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LONGITUDINAL DISTRIBUTION OF VIRTUAL MASS, VIRTUAL MOMENT OF INERTIA, DAMPING FORCE AND DAMPING MOMENT ON A PITCHING

AND HEAVING SHIP
by
M. R. Bottaccini
and
E. F. Schulz

Prepared for S-3 Panel
Hull Structure Committee Society of Naval Architects and Marine Engineers

74 Trinity Place
New York 6, New York

Colorado State University Research Foundation Civil Engineering Section Fort Collins, Colorado

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$$
\begin{aligned}
& \mathrm{Fr}=0.25 \\
& \mathrm{Fr}=0.25 \\
& \mathrm{Fr}=0.25 \\
& \mathrm{Fr}=0.25 \\
& \mathrm{Fr}=0.25 \\
& \mathrm{Fr}=0, \omega=5.00 \\
& \mathrm{Fr}=0, \omega=8.65 \\
& \mathrm{Fr}=0, \omega=11.35 \\
& \mathrm{Fr}=0, \omega=14.00 \\
& \mathrm{Fr}=0.10, \omega=5.00 \\
& \mathrm{Fr}=0.10, \quad=1.65 \\
& \mathrm{Fr}=0.10, \omega=11.35 \\
& \mathrm{Fr}=0.10, \omega=14.00 \\
& \mathrm{Fr}=0.20, \omega=5.00 \\
& \mathrm{Fr}=0.20, \omega=8.65 \\
& \mathrm{Fr}=0.20, \omega=11.35 \\
& \mathrm{Fr}=0.20, \omega=14.00 \\
& \mathrm{Fr}=0.25, \omega=5.00 \\
& \mathrm{Fr}=0.25, \omega=8.65 \\
& \mathrm{Fr}=0.25, \omega=11.35 \\
& \mathrm{Fr}=0.25, \omega=14.00 \\
& \mathrm{Fr} \\
& \mathrm{Fr}=0 \\
& \mathrm{Fr}=0.10 \\
& =0
\end{aligned}
$$

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## NOTATION

| B | Buoyancy |
| :---: | :---: |
| b | Beam |
| C | Equivalent buoyancy spring constant |
| $c_{\theta}$ | Buoyancy moment coefficient |
| d | Square law resistance coefficient |
| $\mathrm{a}_{1}$ | Square law resistance coefficient (upward average) |
| $\mathrm{d}_{2}$ | Square law resistance coefficient (downward average) |
| $\mathrm{d}_{\mathrm{z}}{ }^{\prime}$ | Vertical square law coefficient (dimensionless) |
| F | Net force upward |
| $F^{\prime}$ | Total driving force |
| Fo | Force amplitude |
| $F_{0}$ | Force at $\omega t=0$ |
| $\mathrm{F}_{\pi / 4}$ | Force at $\omega t=\pi / 4$ |
| $F_{\pi / 2}$ | Force at $\omega t=\pi / 2$ |
| $\mathrm{F}_{\pi}$ | Force at $\omega t=\pi$ |
| $f$ | Damping coefficient |
| $f_{\theta}$ | Angular damping coefficient |
| $f_{\theta}{ }^{\prime}$ | Rotational damping coefficient (dimensionless) |
| $\mathrm{f}_{\mathrm{z}}{ }^{\prime}$ | Vertical damping coefficient (dimensionless) |
| J | Apparent moment of inertia |
| $J_{0}$ | Moment of inertia in a vacuum |
| $\mathrm{k}_{1}, \mathrm{k}_{2}$ | Spring constants |
| $\mathrm{k}_{\mathrm{z}}$ | Added mass coefficient (dimensionless) |

## NOTATION (cont'd)

$\mathrm{k}_{\theta} \quad$ Added moment of inertia coefficient (dimensionless)
L Lift
L Length (waterline)
$M \quad$ Net moment ( $M^{\prime}-M_{L}$ )
M' Measured moment
$M_{L} \quad$ Moment due to ship forward motion
Mo $\quad$ Moment at $\omega t=0$
$M_{\pi / 4} \quad$ Moment at $\omega t=\pi / 4$
$M_{\pi / 2} \quad$ Moment at $\omega t=\pi / 2$
$M_{\pi} \quad$ Moment at $\omega t=\pi$
$m \quad$ Apparent mass of ship-water system
$\mathrm{m}_{\mathrm{A}}$
Added mass
$m_{0} \quad$ Mass in a vacuum
$r$ Subscript representing the contribution of one segment
s subscript representing the contribution of the whole ship
$u(t-a)$ Unit step function at $t \geqq a$
$\mathrm{x} \quad$ Moment arm
z Vertical displacement, positive upward
zo Vertical displacement amplitude
$\Delta \quad$ Displacement
$\delta$ Phase shift between force and displacement
$\theta$ Angle in radians (pos. c.c.)
$\theta_{0} \quad$ Amplitude of $\theta$
$\omega \quad$ Frequency

# LONGITUDINAL DISTRIBUTION OF VIRTUAL MASS, VIRTUAL MOMENT OF INERTIA, DAMPING FORCE <br> AND DAMPING MOMENT ON A PITCHING <br> AND HEAVING SHIP 

## INTRODUCTION

This study, sponsored by the S-3 Panel of the Hull Structure Committee, Society of Naval Architects and Marine Engineers was undertaken as an experimental check of the theoretical forces and moments computed by Lewis (1) on a Model T2-SE-Al Tanker. A series of tests were conducted in the Wave Basin at Colorado State University on a model divided into seven segments of equal water-line length. Each segment was suspended in such a fashion that it was structurally independent of other segments. The whole model was then subjected to forced oscillations in pitch or heave and the force on each segment measured by a strain-gage dynamometer. The recorded traces of force as a function of time were used to determine the virtual mass and moment of inertia coefficients and the damping coefficients. To obtain a better fit of measured data to the assumed equation, certain terms proportional to the square of the angular frequency were determined.

## EXPERIMENTAL EQUIPMENT

Model
To afford a direct comparison with tests and computations previously conducted by Lewis (1), a Model T2-SE-Al Tanker hull was chosen for the tests. Constructed according to the lines shown in Figure 1, the model was molded from fiberglass laminate approximately $1 / 8$-inch thick. (Model particulars are shown in Table 1). Since the model was expected to perform forced oscillations of nearly two inch amplitude, its freeboard was made large enough to prevent swamping (3.1 inches).

It was specified by the panel that the distribution of force and moments on the hull were to be studied. Ideally, it would have been necessary to slice the model in many thin segments. Cost and time prevented any extensive subdivision. It was decided to cut the five-foot-long hull into seven equal length segments ( 8.57 inches) as shown in Figure 2. Each hull segment was made water tight by sheet metal bulkheads fitted into the cut ends. (Segment particulars are shown in Table 2).

## Force Balances

A phosphor bronze spring spanned each hull segment fore and aft along the model center line. At the center of each spring, a vertical column was attached firmly and the column was then connected to an aluminum beam or strongback (Figure 3). The segments were mounted in such a fashion that no contact existed with the adjoining segments (Figure 4). As a result of the foregoing precaution, it was assumed that the influence of a hull segment on other hull segments was purely hydrodynamic.















$$
x^{3}
$$

$$
x^{2}
$$

S

-2c-

Photograph

# Fig. 3 Segmental Force Springs, Segmented Model and Aluminum Strongback before Assembly. 

Photograph

Fig. 4 Towing Carriage, Mechanical Oscillator and Segmented Model

The strain of the spring, and hence the force transmitted from the strongback to the hull was recorded by a strain-gage bridge mounted near the root of the cantilever spring. The electrical signal from the strain gages was transmitted through a television cable to a set of 5000 cps carrier amplifiers and then recorded by light beam type high performance galvanometers.

Calibration was done on the completed experimental unit. The model with its attendant instrumentation was mounted in a calibration tank and loaded with known forces and moments. The galvanometer deflection was then correlated with the load. As soon as the model was mounted on the towing carriage, and at frequent intervals thereafter, calibration checks were conducted in situ. The model was moved manually to a known depth or a known angle of pitch. Since the buoyancy forces and moments were known, the galvanometer deflection could be checked against the calibration curves.

## Mechanical Oscillator

The strongback received its motion from a mechanical oscillator which was mounted on an aluminum towing carriage. Driven through a variable speed control and an 11 to 1 speed reducer, the oscillator operated within a range of 5 to 16 radians per second. The oscillator frequency could be maintained within close limits and could be obtained whenever needed.

A worm gear was connected to the variable speed drive with a double ended output shaft. Both ends of the shaft were fitted with an eccentric crank pin, each of which drove one end of the strongback. Heaving oscillations occurred whenever the crank pins were driven in phase while pitching motion resulted whenever the crank pins were driven 180 degrees out of
phase. Amplitude of motion was adjusted by changing the eccentricity of the crank pins. (Both pins had the same eccentricity).

## TEST PROGRAM

Sufficient tests were run to assure coverage of usual prototype behavior and a little beyond to establish trends. In both heave and pitch programs the model was oscillated on the following frequencies:

$$
\begin{array}{rrrrrrr}
\text { CPS } & 0, & 0.8, & 1.2, & 1.4, & 1.6, & 1.8, \\
\mathrm{Rad} / \mathrm{sec} & 0, & 5.0, & 7.5, & 8.8, & 10.0,11.3, & 12.6,
\end{array}
$$

The oscillator was set at one of the test frequencies and then the model was towed through a test program involving at least two runs for each forward speed. The speed runs were made at four speeds as follows:

| Model Speed (fps) | 0, | 1.27, | 2.54, | 3.17 |
| :--- | :--- | :--- | :--- | :--- |
| Froude Number (Fr) | 0, | 0.1, | .0 .2, | 0.25 |
| Fullscale Speed (knots) | 0, | 7.5, | 15.1, | 18.8, |

The previously described experimental pattern was applied to the model three times, each set of tests was conducted with different amplitudes of oscillation:

$$
\begin{array}{llll}
\text { Model (inches) } & 0.5,1.0,2.0 & \text { (1.75 for heave) } \\
\text { Fullscale (feet) } & 4.2, & 8.2,16.4 & \text { (14.3 for heave) }
\end{array}
$$

This amplitude is the half-cycle displacement of the water line in heave and the maximum half cycle vertical displacement of a point on the forward perpendicular in pitch.

In addition to the active run program several calibration static tests were conducted at frequent intervals. In general, the static calibration tests were consulted in reduction of the data.

## BASIC THEORY

## The Mathematical Model

In the past it has been assumed $(2,3,4)$ that the forces and moments acting on a heaving or pitching ship can be represented by a simple mass spring system with damping. Such a simple model has the advantage of permitting easy and rapid reduction of data.

At the onset of this study, there seemed no reason for the authors to believe that the segments of the tanker model would not behave as assumed. Since the ship was constrained to heave or pitch sinusoidally, it seemed reasonable to assume that the two springs in each segment would also impart sinusoidal motion to the segment. Hence it was assumed that for the towed heaving ship, the dynamic constants could be obtained by the simplified model due to Haskind (2).

If the model is forced to go through a vertical displacement $2 z_{o}$ and if the net force of buoyancy is called $B$, then it is assumed that

$$
B=C z
$$

in which $z$ is the vertical distance measured as positive upward from the


Fig. 5 Schematic Diagram of Forces on a Segment.
undisturbed water surface. This assumption of linearity is quite good for most ships. On the T-2 tanker only, the forces on the bow segment deviate from a straight line, and then not markedly.

Setting: $L=$ upward force generated by forward motion of ship
$m$ = apparent mass of ship-water system
$f=$ damping coefficient
C = spring constant (buoyancy)
$F^{\prime}=$ driving force
$z=$ vertical displacement
(The force $F^{\prime}$ is the sum of the forces measured by the forward and aft strain balances of each segment.)

