 102 | 0.164 | 0.0129 | 1.214 | 0.128 | 261.170 | 857 |
| 103 | 0.164 | 0.0152 | 1.299 | 0.140 | 231.115 | 1008 |
| 104 | 0.164 | 0.0187 | 1.413 | 0.159 | 196.888 | 1240 |
| 105 | 0.164 | 0.0024 | 0.585 | 0.049 | 836.494 | 160 |
| 106 | 0.164 | 0.0044 | 0.756 | 0.070 | 533.275 | 295 |
| 107 | 0.164 | 0.0053 | 0.837 | 0.076 | 501.479 | 354 |
| 108 | 0.164 | 0.0069 | 0.934 | 0.089 | 413.855 | 459 |
| 109 | 0.164 | 0.0093 | 0.998 | 0.112 | 279.087 | 618 |
| 110 | 0.164 | 0.0113 | 1.076 | 0.126 | 238.627 | 745 |
| 111 | 0.164 | 0.0137 | 1.160 | 0.141 | 203.873 | 906 |
| 112 | 0.164 | 0.0039 | 0.907 | 0.052 | 1155.106 | 263 |
| 113 | 0.164 | 0.0084 | 1.043 | 0.097 | 390.817 | 558 |
| 114 | 0.164 | 0.0107 | 1.106 | 0.116 | 287.433 | 711 |
| 115 | 0.164 | 0.0125 | 1.146 | 0.131 | 232.515 | 833 |
| 116 | 0.164 | 0.0160 | 1.236 | 0.155 | 179.348 | 1062 |
| 117 | 0.164 | 0.0214 | 1.344 | 0.191 | 128.849 | 1421 |
| 118 | 0.164 | 0.0026 | 0.680 | 0.046 | 1108.368 | 175 |
| 119 | 0.164 | 0.0047 | 0.795 | 0.071 | 544.609 | 314 |
| 120 | 0.164 | 0.0069 | 0.885 | 0.094 | 351.998 | 459 |
| 121 | 0.164 | 0.0097 | 0.981 | 0.119 | 242.364 | 646 |
| 122 | 0.164 | 0.0125 | 1.062 | 0.142 | 185.235 | 833 |
| 123 | 0.164 | 0.0164 | 1.153 | 0.171 | 138.627 | 1089 |
| 124 | 0.164 | 0.0224 | 1.090 | 0.246 | 63.002 | 1484 |
| 125 | 0.316 | 0.0096 | 0.687 | 0.168 | 163.614 | 639 |
| 126 | 0.316 | 0.0237 | 1.006 | 0.283 | 84.715 | 1574 |
| 127 | 0.316 | 0.0429 | 1.254 | 0.410 | 50.487 | 2839 |
| 128 | 0.316 | 0.0612 | 1.502 | 0.489 | 42.495 | 4054 |
| 129 | 0.316 | 0.0785 | 1.586 | 0.594 | 30.492 | 5196 |
| 130 | 0.316 | 0.0955 | 1.601 | 0.716 | 21.180 | 6319 |
| 131 | 0.316 | 0.1117 | 1.600 | 0.838 | 14.454 | 7393 |
| 132 | 0.316 | 0.1300 | 1.601 | 0.974 | 11.438 | 8598 |
| 133 | 0.316 | 0.1637 | 1.600 | 1.227 | 7.202 | 10832 |
| 134 | 0.316 | 0.1788 | 1.589 | 1.350 | 5.910 | 11826 |
| 135 | 0.316 | 0.0019 | 0.777 | 0.029 | 5936.334 | 128 |
| 136 | 0.316 | 0.0049 | 0.701 | 0.085 | 658.074 | 329 |
| 137 | 0.316 | 0.0074 | 0.838 | 0.107 | 495.101 | 495 |
| 138 | 0.316 | 0.0103 | 0.911 | 0.135 | 335.348 | 681 |
| 139 | 0.316 | 0.0138 | 1.015 | 0.163 | 257.697 | 914 |
| 140 | 0.316 | 0.0185 | 1.082 | 0.206 | 172.583 | 1230 |
| 141 | 0.316 | 0.0259 | 1.181 | 0.263 | 115.357 | 1717 |
| 142 | 0.316 | 0.0024 | 0.629 | 0.047 | 1912.662 | 164 |
| 143 | 0.316 | 0.0047 | 0.640 | 0.089 | 547.837 | 314 |
| 144 | 0.316 | 0.0075 | 0.608 | 0.149 | 184.899 | 501 |
| 145 | 0.316 | 0.0094 | 0.735 | 0.154 | 209.870 | 625 |
| 146 | 0.316 | 0.0122 | 0.804 | 0.182 | 164.023 | 809 |
| 147 | 0.316 | 0.0173 | 1.003 | 0.206 | 158.917 | 1144 |

