HYDRAULICS OF

STEADY-FLOW WELL SYSTEMS

of

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SYNOPSIS

A brief summary of the literature regarding the hydraulics of steady-flow wells is presented. Several shortcomings of existing formulas and practices are discussed and suggestions for improvement made.

For unconfined ground-water flow into wells the existence of the seepage surface at the well and the relationship of the magnitude of such a surface to the other elements of the well system are but little understood. Hansen (5) developed in 1949 dimensionless parameters which enabled specific test data to be plotted in general terms. Hansen's work in this regard has been extended herein by adding the theoretical solutions made by Yang (14), also in 1948, using "relaxation" methods. These solutions enable the investigator to estimate the magnitude of the seepage face for a wide range of specific cases of unconfined flow into a well.

Dupuit's classical solution for unconfined flow into a well presumes that all of the discharge flows horizontally into the zone of influence from outside the region under consideration. For a drainage well to relieve lands waterlogged by surface irrigation, flow enters the region of influence by vertical percolation of water falling on or applied to the overlying land surface so that the flow toward the well increases as the well is approached. A theoretical solution is presented herein for this condition.[3] The geometry of the region of influence for this type well of radius, r_w, is the same as for a well of the Dupuit type having the same drawdown, but with radius equal to $\frac{e^{n/2}}{r_w}$ in which n is the ratio of the discharge originating from vertical percolation to the total discharge of the well. With

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3. See symbols and definitions following synopsis.

this transormation, the seepage surface for the unconfined, vertically-recharged well may be found by using the same curves as are used for the horizontally-recharged well.

Critical evaluation of the commonly-used formulas for confined and unconfined ground-water flow into wells leads to the conclusion that they are indeterminate. They are made determinate in practice by introducing the radius of influence, r_e, usually as an arbitrary value, a concept which is somewhat vague at best, if not illogical.

The variables in these formulas include the soil permeability, k, the drawdown, D_w, the thickness of water-bearing material, T, the radius of the well, r_w, the discharge, Q, and the radius of influence, r_e. The independent variables are D_w, k, T, and r_w; Q and r_e are dependent and mutually interdependent. An additional independent variable, q, describing the unit rate at which the influence cone is replenished with water from an external source, is necessary to complete the analysis. These five independent variables are sufficient to fully determine the flow into a well for any particular system. By introducing certain approximations, functional relationships between Q, D_w, k, T, r_w, and q are developed. Dimensional analysis yields the dimensionless parameter $\frac{Q}{k\, r_w^2}$, which includes the dependent variable and which is a function of the well in relation to its hydrologic environment. This parameter is therefore designated the Well-Hydrologic number or simply the W-H number.

A number of illustrative numerical examples are given. Dimensionless quantities presented herein may be used for any system of units providing the same system is used throughout.

Care must be used in calculating the "effectiveness" as defined by Wenzel (12) for wells in unconfined systems or values much too small will result. If the piezometric head is measured at the bottom of the permeable stratum instead of at the water table, reasonable results may be expected. Normal procedure results in considering the head represented by the height of the seepage face as lost head, so that wells in unconfined systems normally appear to be somewhat less effective than similar wells in confined systems. Actually, other things being equivalent, a well in an unconfined system is inherently somewhat more efficient in utilizing available specific energy than a similar well in a confined system.

SYMBOLS AND DEFINITIONS

No.	Symbol	Definition	Dimensions Force-Length-Time
1	C_x	Babbitt-Caldwell variable coefficient	--
2	D_w	Drawdown resulting from pumping, total theoretical	L
3	E_w	Effectiveness of the well	--
4	e	Base of Naperian Logarithms	--
5	F	An unknown function	--
6	H_c	Well-Hydrologic number, $\frac{Q}{kr_w^2}$, confined system	--
7	H_u	Well-Hydrologic number, $\frac{Q}{kr_w^2}$, unconfined system recharged by horizontal flow	--
8	H_v	Well-Hydrologic number, $\frac{Q}{kr_w^2}$, unconfined system recharged by vertical flow	--
9	h	Hydraulic head, $\frac{p}{r_w} + z$	L
10	h_e	Water surface or piezometric elevation at maximum radius	L
11	h_s	Water surface or piezometric elevation just outside the well	L
12	h_w	Water-surface elevation in well	L
13	i_n	Natural slope of the free water table	--
14	k	Permeability of soils to water	L/T
15	L	Length or distance of ground-water flow	L
16	n	The ratio of the discharge of water derived from vertical percolation to the total discharge of the well	--
17	p	Pressure (force per unit area)	F/L^2
18	Q	Total discharge	L^3/T

No.	Symbol	Definition	Dimensions Force-Length-Time
19	Q_r	Quantity of flow per unit time through a cylindrical surface of radius r	L^3/T
20	q	The generalized rate of replenishment for any system	L/T
21	q_v	Rate of replenishment for the vertically-replenished system, quantity of vertical flow per unit area	L/T
22	r	Radial distance from axis of well (a variable)	L
23	r_e	Radius of the circle of influence	L
24	r_w	Radius of well	L
25	r_w'	$r_w' = \frac{e^{n/2}}{r_w}$, equivalent well radius	L
26	T	General value of thickness of water-bearing stratum for any system	L
27	t	Thickness of permeable stratum for confined-flow system	L
28	w	Weight per unit volume of water	F/L^3
29	z	Elevation above datum	L

INTRODUCTION

Darcy's Law

The flow of ground water under conditions of saturation has been widely discussed by many authors (1, 2, 3, 5, 6, 7, 8, 9, 11, and 12). Through isotropic soils the velocity of flow is expressed by the equation of Darcy, which in its simplest and most-used form is:

$$V = k\frac{h}{L} \tag{1a}$$ [4]

in the vector form:

$$V = k \text{ grad } h = k\nabla h \tag{1b}$$

and in the differential form:

$$V = k\frac{\partial h}{\partial s} \tag{1c}$$

where h/L, ∇h, and $\partial h/\partial s$ each represent the hydraulic gradient and V is the velocity of flow.

Soils are usually stratified and therefore the permeability varies with the direction of flow. Further, both sedimentation and pressure of overlying soil materials cause flat particles to be orientated with their longest dimensions horizontal, resulting in a nonisotropic condition with respect to permeability, even though ordinary stratification is not present. Often, however, flow will be parallel to one of the principal directions of permeability, and in this instance k in equation (1) may be treated as constant if measured in the direction of flow. In this report only isotropic cases are treated.

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4. Darcy's equation is sometimes written with a minus sign on the right, $V = -k\ h/L$, to denote that the flow is in the direction of decreasing head.

Equation 1 simply states that direction of flow is parallel to the direction of greatest hydraulic gradient and that the volume of discharge through unit area per unit of time is proportional to the hydraulic gradient.

Equations of Continuity and Laplace

If flow is steady and the fluid is incompressible, from the law of conservation of matter the net flow into and out of any elementary volume of space is zero. This may be expressed mathematically by the equation of continuity:

$$\frac{\partial V_x}{\partial X} + \frac{\partial V_y}{\partial Y} + \frac{\partial V_z}{\partial Z} = 0 \qquad (2a)$$

or in the vector form:

$$\text{div } V = 0 \qquad (2b)$$

Substituting in equation (2) the velocity, V, from Darcy's formula as in equation (1) yields, for isotropic soils, Laplace's equation,

$$\frac{\partial^2 h}{\partial X^2} + \frac{\partial^2 h}{\partial Y^2} + \frac{\partial^2 h}{\partial Z^2} = 0 \qquad (3)$$

where X, Y, and Z are the cartesian coordinates; or in the vector form:

$$\nabla^2 h = 0 \qquad (4)$$

For steady flow of ground water toward a well, cylindrical coordinates are more convenient to use, and equation (3) may be written:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{1}{r^2}\frac{\partial^2 h}{\partial \theta^2} + \frac{\partial^2 h}{\partial Z^2} = 0 \qquad (5)$$

where r, θ, and Z are the cylindrical coordinates.

Equation (1) in cylindrical coordinates will then be:

$$V_r = k\frac{\partial h}{\partial r}, \quad V_\theta = \frac{k}{r}\frac{\partial h}{\partial \theta}, \quad \text{and } V_z = k\frac{\partial h}{\partial Z} \qquad (6)$$

If a mathematical function or expression satisfying equation (3) and reducing to the known values of h and V at the boundaries can be found, this function will describe the hydraulic head at every point in the region of flow.

FLOW OF GROUND WATER INTO WELLS

Confined-Flow Systems

If the highly-permeable water-bearing stratum is bounded above and beneath by impermeable layers (fig. 1), and if the drawdown in the well is less than the vertical distance from the static water table to the top of the permeable stratum, the flow is designated as confined flow.

Simple Case Solution

For a simple case the confined system may be readily solved. Assume

(1) the thickness of the permeable stratum, t, is uniform;

(2) the permeable stratum is horizontal;

(3) the well penetrates the entire depth of the permeable stratum; and

(4) the elevation of the piezometric surface at the uniform maximum radial distance, r_e, has the constant value, h_e.

Confined flow under these assumptions is horizontal, radial, and symmetrical. No component of velocity exists in either the directions z or θ, and thus $\partial h/\partial z$ and $\partial h/\partial \theta$ and all higher partial derivatives of h with respect to z and θ are equal to zero. Consequently, equation 5 becomes

$$\frac{d^2h}{dr^2} + \frac{1}{r}\frac{dh}{dr} = 0 \qquad (7)$$

with boundary conditions as follows:

at $r = r_w$, $h = h_w$; and at $r = r_e$, $h = h_e$.

Equation (7) is in the form of Euler's equation and can be solved by different methods such as inspection, Laplace transformation, or the transormation $w = \ln z$. Any of these solutions gives

$$h = c_1 \ln r + c_2 \tag{8}$$

Evaluating c_1 and c_2 by substituting the boundary values yields

$$h = h_w + \frac{h_e - h_w}{\ln r_e/r_w} \ln \frac{r}{r_w} \tag{9}$$

The discharge of the well can be determined from this equation and the equation of Darcy, which for a steady flow could be written:

$$Q = 2\pi rt\ k \frac{dh}{dr} \tag{10a}$$

Differentiating equation (9) with respect to r, and substituting in equation (10a) the value for dh/dr thus obtained, gives

$$Q = 2\pi rt\ k \frac{h_e - h_w}{\ln r_e/r_w} \tag{10b}$$

The above theoretical analysis is the general method for evaluating groundwater flow. However, equation (10b) could be obtained in a simpler way by directly integrating equation (1) after rearranging its terms, using as limits of integration the boundary values stated previously.

