REPORT ON PROBLEM OF ORESSURE DISTRIBUTION IN OIL BODY CONTAINING FLOWING WATER

for

Petroleum Research Corporation, Denver, Colorado

by

A. T. Corey

Fort Collins, Colorado

February 11, 1957 CER57ATC41

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PROBLEM

The formation under consideration is a confined sand having a $P_D \doteq 0.2$ psi (oil into water) bounded up-slope by a sand having a $P_D = 1.75$ psi. The slope has, in the first case, a value of 100 ft./mile, and in the second case, a value of 20 ft./mile. The S.G. of the water is 0.984 and the S.G. of the oil is 0.716. An oil body extends from the tight sand down-slope for an elevation difference of 40 ft. Assuming that there exists a flow of water down-slope (which is the same throughout the oil body as in the tight sand), what is the distribution of pressure of the fluids within the oil body?

THEORETICAL ANALYSIS

The assumption is made that the k_{rw} varies as the 4th power of the effective saturation to water.* According to this theory, the k_{rw} also varies inversely as the 8th power of the capillary pressure, being 1.0 when $P_c \leqq P_D$.

At the bottom of the oil body, the P_c must be equal to the P_D of the aquifer which is 0.2 psi (the P_D of the tight sand which is retaining the oil body).

*See "The Interrelation between Gas and Oil Relative Permeabilities" by A. T. Corey, Producer's Monthly, November 1954.

A circumspect consideration of the problem indicates that the pressure of the oil, P_o, decreases up-slope from the bottom of the oil body according to the relation,

$$\frac{dP_o}{dZ} = -\gamma_o \qquad (1)$$

where Z is the elevation above the bottom of the oil body and γ_o is the specific weight of the oil. If the water were static, it would decrease in pressure according to the relation,

$$\frac{dP_w}{dZ} = -\gamma_w \qquad (2)$$

Because the water is flowing, the pressure in the water actually decreases at a lesser rate according to the relation,

$$\frac{dP_w}{dZ} = -\gamma_w + f\ (P_c) \qquad (3)$$

where $f(P_c)$ is assumed to be $f(\frac{P_c}{P_D})^8$.

A complete expression for $\frac{dP_w}{dZ}$ is found as follows:

Assuming uniform discharge, and a uniform sand of constant slope at uniform temperature, we have,

$$Ci = C(\frac{P_D}{P_c})^8 \ \frac{d}{d\ Z/S} \ (\frac{P_w}{\gamma_w} + Z) \qquad (4)$$

where S is the slope of the aquifer, C is the conductivity of the aquifer below the oil body, and i is the hydraulic gradient below the oil body.

This gives

$$i = S\ (\frac{P_D}{P_c})^8 \ [\frac{1}{\gamma_w} \frac{dP_w}{dZ} + 1\ ]$$

or
$$\frac{dP_w}{dZ} = \frac{\gamma_{wi}}{S}\left(\frac{P_c}{P_D}\right)^8 - \gamma_w \qquad (5)$$

Since by definition $P_c = P_o - P_w$,

$$\frac{dP_c}{dZ} = \frac{dP_o}{dZ} - \frac{dP_w}{dZ} \qquad (6)$$

and
$$\frac{dP_c}{dZ} = -\gamma_o - \left[\frac{\gamma_{wi}}{S}\left(\frac{P_c}{P_D}\right)^8 - \gamma_w\right]$$

or
$$\frac{dP_c}{dZ} = \Delta\gamma - k\left(\frac{P_c}{P_D}\right)^8 \qquad (7)$$

where $\Delta\gamma = \gamma_w - \gamma_o$ and k is the parameter, $\frac{\gamma_{wi}}{S}$.

This parameter is not an arbitrary constant, however, for reasons which are discussed below.

Equation (7) can most easily be solved by numerical methods. The solution for the given boundary conditions is shown on the attached graph. Because of the nature of Equation (7), only one value of k/P_D^8 is possible for a given set of boundary conditions. In this case, $k/P_D^8 = 1.08 \times 10^{-4}$. This value is such as to make the capillary pressure gradient vanish at the upper end of the oil body. If the value of k/P_D^8 would be larger, P_c would be less than 1.75 at Z = 480", and if the value were less, P_c would be greater than 1.75 at Z = 480".

