SUMMARY OF AN ANALYTICAL APPROACH TO TWO--AND THREE--DIMENSIONAL SCOUR PROBLEMS

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This summary outline of the analytical mechanics of scour is based on:

1. The boundary layer theory as developed by Schlichting and Truckenbrodt (1), (2) and (3).
2. The assumption that for a submerged-diffused jet impinging upon a normal boundary, the Bernoulli Equation is valid in the neighborhood of the stagnation point.
3. Experimental results on the diffusion of a submerged jet by Albertson (4), Homma (5) and Rouse (6).
4. Experimental results on scour by Rouse (7), Thomas (8), Hallmark (9), Doddiah (10), Smith (11), and Laursen (12).

Generalized equations derived by theoretical means are given for the phenomena of scour for both the two-dimensional jet and the three-dimensional jet. For both cases definition sketches are then given to illustrate the various possible types of jet flow causing scour.

I. Two-dimensional Case.

A. Submerged jet.

1. Vertical to the scour bed.

a. Uniform bed material.
b. Non-uniform bed material.
Experimental investigator - Rouse (7).
Dimensionless parameters.

$$\phi\left(\frac{V_o^2}{\left(\frac{\sigma}{\rho}-1\right)g d_s};\ \frac{b}{B_o};\ \frac{r}{b};\ \frac{b+Z_s}{B_o};\ \frac{d_s}{B_o};\ \frac{V_o t}{b}\right) = 0$$

Generalized equations in dimensionless terms

$$\frac{b+Z_s}{B_o} = \left[\left(2m+\frac{29}{9}\right)\xi + \left(\frac{b}{B_o}\right)^{\frac{18m+29}{20}}\right]^{\frac{20}{18m+29}} \qquad (1)$$

or

$$\log\left(\frac{b+Z_s}{B_o}\right) = Y\left[\log\left(\frac{d_s}{B_o}\ \frac{V_o t}{b}\right) + \log Z\right] \qquad (2)$$

2. Inclined to the scour bed.

a. Uniform bed material.
b. Non-uniform bed material.

B. Non-submerged jet.

1. Vertical to the scour bed.

a. Uniform bed material.
b. Non-uniform bed material.

2. Inclined to the scour bed.

a. Uniform bed material.
b. Non-uniform bed material.
Experimental investigator - Thomas (8), Hallmark (9)
Dimensionless parameters

$$\phi_2\left[\frac{\left(V_o \sin\theta\right)^2}{\left(\frac{\sigma}{\rho}-1\right) g\, d_s} ; \frac{b}{B_o \sin\theta} ; \theta ; \frac{r}{b} ; \frac{Z_s}{B_o \sin\theta} ; \frac{d_s}{B_o} \frac{V_o t}{b}\right] = 0$$

Note: $V_o \sin\theta = V_o'$

Generalized equations in dimensionless terms

$$\frac{Z_s}{B_o \sin\theta} = \frac{10}{9(2m+1)\beta} \ln\left(\eta' + 1\right) \tag{3}$$

or

$$\frac{Z_s}{B_o \sin\theta} = Y'\left[\log\left(\frac{d_s}{B_o} \frac{V_o' t}{b}\right) + \log Z'\right] \tag{4}$$

II. Three-dimensional case

A. Finite boundary.

1. Submerged jet.

a. Vertical to scour bed.

(1) Uniform bed material.
(2) Non-uniform bed material.

b. Inclined to scour bed.

(1) Uniform bed material.
(2) Non-uniform bed material.

2. Non-submerged jet.

a. Vertical to scour bed.

(1) Uniform bed material.
(2) Non-uniform bed material.

b. Inclined to scour bed.