Then a force balance (see Fig. 5) yields

$$
\begin{equation*}
\mathrm{m} \ddot{z}+f \dot{z}+C z=F^{\prime}+L=F \tag{1}
\end{equation*}
$$

If z is assumed to vary sinusoidally, it can be easily shown that F will
also be sinusoidal. Indeed if

$$
z=z_{0} e^{i \omega t}
$$

then $F=F_{o} e^{i(\omega t+\delta)}$
in which $\omega=$ driving frequency

$$
t=\text { time }
$$

$\delta=$ phase shift
Substituting these values of $z$ and $F$ in Equation 1 gives

$$
\begin{equation*}
-m z_{0} \omega^{2} e^{i \omega t}+i f \omega z_{0} e^{i \omega t}+C_{0} e^{i \omega t}=F_{0} e^{i(\omega t+\delta)} \tag{2}
\end{equation*}
$$

By separating real and imaginary parts, the following equations are obtained

$$
\begin{gather*}
-m \omega^{2}+C=\frac{F_{0}}{Z_{0}} \cos \delta  \tag{3}\\
f \omega=\frac{F_{0}}{Z_{0}} \sin \delta \tag{4}
\end{gather*}
$$

Since $C$, $\omega$, and $z_{o}$ are known and $F_{o}$ and $\delta$ can be measured, $m$ and $f$ may be easily computed from the oscillographs of driving force as a function of time (Fig. 6).


Fig. 6 Force in Relation to Displacement According to Equation 1 .

## Modification of the Equation

It is evident from Equation 2 that since $z$ is symmetrical about the $z=0$ axis, then $F$ must be symmetrical about the $F=0$ axis. It came as something of a surprise when the authors discovered that in the CSURF tests, F oscillated about a non-zero mean. Summing the forces for the whole ship also did not result in a zero mean force. This displacement of the zero axis could not be explained on the basis of the simple linear equation (1) for if $z$ were symmetrical, so would be $F$.

The displacement of the axis of symmetry can be explained in one of the following ways:
(a) Instrumental error
(b) Unbalanced driving distance
(c) Constant force due to other causes than vertical oscillations (d) Nonlinearity

Considerable care was taken to eliminate possibilities a and b . Although possible, instrumental error seemed unlikely. Similar results were obtained with two different instrumental techniques. Amplifier drift was kept under observation and each amplifier was balanced before each run. Static calibrations were run (by changing the position of the ship manually) at frequency intervals. Unbalanced driving (upstroke different from downstroke) was controlled by careful maintenance of the water level.

It was assumed, therefore, that either $c$ or $d$ or both were responsible for the zero shift. It is well known that a moving ship experiences lift (or suction) along its hull. By examining oscillographs of the model being towed without oscillations, an estimate of this lift force was obtained.

Unfortunately this lift was insufficient to account for the shift and often was small enough to be neglected.

The investigators were forced to concede that there existed nonlinearities. It was assumed that there existed "square law" forces analogous to drag. In fact since the flow regime on the downstroke is completely different from the flow on the upstroke, it was assumed that the vertical square law resistance could be approximated by

$$
\begin{equation*}
\alpha|\dot{z}| \dot{z}=D[a+\cos (\omega t)]|\dot{z}| \dot{z} \tag{5}
\end{equation*}
$$

However, the data was not good enough to obtain $D$ and $a$ with any accuracy. Under the circumstances, it was decided to use a cruder approximation, i.e., to assume that the vertical resistance coefficient was a constant $d_{1}$ on the upstroke and a different constant $d_{2}$ on the downstroke.

$$
\begin{align*}
& d= d_{1}\left[u(t)-u\left(t-\frac{\pi}{2 \omega}\right)+u\left(t-\frac{3 \pi}{2 \omega}\right)-u\left(t-\frac{5 \pi}{2 \omega}\right)+\cdots\right] \\
&+d_{2}\left[u\left(t-\frac{\pi}{2 \omega}\right)-u\left(t-\frac{3 \pi}{2 \omega}\right)+u\left(t-\frac{5 \pi}{2 \omega}+\cdots\right]\right.  \tag{6}\\
& d=d_{1} u(t)+\left(d_{2}-d_{1}\right) \sum_{n=0} u\left[t-\frac{(2 n+1) \pi}{2 \omega}\right]
\end{align*}
$$

in which $u(t-\tau)$ is the unit step function at $t \geqq \tau$.
Then the equation of motion becomes

$$
\begin{equation*}
m \frac{d^{2} z}{d t^{2}}+f \frac{d z}{d t}+d(\text { signum } \dot{z}) \dot{z}^{2}-C z+F^{\prime}-1=F \tag{7}
\end{equation*}
$$

in which $F^{\prime}$ is the driving force, $L$ the "lift", and $F$ the net force.

It is interesting to note that the phase shift-amplitude method does not work at all with Equation 7. The form of $F$ is given by

$$
\begin{equation*}
F=F_{0}(t) e^{i[t+\delta(t)]} \tag{8}
\end{equation*}
$$

Since only two equations can be obtained from Equation 8, it is impossible to determine the four variables $m, d_{1}, d_{2}$, and $f$. A more logical technique is to obtain four values of $F$ and solve by the standard algebraic methods. If $F$ is measured at $\omega t=0, \frac{\pi}{4}, \frac{\pi}{2}$, and $\pi$, the following equations be can easily obtained

$$
\begin{gather*}
m=\frac{C z_{0}-F_{\pi / 2}}{z_{0} \omega^{2}}  \tag{9}\\
f=\frac{2 F_{\pi / 4}-F_{0}-1.414 F_{\pi / 2}}{0.4142 \omega z_{0}}  \tag{10}\\
d_{1}=\frac{1.414 F_{\pi / 2}+1.414 F_{0}-2 F_{\pi / 4}}{0.4142\left(\omega z_{0}\right)^{2}}  \tag{11}\\
d_{2}=d_{1}-\frac{F_{0}+F_{\pi}}{\left(\omega z_{0}\right)^{2}} \tag{12}
\end{gather*}
$$

Coefficients for the Whole Ship (in Heave)
Let the subscript $r$ represent the force contribution of a single segment. Then it is assumed that

$$
\begin{equation*}
m_{r} \ddot{z}_{r}+f_{r} \dot{z}_{r}+d_{r} \dot{z}_{r} \dot{z}_{r}+C_{r} z_{r}=F_{r} \tag{13}
\end{equation*}
$$

If the ship moves in pure heave, it is seen that the total force is simply the sum of the individual forces or

$$
\begin{equation*}
F=\sum_{r=1}^{7} F_{r}=\sum\left(m_{r} \ddot{z}_{r}+f_{r} \dot{z}_{r}+d_{r} \dot{z}_{r} \dot{z}_{r}+C_{r} z_{r}\right) \tag{14}
\end{equation*}
$$

In this instance $z_{1}=z_{2}=\cdots z_{7}$
hence assuming that

$$
F=m_{s} \dddot{z}+f_{s} \dot{z}+d_{s} \dot{z} \dot{z}+C_{s} z
$$

We obtain

$$
\begin{align*}
m_{s} & =\Sigma m_{r}  \tag{15}\\
f_{S} & =\Sigma f_{r}  \tag{16}\\
d_{S} & =\Sigma d_{r}  \tag{17}\\
K_{S} & =\Sigma k_{r} \tag{18}
\end{align*}
$$

i.e., the values of the hydrodynamic constants for the whole ship are simply the sum of the hydrodynamic constants of the individual segments.

## Neglect of Base Line Shift

It was requested by members of the S-3 Panel that some computations be made by assuming that the shift of the zero line was spurious, i.e., by use of Equations $1,2,3$, and 4. Due to the nature of the data, an additional technique had to be introduced. Each segment force was measured at two different points, fore and aft.

Hence, assuming that each force measurement obeyed Equation 1, it was necessary to write

$$
F=F_{O A} e^{j\left(\omega t+\delta_{A}\right)}+F_{o F} e^{j\left(\omega t+\delta_{F}\right)}
$$

or

$$
\begin{equation*}
F=F_{o} e^{j \omega t+\delta}=e^{j \omega t}\left[F_{o A} e^{j \delta_{A}+F_{o F}} e^{j \delta_{F}}\right] \tag{19}
\end{equation*}
$$

$$
\begin{gather*}
F_{O}=\left[\left(F_{O A} \operatorname{\omega s} \delta_{A}+F_{O F} \cos \delta_{F}\right)^{2}+\left(F_{O A} \sin \delta_{A}+F_{O F} \sin \delta_{F}\right)^{2}\right]  \tag{20}\\
\tan \Delta=\frac{F_{O A} \sin \delta_{A}+F_{O F} \sin \delta_{F}}{F_{O A} \operatorname{\omega s} \delta_{A}+F_{O F} \sin \delta_{F}}
\end{gather*}
$$

In the equations above, the subscript A means "aft" while subscript F means "forward."

## Reduction of Data in the Pitch Plane

The moment on each segment may be represented by a mathematical model similar to the previous one

$$
\begin{equation*}
\ddot{\partial}+f_{\theta} \dot{\theta}+\alpha_{\theta}(t)|\dot{\theta}| \dot{\theta}+C_{\theta} \theta=M^{\prime}-M_{L} \tag{21}
\end{equation*}
$$

where $J=$ apparent moment of inertia
$f_{\theta}=$ angular damping coefficient
$\mathrm{d}_{\theta}=\mathrm{drag}$-moment coefficient
$C_{\theta}=$ buoyancy moment coefficient
$\mathrm{M}^{\prime}=$ induced (measured) moment
$M_{L}=$ moment due to ship forward motion
$\theta$ angle in radians
All angles and moments are assumed positive counter-clockwise.
The moment axis is located at a point on the water line midway between station 0 and the stern and is perpendicular to the axis of symmetry of the ship.

If the forces on the aft and fore balances of each segment are $F_{A}$ and $F_{F}$ respectively and the distances to the rotational axis of the ship
from the points of force application are $x_{F}$ and $X_{A}$, then

$$
M^{\prime}=F_{A} x_{A}+F_{F} x_{F}
$$

where $F$ is positive upward, $x$ positive forward, and $M$ positive counterclockwise. Since in this model the angular motion is obtained by driving the bow and the stern linearly but $180^{\circ}$ out of phase, we can write for the elevation of any point.

$$
z=z_{0} \sin (\omega t)
$$

If the distance from the point in question to the axis is $x_{0}$, then (for small angles)

$$
\theta \approx \frac{z_{0}}{x_{0}} \sin (\omega t)=\theta_{0} \sin (\omega t)
$$

It is obvious, therefore, that Equation 21 may be written as

$$
\begin{align*}
& -j \omega^{2} \sin \omega t+f_{\theta} \omega \cos \omega t \\
& \quad+\alpha_{\theta}(t) \omega^{2} \theta_{0} \cos (\omega t) \cos (\omega t)+C_{\theta} \omega \sin \omega t=\frac{M}{\theta_{0}} \tag{22}
\end{align*}
$$

Obviously the solution of this equation for $J, f_{\theta}$, and $\alpha_{\theta}$ yield equations completely similar to Equations 9, 10, 11, and 12 with $M_{0}, M_{\pi / 4}, M_{\pi / 2}, M_{\pi}$ substituted in place of $F_{0}, F_{\pi / 4}, F_{\pi / 2}$, and $F_{\pi}$.

The Moment and Force Parameters for the Whole Ship
Since the moment and force of each segment is referred to the same axis, it is obvious that the stability coefficients for the whole ship can be obtained by summing the coefficients of the individual segments.

## -14-

## Non-dimensional Forms

If the mass of the ship is $m_{0}$, it is possible to write

$$
m_{A}=m-m_{0}
$$

Then we write a dimensionless coefficient

$$
k_{z}=\frac{m_{A} g}{\Delta}
$$

where $g=$ acceleration of gravity
$\Delta=$ segmental or total model displacement at water line.
This term is plotted as a function of "angular Froude number"

$$
f=\omega \sqrt{\frac{b}{g}}=0.144 \omega
$$

where b is the model.'s beam.
The damping term is plotted in the form

$$
f_{z}^{\prime}=\frac{f}{\Delta} \sqrt{g b_{l}}
$$

where $b_{1}$ is the segmental or total beam.
The vertical resistance term is given by

$$
\mathrm{d}_{\mathrm{z}}^{\prime}=\mathrm{d}_{\mathrm{z}} \frac{\mathrm{gb}}{\Delta}
$$

In pitch we obtain

$$
\begin{aligned}
& k_{\theta}=\frac{\left(J-J_{0}\right) g}{\Delta L^{2}} \\
& f_{\theta}^{\prime}=f_{\theta} \sqrt{\frac{g}{\Delta^{2} L^{3}}} \\
& {d^{\prime}}^{\prime}=\frac{d_{\theta} g}{\Delta L^{2}}
\end{aligned}
$$

in which $L$ is the total or segmental length at the water line.

