

## A STOCHASTIC PROGRAMMING MODEL OF SALINITY IN THE COLORADO RIVER BASIN

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### ABSTRACT

Along the Colorado River, naturally occurring salts underlie basin soils. Irrigation water leaches salt from the soil and return flows transport the salt to the river. As a result of salinity, downstream agricultural, municipal, and industrial uses suffer millions of dollars in damages each year. Weather variability can induce large swings in river flow volume, and hence river salinity. During periods of drought, problems due to salinity are worst. Currently, water quality policy decisions are based on average river flows. As a result, river water quality may exceed federal salinity standards during low flow years. This research details an approach for selecting mitigation alternatives to meet or exceed water quality standards under variable river flow conditions. Decisions are based on the value of clean water to downstream agriculture, the cost of mitigation, the variability of river flows, and the risk criteria of policy makers. Regions included in the model are the Grand Valley and Lower Gunnison Basin in Colorado, the Uinta Basin and the Price and San Rafael Regions in Utah, and the Imperial Valley in California.

### INTRODUCTION

Prehistoric seas once covered the area comprising the Colorado River Basin. Although the sea has long since resided, vast salt deposits

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remain beneath basin soils. Shale deposits four to five thousand feet thick underlie soils in the Grand Valley. The salinity of water diverted from the upper reaches of the basin average 300 mg/l. Return flow salinity can exceed 20,000 mg/l. The Colorado River Basin drains over 242,000 square miles of land. From off-farm canals and laterals, and from on-farm ditches and irrigated fields, millions of tons of salt from agricultural irrigation are loaded into the Colorado River each year. Because river water is diverted and used many times, it becomes progressively more saline as it moves downstream.

High costs inhibit upstream farmers from reducing salt loads voluntarily. Transactions costs impede opportunities for arbitrage. As a result, salinity affects 63% of the irrigated acreage in the Lower Basin. Because rights to water quality are not clearly defined, downstream recipients have no recourse. Along the Colorado River, salinity is the most important water quality problem.

#### Previous Work

Predictions by the Bureau of Reclamation of rising salinity through 2010, motivated several economic studies. Moore et al. [1974] simulated Imperial farm production with linear programming and estimated farm losses for salinity levels between 480 and 1920 mg/l. Kleinman and Brown [1980] sought damages to agricultural, municipal and industrial uses for salinity levels up to 1400 mg/l. Gardner [1983] compared the Upper Basin cost of input taxes, discharge penalties, cost share options, and land removal to Imperial Valley production losses for salinity levels between 800 and 1100 mg/l.

Recently, however, the Bureau has revised salinity estimates downward. Based on projected agricultural expansion, water development, and weather patterns, the Bureau predicted in 1985 that Imperial Dam salinity levels would be 1012 mg/l by 2010. In 1989 the Bureau's estimate for year 2010 was 970 mg/l.<sup>3</sup> The federally mandated water quality standard at Imperial Dam is 879 mg/l.

Previous studies have estimated the losses to downstream uses that would result from a rise in salinity to Bureau predicted levels under

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<sup>3</sup>Falling estimates can, in part, be attributed to construction of three Bureau water quality improvement projects. Together, these projects reduce annual Upper Basin salt loading by about 88,800 tons.

static flow assumptions. These studies compared the losses to the cost of avoidance. This study examines the influence of Upper Basin agricultural activities and stochastic river flows on Lower Basin river water quality. The model developed provides an analytical framework for assessing existing water quality standards, for evaluating proposed mitigation alternatives, and for analyzing theoretical risk criteria.

### MODEL DEVELOPMENT

The basin model contains three major components. A salt load component, a hydrology component, and an agricultural production component. Upper Basin salt loads are modelled as functions of water use, acreage planted, and investment in irrigation capital. An equation of motion relates Upper Basin salt loads and river flow volume with downstream water salinity. Regional agriculture is modelled as a function of land, capital, water, and water salinity. A stochastic programming problem comprises these models and in a series of equations that represent the hydrology, the agronomics, and the economics of irrigated river basin agriculture.

#### Modelling Agricultural Salt Load

The primary sources of agriculturally induced salt loads are runoff, field deep percolation, on-farm and off-farm ditches and laterals, and off-farm canals. Federal projects proposed under the Colorado River Basin Salinity Control Act will reduce return flows from these salt sources. Installation of water measuring devices, pipe laterals, and canal lining are examples of the type of projects under consideration. The high capital investment costs and the long term nature of some of the federal projects warrants consideration of alternative, less capital intensive options. Thus, reducing upstream water use, switching to less water intensive crops, and removing land from irrigated production are included in the model as alternative means of improving downstream water quality.