An analogous situation would arise regardless of the functional relationship which one might reasonably assume to hold for k_{rw} vs. P_c. It should be observed, however, that for the assumed range of P_c (0.2 -1.75 psi), Equation (7) would permit a ΔZ greater than 480" with the same value of k/P_D^8. This would mean an extension of the region at the upper end of the oil body at which $\frac{dP_c}{dZ} = 0$. The distance, $\Delta Z = 40$ feet, which has been assumed for this problem, is merely the minimum depth that would give a P_c of 1.75 psi at the upper end with the given value of $\Delta\gamma$ and the assumed value of k/P_D^8. No higher value of k/P_D^8 would permit a P_c of 1.75 psi at the upper end, however, regardless of the depth of the oil body. A smaller value of k/P_D^8, on the other hand, would permit a P_c of 1.75 psi with a lesser value of ΔZ than the one assumed. In this case, a greater ΔZ could not be obtained unless the displacement pressure, P_D, of the containing sand up-slope were greater than 1.75 psi.

Note that the value of the conductivity of the sand cancels out of the final equation. Of course, the conductivity and discharge could be substituted for one of the other variables to obtain a solution.

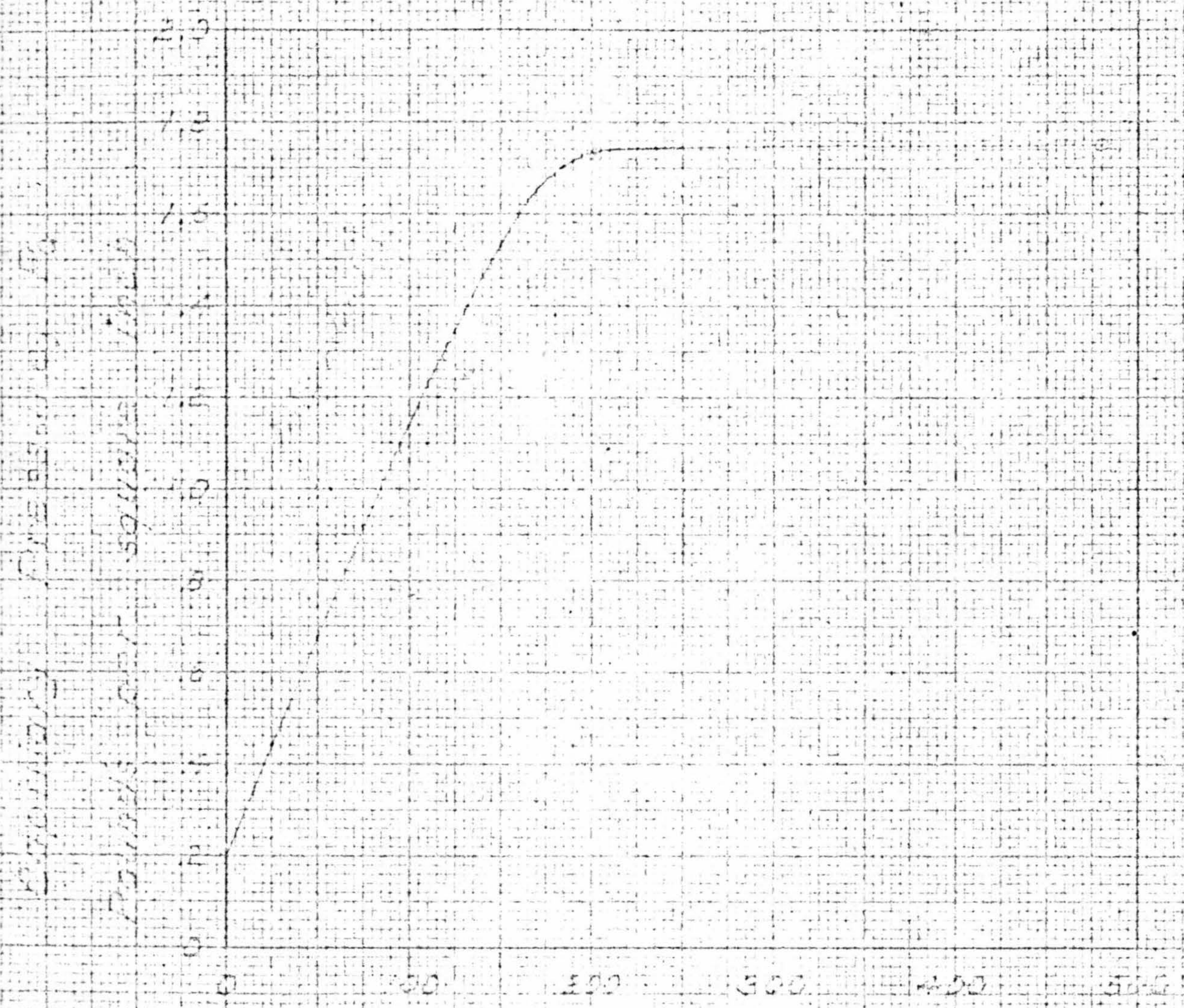
CONCLUSIONS

From the foregoing considerations, it would seem that if one wanted to predict the value of ΔZ from given values of γ_o, γ_w, P_{D_1}, P_{D_2}, S, and i, it would be possible to do this only if the value of $\frac{\gamma_w i}{S P_{D_1}^8}$ were less than a certain critical value. This value would be such as to make $\frac{dP_c}{dZ}$ vanish at the upper end of the oil body. For any greater value of $\frac{\gamma_w i}{S P_{D_1}^8}$, the displacement pressure, P_{D_2}, of the confining sand would not actually be controlling the value of ΔZ, and one would need additional information to obtain a solution, such as the absolute value of the water pressure at the top and bottom of the oil body.