## PRESENTATION OF DATA

## Heave Motion

Figures 8 through 16 indicate the behavior of the added mass coefficients as functions of dimensionless frequency and of Froude number. These data were obtained with Equation 9. Figures 8, 9, 10, and 11 show the variation of the added mass coefficient with frequency and Froude number for the whole ship. As a comparison Golovato's curves (3) have been drawn on Figure 8. It can be seen that the CSURF data are comparable to the DTMB data although consistently lower.

Comparison of individual segment contribution to the added mass of the whole ship is given in Figures 17 through 32 for a representative selection of Froude numbers and of heave frequencies. The graphs are presented as ratios of local added mass to ship added mass. IT MUST BE UNDERSTOOD THAT A CURVE MUST NOT BE DRAWN THROUGH THE POINTS SINCE ALL THAT HAS BEEN PRESENTED IS AN INTEGRATED VALUE OF THE ADDED MASS. The actual distribution of added masses over the segment is unknown.

Damping coefficients for pure heave motions appear in Figure 33 through 40. In these Figures it can be seen that the damping coefficients become negative for the whole ship. This implies that the configuration is dynamically unstable and that the water introduces energy into the model. This result seems far fetched and is not to be trusted.

In Figure 41 through 56 the ratio of local damping to total ship damping is presented. THE SAME PRECAUTIONARY STATEMENT APPLIED TO FIGURE 17 IS VALID HERE.

## Pitch Motion

The added moment of inertia coefficients appears by segment in Figures 57 through 74. Moment of inertia coefficients for the whole ship are given in Figures 57 through 60.

Although the moment of inertia coefficients for each segment have been made dimensionless in terms of the segment characteristics, the center of rotation has been kept as the minor axis of the ship water plane. The measured information was too inaccurate to permit translation of moment characteristics to beam axis on the segment water planes. For comparison with theoretical strip computations, the results should be adequate.

Figures 75 through 90 show a representative collection of approximate longitudinal distributions of added moments of inertia, as a ratio to the added moment of inertia of the whole ship model.

Rotational damping coefficients are given in Figures 91 through 124. Damping coefficients for the whole model are given in Figures 91 through 94. The square law resistance coefficients $d_{z}^{\prime}, d_{G}^{\prime}$ are shown in Figures 125 through 142.

Comparison with Simple Reduction Method
Although the oscillograms did not logically admit use of the simple reduction method, some added masses and damping coefficients were computed by Equations 3 and 4 as a comparison with the more general technique. To carry out the computation, it was assumed that the zero shift did not exist and that the axis of symmetry of the oscillograph was the axis of zero force. Phase and amplitude were measured and the added masses and damping coefficients were computed (Figures 143 through 148).

## VALIDITY OF TESTS

## Instrumentation

Although great care was taken to reduce instrumental errors to a minimum, it was apparent that the instrumentation did not yield the complete information needed. The problem of coupling was not resolved.


## Fig. 7 Actual Mounting of Segment.

Careful examination of oscillograms leads one to believe that if $M$ has a coupling effect, it is small enough to be neglected. However, this conclusion cannot be proved and further tests are needed to clarify the situation.

A major source of lack of confidence was the scatter of force distribution between the two springs of each segment. In some instances where the test conditions were identical, the total forces on both tests
were comparable, but the contribution of each spring to the total varied widely. This seemed to indicate that either the moment system had not yet reached the ultimate state or that even in pure heave, no steady moment existed.

The best way to determine the coupling effects is to run a set of tests with only one degree of freedom movements, i.e., to conduct experiments in which rotation is restrained and then similar experiments in which vertical displacement is discouraged. With the resulting data, it will become possible to end at least a portion of the confusion.

Another major instrumental problem is the separation of the signal from the background noise. In the tests conducted at CSURF, considerable hand fairing of oscillogram curves was needed. It was found that two computists would obtain amplitudes which differed by over 10 per cent and phase shifts separated by at least 15 per cent. This last was quite noticeable at high frequency. Such high degree of error cannot be tolerated since a difference of 15 per cent at high frequencies will make the phase shift over 180 degrees, and hence will make the damping term negative and occasionally will also yield a negative added mass. No solution is possible to this difficulty without a change in instrumental technique. A new filtering system has been developed at CSURF, and it is believed that if applied to the segmented tanker model, more acceptable data would be obtained.

## Mathematical Models

In a problem of this nature the mathematical model chosen is of primary importance. As was indicated in this study, more than one mathematical
model is possible for the physical phenomenon of the oscillating ship. NO CONCLUSION IS POSSIBLE AS TO WHICH MATHEMATICAL MODEL IS VALID. According to the previous section, a complete model would include coupling between displacement and rotation. Evidence of square law damping has been found by several workers (for example, ref. 3). It is entirely possible that square law damping and coupling both exist. Thus, before any conclusions can be drawn from the tests, a clarification of the true mathematical model must be obtained. The authors submit that further repeated tests of the type employed in this study are not useful until it is established that the simple mathematical representation used by most workers in the field is valid.

## Suggestions for Further Work

A. Run through a limited set of runs in heave and pitch with a modified instrumental setup consisting of a one degree of freedom dynamometer and a low pass filter. Great care should be taken to obtain the forces and moments which exist whenever the model is not oscillating and restrained in all modes. From these tests the true location of the zero force axis should be obtained. The existence of non-uniform square law damping may be deduced if the oscillograms exhibit zero shifts which cannot be explained in terms of lift and moment.
B. Measurements of the true lift and moments may be used to determine an approximate distribution per unit length of the forces and moments. This last result cannot be obtained from the
current tests since the moment vs. force relationship is not known.
C. Tests should be made of the model mounted with no driving oscillator but restricted to move freely in only one mode. From such tests, the natural frequency of the entire dynamical system as well as the damping constant should be obtained. A comparison of free vs. forced oscillations should then indicate the adequacy of the simple mathematical model.
D. Run similar tests on simplified models such as the Haskind's ship to permit a generalized theory to be obtained by comparison with an "ideal" theory.

## CONCLUSIONS

The longitudinal distribution of ship forces and moments has been studied. Certain unusual findings were obtained:
(a) Phase shifts were often greater than $180^{\circ}$.
(b) The mean force in heave and the mean moment in pitch were not equal to zero.

As a result of these findings, it was surmised that a non-linear term should be included in the equations. The data were reduced with these modified equations. It was found that added masses and damping for the whole ship conformed to the findings of other researchers $(2,3,4)$ at low frequency. At high frequency of oscillation, the results were doubtful and were disregarded.

It was discovered that although the overall model damping coefficient was positive, the local coefficient was often negative. This was gratifying since P. Kaplan had reported orally that his theoretical results indicated the possibility of local energy sources.

It is obvious that these tests can give only a qualitative check of the validity of the "strip" method since the results are given as integrated values over a non-infinitesimal portion of the hull. Nevertheless, it is expected that the information obtained may be used to obtain an approximation of the relative contribution of each segment to the total ship coefficients. As a final conclusion, it must be admitted that further test are needed to develop a firm comparison of experiment to strip theory.

## REFERENCES

1. Lewis, E.V. "Ship Model Tests to Determine Bending Moments in Waves," Trans. SNAME, Vol. 62, 1954, pp. 426-490.
2. Haskind, M.D. "Oscillation of a Ship on a Calm Sea," Bulletin de l'Academic des Sciences de l'URSS 1946, No. 1. Translation from Russian to English by J. V. Wehausen, Brown University, Providence, Phode Island and published as part of Tech. and Res. Bul. No. 1-12, Society of Naval Architects and Marine Engineers, New York, New York.
3. Golovato, P. "A Study of the Forces and Moments on a Surface Ship Performing Heaving Oscillations," 1957 Taylor Model Basin Report No. 1074, Washington, D. C.
4. Gerritsma, J. "Experimental Determination of Damping, Added Mass and Added Mass Moment of Inertia of a Ship Model," 1957, Netherlands Research Centre TNO for Shipbuilding and Navigation Report No. 25S, Delft, Netherlands.





















































$k_{\theta}$

























|  |  |  |
| :--- | :--- | :--- | :--- |






$f^{\prime}$




















































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TABLE 1.
MODEL PARTICULARS

|  | Model |  |
| :--- | :---: | :---: |
| Scale Ratio | Prototype |  |
| L $_{\text {BP }}$ | $60^{\prime \prime}$ | $503.0^{\prime}$ |
| L $_{\text {WL }}$ | $61.17^{\prime \prime}$ | $512.8^{\prime}$ |
| Beam | $8.11^{\prime \prime}$ | $68^{\prime}$ |
| Draft | $3.58^{\prime \prime}$ | $30^{\prime}$ |
| Displacement, $\triangle$ | 46.5 lb. | 21,770 tons |
|  | (FW) | (SW) |

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TABLE II

## SEGMENT PARTICULARS

| Segment | Weight <br> $($ lb $)$ | Mass <br> $($ slugs $)$ | Buoyancy <br> $(\mathrm{lb} / \mathrm{ft})$ | Av. Beam <br> $(\mathrm{in})$. | Mom. of In.* <br> $\left(\right.$ lb-ft$\left.^{2}\right)$ | Displacement <br> $(\mathrm{lb})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 2.24 | 0.0696 | 10.4 | 2.91 | 0.417 | 2.89 |
| II | 2.86 | 0.0888 | 26.1 | 6.45 | 0.236 | 2.54 |
| III | 3.00 | 0.0937 | 29.8 | 8.09 | 0.086 | 8.58 |
| IV | 3.09 | 0.0960 | 30.2 | 8.10 | 0.106 | 8.96 |
| V | 3.09 | 0.0960 | 30.0 | 8.11 | 0.086 | 8.78 |
| VI | 2.98 | 0.0919 | 26.9 | 8.02 | 0.244 | 7.20 |
| VII | 2.26 | 0.0702 | 15.3 | 3.96 | 0.322 | 2.58 |

* About WL axis through center of ship model


## TABLE III

Observed Forces in Pounds (Heave)