Return flow: Because ground aquifer volumes are large compared to irrigation return flow volume, return flow salinities ( $ec_{gt}$ ) can be assumed constant with respect to both return flow volume ( $RF_{gt}$ ) and irrigation water salinity ( $EC_{gt}$ ). Salinity of runoff water is assumed

equal to the salinity of the water diverted for irrigation, so the net load from runoff is zero.

Upper Basin irrigation practices are assumed to follow those required for long term production. Soils are leached and root zone salinity is in balance with irrigation water salinity. Return flows from fields are proportional to the volume of water applied ( $W_{gt}$ ) up to the amount that maximizes consumptive use. The leached fraction and the water applied in excess of maximum consumptive use deep percolates.

Thus, return flows from fields are linear in acreage planted ( $L_{gt}$ ) and nonlinear in water applied. Land levelling and water measuring devices can reduce salt load from irrigated fields by improving water application uniformity and water use efficiency ( $Z_{gt}$ ).

Through dirt ditches, unlined laterals, and leaky canals, water in transport percolates through the soil and carries salt to the river. Return flow from laterals and ditches are linear in acreage planted and independent of water applied. Laterals and ditches can be lined, head and tailwater ditch structures can be constructed, and pipe laterals ( $Z_{gt}$ ) can be installed to conserve water and reduce return flows. Return flow from canals are independent of applied water and planted acreage.

Salt load: Let  $j$  represent the set [field, ditch, lateral, canal]. Salt load flux is

$$dS_t = \sum_j e c_j dRF(RF_{g,t-1}, W_{gt}, L_{gt}, Z_{gt}) \quad (1)$$

Salt load flux is function of return flow salinity, previous year return flow, water use, irrigated acreage, and investment in salt load reduction capital.

### Modelling Stochastic River Flows

Gunnison River flow past the Grand Valley and Colorado River flow below the Imperial Dam fluctuates with annual deviations in mean precipitation (and evaporation). Though annual precipitation is an independent, random event, river water and salt can be retained in the system for many years in large basin reservoirs. Thus, annual river flows are dependent on the level of precipitation in the current

year and on the level of precipitation in past years. River operation requirements to meet multiple basin uses place lower bounds on and skew the distribution of expected river flows at Grand Junction and below Imperial Dam. For these two particular locations, water quality policy analysis requires estimates of three river flow parameters; the mean ( $\mu_1$ ), variance ( $\mu_2$ ), and degree of skewness ( $\mu_3$ ). The variance and degree of skewness parameters provide additional information regarding the range of possible flows and the probability of a severe drought.

### Modelling Surface Water Quality

A reduction in upstream salt load or a rise in river flow volume will improve downstream water quality. Quantifying the level of improvement is requisite to water quality policy analysis. Existing computer models of Colorado River Basin hydrology (i.e. Udis et al., 1973; and the Bureau's Colorado River Simulation System, 1987) rely on large databases and numerous equations to simulate a wide range of hydrologic scenarios. Incorporating disaggregate hydrologic interactions into a multiregional optimization framework is inherently difficult. Thus, in a previous economic study [Gardner, 1983] river flows were assumed static and the effects of salt load on downstream salinity were approximated with simple conversion ratios.<sup>4</sup> Modelling river flows as static precludes water conservation as a mitigation alternative. Conservation as a means of improving water quality had been considered by Scherer [1977] who modelled a hypothetical stream system in which stream flow could be transferred downstream to dilute salty irrigation water. Flows were assumed to be deterministic. Modelling river flows as deterministic, however, neglects the losses incurred during periods of drought. This section develops a model in which downstream water salinity is stochastic in upstream water use and upstream salt loading.