One of the original specifications for this problem was that the value of i was 1-1/2 feet per mile. It was found that this gradient was not compatible with the given boundary conditions. The solution that has been presented in the attached graph is for values of i (for each of the two given values of S) that give a minimum ΔZ of 40 feet. For a slope of 100 ft./mile, the value of i is 0.3 ft./mile, and for a slope of 20 ft./ mile, the value of i is 0.06 ft./mile.

Capillary Pressure vs. Height Above

Bottom of Oil Body



[illegible]

$Z = 9.313 \times$ [illegible]

$Z = 2.585$ [illegible]

[illegible]

PROBLEM II

PRESSURE DISTRIBUTION

IN OIL BODIES CONTAINING FLOWING WATER

An oil body is trapped in a stratum consisting of two distinct sands in hydraulic contact. The trapping formation is a shaly silt (0.2 md., P_D = 4.65 psi) which is up-slope from the tighter of the two sands and is in hydraulic contact with this sand. The sand down-slope from the tight sand has a permeability of 1,000 md. and a displacement pressure of 0.1 psi. The tight sand extends down-slope from the face of the shale to a depth of 30 feet, and the more permeable sand extends an indefinite depth below this. The oil body is continuous in the tight sand and extends into the more permeable sand to a depth of 50 feet - a total depth of 80 feet below the shale. Assuming that there exists a flow of water down-slope which is uniform throughout the shale and the two sands, what is the distribution of capillary pressure within the oil body? The S.G. of the water is given as 0.984 and that of the oil as 0.716.

THEORETICAL ANALYSIS

The analysis of this problem is similar to that for Problem I. The equation used to obtain the capillary pressure distribution is

$$\frac{dP_c}{dZ} = \Delta\gamma - k \left(\frac{P_c}{P_D}\right)^8 \qquad (1)$$

where k is given by

$$k = \frac{\gamma_w q}{SC}$$

in which γ_w is the specific weight of the brine; q is the volume flux of water; C is the hydraulic conductivity of the sand at a relative permeability of 100%; S is the slope of the formations (assumed to be the same for each); $\Delta\gamma$ is the difference in specific weight between water and oil; P_c is the capillary pressure; Z is the height above the bottom of the oil body; P_D is the displacement pressure (oil into water) of the sand under consideration.

