ON-LINE ADAPTIVE CONTROL FOR COMBINED SEWER SYSTEMS

by

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ABSTRACT

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Urban stormwater management and, in particular, the control of storm runoff within combined sewer systems, is an area of serious environmental concern. The use of in-line or auxiliary storage in conjunction with treatment, has been proposed as a viable solution to the problems of polluting discharges into receiving waters and local street flooding. These overflows result from sewer systems and treatment facilities which are inadequate to effectively contain and treat combined flows during storm events.

On-line computer control for the mechanical features (i.e., pumps, valves, gates, etc.) of a large system of storage facilities is required due to: the highly stochastic storm inputs, the rapid response times of the system, and the high interdependence of control decisions in time and space. Current approaches to the control of urban stormwater are largely reactive or myopic in nature. These approaches may not always be the most suitable for achieving the goal of cost effective control of pollution from combined sewer overflows. An adaptive control algorithm is proposed here which anticipates future values of the stochastic inflows, rather than simply reacting to a current measurement of inflows, and which also deals with the complex large-scale nature of the control problem.

A hierarchical approach to the control problem is proposed here, where the urban area is divided into a number of subbasins which are essentially independent except for their contributions of storm runoff to a common interceptor and treatment facility. The controls for each subbasin are derived separately by the use of a stochastic dynamic

programming formulation. Each subbasin problem, however, is constrained by an upper limit on its releases to the interceptor, which is determined by a master control problem. This master control problem, which ties together the separate subbasin problems, decides how interceptor and treatment capacity should be allocated to the subbasins. It uses a modified cyclic coordinate search algorithm. The inflows are forecasted using an autoregressive-transfer function model which can be updated in real time to respond to new information on the storm event.

A portion of the San Francisco Master Plan for Wastewater Management was used as a case study. The control algorithm was tested for selected design storms which were based upon the historic record. The tests were conducted on a batch-mode computer, but a hierarchy of minicomputers appears to be a more efficient approach to effecting the multilevel optimizations proposed herein.

The results of this work indicate that the large-scale algorithm can converge within the time frame anticipated for real-time control. Controls based upon the stochastic models appeared superior to those based upon forecasts which were assumed deterministic. The adaptive aspects of the model appear to be justified by the superior distribution of the overflows which resulted when overflows were unavoidable. That is, the maximum rate of overflow was lowest for this model. This result is notable in that the forecasting model was deliberately designed to be relatively inaccurate. Total overflows were, however, minimized to a higher degree by a reactive model which was also tested, though the maximum overflow rate was higher. The overall conclusion appears to be that even though the adaptive model with risk is highly dependent on the accuracy of the forecasting model, at least some stormflow anticipation will reduce maximum overflow rates.

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LIST OF SYMBOLS

Symbol	Definition
A	= Matrix of coefficients
A'	= Subsection of A matrix
a_{ij}	= Element of A matrix
$a_j(k)$	= Time varying linear coefficient for location j
a_m	= Routing coefficients
$\underline{b}$	= Vector of constraint values in LP
$\underline{b}'$	= Upper bound constraint values, subset of $\underline{b}$
B^M	= Back Shift Operator
$\underline{C}$	= Cost coefficient vector in LP
$\underline{C}$	= A general aggregation matrix
C_1, C_2, C_3	= Routing coefficients
E	= Statistical expectation
$\underline{e}_j$	= Unit vector in direction of j^{th} vector component
$f_i(\underline{x}_i, \underline{u}_i)$	= General DP objective function
$F_i(\underline{x}_i, \underline{u}_i)$	= DP optimal return function
$F(Q^i_{max})$	= General subbasin objective function value
$F(\underline{Q}^{(\ell)}_{max})$	= Total objective function value for $\underline{Q}^{(\ell)}_{max}$
$\underline{g}$	= Dynamic relation in DP
$\underline{h}_i(\underline{x}_i, \underline{u}_i)$	= Constraints on $\underline{x}_i$ and $\underline{u}_i$
I	= Identity matrix
i	= Subscript denoting location
j	= Subscript denoting location
J_S	= Parameter error cost index
$J(i)$	= Set of pertinent locations adjacent to location i
k	= Time period

Symbol	Definition
L	= Last or furthest downstream detention basin in a subbasin
M	= Number of time intervals
m	= Number of discretizations used in routing technique
N	= Number of detention basins in a subbasin
N_s	= Number of subbasin in system region s
n	= Exponent reflecting nonlinear impact of overflows
$O_j(k)$	= Overflow at detention basin j during time period k
$O^i(k)$	= Overflow at subbasin i during time period k
$\tilde{O}^{*i}(\underline{Q}^i_{max})$	= Optimal value of weighted subbasin releases
P_j	= Probability of j^{th} discrete value of predicted inflow
$\underline{P}_s$	= Prediction parameter correction vector
p	= Autoregressive model order
$Q^i(k)$	= Release from subbasin i during time period k
$[Q^i(k)]^j$	= Contribution of subbasin i to routed flows within interceptor, measured at location j during period k
$[Q_j(k)]'$	= Contribution of detention basin j to downstream routed flows within a subbasin
$\underline{Q}(k)$	= Vector representing all detention basin releases during time period k
$Q^i_{max}(k)$	= Assigned upper limit to throughflows for subbasin i during time period k
Q^i_{max}	= Assigned upper limit to throughflows for subbasin i for all time periods
$\underline{Q}^i_{max}$	= Vector representing $Q^i_{max}(k)$ for k=1,...,M
$\underline{Q}^{(\ell)}_{max}$	= Particular value of vector $\underline{Q}_{max}$ which represents Q^i_{max} for i=1,...,10
Q^{*s}_{total}	= Optimal total releases to treatment from system region s
$Q^{*i}(\underline{Q}^i_{max},k)$	= Optimal release for subbasin i during time period k given $\underline{Q}^i_{max}$

Symbol	Definition
$Q_U(t)$	= Upstream hydrograph function
$Q_D(t)$	= Downstream hydrograph function
Q_{Tmax}	= Total available treatment plant capacity
$\bar{Q}^i_{max}$	= Physical upper limit on releases from subbasin i
$\underline{q}^+,\underline{q}^-$	= Penalty vectors for negative or positive prediction errors respectively
q	= Moving average model order
$R^i(k)$	= Forecasted aggregated inflow to subbasin i during time period k
$R^{ai}(k)$	= *Real* inflow to subbasin i during time period k
$R_i(k)$	= Forecasted inflow to detention basin i during time period k
$\underline{R}$	= Complete vector of inflows for all locations and times
s	= Index indicating a particular region of the total system
$\underline{\hat{S}}^{(\ell)}$	= ℓ^{th} value of steepest descent direction $\underline{\hat{S}}$
t	= Time
T	= Matrix of coefficients representing system continuity (Chapter IV)
T	= Index representing the treatment plant location (Chapter III)
T	= Matrix transpose when seen as superscript in Chapter IV
$\underline{u}$	= General control variable in DP development
U	= Set of all controls $\underline{u}$
$w^i(k)$	= Overflow penalty for subbasin i during time period k
x	= General state variable in DP development
X	= Set of all state variables $\underline{x}$
$\underline{x}$	= Vector of variables
y^+,y^-	= Amount of negative or positive prediction error

Symbol	Definition
$y^i(k)$	= Throughflow credit for subbasin i during time period k
$Z(k)$	= Aggregated state variable during time period k
λ	= Distance traveled in direction $\hat{S}^{(\ell)}$
τ	= Number of time increments considered within routing models
Δ	= Change in value of following variable
$\psi(B)$	= Moving average (MA) model operator
$\phi(B)$	= Autoregressive (AR) model operator
$\Theta(B)$	= Transfer function model operator

Chapter I

INTRODUCTION

A. COMBINED SEWER AND STORMWATER POLLUTION CONTROL

The water pollution potential of combined sewer and stormwater overflows has been well documented [2,3]. Combined sewers, though originally considered an efficient solution to the problem of urban wastewater removal, present a dilemma to those faced with the implementation of the Federal Water Pollution Control Act Amendment of 1972, (PL 92-500) [1]. The *zero discharge* statement describing a national goal of the total elimination of wastewater discharges into navigable waters by 1985, as well as a *softened* interim goal of safe habitat for aquatic life and human recreation by 1983, has provided additional impetus for research into possible solutions. These amendments specifically set the goal of secondary treatment for *all* sewage effluent by 1977.

The implication of this is clear; cost-effective alternatives must be developed for the control and treatment of urban stormwater runoff, whether alone or combined with dry weather flow. Initially, the alternatives proposed to solve this problem included large-scale treatment of the combined effluent, sewer separation, or treatment combined with storage. The first alternative implies the design of treatment plants capable of handling the combined flows resulting from a large design storm such as a 50 or 100 year storm, but appears to be highly unrealistic from a technical viewpoint.

The alternatives to large-scale treatment have focused on two primary areas. The first concerns the improvement of the quality of the overflows. Sewer separation, bulk primary treatment of storm flows prior to overflow, and local storm flow treatment plants constitute the major thrust in this area. The second area deals with the improvement in the staging of flows to the treatment plant. Traditional flood control techniques which decrease peak flows by decreasing total runoff, in-line storage (ambient storage), and off-line storage (auxiliary storage) are the general techniques in this category. The separation of stormwater systems from sewage systems is a viable alternative only where the systems are not very extensive or, ideally, where they are still in the planning stage. The expense and urban disruption involved in converting a combined sewer system into two independent systems is normally prohibitive, as is the expense of having a multiplicity of treatment plants at all locations where sewer overflows are discharged into receiving waters.

McPherson [4] and Lager [5] have extensively discussed advantages of system controls using in-line and/or auxiliary storage, and the U.S. Environmental Protection Agency (EPA) has been supporting considerable research in this area [6]. The idea is to utilize storage in such a way as to temporarily detain the peak flows, while the computer based control phases their subsequent release in order to achieve the system objectives. This is illustrated in Figure I-1 in which the use of auxiliary storage is demonstrated for a single basin. McCuen [20] has discussed the effectiveness of auxiliary or detention storage when used with an *individual site* control strategy which regards the operation of each detention basin independently of any others, versus operation of an entire system of detention basins through a coordinated regional approach. He claims that the *individual site* operation can often lead

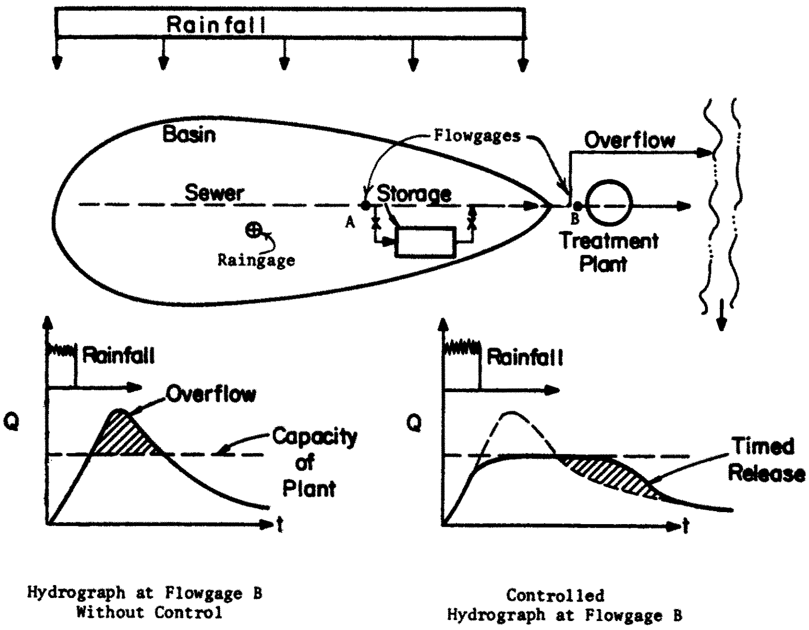

Figure I-1. Overflow Reduction with Auxiliary Storage [21].

to an increased flood threat due to the unintentional in-phase arrival of uncontrolled discharges and independently controlled releases.

Computer control, which appears necessary for a coordinated regional approach to the control of stormwater and combined sewer systems, is new to the field of urban stormwater management, as it is to many other areas of water resources management. As Labadie [7] states, "There is a critical need to take full advantage of current advances in computer technology (hardware and software) and systems engineering." Of primary importance for the establishment of such computer-based control systems is the programmed control logic. Such a control strategy should recognize the interdependence of the system controls in time and space and derive a control policy which achieves, to the highest possible degree, the overall system objectives. These overall system objectives are to: minimize the detrimental impacts of urban runoff on receiving waters, minimize local flooding due to the conveyance limitations of the system or, simply, to minimize the number of overflow events in order to avoid penalties. Without comprehensive strategies, the best the system can do is make decisions based only on local conditions at a particular point in time and space. Systems of this kind may be called reactive or *myopic*, thereby alluding to their short-sightedness.

The work described herein addresses these system objectives and sets sub-objectives which recognize unique features of the situation. These sub-objectives are to: (1) develop and test an algorithm oriented for real-time use which recognizes the interdependence in time and space of system controls; (2) deal with the complexity and large-scale nature of the control problem; (3) attempt to anticipate future behavior and effectively respond to it; and (4) deal with the stochastic nature of the problem. The real-time orientation of the desired algorithm affects the level of attainment possible for all the objectives. This limitation is further accentuated by an implicit constraint upon computer hardware available.

B. REACTIVE VERSUS ADAPTIVE CONTROLS

Adaptive control techniques, which learn from previous errors and thereby, hopefully, improve the subsequent control decision, may provide the needed advantage over reactive methods and enable the development of the desired comprehensive strategies. Current work in the area of computer control for combined sewer systems appears to be mostly reactive rather than adaptive. The Seattle "CATAD" control system, although essentially reactive, does use a simple *look ahead* approach which achieves an advantage over purely reactive systems [19]. This *look ahead* is a rudimentary forecasting of the system inflows in future time periods. Such forecasts are the essence of the adaptive control techniques described herein.

Myopic, set point, or purely reactive-type computer control systems may, in some cases, be an acceptable control approach when viewed from a cost-effectiveness perspective. An adaptive control technique, which uses more extensive forecasts of system inflows to generate control decisions, is illustrated, along with a reactive approach, in Figure I-2. An attempt has been made in this work to offer an initial comparison between these two approaches.

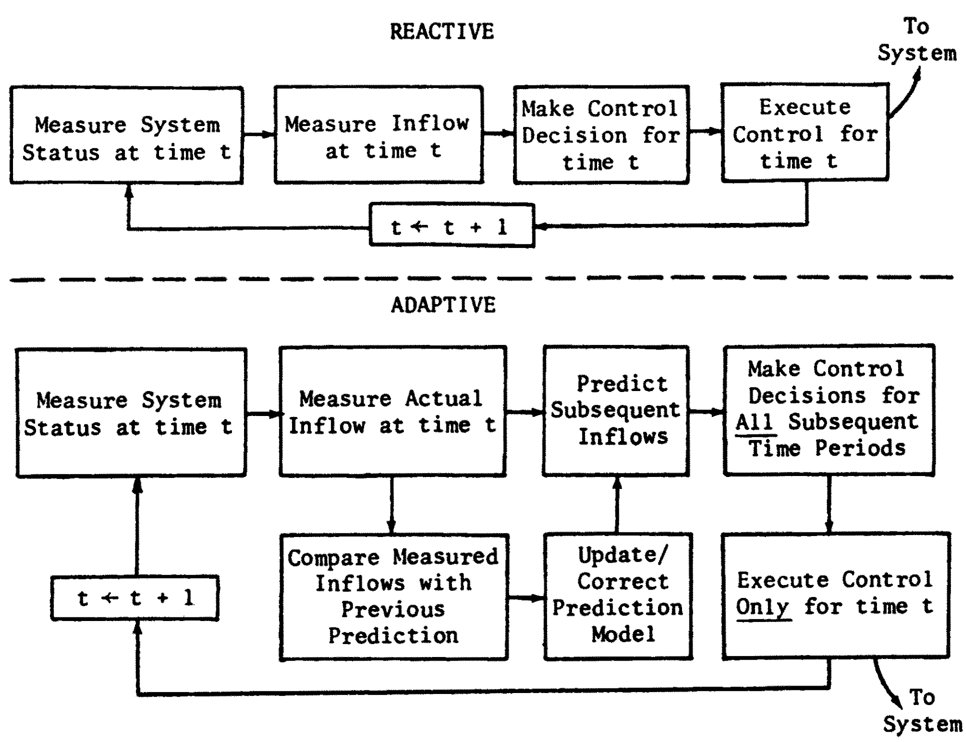

Figure I-2. Reactive Versus Adaptive Derivation of Controls.

System identification techniques, which have been used extensively within the theoretical electrical engineering discipline, can provide insight into the problem of utilizing the differential distribution in time and space of storm flows for the derivation of extensive forecasts. An optimization of controls based entirely on forecasts from averaged hydrologic data (typical depth-duration-frequency curves) will result in controls which cannot anticipate and take advantage of nonuniformities in the rainfall distribution. On-line control would in that case simply react to the current state, and not properly anticipate and take advantage of the characteristics of the event being experienced. System identification and signal theory reduce the problems inherent in the uncertainty of short increment small scale rainfall distribution by permitting the identification of a mathematical structure in the event being experienced, as well as using the historic data as the starting point for the forecast.

A real-time computer control system has been defined as that which "controls an environment by receiving data, processing them, and taking action or returning results sufficiently quick to affect the functioning of the environment at that time" [8]. Adaptive control, or updatable control that benefits from a learning process, can be assumed to be included in the data processing part of the above definition. The term *Management Information and Control Systems* (MICS) has been used to describe control systems associated with industrial and business problems. The term *Metropolitan Water Intelligence Systems* (MWIS) [4] has been coined to describe those MISC which are applicable to urban water systems.

Figure I-3 illustrates the application of a MWIS to the wastewater collection system of a city. The adaptive aspects of such a model would be included in three parts of such a configuration;

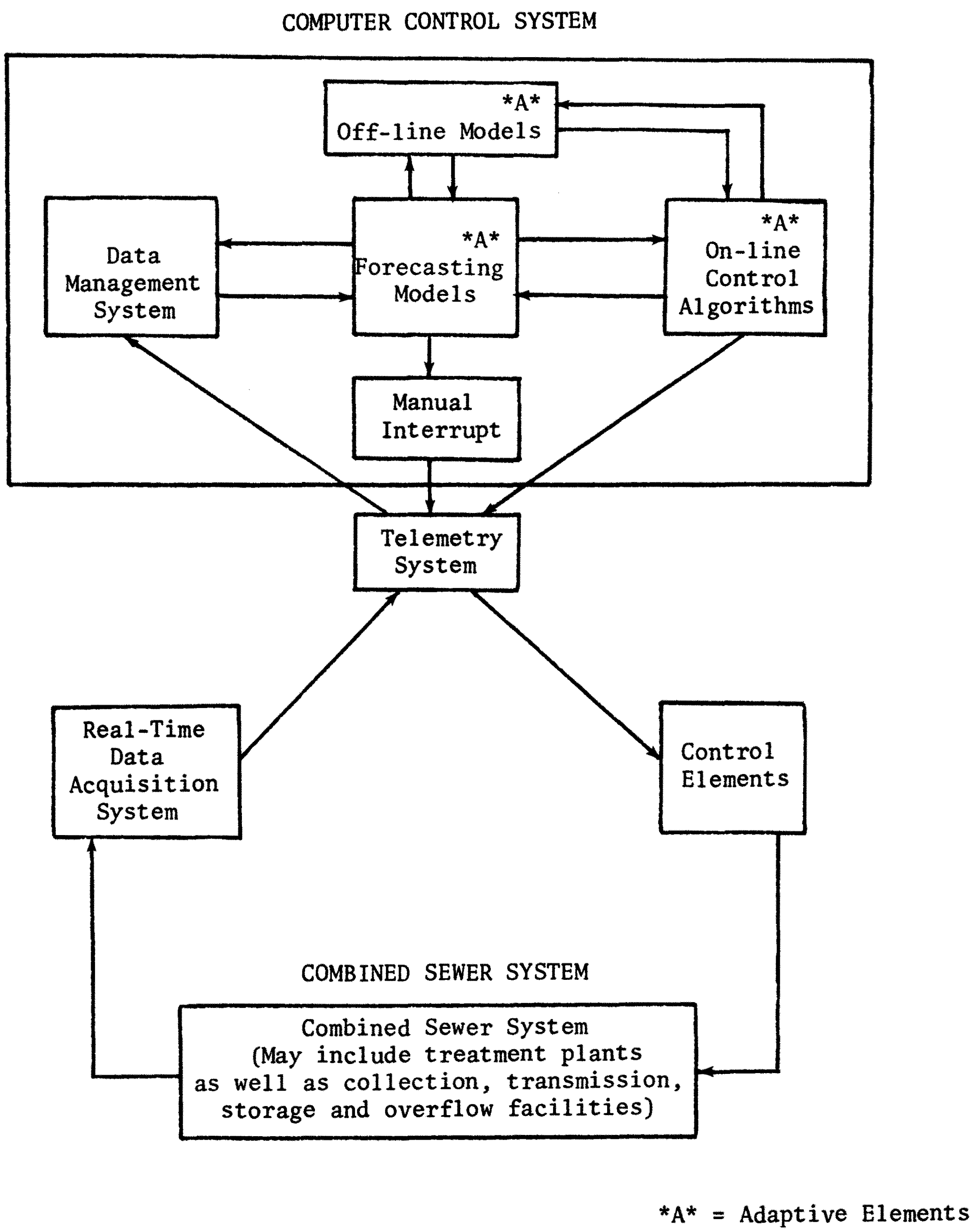

Figure I-3. Flow Diagram for Combined Sewer System in Automatic Control Mode [17].

off-line models, forecasting models, and on-line control algorithms. Long range adaptation would be incorporated into the off-line methods, which analyze past events in a computational framework independent of the physical system and not restricted by time. These models attempt to learn, in an iterative fashion, which can be updated after every event, exactly what forecasting model structure (based on historic data) is best for deriving the initial parameters of the on-line forecasting model. The on-line forecasting model learns from and adapts to the features of the event on hand. Furthermore, the on-line control algorithms, which iteratively calculate the optimal controls, respond to the hopefully improving forecast and converge, as a storm progresses, to the best controls that can be derived on-line in real-time during the limited time available for reaching control decisions.

Real-time control of a system permits changes in decision policies up to and until the control is effected. As more information becomes available, the optimum policy may change. Iterative optimization as a component of adaptive control provides a means of learning the appropriate optimal control for the particular storm being experienced. The storm and resultant system inflows are, however, never fully known until the entire event passes. Consequently, the uncertainty (actually, a better word is *risk*) associated with forecasting, although hopefully decreasing as the event passes, must be incorporated into the optimization, along with the consideration of the risk implicit in the various control policies. It is the problem of on-line adaptive control as compared to reactive control, of the storage capability within a large combined sewer system, to which this report is addressed.

C. SAN FRANCISCO MASTER PLAN AS A CASE STUDY

Bradford [9] has summarized the stormwater projects underway in the U.S. which utilize either on-line or auxiliary storage. Table I-1 compares the current status of these projects and indicates where automatic computer control is anticipated.

Table I-1. Some Cities Considering Combined Sewer Storage Control Systems [9].

City	Type of Storage	Representative Reference
Minneapolis-St. Paul	On-line	[10]*
Detroit	On-line	[11,12]*
Seattle	On-line	[2,13,14)*
San Francisco	Auxiliary (0.10-0.63 inches of runoff in many small underground reservoirs)	[2,15]*
Cleveland	On-line	[18]*
Chicago	Auxiliary (3.14 inches of runoff in three large basins)	[2]
Washington, D.C.	Auxiliary (4.31 inches of runoff in deep tunnel storage, or 1.58 inches at Kingman Lake)	[2]

*Literature explicitly states that computer control is actively being considered.

In particular, the San Francisco Master Plan for Wastewater Management is unique in many respects. The City's often steep topography required a plan which recognized that the resultant short system response time would demand rapid decisions for flow and storage control. The Master Plan calls for a large number of detention basins with an amount of total storage that is relatively low when compared to the systems considered for Chicago and Washington, D.C., as described in the table above. The factors of short system response and low total storage capacity result in the need for effective and efficient control of this proposed highly complex system. It is this proposed system which has been selected as the case study used here.

There are a number of reasons for basing the research reported herein on an actual case study such as the San Francisco Master Plan. The principal ones are as follows, as reported in reference [17]:

"1. A wealth of real data and sophisticated analysis of the system is available.

2. The San Francisco DPW (Department of Public Works) has an innovative automatic rainfall-runoff data collection facility in operation.

3. The San Francisco physical system breaks neatly into total city or subbasin packages.

4. Environmental constraints on the Plan are extremely stringent. (Although derived from poorly defined environmental criteria and objectives.)

5. Public acceptance is an extremely critical factor in San Francisco.

6. The developers of the Master Plan have thoroughly thought out the details of planning, designing, and operating their conceived system and are able to react to suggestions and questions about control strategy."

Each of these factors motivated this research in positive directions. The first two, relating to data availability, has enabled the research to be based from the outset in a real world system description. In particular, actual rainfall-runoff records from significant past events which resulted in flooding and overflows were available. This facilitated the evaluation of the control strategies since these actual events could be used as test cases.

The subbasin packages referred to in point No. 3 above suggest the development of control strategies for portions of the city, culminating in the eventual integration of these strategies for the entire city.

The intuitive appeal of decentralized or decomposed solution techniques, which are suggested by the system configuration proposed in the San Francisco Master Plan for Wastewater Management, increases the potential for acceptance of automated system control; thus, interaction between researcher and project management is encouraged. The factors of environmental constraints and public acceptance have had a direct effect upon the entire development since they both have an impact on the performance criteria needed for the system's evaluation. Defined environmental constraints, rather than the elusive goal of defining economic efficiency, has permitted the solution of the problem independently of nebulous environmental economics. Public acceptance demands the achievement of the environmental goals not only at the minimum cost, but also with minimum total urban disruption.

For a comprehensive report on the proposed San Francisco system, the reader is directed to reference [18], while reference [9] contains a brief overview of the system in its Appendix.

The San Francisco Master Plan presents several alternatives, consisting of various sizes and locations of facilities as well as different plans for staging construction. Alternative C was chosen for study since it contained all the complexity of the largest alternative D, with only 63 percent of the storage capacity of that alternative. It therefore posed a greater challenge, its solution being readily adaptable to lesser plans as well as the larger alternative. Also, since the objectives of this study parallel and extend the work of Bradford [9], the system description used in that study was adapted for this. The Alternative C system is presented in Figure I-4.

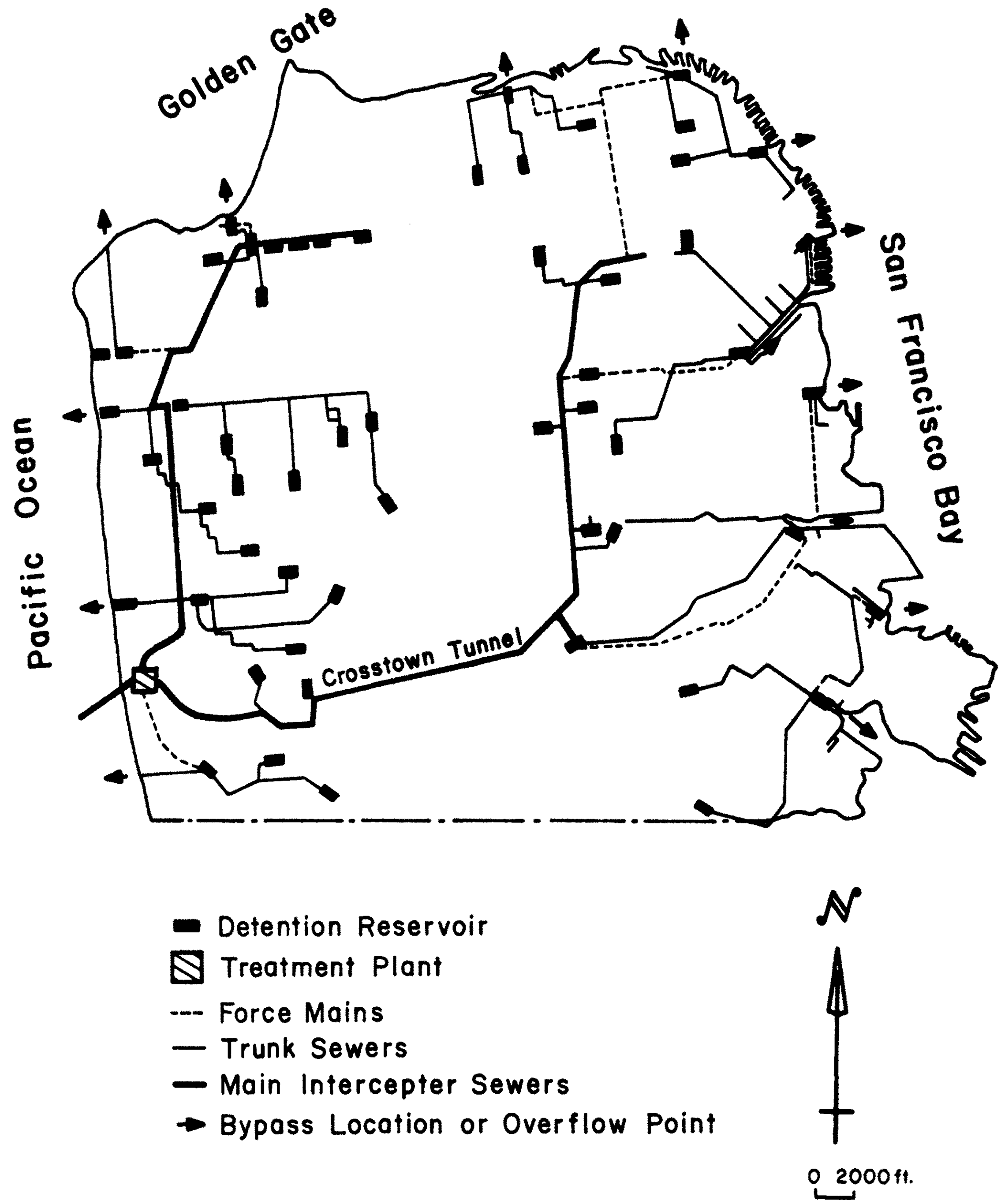

Figure I-4. Proposed San Francisco Combined Sewer System [18].

D. OBJECTIVES

The proper real-time control of the diversions into and out of auxiliary storage within a combined sewer system during storm occurrences over urban areas is the point of concern here. The overall system objectives include:

(1) Minimization of the effect of overflows of untreated sewage into receiving waters,

(2) Elimination or reduction of localized street flooding, and

(3) Full and efficient utilization of the existing and planned conveyance and treatment facilities.

Management of the control system designed to meet these objectives must be carried out within the physical, environmental, economic, and socio-political constraints imposed by the larger encompassing system; namely, the entire urban center. The amount of computer hardware which a city can dedicate to the on-line, real-time control of a combined sewer system is limited. The sophistication of the computer control is consequently constrained by the realities of the urban budget. Recognizing this limitation and working within it has been a basic theme of the development presented herein.

In this study, the real-time control problem is formulated as an optimization problem. Since storm forecasting is not considered to be deterministic, the optimization is carried out under risk. Within the optimization problem the general system goals listed above are represented mathematically in terms of an objective function. The physical constraints of the system, such as the capacities of the treatment plant, the sewer lines, and the detention reservoirs, as well as the laws of physics such as conservation of mass, are modeled explicitly as mathematical constraints. Environmental constraints are, however, handled in an indirect manner due to a lack of definitive models for predicting wastewater quality and its impact on receiving waters. Economic and socio-political constraints, although of great importance within the design phase of the total control system, are of lesser importance in the operational phase since their impact would (hopefully) have already been incorporated in the selection of alternatives for consideration. Once the alternative is selected, system control or operation can only recognize these constraints to the extent that they can be represented as operational guidelines. For example, discouraging the possible adverse hydraulic impacts of repeated and abrupt gate or valve position changes would be an example of an operational guideline reflecting an economic constraint on system maintenance.

It should be noted that the San Francisco Master Plan for Wastewater Management is still a plan, subject to update, review, and design changes. Changes in the political, economic, and environmental climates have potential impacts upon the finalized approved plan of action. It is, therefore, essential that the control strategies developed be flexible and capable of model configuration alterations and parameter changes. Such flexibility in control strategy algorithms would provide an additional tool for the system planners who could then evaluate the actual expected performance of various alternatives and design changes.

There are several factors which contribute to the difficulty of meeting total system objectives. First, the system is large-scale, consisting of many components related in a complex network with controls needed for each component at each point of time. Second, modeling the rainfall, runoff, storage, and transport processes is difficult, particularly when attempting to integrate them into optimal control strategy developments. Third, as the desired system is to operate in real time, the control system response must be rapid. Finally and possibly most important, is the factor of the stochastic nature of the storm flow input.

With these factors in mind, the objectives of this research study were to:

(1) Derive a methodology for achieving control of the flows within a combined sewer system which has controllable storage facilities, in view of the stochastic nature of the stormwater inflows.

(2) Develop a control strategy capable of being implemented in an on-line, real-time mode and one which adapts to an evolving storm pattern by reacting to current data and anticipating future inflows, and compare with reactive-type control.

(3) Develop an overall control strategy integration for the large-scale, city-wide system which permits iterative adjustment of system components to incoming data, where the component solutions taken together will convergently achieve an overall, city-wide optimal control.