Eliminating $h_e - h_w$ from equations (9) and (10b) and solving for $h - h_w$ gives

$$h - h_w = \frac{k}{2\pi tk} \ln \frac{r}{r_w} \tag{11}$$

The total drawdown, D_w, which equals $h_e - h_w$ as obtained from equation (10b) is

$$D_w = \frac{Q}{2\pi tk} \ln \frac{r_e}{r_w} \tag{12}$$

Theoretical Difficulties

Equation (12) presents some theoretical difficulties which are of considerable practical importance. In order to write equation (12) the investigator must assume that the piezometric surface was level at elevation h_e prior to pumping and that its elevation is unaffected at distance in excess of r_e. Thus, h_e becomes also the elevation of the static water level in the well. These assumptions imply no radial flow toward the well from beyond the distance r_e, a condition which cannot exist if the flow into the well is steady. The quantities D_w, k, t, and r_w are independent variables, but Q and r_e are mutually dependent. Equations (8) and (10a) are, therefore, indeterminate except by pumping experiment to determine Q, even though D_w, k, t, and r_w may be known. They may be used to determine k in the field if measurements of h are taken at one or more values of r greater than r_w. By assuming r_e one may estimate Q for a particular value of D_w by using equation (12) if k, r_w, and t are known; conversely k may be estimated if Q, r_w, and t are known. Selection of r_e determines the shape of the drawdown cone. These equations ignore a major factor in well hydraulics; that is the rate at which the region of influence is replenished. It seems that an attempt is made to overcome this by using the arbitrary value r_e as a sort of "fudge factor."

Unconfined-Flow Systems

If there is no impermeable stratum overlying the permeable water-bearing aquifer (fig. 2), or if the drawdown is greater than the depth from the static water level to the confining stratum, the flow system may be classified as unconfined or partially confined. The problem of developing a rational equation to find the drawdown is much more complicated because the position of the boundary of the region of flow is unknown. In addition to this difficulty, the top surface of the flow region intersects the well at an elevation somewhat greater than the elevation of the water in the well. The necessity for the existence of the resulting "seepage surface," AB, fig. 2, between the water surface in the well and the free surface of the flow region, AD, is demonstrated by Muskat (8). This may readily be inferred from a consideration of Kozeny's solution for a porous dam on impermeable foundation reported by Taylor (11) and others which is the corresponding case in cartesian coordinates and in which the seepage face is mathematically demonstrated. The universal economy of nature invariably results in dynamic systems involving minimum expenditure of energy. The development of the free surface at an elevation above the water level in the well is in accordance with this principle.

For the symmetrical gravity or unconfined system (13) the following boundary conditions apply (see fig. 2).

(1) $h = h_w$ along BC ($r = r_w$ and $0 \leq z \leq h_w$)

(2) $h = z$ along AB ($r = r_w$ and $h_w < z \leq h_s$)

(3) $\partial h/\partial z = 0$ along CD ($z = 0$ and $r_w \leq r \leq r_e$)

(4) $h = h_e$ along DE ($r = r_e$ and $0 \leq z \leq h_e$)

(5) $h = z$, $P = 0$, $\partial h/\partial n = 0$ at the piezometric surface where $h_s < z \leq h_e$ and $r_w \leq r \leq r_e$

At any point along the seepage surface, AB, the pressure head is zero (atmospheric) and the piezometric head equals the elevation head. This is also true along the free surface, AD. The component of velocity normal to the free surface, AD, must be zero which implies $\partial h/\partial n = 0$ where 'n' denotes a direction normal to the surface AD. Flow along AD must become parallel in direction to the vertical surface of seepage at point a, fig. 2 (8).

Dupuit Solution

Dupuit, in 1863, introduced the assumptions that flow through any concentric cylinder at radius, r, was horizontal and that the hydraulic gradient at all points on the cylinder surface was equal to the slope of the free surface at its intersection with the cylinder. Since, for steady flow, the entire discharge of the well must pass through each concentric cylinder at any radius, combining the equation of continuity with Darcy's equation yields:

$$Q = 2\pi rhk \frac{dh}{dr}$$ [5]

which may be integrated to obtain

$$\frac{Q}{\pi k} \ln r = h^2 + C$$

By introducing the boundary conditions at r_w, the constant may be evaluated to obtain

$$\frac{Q}{\pi k} \ln \frac{r_w}{r} = h_w^2 - h^2 \qquad (13)$$

or substituting $h = h_e$ when $r = r_e$

$$\frac{Q}{\pi k} \ln \frac{r_e}{r_w} = h_e^2 - h_w^2 \qquad (14)$$

and the equation for the free surface,

$$h^2 = \frac{h_e^2 - h_w^2}{\ln r_e/r_w} \ln \frac{r}{r_w} + h_w^2 \qquad (15)$$

is obtained by eliminating Q from equations (13) and (14).

The Dupuit solution for the position of the free surface gives quite accurate results at fairly large distances from the well but is incorrect in the neighborhood of the well where the assumptions made in developing equation (13) are less applicable. Perhaps the most serious objection to this solution is that the seepage surface, AB, is ignored. Like the solution for the confined case equations (13), (14), and (15) are indeterminate except by experiment. Again the usual procedure is to introduce the radius of influence. The same objections are made to this device as for the confined case.

More Recent Solutions

Yang (14) and Hansen (5) both report the 1932 experiments of Wyckoff, Botset, and Muskat (12) using a sand tank and a sector model of a radial-flow

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5. Equation (9) cannot be used here for evaluating dh/dr as done before since it is valid only for confined flow.

system. These experiments show that equation (15) does not give the correct location of the free surface. On the other hand, the distribution of piezometric head on the surface of the underlying impermeable stratum, CE, (fig. 2) was found to be accurately represented by the Dupuit equation. Hansen (5) demonstrates theoretically that this must be true. Neglecting flow through the capillary zone, Wyckoff, Botset, and Muskat found that the Dupuit equation gave the correct discharge. They noted the existence of the seepage face, but no expression for the position of the free surface was proposed.

Babbitt and Caldwell (1), 1940, using both electrical and sand models, reached the same general conclusions as Wyckoff, Botset, and Muskat regarding the validity of the Dupuit equation. Babbitt and Caldwell plotted the percent of drawdown of the free surface at any distance, r, to the drawdown of the free surface at the well against the ratio r/r_e and found the shape of the free-surface curve to be independent of the physical dimensions of the system. The following equation was proposed for the position of the free surface:

$$h_e - h = \frac{2.3\ Q\ C_x}{k\ h_e} \log \frac{r_e}{0.1\ h_e} \qquad (16)$$

where C_x is the ratio of the drawdown of the free surface at any distance, r, to the maximum possible drawdown when the well is discharging at the maximum value. Hansen (5) proposed the empirical equation

$$C_x = 0.3 \log \frac{r}{r_e} \qquad (17)$$

for values of r/r_e greater than 0.05. Substituting equation (17) in equation (16) gives

$$(h_e - h) = \frac{0.69Q}{kh_e} \log \frac{r_e}{0.1\ h_e} \log \frac{r}{r_e} \qquad (18)$$

which indicates that the elevation of the free surface is a linear function of log r for any value of Q. The values of C_x given by Babbitt and Caldwell are expressed in terms of the drawdown of the free surface curve extended to the center of the well. Yang (14) points out that the curve must theoretically become tangent to the well casing at an elevation of h_s above the bottom of the well and cautions against the use of equation (9) in the region close to the well.

Hansen (5) conducted additional experiments using sand models and gave particular attention to correction for the effects of capillarity. He points out the desirability of expressing the constant of integration in terms of the radius of the well instead of the radius of influence which is an indefinite and hypothetical value at best. Hansen gave particular attention to the extent of the seepage face and using dimensional analysis developed the relation

$$\frac{Q}{kr_w^2} = F\left(\frac{h_s}{r_w}, \frac{h_w}{r_w}\right) \tag{19}$$

The fractional parameters in equation (11) are dimensionless. Using available experimental data the curves shown by fig. 3 were developed.

Yang (13) applied a "relaxation" method of numerical calculation proposed by Southwell (4) in 1940 to theoretical solutions of equation (3) for six particular cases. A great amount of time is required to complete a solution by this method which involves successive approximations.

THE SEEPAGE SURFACE FOR UNCONFINED SYSTEMS

By using the results of Yang's work, Hansen's curves may be extended over a much wider range of values of Q/kr_w^2. Since the Dupuit equation has been found to be correct for the calculation of discharge, one may write, by rearrangement of equation (14),

$$\frac{Q}{kr_w^2} = \frac{\pi(h_e^2 - h_w^2)}{r_w^2 \ln r_e/r_w} \tag{20}$$

Equation (20) makes possible solution for values of the parameter, Q/kr_w^2, from the theoretical computations of Yang. Values of h_s/r_w and h_e/r_w found by Yang may thus be plotted against Q/kr_w^2. These, together with the information compiled by Hansen, form the basis for the curves of fig. 4. Because of the wide range of values of Q/kr_w^2 fig. 4 is plotted on semi-logarithmic paper. The use of these resulting curves is illustrated by example 1 of the seven examples presented at the end of the report.

The height of the seepage surface, AB (fig. 2) should be of particular interest to those designing wells in unconfined aquifers, especially if such wells are for drainage purposes. In the case of example 1 the 30-foot draw-down at the well results in lowering the water table a maximum distance of only six feet. However, wells in unconfined strata are more efficient in utilizing available energy to cause discharge than wells in confined strata. The maximum effectiveness of the well of example 1 calculated as proposed by Wenzel (12) cannot exceed 6/30 (100), or 20 percent.

UNCONFINED FLOW REPLENISHED BY VERTICAL PERCOLATION

Development of Equations

If a valley fill is fully drained by pumped wells little of the well discharge results from water moving horizontally into the region of influence. Water replenishing the soil pore space within the cone of depression percolates vertically downward to be intercepted by the influence region of the well.

Let: q_v = the average flow vertically downward into the zone of influence per unit of horizontal area, volume per unit area per unit time;

n = the ratio of the discharge derived from vertical percolation to the total discharge of the well; and let

Q, r, k, etc., be as before.

Then the flow, Q_r, through any concentric cylinder at radius r, if the Dupuit assumptions are made, is

$$Q_r = Q - \pi q_v (r^2 - r_w^2) \tag{21}$$

As a first approximation, equation (21) may be written

$$Q_r = Q - \pi q_v r^2 \tag{22}$$

and from Darcy's law

$$Q_r = Q - \pi q_v r^2 = 2\pi rhk \frac{dh}{dr}$$

which may be integrated to give

$$\frac{Q}{2\pi k} \ln r - \frac{q_v r^2}{4k} = \frac{h^2}{2} + C \tag{23}$$

When $r = r_w$, $h = h_w$. Evaluating C gives

$$\frac{Q}{2\pi k} \ln \frac{r}{r_w} - q_v \frac{(r^2 - r_w^2)}{4k} = \frac{1}{2} (h^2 - h_w^2) \tag{24}$$

and at the radius of influence

$$\frac{Q}{2\pi k} \ln \frac{r_e}{r_w} - q_v \frac{(r_e^2 - r_w^2)}{4k} = \frac{1}{2} (h_e^2 - h_w^2) \tag{25}$$

by the approximation

$$r_e^2 - r_w^2 = r_e^2$$

and use of the approximate relationship

$$nQ = q_v r_e^2 \tag{26}$$

equation (25) may be written in the forms

$$Q = \frac{\pi k (h_e^2 - h_w^2)}{2.303 \log r_e/r_w - (n/2)} \tag{27}$$

and

$$q_v = \frac{\dfrac{nk\,(h_e^2 - h_w^2)}{r_e^2}}{2.303 \log r_e/r_w - (n/2)} \qquad (28)$$

From equations (24) and (26) the equation for the drawdown curve will be given by the relationship

$$Q = \frac{\pi k\,(h^2 - h_w^2)}{2.303 \log r/r_w - n/2\,(r/r_e)^2} \qquad (29)$$

Substituting the value of Q from equation (27) in equation (29) gives

$$h^2 = (h_e^2 - h_w^2)\;\frac{2.303 \log r/r_w - n/2\,(r/r_e)}{2.303 \log r_e/r_w - n/2} + h_w^2 \qquad (30)$$

Near the well the approximation given by equation (22) may not be justified. Further, it may be necessary to write

$$nQ = \pi\,(r_e^2 - r_w^2)\,q_v \qquad (31)$$

in place of equation (26). Proceeding as before, the more exact equation

$$Q = \frac{\pi k\,(h^2 - h_w^2)}{2.303\left[1 + \dfrac{r_w^2}{(r_e^2 - r_w^2)}\right]\log\,(r/r_w) - n/2\left(\dfrac{r^2 - r_w^2}{r_e^2 - r_w^2}\right)} \qquad (32)$$

may be deduced.