## TABLE III (cont'd)



## TABLE III (cont'd)



## TABLE III (cont'd)



## TABLE III (cont'd)

| 26559 |  |  |  |  | 26609 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\omega$ | $12.56 \mathrm{Rad} / \mathrm{Sec}$ |  |  |  | $\omega$ | $15.7 \mathrm{Rad} / \mathrm{Sec}$ |  |  |  | $\omega$ | 15.7 Rad/ Sec |  | $\mathrm{F}_{\pi / 2}$ | $\mathrm{F}_{\pi}$ |
| Fr |  |  |  |  | Fr | . 1 |  |  |  | Fr | . 1 |  |  |  |
| Segment | $\mathrm{F}_{0}$ | $F_{\pi / 4}$ | $F_{\pi / 2}$ | $F_{\pi}$ | Segment | $\mathrm{F}_{0}$ | $F_{\pi / 4}$ | $F_{\pi / 2}$ | $F_{\pi}$ | Segment | $\mathrm{F}_{0}$ | $F_{\pi / 4}$ |  |  |
| I F | -0.175 | -0.097 | 0.038 | -0.194 | I F | 0.116 | 0.272 | 0.388 | -0.349 | I F | 0.097 | 0.485 | 0.524 | -0.310 |
| I A | -0.460 | -0.314 | -0.063 | 0.021 | I A | -0.376 | 0.125 | 0.543 | 0 | I A | -0.481 | 0.334 | 0.522 | 0.146 |
| II F | -0.352 | -0.124 | 0.310 | -0.041 | II F | -0.248 | 0.662 | 0.828 | -0.580 | II F | -0.145 | 0.911 | 1.076 | -0.476 |
| II A | -0.407 | 0 | 0.660 | 0 | II A | -0.213 | 1.164 | 1.862 | -1.048 | II A | 0.136 | 1.804 | 2.056 | -0.601 |
| III F | -0.508 | 0.102 | 0.365 | -0.386 | III F | 0.203 | 1.380 | 1.360 | -1.340 | III F | 0.974 | 1.644 | 1.806 | -0.995 |
| III A | -0.265 | 0.204 | 0.469 | -0.204 | III A | 0 | 1.652 | 1.673 | -1.489 | III A | 1.102 | 1.714 | 1.958 | -0.816 |
| IV F | -0.102 | 0.636 | 1.025 | 0.410 | IV F | 0.512 | 0.984 | 1.988 | 1.086 | IV F | -1.066 | 1.189 | 2.112 | 0.184 |
| IV A | -0.238 | 0.455 | 0.752 | 0.535 | IV A | -1.208 | 0.752 | 1.584 | 0.713 | IV A | -0.911 | 0.950 | 1.960 | 0.297 |
| V F | 0.240 | 0.960 | 1.120 | -0.160 | V F | -0.280 | 1.840 | 2.180 | -0.520 | V F | 0.660 | 2.580 | 2.600 | 1.020 |
| V A | 0.401 | 0.654 | 0.907 | -0.232 | V A | 0.190 | 1.561 | 2.173 | 0 | V A | 0.232 | 2.384 | 2.469 | -0.717 |
| VI F | -0.062 | 0.144 | 0.391 | -0.206 | VI F | 0.247 | 1.236 | 1.627 | -0.556 | VI F | 0.948 | 1.566 | 1.710 | -0.721 |
| VI A | $0$ | 0.208 | 0.491 | 0.189 | VI A | 0 | 0.850 | 0.850 | -1.190 | VI A | 0.662 | 0.926 | 0.945 | -1.190 |
| VII F | -0.299 | -0.187 | 0.075 | 0.168 | VII F | -0.168 | 0.187 | 0.598 | -0.168 | VII F | -0.411 | 0.447 | 0.598 | -0.262 |
| VII A | 0.116 | 0.116 | 0.155 | 0.155 | VII A | 0.116 | 0.407 | 0.466 | -0.155 | VII A | 0.213 | 0.562 | 0.660 | -0.194 |
| Run |  |  |  |  | Run | 26426 |  |  |  | Run 26449 |  |  |  |  |
| $\omega$ | 5.02 Rad/Sec |  |  |  | $\omega$ | $5.02 \mathrm{Rad} / \mathrm{Sec}$ |  |  |  | $\omega$ | 7.54 Rad/Sec |  |  | $\stackrel{\rightharpoonup}{\square}$ |
| Fr | . 2 |  |  |  | Fr | . 2 |  |  |  | Fr | . 2 |  |  | 1 |
| I F | -0.116 | -0.291 | -0.291 | 0.058 | I F | -0.310 | -0.291 | -0.330 | 0.019 | I F | -0.233 | -0.330 | -0.291 | 0.019 |
| I A | -0.188 | -0.376 | -0.355 | 0.251 | I A | -0.042 | -0.732 | -0.710 | -0.063 | I A | -0.355 | -0.481 | -0.397 | $0.188$ |
| II F | -0.538 | -0.952 | -0.952 | 0.041 | II F | -0.310 | -0.869 | -0.911 | 0.145 | II F | -0.435 | -0.807 | -0.745 | 0.166 |
| II A | -0.854 | -1.436 | -1.261 | 0.136 | II A | -0.621 | -1.067 | -1.183 | 0.369 | II A | -0.718 | -1.222 | -0.989 | 0.291 |
| III F | -0.406 | -0.893 | -0.812 | 0.142 | III F | -0.487 | -0.893 | -0.914 | 0.203 | III F | -0.487 | -0.690 | -0.589 | 0.244 |
| III A | -0.551 | -1.000 | -0.979 | 0.204 | III A | -0.408 | -0.938 | -1.000 | 0.163 | III A | -0.388 | -0.816 | -0.694 | 0.082 |
| IV F | -0.328 | -0.861 | -0.861 | 0.020 | IV F | -0.144 | -0.717 | -0.738 | 0.287 | IV F | -0.308 | -0.697 | -0.676 | 0.266 |
| IV A | -0.337 | -0.832 | -0.990 | -0.079 | IV A | -0.099 | -0.673 | -0.851 | 0.099 | IV A | -0.277 | -0.574 | -0.653 | 0.040 |
| V F | -0.440 | -0.860 | -0.820 | 0.200 | V F | -0.240 | -0.780 | -0.740 | 0.380 | V F | -0.540 | -0.980 | -0.880 | 0.140 |
| V A | -0.549 | -0.950 | -0.928 | 0.211 | V A | -0.422 | -0.992 | -0.928 | 0.422 | V A | -0.549 -0.433 | -0.907 | -0.823 -0.824 | 0.253 0.785 |
| VI F | -0.350 | -0.700 | -0.680 | 0.165 | VI F | -0.515 | -0.989 | -0.927 | 0. | VI F | -0.433 | -0.886 | -0.824 | 0.185 0.057 |
| VI A | -0.416 | -0.850 | -0.832 | 0.284 | VI A | -0.586 | -1.247 | -1.210 | 0.187 | VI A | -0.605 | -1.304 | -1.266 | 0.057 0.112 |
| VII F | -0.150 | -0.299 | -0.299 | 0.187 | VII F | -0.131 0.039 | -0.337 -0.175 | -0.318 -0.252 | 0.187 0.155 | VII F | -0.580 -0.039 | -0.673 -0.233 | -0.561 -0.272 | 0.112 0.155 |
| VII A | -0.058 | -0.136 | -0.194 | 0.078 | VII A | 0.039 | -0.175 | -0.252 | 0.155 |  | -0.039 | -0.233 | -0.272 | 0.15 |

## TABLE III (cont'd)



## TABLE III (cont'd)



## TABLE III (cont'd)



TABLE III (cont'd)



Observed Moments in Foot-Pounds (Pitch)


TABLE IV (cont'd)


## rable IV (cont'd)

| Run | 2737 |  |  |  | Run |  |  |  |  | Run | 27209 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\omega$ | * 15.3 | $6 \mathrm{Rad} / \mathrm{Se}$ |  |  | $\omega$ |  | $\mathrm{Rad} / \mathrm{Sec}$ |  |  | $\omega$ | 5.02 | $\mathrm{Rad} / \mathrm{Se}$ |  |  |
| Fr | 0 |  |  |  | Fr | . 1 |  |  |  | Fr | . 1 |  |  |  |
| Segment | M | $M_{\pi / 4}$ | $M_{\pi / 2}$ | $M_{\pi}$ | Segment | M | $M_{\pi / 4}$ | $M_{\pi / 2}$ | $M_{\pi}$ | Segment | M | $M_{\pi / 4}$ | $M_{\pi / 2}$ | $M_{\pi}$ |
| I F | 0.744 | 0.875 | 0.744 | -1.094 | I F | -0.350 | -0.656 | -0.614 | 0.176 | I F | -0.043 | -0.395 | -0.307 | 0.219 |
| I A | 0.075 | 0.448 | 0.561 | -0.038 | I A | -0.186 | -0.634 | -0.523 | 0.373 | I A | -0.075 | -0.448 | -0.373 | 0.523 |
| II F | 0.739 | 0.998 | 1.036 | -0.666 | II F | -0.332 | -0.666 | -0.554 | 0.296 | II F | -0.148 | -0.443 | -0.407 | 0.407 |
| II A | 0.188 | 0.666 | 0.728 | -0.396 | II A | -0.228 | -0.582 | -0.500 | 0.312 | II A | -0.188 | -0.436 | -0.396 | 0.396 |
| III F | 0.327 | 0.653 | 0.653 | -0.545 | III F | 0 | -0.131 | -0.109 | 0.065 | III F | -0.021 | -0.065 | -0.065 | 0.087 |
| III A | 0.058 | 0.124 | 0.131 | -0.116 | III A | -0.036 | -0.073 | -0.080 | 0.015 | III A | -0.022 | -0.066 | -0.066 | 0 |
| IV F | 0.073 | 0.081 | 0.073 | -0.029 | IV F | 0 | 0.022 | 0.015 | -0.044 | IV F | - 0.022 | -0.029 | 0.036 | -0.044 |
| IV A | -0.014 | 0.050 | 0.099 | -0.042 | IV A | -0.014 | -0.014 | -0.007 | 0.021 | IV A | -0.028 | -0.021 | -0.021 | 0.021 |
| V F | -0.029 | 0.193 | 0.228 | 0.021 | V F | -0.064 | -0.128 | -0.128 | 0.029 | V F | -0.078 | -0.114 | -0.114 | 0.071 |
| V A | 0.271 | 1.063 | 1.041 | -0.316 | V A | -0.294 | -0.385 | -0.316 | 0.204 | V A | -0.385 | -0.452 | -0.362 | 0.136 |
| VI F | 0.110 | 0.795 | 0.773 | -0.309 | VI F | -0.154 | -0.442 | -0.398 | 0.110 | VI F | -0.198 | -0.375 | -0.398 | 0.243 |
| VI A | 0.877 | 1.824 | 1.552 | -0.573 | VI A | -0.877 | -1.114 | -0.979 | 0.371 | VI A | -0.778 | -1.046 | -0.877 | 0.473 |
| VII F | -0.300 | 0.534 | 0.868 | 0.334 | VII F | -0.268 | -0.434 | -0.434 | 0.234 | VII F | -0.300 | -0.568 | -0.500 | 0.268 |
| VII A | -0.092 | 1.008 | 0.824 | -0.458 | VII A | 0.045 | -0.274 | -0.458 | 0.274 | VII A | -0.092 | -0.550 | -0.642 | 0.092 |
| Run | 2723 |  |  |  | Run | 2723 |  |  |  | Run | 2725 |  |  |  |
| $\omega$ | 7.33 | $\mathrm{Rad} / \mathrm{Sec}$ |  |  | $\omega$ | 7.45 | Rad/Se |  |  | $\omega$ | 8.70 | $\mathrm{Rad} / \mathrm{Sec}$ |  | 2 |
| Fr | . 1 |  |  |  | Fr | .l |  |  |  | Fr | . 1 |  |  |  |
| I F | -0.307 | -0.480 | -0.350 | 0.176 | I F | -0.350 | -0.395 | -0.219 | 0.219 | I F | -0.131 | -0.131 | 0 | 0.176 |
| I A | -0.150 | -0.411 | -0.261 | 0.448 | I A | -0.223 | -0.448 | -0.336 | 0.261 | I A | -0.261 | -0.373 | -0.261 | 0.298 |
| II F | -0.073 | -0.407 | -0.332 | 0.407 | II F | -0.259 | -0.370 | -0.186 | 0.332 | II F | -0.148 | -0.296 | -0.073 | 0.221 |
| II A | -0.332 | -0.458 | -0.312 | 0.292 | II A | -0.270 | -0.396 | -0.292 | 0.292 | II A | -0.292 | -0.416 | -0.312 | 0.188 |
| III F | -0.088 | -0.109 | -0.044 | 0.065 | III F | -0.131 | -0.044 | -0.021 | 0.087 | III F | 0.021 | 0 | 0.087 | 0.087 |
| III A | -0.022 | -0.044 | -0.044 | 0.022 | III A | -0.007 | -0.051 | -0.036 | 0.051 | III A | -0.007 | -0.029 | -0.029 | 0.029 |
| IV F | 0.022 | 0.036 | 0.029 | -0.015 | IV F | 0.022 | 0.044 | 0.036 | -0.015 | IV F | 0.051 | 0.066 | 0.051 | -0.007 |
| IV A | -0.021 | 0.007 | 0.015 | -0.021 | IV A | 0 | 0.015 | 0.035 | 0.015 | IV A | -0.007 | 0.028 | 0.056 | 0.021 |
| V F | -0.043 | -0.086 | -0.071 | 0.029 | V F | -0.071 | -0.100 | -0.078 | 0.021 | V F | -0.071 | -0.071 | -0.050 | 0.050 |
| V A | -0.271 | -0.204 | -0.114 | 0.114 | V A | -0.316 | -0.271 | -0.181 | 0.114 | V A | -0.136 | -0.022 | 0.136 | 0.226 |
| VI F | -0.354 | -0.530 | -0.442 | 0.177 | VI F | -0.375 | -0.464 | -0.375 | 0.221 | VI F | -0.221 | -0.309 | -0.198 | 0.331 |
| VI A | -0.607 | -0.843 | -0.641 | 0.539 | VI A | -0.539 | -0.743 | -0.573 | 0.607 | VI A | -0.573 | -0.607 | -0.304 | 0.539 |
| VII F | -0.434 | -0.534 | -0.300 | 0.468 | VII F | -0.500 | -0.602 | -0.334 | 0.368 | VII F | -0.434 | -0.500 | -0.268 | 0.468 |
| VII A | -0.092 | -0.458 | -0.413 | 0.137 | VII A | - | -0.366 | -0.458 | 0.137 | VII A | -0.184 | -0.779 | -0.779 | 0.137 |