PROCEDURE FOR SOLVING EQUATION (1)

Since the problem states that the oil is trapped by the shale, the P_c at the face of the shale is 4.65 psi. At the bottom of the oil body, the P_c cannot be less than 0.1 psi which is the P_D of the cleaner sand. These pressures combined with the given depth of the oil body constitute the boundary conditions for which Equation (1) was solved by numerical methods.

In this case, however, the solution required successive approximations. The procedure was to assume a value of k for one of the sands. Since the value of k for the two sands differs by the inverse ratio of their permeabilities, the value of k for the second sand could be computed. Using these values of k, Equation (1) was solved by plotting the slopes, $\frac{dP_c}{dZ}$, on a graph of P_c vs. Z. By this method, a separate curve was plotted for each sand. The curve for the tight sand was plotted by beginning at the top of the oil body (Z = 960"), and that for the permeable sand was plotted by beginning at Z = 0". Since only one solution is possible with the given boundary conditions, the two curves did not, in general, meet at the junction of the two sands (Z = 600"). By adjusting the values of k, successively better approximations were obtained. When the two curves met within 6 inches of Z = 600", the approximation was considered satisfactory. The result is shown on the graph labeled as Problem II. This result was obtained with a value of $k = 1.2 \times 10^{-9}$ lbs/in^3 for the tight sand.

CALCULATIONS

(1) P_c'	(2) P_c^8	(3) $P_c^8/.43$	(4) (3) $\times 10^{-9}$	(5) .00965 - (4) gives m	(6) 100 x (5) gives m lbs/100 in.^{2}
4.65	2.2×10^5	5.12×10^5	5.12×10^{-4}	.00914	.914
4.6	2.02	4.69			.918
4.55					
4.5	1.68×10^5	3.91×10^5	3.91×10^{-4}	.00926	.926
4.4	1.405	3.27			.932
4.2	9.7×10^4	2.26			.942
4.0	6.5×10^4	1.52			.950
3.8	4.33×10^4	1.006×10^5	1×10^{-4}	.00955	.955
3.0	6.5×10^3	1.51×10^4	1.5×10^{-5}	.009645	.9645

Coarse Sand

(1)	(2)	(3)	(4)	(5)	(6)
P_c	P_c^8	$P_c^8/10^{-8}$	1.3×10^{-11}	.00965 (4) gives m	100 x (5) gives m lbs/100 in.^{2}
.1	10^{-8}				
.2	2.56×10^{-6}				
.4	6.4×10^{-4}				
.5	3.8×10^{-3}				.965
.6	1.8×10^{-2}				.963
.7	5.75×10^{-2}				.958
.8	1.68×10^{-1}				.943
.9	4.32×10^{-1}				.909
1	1				.865
1.1	2.15				.685
1.2	4.3				.405
1.25	5.95				.190
1.285					0

PROBLEM III

1,000 md. Sand $k = 2.24 \times 10^{-10}$

(1)	(2)	(3)	(4)	(5)	(6)
P_c	P_c^{8}	$P_c^{8}/10^{-8}$	$\times 2.24 \times 10^{-10}$	$\frac{.00965 - (4)}{376}$	100 x (5) lbs/100 in.^{2}
.1	10^{-8}		2.24×10^{-10}		.965
.2	2.56×10^{-6}		5.73×10^{-8}		.965
.3	9.3×10^{-5}		2.08×10^{-6}		.965
.4	6.4×10^{-4}		1.43×10^{-5}		.965
.5	3.8×10^{-3}		8.5×10^{-5}		.956
.6	1.8×10^{-2}		4.03×10^{-4}		.925
.7	5.75×10^{-2}		1.29×10^{-3}		.836
.8	1.68×10^{-1}		3.76×10^{-3}		.589
.9	4.32×10^{-1}		9.67×10^{-3}	0	0
.75	1×10^{-1}		2.24×10^{-3}	.00741	.741
.85	2.73×10^{-1}		6.12×10^{-3}	353	.353

13 md. Sand $k = \left(\frac{2.24 \times 10^{-10}}{1.3}\right) \times 10^{2} = 1.725 \times 10^{-8}$

0.9	.43	1	1.725×10^{-7}		.965
1.0					
1.05					
1.10					
1.15					
1.20					
1.25					
1.30					
1.40					
1.45					
1.50	25.6	59.5×10^{1}	1.03×10^{-6}		.965

PROBLEM III (contd.)