Validity of Equations

No experimental data are available for checking the validity of the equations for the unconfined flow toward a well in a system replenished by vertical percolation. The seepage surface which exists at the well has been neglected in the theoretical development. Also the assumption that the flow is entirely horizontal through a vertical cylinder at distance r has been introduced as was done in the classical Dupuit analysis. By inference, equation (18) cannot be expected to give reliable results for the position of the free water surface near the well; however, equation (27) should give reliable values for discharge. Equation (17) may be expected to give reliable results if the measurements of piezometric head are made at points where the assumption of horizontal radial flow through vertical potential surfaces applies. This is exactly true along the impermeable boundary, CE, (fig. 2) and is closely approximated at greater distances above CE as the radius increases. The more exact equation (32) results from a rather minor refinement and its use is considered unnecessary or unjustified in most cases.

Determination of Seepage Surface

Equation (27) may be rewritten

$$Q = \frac{\pi k \left(h_e^2 - h_w^2\right)}{2.303 \log \dfrac{r_e}{e^{n/2}\,(r_w)}} \tag{33}$$

By the transformation, $r_w = r_w'/e^{n/2}$, equation (33) reduces to the form of equation (14) and the geometry is the same as for the Dupuit case for the same discharge but for a well of radius equal to $e^{n/2}\ r_w'$. For the case when n equals 1, the well system replenished entirely by vertical percolation, $r_w' = 1.643 r_w$. Fig. 4 may be used for finding the height of the seepage surface, h_s, by introducing the above transformation for the case of a well in a system replenished by vertical percolation. Example 2 illustrates the use of the foregoing analysis for the purpose of designing an agricultural drainage well of this type.

EFFECT OF REPLENISHMENT

A condition of steady flow implies that the total replenishment of the influence cone equals the discharge of the well. The shape of the influence cone and the discharge, Q, for steady flow for a particular system must be, in general, a function of the drawdown, D_w, the well radius, r_w, the permeability, k, the thickness of the water-bearing stratum, T,[6] and the rate of replenishment to the influence cone, q, having the dimensions L/T. These last five variables are the independent variables and are sufficient to fully determine the flow for any particular system. The weakness of existing formulas for well discharge is that they do not contain the rate of replenishment. They are therefore indeterminate unless an actual test is made or unless some assumption, such as the radius of influence, is introduced. The foregoing statement may be expressed mathematically by

$$F\,(D_w,\ r_w,\ k,\ T,\ q,\ Q) = 0 \tag{34}$$

where F designates an unknown function. Equation (34) involves only the dimensions L and T. Choosing r_w and k as repeating variables one obtains by dimensional analysis

$$F_1\left(\frac{D_w}{r_w},\ \frac{T}{r_w},\ \frac{F}{k},\ \frac{Q}{kr_w^2} = 0\right) \tag{35}$$

or

$$\frac{Q}{kr_w^2} = F_2\left(\frac{D_w}{r_w},\ \frac{T}{r_w},\ \frac{q}{k}\right) \tag{36}$$

as a generalized functional relation for flow into a well.

An examination of equation (36) reveals that the dimensionless parameter. Q/kr_w^2, depends only on the geometry of the well (r_w, D_w) and the hydrology of the ground-water system (k, T, q); in other words, only on the combination of the well and its hydrologic environment. Q/kr_w^2 is therefore designated by H, the Well-Hydrologic number or W-H number. One should note that consideration of replenishment gives the radius of influence real meaning, for within the radius of influence the total replenishment equals the discharge, Q. The following analysis introduces the rate of replenishment and develops

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6. For confined systems the thickness of the water-bearing stratum is represented by t. For unconfined systems the value $h_e + h_w/2$ is a measure of the thickness of the water-bearing stratum. Note that equation 12 reduces to equation 14 by substitution $t = h_e + h_w/2$.

theoretically, using certain approximations, the functional relations expressed by equation (36) for the three cases of steady flow discussed previously.

Unconfined System Recharged by Horizontal Flow

Denoting the W-H number for the unconfined case by the symbol, H_u, and substituting $D_w = h_e - h_w$, equation (36) may be rewritten

$$H_u = F_3 \left(\frac{h_e}{r_w}, \frac{h_w}{r_w}, \frac{q}{k} \right) \qquad (37)$$

Equation (20) may be rewritten in the form

$$H_u = \frac{Q}{k(r_w)^2} = \frac{\pi \left[\left(\frac{h_e}{r_w} \right)^2 - \left(\frac{h_w}{r_w} \right)^2 \right]}{\ln r_e/r_w} \qquad (38)$$

so that all dimensions are expressed in terms of the well radius, r_w. The right side of equation (38) contains only linear dimensions; thus the numerical value of H_u defines the shape of the influence region. If the natural slope of the free water table in the region is i_n, and if no water comes into the cone of influence by vertical percolation, $q = i_n k$ so that q/k in equation (37) may be replaced by i_n. Under conditions of steady flow

$$Q = 2r_e\, k\, h_e\, i_n \qquad (39)$$

and

$$\frac{r_e}{r_w} = \frac{Q}{2\, h_e\, k\, i_n\, r_w}$$

Substituting r_e/r_w in equation (38) yields

$$H_u = \frac{\pi \left[\left(\frac{h_e}{r_w} \right)^2 - \left(\frac{h_w}{r_w} \right)^2 \right]}{\ln H_u - \ln \left(\frac{2\, h_e\, i_n}{r_w} \right)} \qquad (40)$$

Equation (40) contains all of the parameters required by equation (37) and defines the function F_3 for the case at hand. One may observe that H_u

depends upon the radius of the well, the initial and final depths of water in the well, and the natural slope of the water table. One may solve equation (40) implicitly for $\frac{H_u}{(h_e/r_w)^2}$ in terms of $\frac{i_n}{(h_e/r_w)}$ for various ratios of h_w/h_e.[7/] The resulting graphs of the functions are plotted in fig. 5.

Large values of H_u indicate deep, narrow drawdown cones of influence while small values indicate broad, shallow cones. High flows into the zone of influence and/or low permeability cause the cones to become narrow and steep, whereas slow replenishment and/or high permeability cause them to be broad and shallow in terms of the radius of the well.

Confined Systems

The W-H number for the confined system will be designated by H_c. From equation (10b)

$$H_c = \frac{Q}{k\ r_w^2} = \frac{2\pi(D_w/r_w)\ (t/r_w)}{2.303\ \log r_e/r_w} \tag{41}$$

where t is the thickness of the permeable stratum. If i_n is the natural slope of the piezometric surface the flow into the zone of influence is approximated by

$$Q = 2r_e\ k\ t\ i_n \tag{42}$$

Solving equation (42) for r_e and substituting in equation (41) gives

$$H_c = \frac{2\pi\ (D_w/r_w)\ (t/r_w)}{2.303\ \log\ \left(\frac{H_c}{2i_n}\frac{r_w}{t}\right)} \tag{43}$$

Equation (43) satisfies the functional relationship of equation (36) and makes possible computation of values of $\frac{i_n}{D_w/r_w}$ for various values of

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7. Algebraic manipulation of equation (40 for the case of zero drawdown ($h_w/h_e = 1.0$) yields $Q = 2\ i_n\ k\ h_e\ r_w$ instead of zero. The value of Q in this instance is the same as for a well with a radius of influence equal to the radius of the well.

$\frac{H_c}{D_w t/r_w^2}$ to form the curve of fig. 6.

Unconfined System Totally Replenished by Vertical Percolation

Let $H_v = Q/kr_w^2$ define the W-H number for this system. Combining equations (26) and (27) for $n = 1$ yields

$$H_v = \frac{\pi\left[\left(\frac{h_e}{r_w}\right)^2 - \left(\frac{h_w}{r_w}\right)^2\right]}{2.303 \log \frac{1}{r_w}\sqrt{\frac{Q}{q_v \pi}} - 1/2} \qquad (44)$$

Substituting $Q = H_v\, k\, r_w^2$ in the denominator gives

$$H_v = \frac{\pi\left[\left(\frac{h_e}{r_w}\right)^2 - \left(\frac{h_w}{r_w}\right)^2\right]}{2.303 \log \sqrt{\frac{H_v k}{q_v \pi}} - 1/2} \qquad (45)$$

which satisfies the relationship required by equation (36). Equation (45) may be rewritten in the form

$$\frac{H_v}{\left(\frac{h_e}{r_w}\right)^2} = \frac{\pi\left[1 - \left(\frac{h_w}{h_e}\right)^2\right]}{1.151\left[\frac{\log H_v}{\left(\frac{h_e}{r_w}\right)^2} - \frac{\log q_v/k}{\left(\frac{h_e}{r_w}\right)^2} - \frac{\log \pi}{\left(\frac{h_e}{r_w}\right)^2}\right] - \frac{1}{2\left(\frac{h_e}{r_w}\right)^2}} \qquad (46)$$

The ratio h_e/r_w depends entirely on the dimensions of the well while q_v/k is the ratio of the unit replenishment to the permeability. From equation (46) $\frac{H_v}{(h_e/r_w)^2}$ can be computed for values of $\frac{q_v k}{(h_e/r_w)^2}$ for various drawdown ratios, h_w/h_e. Graphs of the results are presented by fig. 7.

Although the value of q_v will no doubt vary at various times during the year and with the seasons, equation (44) should be valuable in planning the design of a system of drainage wells as well as in developing the ultimate ground-water supply of a closed ground-water basin of the unconfined type.

Significance of the W-H Number

If the value of H is known, the effective radius of influence may be established by consideration of the hydrologic factors and the well radius. For the three cases discussed herein the following formulas may be used.

$$H_u = 2 \left(\frac{r_e}{r_w}\right) \left(\frac{h_e}{r_w}\right) \text{ in} \qquad (47)$$

$$H_c = 2 \left(\frac{r_e}{r_w}\right) \left(\frac{t}{r_w}\right) \text{ in} \qquad (48)$$

$$H_v = \left(\frac{r_e}{r_w}\right)^2 \frac{q}{k} \qquad (49)$$

For the confined case $H_c r_w/t$ is linearly proportional to $\dfrac{h - h_w}{r_w \log r/r_w}$.