(4) Derive control strategies which meet the overall system objectives of minimizing overflows and street flooding while maximizing the use of existent facilities for a wide range of probable events.

As mentioned previously, the San Francisco Combined Sewer System is used here as a case study though there is considerable potential for application of these techniques and methodologies to other cities.

E. SUMMARY OF CHAPTERS

The control of discharge from combined sewer systems is a problem of great concern in many large cities. Recent legislation has created additional pressure for the solution of this problem. The use of auxiliary storage has been suggested as a viable solution. The operation of complex systems of auxiliary storage facilities present a challenge to those who operate these auxiliary storage facilities in a manner which best meets the designed purpose. Computer based control algorithms are suggested which adapt to the evolving situation by attempting to anticipate, with the use of a constantly evolving model, the future inflows to the system. Such techniques may overcome some of the problems inherent in myopic strategies. A forecast, however, involves risk and the control algorithm must consider this factor.

The San Francisco Master Plan for Wastewater Management provides an opportunity to develop such adaptive control strategies. The system is complex but fully described with an adequate supply of data for realistic demonstrations. The objectives of this work include the development of adaptive control strategies which use large-scale analysis techniques incorporating risk. These objectives directly address the system goal of minimizing overflows and local flooding.

Chapter II of this work reviews the literature relating to the component areas of adaptive on-line control of combined sewer systems. These areas include: rainfall simulation and forecasting, stochastic optimization, and adaptive on-line control. The literature evidences the convergence of these areas and suggests that this work is in fact timely.

The large-scale problem is posed in Chapter III in which decomposition is chosen as the most worthwhile approach for addressing this large complex problem. The master control problem linking the separate decomposed subproblems is developed and an efficient technique is found for its solution in light of an anticipated subproblem solution strategy.

Chapter IV continues the general development of the subproblems and various deterministic solution strategies are proposed. These subproblem solutions assume a forecasted inflow sequence and solve the control problem without considering the risk of error.

The risk involved in a forecasted inflow sequence is subsequently addressed in this same chapter. Two stochastic optimization techniques are presented for use as subproblem solution strategies. A comparison is then made between the performance of a deterministically derived control policy and a stochastically derived control policy for an example system experiencing equally likely variants of a forecasted event. The stochastically derived policies were seen to be superior by their meeting the objectives to a higher degree, for an illustrative experiment that was developed.

Chapter V describes the adaptive forecast models used for deriving the storm inflow sequences needed. System identification techniques are incorporated for the estimation of parameters needed in the autoregressive-transfer function models used in the forecast.

The entire on-line, real-time process is demonstrated in Chapter VI, where the adaptive forecast model, master control problem, and subproblem algorithms are integrated. Hypothetical events are used for the model identification phase while a historically based event is used as the *real event*. Controls are thus derived for the entire large-scale system by artificially simulating the passage of real-time on the computer. The total algorithm was exercised four times to permit comparisons between system performance for four different approaches to detention basin operation. These approaches ranged from a purely myopic approach, to a stochastic treatment of data forecasted for an entire event. The results indicated that a stochastic optimization over forecasted values was worthwhile for control of the distribution in time and space of system overflows but may not be worthwhile if total overflow reduction is the sole criterion for system performance.

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Chapter II

REVIEW OF LITERATURE

A. INTRODUCTION

The subject of adaptive on-line control of a combined sewer system has many facets and consequently, many areas for literature survey. Some of these are:

1. Control of linked storage facilities.
2. Adaptive control systems.
3. Rainfall forecasting.

The literature associated with the first two areas has been reviewed by Labadie [1], Bradford [2], and Grigg, *et al*., [3], with emphasis on the intersection of these areas with the problem of computer control of combined sewer systems. The use of mathematical programming techniques for solving discrete time formulations within the entire large-scale optimization framework, rather than the application of optimal control theory (maximum principle of Pontryagin) to discrete and continuous time formulations, is suggested in much of this work for a number of reasons. Continuous time formulations are those in which the desired control policy is expressed as a continuous function of time. The system control policy would result from an analysis of: a continuous inflow function, continuity relationships expressed as differential equations, and the system objective which (in this case) would be expressed as an integral. Such formulations have been approached by the direct application of optimal control theory [1]. They have proven, however, to be difficult to solve for control problems of the type considered here [2]. Their application to on-line work seems to be still in the future, although recent efforts by

Bell [57] and Winn and Moore [58] show promise. In addition, the inherent inaccuracies of the system considered herein obviate the advantage of exact continuous solutions. The discrete form of the forecasting models to be developed in a following chapter provide a further rationale for the use of discrete time formulations. Discrete time optimal control theory has been applied by Chan [54], and is discussed later. However, the large-scale approach developed here, despite its reliance on discrete time formulations, could incorporate any viable solution strategy within its structure.

Much of the literature reviewed fails to address the full stochastic nature of the problem, as well as prospects for computer control in an adaptive mode. The stochastic nature of the problem arises from the random distribution in time and space of rainfall and consequent inflows to the combined sewer system. An adaptive capability within a computer control system may enable the on-line control algorithms to dynamically respond to the evolving storm situation by learning from the storm history in real-time.

Both of these features (small-scale spatial and temporal rainfall distributions and adaptive control) come together to the extent that there is a discernible structure associated with the stochastic nature of storm activity. The degree to which the rainfall producing system is understood, both from historic events and information gathered during a particular event, is the extent to which an adaptive learning process or system identification can lead to accurate forecasting. A brief review of the literature relating to the modeling of the internal structure of rainfall events (purely statistical or statistical-conceptual models) is, therefore, worthwhile. This is followed by a review of the state-of-the-art in finding operational strategies for linked reservoir systems where the stochastic nature of the system inflows is considered. Finally, a review of vanguard efforts at effecting full adaptive control for certain water resource systems is presented.

B. RAINFALL MODELING

Research concerned with synthetic generation of the distribution in time and space of small-scale rainfall has evolved dramatically in the past few years. This evolution has taken place in the separate areas of purely statistical modeling and physical process modeling, as well as combinations of these. Purely statistical modeling includes those attempts to statistically simulate the occurrence and distribution of rainfall accumulations on the ground with no regard for the physical characteristics of the phenomena. Physical process modeling, however, attempts to model known relationships describing the mechanisms within rainfall generating events. The combination of these research directions has resulted in the complete stochastic modeling of the three major types of rain storms that exhibit the rapidly varying properties which have frustrated attempts to predict runoff from small urban basins. These three rain storm types are: the squall line resulting from the passage of a cold front; shower activity found in the prewarm frontal area (with these latter two associated with cyclonic activity); as well as the air mass thunderstorm.

Two major obstacles have hampered purely statistical approaches to modeling small-scale and short time increment rainfall. The first is the dramatic change in statistical properties which occur if the time increment considered drops below one hour. Le Cam [4], Pattison

[5], and Grace and Eagleson [6] have demonstrated the complex nature of persistence phenomena found in short time increment rainfall. Their models, relying on Markov chains, have encountered difficulties in simulating both the internal structure of an event and time between events simultaneously. This problem has been addressed by breaking it into various *independent* parts (i.e., time between storms and variation of point rainfall within an event). These parts, modeled separately using Markov chains or Monte Carlo techniques, have enabled many of the problems associated with simulating point rainfall variability to be partially overcome. Time increments in the range of ten minutes were, however, found to be unsuitable for modeling by Markov chains. Thus, other techniques were suggested to simulate serial correlations.

The second major obstacle to statistical modelers has been the simulation of small-scale spatial variability of point rainfall. Wilkinson and Tavares [7] have described the difficulties of trying to use Markov-type models to describe point rainfall variability for more than one location at a time, while properly maintaining appropriate cross correlations. They have proposed the use of a Monte Carlo-type simulation constrained by suitable descriptions of cross and serial correlations of storm parameters. Since the number of correlations needed equals the number of possible combinations of storm parameters, their model was limited to only three statistical descriptors of a storm for each point chosen. This limited description of point rainfall performed adequately for the longer time increments and larger spacings of the three gages found on the modeled river reservoir network. It is doubtful, however, that it would suffice for more densely spaced gages of an urban raingage network with sampling carried out in short increments of time. This is because the number of correlation terms increases rapidly with each additional location and time period desired, as well as with the number of needed statistical descriptors.

Within the past ten years, various researchers have begun to incorporate the growing body of knowledge concerning the atmospheric processes associated with rain storm activity. Building on the fundamental work of the Thunderstorm Project [8] a variety of researchers have identified a multitude of statistical descriptors for distributed rain cell parameters. The rain cell, which is the basic source of erratic rainfall patterns, can be simulated via its orientation, size, growth, and decay cycle, as well as the internal distribution of intensity along its axes.

Sorman and Wallace [9] have developed a model which uses eight statistically based descriptors of rain cell activity in which cells are generated, grow, decay, move (relative to the wind direction), and contribute definable distributed rain intensities. Coordinate frames which move with each major cell sequentially, as well as a stationary frame which relates meteorological activity to raingages on the ground, are utilized. This model has adequately simulated the internal spatial and temporal variability of thunderstorms. It uses assumed probability distributions of the relevant parameters, as well as regression relations with an added *white noise* random component.

Grayman and Eagleson [10] have adopted a more simplified approach to modeling rain cell movement and activity than in the above study. They have designed a model consisting of squares representing distinct discrete levels of more intense rain cell activity. These are in turn nested within larger squares associated with less intense levels of rain. Simple multiples of discrete activity levels are used to approximate the intensity of activities in lower level squares selected by probabilistic switches. This simplified model is capable of simulating the band-like nature of fronts and squall lines which were not considered by Sorman and Wallace. The model also uses a moving frame of reference, though it moves with the entire storm rather than a particular cell. The various levels of activities are probabilistically *switched* on and off according to the assumed distributions and correlations. The passage of a cell is therefore represented as a square wave with discrete steps simulating the rainfall distribution.

More recently, the work of Wu [11] has also simulated the band structure associated with large areas of less intense rainfall in a storm front, as well as the clusters of short-lived rain cells that make up the cyclonic storm systems typical in northern California. This model gives a more accurate description of rainfall intensity variation in a cell than the square wave effects of Grayman and Eagleson's model. It also overcomes the computational disadvantage of the Sorman and Wallace model which sequentially follows one cell at a time. The model developed by Wu uses continuous distributions of rainfall intensities within each cell. These cells are generated, live, and die independent of other cells. In addition, these randomly generated cells can overlap. This creates a full range of intensity patterns which simulate the varied shapes and intensity distributions actually encountered in real data.

Each of the three major modeling attempts described above are simply illustrated in Figure II-1.

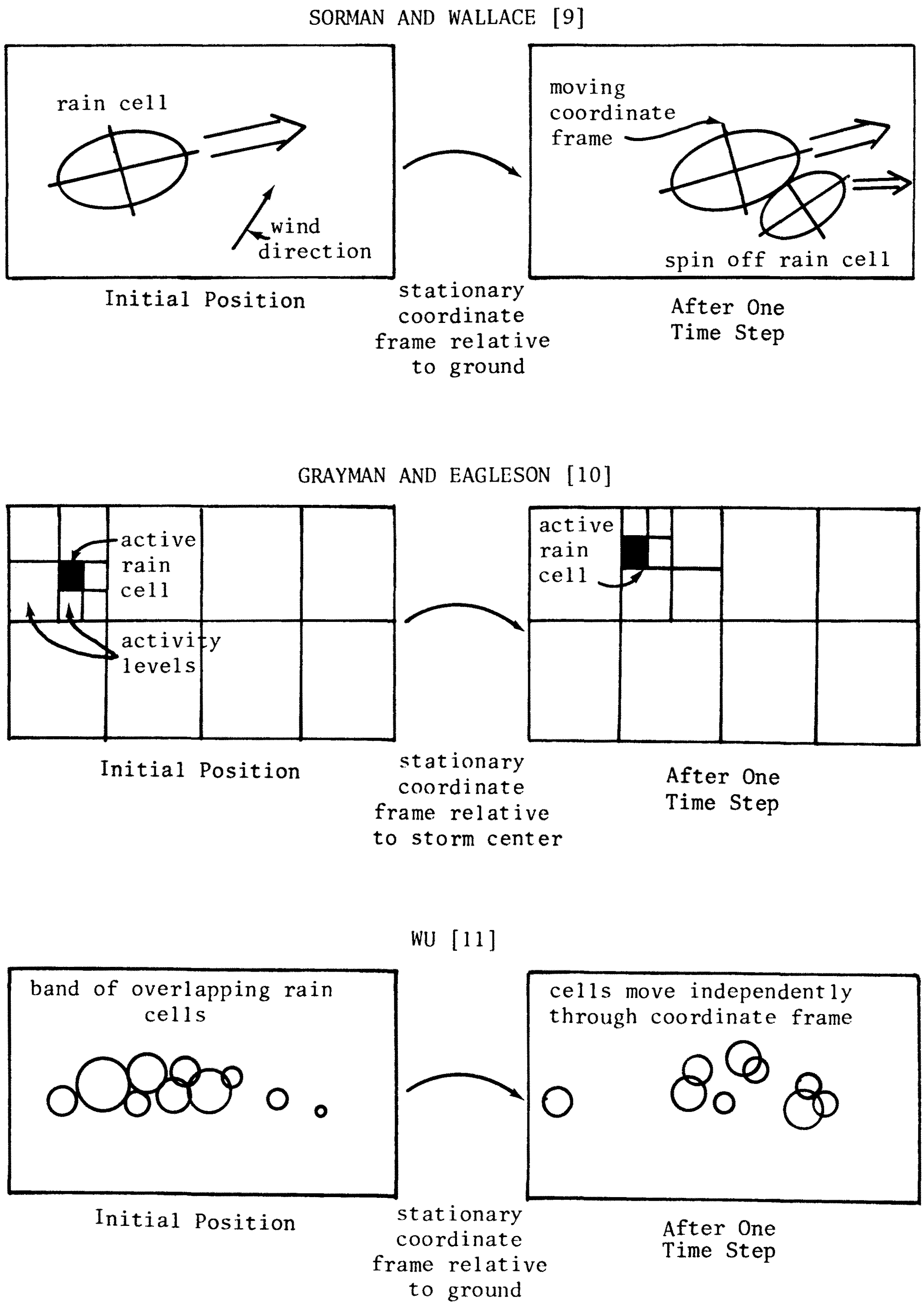

Figure II-1. Rainfall Simulation Models.

These simulation models, although capable of generating storm patterns based on a history of past events, seem to be unwieldy for use as on-line forecasting models. Such on-line models would *learn* the patterns of a particular event in real-time and project it forward. Learning models extrapolate known data into the future rather than using data solely as initial conditions in a random generation of equally likely events. In other words, for real-time control it is not desired to generate an equally likely event, but rather to forecast the actual event taking place. Statistical properties of the particular event on hand must be uncovered in real-time and used within a model capable of some degree of forecasting.

These models are useful for synthetically augmenting inadequate historical data so as to develop an initial predictive model. Their practical application, however, remains primarily in planning studies since the data requirement for modeling physical parameters of rain cells is quite large.

C. RESERVOIR CONTROL WITH STOCHASTIC INPUTS

C.1 Background

Operating policies for complex systems of reservoirs have been derived by intuition, logic and/or experience, and tested on line, or by historic or synthetic data for several years. Simulation, classical statistical approaches, and more recently Bayesian decision theory have been applied to the problem of determining optimal operating policies while considering the uncertainties involved in reservoir operation. The work of Russell [12] is notable for its attempt at integrating Bayesian decision theory into a reservoir operation algorithm. There has, however, been a recent trend toward utilizing mathematical programming techniques for the development of needed operating policies. This is particularly true for large, complex systems. The problem of control for reservoir systems on rivers is similar to the control problem presented by complex networks of detention basins for urban stormwater management. The former, however, are usually multipurpose, whereas the latter are single purpose. A brief review of recent developments in the field of reservoir control through use of mathematical programming is therefore worthwhile.

Mathematical programming techniques for reservoir control have proceeded through three phases of evolution. The first phase involved the derivation of optimal controls for a complex system with assumed inflows and demands. The second phase considered the stochastic nature of the inflows and demands, but often ignored interrelationships in time and space between them. And finally, the most recent efforts comprising the third phase have attempted to consider serial and cross correlations present in time series of random variables. Figure II-2 illustrates these three phases.

The goals of many of the examples to be cited here are different from the goals of this work. They can, however, provide insights contributing to the solution of the problem considered here. For example, some reservoir operation planning studies may seem irrelevant to the desired on-line real-time application considered herein. The mathematical modeling techniques embodied in the development may, however, be applicable. The applicability of a related work in general depends upon various factors. The computational complexity and problem size potentially resulting from a technique's application

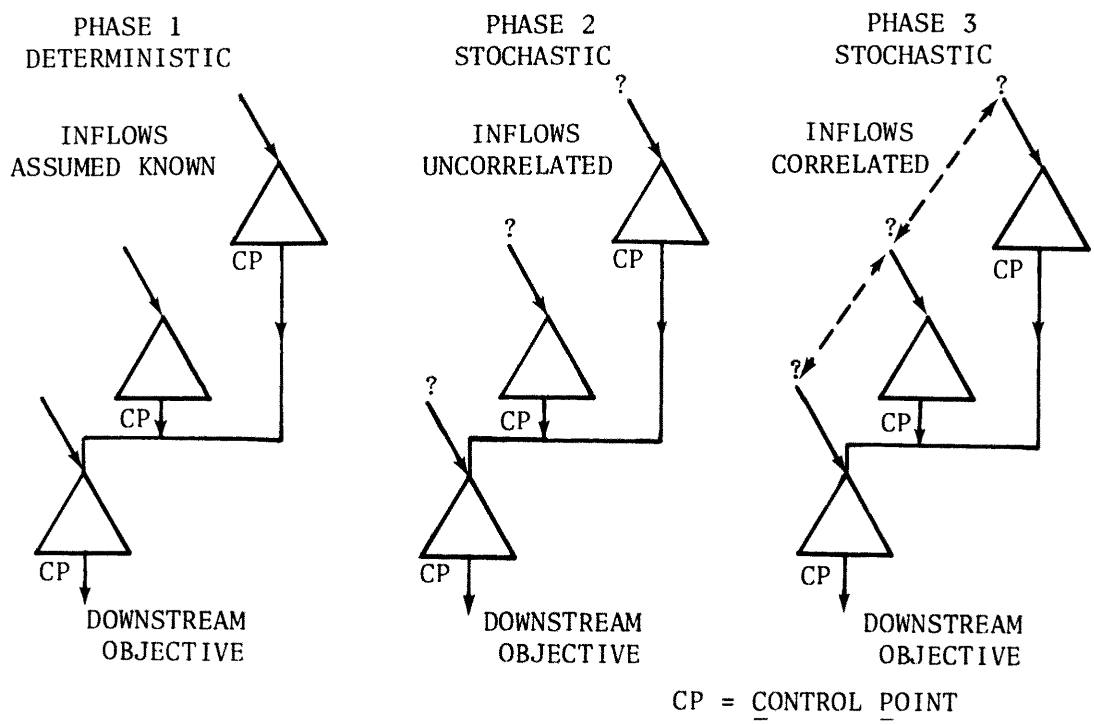

Figure II-2. Three Phases of Reservoir Control Modeling

is of primary importance. Off-line work in which there is no real-time physical interfacing with the system to be operated can utilize techniques too time consuming or computationally involved for consideration in an on-line real-time framework. Similarly, long range, or steady state operating policies are of little direct value here. Long range techniques typically derive controls independent of others far removed in time. In the situation considered here, all controls in the limited number of time intervals are highly related. Steady state controls obviate the entire motivation of real-time control, designed to respond to a changing system impact.

The situation modeled here is complex and multi-faceted, embodying large-scale analysis, stochastic optimization, adaptive forecasting, and hydraulic considerations. A technique seemingly irrelevant because of an assumption or limitation in one area may provide the key insight for another. For example, a planning study for the derivation of long range system controls may provide insight into the problem of maintaining the serial and cross correlative structure in the random system inflows. The related literature is reviewed, therefore, in light of these various factors.

The first phase of these efforts (the deterministic case) is well documented and references cited at the beginning of this chapter refer to many of these purely deterministic techniques. Texts on water resources systems engineering, such as Hall and Dracup [13], introduce linear and dynamic programming techniques available for reducing large-scale deterministic problems into a number of smaller deterministic problems linked together by a master problem are also discussed.

The relative merits and difficulties of using linear programming or dynamic programming for linked reservoir problems have been discussed by Butcher and Fordham [14]. They also mention the less profitable attempts at using steepest ascent methods. Becker and Yeh [15] have recently proposed an approach based on the conjunctive use of dynamic programming and linear programming for efficiently solving the multiple-reservoir deterministic problem. Dynamic programming is used for the selection of an optimal storage policy path through a specified number of policy periods and linear programming is used for the determination of intermediate policy decisions within each period.

The real challenge to researchers in the area of reservoir control, however, has been the inclusion of adequate representations of the stochastic nature of the system inflows. The methods associated with the last two phases of reservoir control modeling (which recognize the stochastic aspects of the problem) can be categorized into explicit and implicit techniques. Explicit techniques are those stochastic techniques which use probability distributions within the optimization, while implicit techniques are those which use some form of sampling from the distributions (Roefs [16]). The essential differences between these techniques is illustrated in Figure II-3.

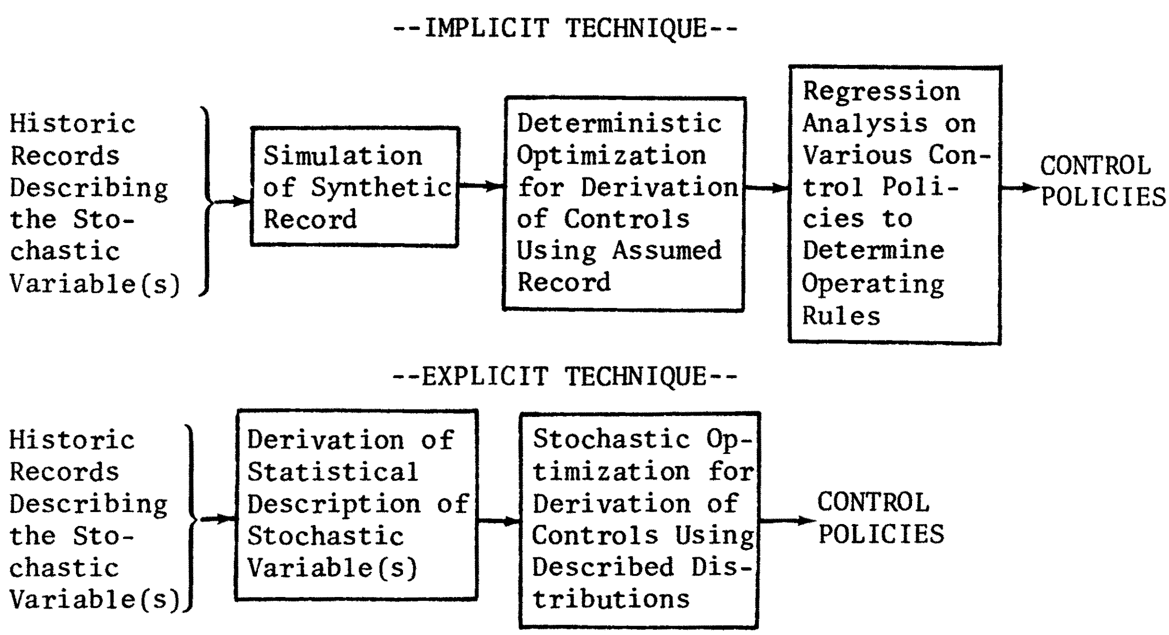

Figure II-3. Implicit Versus Explicit Stochastic Programming

C.2 Implicit Techniques

Implicit techniques are characterized by methodologies such as Monte Carlo Dynamic Programming; first suggested by Hall and Howell [17] and explored further by Young [18]. In this approach, dynamic programming techniques are used to deterministically derive controls for each of a series of simulated inflow sequences. The solutions are then analyzed by regression techniques in order to establish causal factors influencing the optimal policy. This technique permits the use of complex probability distributions of inflows at each point in time and the Monte Carlo type sampling incorporating conditional probability distributions permits the re-creation of the relevant correlations between the elements of the inflow time series. These serial correlations may, thereby, extend beyond the simple one time period correlation implied by often used Markovian relations in which an event in a series is linked to only its predecessor.

Askew, et al., [19] have indicated that Monte Carlo Dynamic Programming techniques are equally applicable to multipurpose multi-reservoir systems. The excessive time required to simulate a sufficiently large set of samples, however, seems only to be justified for off-line planning studies and would not be feasible for use within an on-line system. Regression results could, however, be used for developing on-line control strategies based on rule-curve type algorithms.

An implicit solution to a multireservoir problem which also employs the three steps of streamflow synthesis, deterministic optimization, and regression analysis, has been attempted by Roefs and Bodin [20]. Their work differs from the above approaches, however, in that the solution of the problem was attempted using linear programming. The resulting large-scale linear programming problem was partially solved by Dantzig-Wolfe decomposition. It appears that the work was not completed due to computational intransigence. Fordham [21], taking a similar approach, first summarized the logical development which led from attempts at explicit solutions by stochastic dynamic programming through implicit solutions using Monte Carlo Dynamic Programming. He ultimately rejected dynamic programming all together in favor of a problem framed as a network flow linear programming problem and solved by the *out of kilter* algorithm.

A real-time technique in which forecasts are repeatedly updated using current hydrometeorological data as starting points for simulation was developed by Windsor [22]. The ultimately implemented control policies were a combination of the various control policies derived from each simulation. The optimizations employed linear programming modifications consisting of separable programming and mixed integer programming. Although the simulation is repeated at every decision point, parameters in the simulation remain unchanged. The evolving storm event is, therefore, not capable of influencing the simulation model, and hence the forecast, until the entire event passes.

The above implicit techniques appear to be usable in either of two modes. They can be used for the derivation of steady state control policies, or control policies dependent upon local conditions. For the problem considered herein, the second use is of greater value than the first. Techniques which can adapt to the evolving storm event by iteratively forecasting the inflows are, however, desired for the real-time application considered here.

Croley [23] has developed a modified implicit stochastic optimization technique which can include a degree of this desired adaptation. In this approach, control policies are derived deterministically for simulated sequences. The mean or mode decision for a particular stage or time period is then implemented and the distributions for the random variables updated based upon new available data.

This process repeats itself for the next decision point or time period. This form of repetitive implicit solution strategy is somewhat similar to that of Windsor, though both appear too time consuming for on-line use in systems with short (e.g., minutes) control intervals.

C.3 Explicit Techniques

Explicit techniques, which incorporate probability distributions directly into the optimization, typically use linear programming or dynamic programming as their optimization tools. The orientation of these techniques toward either the derivation of steady state or short range policies determines (as was the case with the implicit techniques) their relevance for on-line real-time control.

C.3.1 Stochastic Linear Programming. There are basically three stochastic linear programming approaches. The method developed by Loucks [26], and Linear Programming Under Uncertainty [40], have been compared and tested by Butcher and Fordham [14] for certain water resources applications. The third method, Chance Constrained Techniques, overcome, to some extent, the extensive computational requirements of the first two approaches. These stochastic linear programming techniques recognize the random aspects of the system inflows, but usually require them to be independent or mildly correlated. They appear to be infeasible for deriving multireservoir operating policies in real time with limited computer hardware since the number of variables and constraints tends to become excessive.

Thomas and Watermeyer [24] were among the first to apply stochastic linear programming to a short range reservoir control problem. Since they used coarse discretizations for the state variable in order to limit the problem size, the usefulness of the solution has been questioned. Dietrich and Loucks [25] developed a stochastic linear programming model which included the serial correlation of the inflows for deriving the optimal control policies for a single multipurpose reservoir. Later, Loucks [26] continued the effort. Operating policies were sought which tended to increase the probabilities of the resultant state being close to the optimum state at that time as defined by the system objectives.

Explicit techniques for the derivation of other than steady state controls seem to begin with the work of Danzig [40] who derived a multistage formulation for a stochastic linear programming model. Research has followed attempting solutions for a simpler form of these multistage programming problems. Linear programming under uncertainty (LPUU), as described by Wets [41], solves such a two stage problem. The first stage problem is a simplified problem without the stochastic constraints, while the second stage deals with the uncertainty. The objective of the two stage problem is to find a vector of decisions which minimizes the first stage costs, plus the expected penalty costs related to the probabilities of constraint violation in the second stage problem. This type of solution strategy has the advantage of using standard linear programming codes as subroutines while considering stochastic aspects of the problem. Correlation structure between the random variables is lost, however, unless it is so strong as to permit a reduction in possible combinations of random variables.

Chance constrained programming (CCP), originally conceived by Charnes, et al. [42], is similar to LPUU in that stochastic constraints can be violated, but differs in that no penalty for the violation is assigned in the objective function. Instead, the probability of a particular level of violation is defined. For problems with more than one random variable, the joint probability (*not* the conditional probability) of all the violations is used as the measure of risk. In general, its advantage in water resource systems analysis is that penalty functions for constraint violations need not be explicitly defined. In some cases, where demand happens to be the stochastic variable, this could be a disadvantage. The CCP formulation, however, need be no larger than the corresponding deterministic case. Also, most CCP techniques become excessively large if serial (or cross) correlations between random variables exist and must be incorporated in the optimization model.

The problem of maintaining serial and/or cross correlations in the random variables has been discussed by Revell et al., [43], Revell and Kirby [44], and Loucks [45]. Their work, however, does not directly address the problem of defining actual conditional probabilities associated with particular levels of constraint violation. They simply maintain that the computational advantage of using decision rules will facilitate incorporation of correlations when expressed as conditional constraints. The linear decision rules suggested by these researchers enable the determination of the optimal control only at that point in time for which the control is needed. The actual control or quantity to be released in the n^{th} period is not chosen until the random inflows in periods 1 through n-1 are observed. This is problematic if all future planned releases are needed for evaluation purposes.

A conditional chance constrained model (CCCP), which incorporated these correlation considerations for the control of a single reservoir, has been developed by Lane [46]. Simply stated, it includes statements about conditional probability distributions, as well as individual (or marginal) probability distributions, in the form of constraints. Handled similarly to the way simple chance constraints are utilized in CCP techniques, CCCP provides a probability of a particular level of total constraint violation for each policy. Again, linear programming is used as the solution tool and as in other techniques, the formulation size increases dramatically with the number of reservoirs.

Lane suggests a variety of ways for reducing the number of constraints caused by the combinatorics of considering joint probability of all combinations of all random variables. Since, however, each suggested technique decreases the solution space in an arbitrary way, their impact on the optimality of the decision is not clear.

One assumption used by Lane is the arbitrary limitation of the effect of random variables to a limited number of time periods. This reduces the number of decision rules and hence the number of primal variables. The further suggestion of solving the dual is of interest, but in general, the strategies still remain too large for on-line use. As a planning tool, however, the technique seems valuable. Curry, et al., [47] have extended chance constrained approaches to systems of reservoirs, and have indicated how serial correlations can be implicitly maintained in the formulation. The chance constraints relate a control at any point in time to the original state (i.e., storage levels), all previous controls (i.e., discharges), and the *sum* of all previous inflows (the random variables). The probability of any such sum implicitly contains serial correlation information. These authors maintain that the chance constrained technique is computationally hampered by the size of the objective function; however, the formulation contains no more constraints than does the deterministic model of the same system. "A primary advantage of the model would be in real-time operation of a linked system of multipurpose reservoirs. If both inflow and water demands could be anticipated through forecasting procedures, the model would provide operational guidelines which could either minimize or maximize the selected objective function" [47]. Assuming computer power is available to model such large systems deterministically, chance constrained techniques of this type add little to the computer load. Cross correlations between the random inputs to the separate reservoirs can also be maintained. This is done by constructing conditional probability distributions against which particular combinations of constraint violations may be measured as in CCCP. Non-linear routing links between reservoirs must, however, be sacrificed and large computers are required for the implementation of these conditional chance constrained techniques.

C.3.2 Stochastic Dynamic Programming. The application of Stochastic Dynamic Programming to reservoir operation was first proposed by Little [27] and later applied by Hall [28], Hall and Buras [29], Hall, Butcher, and Esogbue [30], and Buras [31]. This approach yields the sequential operating policy that will maximize the total expected benefit of the system. Butcher and Fordham [14] extended the work of Fiering [32] and developed a variant of stochastic dynamic programming which maintains the cyclic or periodic behavior of the inflow series as well as the correlation with the previous event. This overcomes some of the problems associated with (stationary) lag one Markov chains which ignore the cyclic behavior in stream records. The sequence of monthly flows is regarded as connected by twelve separate sets of translational probabilities forming a nonstationary (cyclic) Markov Chain. Modeling base flow as deterministic and the residual as a simple Markov Chain also overcomes some of the problem of maintaining a cyclic phenomena with simple recursive relationships.

Dudley and Burt [33] have developed an integrated intraseasonal and interseasonal stochastic dynamic programming approach for design and operation. As applied to a single reservoir system, it functions as a long range planning tool as well as a short range operational tool.

Askew [48] has incorporated the basic idea of CCP into a dynamic programming formulation which incorporates chance constraints as typical dynamic programming constraints and/or as penalty functions within the objective function. In previous work (Askew [49]) it was indicated that, in many cases, optimal operating policies designed to maximize expected net benefits would, if they are followed strictly, allow the system to fail on an appreciable number of occasions. Askew has overcome this *gambler's ruin problem*, prevalent in typical stochastic dynamic programming, by the inclusion of risk. It is admitted, however, that an additional computational load imposed by these modifications can cause difficulties, as they did in CCCP.