f-Re Data on Bermuda grass, after Chen (1976)

| Data number | Bed slope | Discharge (cfs/ft) | Depth (in) | Mean velocity (fps) | Darcy f | Reynolds number |
|---|---|---|---|---|---|---|
| 148 | 0.316 | 0.0211 | 1.027 | 0.247 | 114.099 | 1400 |
| 149 | 0.316 | 0.0038 | 0.792 | 0.058 | 1583.515 | 254 |
| 150 | 0.316 | 0.0069 | 0.736 | 0.113 | 390.015 | 459 |
| 151 | 0.316 | 0.0100 | 0.825 | 0.146 | 259.998 | 667 |
| 152 | 0.316 | 0.0138 | 0.773 | 0.214 | 113.816 | 914 |
| 153 | 0.555 | 0.0184 | 0.528 | 0.419 | 35.720 | 1220 |
| 154 | 0.555 | 0.0266 | 0.500 | 0.639 | 14.547 | 1763 |
| 155 | 0.555 | 0.0059 | 0.708 | 0.100 | 838.192 | 391 |
| 156 | 0.555 | 0.0101 | 0.803 | 0.152 | 412.302 | 6784 |
| 157 | 0.555 | 0.0133 | 0.708 | 0.225 | 165.793 | 881 |
| 158 | 0.555 | 0.0157 | 0.786 | 0.240 | 161.431 | 1044 |
| 159 | 0.555 | 0.0170 | 0.845 | 0.241 | 172.175 | 1126 |
| 160 | 0.555 | 0.0227 | 0.966 | 0.282 | 143.691 | 1506 |
| 161 | 0.555 | 0.0327 | 0.823 | 0.477 | 42.911 | 2168 |
| 162 | 0.555 | 0.0295 | 0.968 | 0.366 | 85.865 | 1954 |
| 163 | 0.555 | 0.0216 | 0.905 | 0.286 | 130.917 | 1432 |
| 164 | 0.555 | 0.0171 | 1.020 | 0.201 | 297.692 | 1135 |
| 165 | 0.555 | 0.0127 | 0.905 | 0.168 | 379.661 | 841 |
| 166 | 0.555 | 0.0098 | 0.821 | 0.144 | 470.430 | 653 |
| 167 | 0.555 | 0.0057 | 0.706 | 0.097 | 880.628 | 381 |
| 168 | 0.555 | 0.0221 | 0.643 | 0.413 | 44.830 | 1466 |
| 169 | 0.555 | 0.0337 | 0.694 | 0.583 | 24.257 | 2233 |
| 170 | 0.555 | 0.0457 | 0.896 | 0.611 | 28.477 | 3023 |
| 171 | 0.555 | 0.0536 | 0.906 | 0.709 | 21.407 | 3549 |
| 172 | 0.555 | 0.0620 | 0.998 | 0.745 | 21.363 | 4106 |
| 173 | 0.555 | 0.0726 | 1.050 | 0.829 | 18.143 | 4803 |
| 174 | 0.555 | 0.0794 | 1.050 | 0.907 | 15.193 | 5253 |
| 175 | 0.555 | 0.0873 | 1.111 | 0.942 | 14.885 | 5777 |
| 176 | 0.555 | 0.0964 | 0.907 | 1.276 | 6.625 | 6380 |
| 177 | 0.555 | 0.1069 | 0.750 | 1.709 | 3.052 | 7071 |
| 178 | 0.555 | 0.1187 | 1.183 | 1.204 | 9.706 | 7854 |
| 179 | 0.555 | 0.1279 | 0.794 | 1.924 | 2.562 | 8461 |
| 180 | 0.555 | 0.1384 | 0.835 | 1.987 | 2.515 | 9156 |
| 181 | 0.555 | 0.1514 | 0.851 | 2.134 | 2.223 | 100178 |
| 182 | 0.555 | 0.1592 | 0.859 | 2.223 | 2.068 | 10533 |
| 183 | 0.555 | 0.1718 | 0.895 | 2.302 | 2.009 | 11363 |
| 184 | 0.555 | 0.1764 | 0.879 | 2.406 | 1.807 | 11671 |
| 185 | 0.555 | 0.1799 | 0.903 | 2.390 | 1.880 | 11904 |
| 186 | 0.555 | 0.0092 | 0.750 | 0.147 | 408.776 | 612 |
| 187 | 0.555 | 0.0187 | 0.584 | 0.385 | 46.884 | 1240 |
| 188 | 0.555 | 0.0264 | 0.530 | 0.599 | 17.586 | 1751 |
| 189 | 0.555 | 0.0337 | 0.932 | 0.434 | 58.832 | 2233 |