This relationship is shown by fig. 8. To use fig 8 if $H_c r_w/t$ is known enter with the values of D_w/r_w to intersect the graph for $H_c r_w/t$. The resulting abscissa is r_e. The value of $\dfrac{h - h_w}{r_w}$ may be found by entering with the value of r and proceeding vertically to intersect the proper curve for $H_c r_w/t$. The resulting ordinate gives the value of $\dfrac{h - h_w}{r_e}$. For example, suppose $H_c = 2,640$, $t/r_w = 30$, and $D_w/r_e = 74$. Then $H_c r_w = 88$. Entering with D_w/r_e to intersect $H_c r_w/t = 88$ gives $r_e/r_w = 190$. At $r/r_w = 110$, $\dfrac{h - h_w}{r_w} = 66.2$. Interpolation for intermediate values of $H_c r_w/t$ may be easily done because the value of the ordinate along the heavy vertical line at $r/r_w = 524$ is equal to the value of $H_c r_w/t$ through that point. Thus, to find the line for $H_c r_w/t = 88$, draw a line through the ordinate 88 on the heavy vertical line and the origin. The relationships for the other two systems are much more complex and further study is needed in order to fully present them.

Typical problems are presented and solved in examples 3, 4, 5, 6, and 7 of the appendix to illustrate the use of the foregoing equations and analyses.

EFFECTIVENESS OF WELLS IN UNCONFINED SYSTEM

The "effectiveness" of a well is defined by Wenzel (12) as

$$E_w = \frac{100\ (h_e - h_s)}{(h_e - h_w)} \tag{50}$$

where

E_w = the "effectiveness" of the well,

h_s = the depth of water immediately outside of the casing, and the other terms are as previously defined.

For a well in a confined system equation (50) has the quality of well efficiency because $h_e - h_s$ is actually the power per unit of weight discharge delivered by the well to the fluid outside of the well whereas $h_e - h_w$ is the power per unit of weight discharge imparted by the pumps to the water inside the well. The difference between the two values represents the power loss per unit of weight discharge through the boundary of the well. "Effectiveness" of wells is a widely-used term describing the condition of wells from the point of view of their efficiency as a power-transferring device. The draw-down in the well, $h_e - h_w$, may be quite easily measured; however, greater difficulty is encountered in the measurement of $h_e - h_s$. The head loss in the simple, confined case of radial flow is a linear function of the logarithm of the radius (equation (10b)). The elevation of the piezometric surface observed at various distances from the well may therefore be plotted against log r and the resulting straight line extended to the casing to determine h_s of fig. 9.

For the unconfined case considerable care must be exercised in applying Wenzel's procedure or misleading results will be obtained. If the piezometers extend only a short distance into the saturated media the elevations represent the elevation of the free water surface. Following this procedure for the unconfined system will always yield values of effectiveness less than 100 percent because the head represented by the height of the seepage surface is counted as lost. The total available energy per unit weight of water is equal to D_w or $h_e - h_w$. Actually the unconfined system is inherently more efficient in utilizing the available specific energy than is the confined system. This conclusion may be readily deduced. The available specific energy in a confined system is, from equation (12)

$$h_e - h_w = \frac{Q \ln (r_e/r_w)}{2 \pi k t}$$

and in an unconfined system is, from equation 14

$$h_e - h_w = \frac{Q \ln (r_e/r_w)}{k \pi (h_e + h_w)}$$

For the same value of available specific energy, permeability, r_w and r_e

$$\frac{Q_u}{Q_c} = \frac{\frac{h_e + h_w}{2}}{t} \qquad (51)$$

where Q_u is the discharge for the unconfined system and Q_c is the discharge for the confined system. Since t must be less than h_w, Q_u must always exceed Q_c providing both wells are equally efficient. The following modification of procedure for determining the effectiveness of wells in unconfined media is suggested: The piezometer pipes should be unperforated with an open end. The pipe should be extended to within a short distance of the bottom impermeable stratum. Theoretically the squares of the differences between the piezometric level and the elevation of the bottom impermeable stratum should form a straight line when plotted against the logarithm of the radial distance from the well. The value of h_s may be found by extending the plotted line to the position of the well casing. Actually no experimental information regarding this procedure is available and it needs to be checked in the field.

All of the examples have assumed the well to be 100 percent effective. If the effectiveness is less than 100 percent, the value of h_s should be used

EXAMPLES

Example 1

Given: A gravity well discharges 2.0 cfs and the permeability of the material is determined as 10^{-3} ft/sec. The original depth of water in the well, h_e, is 50 ft and the drawdown is 30 ft, making $h_w = 20$ ft. What will be the height of the seepage face (AB, fig. 2) if the well is 24 in. in diameter?

Solution:

$$\frac{Q}{kr_w^2} = \frac{2.0}{(0.001)\ (1)^2} = 2{,}000.$$

$\dfrac{h_w}{r_w} = \dfrac{20}{1} = 20$, and from fig. 4 $\dfrac{h_s}{r_w} = 44$. Therefore, $h_s = \dfrac{44}{1} = 44$ ft

and the height of the seepage face is (44 - 20) or 24 ft = Ans.

Example 2

Given: Excess irrigation water of 1 foot depth per year is to be removed by steady pumping. The diameter of the well is 24 in., and the sub-soil permeability, k, is 10^{-3} ft/sec. The depth of the saturated permeable overburden, h_e, is 50 ft and the casing is perforated for the entire depth. What will be the drawdown if an area of radius 2,000 ft is to be drained by one well? What will be the height of the seepage surface?

Solution:

From equation (28) with n = 1

$$h_e^2 - h_w^2 = \frac{1}{(365)\ (24)\ (3600)}\ \frac{(2000)^2}{(0.001)}\ \left[2.3 \log \frac{2000}{1} - 1/2\right]$$

$$= (127)\ \left[(2.303)\ (3.301 - 0.500)\right] = 902$$

$$h_w = \sqrt{2500 - 902} = 40.9 \text{ ft}$$

From equation (26) $nQ = q_v \pi r_e^2$

$$Q = \frac{1}{(365)\ (24)\ (3600)}\ (2000)^2 \pi = 0.4 \text{ cfs}$$

$r_w' = (1)\sqrt{e} = 1.643.$

$$\frac{Q}{kr_w'^2} = \frac{0.4}{(0.001)(1.643)^2} = 148, \quad \frac{h_w}{r_w'} = \frac{40.0}{1.643} = 24.4$$

and from fig. 4 $h_s/r_w = 25.8$; therefore,

$h_s = (1.643)(25.8) = 42.3$ ft and the height of the seepage surface

$h_s - h_w = 42.3 - 40.9 = \underline{1.4 \text{ ft}} =$ Ans.

Example 3. Yield of Confined-Flow System

Given: A permeable confined aquifer 30 ft thick is at a depth of 100 ft below the ground surface. The permeability of the gravels is estimated at 2×10^{-3} ft/sec. The natural slope of the piezometric surface is 0.01. The water stands at a depth of 20 ft below the ground surface in a 12-in. diameter well. Assuming that the water surface may be drawn down to a depth of 70 ft, what steady production may be expected from the well?

Solution:

Equation (43) and the graph of fig. 6 apply

$$\frac{i_n}{D_w/r_w} = \frac{0.01}{50/0.5} = 0.0001$$

and from fig. 6a

$$\frac{H_c}{D_w t/r_w^2} = 0.764$$

hence, $H_c = \dfrac{Q}{kr_w^2} = (0.764)\dfrac{(50)(30)}{(0.5)^2} = 4584$

and

$$Q = (4584)(2)(10^{-3})(0.5)(0.5) = 2.29 \text{ cfs}$$

Example 4. Permeability of the Confined System

Given: A confined sand layer 10 ft thick yields 0.2 cfs steady discharge when pumped under a drawdown of 30 ft. The natural slope of the piezometric surface is 5 ft/hundred and the diameter of the well is 24 in. Estimate the permeability of the sand.

Solution:

$$\frac{i_n}{D_w/r_w} = \frac{0.05}{30/1} = 0.00167$$

and from fig. 6,

$$\frac{H_c}{D_w t/r_w^2} = 1.07; \quad H_c = 321 = \frac{Q}{kr_w^2}$$

from which

$$k = \frac{0.2}{(321)\ (1)\ (1)} = 6.23 \times 10^{-4}\ \text{ft/sec}$$

Example 5

Drawdown Required to Discharge 1 cfs in Confined System

Given: The permeability of a 20-ft gravel stratum is estimated at 1.0×10^{-3} ft/sec. The well diameter is 24 in. and i_n is 10 ft/hundred. What draw down will be required to produce a discharge of 1.0 cfs?

Solution:

This problem may be solved by trial and error using fig. 6.

$$H_c = \frac{Q}{kr_w^2} = \frac{1.0}{(10^{-3})\ (1.0)^2} = 1000$$

$$i_n = 0.1$$

Try $D_w/r_w = 20$;

$$\frac{H_c}{D_w t/r_w^2} = \frac{1000}{(20)\ (20)} = 2.5 \text{ and } \frac{i_n}{D_w/r_w} = 0.01, \text{ entering fig. 6 with}$$

$$\frac{H_c}{D_w t/r_w^2} = 2.5$$

the corresponding value of $\frac{i_n}{D_w/r_w} = 0.10$.

Try $D_w/r_w = 50$;

$\frac{H_c}{D_w t/r_w^2} = 1.0$, $\frac{i_n}{D_w/r_w} = 0.002$, entering fig. 6a with $\frac{H_c}{D_w/r_w} = 1.0$ gives

$$\frac{i_n}{D_w/r_w} = 0.00106$$

Therefore, the correct value of D_w/r_w is between 10 and 50 and much nearer 50.

Try $D_w/r_w = 40$;

$\frac{H_c}{D_w/r_w} = \frac{50}{40} = 1.25$, $\frac{i_n}{D_w/r_w} = 0.0025$, repeating the preceeding steps gives

$\frac{i_n}{D_w/r_w} = 0.00385$. The correct value of D_w/r_w appears to be about 45, and the required drawdown is therefore estimated to be 45 feet.

Example 6a. Yield of Unconfined System

Given: A ground-water survey of an unconfined-flow aquifer indicates that the average depth of water in the permeable layer is 80 ft, that the permeability is 5×10^{-3} ft/sec, and that the natural slope of water table is 2.5 ft per thousand. If the economic lift enables the water table to be drawn down 60 feet at the well what will be the estimated theoretical yield of the 12-in. diameter well? Estimate the desirable well spacing.

Solution:

$\frac{i_n}{h_e/r_w} = \frac{.0025}{80/.5} = .0000156$, $\frac{h_w}{h_e} = \frac{80 - 60}{80} = 0.250$, from fig. 5

$\frac{H_u}{(h_e/r_w)^2} = 0.331 \qquad H_u = (0.33)\ (160)^2 = 8,450$

$$\therefore Q = H_u\ r_w^2\ k = (8450)\ (0.5)^2\ (.005) = 10.6 \text{ cfs}$$

$$r_e = \frac{Q}{2k\ h_e\ i_n} = \frac{10.6}{(2)\ (.0015)\ (80)\ (0.0025)} = 5300 \text{ ft}$$

Wells should be spaced somewhat less than every two miles for greatest economy and full coverage.

Example 6b

Given: The same data as in example 6a but using the gallon as the unit of volume and the minute as the unit of time.

Solution:

In this case k = 2.24 gal/sq ft/min corresponds to 5×10^{-3} ft/sec. As before $H_u = 8,450$ and $Q = (8450)\ (0.5)^2\ (2.24) = 4,770$ gpm

$$r_e = \frac{4770}{(2)\ (2.24)\ (80)\ (.0025)} = 5,300 \text{ ft}$$

Example 7. Design of Drainage Well

Given: During the summer months, May to October, an average excess of one ft of irrigation water is applied each month. This is all to be removed by pumped drainage using 12-in. wells. The depth of the permeable overburden is 65 ft and the water table is to be maintained at a depth of at least 15 ft below the ground surface. The permeability of the material is estimated at 10^{-4} ft/sec. In order to benefit by special power rates, pumps will be operated only during the irrigation season. What discharge may be expected and what should be the well spacing if the average lift (from water surface in the well to the ground surface) is maintained at 50 ft?