TABLE IV (cont'd)


## TABLE IV (cont'd)




## TABLE IV (cont'd)



## TABLE IV (cont'd)



## TABLE IV (cont'd)



TABLE IV (cont'd)


| Run | 27351 |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\omega$ | 15.5 | $\mathrm{Rad} / \mathrm{Se}$ |  |  |
| Fr | . 25 |  |  |  |
| Segment | M | $\mathrm{M}_{\pi / 4}$ | $M_{\pi / 2}$ | $\mathrm{M}_{\pi}$ |
| I F | 0.395 | 1.137 | 1.225 | -0.568 |
| I A | -0 0373 | 0.597 | 0.970 | 0.112 |
| II F | 0.186 | 1.330 | 1.479 | -0.111 |
| II A | 0.291 | 0.707 | 0.748 | -0.374 |
| III F | 0.740 | 0.805 | 0.761 | -0.500 |
| III A | 0153 | 0.138 | 0.029 | -0.197 |
| IV F | 0.176 | 0.234 | 0.205 | -0.198 |
| IV A | -0.156 | -0.056 | 0.156 | 0.191 |
| V F | -0.214 | -0.036 | 0.243 | 0.278 |
| V A | -0.679 | 0.860 | 1.290 | 0 |
| VI F | -0.287 | 0.662 | 0.861 | -0.110 |
| VI A | 0.102 | 1.447 | 1.991 | -0.338 |
| VII F | -0.168 | 0.802 | 0.734 | -0.368 |
| VII A | 0.229 | 1.008 | 0.961 | -0.366 |

TABLE V
ADDED MASS COEFFICIENTS $\left(\mathrm{k}_{\mathrm{z}}\right)$

| $\omega$ | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.57 | 15.71 | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.56 | 15.70 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Seg \# |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| I | 0.284 | 0.164 | 0.174 | 0.177 | 0.196 | 0.200 | 0.190 | 2.19 | 0.048 | 0.064 | 0.209 | 0.274 | 0.306 | 0.306 |
| II | 0.249 | 0.431 | 0.515 | 0.586 | 0.661 | 0.701 | 0.686 | 1.76 | 0.525 | 0.537 | 0.611 | 0.685 | 0.704 | 0.704 |
| III | 1.254 | 0.679 | 0.755 | 0.832 | 0.849 | 0.836 | 0.758 | 2.42 | 0.917 | 0.786 | 0.732 | 0.743 | 0.748 | 0.765 |
| IV | 1.140 | 0.625 | 0.642 | 0.733 | 0.777 | 0.811 | 0.766 | 1.92 | 0.695 | 0.670 | 0.690 | 0.742 | 0.784 | 0.794 |
| V | 0.693 | 0.499 | 0.587 | 0.739 | 0.821 | 0.878 | 0.913 | 2.27 | 0.630 | 0.604 | 0.742 | 0.922 | 0.991 | 0.959 |
| VI | 0.509 | 0.540 | 0.589 | 0.614 | 0.666 | 0.701 | 0.670 | 2.15 | 0.550 | 0.288 | 0.321 | 0.583 | 0.694 | 0.655 |
| VII | -0.343 | 0.282 | 0.343 | 0.416 | 0.461 | 0.464 | 0.462 | 3.18 | 1.193 | 0.759 | 0.524 | 0.452 | 0.434 | 0.542 |
| SHIP | 0.689 | 0.518 | 0.577 | 0.657 | 0.708 | 0.735 | 0.710 | 2.17 | 0.660 | 0.565 | 0.598 | 0.696 | 0.739 | 0.738 |
| $F r=0.20$ |  |  |  |  |  |  |  | $\mathrm{Fr}=0.25$ |  |  |  |  |  |  |
| $\omega$ | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.72 | 15.47 | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.56 | 15.47 |
| Seg \# $\#$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| I | 0.071 | 0.119 | 0.192 | 0.245 | 0.292 | 0.359 | 0.321 | -0.778 | -0.071 | 0.158 | 0.258 | 0.333 | 0.258 | 0.396 |
| II | 0.739 | 0.026 | 0.217 | 0.434 | 0.657 | 0.776 | 0.686 | -0.381 | 0.298 | 0.197 | 0.296 | 0.444 | 0.548 | 0.681 |
| III | 0.810 | 0.680 | 0.827 | 0.913 | 0.908 | 0.869 | 0.764 | 1.883 | 1.237 | 1.078 | 0.945 | 0.884 | 0.858 | 0.690 |
| IV | 0.972 | 0.620 | 0.682 | 0.733 | 0.793 | 0.845 | 0.834 | 1.982 | 0.863 | 0.783 | 0.733 | 0.759 | 0.783 | 0.778 |
| V | 1.044 | 0.539 | 0.588 | 0.652 | 0.751 | 0.863 | 0.916 | 1.044 | 0.539 | 0.576 | 0.630 | 0.717 | 0.779 | 0.844 |
| VI | 0.122 | 0.186 | 0.322 | 0.473 | 0.610 | 0.678 | 0.521 | 1.849 | 0.551 | 0.435 | 0.495 | 0.644 | 0.684 | 0.548 |
| VII | 2.93 | 0.311 | 0.363 | 0.399 | 0.412 | 0.435 | 0.376 | 2.396 | 0.629 | 0.421 | 0.325 | 0.320 | 0.331 | 0.356 |
| SHIP | 0.838 | 0.404 | 0.511 | 0.613 | 0.703 | 0.762 | 0.706 | 1.231 | 0.659 | 0.590 | 0.592 | 0.653 | 0.684 | 0.675 |

TABLE VI
VERTICAL DAMPING COEFFICIENIS ( $f_{z}^{\prime}$ )

| $\omega$ | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.56 | 15.70 | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.56 | 15.70 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Seg \# |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| I | 1.90 | 1.05 | 0.671 | 0.536 | 0.316 | 0.158 | -0.0424 | 2.45 | 1.06 | 1.01 | 1.37 | 0.953 | 0.395 | -0.982 |
| II | 1.09 | 0.754 | 0.566 | 0.903 | 0 | -1.80 | -1.56 | 4.98 | 2.37 | 1.62 | 1.36 | 0.843 | -0.242 | -2.31 |
| III | 3.02 | 1.18 | -0.443 | 0.709 | -0.812 | -2.40 | -2.77 | 3.94 | 7.96 | 0.243 | -0.016 | 1.07 | 1.14 | -1.14 |
| IV | 2.38 | 0.851 | -0.766 | -0.373 | -0.170 | -0.652 | -0.789 | 5.52 | 0.886 | 0.927 | 1.98 | 1.61 | -0.445 | 1.30 |
| V | 2.61 | 2.17 | 1.31 | 0.869 | -1.66 | -3.21 | -3.34 | 6.53 | 1.38 | 1.38 | 1.09 | 2.06 | -2.83 | -4.03 |
| VI | 7.70 | 4.28 | 5.21 | 3.52 | 1.06 | -0.532 | -0.991 | 4.38 | 1.60 | 1.65 | 1.27 | 0.358 | -0.039 | -2.36 |
| VII | 3.23 | 1.86 | 0.155 | 4.54 | 1.51 | 0.885 | 2.17 | 10.71 | 1.10 | 2.80 | 1.51 | 0.756 | 1.15 | -1.08 |
| SHIP | 3.34 | 1.84 | 1.00 | 1.34 | -0.180 | -1.50 | -1.56 | 5.71 | 2.22 | 1.36 | 1.29 | 1.26 | -0.327 | -1.69 |
|  | $\mathrm{Fr}=0.20$ |  |  |  |  |  |  | $\mathrm{Fr}=0.25$ |  |  |  |  |  | $\begin{aligned} & \stackrel{\vdots}{\circ} \\ & \substack{\infty \\ \hline} \end{aligned}$ |
| $\omega$ | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.72 | 15.47 | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.56 | 15.48 |
| I | 2.96 | 1.71 | 1.97 | 2.23 | 2.10 | 1.76 | -0.472 | 2.39 | -0.626 | 0.106 | 0.973 | 2.15 | 1.51 | 1.37 |
| II | 7.91 | 1.22 | 0.904 | 1.46 | 2.66 | 2.76 | -1.68 | 3.67 | 2.58 | 0.799 | 1.73 | 3.87 | 4.96 | 2.78 |
| III | 1.34 | 0.471 | 1.19 | 2.08 | 1.41 | -0.087 | -0.471 | 4.17 | 4.77 | 0.184 | 1.01 | 0.173 | -0.833 | -3.25 |
| IV | 1.2 | 0.632 | 1.03 | 0.337 | -1.35 | -1.00 | 1.85 | 3.85 | -1.44 |  | 1.64 | 1.94 | -0.513 | 1.06 |
| V | 2.94 | 2.74 | 2.17 | 1.30 | 0.244 | -1.20 | -7.00 | 6.23 | 0.668 | 0.058 | -0.085 | 0.694 | 1.44 | -0.684 |
| VI | -0.976 | 3.45 | 2.55 | 1.87 | 0.905 | 1.91 | -1.20 | 4.04 | 0.397 | 0.241 | 0.827 | 1.93 | 2.40 | 0.605 |
| VII | 4.85 | 0.605 | 1.26 | 1.52 | 5.52 | 1.63 | -1.36 | 1.98 | 2.80 | 1.60 | 0.038 | 0.252 | 1.29 | 3.98 |
| SHIP | 3.02 | 1.70 | 1.69 | 1.60 | 1.32 | 0.670 | -1.695 | 4.38 | 1.213 | 0.607 | 1.032 | 1.778 | 2.004 | 0.530 |

## ADDED MOMENT OF INERTIA ( $\mathrm{k}_{\theta}$ )

|  | $\mathrm{Fr}=0$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\omega$ | 5.00 | 7.35 | 8.65 | 10.10 | 11.35 | 14.00 | 15.50 |  |
| Seg \# |  |  |  |  |  |  |  |  |
| II | 6.78 | 3.51 | 2.42 | 1.48 | 0.806 | -0.349 | -0.871 |  |
| III | 2.98 | 2.85 | 2.68 | 2.49 | 2.31 | 1.95 | 1.76 |  |
| II | 2.41 | 1.32 | 1.075 | 0.914 | 0.825 | 0.707 | 0.649 |  |
| V | -0.612 | -0.576 | -0.570 | -0.570 | -0.577 | -0.584 | -0.591 |  |
| VI | 1.876 | 0.409 | 0.539 | 0.660 | 0.740 | 0.834 | 0.862 |  |
| VII | 7.65 | 17.2 | 3.38 | 3.40 | 3.13 | 2.80 | 2.62 |  |
| SHIP | 0.0765 | 0.0479 | 0.2 | 10.25 | 8.38 | 6.89 | 6.46 |  |


| $\mathrm{Fr}=0.20$ |  |  |  |  |  |  |  | $\mathrm{Fr}=0.25$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\omega$ | 5.00 | 7.35 | 8.65 | 10.10 | 11.35 | 14.00 | 15.50 | 5.00 | 7.35 | 8.65 | 10.10 | 11.35 | 14.00 | $15.00{ }^{\prime}$ |
| Seg \# |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| I | 7.56 | 3.47 | 2.33 | 1.33 | 0.698 | 0.262 | -0.327 | 6.52 | 3.73 | 2.94 | 2.27 | 1.87 | 1.11 | 0.698 |
| II | 3.18 | 2.59 | 2.44 | 1.98 | 2.26 | 2.10 | 2.02 | 4.28 | 2.95 | 2.70 | 2.51 | 2.41 | 2.24 | 2.14 |
| III | 2.87 | 1.29 | 0.927 | 0.707 | 0.611 | 0.530 | 0.523 | 2.69 | 1.24 | 0.950 | 0.082 | 0.729 | 0.684 | 0.670 |
| IV | - 0.373 | - 0.514 | - 0.549 | - 0.570 | -0.577 | -0.577 | -0.577 | - 0.584 | - 0.577 | - 0.570 | -0.0570- | -0.577 | -0.577 | -0.584 |
| V | 3.02 | 0.287 | 0.596 | 0.817 | 0.491 | 1.02 | 1.01 | - 1.40 | 0.495 | 0.869 | 1.06 | 1.14 | 1.16 | 1.13 |
| VI | 4.19 | 7.29 | 3.03 | 2.84 | 2.71 | 0.398 | 2.80 | 3.77 | 3.45 | 3.24 | 3.05 | 2.93 | 3.09 | 3.24 |
| VII | 34.9 | 16.2 | 11.9 | 9.19 | 7.94 | 0.909 | 5.37 | 36.6 | 15.5 | 10.5 | 7.47 | 10.39 | 5.81 | 5.90 |
| SHIP | 0.088 | 0.055 | 0.035 | 0.030 | 0.028 | 0.025 | 0.023 | 0.073 | 0.044 | 0.037 | 0.032 | 0.034 | 0.028 | 0.028 |