$k = 1.62 \times 10^{-4}$

(1) P_c	(2) P_c^8	(3) $P_c^8/.43$	(4) $\times 1.62 \times 10^{-4}$	(5) .00965 -(4)	(6) 100 x (5) $lbs/100\ in.^2$
0.9	.43	1	1.62×10^{-4}	.00949	.949
1.0	1.0	2.32	3.77×10^{-4}	.00927	.927
1.05	1.478	3.44	5.58×10^{-4}	.00909	.909
1.10	2.145	5.0	8.10×10^{-4}	.00884	.884
1.15	3.06	7.1	1.15×10^{-3}	.00850	.851
1.20	4.3	10.0	1.61×10^{-3}	.00804	.804
1.25	5.95	13.85	2.25×10^{-3}	.00740	.742
1.30	8.2	19.1	3.09×10^{-3}	.00656	.658
1.35	11.0	25.6	4.15×10^{-3}	.00550	.552
1.40	14.8	34.4	5.58×10^{-3}	.00407	.410
1.45	19.5	45.4	7.35×10^{-3}	.00230	.234
1.50	25.6	59.5	9.65×10^{-3}	00	000

$k = 2.12 \times 10^{-6}$

P_c	P_c^8	$P_c^8 \times 2.12 \times 10^2$	.00965 -(4)	100 x (5) $lbs/100\ in.^2$
0.1	10^{-8}	2.12×10^{-6}	.00965	.965
.2	2.56×10^{-6}	5.4×10^{-4}	.00911	.911
.4	6.4×10^{-4}	1.35×10^{-1}		000
.5	3.8×10^{-3}			
.6	1.8×10^{-2}			
.7	5.75×10^{-2}			
.8	1.68×10^{-1}			
.9	4.32×10^{-1}			

$$\frac{dP_c}{d_Z} = 0 = .00965 - 2.12 \times 10^2\ P_c^8$$

$$P_c^8 = \frac{9.65 \times 10^{-3}}{2.12 \times 10^2} = 4.55 \times 10^{-5}$$

$$4.55 \times 10^{-7}$$

Problem II

$k = 1.2 \times 10^{-8}$

PROBLEM III

PRESSURE DISTRIBUTION IN OIL BODIES UNDER HYDRODYNAMIC CONDITIONS

This problem is the same as Problem II except that the silt which confines the oil has a P_D = 1.5 psi instead of 4.65 psi. Consequently, the oil body is much smaller and is in two parts. The first part extends 20 feet below the silt, and the second extends 40 feet below the junction of the two sands and is confined by the upper sand.

THEORETICAL ANALYSIS

It is immediately evident that if the value of "q" is considered constant throughout both oil bodies, and the values of P_c at the boundaries are specified, the depths of the oil bodies are not independently variable. Consequently, it is very improbable that Equation (1) as given in Problem II would have a solution satisfying the given boundary conditions. The procedure explained below was followed with this fact in mind. It will be noted that if one assumed the values of k found to apply for Problem II, the depth of the upper oil would be only 5' and the lower oil about 8'. This would be virtually a hydrostatic distribution. It is characteristic of Equation (1) that values of k appreciably less than a certain critical value will give depths of oil not much different than hydrostatic conditions, considering the limiting P_c. Values greater than the critical k will not permit the P_c to build up to its limiting value regardless of the depth of oil. In fact, if a depth of oil is specified which is appreciably greater than would be possible under hydrostatic conditions, it is sufficient to assume that k is exactly critical. The resulting plot of P_c vs. Z will be as accurate as is possible to obtain by graphical methods. Theoretically, however, the value of k is slightly less than critical because the critical k is that which would permit the P_c to reach its limiting value at $Z = \infty$.