Solution: $q_v/k = \dfrac{1/.0001}{(30)\ (3600)\ (24)}$, $\dfrac{h_e}{r_w} = \dfrac{50}{0.5} = 100$

$$\frac{q_v/k}{(h_e/r_w)^2} = \frac{10000}{(30)(3600)\ (24)\ (100)^2} = 3.68 \times 10^{-7} \text{ and } h_w/h_e = 15/50 = .3$$

from fig. 7

$$\frac{H_v}{(h_e/r_w)^2} = 0.472, \quad H_v = 4720$$

$$Q = (4720)\ (0.0001)\ (0.5)^2 = 0.118 \text{ cfs}$$

$$r_e = \frac{Q}{q_v\,\pi} = \frac{(0.118)\ (3600)\ (24)\ (30)}{(\pi)\ (1)} = 312 \text{ ft}$$

If the spacing of the wells is made $2r_e$ there will be a small undrained area because of the circular shape of the drained area. To allow a full coverage of drainage the points of tangency of the circular areas should be brought to a common point. Thus, for an arrangement of wells where there are three tangency points, the spacing should be made $r_e\sqrt{3}$, and for four tangency points, $r_e\sqrt{2}$. In order to find the seepage face an equivalent well having a radius of (1.643) (0.5) or 0.821 ft must be assumed. The new values of H and h_w/r_w are 1740 and 18.3 respectively. From fig. 4, $h_e/r_w = 40.8$, therefore $h_s = (40.8)\ (0.821) = 32.8$ ft. The seepage surface is therefore $32.8 - 15 = 17.8$ ft high.

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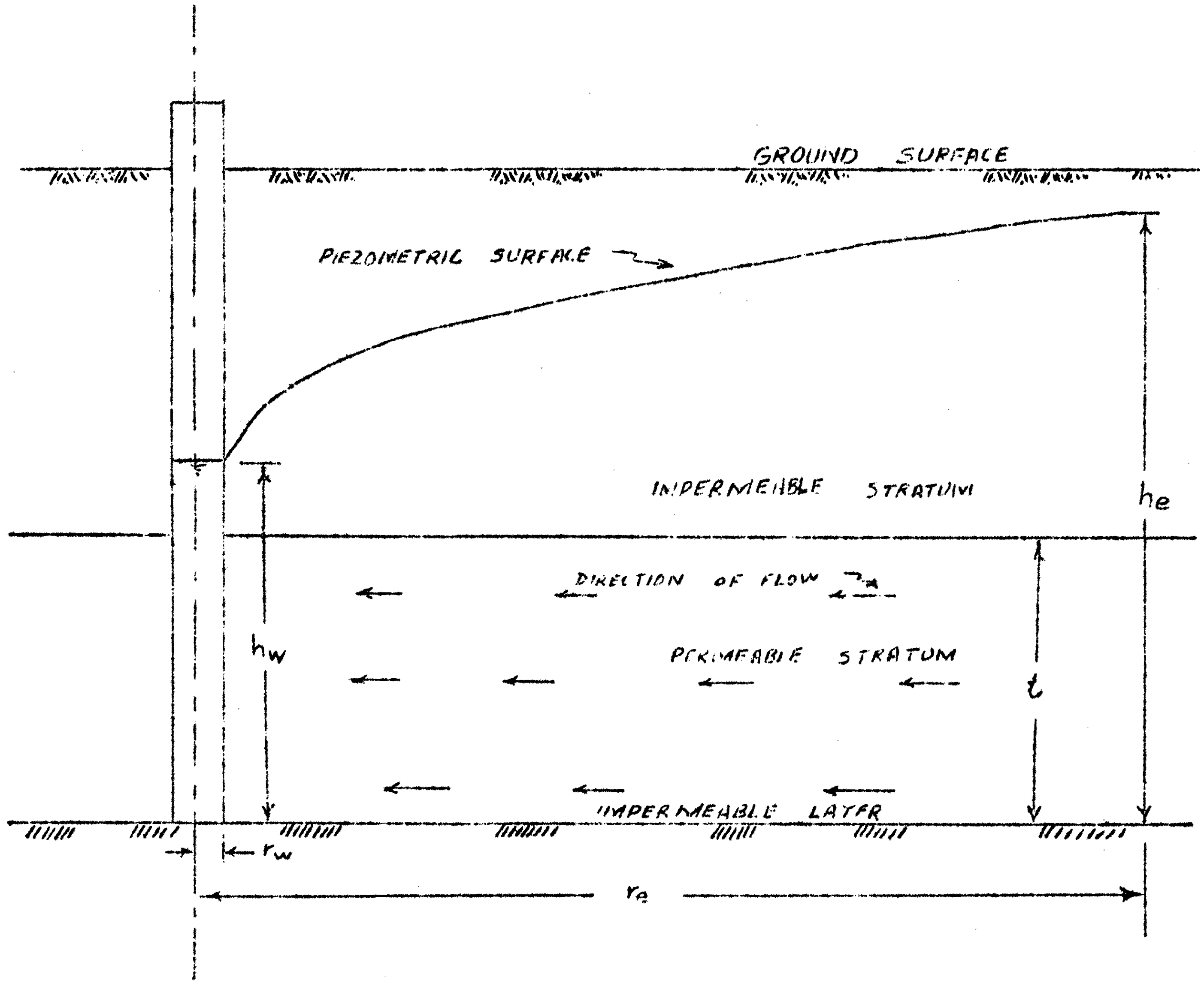

Fig. 1. Flow of confined ground-water to a well.

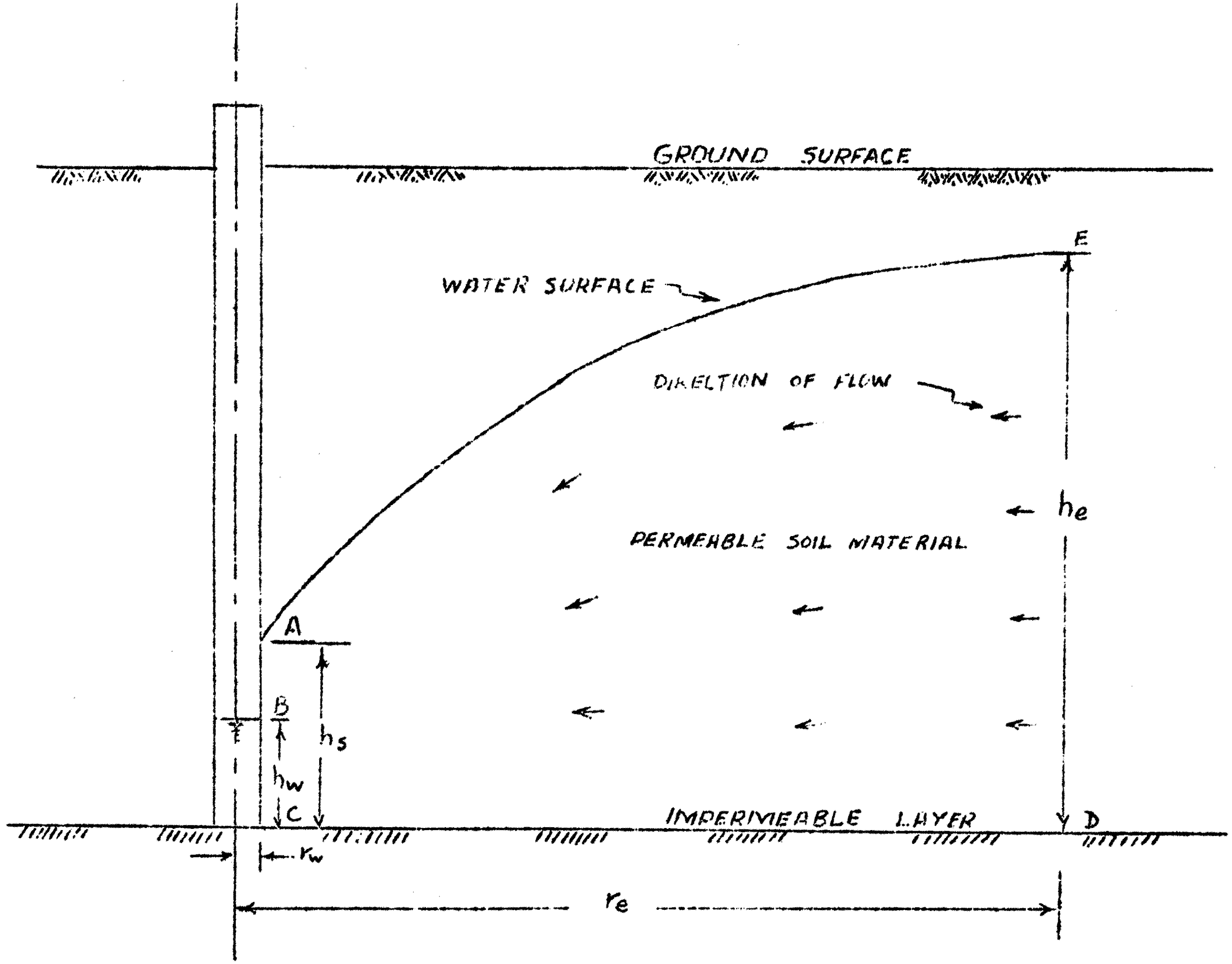

Fig. 2. Flow of unconfined ground-water to a well in highly-permeable soils overlying impermeable material.

