A DIMENSIONLESS RATING CURVE

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Studies of the hydraulics and morphology of rivers point toward the concept that the river channel is formed and maintained by what in irrigation practice is called the dominant discharge. This effective discharge is assumed to correspond approximately to the bankfull condition. For comparison of one river with another the characteristics of the bankfull condition are of considerable interest.

The consistency of the hydraulic geometry of natural channels suggests that there should be a similar consistency among the characteristics of bankfull stage, but attempts to make such an analysis have been frustrated by the fact that no reproducible method of obtaining a value for the bankfull stage had been devised. Considerable work went into obtaining an average cross section and we used that average to find the bankfull stage, but there is sufficient variability, owing to the occurrence of pools and riffles and of low terraces near the flood plain, that an average condition was not possible to specify.

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A different approach to the problem of the average condition was taken by L. B. Leopold and R. M. Myrick who found that comparable results could be obtained by different operators if the mean bed elevation and the mean height of banks were determined by two longitudinal profiles, one down the center of the stream and one down the adjacent flood plain. These profiles in connection with a simple planimetric map near a gaging station are sufficient to provide the required measurements.

Through Surface Water Branch memorandum No. 60.12 (August 13, 1959) the cooperation of the field offices was obtained to make a survey of gaging stations on streams of moderate size which had a sufficiently long record and certain physical characteristics, including a well-developed flood plain. The District Offices classified approximately 100 stations as initial possibilities for such surveys and in 1959 a small amount of money was available for allocation to obtain such surveys at 13 of the selected sites. The present paper reports on certain aspects of the data obtained from these 13 stations.

Following a suggestion by Walter B. Langbein the data were plotted as a dimensionless graph of depth to discharge, specifically the ratio of depth to bankfull depth is plotted against the ratio of discharge to bankfull discharge (see Fig. 1). The method of analysis dictates that the curve pass through the point where H/H_b and Q/Q_b equal one. At depths greater than bankfull stage the flow is no longer confined to the channel and the depth ratio is shown to increase less rapidly as discharges exceed that of bankfull.

Bankfull stage was determined by projecting longitudinal profiles of the flood plain and channel bed to the location of the gaging station. The intersection of these profile lines with the cross-section of the gaging station constituted the basis for determining the gage height of bankfull stage. At most of the stations these lines were parallel and the vertical distance separating them is the height of the stream banks. From this information discharges at various ratios of bankfull stage were obtained from the station rating curve. The ratio of these discharges to the bankfull discharge and the ratio of corresponding depths to bankfull depth are the points plotted in Fig. 1. Each station is represented by an equal number of plotted points. For discharges and stages less than bankfull the suggested relationship between stage and discharge is

$$\frac{D}{D_b} = \left(\frac{Q}{Q_b}\right)^{0.5}$$

and for flows equal or greater than bankfull

$$\frac{D}{D_b} = \left(\frac{Q}{Q_b}\right)^{0.4}$$

Values of the discharge ratio were entered into flood frequency curves of the stations and average values of recurrence interval are also shown in Fig. 1. These data can be cross-plotted (see Fig. 2) to yield an expression for the frequency at which a certain overbank flow may be expected to occur. The available data suggest the following relation:

$$\text{R.I. (Recurrence interval, years)} = 1.5\left(\frac{D}{D_b}\right)^{6.0}$$

The data from these stations confirm a previous finding that on the average bankfull stage occurs once every 1.5 years, but the value varies between about 1.2 to 2.0..

That a general curve can satisfy the data for stages above bankfull indicates that on the average valley width as well as stream width is related to the size of the drainage area.

The frequency at which a large stream attains a given depth of water over the flood plain is greater for large rivers than for small streams. For example, if the flood-damage stage is 4 feet above bankfull the recurrence interval for the Green River at Munfordville, Ky. (D.A. = 1790 sq. mi.) is 3.3 years while for Pymatuning Creek near Orangeville, Penn. (D.A. = 169 sq. mi.) is 24 years. This is generalized in Figure 2.

It will be understood that the generalized relationships presented here are derived from a small number of examples. Field officers are urged to try this curve, using their own data, to see whether it is useful under a variety of conditions. It must be understood, however, that consistency of the present points depends greatly on a consistent method of determining the bankfull stage. The same curve applied previously to a variety of field data in which the present method for determination of bankfull stage was not used showed a large amount of scatter.

For comparison with the present curve one practical way of determining the bankfull stage in the office would be to assume that the bankfull stage corresponds to that depth which has a recurrence interval of 1.5 years. What is needed to give a further test of the present relation is a larger number of stations for which the surveys described in Surface Water Branch memorandum No. 60.12 are available.

The authors would be very pleased to have comments on the suggestions made here either by letters to us or to the Editor, Water Resources Bulletin.