It should also be noted that because of these characteristics of Equation (1) and the graphical solutions, it would not be practical to attempt a determination of Z from given values of k and a limiting P_c unless the distribution of pressure was nearly hydrostatic. Theoretically, the value of Z is determined so long as k is less than critical, but when k approaches the critical value, Z is extremely sensitive to small differences in k. On the other hand, if a value of Z is given that is appreciably larger than hydrostatic, very accurate plots of P_c vs. Z can be made by assuming a critical value of k.

PROCEDURE

Two different solutions are shown, neither of which satisfies all of the given boundary conditions. Solution (a) was obtained by assuming a critical k in the more permeable sand. The critical k was computed by setting $\frac{dP_c}{dZ} = 0$ at the limiting P_c which was 0.9 psi. This gave

$$\Delta\gamma \left(\frac{P_D}{P_c}\right)^8 = k = 2.24 \times 10^{-10} \text{ lbs/in.}^3$$

for the lower sand. The corresponding k for the upper sand is

$$(2.24 \times 10^{-10}) \left(\frac{1000}{13}\right) = 1.725 \times 10^{-8} \text{ lbs/in.}^3$$

The resulting solution satisfies all boundary conditions except that the depth of the upper oil body is virtually hydrostatic.

Solution (b) was obtained by assuming k to be critical in the upper sand. Under these conditions, P_c cannot reach its limiting value of 0.9 psi in the lower sand, but all other boundary conditions are satisfied.

As previously noted, a value of k appreciably less than critical for the lower sand would result in the pressure distributions being virtually hydrostatic for both the lower and upper sands. A value of k greater than critical in the upper sand would prevent P_c from reaching its limiting value in either sand.

DISCUSSION AND CONCLUSIONS

It would appear from the foregoing that the equation

$$\frac{dP_c}{dZ} = \Delta\gamma - k \left(\frac{P_c}{P_D}\right)^8 \qquad (1)$$

is very useful for predicting pressure distributions when certain boundary conditions are known. In many cases, it is necessary to know the depth of the oil body for graphical solutions to be practical under hydrodynamic conditions. Except in situations where the distribution of pressure differs only slightly from hydrostatic conditions, it will be difficult to predict the depth of an oil body from the displacement pressure of the confining rock and the hydraulic gradients. This is because the capillary pressure gradient is so extremely small for a great distance below the confining rock.