**Stochastic Mass Transport Model:** Where  $T_{Gt}$  is the flow of salt in the river past location  $G$  at time  $t$  and  $V_{Gt}$  is the volume of river

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<sup>4</sup> Assuming mean river flows below Imperial Dam of eight million acre-feet per year, a 10,000 ton salt load reduction will lower Imperial Valley water salinity by 1.01 mg/l.

flow past location G at time t, the change in the concentration of salts in the river at location G in time t ( $dEC_{Gt}$ ) can be expressed

$$dEC_{Gt} = (EC_{G,t-1}, dT_{Gt}, dV_{Gt}) \quad (2)$$

Let  $WD_{gt}$  represent the volume of water diverted from the river upstream of G at location g. Salt load flux at G is expressed

$$dT_{Gt} = dT(T_{G,t-1}, dV_{gt}, dWD_{gt}, dS_{gt}, EC_{g,t-1}, dEC_{gt}) \quad (3)$$

Let  $d\omega_{Gt}$  measure the deviation from mean precipitation. The fluctuation in river flow volume at G is modelled

$$dV_{Gt} = dV(V_{G,t-1}, dV_{gt}, dWD_{gt}, dRF_{gt}, d\omega_{Gt}) \quad (4)$$

Application to the Colorado River Basin: Hydraulically, the Lower Gunnison Basin, the Grand Valley, and the Imperial Valley are in series. The Grand Valley, the Uinta Basin, the Price River Basin, and the San Rafael River Basins are parallel to each other and are in series with the Imperial Valley.

Substitute Eqs. (3) and (4) into Eq. (2). Equation (5) models the hydrologic link between water use and salt loading at the Lower Gunnison River Basin ( $g_0$ ) and river salinity at Grand Valley ( $g_1$ ). The subscript  $\tau$  denotes the time that it takes salts to travel the Lower Gunnison River from the Lower Gunnison Basin to the Grand Valley. Salinity flux at the Grand Valley is

$$dEC_{g_1,t} = dEC(EC_{g_1,t-1}, V_{g_1,t-1}, dV_{g_0,t-\tau}, dWD_{g_0,t-\tau}, dRF_{g_0,t-\tau}, EC_{g_0,t-1-\tau}, dEC_{g_0,t-\tau}, eC_{g_0}, d\omega_{g_1,t}) \quad (5)$$

Salinity flux below the Imperial Dam is modelled as a function of the changes in water use and salt loading from the Lower Gunnison Basin ( $g_0$ ), Grand Valley ( $g_1$ ), Uinta Basin ( $g_2$ ), Price River Basin ( $g_3$ ), and San Rafael River Basin ( $g_4$ ). To reduce notation, bolded variables are 5x1 vectors. The vector elements represent the variable values for all five of the Upper Basin regions. For example,  $dV_t' = [dV_{g_0,t} \ dV_{g_1,t} \ dV_{g_2,t} \ dV_{g_3,t} \ dV_{g_4,t}]$ . The subscript  $\tau$  denotes the

time required for salts to travel between the Upper Basin regions and the Imperial Dam. Equation (6) links Imperial Dam (G) salinity with water use and salt loading from the five Upper Basin regions.

$$dEC_{G,t} = dEC(EC_{G,t-1}, V_{G,t-1}, dv_{t-\tau}, dWD_{t-\tau}, dRF_{t-\tau}, EC_{t-1-\tau}, dEC_{t-\tau}, ec, d\omega_{G,t}) \quad (6)$$

Equations (5) and (6) model the physical links between the spatially separated producing regions. More explicit detail of hydrology model appears in Lee, et al. [1989].

### Modelling Regional Production

Salt from irrigation water raises the soil osmotic potential. As salinity levels rise, the rate of evapotranspiration falls off and plant growth diminishes. Plants under severe osmotic stress are often stunted and appear to suffer from drought. Salt sensitivity, as measured by yield decline, varies widely across crops and growing conditions. Cotton and barley are naturally tolerant to salinity. Alfalfa by comparison is salt sensitive.

Salinity of Grand Valley irrigation water averages 500 mg/l. Leaching prevents salts from accumulating in the soil. Water from the Imperial Dam arriving at the Imperial Valley averages 756 mg/l. Tiles drain over 90% of the irrigated acreage in the Imperial Valley. Because, abundant irrigation water and good drainage typifies production in both areas, root zone salinity is assumed in balance with irrigation water salinity.

To model the relationship between river salinity and irrigated agriculture, regional production is expressed as a function of water quality. Let  $Y_{gt}$  denote the production vector of crops grown in region  $g$  at time  $t$ . Inputs to production are land ( $L_{gt}$ ), capital ( $K_{gt}$ ), and irrigation water ( $W_{gt}$ ). Regional production as a function traditional inputs and irrigation water salinity is

$$Y_{gt} = Y_g(L_{gt}, K_{gt}, W_{gt}, EC_{gt}) \quad (7)$$

### Model Objective

The program chooses factor inputs,  $X_g' = [L_{gt} K_{gt} W_{gt}]$  and salt load reduction capital,  $Z_{gt}$ , to maximize returns to basin agriculture. Agricultural production ( $f$ ), regional resource constraints ( $h$ ), irrigation water quality ( $v$ ), and water quality criteria ( $q$ ) restrict the solution.