Explicit techniques which yield steady state controls, although not immediately relevant here yield insight into methods of handling the uncertainties within a mathematical programming effort and are consequently worth a brief review. Bellman [34], Buras [35], and Hall [28] are the first to have utilized a value iteration algorithm for modifying steady state control strategies via stochastic dynamic programming.

The Policy Iteration Models of Howard [36] consist of separate value determination and policy improvement stages. They are methods which apply when there are many time periods or cycles and the system's condition at any particular point in time is independent of the particular time period. Such a system is described as *ergodic*, and the derivation of steady state policies is the desired result. The first stage of this two stage process, the value iteration stage, selects an initial value for a penalty cost associated with the total violation of the chance constraints. The second stage seeks a steady state policy which achieves the selected penalty cost. In this technique, the penalty cost is continually decreased (value iteration) while the control strategies improve (policy iteration) until the problem converges to a solution for which no improvement can be found. The advantage of Howard's techniques lie in an ability to solve problems with many time periods more efficiently than other techniques which must model each time period explicitly. Its value as a technique is increased by studies which have extended its application [14] to problems of more than one reservoir via a decomposition approach. Schweig and Cole [37] were the first to develop such techniques fully for more than one reservoir. This work recognized the serial correlations of month to month flows and represented them in a distribution free way by classifying inflow data according to whether an inflow was preceded by an inflow higher or lower than mean for the antecedent month. Cross correlations between locations were not modeled. Iterative dynamic programming in the form of policy improvement, was the characteristic optimization technique used.

D. ADAPTIVE CONTROL OF LINKED RESERVOIRS

The effectiveness of adaptive computer based forecasting models has been demonstrated by Hu and Root [50]. An adaptive learning model was created which was to simply forecast *rain* or *no rain* conditions in the San Francisco Bay area for three successive 12 hour periods (today, tonight, and tomorrow). This model provided a percent probability for any forecast, with a rain probability of 50 percent or higher considered as *rain*. The objective of this decision format was to provide a forecast in the same form as the Weather Bureau forecasts in order to compare the performance of the two approaches.

The adaptive model developed in their work was a simple linear model in which meteorologic data, as system inputs, were related to the forecasted output by coefficients that were adjusted over time in proportion to the error encountered in the series of forecasts. Such a linear input-output model, relating meteorologic measurements to a forecast, requires no elaborate thermodynamic models of weather producing systems. It does, however, recognize the meteorologic factors involved, and is capable of a better forecast than that which a simple time series analyses of rainfall events could produce.

Although many researchers have pointed out the need for detailed knowledge of time and space variability of precipitation for urban water problems, Thomasell [51] may have been one of the first to suggest an adaptive or learning-type pattern extrapolator model. In outlining research needs for improved operation of urban water drainage systems, he notes the importance of real-time forecasting of precipitation while also describing so-called *objective analysis* techniques in existence that are capable of *data translation*. These data translation techniques, derived by Thomasell and Welsh [52] for the prediction of temperature fields in the ocean, are suggested for use by urban hydrologists. A discussion of computer graphics and data translation techniques lead to the suggested use of forecasting error in updating the data translation models. Automatic updating or learning capability can be called adaptive modeling. However, since the simulation studies (especially those by Sorman and Wallace, Grace and Eagleson, and Wu) had not as yet identified the relevant linkages between parameters in the rainfall process, the suggestion may have been premature. (The mechanisms of heat transfer in the ocean, complicated as they may be, are more understood than are the meteorologic mechanisms producing rainfall patterns.) It is again worth noting that the adaptive forecasting model developed by Hu and Root needs no such process model despite its reliance on relevant physical parameters. On the other hand, the Thomasell model looks for a process model for small scale forecasting purposes. Both models, however, point in the direction of on-line adaptive models for forecasting.

More recent work by Moore and Brewer [53] describes the general application of *filtering* techniques to water resources systems. Filtering techniques imply the identification of correlative structures between inputs and outputs of a system model. The model, however, need not incorporate known physical linkages. That is, the model need not be a *process* model. On the other hand, the model need not be a simple time series analysis of an isolated record. Models based on filtering techniques can grow in order and complexity until the error series generated by a comparison between the model's forecasted output and the actual measured data is reduced or *filtered* to *white noise*. By this, it is meant that all of the information available in the input for deriving the output has been identified. The model developed by Hu and Root was a simple filtering attempt. Advances in identification theory now enables researchers to develop models which can incorporate more data in an on-line adaptive mode.

Chan [54] has modeled a small urban combined sewer network using optimal control theory in conjunction with an estimator model. Limited to only two subcatchments, the storm inflows were modeled as Gaussian white noise with a time varying mean. The inflow sequences for the two subcatchments were assumed independent. Modeling inaccuracies and measurement error were considered as uncorrelated random variables. The separation theorem was applied to divide the problem into two parts: estimation and control. Using a Kalman-type filter as the basis for random variable estimates, the control objective of optimally utilizing the system capacities while minimizing overflows is similar to the problem addressed herein. The disregard of any cross correlative structure in the inflows and the extensive analysis of a rather small system decrease the usefulness of this work for the application considered here.

Real-time forecasts of the stochastic aspects of a system in conjunction with the derivation of controls for that system brings together the two major features of the work considered herein. Such a blending was incorporated in the development of Jamieson and Wilkinson [56]. Building on previous efforts (Jamieson, Wilkinson and Ibbitt [55]), the use of identification procedures for the on-line derivation of a rainfall forecasting model was demonstrated. The parameters of the autocorrelative model used were modified by the error series describing the differences between previous forecasts and subsequent measurements. The forecasted rainfall was then used as inputs to a watershed model for the prediction of system inflows. In the subsequent study, a short time control strategy for multipurpose reservoir systems was derived for a network of reservoirs on a river system.

Their work differs from the large body of literature concerning reservoir operation in three ways that are of significance here. First, the time horizon of the control strategy and consequently the control intervals are much shorter, being days and hours respectively, than that of most reservoir control studies. Second, the computer algorithm is used in an on-line manner, adapting to experienced conditions, rather than in the off-line preparation of rule curves or operational guides. Third, a forecasting model is incorporated to extrapolate the trends identified in the rainfall. Commenting on other studies (including Moore and Brewer [53] mentioned above) which used filtering techniques to simulate or predict system inputs (runoff) solely on the bases of telemetered rainfall, the authors (Jamieson, Wilkinson and Ibbitt [55]) state:

> "If the forecast (of system inflows) is based solely on telemetered values of rainfall, the implicit assumption is that there will be no subsequent rainfall from the time of forecasting; this assumption must be the worst one possible in the middle of a severe storm. Clearly, some other assumption is desirable, but in the absence of quantitative rainfall forecasts it is not obvious what it should be. An attempt has been made to fit a typical storm to assess the possibility of using such an approach in forecasting rainfall amounts. The analysis followed the methodology of Box and Jenkins (1970)."

Using a first order linear autoregressive model, subsequent inputs to a watershed runoff model were forecasted. The resultant runoff was then routed down the tributaries and through the reservoirs. Dynamic programming, incorporating a multi-objective return function, was used to derive the controls at each reservoir. The routed releases from each reservoir were used as inflows to downstream reservoirs. This large-scale complex system, while very similar in outward appearances to the detention storage problem addressed herein, has some significant differences despite its on-line use of (1) realistic non-linear watershed and routing models, (2) forecasting of rainfall, and (3) multi-objective return functions. These differences all addressed in the chapters to follow are as follows:

1. The models used for forecasting rainfall were simple autoregressive models based on each separate location's history. The parameters, although initially based on historic events, were modified by the real incoming data. This resulted in a learning or adaptive model similar to those developed herein. No consideration of the spatial distribution of the rainfall was, however, made. More comprehensive models similar to the ones developed here could relate the rainfall (or inflow) at one location to the measured previous values at a number of other locations, as well as possibly other related meteorologic phenomena. The efficient algorithms needed for the on-line evolution of such comprehensive models have only recently been developed, however, and may not have been available for use by Jamieson and Wilkinson.

2. The realities of the system considered and its existent operational constraints reduced the dimensionality of the resultant dynamic programming strategy considerably. The operation of the entire system pivoted around two main reservoirs. Thus, operating policies of these determined the solution of the other components. The problem as originally formulated included two decision variables and five state variables. This was then further reduced by the adoption of fixed operational policies for the upstream reservoir. This reduced the size of the decision algorithm considerably. These simplifications enabled the researchers to avoid any need for large-scale analysis techniques (i.e., decomposition) which have been unavoidable in the work described herein.
3. The risks of the forecasted flows being in error were not taken into account in the decision phase of the model. The dynamic programming algorithm developed regarded the inflows as deterministic values rather than as stochastic values. An attempt at dynamic programming under risk is incorporated here.

The above comments, not intended to criticize Jamieson and Traveres, are meant, rather, to illustrate the state of the art in this area and the advances attempted in the work described here.

Adaptive on-line control has also been applied in water distribution systems with some success. The differences in the problems encountered, however, seriously reduce its applicability to the problem addressed herein. Pressure flow is easier to model than is open channel flow. The demand, although stochastic, is very periodic and hence easier to forecast. System response times are very fast and consequently, controls and their effects are readily measurable for feedback purposes. The problem, then, is similar to a complex electrical network for which much directly applicable theory is available for use in system control (i.e., network analysis).

E. SUMMARY

The concept of on-line adaptive control of a combined sewer system is well supported by the developments described in the literature. The deterministic and stochastic control of linked storage facilities has evolved to the point where on-line applications are feasible. Their use in the past was generally restricted to planning efforts. Rainfall simulation studies have progressed through the three phases of purely statistical modeling, process simulation modeling, and now, adaptive-forecast modeling. Adaptive modeling produces forecasts which can be iteratively generated from current conditions. Previous forecast errors are subsequently used for modifying the model.

The combination of the advances in stochastic approaches to the control of linked storage facilities, and adaptive forecast modeling, has enabled the development of a system for the on-line adaptive control of combined sewer system using detention storage facilities. Such a system is described in the chapters to follow.

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Chapter III

DECOMPOSITION OF THE LARGE-SCALE CONTROL PROBLEM

A. INTRODUCTION

The development of a total algorithm designed to derive needed controls for a linked reservoir system as large and complex as that proposed for San Francisco's planned wastewater system (illustrated in Figure I-4) is a challenging goal. The challenge is amplified further by the assumption of limited computer hardware. Under this restriction, the techniques discussed in the previous chapter are generally limited to a small number of system components. Methods must therefore be applied which can decompose the large-scale system of reservoirs into a number of mildly linked subsystems of manageable size. Such methods are available [1,2], and their use in water resources problems are increasing [3]. The goal of this chapter is, therefore, to describe such a methodology which allows the resulting subproblems for each subsystem to be solved by the techniques discussed in Chapter II.

The intent here is to demonstrate the feasibility of large-scale adaptive control by simulating the real-time environment envisioned as the prototype. The algorithm developed can be applied to the entire city, but it was decided that it would be more efficient to focus on a portion of the city for computational experience. It will be demonstrated how the derived techniques can be extended to the entire city.

Figure III-1 illustrates conceptually the total strategy for on-line real-time hierarchical computer control of the entire system that will be developed here. Four separate levels of control are identified. The highest level is the conceptual juncture of three

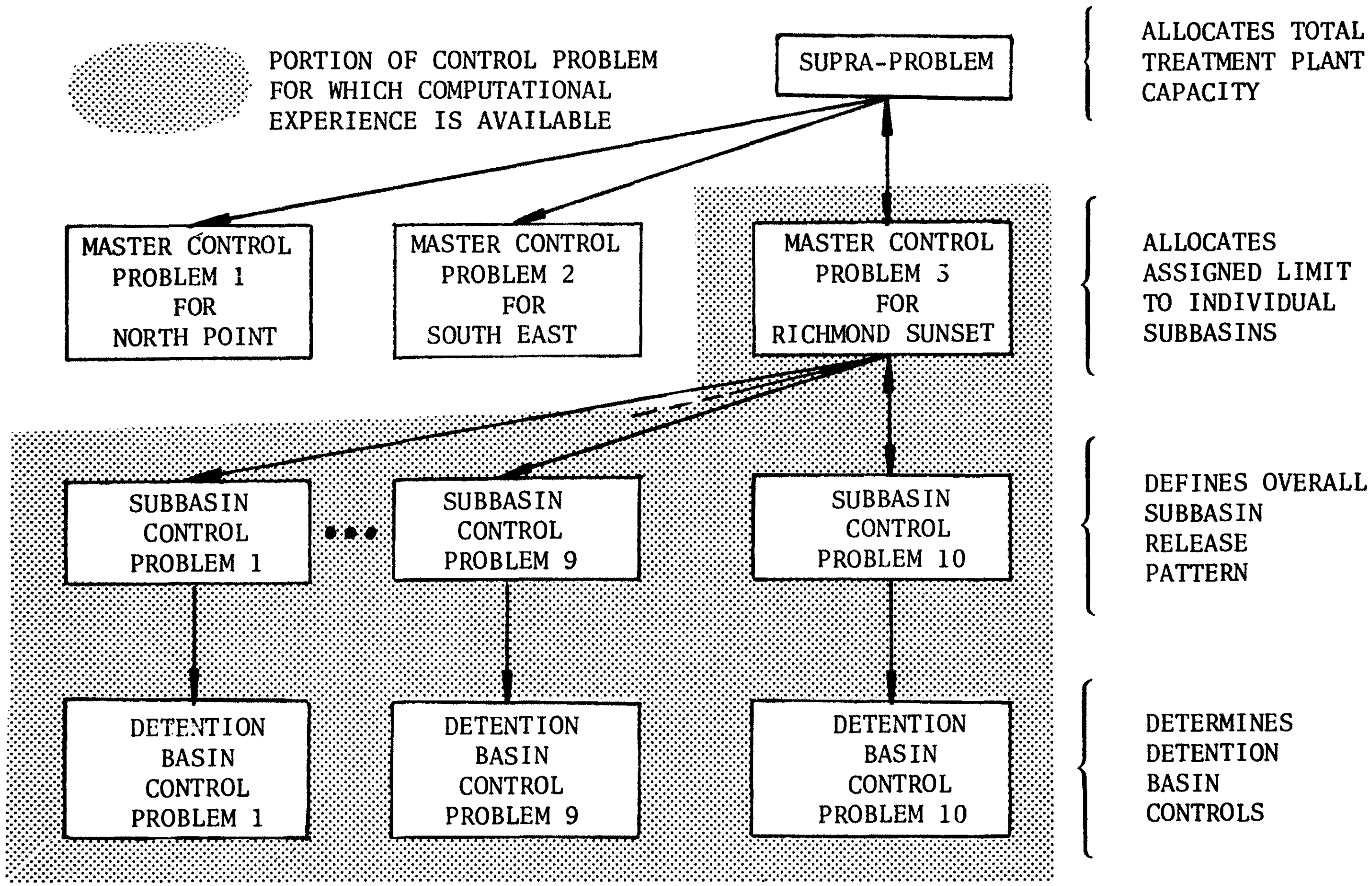


Figure III-1. Total Strategy for On-Line Hierarchical Computer Control of San Francisco Sewer System.

separate master control problems for the three major regions of the city: Richmond Sunset, North Point and South East. It may, however, be possible to reduce this to one level in which one master control problem manages and integrates all the subbasins within the city. The Richmond Sunset area will be isolated in order to more conveniently illustrate the use of the large-scale systems techniques envisioned for the entire city. It provides a physically discrete subsection which is large enough to necessitate the multi-level decision making which characterizes the city-wide approach. The techniques developed, consequently, are applicable to the entire city.

Associated with the Richmond Sunset area is a master control problem which integrates and influences the demands put upon the interceptor sewers and treatment plant by the component subbasins. Each subbasin has two subproblems identified with it. The interactive subproblem is shown directly below the master control problem in Figure III-1. It provides tentative performance data based upon parameters provided by the master control problem. The performance data, in turn, influence subsequent values of the parameters chosen by the master control problem. The emphasis in this interactive subproblem is computational speed, at the expense of high accuracy, since many iterations between the master control and interactive subproblem may be required within a control time interval. Since the control interval may be as short as five minutes in length, the need for fast communication is clearly evident. A more detailed subbasin control problem lies below this interactive subproblem in the illustration. It carries out the detailed computations necessary for finding the actual system controls to be implemented, but need be run only once per control time interval, after necessary information from the above interactive process has been obtained.

The Richmond Sunset area of the city is divided into ten separate subbasin areas, which are illustrated in Figure III-2. This division was motivated by the need to create many small subproblems suitable for independent solution. As divided, each subbasin area contributes directly to the interceptor. In this way, upstream interconnections between subbasins or subproblem areas are avoided, thereby facilitating the formulation of the master problem. Although not all of the subbasin areas so delineated have overflow bypass facilities, each is modeled as having a distinct overflow point. This representation is by no means unique, and there are many other possibilities for decomposing a large-scale sewer system. In most other cities, however, it should be possible to define tributary subareas or subbasins to the interceptor sewers in much the same way as done here. A full variety of subbasin configurations are represented in this division. Three subbasins consist of only one detention basin, while others contain as many as seven. Also represented are subbasins with differing hydraulic characteristics caused by the range of slopes encountered within this part of the city.

This multi-level or hierarchical approach to the total system was motivated by the need to provide optimal control algorithms for a large-scale system constrained by both the demands of a real time operating environment as well as an implicit limitation on total computer hardware.

Decomposition represents a class of techniques whereby a large system to be optimized in some way is decomposed into several mildly

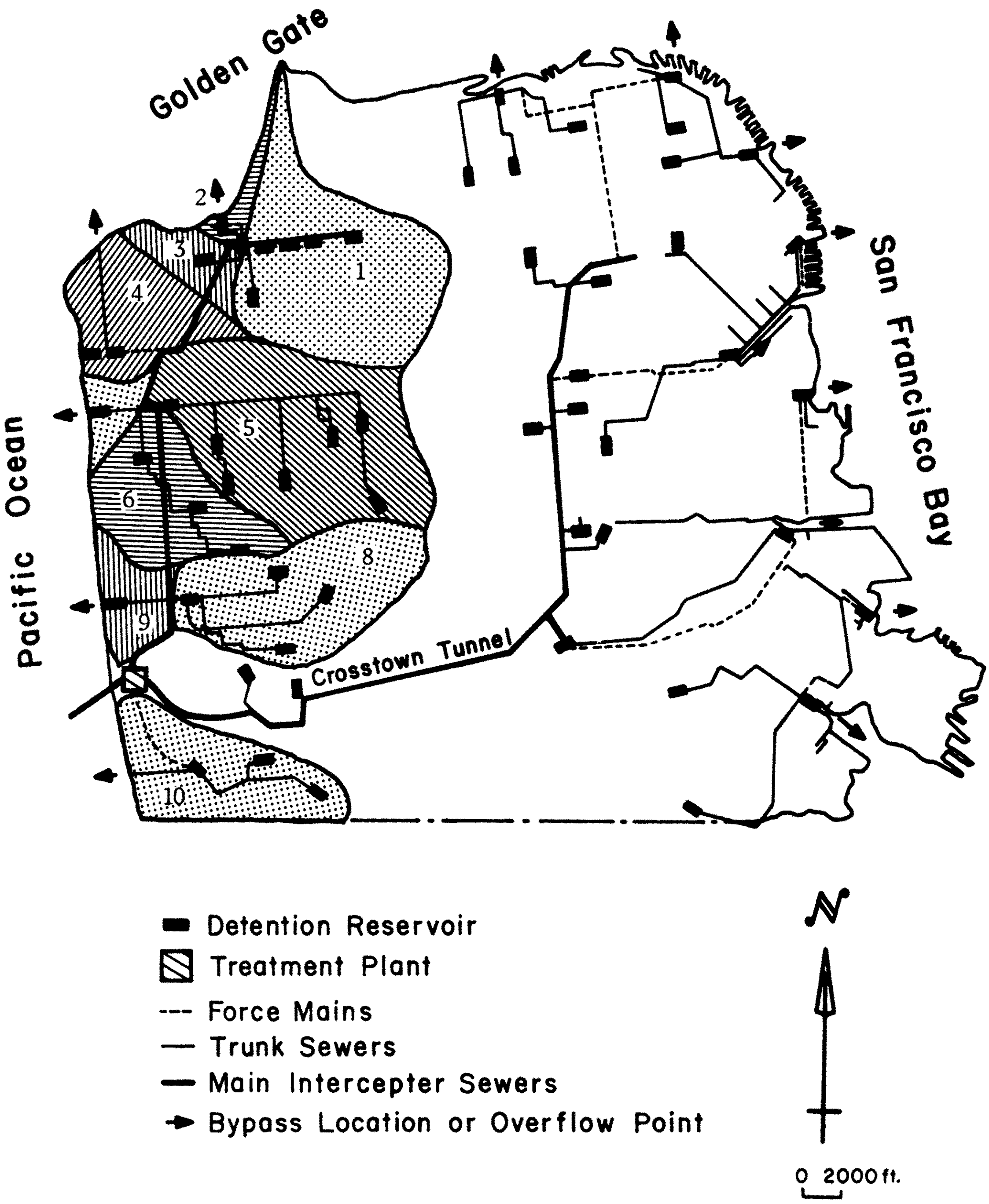


Figure III-2. Ten Subbasin Areas Within Richmond Sunset Region of the Proposed San Francisco Master Plan interlinked subsystems. The subsystems are treated independently and then recombined by a master control program in such a way as to achieve an overall optimum strategy. Decomposition as applied to a sewer system of this type involves the separate determination of optimal controls for each subbasin. The master problem then checks to see if the interlinking constraints and optimality conditions are satisfied. If they are not, another iteration of the master problem takes place in which the constraints, or influence, imposed by the master problem on the subbasin problems would be adjusted. Iterations continue until an optimal solution for the entire system is determined. Three important advantages of decomposing a large-scale system have been listed by Labadie [3]:

1. Greater conceptual understanding of the behavior of the system is attained when subsystems or subparts of the large-scale system are identified and analyzed separately.
2. Mathematical programming techniques are available [1] which enable subsystems to be temporarily severed from the total problem, optimized as smaller problems, and recomposed by an iterative master problem which achieves overall system objectives.
3. Required computational storage (e.g., rapid-access core memory) is decreased at the expense of total required computation time.

It is noted, however, that the impact of the additional time can be lessened by using a hierarchy of computers capable of simultaneous, or parallel, optimizations of the various subsystems defined for each particular level.

B. DEFINITION OF THE LARGE-SCALE CONTROL PROBLEM

The major objective for real-time control of the entire system is to minimize the expected effect of overflows, as well as localized flooding, for any storm event which might occur. The variation in quality of overflows and their consequent impact on receiving waters greatly complicates this objective. Adequate water quality models are, however, as yet unavailable and would be difficult to incorporate into the necessary control algorithms if they were available. Furthermore, since control decisions must be made before the storm event is realized, some measure of risk, or the probability of the forecasted inflows and, hence, controls being in error, must also be reflected in the objective used for comparing various control strategies. Thus, for the analysis, the objective is to minimize total *expected* detrimental overflow impacts. The use of expectation is justified by the noncatastrophic nature of an overflow event. Were this not the case, expectation would be an inappropriate measure.

Figure III-3 illustrates schematically the ten subbasins identified within the Richmond Sunset area of the San Francisco system. In this figure, $O^i(k)$ represents the overflow anticipated from subbasin i during period k. This overflow is either conveyed directly to the receiving waters by a suitable bypass structure, or results in local street flooding where no bypass conveyance exists. The distinction between these two will be made later when the effects of these overflows are considered.

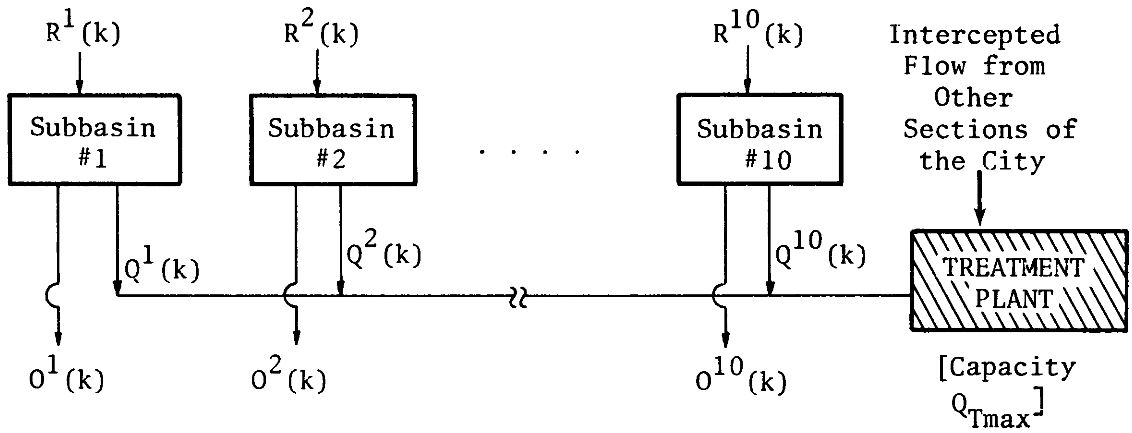


Figure III-3. Schematic Illustration of Subbasin Interaction.

The total forecasted runoff into the sewer system of subbasin i during time period k is represented as $R^i(k)$, while $Q^i(k)$ represents the quantity of flow to be discharged into the interceptor sewer from the subbasin. The random nature of $R^i(k)$ is discussed in subsequent chapters where methods of forecasting $R^i(k)$ are developed and where the risks associated with the forecast are considered. The variable Q_{Tmax} is the upper limit on total interceptor flows delivered to the treatment plant from the Richmond Sunset area. As noted in the previous section, the Richmond Sunset area is only one of three major regions of the city contributing to the treatment plant. It has been assumed, therefore, that the capacity of the treatment plant will somehow be allocated among the three branches. This allocation, conceptually the supra problem, would be based upon either discrete real-time decisions or entire policies based on on-line optimizations and would be carried out, at least conceptually, by the supra problem.

B.1 Control Decision Criterion

The *effect* of overflows from each subbasin on the receiving water may not be the same and consequently their relative impact must be gaged for the derivation of optimal controls. Weighting factors are used to establish this relative impact for: (1) various time periods defined within the storm event, (2) various locations throughout the city, and (3) differing quality of sewer flows. These weighting factors relate the detrimental impact of overflows to the quantity of overflows. The effect of an anticipated overflow event of quantity $O^i(k)$ from subbasin i during time period k is assumed directly proportional to that volume of overflow. The weighting factors $w^i(k)$ are defined for each time period and subbasin. Overflow effect is, therefore, represented as: $w^i(k) \times O^i(k)$.

Measuring the effectiveness of a control policy for the imminent and developing event requires a forecast represented in probabilistic terms. Operating policies may then be judged using the expected overflow measure. That is, at any point in real time, the expected sum of the weighted overflows for a forecasted inflow sequence under risk for the remainder of the event can be used as an objective function for the selection of controls:

$$E\left[\sum_{i=1}^{10} \sum_{k=1}^{M} w^i(k)\, O^i(k)\right] \tag{1}$$

where E is the statistical expectation and M represents the expected number of time periods required for the effects of a current event to cease, where $k=1$ corresponds to any point in real time.

The system, however, has not been planned to minimize overflows for an isolated storm event, but rather to minimize overflows occurring from any sequence of events. If a single event criterion was used, there would be danger that the operating policies would not effectively utilize storage, interceptor, and treatment capacity in such a way that a rapidly following storm event would be properly anticipated.

Initial experience with techniques designed to solve the control problem indicates that derived controls for simple overflow minimization do not fully utilize the capacities of the interceptor sewer and treatment plant [5]. The unnecessary storage of storm flows in detention basins throughout the entire modeled time horizon often results when such a criterion is used. This has occurred despite available system capacity for the transport of these flows. The rapid drainage of detention basins is seen as an important system operating policy. This rapid drainage can be encouraged by favoring control policies which maximize throughflow into the interceptor sewer. This enables the system to be at a maximum state of readiness for subsequent events which may rapidly follow the immediate event.

An improved criterion for selection of controls was, therefore, required in which the order of priority of allocation of sewage would be: (1) send discharge to the treatment plant until capacity is reached, (2) divert as much of the excess as possible to temporary detention storage, and then (3) overflow any necessary quantities at the least sensitive location and at an appropriate time, based on anticipation of future storm flows [4,5]. A simple addition to the previous expression of overall system performance enables the attainment of these control decision priorities. Letting $y^i(k)$ be a weighting factor (or credit) for throughflow allows the criterion to be written as:

$$E[\sum_{i=1}^{10} \sum_{k=1}^{M} w^i(k)\ O^i(k) - y^i(k)\ Q^i(k)]$$

or,

$$\sum_{i=1}^{10} E[\sum_{k=1}^{M} w^i(k)\ O^i(k) - y^i(k)\ Q^i(k)] \qquad (2)$$

The detrimental impact of overflows may, however, not be best represented in terms of total overflows. For example, a surge of overflow may be more harmful than a continuous flow. Adding an appropriate exponent to the first term would encourage operating policies generating less variable overflow discharge rates over time:

$$\sum_{i=1}^{10} E[\sum_{k=1}^{M} w^i(k)(O^i(k))^n - y^i(k)\ Q^i(k)] \qquad (3)$$

With the criterion thus formulated, it is now assumed that given a forecasted inflow sequence in probabilistic terms, an operating policy for a subbasin can be found which minimizes the criterion for that subbasin as a function of some specified maximum subbasin release $Q^i_{max}(k)$ to the interceptor sewer for $k=1,\ldots,M$. The optimal value of the subbasin criterion, as a function of this local release limit, can be designated for subbasin i as:

$$\tilde{O}^{*i}(\underset{\sim}{Q}^i_{max}) \qquad (4)$$

where $\underset{\sim}{Q}^i_{max} = (Q^i_{max}(1),\ldots,Q^i_{max}(M))$.

The algorithms for finding the subbasin operating policies needed to achieve these minimal weighted overflows are discussed in the following chapter. A release relating to the actual optimal inflow to the interceptor, for given $\underset{\sim}{Q}^i_{max}$, can be designated as $Q^{*i}(\underset{\sim}{Q}^i_{max},k)$

for a particular time period. With the details of the subbasin control strategy which results in these releases relegated to another chapter, the overall criteria relating to the expected overflow impact and throughflow credit may be simply represented by:

$$\sum_{i=1}^{10} \tilde{O}^{*i}(\underline{Q}^{i}_{max}) \tag{5}$$

B.2 <u>System Constraints</u>

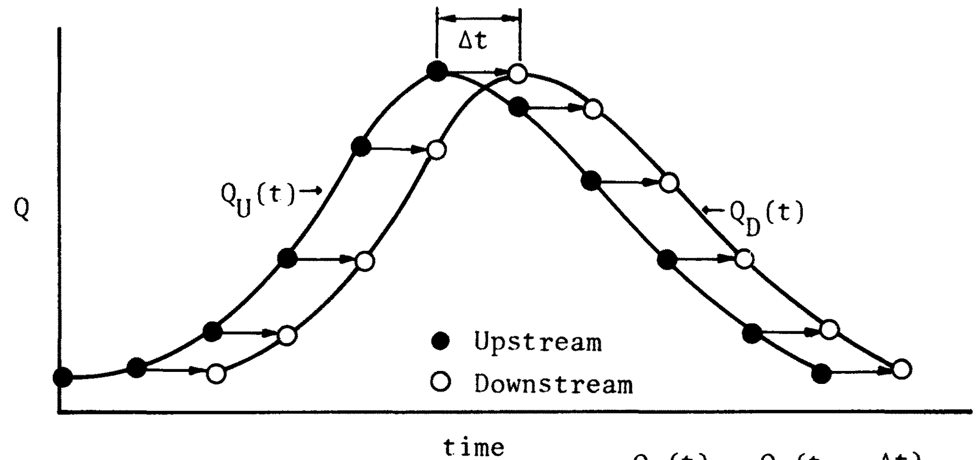
Having derived a measure of system performance, models are now needed to predict the system behavior. Such models incorporating the system constraints must, however, remain simple for real-time use. It has been noted that the major cause of overflows at the subbasins is the limited allocated quantity of treatment plant capacity Q_{Tmax} which has been assigned to the section of the city being considered here for purposes of illustration. Therefore, any model of system performance must consider the routing or translation of subbasin discharges into the interceptor down to the treatment plant.

Routing techniques available from the field of fluid mechanics and open channel hydraulics cover a wide spectrum of accuracy vs. facility. The full solution of the St. Venant equations describing unsteady flow is approached using techniques such as the method of characteristics and various finite difference schemes. These equations, which include continuity, momentum, and energy principles, require significant quantities of information describing the flood wave, boundary conditions (upstream and downstream depth), and the physical characteristics of the channel. Assuming availability of this information, these techniques would still be inappropriate for on-line work due to the excessive amounts of computer time and capacity required for their solution. The iterative nature of the optimizations, in both the master control problem and the subbasin analysis, considerably compounds the problem of the excessive time needed for these routing algorithms.

Kinematic wave techniques, such as those used for operational studies on the Seattle CATAD stormwater management system [6], describe the essence of wave propagation using only continuity considerations, so that they are a significant departure from the solution of the full equations. Computational advantages over the more complex techniques tends to compensate for any loss in accuracy. Computational requirements of this technique still, however, appear large when viewed in the context of the repeated computations anticipated during real-time optimizations.