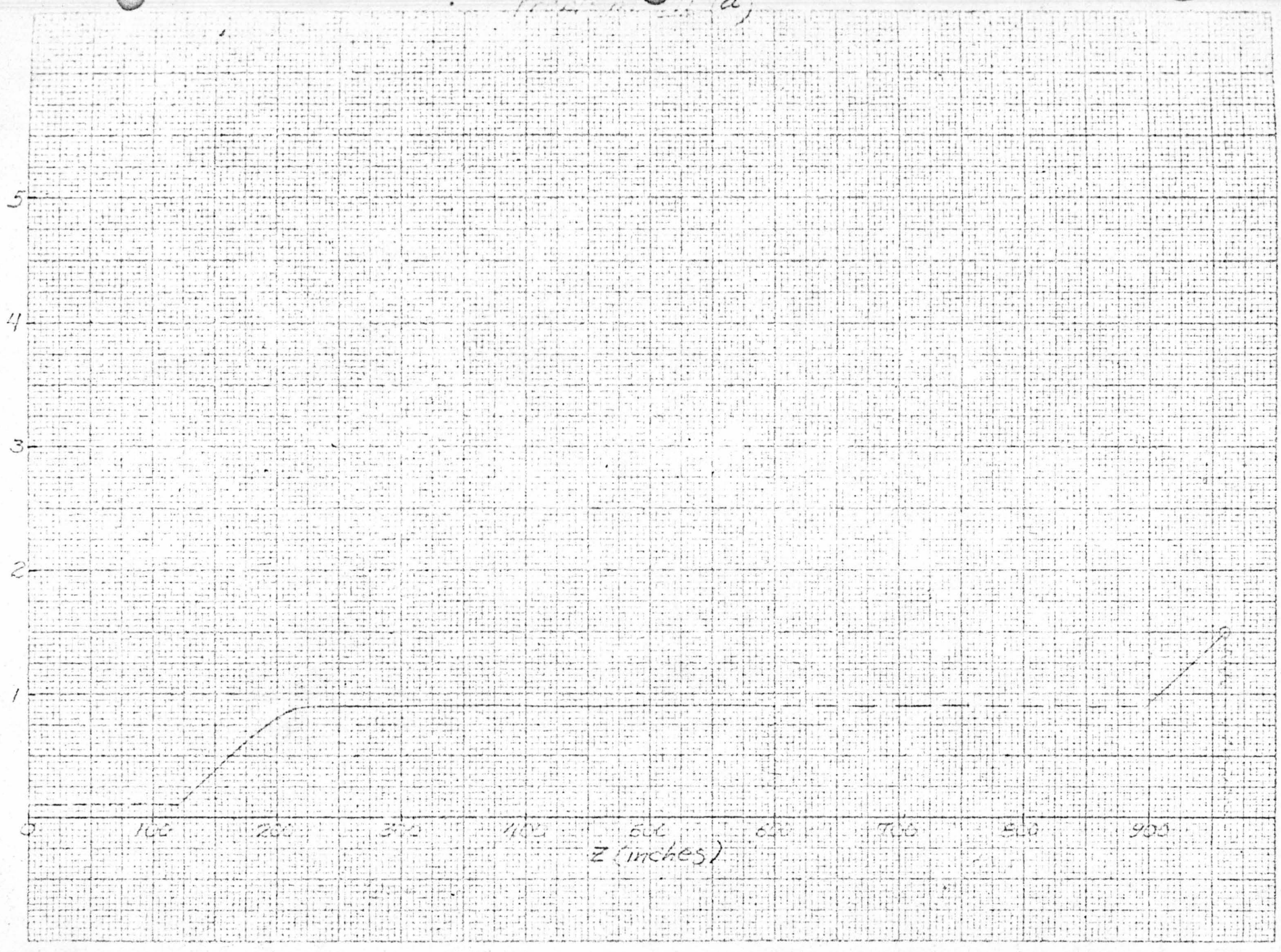
The value of the parameter, k, which gives

$$\frac{dP_c}{dZ} = 0$$

for a particular limiting value of P_c has been called the critical k in the foregoing discussion. This value is significant because for any system wherein the oil body is substantially deeper than could exist under static conditions, the value of k will be only slightly less than critical, and for the purpose of solving Equation (1) to obtain pressure distributions, it is sufficient to assume k to be critical.

There are certain geological and hydrodynamic considerations which indicate that a value of k approaching the critical value would often exist in nature. Consider, for example, the situation that exists in a formation of a given slope wherein water enters at a given elevation and leaves at a certain lower elevation. The potential energy available for water movement would therefore be fixed. If oil migrates into such a system and is trapped by a tight sand up-slope, the rate of flow of water would be reduced because of the added resistance. Consequently, the value of k would be gradually reduced as more and more oil migrated into the formation. As k is reduced, the maximum capillary pressure which occurs at the face of the confining rock would increase. If k exceeded a critical value, however, the oil would break through the confining rock and the accumulation of oil would cease. It would seem, therefore, that k would always approach a critical value if there were a sufficient supply of hydrocarbons in the beginning and if the potential energy available for water flow were sufficiently large that the oil accumulation could be materially greater than under static conditions.

According to the foregoing theory, if sands of varying displacement pressures are in series in a continuous hydrodynamic system, oil will accumulate preferentially in the less tight sands. Hence, the sands which have the greater permeability will tend to have the least effective permeability to water, and the flow of water will be controlled by them. As a result, the distribution of pressure in the tighter sands will ordinarily differ only slightly from the static case. This, of course, presupposes that the flow of water is the same for all the sands.


5
4
3
2
1
0
100
200
300
400
500
600
700
800
900
Z (inches)

Problem II (b)