Table 1.- Compilation of Bankfull Stage Data
13 Selected Gaging Station Sites

River and location	Drainage area (sq. mi.)	Discharge (cfs)	Height of banks (ft)	Mean* Depth (ft)	Width (ft)	Mean Velocity (fps)	Recurrence interval (yrs)
Ipswich River S.Middleton, Mass.	43.4	550	7.4	4.0	54	2.55	NA
Sewee Creek Decatur, Tenn.	117	4,900	15.0	8.0	107	5.72	1.38
Platte River Rockville, Wisc.	139	3,700	7.5	4.84	225	3.44	1.40
Pymatuning Creek Orangeville, Penn.	169	1,070	6.8	2.6	56	7.35	1.01
S. Chickamauga Creek, Ga.-Tenn. Line	249	2,250	9.0	8.9	93	2.72	1.01
Taunton River State Farm, Mass.	260	1,820	10.2	9.4	110	1.76	NA
Heart River S. Heart, N. Dak.	315	2,950	19.2	10.4	110	2.56	3.85
Red Cedar River, Colfax, Wisc.	1,100	4,200	5.8	5.5	145	5.27	1.45
Elk River Prospect, Tenn.	1,784	23,600	29.0	19.5	225	5.39	1.13
Green River Munfordville, Ky.	1,790	20,500	28.3	23.0	325	2.74	1.40
Licking River Catawba, Ky.	3,250	59,000	35.1	26.5	333	6.69	3.50
Rock River Afton, Wisc.	3,300	10,200	11.1	8.5	240	5.00	7.0
James River Forestburg, S. Dak.	13,810	1,570	8.6	7.5	190	1.10	2.18