## TABLE VIII

## ROTARY DAMPING COEFFICIENTS ( $\mathrm{f}_{\theta}{ }^{\prime}$ )

$\mathrm{Fr}=0$
$\mathrm{Fr}=0.10$

| $\omega$ | 5.00 | 7.35 | 8.65 | 10.10 | 11.35 | 14.00 | 15.50 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Seg \# |  |  |  |  |  |  |  |
| I | 4.51 | 5.85 | 5.52 | 2.98 | -2.42 | -4.48 | 4.09 |
| II | 3.14 | 5.65 | 4.97 | 3.41 | 0.263 | -0.183 | 1.83 |
| III | 0.522 | 1.48 | 0.763 | 0.161 | -0.552 | 0.090 | 0.612 |
| IV | 0.567 | -0.260 | -0.183 | -0.096 | -0.029 | 0.048 | 0.183 |
| V | 0.324 | 1.39 | 0.334 | -0.226 | $-0.481-1.37$ | -2.32 |  |
| VI | 2.06 | 6.37 | 4.40 | 2.55 | $0.279-2.78$ | -2.49 |  |
| VII | -4.38 | -1.20 | 9.74 | 17.1 | 16.7 | -10.8 | -8.90 |
| SHIP | 0.0609 | 0.1441 | 0.1365 | 0.1103 | $0.0357-0.0843$ | -0.0330 |  |


| 5.00 | 7.35 | 8.65 | 10.10 | 11.35 | 14.00 | 15.50 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
| 28.0 | 9.79 | 12.9 | 12.4 | 17.85 | 1.363 | -0.478 |
| 9.26 | 3.61 | 2.09 | 1.62 | 1.66 | 0.709 | 0.985 |
| 2.70 | -0.303 | -0.276 | -0.201 | -1.07 | -0.176 | -0.331 |
| -0.132 | 0.189 | 0.131 | 0.062 | -0.0083 | -0.118 | -0.102 |
| 0.417 | 0.040 | -0.397 | -0.528 | -0.0766 | -1.933 | -1.90 |
| 1.94 | 1.09 | 0.176 | -0.625 | -1.64 | -2.222 | -0.939 |
| 51.8 | 77.5 | 31.3 | 14.2 | 7.22 | -1.808 | -9.64 |
| 0.376 | 0.304 | 0.151 | 0.088 | 0.064 | -0.046 | -0.071 |

$\mathrm{Fr}=0.20$

| $\omega$ | 5.00 | 7.35 | 8.65 | 10.10 | 11.35 | 14.00 | 15.50 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 34.3 | 10.1 | 5.61 | 2.08 | 0.257 | 2.57 | 2.24 |
| II | 10.7 | 4.99 | 3.52 | 2.19 | 1.47 | 0.068 | -0.264 |
| III | 1.61 | 0.214 | -0.10 | -0.294 | -0.619 | -0.901 | -0.329 |
| IV | -0.416 | 0.287 | 0.429 | 0.259 | -0.186 | 0.961 | .0 .406 |
| V | 1.08 | 1.79 | 1.63 | 1.10 | 0.365 | -2.00 | -0.345 |
| VI | 4.82 | 4.38 | 4.96 | 4.96 | 3.31 | -1.95 | -5.37 |
| VII | 8.02 | 3.06 | 6.01 | 3.92 | 4.23 | -4.37 | -15.0 |
| SHIP | 0.296 | 0.147 | 0.130 | 0.090 | 0.050 | -0.040 | -0.127 |

$\mathrm{Fr}=0.25$

| 5.00 | 7.35 | 8.65 | 10.10 | 11.35 | 14.00 | 15.50 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 24.2 | 8.22 | 4.17 | 0.868 | -0.491 | -3.34 | -4.54 |
| 5.55 | -0.051 | -0.165 | 0.398 | 0.529 | -0.476 | -1.69 |
| 1.17 | -0.476 | -0.530 | -0.294 | -0.126 | 0.194 | 0.413 |
| -0.277 | 0.189 | 0.223 | 0.112 | -0.025 | -0.398 | -0.674 |
| 1.73 | 2.59 | 5.53 | 2.01 | 1.29 | 2.94 | 1.40 |
| 6.79 | 2.55 | 8.69 | 2.77 | 0.705 | -0.655 | 2.97 |
| 12.9 | 7.59 | 23.8 | 6.69 | 28.8 | -3.10 | -14.0 |
| 0.252 | 0.095 | 0.210 | 0.068 | 0.106 | $-0.0023-0.037$ |  |

## TABLE IX

## SQUARE LAW VERTICAL RESISTANCE

(UP STROKE $\mathrm{d}^{\prime}{ }_{\mathrm{z}_{1}}$ )

| $\omega$ | 5.00 | 6.00 | 8.00 | 10.00 | 12.00 | 14.00 | 15.47 | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.56 | 15.70 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Seg}_{I} \#$ | - 7.50 | - 4.75 | - 2.80 | - 2.24 | -0.675 | 0.405 | 0.675 | - 8.85 | $-5.32$ | -2.89 | -3.27 | $-2.62$ | $-1.39$ | 0.262 |
| II | $-13.5$ | - 9.62 | - 5.57 | - 3.89 | -0.985 | 5.29 | 3.16 | -26.0 | - 9.85 | -7.12 | -5.22 | -2.93 | $-0.788$ | $2.36$ |
| III | -17:7 | - 9.36 | - 2.56 | - 2.61 | 0.916 | 6.23 | 3.86 | -19.9 | -14.2 | 2.67 | -1.40 | -1.95 | -1.94 | 2.42 |
| IV | -14.8 | - 7.36 | - 1.64 | - 1.95 | -1.55 | 0.099 | 0.588 | -29.5 | - 5.44 | -3.94 | -6.48 | -4.02 | -0.724 | -3.58 |
| V | -22:2 | -16.4 | - 8.94 | - 4.72 | 2.06 | 10.5 | 6.15 | -37.2 | - 9.52 | -6.42 | -4.24 | -4.61 | 5.46 | 5.83 |
| VI | -43.2 | -23.9 | -19.2 | -10.7 | -3.35 | 1.80 | 2.20 | -29.8 | -11.2 | -9.52 | $-6.25$ | -0.900 | 0.112 | 1.88 |
| VII | -18:8 | -11:7 | - 4.91 | - 9.30 | -3.37- | $-3.24$ | -2.66 | - 7.07 | - 4.15 | -7.44 | -4.07 | -1.81 | -2.01 | -0.029 |
| SHIP | $-24.3$ | -14.6 | - 7.83 | - 5.71 | -0.923 | 4.55 | 3.00 | -29.8 | -11.4 | -6.91 | -5.46 | -3.50 | -0.190 | 1.80 |

[^0]$\mathrm{Fr}=0.25$


| $\omega$ | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.72 | 15.47 | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.56 | 15.48 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Seg \# |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| I | -12.8 | - 6.37 | -5.37 | -4.91 | $-3.78$ | -3.54 | -0.032 | -17.2, | - 2.89 | -3.62 | -3.24 | - 3.86 | $-3.32$ | -2.15 |
| II | -45.8 | -10.3 | -7.24 | -7.21 | -7.14 | -5.98 | 1.46 | -34.1 | -17.1 | -8.52 | -8.34 | -10.5 | -10.8 | -3.66 |
| III | -15.2 | -5.40 | -4.91 | -4.93 | -2.67 | 0.179 | 2.26 | -21.8 | -14.1 | -3.86 | -2.57 | - 0.640 | 1.13 | 4.13 |
| IV | -11.8 | - 5.49 | -4.79 | -3.00 | -0.574 | 0.668 | -3.80 | -19.9 | - 0.208 | -5.54 | -5.44 | - 5.86 | - 0.339 | -3.15 |
| V | $-24.7$ | - 9.55 | -6.74 | -5.31 | -2.30 | 2.65 | 9.13 | -37.0 | -8.78 | -3.42 | -2.83 | - 2.50 | - 1.80 | 2.85 |
| VI | $-12.3$ | -17.1 | -7.82 | -6.32 | -2.34 | -2.72 | 2.79 | -29.4 | -11.8 | -5.12 | -4.62 | - 3.47 | - 2.21 | 1.61 |
| VII | -18.1 | - 6.70 | -5.38 | -3.86 | -8.01 | -2.19 | 1.85 | -11.5 | -12.5 | -5.30 | -1.64 | - 0.477 | - 1.73 | -2.92 |
| SHIP | -26.1 | -10.8 | -7.61 | -6.51 | -4.57 | -1.84 | 2.47 | -31.9 | -12.3 | -6.40 | -5.37 | - 5.16 | - 3.45 | -0.275 |

TABLE X
SQUARE LAW VERTICAL RESISTANCE
(DOWN STROKE $d^{\prime} z_{2}$ )
$\mathrm{Fr}=0$
$\mathrm{Fr}=0.10$

| $\omega$ | 5.00 | 6.00 | 8.00 | 10.00 | 12.00 | 14.00 | 15.00 | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.56 | 15.70 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Seg \# |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| I | - 6.09 | - 3.56 | - 2.13 | - 1.21 | -0.378 | 0.248 | 0.432 | -15.4 | - 1.84 | -2.54 | -2.46 | -1.32 | -0.513 | 1.17 |
| II | - 7.28 | - 5.77 | - 3.87 | - 3.00 | -0.595 | 3.96 | 1.82 | -40.2 | - 9.92 | -6.27 | -4.49 | -1.62 | -0.062 | 4.49 |
| III | -12.4 | - 6.16 | - 1.32 | - 1.88 | 1.19 | 6.23 | 3.74 | -25.7 | -13.5 | -2.05 | -0.98 | -1.48 | -0.423 | 3.78 |
| IV | -13.4 | - 5.61 | 0.109 | - 0.767 | -1.21 | -1.52 | -0.929 | -36.3 | -5.81 | -5.81 | -5.61 | -3.78 | -1.29 | -3.29 |
| V | -19.3 | -15.2 | - 8.162 | - 3.29 | 2.41 | 8.12 | 4.12 | -50.7 | - 9.80 | -6.37 | -3.32 | -4.12 | 5.11 | 5.41 |
| VI | -38.7 | -23.6 | -19.7 | - 9.96 | -3.03 | 0.0412 | 0.735 | -40.0 | -11.1 | -6.14 | -4.41 | -1.92 | 1.06 | 3.41 |
| VII | -16.2 | -10.8 | - 5.37 | -10.4 | -4.15 | -3.69 | -3.00 | -19.4 | - 7.68 | -8.06 | -5.42 | -2.26 | -2.12 | 1.20 |
| SHIP | -19.7 | -12.2 | - 6.71 | - 4.79 | -0.634 | 3.16 | 1.71 | -42.0 | -11.14 | -6.41 | -4.59 | -2.97 | 0.253 | 3.80 |
| $\mathrm{Fr}=0.20$ |  |  |  |  |  |  |  | $\mathrm{Fr}=0.25$ |  |  |  |  |  |  |
| $\omega$ | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.72 | 15.47 | 5.02 | 7.54 | 8.80 | 10.03 | 11.30 | 12.56 | 15.48 |
| Seg \# |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| I | - 9.67 | - 4.37 | -4.29 | -4.21 | -3.35 | -2.38 | 0.084 | - 9.50 | $-2.34$ | -1.60 | -3.16 | -3.78 | $-2.52$ | -1.33 |
| II | -34.6 | - 4.97 | -4.81 | -4.17 | -6.32 | -4.95 | 2.54 | -23.6 | - 9.96 | -4.99 | -4.90 | -7.26 | -8.36 | -2.05 |
| III | -8.54 | - 2.79 | -3.70 | -4.29 | -2.88 | 1.10 | 2.01 | -24.4 | -13.2 | -1.64 | -2.14 | -0.060 | 1.78 | 5.77 |
| IV | - 7.70 | - 3.51 | -4.31 | -3.27 | 0.179 | -0.406 - | $-3.82$ | -29.4 | - 0.576 | -4.94 | -5.37 | -4.98 | -1.57 | -1.70 |
| V | -21.5 | - 7.36 | -7.29 | -3.72 | -0.040 | 2.78 | 10.1 | -42.6 | - 7.27 | -2.73 | 0.506 | -1.18 | -0.255 | 4.34 |
| VI | 0.106 | -11.6 | $-7.77$ | -4.85 | -1.58 | -1.60 | 4.06 | -34.4 | - 4.35 | -1.82 | -2.97 | -2.88 | -1.39 | 3.88 |
| VII | -21.9 | - 4.60 | $-4.36$ | -4.77 | -7.81 | -2.08 | 2.17 | -21.3 | -10.2 | -6.12 | -2.11 | -1.34 | -1.70 | -2.07 |
| SHIP | -18.7 | - 6.85 | $-6.53$ | $-5.24$ | $-3.63$ | -1.20 | 3.06 | -33.8 | - 8.90 | $-4.13$ | -3.89 | -3.93 | -2.47 | 1.60 |