f-Re Data on Bermuda grass, after Ree and Palmer (1949)

Trapezoidal Shape, Bottom Width 1.5 ft, Bed Slope 24%

| Data number | Discharge (cfs) | Velocity (fps) | Effective slope | Darcy f | Reynolds number |
|---|---|---|---|---|---|
| 1 | 0.950 | 3.090 | 0.2345 | 0.921 | 37300 |
| 2 | 1.850 | 4.300 | 0.2308 | 0.607 | 67200 |
| 3 | 2.900 | 5.300 | 0.2276 | 0.469 | 98600 |
| 4 | 3.750 | 5.580 | 0.2346 | 0.481 | 114000 |
| 5 | 4.900 | 6.200 | 0.1932 | 0.356 | 141000 |
| 6 | 2.900 | 5.320 | 0.2262 | 0.461 | 98500 |
| 7 | 5.020 | 6.620 | 0.2135 | 0.321 | 140000 |
| 8 | 3.030 | 5.660 | 0.2350 | 0.447 | 103000 |
| 9 | 5.320 | 7.540 | 0.2287 | 0.291 | 161000 |
| 10 | 7.320 | 7.730 | 0.2307 | 0.346 | 202000 |

Trapezoidal Shape, Bottom Width 1.5 ft, Bed Slope 20%

| Data number | Discharge (cfs) | Velocity (fps) | Effective slope | Darcy f | Reynolds number |
|---|---|---|---|---|---|
| 1 | 4.200 | 5.010 | 0.1926 | 0.567 | 139000 |
| 2 | 6.500 | 6.660 | 0.1944 | 0.357 | 205000 |
| 3 | 9.850 | 7.770 | 0.1954 | 0.316 | 286000 |
| 4 | 13.400 | 8.640 | 0.1931 | 0.294 | 377000 |
| 5 | 17.300 | 9.480 | 0.1974 | 0.276 | 419000 |
| 6 | 21.600 | 9.880 | 0.1964 | 0.281 | 493000 |
| 7 | 21.300 | 10.000 | 0.2049 | 0.277 | 485000 |
| 8 | 27.300 | 9.310 | 0.1940 | 0.362 | 415000 |
| 9 | 9.510 | 2.120 | 0.1954 | 0.530 | 27400 |
| 10 | 30.200 | 4.120 | 0.1979 | 0.541 | 67500 |
| 11 | 4.680 | 5.040 | 0.1961 | 0.415 | 94000 |
| 12 | 9.400 | 6.720 | 0.1994 | 0.299 | 158000 |
| 13 | 14.260 | 7.980 | 0.2012 | 0.252 | 226000 |
| 14 | 19.170 | 8.850 | 0.1990 | 0.225 | 279000 |
| 15 | 23.650 | 9.890 | 0.2062 | 0.194 | 337000 |
| 16 | 29.310 | 10.080 | 0.1977 | 0.204 | 378000 |
| 17 | 4.570 | 4.080 | 0.1978 | 0.728 | 89800 |