(1) Uniform bed material.
(2) Non-uniform bed material.
Experimental investigator - Bureau of Public Roads. (Project Study No. 735)
Dimensionless parameters

$$\phi_3\left[\frac{(W_o \sin\theta)^2}{\left(\frac{\sigma}{\rho}-1\right) g\, d_s} ; \frac{b}{d \sin\theta} ; \theta ; \frac{r}{b} ; \frac{Z_s}{d \sin\theta} ; \frac{d_s}{d \sin\theta} \frac{W_o' t}{b}\right] = 0$$

Note: $W_o \sin\theta = W_o'$

Generalized equations in dimensionless terms

$$\frac{Z_s}{d \sin\theta} = \frac{10}{9(2m+1)\beta'} \ln(\eta' + 1) \quad (5)$$

or

$$\frac{Z_s}{d \sin\theta} = Y''\left[\log\left(\frac{d_s}{d \sin\theta} \frac{W_o' t}{b}\right) + \log Z''\right] \quad (6)$$

B. Infinite boundary.

1. Submerged jet.

a. Vertical to scour bed.

(1) Uniform bed material.
(2) Non-uniform bed material.

b. Inclined to the scour bed.

(1) Uniform bed material.
(2) Non-uniform bed material.

2. Non-submerged jet.

a. Vertical to the scour bed.

(1) Uniform bed material.
(2) Non-uniform bed material.

Experimental investigator - Doddiah (10)
Dimensionless parameters

$$\phi_4\left[\frac{W_o^2}{\left(\frac{\sigma}{\rho}-1\right)g\,d_s};\ \frac{b}{d};\ \frac{r}{b};\ \frac{Z_s}{d};\ \frac{d_s}{d}\ \frac{W_o t}{b}\right]=0$$

Generalized equations in dimensionless terms.

$$\frac{Z_s}{d}=\frac{10}{0.137(18m+9)}\ \ln(\eta+1) \qquad (7)$$

or

$$\frac{Z_s}{d}=Y'''\left[\log\left(\frac{d_s}{d}\ \frac{W_o t}{b}\right)+\log Z'''\right] \qquad (8)$$

b. Inclined to the scour bed.

(1) Uniform bed material.
(2) Non-uniform bed material.
Experimental investigator - Bureau of Public Roads (Project 735)

<u>Note:</u> This study is now under investigation to determine extent of influence of channel boundaries on rates of scour. Dimensionless parameters, etc., are considered the same as for the case of the finite boundary.