## TABLE XI

## SQUARE LAW ROTATIONAL RESISTANCE

(UP STROKE $\mathrm{d}^{\prime} \theta_{1}$ )

$$
F r=0
$$

| $\omega$ | 4.99 | 7.35 | 8.65 | 10.11 | 11.35 | 13.80 | 15.43 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Seg \# |  |  |  |  |  |  |  |
| I | -58.9 | -35.3 | -15.7 | -41.4 | 50.4 | 40.1 | -114.2 |
| II | -24.4 | -73.1 | -59.6 | -37.7 | 1.52 | -20.2 | -48.9 |
| III | 25.2 | -17.7 | -5.84 | 0.515 | 9.57 | -9.05 | -19.5 |
| IV | -22.8 | 2.34 | 3.89 | 0.620 | 1.77 | -1.30 | -1.11 |
| V | 37.2 | -1.68 | 1.04 | 10.8 | 3.66 | 17.5 | 27.1 |
| VI | 240.0 | -35.9 | -8.64 | 5.40 | -39.1 | 42.2 | 16.1 |
| VII | 521.8 | 274.4 | -11.2 | -150.9 | -411.6 | 213.6 | 73.0 |
| SHIP | 1.37 | -0.181 | -0.238 | -0.291 | -0.473 | 0.396 | -0.162 |

$$
\mathrm{Fr}=0.10
$$

| $\omega$ | 5.02 | 7.33 | 8.72 | 10.12 | 11.49 | 13.81 | 15.49 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Seg \# |  |  |  |  |  |  |  |
| I | -872.0 | -81.1 | -202.5 | -194.2 | -300.8 | -93.1 | -52.3 |
| II | -236.9 | -33.5 | -8.35 | 3.10 | 30.2 | -38.2 | -48.5 |
| III | 98.6 | 23.6 | 7.08 | -0.780 | 3.80 | -10.4 | -17.7 |
| IV | 7.88 | -6.57 | -6.77 | -1.80 | -1.23 | 1.46 | 1.10 |
| V | 89.0 | 40.8 | 29.4 | 27.9 | 23.8 | 41.7 | 34.1 |
| VI | 238.3 | 106.9 | -76.0 | 73.8 | 55.5 | 50.8 | -18.6 |
| VII | -1818 | -1913 | -514.5 | -83.3 | -10.6 | 125.4 | 151.4 |
| SHIP | -3.26 | -1.86 | -0.524 | 0 | -0.018 | 0.189 | -0.046 |

## TABLE XI (cont'd)

$$
\mathrm{Fr}=0.20
$$

| $\omega$ | 5.00 | 7.43 | 8.72 | 10.18 | 11.48 | 13.00 | 15.44 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Seg \# |  |  |  |  |  |  |  |
| II | -981.0 | -14.2 | -23.8 | 52.8 | 42.3 | -24.2 | -58.4 |
| II | -262.0 | -49.5 | -33.9 | -19.8 | -22.8 | -16.0 | -30.3 |
| III | -54.6 | -0.611 | -11.0 | -7.51 | -4.63 | -1.21 | -22.0 |
| IV | -18.7 | -11.0 | -13.9 | -5.53 | 3.64 | -14.4 | 3.61 |
| V | 39.6 | -0.596 | 4.94 | 2.22 | 9.76 | 21.2 | 58.1 |
| VI | -36.6 | 8.17 | -40.6 | -30.6 | -31.7 | 36.4 | 53.8 |
| VII | 164.5 | 169.3 | 101.9 | 68.6 | 24.1 | 32.8 | 195.3 |
| SHIP | -2.24 | -0.013 | -0.239 | -0.061 | -0.061 | 0.089 | 0.379 |


| $\omega$ | 5.00 | 7.48 | 8.64 | 10.05 | 11.35 | 13.31 | 15.44 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Seg \# |  |  |  |  |  |  |  |
| I | -503.6 | 18.2 | 84.4 | 93.7 | 55.8 | 67.1 | 66.3 |
| II | - 69.6 | 78.2 | 75.3 | 1.84 | - 9.12 | -16.2 | 60.4 |
| III | - 18.5 | - 9.20 | -. 6.90 | - 7.07 | - 15.4 | -25.5 | - 29.7 |
| IV | 6.59 | - 7.53 | - 8.66 | - 2.36 | 0.098 | 4.49 | 7.39 |
| V | 30.7 | -23.5 | - 96.2 | -10.6 | - 5.66 | -20.4 | 40.1 |
| VI | -110.4 | 35.9 | -1.50.7 | - 8.85 | 13.7 | 32.4 | 42.7 |
| VII | -200.2 | -19.9 | -313.6 | 1.94 | -431.2 | 53.2 | 191.6 |
| SHIP | - 1.40 | 0.230 | - 0.925 | 0.023 | - 0.492 | 0.038 | 0.538 |

TABLE XII
SQUARE LAW ROTATIONAL RESISTANCE
(DOWN STROKE ${ }^{2} \dot{\theta}_{2}$ )

$$
\mathrm{Fr}=0
$$

| Seg $\#$ | 4.99 | 7.35 | 8.65 | 10.11 | 11.35 | 13.80 | 15.43 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Seg}_{\mathrm{I}}^{\#}$ | 102.9 | 42. | -44.2 |  |  |  |  |
| II | - 32.0 | - 47.4 | -44.2 -78.5 | -22.7 -39.8 | - 61.9 <br> - 0.469 | -39.4 $-\quad 2.61$ | -128.2 |
| III | - 3.97 | - 22.5 | - 1.59 | - 1.82 | 10.3 | - 11.3 | - 21.7 |
| IV | - 22.8 | - 3.40 | 4.06 | 2.11 | 2.26 | - 0.774 | - 3.77 |
| V | - 11.1 | - 6.94 | 9.33 | 7.75 | 1.35 | 6.59 | 18.4 |
| VI | - 36.9 | - 47.3 | - 17.6 | 8.50 | - 31.8 | 31.3 | - 17.2 |
| VII | 592.9 | 264.6 | 19.7 | -187.7 | -436.1 | 210.4 | 95.3 |
| SHIP | 0.441 | - 0.072 | - 0.308 | - 0.323 | - 0.473 | 0.365 | P $-\quad 0.317$ |

$$
\mathrm{Fr}=0.10
$$

| $\omega$ | 5.02 | 7.33 | 8.72 | 10.12 | 11.49 | 13.81 | 15.49 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Seg \# |  |  |  |  |  |  |  |
| I | -619.1 | - | 67.6 | -177.4 | -192.3 | -296.5 | -79.8 |
| II | -159.0 | - | 6.28 | -1.33 | -12.6 | -22.9 | -29.5 |
| III | -86.8 | 22.1 | 15.2 | 1.04 | 10.2 | -11.4 | -18.8 |
| IV | 0.507 | - | 7.25 | -3.39 | -2.36 | -0.246 | 13.8 |
| V | 40.1 | 15.5 | 30.8 | 22.8 | 19.5 | 28.0 | 0.845 |
| VI | 110.4 | 82.8 | 91.1 | 62.2 | 57.6 | 37.0 | 23.2 |
| VII | -1703 | -1896 | -521.8 | -155.8 | -26.5 | 128.9 | 103.6 |
| SHIP | -1.13 | -1.92 | -0.388 | -0.187 | -0.192 | 0.138 | -0.037 |

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## TABLE XII (cont'd)

| $\mathrm{Fr}=0.20$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\omega$ | 5.00 | 7.43 | 8.72 | 10.18 | 11.48 | 13.00 | 15.44 |
| Seg \# |  |  |  |  |  |  |  |
| I | -839.3 | -0.371 | 45.1 | 61.7 | 31.4 | -25.3 | 74.1 |
| II | -263.6 | -62.2 | -39.2 | -33.6 | -31.6 | -15.4 | 25.4 |
| III | -29.7 | -4.05 | -12.8 | -14.5 | -7.04 | -5.61 | 14.9 |
| IV | -19.8 | -7.60 | -10.5 | -6.64 | 2.60 | -14.8 | - |
| V | 34.4 | -13.8 | -15.9 | -0.696 | 15.0 | 19.4 | -40.1 |
| VI | 134.9 | -24.2 | -48.8 | -64.7 | -25.2 | 23.5 | -47.2 |
| VII | -63.4 | 75.5 | 46.6 | 78.6 | 43.4 | 46.8 | -138.4 |
| SHIP | -1.68 | -0.303 | -0.325 | -0.024 | -0.055 | 0.040 | -0.249 |


|  | $\mathrm{Fr}=0.25$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\omega$ | 5.00 | 7.48 | 8.64 | 10.05 | 11.35 | 13.31 | 15.44 |
| Seg \# |  |  |  |  |  |  |  |
| I | -479.6 | 18.5 | 100.7 | 87.6 | 77.6 | 49.9 | 21.4 |
| II | -35.6 | 74.5 | 50.9 | 8.79 | -1.68 | -13.2 | 48.0 |
| III | -35.5 | -2.94 | -5.67 | -12.7 | -14.7 | -26.5 | -27.8 |
| IV | 1.01 | -6.54 | -14.91 | -3.10 | 0.049 | 4.82 | 8.38 |
| V | -9.55 | -30.9 | -93.3 | -15.2 | 1.51 | -22.2 | 27.9 |
| VI | -56.8 | 22.8 | -134.0 | -13.9 | 10.2 | 25.9 | 26.1 |
| VII | -178.1 | 12.6 | -460.6 | -57.3 | -409.2 | 62.0 | 114.7 |
| SHIP | -1.30 | 0.205 | -1.07 | -0.080 | -0.405 | 0.005 | 0.297 |