Trapezoidal Shape, Bottom Width 1.5 ft, Bed Slope 10%

| Data number | Discharge (cfs) | Velocity (fps) | Effective slope | Darcy f | Reynolds number |
|---|---|---|---|---|---|
| 1 | 4.650 | 4.090 | 0.0916 | 0.509 | 141000 |
| 2 | 7.120 | 4.970 | 0.0906 | 0.380 | 192000 |
| 3 | 10.000 | 5.660 | 0.0907 | 0.333 | 257000 |
| 4 | 13.500 | 6.400 | 0.0906 | 0.288 | 322000 |
| 5 | 17.900 | 7.070 | 0.0884 | 0.246 | 381000 |
| 6 | 23.000 | 7.800 | 0.0874 | 0.216 | 449000 |

Trapezoidal Shape, Bottom Width 1.5 ft, Bed Slope 10%

| Data number | Discharge (cfs) | Velocity (fps) | Effective slope | Darcy f | Reynolds number |
|---|---|---|---|---|---|
| 7 | 28.100 | 8.060 | 0.0845 | 0.217 | 512000 |
| 8 | 26.100 | 8.510 | 0.0842 | 0.187 | 362000 |
| 9 | 25.900 | 8.740 | 0.0872 | 0.178 | 363000 |
| 10 | 26.300 | 8.700 | 0.0880 | 0.184 | 372000 |
| 11 | 26.300 | 8.820 | 0.0846 | 0.171 | 369000 |
| 12 | 26.100 | 8.900 | 0.0857 | 0.168 | 349000 |
| 13 | 1.040 | 0.940 | 0.1024 | 7.150 | 20300 |
| 14 | 2.960 | 1.940 | 0.1009 | 1.970 | 46200 |
| 15 | 4.940 | 2.650 | 0.0988 | 1.120 | 73900 |
| 16 | 9.840 | 3.900 | 0.0985 | 0.606 | 126000 |
| 17 | 15.120 | 4.980 | 0.0982 | 0.411 | 183000 |
| 18 | 20.820 | 5.930 | 0.0966 | 0.304 | 234000 |
| 19 | 25.840 | 6.560 | 0.0974 | 0.266 | 284000 |
| 20 | 30.440 | 7.070 | 0.0964 | 0.237 | 306000 |
| 21 | 35.460 | 7.510 | 0.0980 | 0.214 | 333000 |
| 22 | 0.979 | 1.580 | 0.1010 | 1.770 | 19300 |
| 23 | 2.820 | 2.960 | 0.1012 | 0.649 | 47100 |
| 24 | 4.710 | 3.900 | 0.1000 | 0.432 | 74300 |
| 25 | 9.930 | 5.670 | 0.0984 | 0.241 | 117000 |
| 26 | 14.700 | 6.560 | 0.0980 | 0.204 | 151000 |
| 27 | 19.800 | 7.440 | 0.0999 | 1.783 | 180000 |
| 28 | 24.600 | 8.060 | 0.0977 | 0.155 | 217000 |
| 29 | 29.800 | 8.740 | 0.1002 | 0.145 | 292000 |