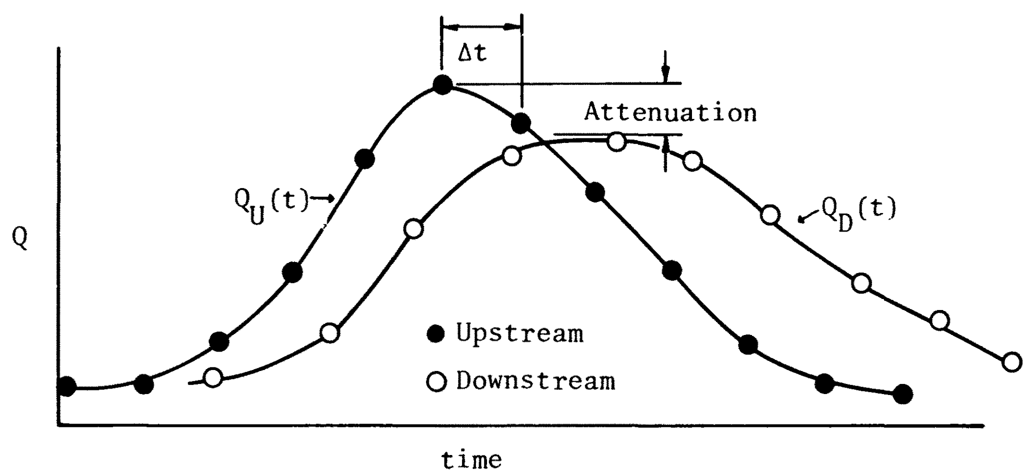
Empirical routing techniques depart further from the full equations. They result, however, in more rapid methods for routing a hydrograph or sequences of reservoir releases downstream and are amenable for use in optimization schemes [7]. Successive averaging approaches based upon concepts of wedge and prism storage, such as the Muskingum method, are widely used. Such methods approximate the attenuation and dispersion of natural flow waves. Simpler approaches, which simply lag flows by an appropriate travel time are compared to successive averaging techniques in Figure III-4. In this figure, $Q_U(t)$ represents the upstream hydrograph, while $Q_D(t)$ represents the downstream routed hydrograph. The discretization of time, t, is represented by the interval Δt. Simple lag techniques may, however, suffice where slopes are high and reaches are short. These conditions

LAG ROUTING



$$Q_D(t) = Q_U(t - \Delta t)$$

SUCCESSIVE AVERAGING ROUTING



$$Q_D(t) = C_1 Q_D(t - \Delta t) + C_2 Q_U(t) + C_3 Q_U(t - \Delta t)$$

Where: $C_1 + C_2 + C_3 = 1$

Figure III-4. Lag and Successive Average Routing are particularly prevalent in the San Francisco sewer system, where the steep gradients tend to decrease the time available for the release profile to change its shape significantly. Successive average routing can be generally represented by

$$Q_D(t) = \sum_{m=0}^{\tau} a_m Q_U(t-m\Delta t) \tag{6}$$

where a_m, $m=0,\ldots,\tau$, are routing coefficients, and $\tau\Delta t \leq t$.

A number of factors inhibit the use of sophisticated routing techniques within this large-scale, real-time control problem. Of primary importance to all levels of optimization is the computer time required for the selected technique. A real-time application with limited computer hardware demands a fast routing technique. Many control policies are generated during the search for optimal controls, resulting in many different release sequences to be routed.

The availability of data is an additional factor affecting the selection of the routing technique. More sophisticated techniques demand certain data for calibration purposes that are difficult to obtain, such as boundary conditions (upstream and downstream flow levels), physical and hydraulic characteristics of the sewer system, as well as a full description of the flow sequence to be routed. Successive averaging techniques do not require boundary data, while lag techniques permit the routing of parts of flow sequences independently of the entire sequence. Despite the lack of elegance in the methods, such features are important for facilitating the use of optimization techniques for finding control policies. Routing is obviously critical, and considerably more research is required for evaluating tradeoffs between computational speed and accuracy for the various methods. For our purposes, it will be assumed that empirical techniques are adequate.

In order to represent routing effects upon the interceptor capacity constraint in a simple straightforward manner, and to facilitate the following developments, two assumptions are made. First, it is assumed that the predicted flows from each subbasin contributing to the interceptor can be routed independently of the others. Therefore, downstream flows are the superposition of separately routed upstream flows. Second, it is assumed that routed flow quantities during finite intervals are linear functions of previous flow quantities.

Representing time by discrete intervals k, a back shift operator B^m is defined such that

$$B^m Q^i(k) = Q^i(k-m) \tag{7}$$

It is convenient to represent a release from subbasin i routed to a downstream release point for subbasin j (where the subbasins are ordered consecutively downstream, as shown in Figure III-3) as [for $\tau \leq k-1$]:

$$[Q^i(k)]^j = \sum_{m=0}^{\tau} a_m^{ij} \; B^m \; Q^i(k) \tag{8}$$

where τ is the number of time intervals for which the effects of an upstream release at a particular time significantly affect the flow at point j. Therefore, the summation of all upstream releases routed to point j is [where $a_0^{jj} = 1$ and $a_1^{jj} = a_2^{jj} = \ldots = a_\tau^{jj} = 0$]:

$$\sum_{i=1}^{j} [Q^i(k)]^j = \sum_{i=1}^{j} [\sum_{m=0}^{\tau} a_m^{ij} \; B^m \; Q^i(k)] \tag{9}$$

(for $j=1,\ldots,10$)

At each point j along the interceptor, the flows are constrained by the interceptor capacity at that point $\bar{Q}^i_{max}$. This interceptor capacity is the physical upper limit on flows at that point. This constraint can be written as

$$\sum_{i=1}^{j} [Q^i(k)]^j \leq \bar{Q}^i_{max} \tag{10}$$
$$(\text{for } j=1,\ldots,10)$$

In addition, an upper limit exists on total flows to the treatment plant, represented as point T, which is downstream from the release point of subbasin 10. This results in the additional constraint

$$\sum_{i=1}^{10} [Q^i(k)]^T \leq Q_{Tmax} \tag{11}$$

The complete large-scale optimal control problem may now be written using the above constraints and the criterion function of Equation (5) presented previously. The flows that are routed, however, are the $Q^{*i}(\underline{Q}^i_{max},k)$ computed by the subbasin control problems. Assuming forecasted inflow sequences with probabilistic information for each subbasin are given, as well as a treatment capacity allocation Q_{Tmax} by the supra problem, the large-scale control problem can be expressed in a convenient, concise way as

$$\underset{\underline{Q}^1_{max},\ldots,\underline{Q}^{10}_{max}}{\text{Minimize}} \qquad \sum_{i=1}^{10} \tilde{O}^{*i}(\underline{Q}^i_{max}) \tag{12}$$

Subject to:

$$\sum_{i=1}^{j} [Q^{*i}(\underline{Q}^i_{max},k)]^j \leq \bar{Q}^i_{max} \tag{13}$$

$$\sum_{i=1}^{10} [Q^{*i}(\underline{Q}^i_{max},k)]^T \leq Q_{Tmax} \tag{14}$$

$$(j=1,\ldots,10; \quad k=1,\ldots,M)$$

The above formulation assumes that the subproblems have properly allocated their releases in such a way that the minimum weighted overflow $\tilde{O}^{*i}(\underline{Q}^i_{max})$ is obtained for a given $\underline{Q}^i_{max}$. It is obvious that it would be nonoptimal for more capacity $\underline{Q}^i_{max}$ to be allocated than is needed. Thus, in general, when the assigned capacity is fully utilized,

$$Q^{*i}(\underline{Q}^i_{max},k) = Q^i_{max}(k)$$

The constraints could therefore be equivalently written as

$$\sum_{i=1}^{j} [Q^i_{max}(k)]^j \leq \bar{Q}^i_{max} \tag{15}$$

and

$$\sum_{i=1}^{10} [Q^i_{max}(k)]^T \leq Q_{Tmax} \tag{16}$$

$$(j=1,\ldots,10; \quad k=1,\ldots,M)$$

C. DECOMPOSITION INTO MASTER AND SUBBASIN PROBLEMS

Reconsidering the large-scale problem formulated in the previous section, it is evident that decomposition is implicit in its structure. The constraints on total flows routed or translated through the interceptor and treatment plant constitute the only link between the subbasins in the mathematical formulation. If this constraint were eliminated, each subbasin would minimize its criterion by releasing as much water as available or physically possible. The result would be no subbasin overflows; however, the imposed limit on interceptor and treatment capacity would be exceeded. The subbasin problems have, however, been presented as solved for given values of $\underset{\sim}{Q}^{i}_{max}$. The solution of these subbasin problems yields not only $\tilde{O}^{*i}(\underset{\sim}{Q}^{i}_{max})$, but also $Q^{*i}(\underset{\sim}{Q}^{i}_{max},k)$. Therefore, using the $\underset{\sim}{Q}^{i}_{max}$ terms as decision variables in the master problem and the routed values of the releases $[Q^{*i}(\underset{\sim}{Q}^{i}_{max},k)]^{j}$, for use within the linking constraint, results in the needed decomposition.

Figure III-5 illustrates the decomposed nature of the large-scale problem previously formulated, where it is divided into independent subproblems tied together by a master control problem. The master control problem trades off the subbasin solutions to achieve optimality. Optimality for the large-scale problem is simply seen as filling the assigned capacity with flows that carry the most penalty, as determined by the criterion functions.

To facilitate the rapid search for optimal controls on-line, perhaps at the expense of finding the best or *global* optimal controls, the following assumptions are made:

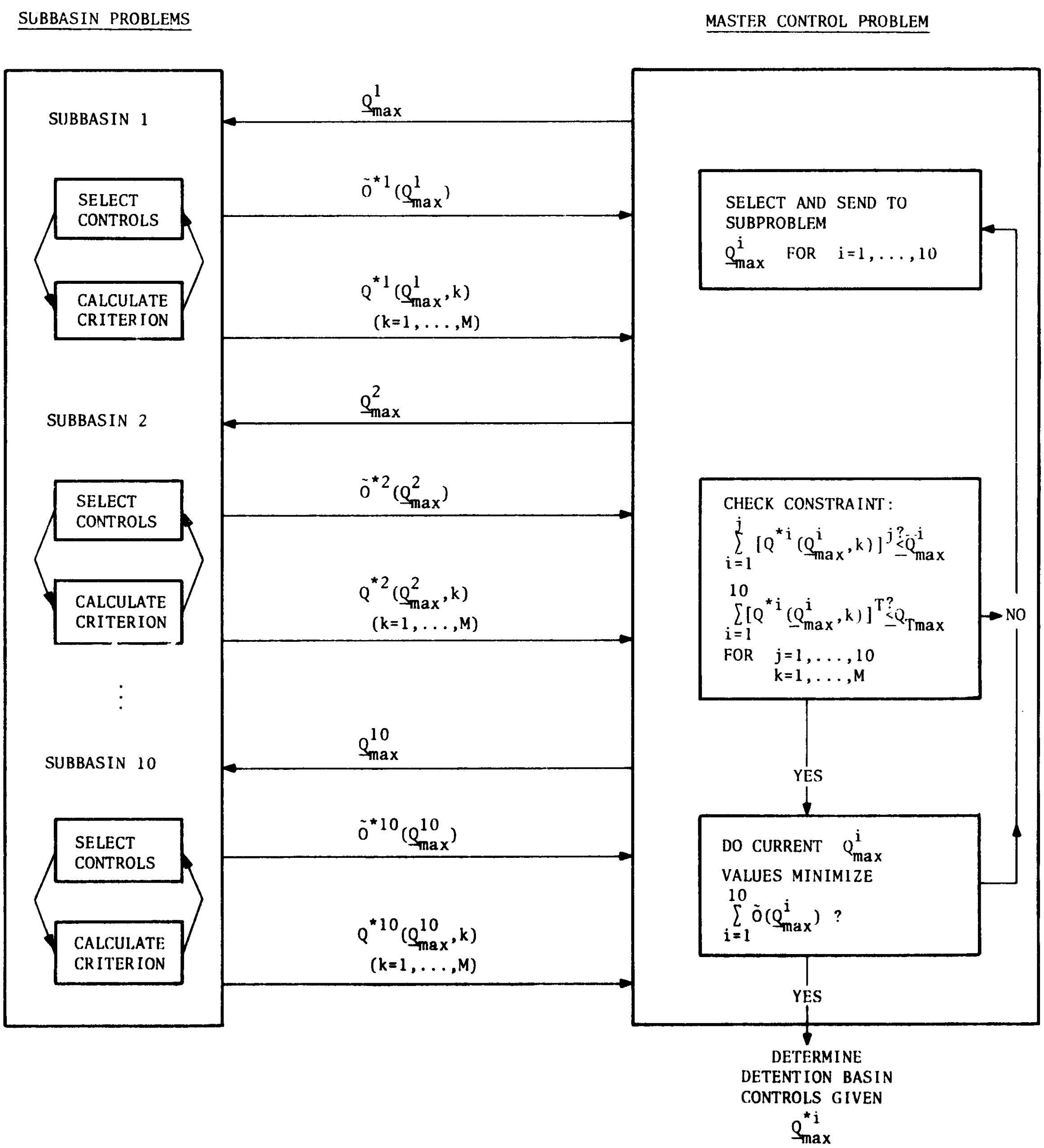


Figure III-5 Interaction between Master Control Problem and Individual Subbasin Problems.

1. Since new optimal control problems are solved during succeeding time intervals, there is the opportunity to change the values of Q^i_{max} during each control interval. Thus, it would seem reasonable to use a time invariant Q^i_{max} for the master control at any particular time interval, since it will be updated with each succeeding time interval k when new master controls are computed.
2. It has been noted by Bradford [5] that the upstream capacity constraints for the interceptor are unnecessary for the case study being considered. They are, therefore, eliminated from further consideration in this study, though the following algorithm can be modified to include them if necessary.

These assumptions simplify the master control problem and considerably reduce the number of decision variables, thus facilitating solution of the master problem on the limited hardware of a real-time computer system. The result is the following problem:

<u>Master Control Problem</u>

$$\underset{Q^1_{max},\ldots,Q^{10}_{max}}{\text{Minimize}} \sum_{i=1}^{10} \tilde{O}^{*i}(Q^i_{max}) \tag{17}$$

Subject to:

$$\sum_{i=1}^{10} [Q^{*i}(Q^i_{max},k)]^T \le Q_{Tmax} \qquad \text{(for } k=1,\ldots,M) \tag{18}$$

Decomposition of the large-scale problem has been achieved by use of individual constraints upon the maximum release from each sub-basin. The values of the constraint are then used as the decision variables in the master control problem. It is important to note that under the above assumptions, the alternative constraint of Equation (16) no longer applies. The actual flows must be used, as in Equation (14), since it is no longer assured that $Q^{*i}(Q^i_{max},k) = Q^i_{max}$. This is illustrated in Figure III-6.

D. ALGORITHM FOR MASTER CONTROL PROBLEM

The optimization which has been applied to the master problem stated above is a discrete direction steepest descent technique. The form of the master problem is suitable for solution by dynamic programming (DP) but the discrete direction steepest descent method (DDSD) requires fewer iterations to achieve an improved, though not necessarily global, solution. This is important, since the real-time environment may demand that a reasonable solution be implemented before a global solution can be found. In the DDSD method, improvements in the total objective function are sought by modifying the upper limit on releases Q^i_{max} for the subbasin problem which yields the best improvement in the total objective function. This subbasin problem will have its constraint on releases relaxed until it no longer produces the greatest improvement in the objective function. At that time, another subbasin problem is selected and its constraint is relaxed. This process continues until the total assigned level Q_{Tmax} is reached.

Steepest descent methods, all traceable to the original work by Cauchy [2], are based upon the fact that the gradient of the objective function at any point is a vector in the direction of the greatest local rate of increase in that objective function. Therefore, the negative of that gradient will be the direction of steepest descent.

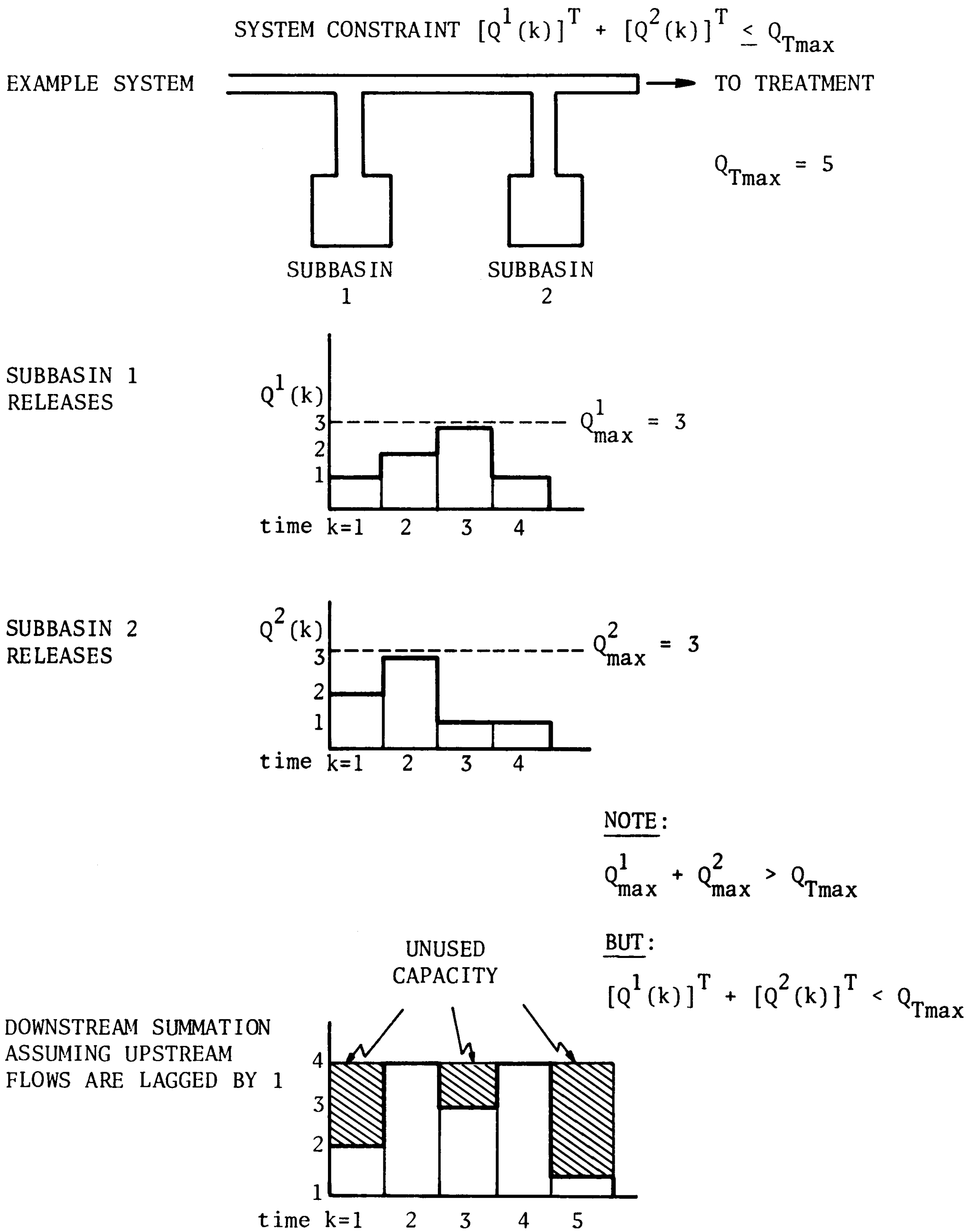


Figure III-6. Illustration of Suboptimality of $\sum Q^1_{max} \le Q_{Tmax}$ as System Constraint

Let

$$\underset{\sim}{Q}_{max} = [Q^1_{max}, Q^2_{max}, \ldots, Q^{10}_{max}] \qquad (19)$$

Particular values of the components of the vector at any iteration ℓ can be represented as $Q^{(\ell)i}_{max}$. Furthermore, let $F(\underset{\sim}{Q}^{(\ell)}_{max})$ represent the total value of the objective function at that point. That is,

$$F(\underset{\sim}{Q}^{(\ell)}_{max}) = \sum_{i=1}^{N} \tilde{O}^{*i}(Q^{(\ell)i}_{max}) \qquad (20)$$

The direction of steepest descent would then be defined as

$$\underline{\hat{S}}^{(\ell)} = \nabla F(\underset{\sim}{Q}^{(\ell)}_{max}) \qquad (21)$$

and proceeding in that direction, a new point is reached, $\underset{\sim}{Q}^{(\ell+1)}_{max}$, where

$$\underset{\sim}{Q}^{(\ell+1)}_{max} = \underset{\sim}{Q}^{(\ell)}_{max} + \lambda \underline{\hat{S}}^{(\ell)} \qquad (22)$$

where step size λ is chosen such that $F(\underset{\sim}{Q}^{(\ell+1)}_{max}) < F(\underset{\sim}{Q}^{(\ell)}_{max})$. Attempts can be made to find an optimal step size λ^* along the direction $\underline{\hat{S}}^{(\ell)}$, but this process can be computationally time consuming on-line. In addition, since the gradient $\nabla F(\underset{\sim}{Q}^{(\ell)}_{max})$ cannot be determined analytically, it must be estimated numerically. This is also computationally time consuming, since perturbation around $\underset{\sim}{Q}^{(\ell)}_{max}$ must be carried out in _all_ component directions, implying that all subbasin control problems must be solved simply to find the gradient direction. All of them would again have to be solved in order to move along the gradient direction determined. Of course, the actual direction chosen would be that direction closest to the numerically approximated gradient direction such that the step size λ along that direction falls on a discrete point. In addition, it must be a feasible direction such that there exists a $\lambda > 0$ such that Eq. (18) is not violated.

A less time consuming approach would be to choose as the next point an adjacent feasible point which lies in a coordinate direction yielding the fastest rate of change of the objective, as illustrated in Figure III-7. That is, choose a coordinate direction $m \in \{1,\ldots,10\}$ (corresponding to a particular subbasin m) such that

$$\underset{\sim}{Q}_{max}^{(\ell+1)} = \underset{\sim}{Q}_{max}^{(\ell)} + \underset{\sim}{e}_m \, \Delta Q_{max}^m \tag{23}$$

where

$$\frac{F(\underset{\sim}{Q}_{max}^{(\ell)} + \underset{\sim}{e}_m \, \Delta Q_{max}^m) - F(\underset{\sim}{Q}_{max}^{(\ell)})}{\Delta Q_{max}^m}$$

$$\leq \frac{F(\underset{\sim}{Q}_{max}^{(\ell)} + \underset{\sim}{e}_j \Delta Q_{max}^j) - F(\underset{\sim}{Q}_{max}^{(\ell)})}{\Delta Q_{max}^j} \leq 0 \tag{24}$$

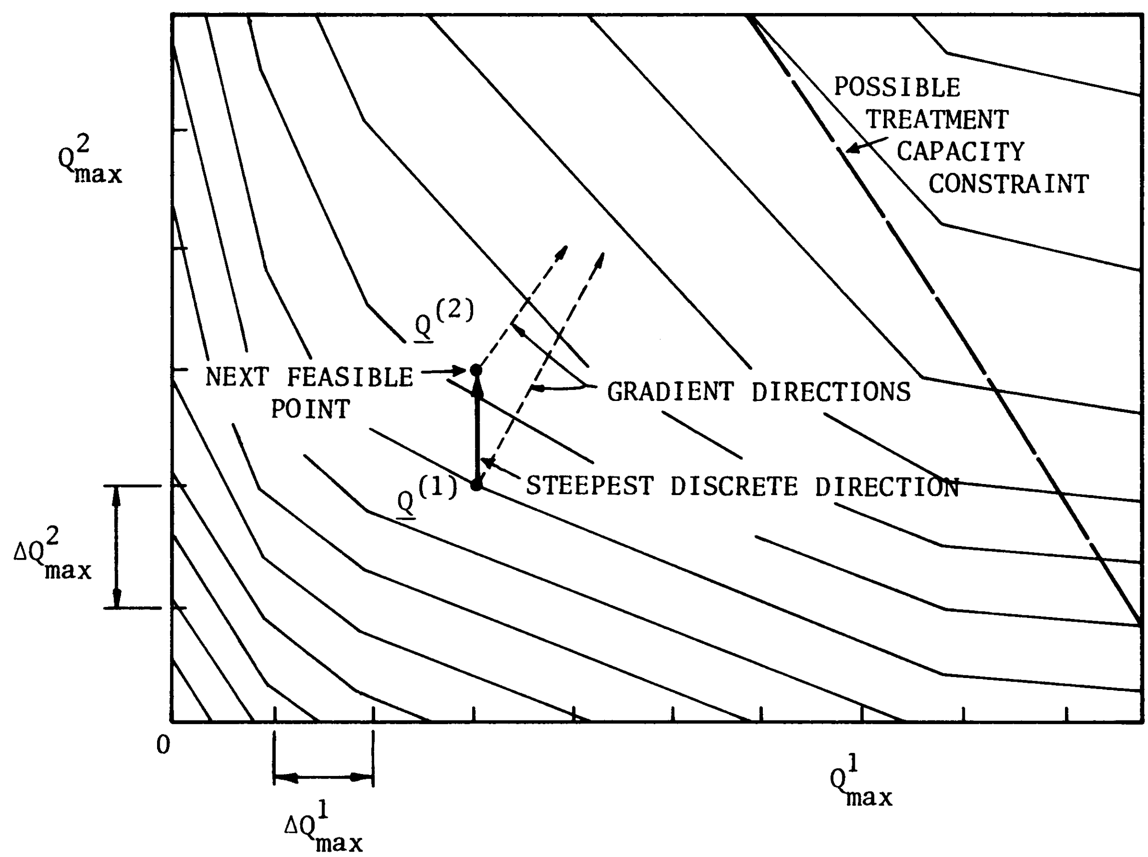
(for all $j=1,\ldots,10$; $j \neq m$, such that (24) is valid)

where

$$\underset{\sim}{e}_j = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (j^{th} \text{ component}) \tag{25}$$

so that $\lambda \triangleq \Delta Q_{max}^j$, $j=1,\ldots,10$.

DISCRETE DIRECTION STEEPEST DESCENT METHOD



Contours represent equal valued combinations of Q^1_{max} and Q^2_{max} within the total objective function of the Master Control Problem

Figure III-7. Illustration of the Master Control Search Algorithm

This method requires only one subproblem evaluation (subbasin optimization) for each step since each coordinate direction is related to a particular subproblem parameter. In addition, no calculation is necessary to locate the next feasible point along the gradient direction.

For the ten subbasin system used for illustrative purposes here, the objective function is the summation of the results of ten subbasin optimizations. Therefore, it is a ten variable (or dimension) master control problem. The discretization used for a subproblem associated with a particular subbasin or equivalently, the step size used in each of the directions relating to each dimension of the master problem, was chosen to be one-tenth of the upper physical limit $\bar{Q}^i_{max}$ on releases from that particular subcatchment. That is:

$$\Delta Q^i_{max} = \frac{\bar{Q}^i_{max}}{10} \tag{26}$$

This was chosen to correspond roughly with the anticipated precision of control in the actual system, as well as for facilitating the subproblem solution. The next adjacent point in the search routine is determined by a simple comparison of objective function improvements for each of the ten subproblems. Computational experience with the algorithm is given in Chapter VI. It is shown that convergence of the technique was quite rapid, despite the simplifying assumptions used. Figure III-8 illustrates the algorithm.

A formulation for the city-wide optimal control problem is now easily developed. Let the three major sections of the city be represented by an index $s = 1,2,3$. Assume that optimal $\underline{Q}^*_{max}$ have been

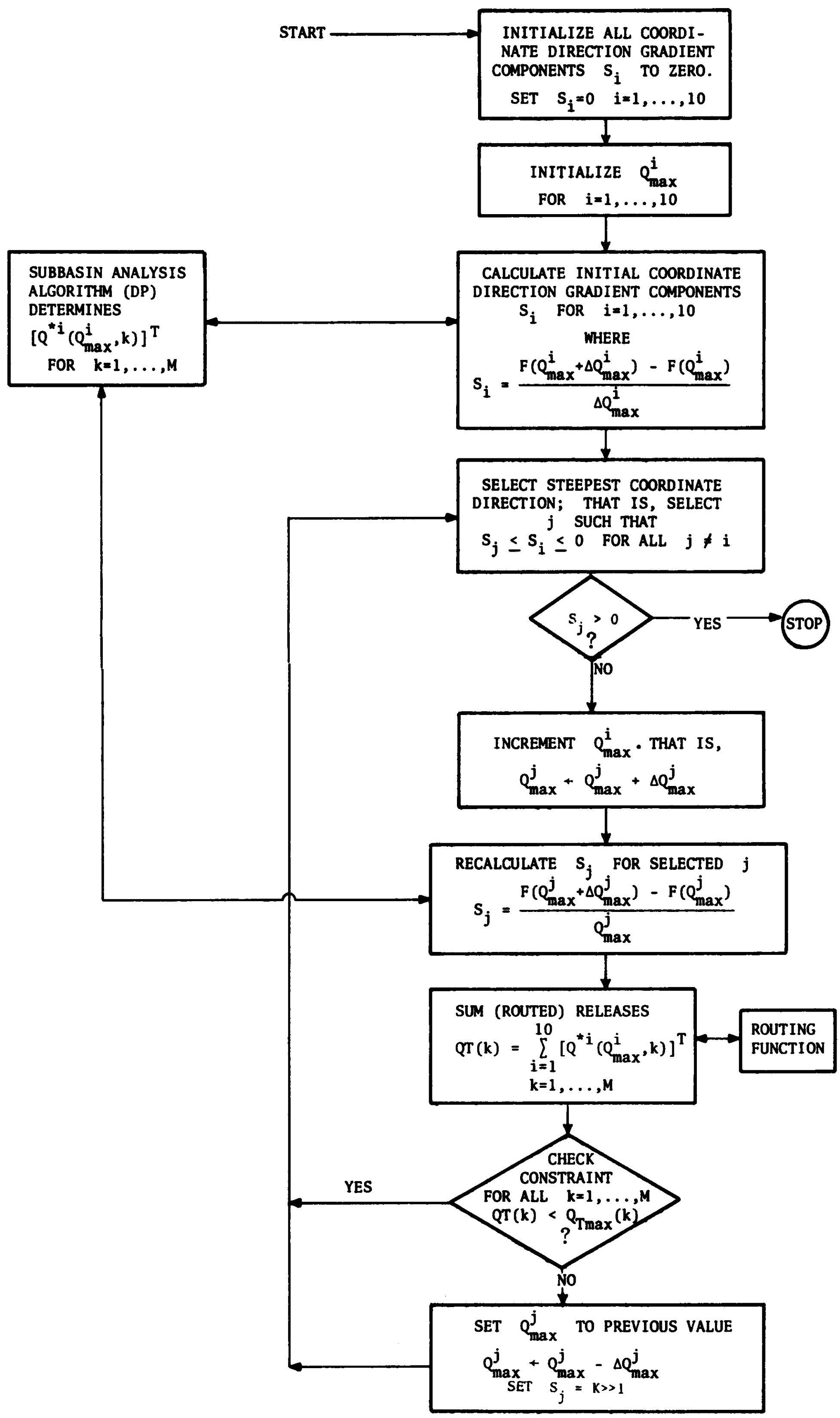


Figure III-8 Master Control Algorithm Flow Chart found for each section of the city, which will hereafter be represented as Q^{*s}_{max}, for specified levels of total treatment plant capacity Q^{s}_{Tmax}. Using the definition of Equation (20), let

$$F^{s}(Q^{s}_{Tmax}) = \sum_{i=1}^{N_s} \tilde{O}^{*i}(Q^{*s}_{max}) \tag{27}$$

It is now possible to write the supra problem for the entire city as:

<u>Supra Problem</u>

$$\underset{Q^{s}_{Tmax},\ s=1,2,3}{\text{Minimize}} \quad \sum_{s=1}^{3} F^{s}(Q^{s}_{Tmax}) \tag{28}$$

Subject to:

$$\sum_{s=1}^{3} Q^{*s}_{total}\ (Q^{s}_{Tmax},k) \leq \overline{Q}_{Tmax} \tag{29}$$

$$(\text{for} \quad k=1,\ldots,M)$$

where

$$Q^{*s}_{total}(Q^{s}_{Tmax},k) = \sum_{i=1}^{N_s} [Q^{*i}(Q^{i}_{max},k)]^{T} \tag{30}$$

$$(\text{for} \quad k=1,\ldots,M)$$

$\overline{Q}_{Tmax}$ = total treatment plant capacity

The index N_s indicates the number of subbasins found in section s. This problem is solvable by the application of the same assumptions and methods used for the master control problem for Richmond-Sunset.

Such a formulation, however, may not be capable of finding the global optimal solution for the entire city. The assumption that each Q^{s}_{Tmax} term is time invariant puts an additional constraint upon the total solution. Such a constraint may eliminate a possibly better solution which is based upon time varying Q^s_{Tmax} terms. Replacing each Q^s_{Tmax} term with a vector $\underline{Q}^s_{Tmax}$ containing values of Q^s_{Tmax} for each period of time would permit the retention of the basic form of the above supra problem formulation. That is, let $\underline{Q}^s_{Tmax} = (Q^s_{Tmax}(1), \ldots, Q^s_{Tmax}(M)$. If this was done, however, the dimensionality of the master problem would be increased significantly since separate Q^i_{max} values would then have to be derived for each time period within each subbasin as well as for each subbasin location. For this development, therefore, a time invariant Q_{Tmax} term has been assumed for the master problem while the solution of the city wide problem was not attempted.