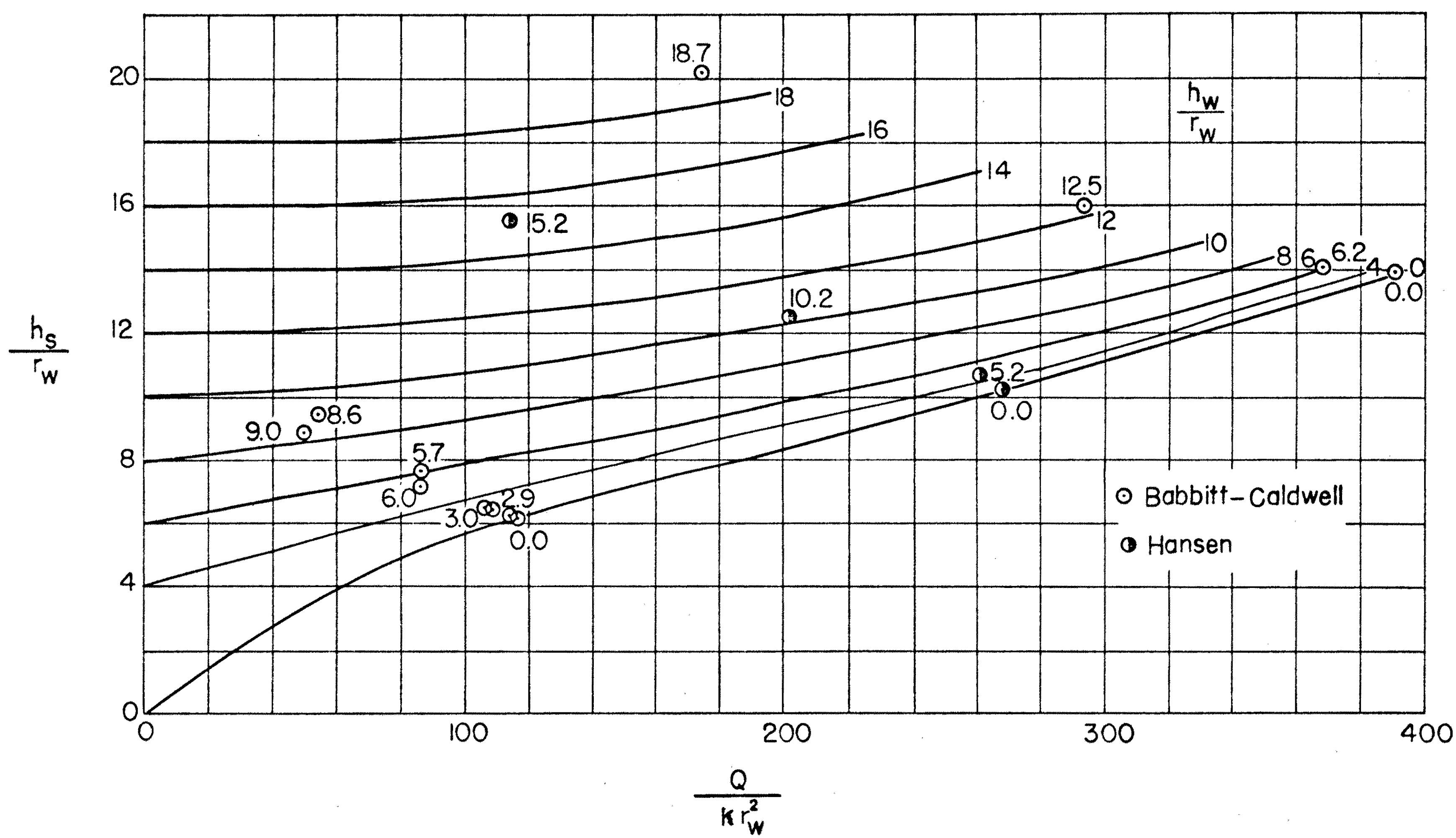

Fig. 3. The relation of seepage surface to well parameters of small magnitudes.

Taken from Vaughn E. Hansen's Doctor's Thesis, June 1949 — Fig. 19.

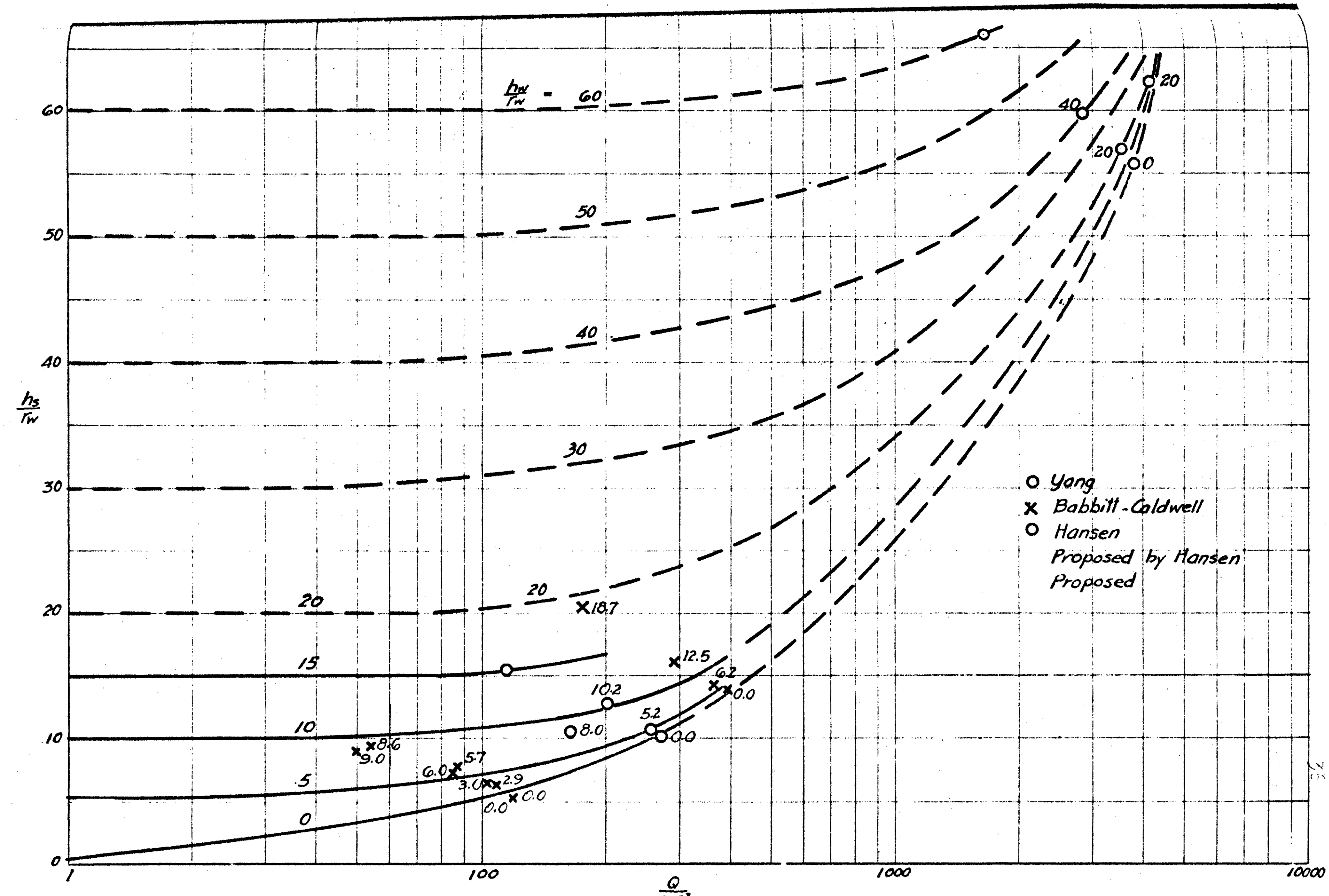

Fig. 4. The relation of seepage surface to well parameters of large magnitudes.

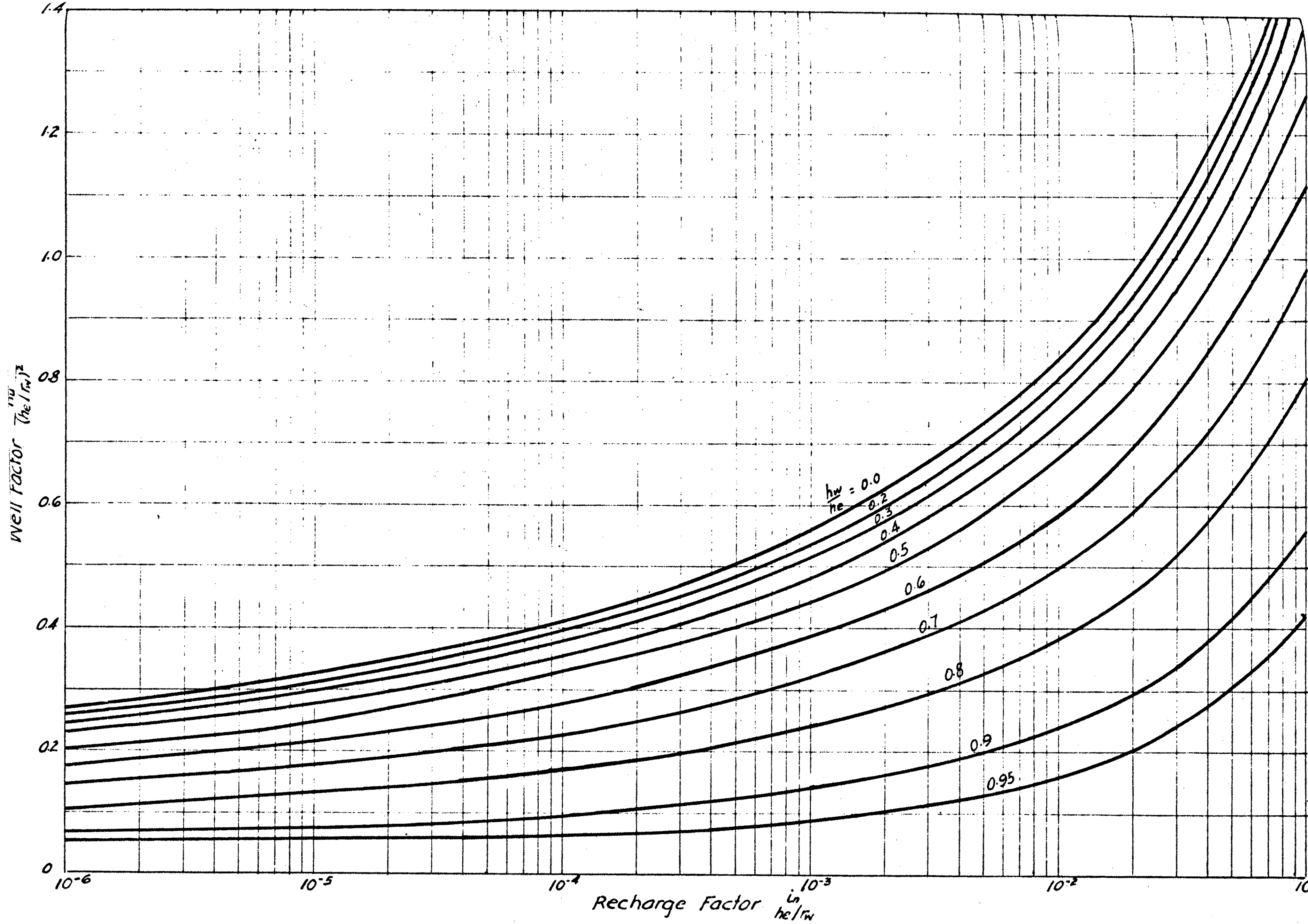

Fig. 5. Representing the [illegible] for [illegible] horizontally-recharged well in unconfined stratum.