Trapezoidal Shape, Bottom Width Varies , Bed Slope 3%

| Data number | Discharge (cfs) | Velocity (fps) | Effective slope | Darcy f | Reynolds number |
|---|---|---|---|---|---|
| 1 | 4.090 | 1.630 | 0.0324 | 1.370 | 68300 |
| 2 | 4.090 | 1.690 | 0.0322 | 1.230 | 72600 |
| 3 | 6.870 | 2.340 | 0.0319 | 0.729 | 114000 |
| 4 | 9.760 | 2.790 | 0.0318 | 0.575 | 145000 |
| 5 | 14.000 | 3.500 | 0.0318 | 0.397 | 198000 |
| 6 | 18.800 | 4.100 | 0.0318 | 0.314 | 244000 |
| 7 | 23.300 | 4.540 | 0.0323 | 0.279 | 289000 |
| 8 | 29.000 | 5.030 | 0.0312 | 0.235 | 3580900 |
| 9 | 0.093 | 0.226 | 0.0319 | 18.400 | 1990 |
| 10 | 0.215 | 0.301 | 0.0320 | 16.000 | 4100 |
| 11 | 0.356 | 0.363 | 0.0327 | 14.700 | 6090 |
| 12 | 0.561 | 0.432 | 0.0323 | 12.600 | 9430 |
| 13 | 0.748 | 0.530 | 0.0314 | 8.650 | 11500 |
| 14 | 1.040 | 0.687 | 0.0308 | 5.320 | 17000 |
| 15 | 1.760 | 1.050 | 0.0312 | 2.510 | 28300 |
| 16 | 2.690 | 1.440 | 0.0321 | 1.470 | 42300 |
| 17 | 4.380 | 2.030 | 0.0329 | 0.845 | 66500 |
| 18 | 3.890 | 1.880 | 0.0337 | 0.949 | 62500 |
| 19 | 1.050 | 0.990 | 0.0322 | 1.940 | 19800 |
| 20 | 2.960 | 2.080 | 0.0328 | 0.560 | 53100 |

Trapezoidal Shape, Bottom Width Varies , Bed Slope 3%

| Data number | Discharge (cfs) | Velocity (fps) | Effective slope | Darcy f | Reynolds number |
|---|---|---|---|---|---|
| 21 | 4.920 | 2.820 | 0.0332 | 0.362 | 75300 |
| 22 | 9.860 | 4.140 | 0.0352 | 0.223 | 128000 |
| 23 | 14.940 | 4.940 | 0.0356 | 0.187 | 163000 |
| 24 | 20.630 | 5.630 | 0.0355 | 0.161 | 226000 |
| 25 | 25.940 | 6.070 | 0.0354 | 0.151 | 272000 |
| 26 | 28.470 | 6.260 | 0.0354 | 0.148 | 294000 |
| 27 | 35.420 | 6.720 | 0.0348 | 0.137 | 337000 |
| 28 | 3.990 | 2.320 | 0.0359 | 0.753 | 93900 |
| 29 | 6.510 | 2.930 | 0.0361 | 0.546 | 141000 |
| 30 | 9.910 | 3.570 | 0.0370 | 0.426 | 193000 |
| 31 | 13.700 | 4.130 | 0.0365 | 0.347 | 253000 |
| 32 | 18.500 | 4.650 | 0.0354 | 0.288 | 313000 |
| 33 | 24.200 | 5.080 | 0.0352 | 0.264 | 377000 |
| 34 | 30.300 | 5.370 | 0.0352 | 0.260 | 439000 |
| 35 | 3.950 | 2.460 | 0.0350 | 0.618 | 91300 |
| 36 | 1.090 | 1.720 | 0.0318 | 0.693 | 38000 |
| 37 | 2.930 | 2.850 | 0.0335 | 0.346 | 82700 |
| 38 | 4.860 | 3.350 | 0.0346 | 0.312 | 118000 |
| 39 | 9.850 | 4.460 | 0.0340 | 0.217 | 205000 |
| 40 | 15.200 | 5.180 | 0.0341 | 0.192 | 279000 |
| 41 | 20.200 | 5.680 | 0.0344 | 0.179 | 341000 |
| 42 | 24.600 | 6.040 | 0.0342 | 0.169 | 389000 |
| 43 | 29.800 | 6.300 | 0.0348 | 0.170 | 379000 |
| 44 | 34.800 | 6.570 | 0.0348 | 0.166 | 433000 |
| 45 | 0.939 | 0.660 | 0.0314 | 5.180 | 17600 |
| 46 | 2.980 | 1.420 | 0.0310 | 1.400 | 458700 |
| 47 | 4.680 | 1.840 | 0.0312 | 0.919 | 64200 |
| 48 | 9.440 | 2.640 | 0.0315 | 0.539 | 109000 |
| 49 | 14.390 | 3.290 | 0.0312 | 0.384 | 153000 |
| 50 | 19.660 | 3.780 | 0.0312 | 0.315 | 174000 |