E. IMPACT OF ALGORITHMS UPON COMPUTER HARDWARE

It has been mentioned earlier that an implicit constraint upon the choice of control algorithms has been the computer power available. In the past, computer process control has been most often carried out in a centralized manner using a single computer. "The trend now, however, seems to be toward using many small scale computers, perhaps tied together by a large central processor, rather than to centralize all computer control operations for an entire plant within a single large-scale process computer" [7]. In research efforts directly related to work presented herein, Trimble [8] has summarized the basic advantages of distributed computer power. Some of the more significant points are as follows:

1. Studies have indicated that a distributed computer system, equivalent in computer power to a single large-scale computer, can cost 20 to 30 percent less than the large-scale system [9].

2. The cost impact of necessary system redundancy is more severely felt with a single large-scale computer facility than with a distributed system. A single computer must be duplicated for redundancy whereas with distributed facilities, a local failure does not threaten the entire system, and consequently, the system requires a lower total level of redundancy.
3. Distributed computational power permits parallel computations. The possibility of having separate computers for each subsystem enables the simultaneous solution of subsystem problems. To duplicate the resultant speed of convergence for the master control problem within a large central computer facility would require a memory larger than the total memory of an equivalent distributed system. Computer word size and processor speed would need to be increased to manage this larger core and match the speed of the distributed system.
4. Distributed processing configurations to achieve hierarchical control may either be geographically distributed or centralized. Tradeoffs between the necessary system communications costs (which are greater with a centralized facility) and vulnerability-backup considerations (which are more significant with a geographically distributed system) must be made.

For additional, more specific recommendations on computer hardware configurations and further discussion on the implications of hierarchical computer control, the reader is referred to Grigg, _et al_. [7].

Based upon the above discussion, the decomposition techniques developed here, although probably not the only way of disassociating various parts of the control strategy problem for the San Francisco master plan, seem well suited for distributed computer control. Some reasons for this are as follows:

First, the division into distinct levels of optimization is paralleled exactly by the physical divisions found in the system. In addition, the hierarchical strategy presented in the previous pages is ideally suited for inclusion within a distributed computer system. The dendritic cascade of problems and subproblems down to the level of detailed decisions determining the actual system physical controls, points to a parallel computer hardware hierarchy. A large-scale approach based on temporal aggregations, for example, in which finer solutions corresponded to increasingly finer divisions of time, would not lend itself to a parallel physical differentiation; hence, its intuitive appeal would be less.

Second, since the divisions correspond to physical subsections of the city, the possibility exists for the optimizations to be carried out locally by computer hardware located at or near the subbasin. This will decrease the extent of the needed data link systems between the components being optimized and the computer doing the optimization. This is of particular importance to a system as large as that planned in San Francisco and for one in which constant remote sensing of system status is needed for efficient real-time control.

Third, as each subbasin could have its own computer, failures in components are easier to handle. The remainder of the system can continue to operate despite a local failure.

Fourth, the decision speed necessary in a real-time control environment will demand that optimizations be carried out in parallel across the entire system and not in series. That is, with the distributed computational power suggested above many phases of the total system optimization can go on simultaneously. This would effect a tremendous time savings over other large-scale approaches which sequentially optimize the entire system using first large time increments (or flow discretizations) and then finer divisions for the final solution.

And finally, the hierarchical control suggested above will permit the tailoring of computer needs to the exact job to be done. Simple minicomputers are envisioned for many of the lower levels of optimization.

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Chapter IV

SUBBASIN CONTROL ALGORITHMS

A. INTRODUCTION

In the previous chapter, it was shown how the large-scale control problem could be decomposed into master problems and subbasin problems. It is the solution of these subbasin problems in a manner amenable to on-line use that is to be considered in this chapter. The system inflows are again assumed to have been previously forecasted, with probabilistic information on the forecasts also given. The risks inherent in these forecasts are, therefore, considered. Two subbasin optimal control algorithms are presented. One is a rough, aggregated approach intended to be used for purposes of dialogue with the master control problem. It is required for facilitating the discrete direction steepest descent procedure. It enables rapid interaction with the master control problem so as to obtain an approximate city-wide control within the time allotted. The solution of this simplified problem then forms the basis for a more detailed algorithm for derivation of the needed system controls, which need only be computed once per time interval. Stochastic dynamic programming is used to solve the simplified (aggregated) problem, while it is proposed that the detailed subbasin control algorithm use a stochastic linear programming approach.

A typical subbasin problem is formulated first as a dynamic programming (DP) problem and then as a linear programming (LP) problem in order to illustrate the rationale for the above described division of roles and to justify the selection of techniques for each role. Both techniques are subsequently extended to include the consideration of risk and the resultant formulations are compared. Methods capable of reducing the problem size are applied to both techniques and the effect upon the solution is observed.

In general, the completely detailed DP formulation is too time consuming for on-line work, with or without the consideration of risk. The completely detailed LP formulation is, however, often too large for the limited computer facilities envisioned. A *simplified* stochastic DP which considers the inherent risk is consequently developed which is fast enough to permit the interaction needed with the master control problem. This interactive stochastic optimization is illustrated in Figure IV-1. The final solution of this problem, after convergence of the master control problem, decreases the number of unknowns in the total detailed problem. This reduced problem can then be efficiently solved once by a stochastic LP algorithm, yielding the needed system controls. The detailed stochastic optimization is illustrated below the interactive stochastic optimization in Figure IV-1. The communication of the interactive problem with the master control problem was illustrated in Figure III-5 in the previous chapter.

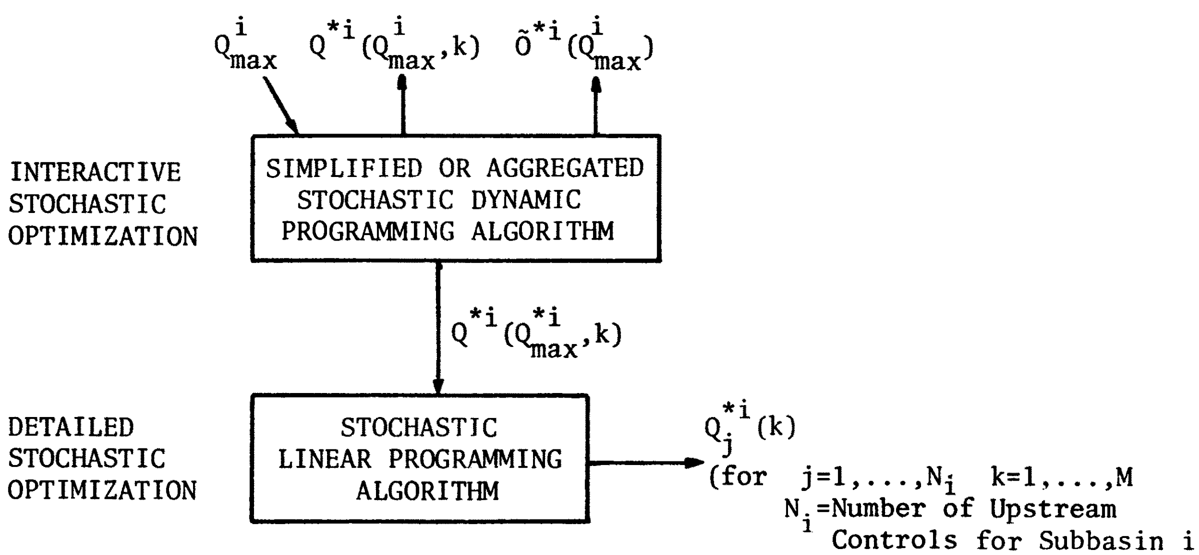


Figure IV-1. Subbasin Analysis Hierarchy.

B. EXAMPLE SUBBASIN CONTROL PROBLEM

A typical subbasin consisting of four detention basins is chosen to facilitate the exposition of the general optimal control problem pertinent to all the subbasins. Figure IV-2 illustrates the configuration of detention basins found in subbasin 10 defined in Figure III-2. It is located at the southernmost extreme of the Richmond-Sunset area along Brotherhood Way, south of Lake Merced. The component detention basins have been labeled 1, 2, and 3 for simplicity; however, Table VI-5 provides the corresponding identification used in the San Francisco Master Plan.

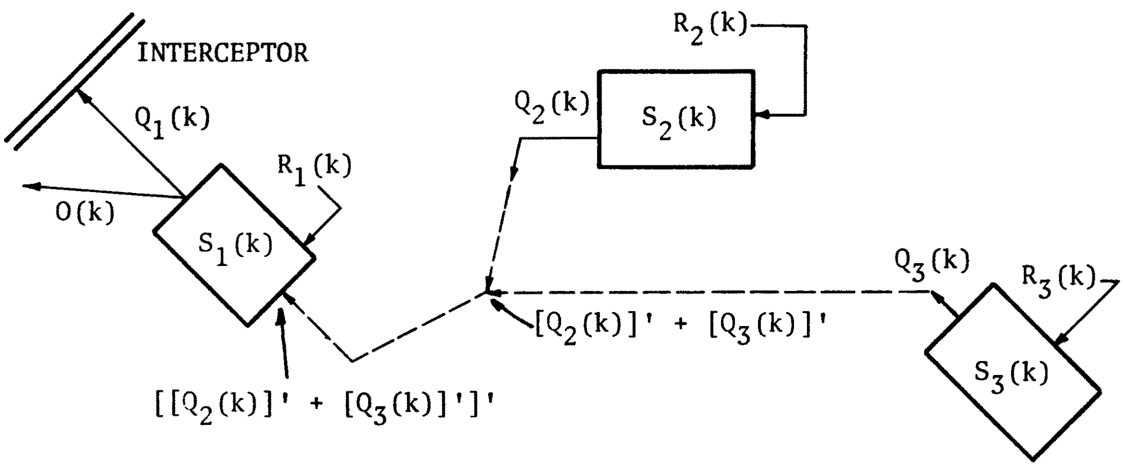


Figure IV-2. Example Subbasin.

In the above figure, the superscript indicating the particular subbasin has been dropped in order to facilitate the following discussion. The subscripts indicate the element within the subbasin to which the variable refers. The variables shown in the above figure have the following meanings:

$Q_j(k)$	= Average rate of throughflow during period k from detention basin j with $Q_1(k)$ entering the interceptor sewer as $Q^i(k)$, from subbasin i
$S_j(k)$	= Storage in detention basin j at beginning of time k
$[Q_j(k)]'$	= Average rate of routed or translated throughflow from detention basin j during time k
$R_j(k)$	= Average rate during time k of stormflow inflow (runoff) to detention basin j
$O_j(k)$	= Average rate of overflow to receiving waters from detention basin j during time period k
$\dashrightarrow$	= Releases routed downstream
$[Q_2(k)]'+[Q_3(k)]'$	= Routed flows summed at juncture

where

$$[Q_i(k)]' = \sum_{m=0}^{\tau} a_m^i B^m Q_i(k), \text{ for } \tau \leq k-1$$

Each of these above variables has an upper limit determined by the design of the physical system. Table IV-1 shows the upper limits S_{jmax} and Q_{jmax} for variables $S_j(k)$ and $Q_j(k)$ respectively, which are derived from the description of Alternative C of the San Francisco Master Plan.

Table IV-1. Data for Subbasin #10.

Detention Basin No.	S_{jmax} $ft^3 \times 10^{-6}$	Q_{jmax} cfs
1	0.24	107
2	0.27	380
3	0.36	432

The system dynamics for such a configuration are easily expressed by the continuity relationships for each detention basin, which link the reservoir contents in time.

Continuity Relationships:

$$
\begin{aligned}
S_1(k+1) &= S_1(k) + R_1(k) + [[Q_2(k)]' + [Q_3(k)]']' - Q_1(k) - O(k) \\
S_2(k+1) &= S_2(k) + R_2(k) - Q_2(k) \\
S_3(k+1) &= S_3(k) + R_3(k) - Q_3(k) \\
&\qquad (k=1,\ldots,M)
\end{aligned}
\tag{1}
$$

where there are also

Initial Conditions:

$$
S_j(1),\ Q_j(1),\ [[Q_2(1)]' + [Q_3(1)]']' \qquad (j=1,\ldots,N) \tag{2}
$$

and,

Upper and Lower Bounds:

$$
\left.
\begin{aligned}
0 &\le S_j(k) \le S_{jmax} \\
0 &\le Q_j(k) \le Q_{jmax} \\
0 &\le [Q_j(k)]' \le Q_{jmax} \\
O(k) &\ge 0
\end{aligned}
\right\} \quad (j=1,2,3;\ \ k=1,\ldots M) \tag{3}
$$

The general control problem for such a subbasin would be to minimize the expected effects of overflows from the sewer system while satisfying as much as possible the interceptor capacity, Q^i_{max}, for that subbasin. Such an objective has been formulated in the previous section as the total subbasin objective function. The problem, then, is written

$$\underset{\substack{S_j(k+1),\ Q_j(k),\ O(k), \\ k=1,\ldots,M;\ \ j=1,2,3}}{\text{Minimize}} \sum_{k=1}^{M} [w(k)(O(k))^n - y(k)Q_1(k)] \tag{4}$$

subject to: the continuity constraints, Equation (1); initial conditions, Equation (2); and limits on appropriate variables, Equation (3).

C. INTERACTIVE STOCHASTIC OPTIMIZATION

Dynamic programming is a mathematical programming technique that can be valuable for solving sequential decision problems where the objective function and the constraints are nonlinear and/or when the solution space is nonconvex. The subbasin control problem belongs to this class of problems since it involves sequences of decisions in time and space. More important for this study, however, is that dynamic programming is valuable in some cases for solving certain linear problems when an approximate discretized solution is adequate. That is, LP will find exact solutions, whereas DP involves discretization of the variables, which may save computer time considerably. If the objective function can be framed as a series of separable terms, each term of which depends on at most one state variable, and the process dynamics can be described by a first-order Markovian relationship, then the problem may be solved as a one-dimensional problem despite the possible existence of many control variables. Furthermore, constraints do not add to the complexity of the problem, but rather often tend to reduce the computational load by eliminating the consideration of infeasible solutions. Figure IV-3 conceptually illustrates sequential decision problems of this type.

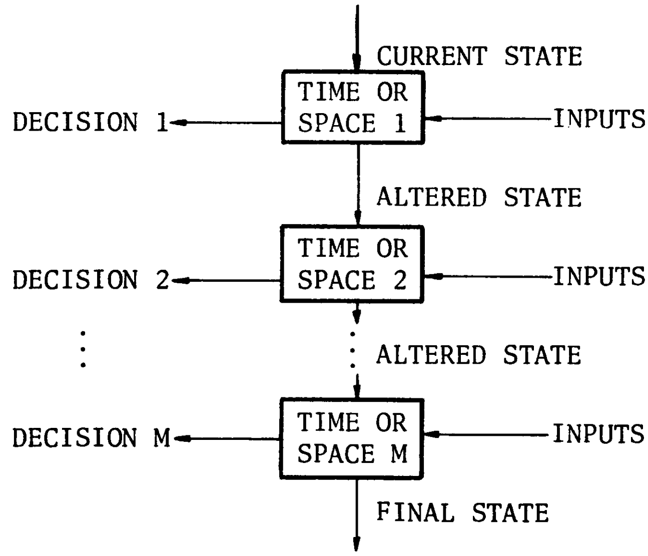


Figure IV-3. Sequential Decision Problem.

A general format for initial-value sequential decision problems solvable by dynamic programming is:

$$\min_{\substack{\underline{u}_i,\ \underline{x}_{i+1} \\ i=1,\ldots,M}} \sum_{i=1}^{N} f_i(\underline{x}_i, \underline{u}_i) + \phi(\underline{x}_{N+1}) \qquad (5)$$

where:

$$\begin{aligned} &\underline{x}_1 = \underline{c} \text{ (given)} \\ &\underline{x}_{i+1} = \underline{g}_i(\underline{x}_i, \underline{u}_i) \\ &\underline{x}_{i+1} \in X_{i+1} \\ &\underline{u}_i \in U_i \\ &\underline{h}_i(\underline{x}_i, \underline{u}_i) \leq 0 \\ &(i=1,\ldots,N) \end{aligned} \qquad (6)$$

In the above formulation, $\underline{u}_i$ represents a vector of decision variables at stage i and $\underline{x}_i$ represents the vector of state variables at stage i; however, stage may be defined in time or space.

The initial state $\underline{x}_1$ is assumed given as some constant vector $\underline{c}$. The effect of the decision vector on the subsequent state vector is represented by $\underline{g}_i(\underline{x}_i,\underline{u}_i)$; while $\underline{h}_i(\underline{x}_i,\underline{u}_i)$ represents constraints upon $\underline{x}_i$ and $\underline{u}_i$ in combination. Sets U and X simply specify the sets individually constraining the decisions and states considered.

Associated with such sequential decision problems are recursive relations which enable an optimal solution to be found in a systematic way, thereby avoiding the necessity of searching through all possible sequences of decisions. Such a relationship in general form is [12]:

$$F_j(\underline{x}_j) = \min_{\underline{u}_j \in U_j} \{f_j(\underline{x}_j,\underline{u}_j) + F_{j+1}(\underline{x}_{j+1})\} \tag{7}$$

$$\underline{x}_{j+1} = \underline{g}_j(\underline{x}_j,\underline{u}_j) \in X_{j+1} \tag{8}$$

$$\underline{h}_j(\underline{x}_j,\underline{u}_j) \leq 0$$

$$F_{N+1}(\underline{X}_{N+1}) = \phi(\underline{X}_{N+1})$$

where $F_j(\underline{x}_j)$ is the optimal (or minimum) objective value for the sequential decision process, starting with decision point (stage) j and a known state vector $\underline{x}_j$ at that stage. It will now be shown that the above terms have equivalent terms associated with the subbasin control problem:

i — In the above standard format, the index i signifies the *stage* of the decision making process. For the control of the linked detention reservoirs, this can be either the particular time interval k in which the control decisions are made for all locations *or* the location of a particular detention reservoir for which decisions for all time intervals are made.

$\underline{x}_i$ — The state vector $\underline{x}_i$ in the above formulation has direct counterparts in the subbasin problem; namely $\underline{S}_i$, representing contents for all times at location i, *or* a vector $\underline{S}(k)$, representing contents for all locations at a particular time k.

$\underline{u}_i$ The control vector variable $\underline{u}_i$, like the state vector, takes on slightly different meaning, depending on the definition of the state variable. Again, as with the states above, this variable is a vector of release decisions in a multi-detention basin subbasin problem, represented as vectors $\underline{Q}_i$ or $\underline{Q}(k)$.

f_i The terms in the objective function f_i are expressed above as a function of the state and control vectors. In the sub-basin problem, the objective function described previously is recalled as being a linear function of the downstream release $Q_1(k)$, which is the subbasin release to the interceptor, and $O(k)$ represents overflows. If a new $Q_1(k)$ is defined as the previous $Q_1(k)$ plus $O(k)$, and $O(k)$ is defined as,

$$O(k) = 0, \text{ if } Q_1(k) \leq Q_{1max} \tag{9}$$

and $$O(k) = Q_1(k) - Q_{1max} \text{ if } Q_1(k) > Q_{1max}$$

then the objective function can be expressed in terms of control variable $Q_1(k)$.

$\underline{g}_i$ The dynamic relationship $\underline{g}_i$ relates the state vector at one stage to that at the next. If stages are chosen as steps in time, then the continuity equations as expressed earlier suffice. If, however, the stage is chosen as location, different equations must be written relating the reservoirs in space.

X_{i+1}, U_i represent the sets of all possible state levels and control decisions, respectively. For the subbasin problem, they are upper and lower bounds on storage and release, respectively.

The previous discussion has left open the question of which stage definition to use. In problems with many detention basins and relatively few time periods, it is better to use location as stage and thereby reduce the dimensionality of the state vector, which would then extend in the time domain. The number of stages affects problem solution linearly, while the number of states has an exponential effect on solution time requirements. For subbasin analysis in general, it appears that the number of time periods will usually exceed the number of detention basins, so that time is best considered as stage. This permits the retention of the original argument k as the stage indicator. That is

$\underline{S}(k)$ = vector of all detention basin levels at time period or stage k within a particular subbasin

$\underline{Q}(k)$ = vector of all release decisions at time or stage k within a particular subbasin.

The criterion function for stage k is now easily defined as:

$$f_k(Q_1(k)) = w(k)(Q_1(k) - Q_{1max})^n - y(k)\ Q_{1max} \qquad \text{if } Q_1(k) > Q_{1max} \tag{10}$$

and

$$f_k(Q_1(k)) = -y(k)\ Q_1(k) \qquad \text{if } Q_1(k) \leq Q_{1max} \tag{11}$$

The total objective function then becomes,

$$\underset{\substack{\underline{Q}(k)\ \underline{S}(k+1) \\ k=1,\ldots,M}}{\text{Min}} \sum_{k=1}^{M} f_k(Q_1(k)) \tag{12}$$

The dynamic relationship relating the state vector at one stage to the state vector at the subsequent stage remains to be defined for the completion of the dynamic programming formulation. Its functional form in the no routing case is simple:

$$\underline{S}(k+1) = \underline{g}(\underline{S}(k),\ \underline{Q}(k),\ \underline{R}(k)) \tag{13}$$

The actual form of the component functions, however, are dependent upon the subbasin configuration chosen for analysis, and if routing effects are considered. The continuity relationships developed earlier for the example problem Equation (2) can be rewritten in the no routing case as:

$$S_1(k+1) = S_1(k) + R_1(k) + Q_2(k) + Q_3(k) - Q_1(k) - O(k)$$

$$S_2(k+1) = S_2(k) + R_2(k) - Q_2(k) \qquad (14)$$

$$S_3(k+1) = S_3(k) + R_3(k) - Q_3(k)$$

Again, redefining the downstream release decision $Q_1(k)$ as the sum of the release to treatment and the overflow amount, the first equation can be rewritten as:

$$S_1(k+1) = S_1(k) + R_1(k) + Q_2(k) + Q_3(k) - Q_1(k) \qquad (15)$$

This is a simple Markovian relationship well suited for dynamic programming. The recursive relation for the dynamic programming solution to the optimization problem can then be written (omitting the constraints on $Q(k)$ to simplify the presentation) as:

$$F_k(\underline{S}(k)) = \min_{\underline{Q}(k)} [f_k(Q_1(k)) + F_{k+1}(\underline{S}(k+1))] \qquad (16)$$

The inclusion of even simple routing, however, adds significantly to the problem size. Consider a simple routing in which the inflow to reservoir 1 from the upstream detention basins during time k is not simply $Q_2(k) + Q_3(k)$, but actually the weighted average of the releases at time k and the previous time $k-1$. The inflow into detention basin 3 would then be,

$$a_1(Q_2(k) + Q_3(k)) + a_2(Q_2(k-1) + Q_3(k-1)) \qquad (17)$$

where, $a_1 + a_2 = 1$

The dynamic relationship for the first detention basin is therefore

$$S_1(k+1) = S_1(k) + R_1(k) + a_1[Q_2(k) + Q_3(k)] + a_2[Q_2(k-1) + Q_3(k-1)] \tag{18}$$

or, in general, for the entire subbasin

$$\underline{S}(k+1) = \underline{g}_k(\underline{S}(k), \underline{Q}(k), \underline{Q}(k-1), \underline{R}(k)) \tag{19}$$

This non-Markovian relationship spanning three time periods complicates matters. In order to overcome the linking of three time periods, an added state vector is defined.

$$\underline{P}(k) \triangleq \underline{Q}(k-1) \tag{20}$$

This enables the creation of two first-order Markovian dynamic relationships replacing the previous relationship. They take the form,

$$\underline{S}(k+1) = \underline{g}_k(\underline{S}(k), \underline{Q}(k), \underline{P}(k), \underline{R}(k)) \tag{21}$$

$$\underline{P}(k+1) = \underline{Q}(k)$$

The dimensionality of the problem, therefore, grows with even simple linear routing techniques. As demonstrated here, a new state variable is added for each routed flow and for each step back in time the routing includes. The recursive relation for such a formulation is therefore written as:

$$F_k(\underline{S}(k), \underline{P}(k)) = \min_{\underline{Q}(k)} [f_k(\underline{Q}(k) + F_{k+1}(\underline{S}(k+1), \underline{P}(k+1))] \tag{22}$$

The consistent simplicity of the control terms, despite the growing list of state variables, suggests the application of some form of incremental dynamic programming [5] with its requirement of *system invertability*. That is, there must exist functions $\underline{g}^{-1}$ such that

$$\underline{Q}(k) = \underline{g}_k^{-1}(\underline{S}(k),\ \underline{S}(k+1),\ \underline{P}(k),\ \underline{R}(k)) \tag{23}$$

The recursive relation in this case would take the form:

$$F_k(\underline{S}(k),\ \underline{P}(k)) = \underset{\underline{S}(k+1)}{\text{Min}} \quad [f_k(\underline{Q}(k) + F_{k+1}(\underline{S}(k+1)\ \underline{P}(k+1))] \tag{24}$$

Without this, time consuming successive approximation procedures are required. Such a formulation, however, depends on the existence and uniqueness of $\underline{g}^{-1}$. Flow hydraulics indicate that $\underline{g}^{-1}$ is often not easily defined, since it includes upstream routing or the reconstruction of an upstream hydrograph from its downstream counterparts. Thus, incremental dynamic programming is difficult to apply when routing is included.

Heidare, et al., [6] have applied a more general form of incremental dynamic programming called *discrete differential dynamic programming* to a linked reservoir problem. They overcome the invertability problem by neglecting routing in their demonstration problem while suggesting an iterative successive approximation method for cases where invertibility does not exist.

The dimension of the resultant DP subbasin problems formulated to solve this sequential decision problem has remained a problematic area for on-line considerations. The *curse of dimensionality* has been approached by attempts to further decompose the subbasin problems into separate one-dimensional problems through use of orthogonal polynomials which approximate the upstream releases. A master or *outer* optimization problem adjusts the coefficients of the polynomials until convergence to the separately derived control policies (releases) is achieved. This *flow projection technique* suggested by Labadie et al., [7] is, however, still too time consuming for on-line work.

C.1 Aggregated Subbasin Solution

Various techniques are available for disassociating complex systems into sets of smaller problems. The two approaches used in this work are decomposition and aggregation. They differ basically in the orientation and purpose of the subproblems. Decomposition has been successfully applied when a means exists to decouple the system elements, thereby enabling individual solutions to subproblems. Such a means exists when considering subbasin interaction based entirely upon the interceptor constraint. Such considerations led to the master problem developed in the previous chapter. Within the individual subbasins, however, there is an intimate relationship between all the control and state variable. No efficient means has been found for decoupling the individual elements of such systems. Aggregation, however, provides a way for solving increasingly more detailed problems by using the solution of simpler less detailed system models as overall solution envelopes.

Bradford [8] applied an aggregation technique, based in part on the theoretical work of Aoki [9], for operation of the entire system of reservoirs associated with the San Francisco Master Plan. Aoki provides a mathematical basis for the intuitively clear concept of solving more finely detailed problems based on the solutions of higher level *aggregated* problems. Bradford applied aggregation repeatedly in a multi-level procedure for determining the control for this large system of reservoirs considered here. The storage capacities of detention basins in localized areas were aggregated to create larger conceptual detention basins for which controls were more easily derived. Mathematically, the initial step of aggregation is straightforward. Letting $\underline{S}(k)$ be the complete vector of state values or detention reservoir contents at time k, with a dimension N, the aggregation is achieved by multiplying this vector by a constant matrix C of dimension LxN, where L is the dimension of the aggregated state vector, represented by $\underline{Z}(k)$. Therefore:

$$\underline{Z}(k) = C\ \underline{S}(k) \tag{25}$$

For example, aggregating a six reservoir system illustrated in Figure IV-4 into a three aggregated reservoir system is accomplished by defining C as,

$$C \triangleq \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \tag{26}$$

Multiplying the six element state vector $\underline{S}(k)$ results in the three element aggregated state vector $\underline{Z}(k)$. That is,

$$\underline{Z}(k) = \begin{bmatrix} Z_1(k) \\ Z_2(k) \\ Z_3(k) \end{bmatrix} = \begin{bmatrix} S_1(k) + S_3(k) \\ S_2(k) \\ S_4(k) + S_5(k) + S_6(k) \end{bmatrix} \tag{27}$$

Note that one of the aggregated systems contains only one of the original system components while another contains three. The choice of the aggregation matrix C would be determined by the system configuration as well as the strategy anticipated for the disaggregation of the solution. In this example, once a control policy is derived for each of the aggregated detention basins, an additional strategy must be developed for deriving controls for the actual distributed (disaggregated) system components if, of course, they have more than one element.

Dynamic programming was chosen for the overall subbasin analysis problem since a one detention basin stochastic DP problem is readily solvable on-line. This requires the complete aggregation of subbasin components into a single conceptual reservoir.

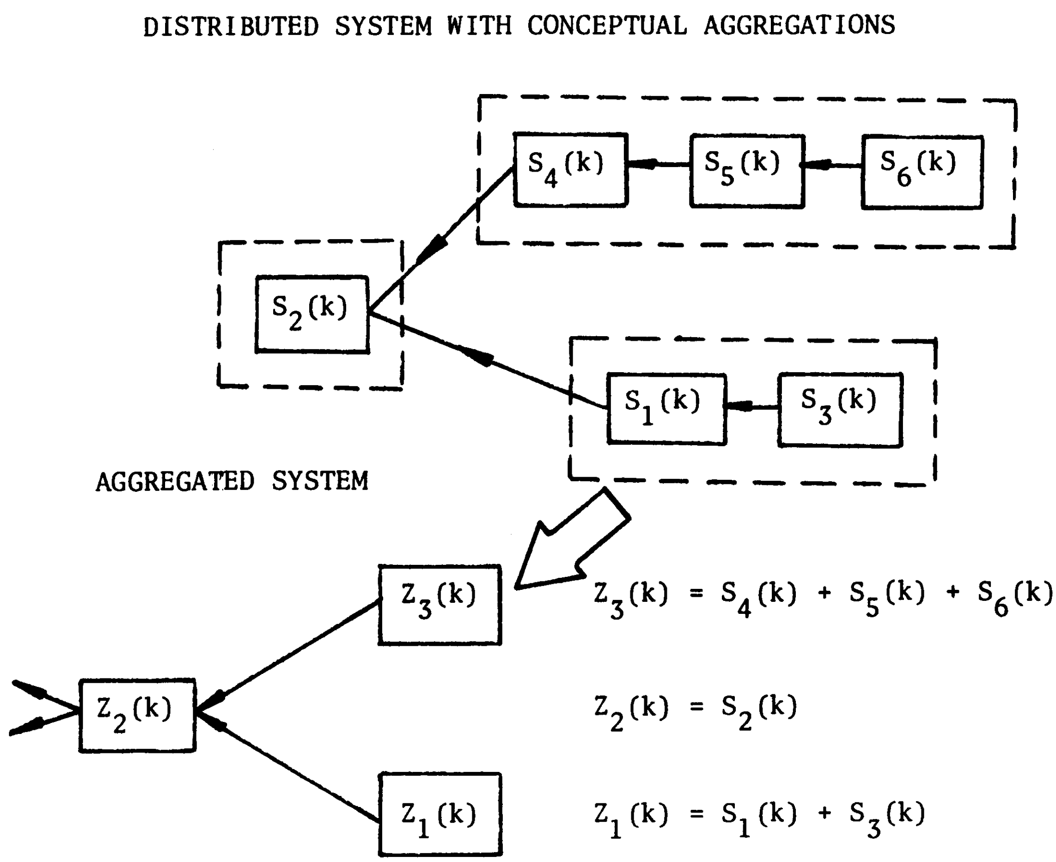


Figure IV-4. Aggregation of System Components.

This approximation, although rough, is justified by its use for interaction or dialogue with the master control problem. The detailed solution for the component releases is achieved by the use of an additional optimization after the interactive phase is completed. In the example above, the needed aggregation is illustrated by using a C matrix defined as,

$$C \triangleq [1\ 1\ 1\ 1\ 1\ 1]$$

Therefore, only one aggregated detention basin results, which is synonymous with the entire subbasin. Its state and control, after again dropping the subbasin designation superscript, are defined by

$$\begin{aligned} S(k) &\triangleq \sum_{j=1}^{N} S_j(k) \\ Q(k) &\triangleq Q_1(k) \end{aligned} \tag{28}$$

where

j = index on particular detention basin within the subbasin

N = number of detention basins in the subbasin

A dynamic programming solution to this aggregated problem is easily carried out. Since routing effects between the detention basins are neglected in this aggregation, the entire problem is simply a one-dimensional DP that is identical to the previous developments, with the vector notation dropped. The objective function is identical, being simply

$$\underset{\substack{S(k+1),\ Q(k)\\ k=1,\ldots,M}}{\text{Minimize}} \quad \sum_{k=1}^{M} f_k(Q(k)) \tag{29}$$

Subject to

$$0 \leq Q(k) \leq Q^i_{max}, \quad k=1,\ldots,M$$

The dynamic relation becomes simply

$$S(k+1) = S(k) - Q(k) + R(k), \quad k=1,\ldots,M \tag{30}$$

since all upstream effects are contained in the solution. And, the recursive relation is

$$F_k(S(k)) = \underset{Q(k)}{\text{Min}} \left[f_k(Q(k)) + F_{k+1}(S(k+1))\right] \tag{31}$$

The solution of this one-dimensional dynamic programming problem yields the sum of the weighted overflows $\tilde{O}^i(Q^i_{max})$ and the optimal subbasin release pattern $Q^{*i}(Q^i_{max},k)$ for a given value of Q^i_{max}. The superscript indicating the particular subbasin has been restored to emphasize the relationship between this subbasin problem and the decomposition approach discussed in the previous chapter.