$$\begin{aligned}
 &\text{Choose } X_{gt}, Z_{gt} \text{ to} \\
 &\text{Max } \sum_g p_g' Y_g - c_{xg}' X_g - c_{zg}' Z_g \quad \text{for } g = g_0 \dots g_4, G \\
 &\text{Subject to} \\
 &\quad f(X_{gt}, Z_{gt}) = Y_g \quad \text{for } g = g_0, g_2, g_3, g_4 \\
 &\quad f(X_{gt}, Z_{gt}, EC_{gt}, f(X_{g0}) \dots f(X_{g4})) = Y_g \quad \text{for } g = g_1, G \\
 &\quad h(X, Z) \leq 0 \\
 &\quad v(X, Z) \leq 0 \\
 &\quad q(X, Z) \leq 0 \quad (8)
 \end{aligned}$$

### EMPIRICAL APPROACH

Because of the long retention time of water and salts in the river, decisions to reduce upstream salt loads must be made well before actual flow levels are realized. Mitigation alternatives to control water quality can be undertaken to meet water quality standards, but because river flows are stochastic, water quality standards can be met only in probability. All decisions regarding water quality therefore assume a level of risk ( $\alpha$ ). The realized level of water quality  $EC_t$  will meet water quality standard  $EC^* 100(1-\alpha)\%$  of the time. In other words, river water salinity will exceed the standard  $100\alpha\%$  of the time.

$$\Pr\{ EC^* \geq EC_t \} \geq 1-\alpha \quad (9)$$

If the desired level of water quality is very high (small  $EC^*$ ) or if the selected level of risk is very low, then meeting the objectives will cost more than less stringent standards. Model simulations provide information regarding the costs and expected benefits of various policy criteria under different river flow scenarios. Five model scenarios for salinity at Imperial Dam are described below.



**Baseline:** The baseline model simulates agriculture and river flows for the scenario in which no additional water quality improvement is undertaken. Under various low flow conditions, the baseline model will provide the worst case scenario (in terms of water quality).

**Model 1:** This model chooses the least cost mitigation alternatives to meet the mandated 879 mg/l standard at Imperial Dam 90% of the time. ( $EC^*=879$ ,  $\alpha=.10$ ).

**Model 2:** Model 2 minimizes the cost of meeting federal water quality standards 95% of the time. Under this scenario, standards will be exceeded only once every 20 years. ( $EC^*=879$ ,  $\alpha=.05$ ).

**Model 3:** This model relaxes the risk criteria and chooses the least cost mitigation alternatives necessary to meet the 879 mg/l standard at Imperial Dam 75% of the time. ( $EC^*=879$ ,  $\alpha=.25$ ).

**Model 4:** Model 4 solves for the level of water quality that maximizes expected net returns to basin water uses.

## DISCUSSION

The baseline model presents the worst case scenario for water salinity during drought years. It also provides a lower bound for net economic returns to basin agriculture. For example, an overly stringent water quality policy could lower net economic returns to agriculture. A policy of this sort would improve lower basin agriculture productivity at a cost greater than is warranted by the downstream benefits.

From a legislative standpoint, meeting legal water quality standards with a high degree of probability is desirable. Results from Models 1 through 3 can indicate whether the expense is warranted by providing information about the marginal cost of risk aversion.

Results from Model 4 provide an additional measure of comparison regarding the economic efficiency of existing water quality standards. If existing water quality standards are too stringent, Model 4 will prescribe a lower standard (higher  $EC^*$ ) which is equivalent to

recommending a lower rate of compliance. If net economic gains are available from a higher standard of water quality, then Model 4 will suggest a lower EC\*, which is essentially the same as increasing the frequency of compliance.

In the Colorado River Basin, river salinity is worst during periods of drought. Water quality policy decisions are currently based on mean river flows. As a result, salinity levels will on the average be in compliance with existing water quality standards. In a given year, however, actual river salinity may exceed the legal standard. Consequently, failure of compliance may occur more frequently than is tolerable. This research provides the framework for evaluating water quality policy. Within the model, politically acceptable compliance rates can be specified directly. The model then solves for the required level of mitigation. In addition, the model can provide policy makers with economic information for selecting water quality criteria and the level of mitigation necessary for meeting those standards.

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