Trapezoidal Shape, Bottom Width 1.5 ft, Bed Slope 1%

| Data number | Discharge (cfs) | Velocity (fps) | Effective slope | Darcy f | Reynolds number |
|---|---|---|---|---|---|
| 1 | 0.980 | 0.606 | 0.0098 | 2.080 | 13100 |
| 2 | 2.820 | 1.070 | 0.0097 | 0.845 | 29700 |
| 3 | 4.740 | 1.400 | 0.0103 | 0.604 | 46500 |
| 4 | 9.920 | 2.050 | 0.0108 | 0.359 | 74700 |
| 5 | 14.630 | 2.460 | 0.0111 | 0.285 | 97800 |
| 6 | 19.680 | 2.820 | 0.0101 | 0.211 | 116000 |
| 7 | 24.670 | 3.090 | 0.0102 | 0.190 | 140000 |
| 8 | 30.000 | 3.400 | 0.0102 | 0.163 | 189000 |

Rectangular Shape, Bottom Width Varies , Bed Slope 3%

| Data number | Discharge (cfs) | Velocity (fps) | Effective slope | Darcy f | Reynolds number |
|---|---|---|---|---|---|
| 1 | 0.099 | 0.643 | 0.0297 | 2.850 | 6020 |
| 2 | 0.306 | 1.410 | 0.0301 | 0.850 | 15900 |
| 3 | 0.471 | 1.910 | 0.0294 | 0.512 | 29500 |
| 4 | 0.694 | 2.420 | 0.0290 | 0.364 | 44500 |
| 5 | 1.130 | 3.340 | 0.0278 | 0.218 | 71900 |
| 6 | 1.430 | 3.580 | 0.0278 | 0.222 | 91300 |
| 7 | 1.660 | 3.840 | 0.0279 | 0.210 | 106000 |
| 8 | 2.120 | 4.400 | 0.0278 | 0.179 | 137000 |
| 9 | 2.920 | 5.170 | 0.0276 | 0.150 | 193000 |
| 10 | 4.880 | 6.450 | 0.0267 | 0.126 | 328000 |
| 11 | 6.370 | 7.050 | 0.0264 | 0.124 | 405000 |
| 12 | 7.810 | 7.680 | 0.0270 | 0.121 | 505000 |
| 13 | 0.900 | 0.363 | 0.0298 | 7.340 | 2780 |
| 14 | 0.304 | 0.896 | 0.0302 | 1.660 | 9380 |
| 15 | 0.689 | 1.620 | 0.0304 | 0.644 | 22400 |
| 16 | 0.440 | 2.520 | 0.0300 | 0.354 | 46600 |
| 17 | 2.120 | 3.200 | 0.0298 | 0.254 | 69900 |
| 18 | 2.900 | 3.740 | 0.0298 | 0.217 | 97600 |
| 19 | 4.860 | 4.840 | 0.0303 | 0.169 | 167000 |
| 20 | 6.450 | 5.490 | 0.0304 | 0.154 | 211000 |
| 21 | 7.850 | 6.000 | 0.0309 | 0.145 | 261000 |
| 22 | 10.600 | 6.820 | 0.0310 | 0.138 | 345000 |
| 23 | 13.400 | 7.500 | 0.0307 | 0.129 | 435000 |
| 24 | 0.300 | 0.568 | 0.0298 | 3.610 | 5250 |