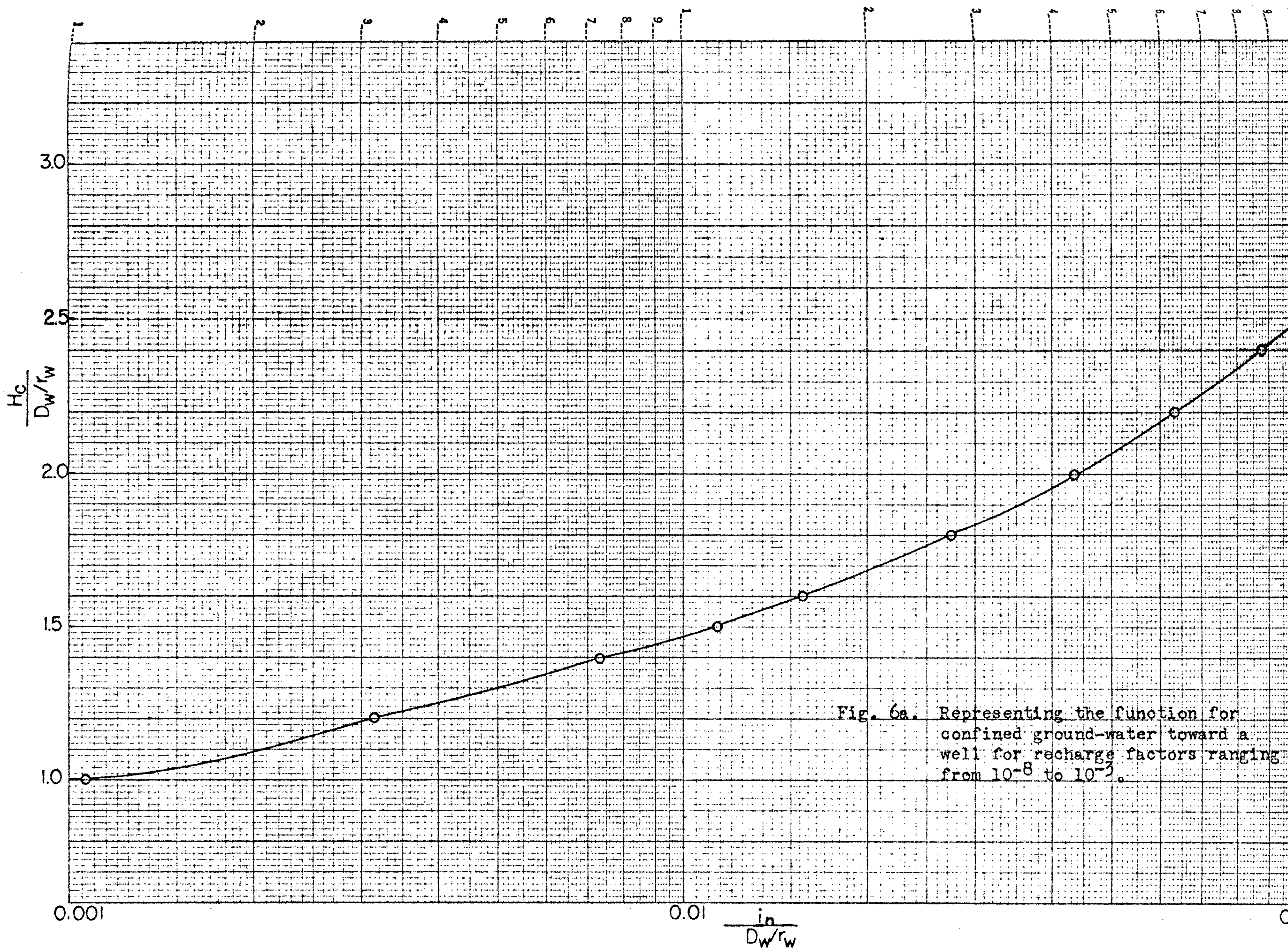

Fig. 6a. Representing the function for confined ground-water toward a well for recharge factors ranging from 10^{-8} to 10^{-3}.

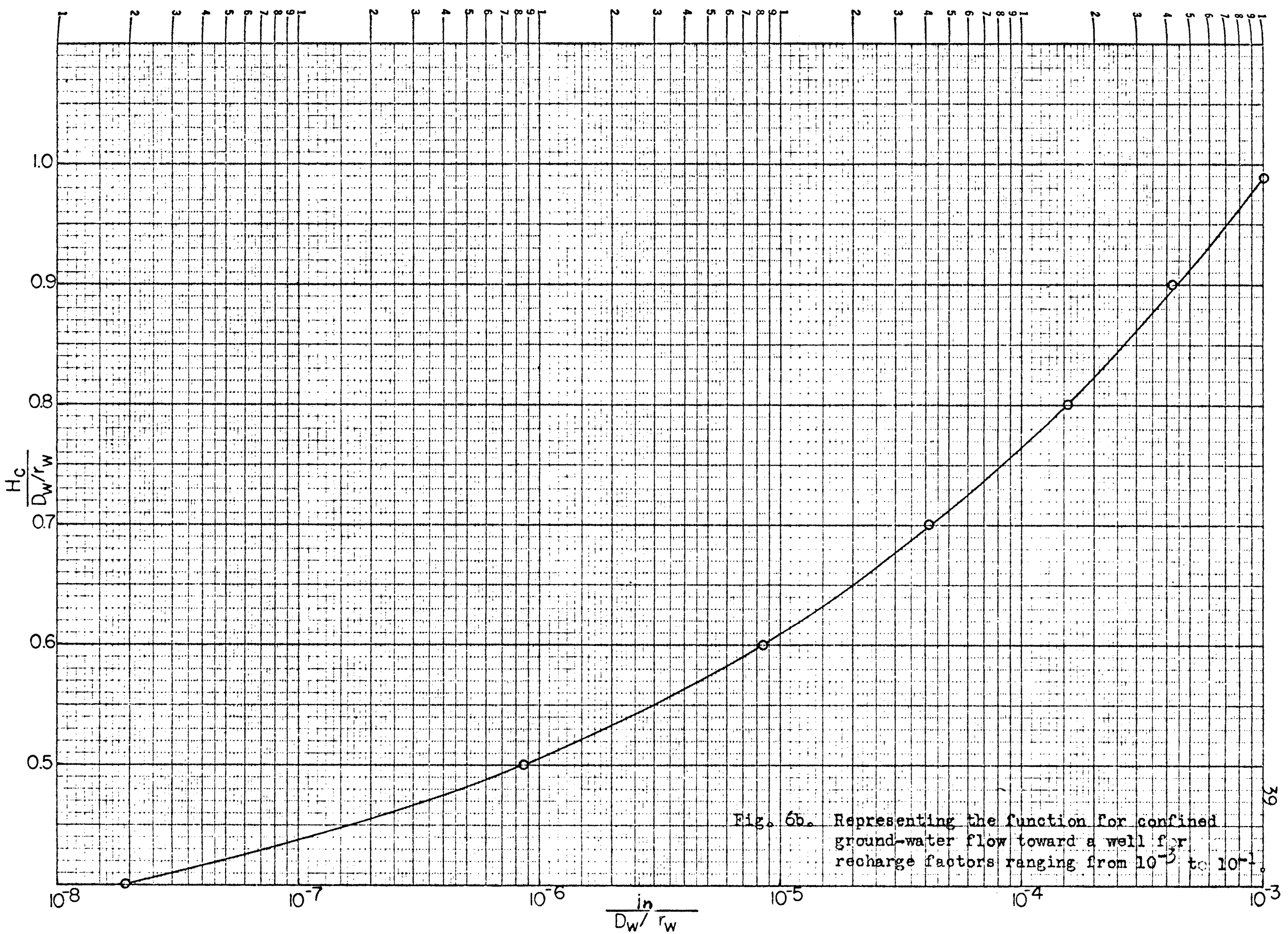

Fig. 6b. Representing the function for confined ground-water flow toward a well for recharge factors ranging from 10^{-3} to 10^{-1}.

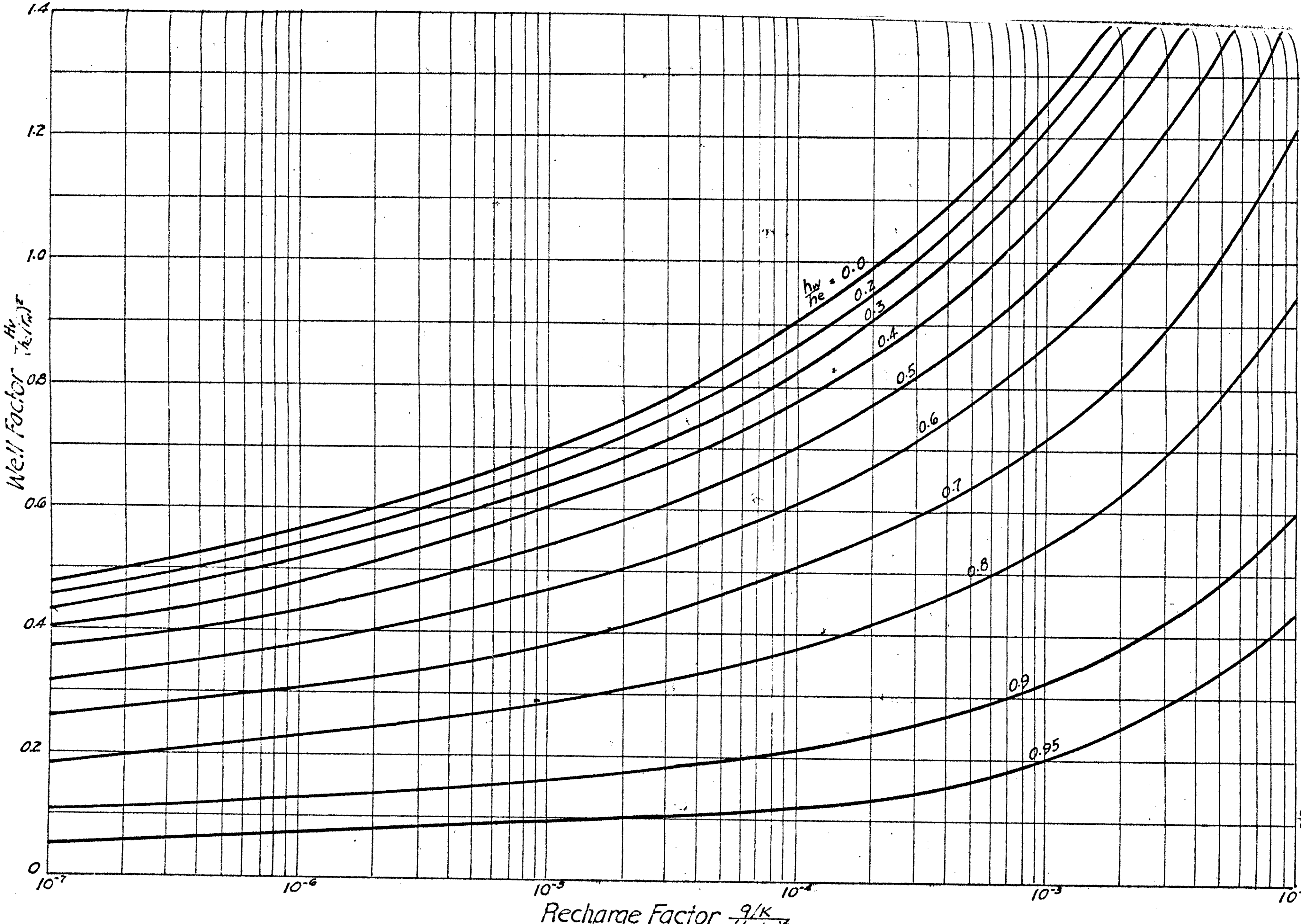

Fig. 7a. Representing the function for vertically recharged, and unconfined ground-water flow to a well for recharge factors ranging from 10^{-7} to 10^{-2}