TWO-DIMENSIONAL JET

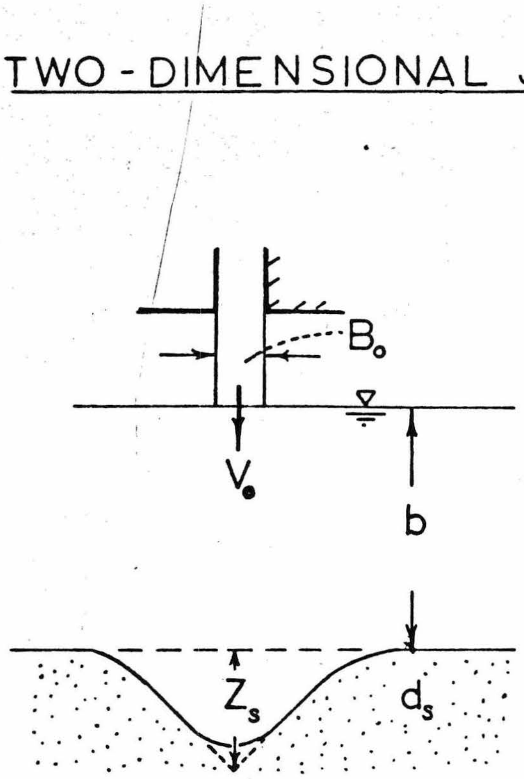

FIG 1: DEFINITION SKETCH OF A VERTICAL JET

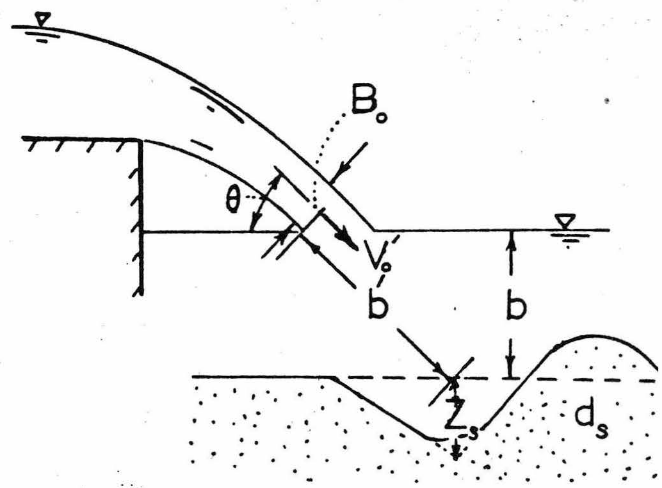

FIG.2: DEFINITION SKETCH OF AN INCLINED JET

THREE-DIMENSIONAL JET

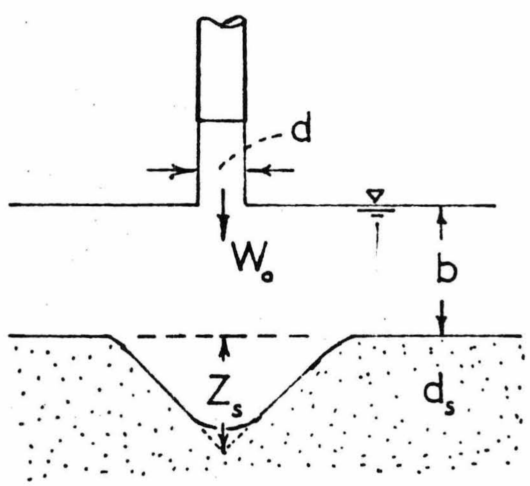

FIG 3: DEFINITION SKETCH OF A VERTICAL JET

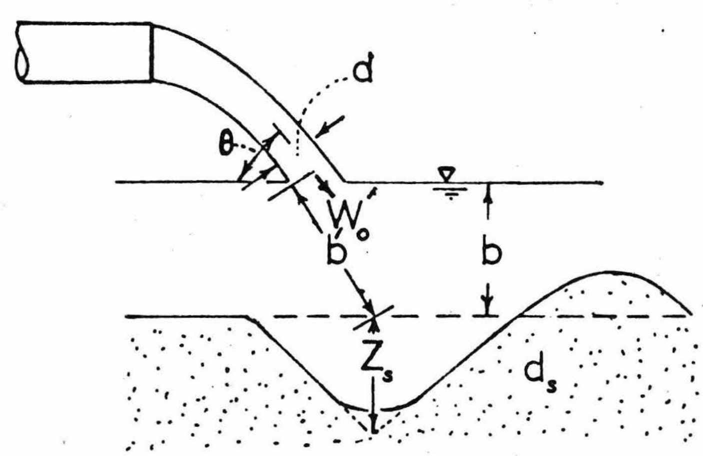

FIG 4: DEFINITION SKETCH OF AN INCLINED JET

LIST OF SYMBOLS

b = tailwater depth - ft.

B_o = diameter of the two-dimensional jet at the tailwater surface - ft.

d = diameter of the three-dimensional jet at the tailwater surface - ft.

d_s = the statistical mean diameter of the bed material - ft.

g = acceleration due to gravity, 32.2 ft/sec^2.

m = exponent giving q_s -- value between 1 and 2.

r = radial coordinate parallel to bottom.

t = the time of scour - hr.

V_o, V_o' = velocity of the two-dimensional jet at the tailwater surface.