The aggregated solution assumes instant availability of upstream inflows. This then eliminates the advantage of in-system delays of flows caused by upstream storage and routing effects. The maximum release from the actual system will, therefore, be less than or equal to that maximum release derived for the aggregated system. The actual solution, then, of the total subbasin problem will be within the envelope of the aggregated subbasin solution described above.

The derivation of the actual system controls for the aggregated problem is addressed in a following section.

C.2 Aggregated Stochastic Dynamic Programming

In the previous formulations describing dynamic programming approaches to subbasin analyses, the risk inherent in a particular control decision has not been included. The dynamic programming solution to the detailed subbasin problem has been shown to be unsuitable for on-line use due to the excessive time requirement implied by the dimensionality of the resultant problem. The inclusion of risk would tend to accentuate this problem.

The smaller, aggregated or lumped parameter model, however, is well suited for on-line use. Simple considerations of risk can, therefore, be introduced into this model.

The original subbasin objective was to minimize the expected value of weighted releases (i.e., throughflows and overflows). This is simply included in the DP objective function for the aggregated problem as

$$\min_{\substack{Q(k),\ S(k+1)\\ k=1,\ldots,M}} \ \underset{R(k)}{E}\left[\sum_{k=1}^{M} f_k(Q(k))\right] \tag{32}$$

The recursion relation is

$$F_k(S(k)) = \min_{Q(k)} \ \underset{R(k)}{E}\left[f_k(Q(k)) + F_{k+1}(S(k+1))\right] \tag{33}$$

The expectation is now seen to be taken over the various values of a particular random component of the inflow series. The random components of the inflow series have thereby been considered independent. It has been assumed that the forecasting of inflows has incorporated the correlation in time and space between the various inflows. With the basic correlative structure between the random variables thereby built into the forecast, it has been further assumed that the residual randomness in the individual inflows is independent.

The expectation can be explicitly written in terms of the probabilities of the possible inflow variations around the forecasted value.

$$F_k(S(k)) = \underset{Q(k)}{\mathrm{Min}} \sum_{j=1}^{m} [P_j(R^j(k))(f_k(Q(k) + F_{k+1}(S(k+1))] \qquad (34)$$

where

j = index on possible discrete outcomes of the random variable $R(k)$

m = number of discretizations used in probability distributions describing random variable $R(k)$

P_j = probability of the inflow being within the discretized range of $R^j(k)$

The probability distribution has been assumed known for the random variables at each location and time. The description of the probability distribution is based upon the assumed accuracy of the forecast. For example, if a particular forecasted value is only assumed to be within 50 percent of the actual value, then a flat (uniform) distribution between 1/2 and 1-1/2 of the forecasted value would be a suitable distribution describing the probabilities. It can be seen that the criterion function term at stage k, for a known state and selected release, is independent of the random variable $R(k)$. The resultant state is not, however, independent of $R(k)$, being related through the dynamic relationship. That is,

$$S(k+1) = S(k) - Q(k) + R(k) \qquad (35)$$

Therefore, since the expected value of the sum of a random term and a deterministic term is simply the expected value of the random term added to the deterministic term, Equation (7) can be rewritten as:

$$F_k(S(k)) = \underset{Q(k)}{\mathrm{Min}} \left[f_k(Q(k)) + \underset{R(k)}{E} F_{k+1}(S(k+1))\right] \tag{36}$$

Such a formulation will only yield optimal $Q^*(1)$, assuming $S(1)$ is given. The optimal $Q^*(k)$, k=2,...,M, cannot be determined because succeeding values of $S(k)$, k=2,...,M, are not known, a priori. That is, even though optimal $Q^*(k)$ have been stored as a function of $S(k)$, the values of $S(k)$ for $k=2,\ldots,M$ are only known probabilistically. The master control problem determining Q^i_{max} for each subbasin must, however, check the downstream summation of flows for all remaining time periods. Consequently, a tentative control decision for all remaining times $k=2,\ldots,M$ must be made within the subbasin analyses. These tentative decisions can be seen as very short range planning tools and need not be the actual controls which will be effected. The necessary intermediate states can all be observed prior to choosing or applying any actual control. These tentative subsequent controls can be calculated by using the expected value of the subsequent states.

For a particular intermediate stage, the expected resultant state may be written as

$$E[S(k+1)] = \underset{R(k)}{E} [S(k) - Q(k) + R(k)] \tag{37}$$

or

$$E[S(k+1)] = S(k) - Q(k) + E[R(k)] \tag{38}$$

Thus, having computed $Q^*(1)$ by the stochastic DP algorithm, and stored all $Q^*(k)$ policies as a function of $S(k)$, for $k=2,\ldots,M$, the value of $Q^*(k)$ associated with $E[S(k)]$ $(k=2,\ldots,M)$ is found using Equation (38) and these stored optimal policies. Even though these $Q^*(k)$, $k=2,\ldots,M$, are not stochastically optimal in the true sense, they serve to aid the master controller in allocating treatment capacity, in anticipation of forecasted inflows. They are, of course, updated as the stochastic control process continues. The algorithm developed to solve this stochastic DP problem is illustrated in Figure IV-5.

C.3 A Note on Simple Operating Rules

The subbasin control policy obtained by the application of dynamic programming may at times be identical to the policy derived using simple operating rules. If the credit on throughflows and the penalty on overflows are time invariant, in addition to the exponent n used in the subbasin criterion being equal to one, then a simple operating rule can replace the aggregated stochastic DP. In this case, anticipation of inflows carries no advantage, and reactive or myopic policies using such operating rules work just as well as adaptive policies.

Consider *any* aggregated subbasin (we can, therefore, delete the superscript i) with a forecasted (or measured) inflow sequence $R(k)$, $k=1,\ldots,M$. If the credit for throughflow is time invariant and the total penalty for overflows is linear with overflow, then the optimal decision at any decision point is to release as much as possible until Q^i_{max} is reached. That is,

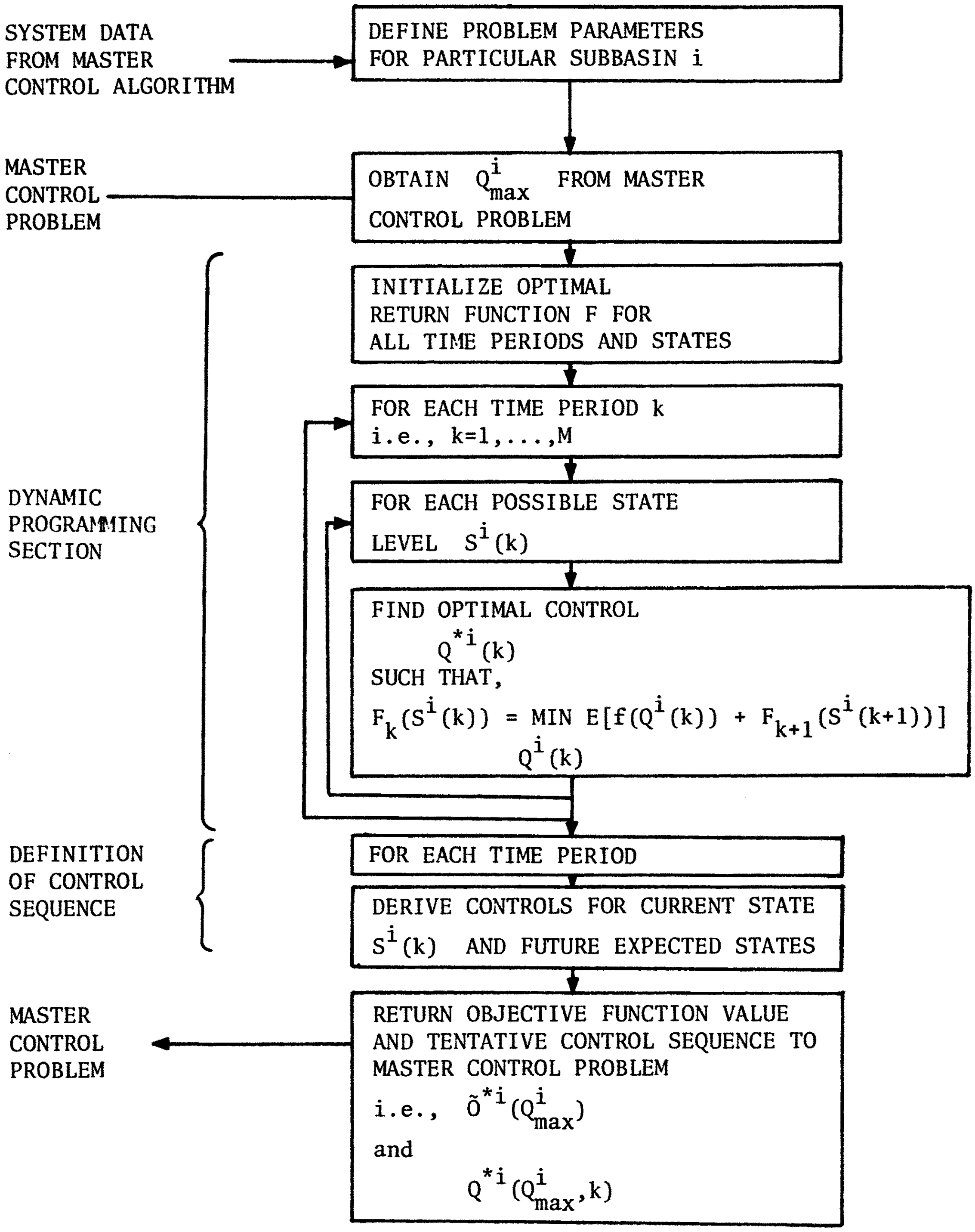


Figure IV-5. Stochastic DP Algorithm for Subbasin Problems

$$Q^*(k) = Q_{max} \qquad \text{if} \quad S(k) + R(k) \geq Q_{max} \tag{39}$$

$$Q^*(k) = S(k) + R(k) \qquad \text{if} \quad S(k) + R(k) < Q_{max} \tag{40}$$

where

$$S(k+1) = S(k) - Q^*(k) + R(k)$$
$$S(1) \quad \text{given.}$$

When credits and penalties are time invariant, storage is used only when overflows are imminent. Consequently, overflows occur only when storage is exhausted. That is,

$$O^*(k) = 0 \qquad \text{if} \quad S_{max} - S(k) + Q_{max} \geq R(k) \tag{41}$$

where $S_{max} - S(k)$ represents the remaining storage capacity; and,

$$O^*(k) = R(k) - Q_{max} - (S_{max} - S(k)) \qquad \text{if} \quad S_{max} - S(k) + Q_{max} < R(k) \tag{42}$$

Such simple control rules, which treat each time period independently, are equally suited for: the short range planning of controls based upon forecasted inflow values, or real-time, on-line control based upon actual data.

However, time varying credits and penalties, although not used in the demonstration, are anticipated in the proposed system. In addition, it may ultimately be decided that surges of overflows are more dangerous than a moderated level over a longer time. Consequently, n may be greater than one. The DP algorithm is well suited for solving the subproblems with time varying weighting factors and a non-linear objective function.

D. LINEAR PROGRAMMING APPROACHES TO DETAILED SUBBASIN SOLUTIONS

The subbasin technique discussed in the previous section results only in the overall release pattern for the subbasin. The aggregated or lumped dynamic programming approach was chosen for the on-line interactive portion of the subbasin analysis because of its speed. Once, however, the master problem in communication with the aggregated problems selects Q^{*i}_{max} for each subbasin, then the individual controls for each detention basin must be derived. The techniques capable of working out these desired detention basin controls are identical to the techniques originally considered for the interactive role. Dynamic programming was, however, judged infeasible for the detailed subbasin analysis due to the resultant dimensionality of the multi-detention basin DP formulations. Alternatively, LP techniques with the inclusion of risk appear adequate for the noninteractive role.

The passive role of the detailed solution may be viewed in several ways:

1. The desired Q^{*i}_{max} for a subbasin can be used as a constraint on the downstream releases for the solution of the complete subbasin problem.
2. The selected subbasin controls $Q^{*i}(Q^{*i}_{max},k)$ derived in the aggregated problem for the given Q^{*i}_{max} can be used as upper bounds for the controls to be derived in the detailed solution. Or,

3. The selected subbasin controls derived in the aggregated problem can be assumed fixed as constants and consequently are no longer subbasin variables.

Figure IV-6 illustrates these three alternatives. In this figure, subscript 1 indicates the furthest downstream control point in a particular subbasin i while the subscript j refers to upstream control points in subbasin i. Releases from this control point enter the interceptor. The number of detention basins in a particular subbasin is indicated by N_i .

Each of these methods, however, modifies the needed LP problem only slightly. Modifications of the basic LP formulation are discussed following the general LP development for detailed subbasin control.

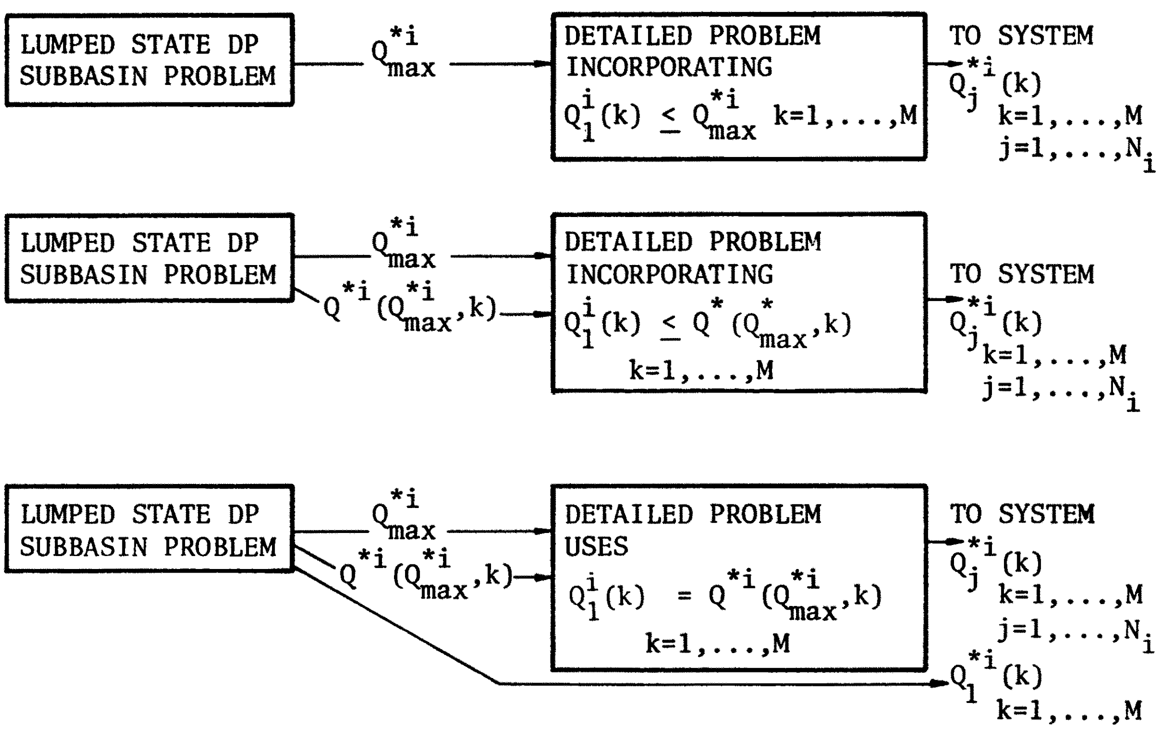


Figure IV-6. Alternative Relationships between Aggregated and Detailed Problems.

D.1 Deterministic Linear Programming Formulations

The problem presented in the previous sections can be formulated in a manner which leads directly to a set of linear equations solvable by the well-known simplex method for linear programming problems, and its variant.

The linear programming approaches explored here were originally derived in [10] and [11]. The basic linear programming (LP) problem in standard form can be expressed as:

$$\begin{aligned} &\text{Minimize} \quad z(\underline{x}) = \underline{c}\ \underline{x} \\ &\text{Subject to} \quad A\ \underline{x} = \underline{b} \\ &\text{and} \qquad\qquad \underline{x} \geq \underline{0} \end{aligned} \tag{43}$$

where

$$A = \begin{bmatrix} a_{11} a_{12} \cdots a_{1n} \\ a_{21} a_{22} \cdots a_{2n} \\ \vdots \\ a_{m1} a_{m2} \cdots a_{mn} \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}, \quad \underline{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{44}$$

and

$$\underline{c} = [c_1, c_2, \ldots, c_n]$$

For the subbasin analysis desired, the problem must be expressed in this form for application of linear programming (LP) techniques.

The variables in the LP formulation include the needed controls $Q_j(k)$ as well as the states or contents of each detention basin $S_j(k)$. The subscript j indicates a particular detention basin within the subbasin area which is being considered as an example in this development. Overflows can be considered at each detention basin or only at the downstream control point where flows enter the interceptor. It was assumed for the purpose of this work that the subbasins were adequate for deferring any overflows to the downstream interceptor bypass. Overflows at this location are simply designated as $O(k)$. As this development could be for any subbasin, the superscripts indicating the particular subbasin have been deleted. Each of these variables must be represented for each time period modeled. The vector of variables for the LP formulations is therefore represented by (with superscript T denoting transpose)

$$\underline{x} = [Q_j(k),\ S_j(k),\ O(k):\quad j=1,\ldots,N;\ k=1,\ldots,M]^T \tag{45}$$

The state and control variables for each time period and location have upper limits. That is,

$$\left.\begin{aligned} Q_j(k) &\leq Q_{jmax} \\ S_j(k) &\leq S_{jmax} \end{aligned}\right\} \quad \text{for} \quad j=1,\ldots,N; \quad k=1,\ldots,M \tag{46}$$

The overflow variable may also have an upper limit expressed as a constraint. Although conceptually there is no limit to overflow, there may be an upper limit on the bypass conveyance. Using this as a constraint insures the distribution through time of any needed overflows which would otherwise be too large for the overflow conveyance. Such an event, although unlikely, would have disproportionally greater environmental impacts and is therefore modeled as infeasible. Each of the above constraints can be expressed in equality form by the addition of slack variables which represent the difference between the constraint value and the variable value. The top half of the A matrix designated as A' can therefore be represented as

$$A'\underline{x} = \underline{b}' \tag{47}$$

where $\underline{b}'$ is the vector of values representing the constraining values of the variables.

In a similar manner, the continuity relationships can be written as constraints. Rewriting Equation (1) with the inflows on the right side, results in a formulation which can be expressed in matrix form as:

$$D\,\underline{x} = \underline{R} \tag{48}$$

where $\underline{R}$ is a vector of all inflows for all locations and times. That is,

$$\underline{R} \triangleq [R_j(k):\;\; j=1,\ldots,N;\; k=1,\ldots,M]^T \tag{49}$$

D is a matrix of constants representing the continuity relationships. D *transforms* the selected values of controls, states and overflows for each time period into inflows consequent from mass balance considerations. These values are constrained to equal the particular forecasted $R_j(k)$ to which they correspond.

If some form of linear routing is included, the coefficients which represent the fraction of each previous upstream release that contributes to the particular downstream control release are included in D.

The objective function, Equation (4), is also written in terms of decision variables and constants and can, hence, also be written in vector form as

$$z(\underline{x}) = \underline{c}\ \underline{x} \tag{50}$$

where $\underline{c}$ is a vector containing the values of $w(k)$ and $y(k)$ for each time period. Considering the stochastic nature of the problem, this is written as

$$z(\underline{x}) = E[\underline{c}\ \underline{x}] \tag{51}$$

In deterministic solutions, however, the first form is used. The entire subbasin problem can now be written as

$$\begin{aligned} &\text{Minimize} \quad z(\underline{x}) \\ &\text{Subject to} \quad A'\underline{x} = \underline{b} \\ &\text{and} \qquad\quad D\ \underline{x} = \underline{R} \end{aligned} \tag{52}$$

The A' and D matrices have been developed separately here to facilitate the exposition of the linear programming under uncertainty technique which follows the discussion of the deterministic approaches to the subbasin problem. If the forecasted inflow sequence can be assumed a deterministic variable, the two constraint equations can be combined, resulting in the desired standard LP form. That is, in the deterministic case, let

$$A = \begin{bmatrix} A' \\ \\ T \end{bmatrix} \tag{53}$$

and

$$b = \begin{bmatrix} \underline{b}' \\ \\ \underline{R} \end{bmatrix} \tag{54}$$

The constraints therefore can be written as:

$$A \underline{x} = \underline{b} \tag{55}$$

The size of the resultant LP solutions for subbasin analysis becomes apparent by considering the chosen example subbasin with three detention basins modeled for ten time periods. The three dynamic equations developed in the previous section for the example subbasin can be written explicitly for each time period. This results in 30 equations with 70 variables representing the system continuity. If some form of linear routing scheme is considered, it can be incorporated, as described earlier, directly into these continuity relationships. The 30 equations derived for the modeled system are illustrated as lines 72 through 101 in Table IV-2, which illustrates the total LP problem format. Initial states, assumed zero, are implicit in these equations.

In addition to these dynamic equations, upper bounds for each original variable (i.e., $Q_j(k)$, $S_j(k)$, $j=1,2,3$ and $O(k)$ for all $k=1,\ldots,10$) must be explicitly represented for each time period. This creates 70 additional equations. These 70 equations are represented in Table IV-2 as lines 2 through 71. The objective function is represented as the first line where the $w(k)$ and $y(k)$ terms are weighting factors on overflows and throughflows respectively. The original system variables are tabulated across the top of the matrix chart along with the slack variables. The upper bounds for the constraints along with the system inflows are represented in the right-hand column as the $\underline{b}$ vector.

Solving this problem using standard LP codes requires the use of the deterministic objective function Equation (50) and an a priori knowledge of the inflows. The deterministic use of the forecasted

TABLE IV-2

EXAMPLE SUBBASIN IN STANDARD LP FORMAT FOR SOLUTION USING SIMPLEX

DETAILED COEFFICIENT MATRIX

	Time Period k=1 $Q_3(1)$ $S_3(2)$ $Q_2(1)$ $S_2(2)$ $Q_1(1)$ $O_1(1)$ $S_1(2)$	2 $Q_3(2)$ $S_3(3)$ $Q_2(2)$ $S_2(3)$ $Q_1(2)$ $O_1(2)$ $S_1(3)$	. . .	10 $Q_3(10)$ $S_3(11)$ $Q_2(10)$ $S_2(11)$ $Q_1(10)$ $O_1(10)$ $S_1(11)$	+70 Slack Variables	$\underline{b}$
OBJECTIVE FUNCTION	y(1) w(1)	y(2) w(2)		y(10) w(10)		
70 UPPER BOUND CONSTRAINTS	1 1 1 1 1 1 1				1 1 1	Q_{max3} S_{max3} Q_{max2} S_{max2} Q_{max1} O_{max1} S_{max1}
		1 1 1 1 1 1 1				. . .
			1			. . .
				1 1 1 1 1 1 1		. . .
30 CONTINUITY CONSTRAINTS	1 1 -1 1 1 -1 1 1 1	[0]	[0]	[0]	[0]	$R_3(1)$ $R_2(1)$ $R_1(1)$
	-1 -1 -1	1 1 -1 1 1 -1 1 1 1	[0]	[0]	[0]	$R_3(2)$ $R_2(2)$ $R_1(2)$
	[0]	[0]	. . .	[0]	[0]	. . . .
	[0]	[0]		1 1 -1 1 1 -1 1 1 1	[0]	$R_3(10)$ $R_2(10)$ $R_1(10)$

inflows implies an important assumption. It is assumed that the optimal solution based upon the expected inflow sequence (i.e., the forecasted inflows) is equivalent to the solution which minimizes the expected value of the objective function. This assumption is discussed in the section to follow where control strategies which incorporate this assumption are compared to ones derived without it.

D.2 Compacted LP Techniques

The LP problem size for a typical subbasin tends to be large. For the example problem presented in the preceding section, the matrix of coefficients contained 14,000 elements. The modeling of larger subbasins or more time periods quickly expands this storage requirement. The selection of the total number of time discretizations (M) is the most obvious source of problem size once the number of reservoirs (N) is established for a particular subbasin. In addition, the objective function contains a term describing the overflow and throughflow for each time and location. The unknowns in the problem are the releases and the states (i.e., detention basin contents) for each location and time period; as well as the overflow from the subbasin for each period. Therefore, the total number of variables in the LP is calculated as follows:

Number of State Variables for Each Period	N	
+ Number of Control Variables for Each Period	$+N$	
+ Number of Overflow Variables for Each Period	$+ 1$	
= Total Number of Variables for Each Period	$2N + 1$	
x Number of Time Periods	xM	
= Total Number of Variables	$(2N+1) \times M$	(56)

The total number of constraints can be calculated in a similar manner.

Number of Variables Needing Upperbound Constraints (all variables)	(2N+1) x M	
+ Number of Continuity Relationships as Constraints	+ N x M	
= Total Number of System Constraints	(3N+1) x M	(57)

Considering the example subbasin with three reservoirs discussed previously illustrates these points. We have in the example problem,

$$M = 10 \quad \text{and} \quad N = 3$$

Applying Equation (56) above for the total number of variables yields 70, while Equation (57) describing constraints yields 100. Adding an equivalent number of slack variables results in the 140 variables defined earlier, and an A matrix with dimensions 100 x 140.

These relationships help in understanding the dimensionality problem associated with linear programming solutions, and in devising ways to overcome the problem. Considering for on-line analysis a 6-hour storm for which controls are desired every 15 minutes demonstrates how not uncommon problems can get out of hand. Such a problem with 24 control periods (i.e., M = 24) results in an A matrix with over 40,000 elements.

It has probably been observed that the vast majority of the elements in these A matrices are zero values. This feature leads to the suggestion of utilizing techniques which capitalize upon this feature and more efficiently use computer space for solving large LP problems. Product form LP and upper bounding codes, which eliminate much of the storage requirements, still require large amounts of computer space and often trade off computer space for computer time. In the on-line real-time problem considered here, neither of these can be spared and additional techniques are, therefore, sought which reduce the entire problem to a size manageable on-line within, possibly, a mini-computer.

In efforts to reduce the problem size without significantly compromising the solution, the following simplifications are possible. They incorporate aggregations of time periods, control intervals, total reservoirs, or reservoir states.

1. The size of the matrix of constraint coefficients is directly proportional to the square of the number of time increments (M). That is,

$$\underset{\text{Constraints x Variables}}{(3NM + M) \times (2NM + M)} = M^2(6N^2 + 5N + 1) \tag{58}$$

   Hence, the most direct simplification is to increase the length of the time increments and therefore reduce the number of increments, M.

2. A less direct simplification produces a smoother control by decreasing the number of control intervals while maintaining the number of time intervals. In this technique, a control is assumed to be constant for two or more periods of time. This technique permits the a priori assignment of control sensitivity by permitting more control intervals per unit of time during certain phases of the event and less during others.

   Aggregation of controls is simply carried out by assuming: $Q_i(k+1) = Q_i(k)$ for L different values of k. The number of original variables is, therefore, decreased by L. The size of the matrix of coefficients is then equal to:

   (Total Number of Original Constraints - L)

   x (Total Number of Original Variables - L)

   or

$$[(2N + 1) \times M - L] \times [(3N + 1) \times M - L] \tag{59}$$

   which is quadratic in L. In this manner, a decrease in the number of control intervals decreases total problem size, but not as quickly as does the total number of time intervals discussed above.

3. Aggregations of reservoirs with the consequent aggregation of controls (in space, not time) has an effect similar to reductions in the number of time intervals. This technique was discussed in the previous section on DP approaches and was chosen for use within the interactive portion of subbasin analysis. In the logical limit, a one-reservoir problem is rather small. That is, considering Equation (58) it is obvious that when $N = 1$, the A matrix decreases in size to $12M^2$.

4. Aggregations of state values, with the maintenance of independent controls for the separate detention basins has an effect upon the problem size which is similar to the effect of control aggregations. The number of state values aggregated in time or space decreases the size of the coefficient matrix in a way fully described by Equation (56) where L represents the total number of state variables removed from the formulation.

   Aggregation of states in time is simply carried out by assuming: $S_i(k+1) = S_i(k)$ for various values of k. Aggregations in space assume: $S_i(k) = a \times S_j(k)$ for selected values of i and j. Where 'a' is a constant valued ratio relating the state at one location to another. Once such dependence is established between state values, one can be substituted for the other, reducing the number of variables to be explicitly represented in the formulation.

The aggregation of states in space achieved by assuming a constant valued ratio between the state values of the various detention basins within a subbasin was selected for further considerations for two reasons. The detention basins within the San Francisco system are sized equally proportionally to their respective drainage areas, which suggests their equally proportional use. And, the locally defined subbasin areas are small enough that the assumption of uniform rainfall over the entire basin, is acceptable.

To gain insight into the effect of these aggregation techniques upon the attainment of the system objectives, the example subbasin problem was solved considering two different storm inflow sequences and three levels of state aggregation.

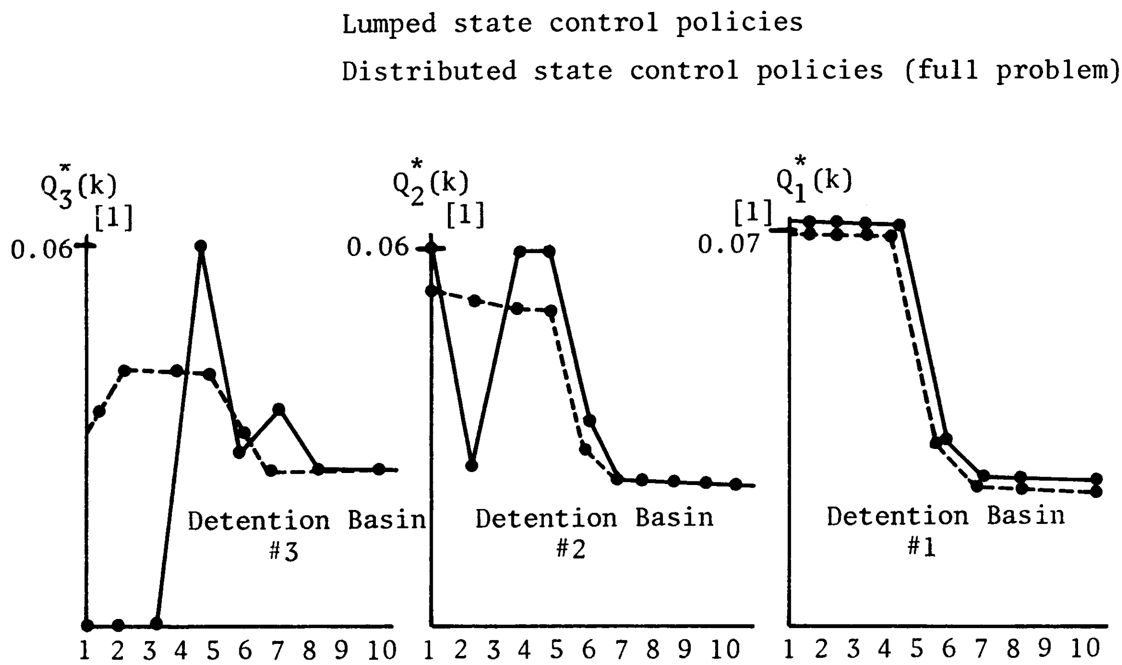
The two storms differed significantly. The first assumed uniform rain over the entire subbasin resulting in system inflows equally proportional to the detention basin sizes. The second storm assumed inflows which were not equally proportional to the drainage areas within the subbasin.

The three levels of aggregation were none (i.e., the full problem), state aggregation only (i.e., independent controls), and total state and control aggregation (i.e., *lumped parameters*).

The results of this experiment indicated that a totally *lumped parameter* problem can be expected to result in the optimal solution only if the inflows are equally proportional to the detention basin capacities. It was observed that with uniform inflows, all three techniques resulted in the same controls. The distributed control problem with the distributed storm (i.e., nonuniform inflows) yielded an equivalent, though different solution than the full problem, while the totally aggregated problem yielded an inferior policy. Figure IV-7 illustrates these equivalent but different solutions which resulted in a smoother control policy for the lumped state problem.

It appears then that the use of a totally lumped parameter subbasin solution strategy is adequate for use within on-line work if and only if the assumption of locally uniform rain is made.

The need for drastic reductions in subbasin problem size has been reduced by the solution framework chosen in this work. The additional time and storage requirements imposed by the use of stochastic LP techniques discussed in the following sections, however, maintain the importance of such reductions. Time constraints upon the solution for the actual detention basin controls (i.e., the detailed problem) are



NOTE: [1] values indicate maximum releases in $ft^3 x10^{-6}$ over 20 minute control intervals.

Figure IV-7. Comparison between Full Problem Control Policies and Lumped State Control Policies.

relieved by the noninteractive mode of the detailed solution algorithm and by the possibility of distributed computer power which would enable parallel subbasin solutions. Computer storage requirements, however, remain problematic. The availability of computer power at the various optimization levels determines the degree of problem size reduction ultimately needed.

D.3 Linear Programming with Risk and Uncertainty

Thus far, the linear programming techniques developed have assumed that the use of the forecasted inflow values within a deterministic optimization framework were sufficient for the derivation of control policies. These forecasted inflow sequences, the expected values of complexly related random variables, are however only imperfectly known.