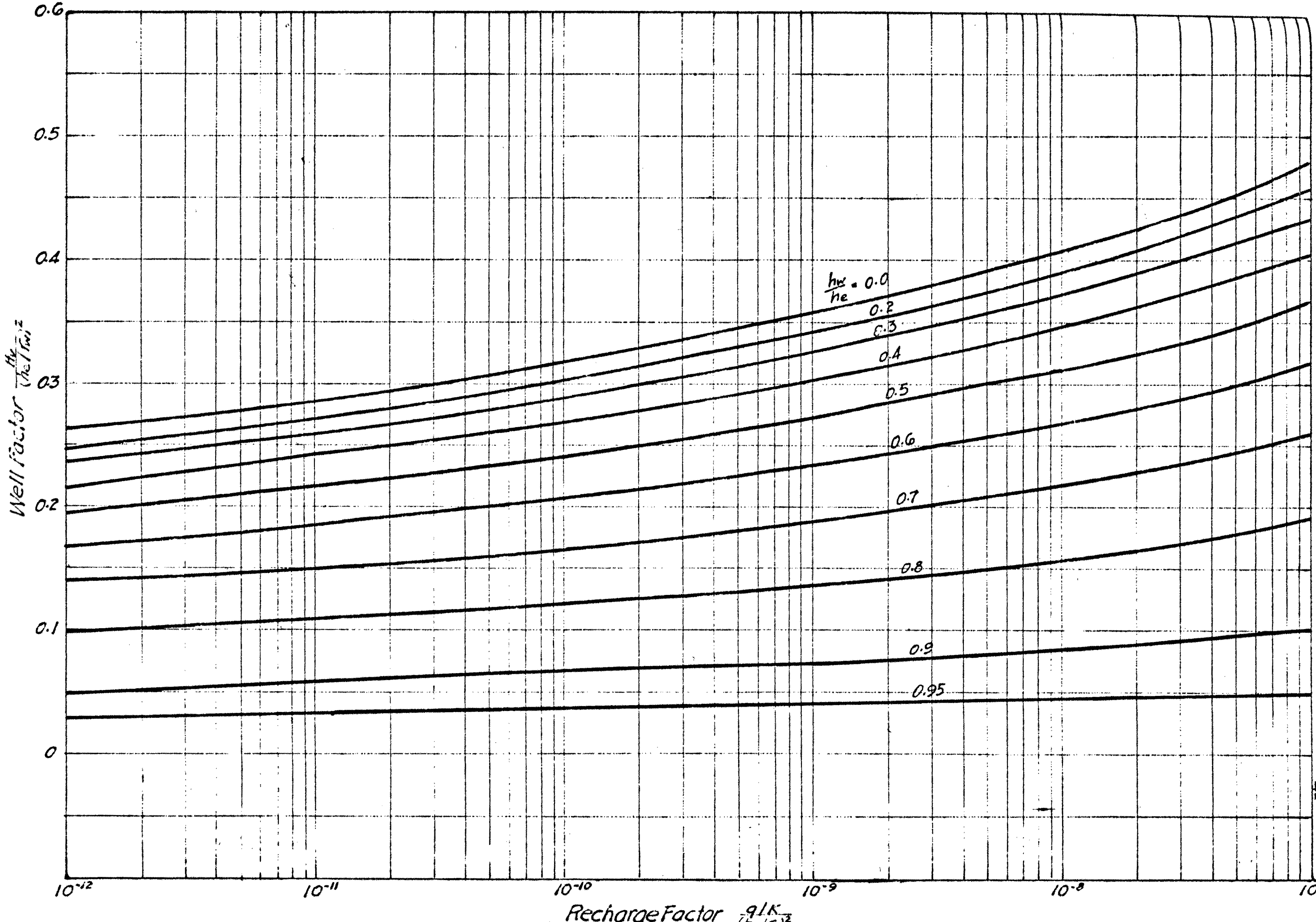

Fig. 7b. Representing the function for vertically recharged and unconfined ground-water flow to a well for recharge factors ranging from 10^{-12} to 10^{-7}.

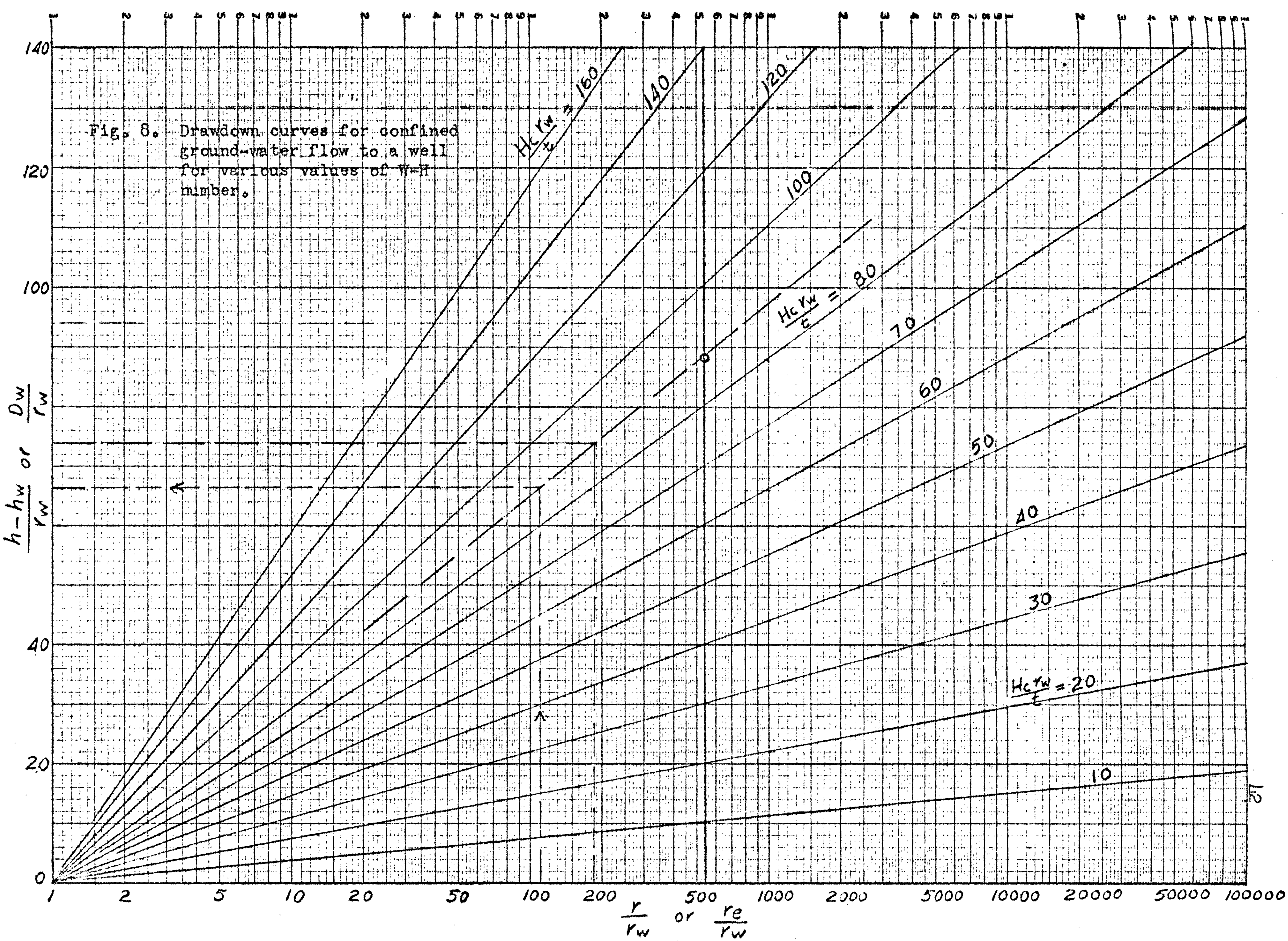

Fig. 8. Drawdown curves for confined ground-water flow to a well for various values of W-H number.

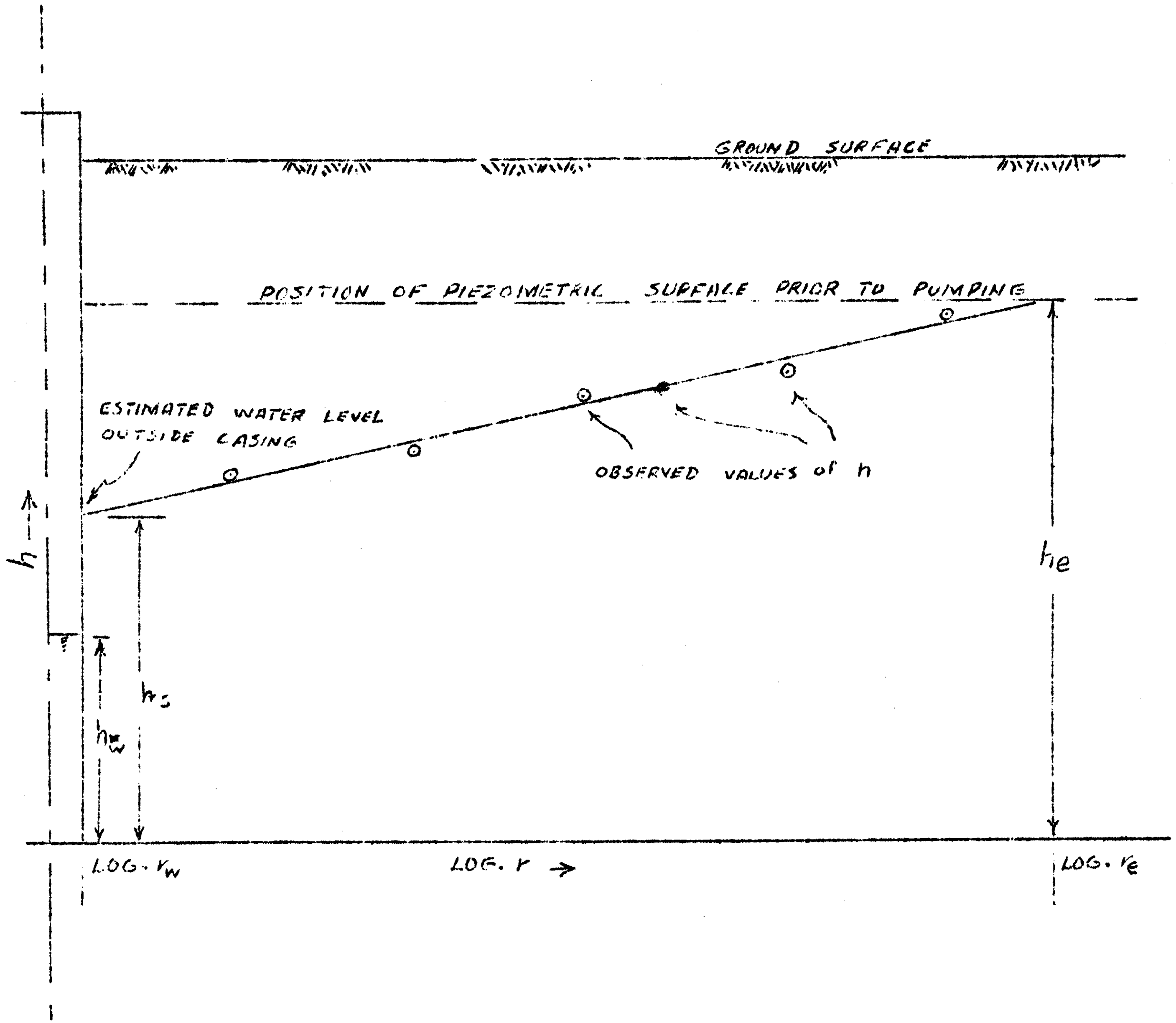

Fig. 9. Illustrating the estimation of water level outside a well.