W_o, W_o' = velocity of the three-dimensional jet at the tailwater surface.

x = horizontal distance from centerline of the jet.

$$Y = \frac{20}{18m + 29}$$

$$Y' = \frac{10}{9(2m + 1)\beta}$$

$$Y'' = \frac{10}{9(2m + 1)\beta'}$$

Z_s = the apparent depth of scour - ft.

$$Z = \frac{(b + Z_s)^{-Y}}{B_o}$$

$$Z' = \left(\frac{Z_s}{B_o \sin\theta}\right)^{-Y'}$$

β (beta) = constant for law of diffusion along the centerline of a two-dimensional jet.

β' (beta prime) = constant for law of diffusion along the centerline of a three-dimensional jet.

η (eta) = a theoretical expression for the following parameters:

$$\frac{W_o^2}{\left(\frac{\sigma}{\rho}-1\right) g d_s} ; \frac{b}{d} ; \frac{r}{b} ; \frac{d_s}{d} \quad \frac{W_o t}{b} ; \text{and } \frac{x}{b}$$

η' (eta prime) = a theoretical expression for the following parameters:

$$\frac{\left(V_o' \sin \theta\right)^2}{\left(\frac{\sigma}{\rho}-1\right) g d_s} ; \frac{b}{B_o \sin \theta} ; \frac{r}{b} ; \frac{d_s}{B_o} \quad \frac{V_o' t}{b} ; \text{and } \frac{x}{b}$$

η'' (eta double prime) = the same as η' except V_o' is replaced by W_o'.

θ (theta) = angle of impingement of jet with tailwater surface-degrees.

ν (nu) = kinematic viscosity of fluid - ft^2/sec.

ξ (xi) = a theoretical expression of the following parameters.

$$\frac{V_o^2}{\left(\frac{\sigma}{\rho}-1\right) g d_s} ; \frac{b}{B_o} ; \frac{r}{b} ; \frac{d_s}{B_o} \quad \frac{V_o t}{b} ; \text{and } \frac{x}{b}$$

ρ (rho) = specific gravity of water.

σ (sigma) = specific gravity of sediment ($\frac{\sigma}{\rho} = 2.65$).

BIBLIOGRAPHY

1. Schlichting, H. Boundary layer theory. McGraw-Hill Book Co., Inc. New York, 1955. p. 73.

2. Schlichting, H. and Truckenbrodt, E. Die Strömung an einer angeblasenen rotierenden Scheibe. ZAMM Band 32. Heft 4/5. April/Mai 1954. S. 150-162.

3. Truckenbrodt, E. Die trubulent Strömung an einer angeblasenen rotierenden Scheibe. ZAMM, Band 34. Heft 4/5. April/Mai 1954. S. 150-162.

4. Albertson, M. L., Dia, Y. B., Jensen, R. A. and Rouse, H. Diffusion of submerged jets. Trans. ASCE. Vol. 115, 1950 pp. 639-697.

5. Homma, M. An experimental study on water fall. Proc. of Minnesota International Hydraulic Convention. Sept. 1953 pp. 477-481.

6. Rouse, H. Seven exploratory studies in hydraulics. (Study entitled "Deflection of a jet by a normal wall," by Murray, Nelson and Steven, pp. 26-29.) Proc. ASCE Vol. 82. Separate No. 1038. August 1956.

7. ________. Criteria for similarity in the transportation of sediment. Proc. Hydraulics Conf. University of Iowa Studies in Engineering, Bulletin 20, 1940, pp. 33-49.

8. Thomas, R. K. Scour in a gravel bed. M. S. Thesis, Colorado A. and M. College, 1953.

9. Hallmark, D. E. Scour at the base of a free overfall. M. S. Thesis. Colorado A. and M. College, 1955.

10. Doddiah, D. Comparison of scour caused by hollow and solid jets of water. M. S. Thesis, Colorado A. and M. College, 1950.

11. Smith, G. L. Scour and energy dissipation below culvert outlets. Report. Dept. of Civ. Eng., Colorado A. and M. College. April 1957.

12. Laursen, E. M. Observations on the nature of scour. Proc. Fifth Hyd. Conf., University of Iowa Studies in Engineering, Bulletin 34, 1953, pp. 179-197.