An adequate consideration of the uncertainties involved in these forecasts could result in operating policies which more closely achieve the goal of minimizing the expected impact upon the environment. The following chapter describes how an on-line forecast of these inflow sequences is made, however, it is the subbasin analysis which must take into account the risk of these forecasts. It has been pointed out that the uncertainty of a forecast increases as the time in the future to which it applies increases. Considering the discrete forecasts used in this development, this increasing uncertainty can be represented as illustrated in Figure IV-8, in which the distributions assumed to describe this uncertainty are seen to flatten out with increasing variance as the forecast moves further into the future.

It is desired then, in light of the uncertainty, to develop operating policies which, while addressing this uncertainty, minimize the risk inherent in the selected policy. That is, a trade-off has to be made between policies which achieve to a lower degree the system objectives, but are reliable and can be counted upon to perform at this level for a wide variety of events, versus policies which achieve a high degree of the system objectives, but for only a narrow range of possible events.

The development in this section assumes that only the probability distributions are known for each inflow at each point in time, and demonstrates how these probability distributions may be incorporated into the optimization, replacing the expected inflow values used in the previous section. The process of controlling this system, in light of the uncertainty inherent in the forecasts and the risk involved, may be called a *discrete stochastic process*. Bellman [12] has defined the

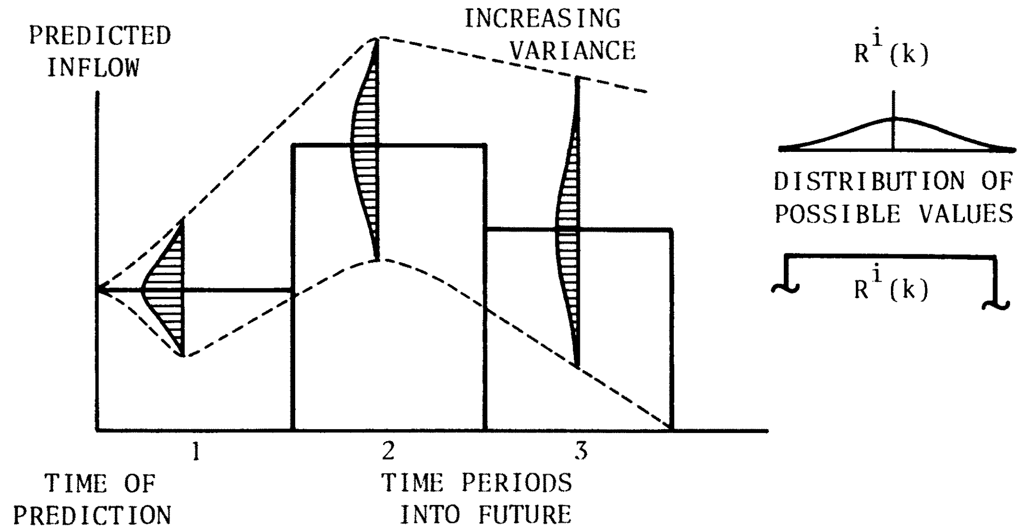


Figure IV-8. Increasing Prediction Uncertainty.

term *discrete stochastic process* in which discrete "signifies a process in which the transformations occur at a finite, or, at worst, enumerable set of time," and where stochastic implies that a decision "determines not a unique outcome, but a set of possible outcomes." This can be extended to multistage processes in which decisions are to be made at various points within the ongoing process. It follows that given an initial state of the subbasin where an initial decision is made concerning controls, the resultant state is a stochastic state, with a known range of values. The range of values in subbasin problem is determined by the inflows. Bellman continued to differentiate between two classes of *discrete stochastic processes*. The first class is that type of process for which the initial state at each decision stage is known. That is, although the resultant state is a stochastic variable, it is assumed that it will be shortly defined and each subsequent decision will in turn have known initial state conditions. The second class which Bellman identifies are those in which the state of the system is only imperfectly known at each stage of the process. This more complicated second class contains the subbasin problem developed herein, once the inflow is treated as a stochastic variable.

Wets [13] has developed an algorithm for solving linear programming problems with stochastic constraints known as LPUU, Linear Programming Under Uncertainty. It can be applied directly to the discrete stochastic process described by the subbasin problem as the transformation from one state to the next can be expressed as a constraint containing a stochastic variable. It addresses the *complete* problem in linear programming under certainty in which the risk involved is included by the use of penalty terms for errors in policy decisions, and in which the stochastic nature of the continuity constraints is considered. Wets presents the standard form of the two stage linear program as:

$$\text{Minimize} \quad z(\underline{x}) = E_{\underline{R}}\{\underline{cx} + \underline{q}^{+}\underline{y}^{+} + \underline{q}^{+}\underline{y}^{+}\}$$

$$\text{Subject to} \quad A\underline{x} = \underline{b}$$

$$D\underline{x} + I\underline{y}^{+} - I\underline{y}^{+} = \underline{R} \tag{60}$$

$$\underline{x} \geq \underline{0},\ \underline{y}^{+} \geq \underline{0},\ \underline{y}^{-} \geq \underline{0}$$

where

A is a matrix $m \times n$

D is a matrix $\bar{m} \times n$

$\underline{R}$ is a random vector (with a continuous distribution function)

E is the statistical expectation

y^{+} and y^{-} are the errors implied by the selection of a set of values for x which can be represented as:

$$I\underline{y}^{+} - I\underline{y}^{-} = \underline{R}^{a} - D\underline{x} \tag{61}$$

The above equation simply relates the fact that the error term is a vector, some parts of which are positive $\underline{y}^+$ and some are negative $\underline{y}^-$. Disaggregated into two equal dimensional vectors with positively signed components in mutually exclusive locations the penalty for error can be assigned according to the individual sense of the error components. That is,

$$q^+\underline{y}^+ + q^-\underline{y}^- = \text{error penalty function}$$

where

$\underline{q}^+$ = penalty vector for under anticipating inflow by amount $\underline{y}^+$

$\underline{q}^-$ = penalty vector for over anticipating inflow by amount $\underline{y}^-$

The development also requires the following definition and assumptions:

Define $\underline{R}^o = D\underline{x}$

Assume:

(1) The initial problem, Equation 60, is solvable

(2) $\underline{q}^+ + \underline{q}^- \geq \underline{0}$

(3) $K = \{\underline{x} \mid A\underline{x} = \underline{b},\ \underline{x} \geq \underline{0}\}$ (62)

Has a nonempty interior

(4) $E_{\underline{R}}\{\underline{R}\}$ exists (expected value of the distribution)

also, for this subbasin problem, the following are needed.

$\underline{R}^o$ represents the transformation of the decision variables into anticipated inflow via strict mass balance considerations.

$\underline{R}^a$ represents the vector of actual inflows that will be experienced.

$\underline{R}$ represents a random vector of possible inflows.

$\underline{y}^+$ & $\underline{y}^-$ represent the error, the difference between the actual experienced inflows and the anticipated inflows based on the transformation of solution variables.

The original problem may, however, be rewritten as:

$$\text{Minimize} \quad z(\underline{x}) = \underline{c}\ \underline{x} + E_{\underline{R}}\{\min q^+\underline{y}^+ + q^-\underline{y}^-$$
$$\text{where} \quad \underline{y}^+ - \underline{y}^- = \underline{R}^a - \underline{R}^o,\ \underline{y} \geq 0\} \tag{63}$$

or simply

$$z(\underline{x}) = \underline{c}\ \underline{x} + E_{\underline{R}}\{\min q^+\underline{y}^+ + q^-\underline{y}^- | \underline{R}^a\}$$

$$\text{subject to} \quad A\underline{x} = b$$
$$D\underline{x} = \underline{R}^a$$

This new problem formulation has simply removed the term $\underline{c}\ \underline{x}$ from the expectation operation as this term is not directly dependent upon stochastic variable. The above problem is however, still impossible to solve in the *here and now* as the error terms can only be known after the considered event passes and $\underline{R}^a$ is known. Also, the $\underline{x}$ values cannot be solved for directly as they must satisfy the transformation constraint matrix,

$$D\underline{x} = \underline{R}$$

where $\underline{R}$ is a random vector. An attempt can be made, however, to achieve a set of values for $\underline{x}$ which minimize a combination of the original objective and a new component reflecting a penalty for a solution based on a partially incorrect assumption about $\underline{R}^a$. This type of formulation is called the *complete* problem of linear programming under uncertainty because the *chance* constraints (i.e., the equations equated to the random variable) when violated are reflected in the objective function. The following formulation, developed by Wets has been proved to have an identical set of feasible solutions to the original problem. However, it is independent of the a priori unknowable error; including, rather, probabilities of various levels of error. First defining:

$$\pi_i(\underline{R}^o{}_i) = q_i^+(\text{prob}\{\underline{R}^a > \underline{R}^o{}_i\}) + q_i^-(\text{prob}\{\underline{R}^a < \underline{R}^o{}_i\}) \quad (64)$$

Wets formulates a new linear program as:

$$\begin{aligned} &\text{Minimize} \quad (c - \pi(\underline{R}^o))\,\hat{\underline{x}} \\ &\text{Subject to} \quad A\hat{\underline{x}} = \underline{b} \\ &\qquad\qquad\quad \hat{\underline{x}} \geq 0 \end{aligned} \quad (65)$$

Using this problem's solution and an initial feasible point a direction of descent is defined and a one-dimensional search along this line results in the next improved solution. A feasible point in the solution space containing any better solution and bounded by a hyperplane containing the initial point as its minimum will yield a new direction along which a new, better solution will be encountered. This iterative process is continued until no new directions are encountered which yield an improvement over the base point solution.

This technique worked adequately for small unaggregated subbasins, but consumed excessive time, 3-4 minutes, for detailed problem formulations. It uses, however, an initial solution based upon the deterministic formulation discussed in a previous section. Each iteration thereafter is an improvement over previous solutions. Convergence of this technique, therefore, is not judged to be essential. Intermediate solutions are available for system control should computation time constraints prohibit convergence. The tradeoff between intermediate solutions of the detailed problem and a final solution of a simplified problem was not addressed in this work, although this author favors the former. Such an intermediate solution forms the basis of the following discussion.

D.4 Stochastic Versus Deterministic Control Strategies

Control strategies based upon optimization techniques, which recognize the uncertainties in the forecasted inflows, are superior in many cases to control strategies based on the assumption of a deterministic knowledge of the inflows. This has been demonstrated for a simplified problem based on real San Francisco subbasin data and using a limited number of time intervals. Figure IV-3 presented earlier illustrates the subbasin configuration chosen for the demonstration and Figure IV-9 illustrates the forecasted inflow sequence chosen for the demonstration.

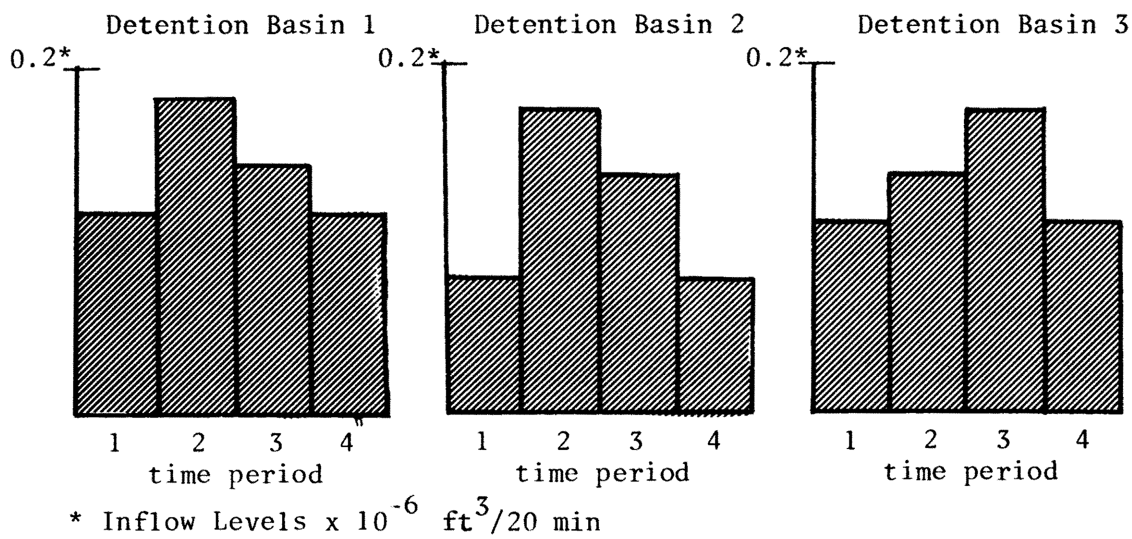


Figure IV-9. Forecasted Inflow Sequences.

This inflow sequence was purposely chosen to create a situation where overflows could be avoided entirely but for which mass balance considerations indicate the system is at or near its capacity. That is, the sum of all the inflows was slightly less than the total available storage combined with the maximum possible release (without overflows) for the four time periods. Table IV-1 presents the system data and 20 minute control intervals were used. The above described situation can be represented as:

$$\sum_{j=1}^{3} \sum_{k=1}^{4} R_i(k) \cong \sum_{j=1}^{4} S_{j\max} + 4(Q_{1\max}) \tag{66}$$

This *tight* situation enhances the demonstration of control strategy superiority. The policy which avoids the most overflows for equally likely variants of the forecasted inflow sequence is judged to be a superior policy.

First, the forecasted inflows were assumed to be deterministically known values and the entire problem was modeled and solved via linear programming. Second, the forecasted inflows were used as the means of a uniform probability distribution (i.e., the forecasted value is the expected value of the distribution, but events higher and lower are equally likely). Thus, the assumption that the real value of inflow will be within 50 percent of the forecasted inflow, and that all values within that margin of uncertainty are equally likely is easily expressed as a probability distribution. Such a probability distribution is illustrated below in Figure IV-10. These distributions were then used in the LPUU algorithm for the determination of the stochastic policies.

Third, the resultant policies were applied to the modeled subbasin using inflow sequences which were equally likely variants of the original sequence. The resultant overflows were then compared.

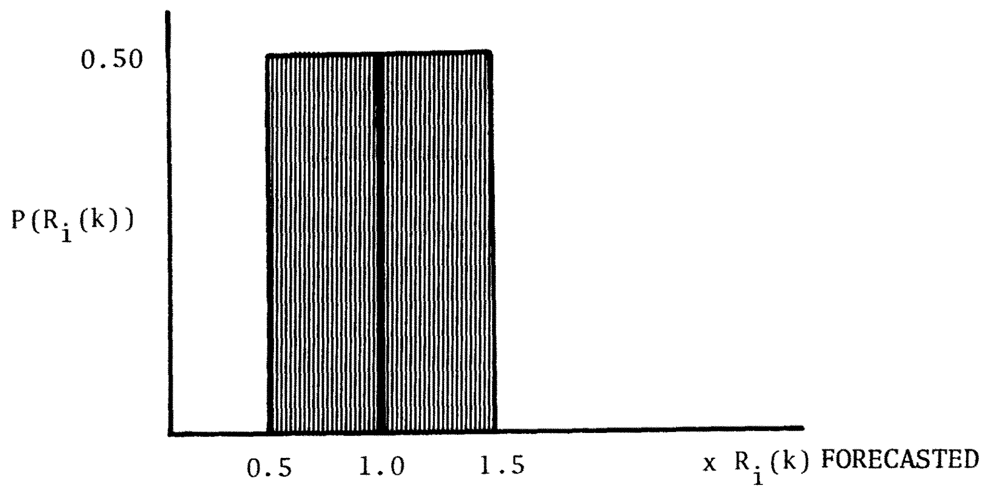


Figure IV-10. Assumed Probability Distribution.

Figure IV-11 illustrates the mass balance summary of the analysis. The abscissa indicates the ratio of the average total inflows for the variants to the total forecasted inflow. The ordinate indicates the percent *reduction* of overflows for all variants with the same average total inflow. For example, assume each of the 12 elements in the rainfall inflow sequence (i.e., 3 retention basins, 4 time periods, therefore 12 forecasted inflow values) is 40 percent greater than predicted (i.e., FI = 0.4) with a 0.5 probability. The average of the sums of all the combinations of the 12 elements possibly increased by up to 40 percent is 20 percent higher than the sum of the forecasted inflow sequence. Figure IV-11 shows that on the average, sequences of this sort resulted in approximately 70 percent less overflows when controlled by the stochastically derived policy than when controlled by the deterministically derived policy.

This situation is only one of many different subbasin configurations and overall storm patterns, however it illustrates the

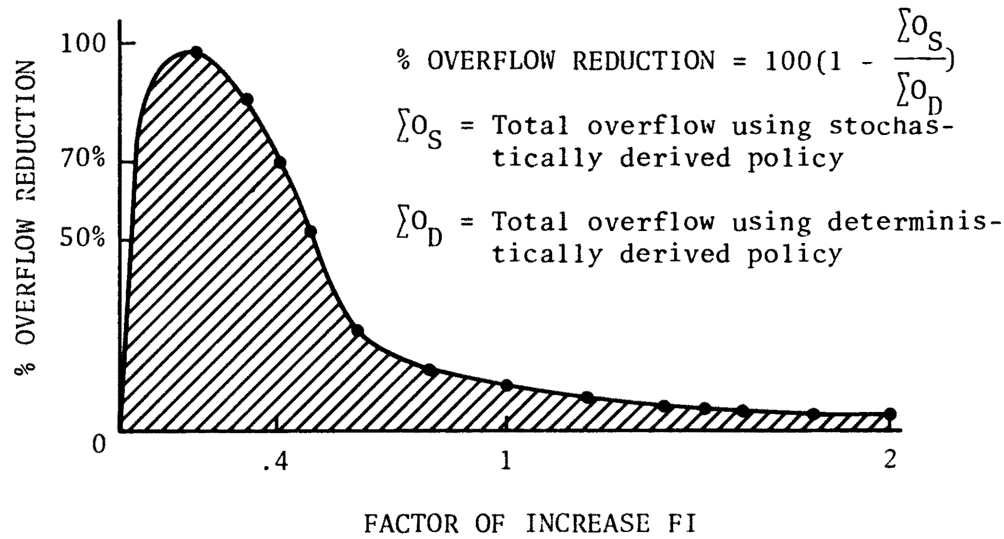


Figure IV-11. Performance of Stochastically Derived Control versus Performance of Deterministically Derived Controls for Variants of Original Inflow Forecast.

increased safety of stochastically derived control policies. These equally likely sequences were generated in an exhaustive manner using the forecasted storm as a basis. As the control policies derived by both techniques eliminated overflows completely when the forecasted storm was used, storms of lesser total magnitude were not considered. A uniform fraction of the forecasted inflow was added to all combinations of from 1 to 12 of the inflow values. For a particular fraction used, therefore, there were 4096 (i.e., 2^{12}) different combinations of the sequence in which some elements might have been increased by the fraction. As each element in the series had a 0.5 probability of being increased by the fraction, on the average the entire storm inflow was increased by 0.5 times the fraction. The process then can be represented as:

$$R_j^{(v)}(k) = (1 + FI \times RN) \times R_j^{(f)}(k) \tag{67}$$

where

(v) indicates variant of inflow value

(f) indicates forecasted inflow value

FI = Factor of Increase

RN - Random Number = 0.0 or 1.0 with uniform p.d.f.

Using this technique, all the variants of storms with factors of increase ranging from 0 to 2 were used to compare the two techniques for the derivation of the system control policies.

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Chapter V

STORM INFLOW FORECASTING

A. PREDICTION VERSUS FORECASTING MODELS

The previous chapters have made liberal reference to *forecasted inflows*. The model generating these forecasts are to be addressed in this chapter. A distinction is often made, however, between prediction and forecasting, but there seems to be little agreement on what the distinction is. Johnson [1] describes prediction as being those attempts at anticipating the future value of a time series which include subjective, qualitative estimates of future system perturbations and their effect upon the time series. According to this definition, prediction requires skill, experience, and judgment, as well as quantitative information. On the other hand, the output of mathematical models simulating the behavior of dynamic systems is often referred to as a *predicted* output. The distinction, based upon the use of either quantitative or qualitative judgments, between forecasting and prediction is, consequently, unclear. Of greater use for descriptive purposes is a distinction based upon the degree of knowledge assumed about the system.

Using the nature of the model developed as the basis for the choice of terms results in a clearer division. This basis enables a clearer distinction between system models attempting to describe the underlying physical phenomena and those which regard the system as a *black box*. Black box models simply analyze time series and trends. They attempt no modeling of the internal process structure. They are, henceforth, referred to as forecasting models in this development.

Stream records are often analyzed with such time series analysis techniques which regard the causal process as a black box. Such models are used to: forecast extreme events and their probabilities, generate missing records, and synthesize future records.

Process models which attempt to incorporate mathematical descriptions of the phenomena are referred to as prediction models in this development. Such models attempt to predict the outcome of an event by describing the relevant mechanisms and system inputs, which taken together constitute a mathematical simulation model. River flow routing models discussed earlier, are such process models used for prediction of downstream flows. The upstream hydrographs are used as model inputs. The mathematical relations describing conservation of mass, energy, and momentum are incorporated into the model, simulating the phenomena which govern the process.

With the above distinction between forecasting and prediction having been made, it is evident that subjective or qualitative inputs can be incorporated into either forecasting models or prediction models. Such subjective judgments tend to compensate for omissions in either the process model (i.e., prediction) or black box statistical models (i.e., forecasting).

The storm-related inflows into a sewer system are a consequence of two complex phenomena. Both the rainfall process itself, resulting from the storm activity; and the conveyance of the rainfall to the sewer system, as affected by the topography of the area, influence the character in time and space of the storm-related inflows. Both of these phenomena influence the form of the model chosen for anticipating the system inflows.

It was pointed out in Chapter II that the complex meteorologic processes relating to the distribution of rainfall in time and space make the development and use of a *prediction* type model difficult, particularly for on-line use. Forecasting models, based on time series analyses are, however, adequate for on-line work, despite their omission of the mechanics of the process. Forecasting models can be sized according to the needs and requirements of the system they are designed to serve. The justification for these models is based upon the assumption that the highly correlated storm inflows can be conceived as being generated by a series of statistically independent *disturbances*. Such a sequence of disturbances is called a *white noise* process. This white noise is then assumed responsible for the error series between the actual realized values of the variables and the forecasted values. Previous values of the time series are considered as inputs in such a forecasting model. This is illustrated in Figure V-1, where the entire

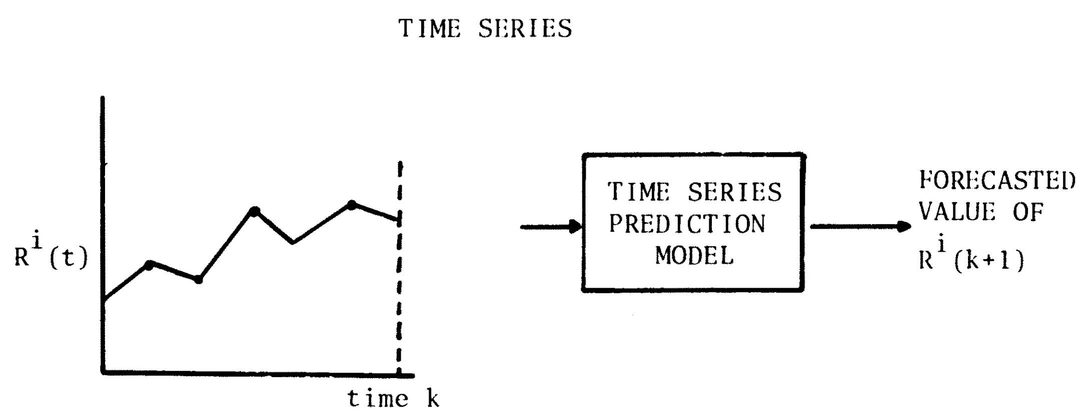

Figure V-1. Time Series Forecasting Model.

time series describing the inflow at a point in time is used as the input to a model which forecasts the subsequent value of the time series. The analysis of a time series and the use of that analysis for a forecast can be followed by qualitative judgments about the impact of other variables upon the series. The inflow at a particular location may be related to other local inflows. The inclusion of this information, either qualitatively or analytically, adds additional degrees of accuracy to a forecast based upon time series analysis. The derivation of the relevant parameters, however, is complicated. Figure V-2 illustrates a forecasting model where the time series representing inflows to each of the ten subbasins considered in this work are used to forecast subsequent values of inflow for each of these locations.

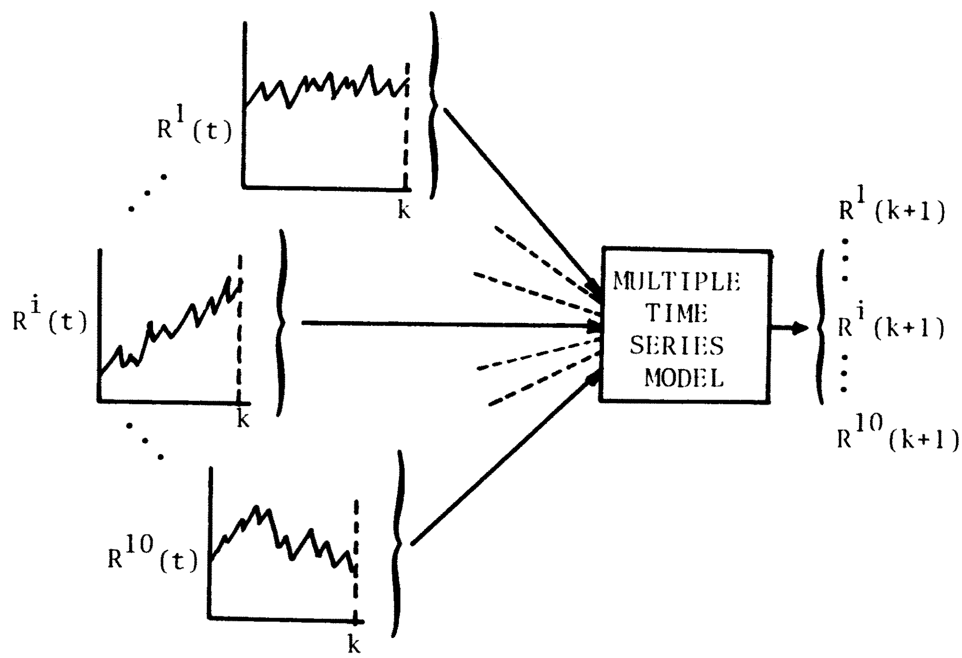

Figure V-2. Multiple Input Multiple Output Model.

The use of multiple time series analysis for deriving information about the future of one or all of the time series is the subject of a large body of literature. Various techniques are available for identifying the relevant structure of the forecasting model. Identification algorithms are available for finding or identifying the needed forecasting model parameters in an on-line setting, as well as updating them as needed. Such an algorithm will be discussed in a subsequent section of this chapter.

Since a storm event is a limited duration phenomenon, there is no opportunity within its short time span to develop an adequate forecasting model capable of fully anticipating the storm behavior. That is, the fact that storms always end is not apparent in the developing time series of a particular event. A model capable of duplicating this behavior must learn the pattern from a history of past events of like character. Such a simulation model based entirely upon past events would, however, be only capable of forecasting the average storm based upon past records. The variation in such records is significantly high, rendering the use of average storm data a very risky endeavor [4].

The resultant dilemma between the use of historic records resulting in average storms and use of developing records incapable of forecasting the characteristic storm behavior is resolved by blending the two. A model derived in such a way that historic storms form a basis of prediction embodying the characteristic storm behavior, while the occurring event changes the model to a moderate degree, is an on-line adaptive model.

Such a model is illustrated in Figure V-3. In the illustration, the current data are shown influencing the baseline model as well as the on-line adaptive model. The base-line model integrates all past events while the adaptive model dynamically responds to the occurring situation. As time goes on, data from the evolving event are used to modify the model. This in turn results in an updated forecast on future system inputs. These inputs are then used for the next iteration of the entire large scale analysis process described in the previous two

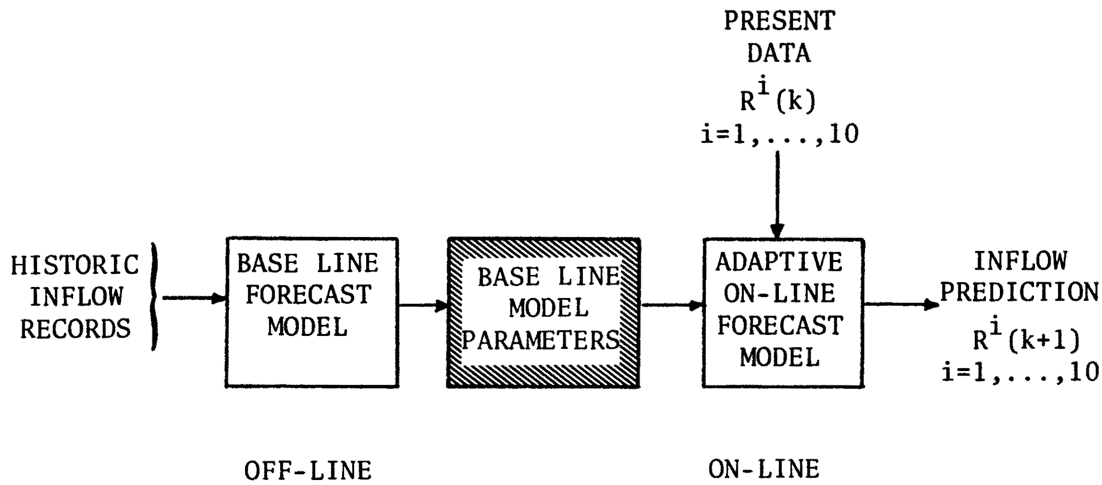

Figure V-3. On-Line Adaptive Forecasting Model.

chapters and illustrated in Figure III-1. This then results in a totally on-line adaptive control system.

B. RUNOFF VERSUS RAINFALL FORECASTING

Many adequate models are available for calculating an estimate of runoff from a mathematically simulated basin or catchment, and given rainfall hyetographs as inputs. With such models available, it might appear that the missing part of the complete system control is a rainfall forecasting model. It is, however, the well developed nature of available deterministic models that predict runoff from given rainfall (or rainfall-runoff models) that enables a more accurate forecast of system inflows to be carried out directly.

Any real-time operating policy derived from stochastic optimizations based upon statistical properties of historic events, would seem to be suboptimal in proportion to the degree of variability found in the historic events. The variance of parameters describing micro-scale rainfall hydrologic events have been shown to be significant [2,3,4]. It is well known, however, that a catchment or watershed can have a convoluting or smoothing effect on the rainfall input, as evidenced by the smoother output hydrographs. This, of course, assumes that there are no major discontinuities in the storage-outflow relation for the catchment. Consequently, it appears that runoff data may be more amenable to the statistical analysis required for forecasting. Figure V-4 illustrates this natural smoothing effect of the runoff process.

There are many ways of smoothing data. A simple example would be the averaging of adjacent measurements, or the using of the log of the data as a surrogate measure. These smoothing techniques could then be applied to historic rainfall data, thereby resulting in significantly lower variances. These averaged data could then be used for the derivation of a rainfall forecasting model. The output forecasts of this model could be then applied to a rainfall runoff model, resulting in the needed inflow forecasts. Such a process, however, ignores important

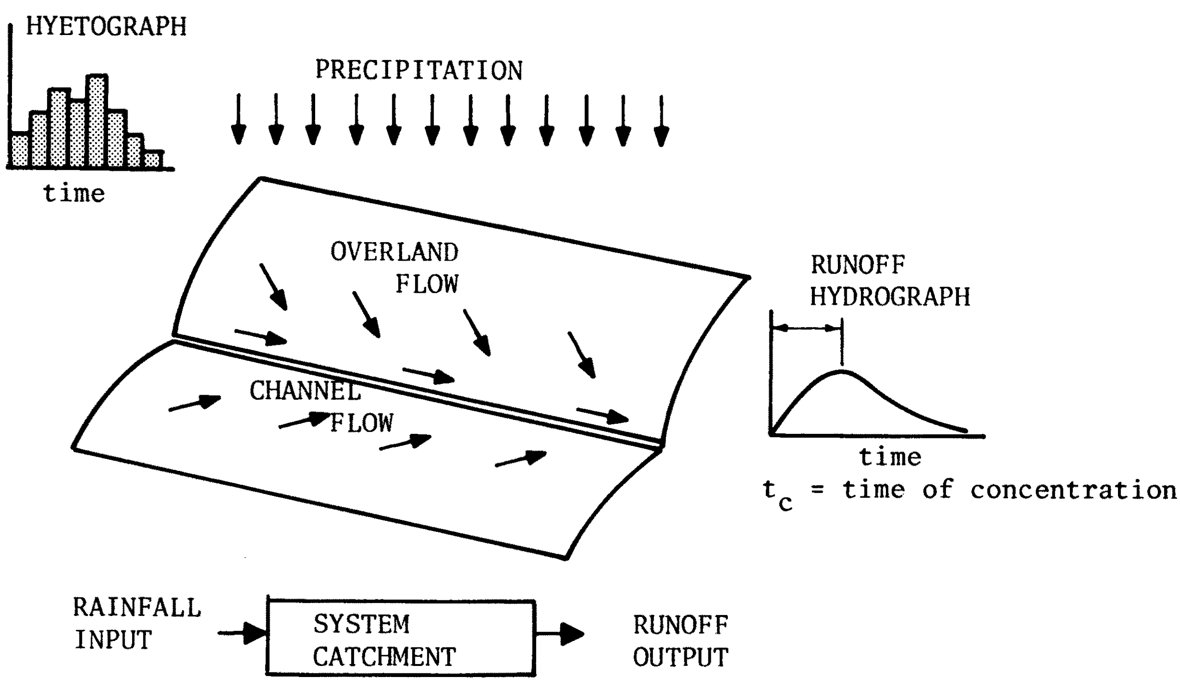

Figure V-4. The Rainfall Runoff Smoothing Process.

information in the rainfall record, thereby ultimately resulting in less accurate forecasts. The approach taken here is to use the full record of runoff generated by the entire rainfall record as the basis for a forecasting model. It is assumed that this results in a more accurate model, which capitalizes only upon the natural smoothing effect of the basin.