| 25 | 0.686 | 1.090 | 0.0296 | 1.160 | 12600 |
| 26 | 1.440 | 1.780 | 0.0296 | 0.560 | 26300 |
| 27 | 2.120 | 2.270 | 0.0295 | 0.398 | 39600 |
| 28 | 4.860 | 3.660 | 0.0294 | 0.217 | 93800 |
| 29 | 7.790 | 4.690 | 0.0300 | 0.168 | 148000 |
| 30 | 13.450 | 6.120 | 0.0300 | 0.131 | 248000 |
| 31 | 17.100 | 6.840 | 0.0294 | 0.117 | 313000 |
| 32 | 21.950 | 7.360 | 0.0284 | 0.117 | 424000 |
| 33 | 24.000 | 7.480 | 0.0270 | 0.115 | 473000 |
| 34 | 0.300 | 0.363 | 0.0292 | 7.910 | 3090 |
| 35 | 1.140 | 1.000 | 0.0294 | 1.390 | 11700 |
| 36 | 2.120 | 1.590 | 0.0293 | 0.669 | 22800 |
| 37 | 4.840 | 2.680 | 0.0294 | 0.315 | 55400 |
| 38 | 10.800 | 4.250 | 0.0305 | 0.184 | 115000 |
| 39 | 19.100 | 66.150 | 0.0332 | 0.118 | 207000 |
| 40 | 22.000 | 6.020 | 0.0316 | 0.137 | 251000 |
| 41 | 23.900 | 6.200 | 0.0312 | 0.135 | 277000 |

Trapezoidal Shape, Bottom Width 4.0 ft, Bed Slope .2%

| Data number | Discharge (cfs) | Velocity (fps) | Effective slope | Darcy f | Reynolds number |
|---|---|---|---|---|---|
| 1 | 1.180 | 0.353 | 0.0188 | 2.150 | 19400 |
| 2 | 1.290 | 0.372 | 0.0174 | 1.780 | 17100 |

Trapezoidal Shape, Bottom Width 4.0 ft, Bed Slope .2%

| Data number | Discharge (cfs) | Velocity (fps) | Effective slope | Darcy f | Reynolds number |
|---|---|---|---|---|---|
| 3 | 2.750 | 0.597 | 0.0202 | 0.957 | 39100 |
| 4 | 4.720 | 0.813 | 0.0208 | 0.624 | 61300 |
| 5 | 4.750 | 0.814 | 0.0192 | 0.565 | 51600 |
| 6 | 10.100 | 1.180 | 0.0213 | 3.369 | 114000 |
| 7 | 14.900 | 1.400 | 0.0199 | 0.282 | 157000 |
| 8 | 15.100 | 1.400 | 0.0185 | 0.269 | 124000 |
| 9 | 20.200 | 1.550 | 0.0178 | 0.229 | 192000 |
| 10 | 25.100 | 1.640 | 0.0141 | 0.177 | 214000 |
| 11 | 24.700 | 1.620 | 0.0144 | 0.179 | 173000 |
| 12 | 30.400 | 1.710 | 0.0127 | 0.158 | 251000 |
| 13 | 34.800 | 1.710 | 0.0111 | 0.144 | 272000 |
| 14 | 35.700 | 1.700 | 0.0108 | 0.149 | 214000 |