*Mean Depth = Flow Area/Width

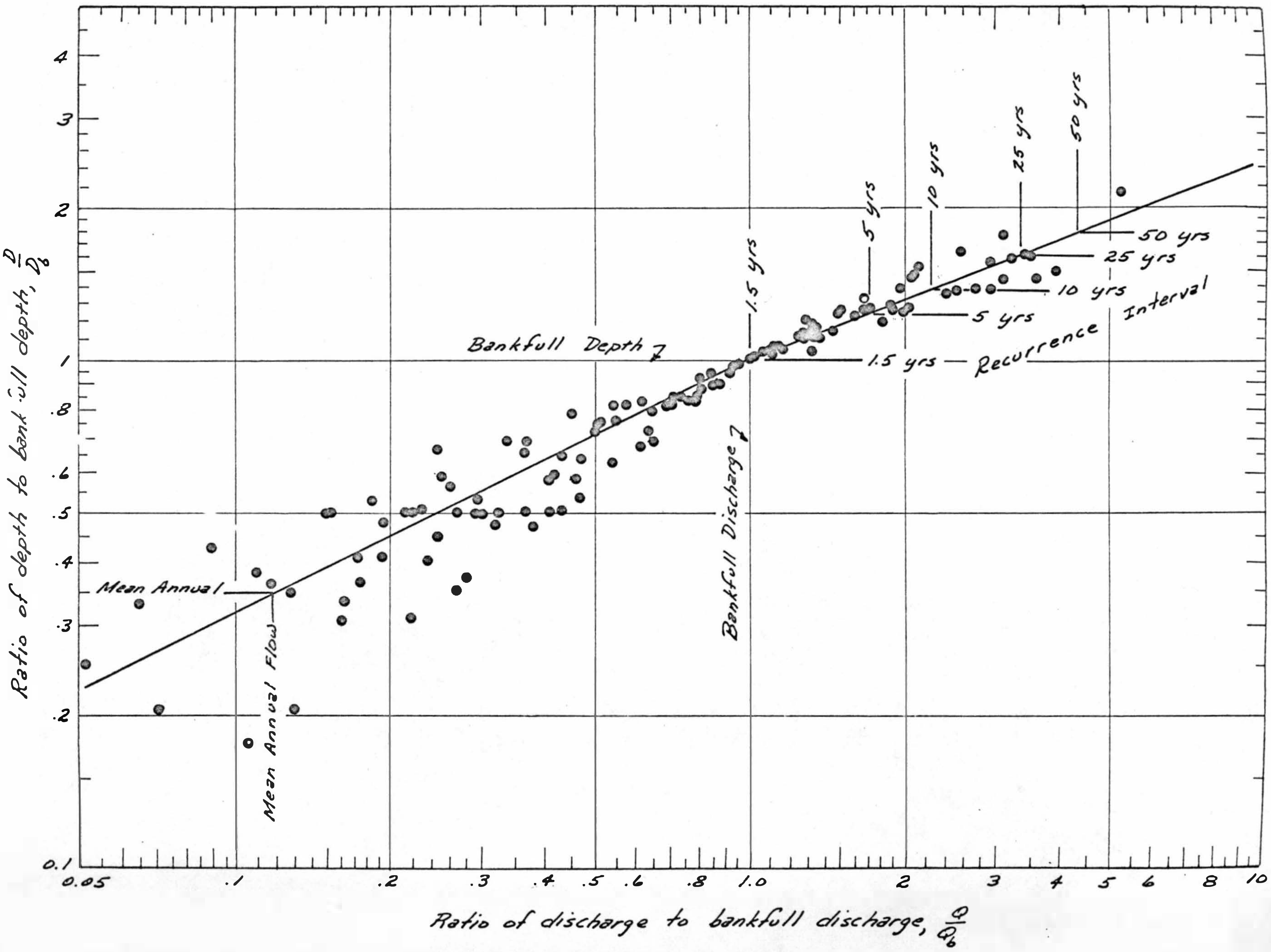

Fig. 1: Dimensionless Rating Curve.

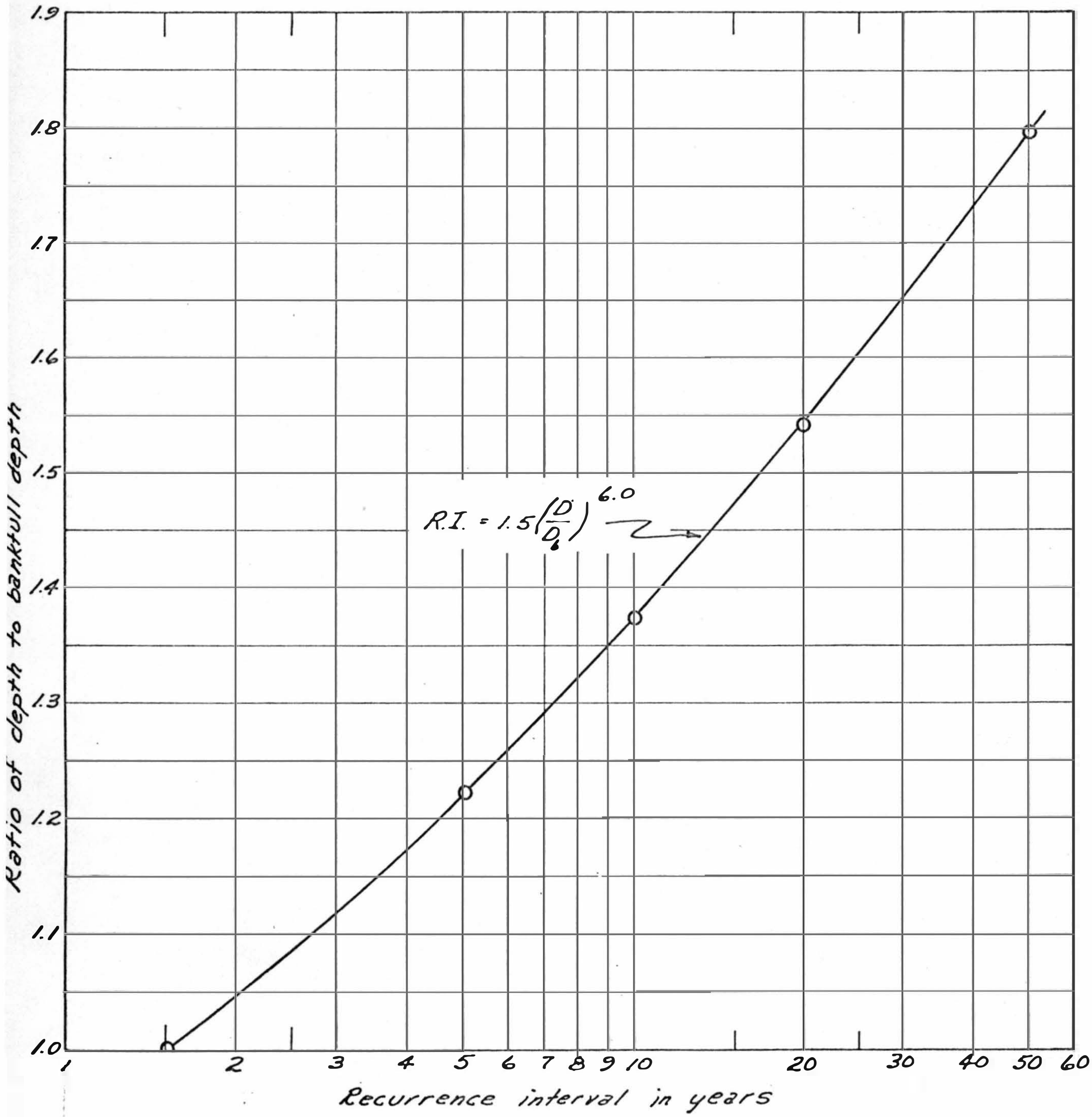

Fig. 2- Recurrence interval as a function of the ratio of depth to bankfull depth.