## TABLE XIII

AMPLITUDE IN POUNDS AND PHASE SHIFT IN DEGREES

| Ru |  | 26407 |  | 26410 |  | 26455 |  |  |  | 26462 |  | 26465 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | y (fps) | 0 |  | 0 |  | 1.268 |  | 1.268 |  | 0 |  | 0 |  |
|  | ( $\mathrm{ad} / \mathrm{sec}$ ) | 5.02 |  | 5.02 |  | 7.00 |  | 7.00 |  | 7.54 |  | 7.54 |  |
|  |  | $\delta$ | A | $\delta$ | A | $\delta$ | A | $\delta$ | A | $\delta$ | A | $\delta$ | A |
| I | F | -14.9 | 0.600 | - 0 | 0.660 | 10.9 | 0.525 | -56.5 | 0.505 | -10.9 | 0.407 | -10.8 | 0.330 |
|  | A | -28.4 | 0.815 | -23.4 | 0.835 | -13.0 | 0.722 | -43.5 | 0.754 | -21.4 | 0.627 | -19.5 | 0.670 |
| II | F | -48.4 | 1.40 | -14.6 | 1.45 | -32.6 | 1.26 | -43.5 | 1.08 | -8.68 | 1.035 | - 4.35 | 1.055 |
|  | A | -26.9 | 2.14 | -24.9 | 1.94 | -36.9 | 1.77 | -56.5 | 1.65 | -43.4 | 1.34 | -39.1 | 1.36 |
| III | F | -43.4 | 1.32 | -35.1 | 1.26 | -52.1 | 1.30 | -63.0 | 1.24 | -43.4 | 1.30 | -32.6 | 1.44 |
|  | A | -31.4 | 1.41 | -35.1 | 1.59 | -34.8 | 1.24 | -50.0 | 1.24 | -34.7 | 1.12 | -30.4 | 1.30 |
| IV | F | -22.4 | 1.48 | -13.2 | 1.56 | -30.4 | 1.11 | -30.4 | 1.11 | -39.1 | 1.19 | -21.7 | 1.39 |
|  | $\stackrel{\text { A }}{\text { F }}$ | -31.4 -25.4 | 1.35 1.82 | -27.8 | 1.63 | -30.4 | 1.07 | -30.4 | 1.13 | -56.6 | 1.25 | -50.0 | 1.37 |
| v | F | -25.4 -26.9 | 1.82 2.00 | -27.8 -24.9 | 1.82 1.96 | -54.3 -45.6 | 1.56 1.67 | -54.3 -54.3 | 1.60 1.79 | -41.2 | 1.38 1.54 | -50.0 | 1.46 1.54 |
| VI | F | -26.9 | 1.65 | -24.9 | 1.96 1.71 | -45.6 | 1.67 1.54 | -54.3 -46.5 | 1.79 1.52 | -39.1 | 1.54 1.38 | -23.9 | 1.54 1.51 |
|  | A | -26.9 | 2.20 | -24.9 | 2.12 | -47.8 | 1.61 | -36.9 | 1.68 | -28.2 | 1.70 | -45.5 | 1.87 |
| VII | F | -37.4 | 1.12 | -39.5 | 1.08 | -67.4 | 0.880 | -50.0 | 0.758 | -34.7 | 0.897 | -30.4 | $1.03$ |
|  | A | -14.9 | 1.12 | -13.2 | 1.14 | -13.0 | 0.776 | 4.35 | 0.795 | -39.0 | 0.815 | -28.2 | $0.911$ |

TABLE XIII (cont'd)


| Ru |  | 26556 |  | 26559 |  | 26562 |  | 26565 |  | 26603 |  | 26606 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ty (fps) | 1.2 |  | 1.2 |  | 0 |  | 0 |  |  | 0 |  | 0 |
| Freq. (rad/sec) |  | 12.56 |  | 12.56 |  | 12.56 |  | 12.56 |  | 15.70 |  | 15.70 |  |
|  |  | $\delta$ | A | $\delta$ | A | $\delta$ | A | $\delta$ | A | $\delta$ | A | $\delta$ | A |
| II | F | -178 | 1.24 | -112 | 0.640 | -193 | 0.446 | -170 | 0.270 | -171 | 1.03 | -299 | 1.03 |
|  | A | -131 | 0.962 | - 73.5 | 0.878 | -136 | 0.606 | -148 | 0.560 | -148 | 1.23 | -192 | 1.27 |
|  | F | -150 | 1.38 | -128 | 1.55 | -163 | 1.20 | -170 | 1.18 | -144 | 2.42 | -229 | 2.41 |
|  | A | -190 | 2.14 | -151 | 2.52 | -159 | 2.04 | -186 | 1.90 | -158 | 4.50 | -202 | 4.20 |
| III | F | -174 | 1.81 | -155 | 2.42 | -170 | 2.22 | -178 | 1.89 | -148 | 3.60 | -215 | 3.40 |
|  | A | -202 | 2.53 | -159 | 2.32 | -185 | 2.59 | -194 | 2.61 | -158 | 4.50 | -210 | 4.41 |
| IV | F | -170 | 2.87 | $-182$ | 2.09 | -155 | 2.40 | -170 | 2.25 | -131 | 3.95 | -178 | 5.09 |
|  | A | -170 | 2.48 | $-182$ | 2.48 | -155 | 2.34 | -159 | 2.31 | -131 | 3.35 | -187 | 4.75 |
| V | F | -190 | 3.22 | -197 | 3.28 | -159 | 2.76 | -163 | 2.66 | -144 | 5.86 | -202 | 5.00 |
|  | A | -190 | 3.23 | -201 | 3.06 | -155 | 2.45 | -178 | 2.47 | -165 | 4.63 | -206 | 4.22 |
| VI | F | -186 | 1.81 | -267 | 1.85 | -151 | 1.36 | -186 | 1.38 | -165 | 2.79 | -215 | 2.93 |
|  | A | -238 | 1.70 | -205 | 1.89 | -163 | 1.52 | -182 | 1.38 | -144 | 2.60 | -215 | 2.78 |
| VII | F | - 95.0 | 0.749 | $-136$ | 0.730 | - 98.5 | $0.785$ | -166 | $0.935$ | $-131$ | $1.46$ | $-183$ | $1.52$ |
|  | A | -210 | 0.252 | ? | 0.194 | -159 | 0.136 | -110 | $0.116$ | -152 | $0.74$ | -220 | 0.95 |
| Run |  | 26609 |  | 26612 |  | 27006 |  | 27009 |  |  |  |  |  |
| Velocity (fps) |  | 1.279 |  | 1.279 |  | 1.272 |  | 1.274 |  |  |  |  |  |
| Freq. ( $\mathrm{rad} / \mathrm{sec}$ ) |  | 15.70 |  | 15.70 |  | 5.02 |  | 5.02 |  |  |  |  |  |
| Segment |  | 8 | A | $\delta$ | A | ठ | A | $\delta$ | A |  |  |  |  |
| I | F | -164 | 1.71 | -159 | 1.24 | -58.0 | 0.601 - | 29.9 | 0.68 |  |  |  |  |
| II | A | -146 | 1.90 | -138 | 1.67 | -56.5 | $0.752-$ | 40.7 | 0.79 |  |  |  |  |
|  | F | -178 | 3.23 | -156 | 2.68 | -36.2 | 1.68 - | 38.0 | 1.68 |  |  |  |  |
|  | A | -159 | 5.76 | -159 | 3.82 | 36.2 | 2.16 - | 53.0 | 2.16 |  |  |  |  |
| III | F | -195 | 4.23 | -167 | 3.27 | -49.3 | 1.60 - | 47.5 | 1.56 |  |  |  |  |
|  | A | -196 | 4.84 | -167 | 4.77 | -33.4 | $1.67-3$ | 3616 | 1.67 |  |  |  |  |
| IV | F | -159 | 5.10 | -178 | 2.62 | -49.3 | 1.60 - | 28.5 | 1.46 |  |  |  |  |
|  | A | -105 | 4.85 | -138 | 2.74 | -30.4 | 1.62 - | 31.2 | 1.58 |  |  |  |  |
| V | F | $-182$ | 6.86 | -156 | 3.80 | -42.0 | 1.62 - | 42.1 | 1.80 |  |  |  |  |
|  | A | -196 | 5.68 | -159 | 5.06 | -46.5 | $1.84-$ | 53.0 | 1.88 |  |  |  |  |
| VI | F | -192 | 4.05 | -167 | 3.44 | -39.2 | $1.34-$ | 53.0 | 1.44 |  |  |  |  |
|  | A | -205 | 3.86 | -171 | 2.97 | -30.4 | 1.87 - | 43.5 | 1.99 |  |  |  |  |
| VII | F | -150 | 1.76 | -149 | 1.70 q | -45.0 | $0.805-$ | 40.7 | 0.82 |  |  |  |  |
|  | A | -196 | 1.11 | $-182$ | 1.07 | $-34.8$ | 0.970 - | 29.9 | 1.05 |  |  |  |  |

## TABLE XIV

## ADDED MASS COEFFICIENTS BY

 SIMPLIFIED METHOD$$
F r=0
$$

| $\omega$ | 5.00 | 7.26 | 8.61 | 9.97 | 11.30 | 12.70 | 15.70 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Seg \# |  |  |  |  |  |  |  |
| I | 0.201 | 0.246 | 0.141 | -0.035 | 0.052 | 0.138 | 0.230 |
| II | 1.93 | 1.01 | 1.05 | 0.791 | 0.588 | 1.07 | 0.939 |
| III | 2.13 | 0.814 | 0.745 | 0.916 | 0.476 | 0.982 | 0.733 |
| IV | 1.41 | 0.946 | 0.597 | 0.765 | 0.867 | 0.903 | 0.735 |
| V | 1.67 | 0.437 | 0.668 | 0.651 | 0.558 | 0.910 | 0.895 |
| VI | 0.805 | 0.559 | 0.859 | 0.695 | 0.842 | 0.769 | 0.592 |
| VII | 0.610 | 0.561 | 0.672 | 0.195 | 0.262 | 0.592 | 0.222 |
| SHIP | 1.32 | 0.778 | 0.697 | 0.684 | 0.598 | 0.840 | 0.689 |

$\mathrm{Fr}=0.10$

| $\omega$ | 4.88 | 7.00 | 8.68 | 9.85 | 11.40 | 12.80 | 15.40 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Seg \# |  |  |  |  |  |  |  |
| I | 1.12 | 0.269 | 0.263 | 0.141 | 0.511 | 0.294 | 0.499 |
| II | 1.84 | 1.10 | 0.924 | 0.914 | 0.698 | 1.10 | 1.19 |
| III | 2.53 | 1.19 | 0.706 | 0.875 | 0.825 | 0.883 | 0.904 |
| IV | 1.92 | 1.03 | 0.813 | 0.506 | 0.948 | 0.964 | 0.576 |
| V | 2.20 | 1.30 | 0.824 | 0.906 | 0.532 | 1.16 | 1.23 |
| VI | 2.60 | 0.787 | 0.301 | 0.304 | 0.787 | 0.514 | 0.865 |
| VII | 2.60 | 1.19 | 0.412 | 0.900 | 0.775 | 0.469 | 0.737 |
| SHIP | 2.08 | 1.00 | 0.646 | 0.643 | 0.713 | 0.944 | 0.858 |

## TABLE XV

## DAMPING COEFFICIENTS BY SIMPLIFIED METHOD

$$
\mathrm{Fr}=0
$$

| $\omega$ | 5.00 | 7.26 | 8.61 | 9.97 | 11.30 | 12.70 | 15.70 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Seg \# |  |  |  |  |  |  |  |
| I | 2.32 | 0.816 | 1.27 | 2.33 | 0.919 | 0.588 | 0 |
| II | 3.24 | 1.59 | 1.16 | 0.855 | 1.12 | 0.481 | 0.13 |
| III | 3.75 | 2.49 | 1.67 | 1.32 | 1.98 | 0.303 | -0.195 |
| IV | 2.92 | 2.52 | 0.879 | 1.53 | 1.62 | 1.38 | 1.75 |
| V | 3.57 | 2.73 | 1.68 | 1.48 | 3.07 | 1.26 | 0.610 |
| VI | 4.34 | 4.02 | 2.24 | 2.68 | 1.53 | 0.628 | 0.455 |
| VII | 5.07 | 4.22 | 2.12 | 1.78 | 2.51 | 1.15 | 0.630 |
| SHIP | 3.01 | 2.21 | 3.42 | 1.28 | 1.57 | 0.707 | 0.330 |

$$
\mathrm{Fr}=0.10
$$

| $\omega$ | 4.88 | 7.00 | 8.68 | 9.85 | 11.40 | 12.80 | 14.40 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Seg \# |  |  |  |  |  |  |  |
| I | 4.13 | 1.32 | 1.27 | 1.90 | 1.66 | 1.75 | 1.11 |
| II | 5.92 | 3.17 | 2.45 | 2.34 | 2.73 | 1.24 | 0.921 |
| III | 4.95 | 2.88 | 4.06 | 1.87 | 2.56 | 1.32 | -0.319 |
| IV | 3.95 | 1.74 | 1.65 | 1.94 | 1.75 | 0.327 | 2.37 |
| V | 6.05 | 2.77 | 1.59 | 2.06 | 2.46 | 1.37 | -0.183 |
| VI | 3.90 | 4.28 | 2.03 | 1.48 | 1.76 | 1.28 | -0.772 |
| VII | 5.90 | 2.88 | 2.82 | 3.10 | 2.76 | 1.27 | -0.608 |
| SHIP | 5.68 | 3.15 | 2.23 | 2.33 | 2.56 | 1.29 | 1.33 |


[^0]:    $\mathrm{Fr}=0.20$