Figure IV-5 illustrates these two approaches to inflow forecasting. The basic structures of the forecasting models used in each approach are identical. The only difference is that one predicts rainfall, and then uses a deterministic rainfall-runoff model to predict runoff, while the other receives rainfall data which has been transformed by a rainfall-runoff model, and then directly predicts runoff. The latter approach was chosen for this study, though considerably more research is required to determine which approach is superior.

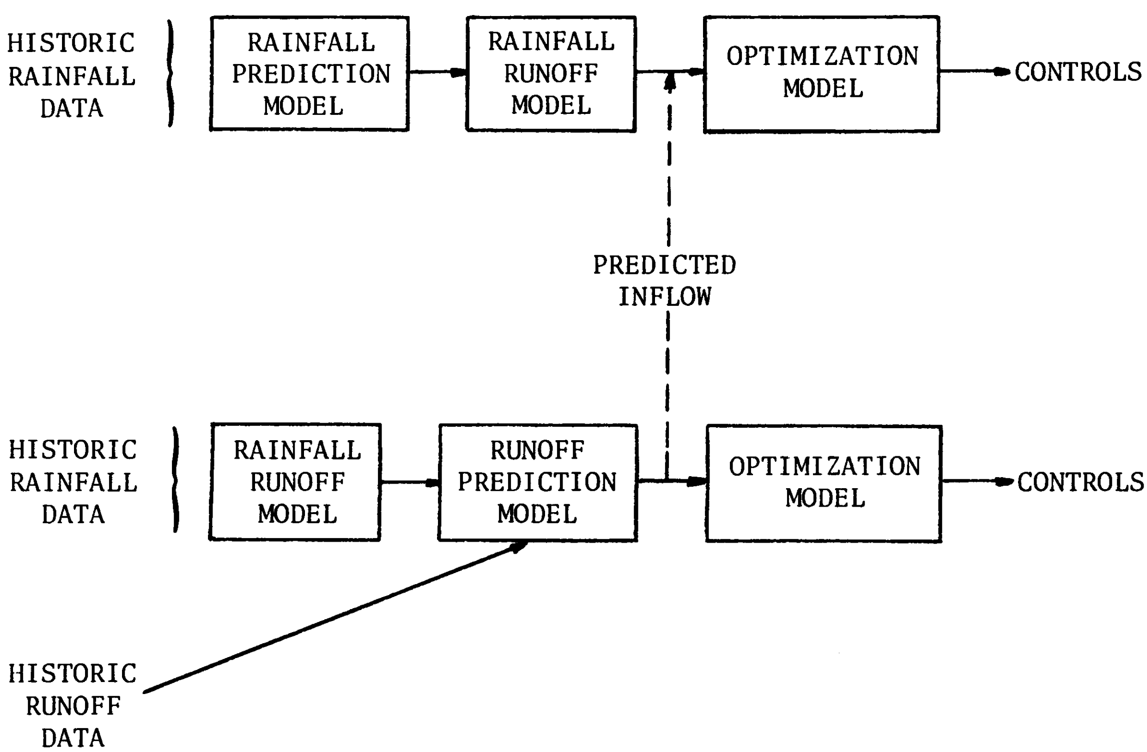

Figure V-5. Two Approaches to Inflow Forecasting.

C. AUTO-REGRESSIVE TRANSFER FUNCTION MODEL

Successive observations or measurements of the time series describing runoff are highly correlated. Time series regression analysis techniques, which attempt to account for dependencies between time series elements, are available and are generally referred to as Box-Jenkins [5] models.

As stated previously, such models assume that a time series in which successive observations are dependent can be modeled as a linear combination of independent random disturbances or *shocks* drawn from a stable distribution. Such a series of disturbances is called a white noise process, as previously mentioned. Let $u(k)$, $u(k-1)$, $u(k-2), \ldots$ represent these random components and $a_0, a_1, a_2, \ldots$ represent the weighting coefficients associated with them. The dependent sequences of inflows $R^i(k)$ at location i and time k can then be represented

$$R^i(k) = a_0 \, u(k) + a_1 \, u(k-1) + a_2 \, u(k-2) + \ldots \tag{1}$$

Such a stochastic model process is usually called a *linear filter*. Successive observations of $R^i(k)$ are dependent because they are drawn from the same previous realizations of $u(k)$. Such a time series model transforms a dependent time series into a white noise process [6]. Models of this sort are gaining wide acceptance for use as forecasting models.

Models derived from such white noise process models are capable of representing both *stationary* and *nonstationary* time series. Stationary processes are those in which the series fluctuates around some constant mean level while nonstationary processes have no such mean level.

The above model has, however, an infinite number of terms in its definition and consequently is of little use. Various approaches are available, however, to find an efficient or *parsimonious* model which adequately represents the process for the purpose of forecasting. Box and Jenkins have defined a *moving average* forecasting model of order q which is called an MA(q) model as,

$$R^i(k) = a_0\, u(k) + a_1\, u(k-1) + \ldots\, a_q\, u(k-q) \tag{2}$$

where the coefficients are unique for each location: This model differs from the previous in that only a finite number of terms are necessary for the forecast. Defining a back shift operator B^j such that

$$B^j u(k) \triangleq u(k-j) \tag{3}$$

and the function $\psi(B)$ defined as

$$\psi(B) = a_0 + a_1 B^1 + \ldots + a_q B^q \tag{4}$$

permits a simple representation of the MA(q) forecast model as

$$R^i(k) = \psi(B)\, u(k) \tag{5}$$

Autoregressive models assume that the independent random variables are the previous members of the considered series or their deviation from a constant. Such a model using p terms back in time is abbreviated as AR(p). It is written as

$$R^i(k) = b_1 R^i(k-1) + b_2 R^i(k-2) + \ldots + b_p R^i(k-p) + u(k) \tag{6}$$

where $u(k)$ is white noise and the coefficients are again unique. Using back shift notation again and the function $\phi(B)$ defined as

$$\phi(B) = b_1 + b_2 B + \ldots + b_{p+1} B^p \tag{7}$$

enables the AR(p) forecast model to be written as

$$R^i(k) = \phi(B)\ R^i(k-1) + u(k) \tag{8}$$

Another type of model which can be used for forecasting might be termed a *transfer function* or *input-output* model. In this case, $R^i(k)$ is not correlated with past values of itself or independent random shocks, but rather to *other measurable inputs*. For the situation considered herein, these other measurable inputs might be inflow measurements at each location j adjacent to location i, for various past times. The structure of such a forecasting model would be:

$$R^i(k) = \sum_{j \varepsilon J(i)} \sum_{\ell=1}^{S} c_{j\ell}[R^j(k-\ell)] \tag{9}$$

where the coefficients are unique for each location, and

$J(i)$ = set of pertinent locations adjacent to i. The number of elements in each set is assumed equal to r.

S = number of time periods backward which the model considers.

Introducing the back shift function $\Theta(B)$ such that

$$\Theta^j(B) = \sum_{\ell=1}^{S} c_{j\ell} B^{\ell} \tag{10}$$

enables a simpler representation of the forecasting model. That is,

$$R^i(k) = \sum_{j \varepsilon J(i)} \Theta^j(B) R^j(k-1) \tag{11}$$

The order of this model, which describes the total number of terms considered, is then $r \times s$.

Finally, a *mixed autoregressive moving-average transfer function* model would combine the features of the above three models and take the form:

$$R^i(k) = \phi(B)\ R^i(k-1) + \sum_{j\varepsilon J(i)} \Theta^j(B)\ R^j(k-1) + \psi(B)\ u(k) \qquad (12)$$

Such a model is needed for each location. Assuming the parameters are available, or can be derived, it is a straightforward matter to use such a model for forecasting purposes. The model used within this work does not include random disturbances or *shocks* and hence, does not incorporate the moving average terms. They could, however, be incorporated readily. Seasonal factors and some nonstationarities could be dealt with by the use of a differencing operator $\nabla^d R^i(k)$ where

$$\nabla^d R^i(k) = R^i(k) - R^i(k-d) \qquad (13)$$

This technique, however, was not incorporated in the model used herein.

The variance of a forecast made by linear models of the type described above has been shown by Brown [7] to generally increase as a linear function of the forecast time interval. This increasing risk of a poor forecast must, therefore, be incorporated into the optimization model which uses the forecast to derive the system controls. Graupe [8] has derived probability limits for these risks based upon the length and variance of the historic record.

An inflow forecasting model such as the one derived above should, if it is based upon the optimal set of previous measurements and linear coefficients, converge to the best forecast possible. The error series generated by the difference between the best forecast and the actual occurrence will then be white noise if a sufficient number of terms are included in the model. Various techniques are available for deriving the best order. Graupe [8] as well as Box and Jenkins [5] have discussed techniques for identification of the most efficient model order.

The above linear models have assumed that the random noise system inputs, which are responsible for the error series between the forecasted and observed inflows, are normally distributed. In the future, better models may be developed which will incorporate nonlinear terms if the assumption is proven false. If such an improved model were necessary and used, the error series generated by its forecast would not only be white noise but statistically independent as well. (Note that in the Gaussian case, any white noise is statistically independent.) There is now no way, however, to find the optimal nonlinear forecasting model without extensive a priori knowledge of the general nature of the system. Hence, a linear optimal forecasting model or a nonlinear suboptimal forecasting model must be used. The linear optimal model will result in an error series which is white noise despite the fact that it may be larger than the possibly non-white error series which the nonlinear suboptimal model produces.

Once it is established that such models are useful for forecasting the subsequent events in a time series, the question arises concerning how to identify or estimate the needed parameters. In the work addressed herein, the question is extended further to considering how to identify the parameters in an adaptive, on-line model.

The model used in the case study considered herein was constrained by the implicit limitations on computer power as well as by data available for parameter estimation. The model used takes the form

$$R^i(k) = \phi(B)\ R^i(k-1) + \sum_{j \in J(i)} \Theta^j(B)\ R^j(k-1) \tag{14}$$

for all locations $i = 1,\ldots,10$.

It is noted that each location for which a forecast is needed requires a separate model.

It was decided to include the data from each of the other nine locations as the *other measurable inputs* within the transfer function part of each of the ten models. This decision was based in part upon the small scale, and hence close proximity of the ten subbasin locations, as well as the lack of any sound criteria for excluding locations. Data from each location for the time periods corresponding to the time periods used in the autoregressive part of the model were included. This decision was based upon the assumption that due to the dynamic nature of a storm event, the previous measurements at adjacent or local locations were probably as important for the forecast as the previous measurements of the location being considered. Therefore, the order rs of the transfer function part of the model was $9\ p$. There were nine locations $(r = 9)$, and s equals the order of the AR model p.

The order p of the autoregressive part of the model was, therefore, selected while considering its impact upon the total model size. For each model needed, there were consequently $9p + p$ or $10p$ parameters to be estimated. For ten locations, this resulted in $100p$ parameters. With the data available, it was observed that when $p=1$, the model failed to forecast the consistent aspects of the data. With $p=2$, the forecasts improved considerably. The limited further improvement in forecasts with $p=3$, however, did not seem to warrant the additional 100 parameters. For the demonstration, therefore, $p=2$, which resulted in the need to identify 200 parameters.

In future work, it should be possible to eliminate extraneous model components while adding other relevant measurements (i.e., windspeed, direction, temperature change, etc.) to the transfer function part of the model. In addition, an increase in the data base used for parameter estimation might enable further significant improvements with an increase in the AR order above 2.

D. ON-LINE IDENTIFICATION OF MODEL PARAMETERS

If a priori knowledge of the process to be forecasted is not available, then the identification of the needed parameters is a prerequisite to forecasting. For on-line work in which the evolving situation is a unique section of the continuous time series, the identification of parameters may be an ongoing process corequisite with prediction. A theory of identification has been developed which deals with the estimation of system parameters from a history of measurements in which the identification is updated as additional data are obtained. Forecasting errors generated by the identified model are employed to improve the model. Graupe [9] distinguished six categories of systems that call for different techniques of parameter identification, (1) linear vs. nonlinear, (2) stationary vs. nonstationary, (3) discrete or continuous, (4) single vs. multiple input, (5) deterministic vs. stochastic and (6) degree of prior knowledge about the system's structure. The various combinations of these descriptors determine not only the method to be used, but are also indicators of the degree of difficulty to be expected in the subsequent identification. It was decided, for the scope of the work undertaken, that the assumption that any parameter nonstationarities in the inflow forecast model would be small compared to the *lead time* used within the model. Lead time refers to the distance into the future for which a forecast is desired. This assumption, in turn, permits the identification to proceed in a sequential manner, with the avoidance of the matrix inversions typical of parameter identification. The sequential nature of the identification lends itself well to the real world problem faced in which new data are constantly becoming available and their incorporation into the identification is desired with the least possible computational effort. The elimination of the need for *matrix inversion* in these approaches permits faster updating of the model with less computer power required and consequently a more comprehensive model can be designed with the same computer facilities.

Since the data base upon which the sequential regression identification is constantly growing, one might expect that the process may be restricted only to purely stationary processes. However, as sequential regression estimates converge to those of nonsequential regression techniques after a number of iterations of the order of the number of parameters being identified, stationarity must be assumed for only that number of intervals. As *slow nonstationarities* will cause different parameters depending on which section of data are used from a continuous record, it would be worthwhile to disaggregate the data record into categories that reflect fairly obvious shifts in systems structure. For the case at hand, the inflow forecasting model could be broken into a series of models to reflect the more obvious hydrologic nonstationarities of seasons, storm types and perhaps prevailing wind direction categories. Other factors which could cause less obvious nonstationarities could be measured and incorporated into the regression model as additional parameters. Again, for this study, barometric pressure, relative humidity, air temperature as well as possibly radar derived data (e.g., rain cells) might illustrate this point.

The identification or parameter estimation of the previously described model is carried out as follows. For notational convenience, let all the variables assumed to be correlated with forecasted inflows be designated as u_i, i=1,...,20, where the model order equals 10 (i.e., ten locations where p is assumed to equal two) discussed previously when p equals two. These 20 variables, it is recalled, are AR model component terms plus transfer function component terms. Recalling Figure V-6, it is seen that the multiple time series analysis discussed, and subsequently developed above, uses all the relevant time series data as inputs for a particular forecast. Since a separate forecasting model was needed to each location, the index i is added and the total model written as

$$\begin{aligned} R^1(k) &= a_{1,1}u_1 + \ldots a_{1,j}u_j + \ldots a_{1,20}u_{20} \\ &\vdots \\ R^i(k) &= a_{i,1}u_1 + \ldots a_{i,j}u_j + \ldots a_{i,20}u_{20} \\ &\vdots \\ R^{10}(k) &= a_{10,1}u_1 + \ldots a_{10,j}u_j + \ldots a_{10,20}u_{20} \end{aligned} \tag{15}$$

or, in vector form:

$$\underline{R}(k) = A\underline{u} \tag{16}$$

where

$$\underline{R}(k) \triangleq (R^1(k)\ldots R^{10}(k))^T \qquad A \triangleq \begin{bmatrix} a_{1,1} & \cdots & a_{1,20} \\ \vdots & \ddots & \vdots \\ a_{10,1} & \cdots & a_{10,20} \end{bmatrix} \qquad \underline{u} \triangleq (u_1 \ldots u_{20}) \tag{17}$$

The a_{ij}'s then are the coefficients to be identified. Isolating the j^{th} row, it may be written as:

$$R^i(k) = [\underline{a}^i]^T \underline{u} \tag{18}$$

where

$$[\underline{a}^i]^T \triangleq [a_{i1}a_{i2}\ldots a_{ij}\ldots a_{i20}] \tag{19}$$

Hence, 10 separate identification problems are thereby isolated with each set of parameters, $(\underline{a}^i)$, being identifiable by sequential regression methods. The estimation of parameters is performed such that the estimated parameter vector, $\hat{\underline{a}}^i$, minimizes the cost index J_s (the s denoting the estimation iteration) defined by the equation:

$$J_s = \sum_{k=1}^{s} q_k (R^i(k) - [\hat{\underline{a}}_s^i]^T u_k)^2 \tag{20}$$

where q_k are weighting factors on measurement error.

An algorithm has been developed and is presented by Graupe for the sequential identification of the parameters, and the reader is referred to his text for its full development. It is shown that for each model (dropping the index on the parameters indicating location):

$$\hat{\underline{a}}_s = \hat{\underline{a}}_{s-1} + P_s q_s \underline{u}_s (R^i(s) - \underline{u}_s^T \hat{\underline{a}}_{s-1}) \tag{21}$$

where

$$P_s^{-1} \triangleq \sum_{k=1}^{s} q_k (\underline{u}_k \underline{u}_k^T) \tag{22}$$

"Hence," according to Graupe, " $\hat{\underline{a}}_s$ may be derived sequentially from the previous estimate $\hat{\underline{a}}_{s-1}$ and from the measurements and weights $R^i(s)$, $\underline{u}_s$, q_k, provided that P_s may also be sequentially obtained." The matrix P_s is shown to be sequentially obtained according to the relation:

$$P_s^{-1} = P_{s-1}^{-1} + q_s \underline{u}_s \underline{u}_s^T \tag{23}$$

where

$$P_o^{-1} \triangleq 0 \tag{24}$$

Instead of inverting the matrix P_s, the matrix inversion lemma (14) is used to facilitate the recursive derivation of P_s, yielding:

$$P_s = P_{s-1} - P_{s-1} v_s (1 + \underline{v}_s^T P_{s-1} \underline{v}_s)^{-1} \underline{v}_s^T P_{s-1} \tag{25}$$

where

$$v_s \triangleq \sqrt{q_s}\, u_s \quad \text{such that} \quad v_s v_s^T = q_s u_s u_s^T$$

As $(1 + \underline{v}_s^T P_{s-1} \underline{v}_s)$ is scalar no matrix inversion is involved in deriving P_s and consequently $\hat{\underline{a}}_s$.

Looking back on the i^{th} row of Equation (15), it is evident that the form of each of the ten separate forecasting models mentioned can be illustrated by the same illustration presented in a previous section as Figure V-2.

E. ON-LINE AND OFF-LINE IDENTIFICATION/FORECASTING MODELS

The inflow forecasting model has three main parts, the off-line historic base line parameter identification model, the on-line adaptive (present event) parameter modification model, and the inflow forecasting model. The relationship between these functions is illustrated in Figure V-6.

The off-line, historic, base line parameter identification model analyzes past storm records and estimates the parameters of the inflow forecasting model. These parameters could be estimated in such a way that more relevant storms are given a higher weighting. Relevance, in this case, would be defined by the newness of the record and the similarity of current meteorologic conditions to those existent at the time of the historic event (e.g., season, wind direction, barometric pressure, etc.). The off-line, base line parameter identification model would, consequently, be run periodically to adjust the base line parameters to more currently relevant conditions. In the algorithm demonstration, within this work, ten variations of a *particular* historic event were used as the historic data. Equal weighting was assigned to all the simulated inflow events.

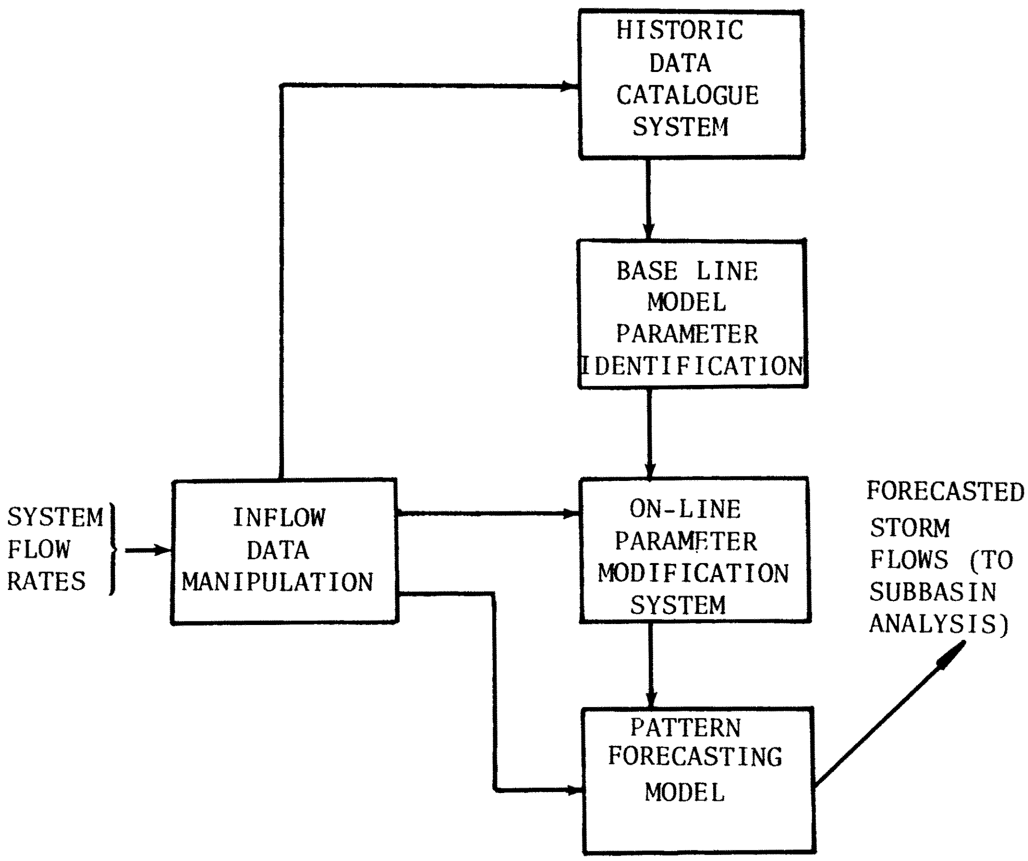

Figure V-6. Inflow Forecasting Model

As data describing the current storm event in progress becomes available, the on-line parameter estimation model can modify the base line parameters. The weighting of this data will depend upon the sensitivity of the model. A weighting factor on current data which is too high will result in an erratic model which ignores trends identified in the historic data. Weighting factors on current data which are too low will, however, ignore the evolving structure of the storm currently being experienced. The determination of the proper balance will require extensive testing on a prototype system.

The pattern predictor simply uses the previously identified parameters and available data which describes the unfolding event and forecasts the subsequent inflows for the chosen lead time. The estimated coefficients of the autoregressive transfer function forecasting model are simply multiplied by the appropriate value of the previous inflows (or previously forecasted inflows) to obtain the forecasted inflow for the desired location.

For the demonstration development, data from the ten locations were used to generate ten separate forecasting models. The identification model which identified the parameters needed for each forecasting model was the same algorithm used in the on-line parameter modification system since they both must be of the same order or dimension. They are illustrated as separate, however, to accentuate the fact that the off-line identification may be used experimentally by the systems operators. The off-line use facilitates the search for the most efficient model configuration. The off-line model must, however, retain in some storage facility the appropriate information needed by the current on-line model to be used during storm events. Except for the order and dimension, then, the logic flow for both models can be illustrated as shown in Figure V-7. The algorithm developed simply effects the iterative identification procedure developed in the previous section after appropriately arranging the needed data into prediction model inputs and outputs. In this figure model inputs are again designated as a vector $\underline{u}_j$, indicating the j^{th} value of the vector representing inflows. It includes previous $R^i(k)$ values for the i^{th} location as well as for the other locations. After the parameters have been defined to the extent possible using the historic data and any data available on the current event, the inflow forecast part of the model then forecasts the remainder of the event at a particular location. This is accomplished by the repeated application of the model generated for that location. Recalling Equation (18) that model is simply:

$$R^i(k) = [\underline{a}^i]^T \underline{u} \qquad (26)$$

$\underline{a}_i$ and $\underline{u}_i$ are the model parameters and inputs (both vectors). The parameters are the updated parameters from the identification model, and the inputs are the real values of inflows as measured and/or those inflows which were previously forecasted. The algorithm which accomplishes this is illustrated in Figure V-8 below.

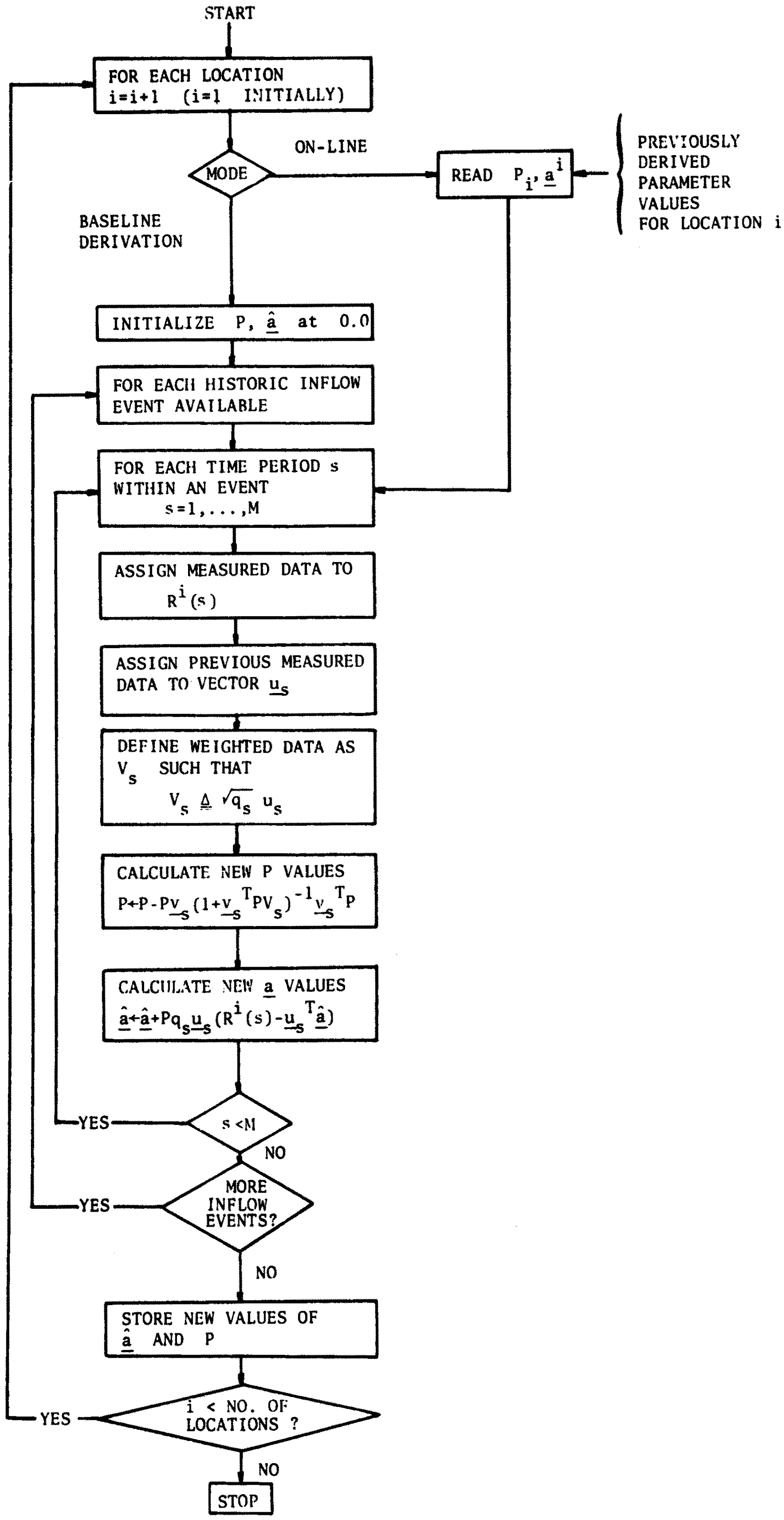

Figure V-7. Sequential Regression Parameter Identification Algorithm.

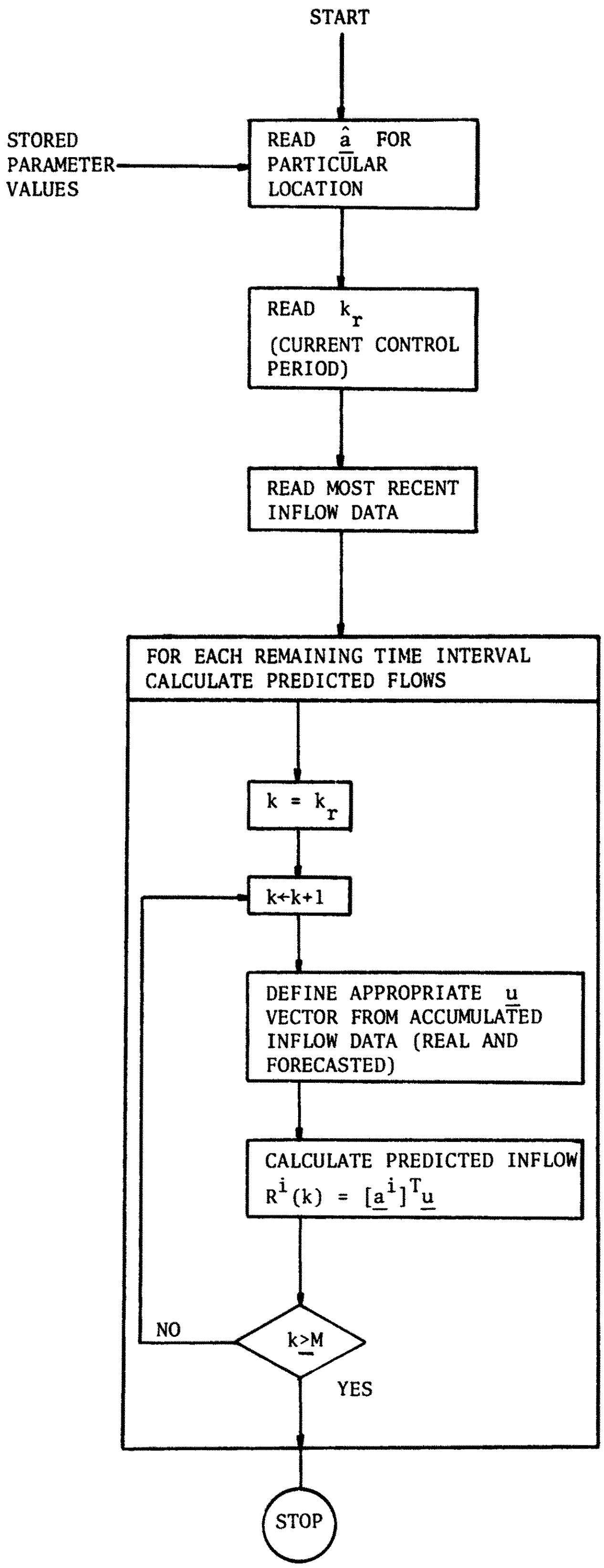

Figure V-8. Inflow Pattern Predictor

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Chapter VI

COMPUTATIONAL EXPERIENCE

A. OVERVIEW

The approach discussed in the previous chapters for the on-line real time control of the complex system used herein as a case study (i.e., the system described in the San Francisco Water Plan for Wastewater Management) consists, essentially, of three parts. In their order of anticipated on-line application they are: an adaptive inflow forecasting model, a master control algorithm, and subbasin control algorithms.

The adaptive inflow forecasting model forecasts the system inflows based upon previous off-line analyses which estimate base line parameters, as well as any available data describing the event in progress. Chapter V described the forecasting model used in this study and discussed the sequential parameter identification model which permits efficient on-line real-time adaptation of the model to the event in progress.

The master control algorithm simply allocates the treatment capacity among the subbasins. The decisions of the master control algorithm are based upon subbasin performance and are limited by a constraint upon the total releases selected by the subbasins. Chapter III described the discrete direction steepest descent method adopted for this decision level. The optimal values of the assigned subbasin release limits, represented by the vector $\underline{Q}^*_{max}$, result in subbasin controls which yield the minimum expected total weighted overflow, while they tend to maximize total expected throughflows to the treatment plant.

Subbasin control is achieved by the use of two complementary algorithms which were discussed in Chapter IV. The overall release pattern of each subbasin as a whole is worked out iteratively by a continuous interaction with the master control problem. The technique chosen for this repetitive subbasin optimization was the aggregated stochastic dynamic programming technique. The overall solution achieved by the aggregated problem is then applied as a series of guidelines (constraints) to noniterative, detailed solutions of the subbasin control problems. This is done after convergence of the master problem is achieved or as the necessities of the real time operating environment demand. These detailed solutions are then to be solved by the use of linear programming or, if time permits, stochastic linear programming.

These three major control components interact in a real-time setting and adapt to current conditions and new data. With each iteration of the master control problem, the subbasin problems are provided with current system characteristics (i.e., actual subbasin contents). In addition, with the passage of real time, the forecasting model receives more data with which to make its forecast. Thus, the forecasts should improve as time goes on.

B. SYSTEM DESCRIPTION

The system which was used for the demonstration is based upon the San Francisco Master Plan for Wastewater Management, as was previously mentioned. In particular, the Alternative C plan has been adopted for this demonstration. It contains all the complexity of the largest plan (Alternative D) while having somewhat less total storage. Table VI-1 presents relevant system data which describes the component detention

TABLE VI-1

SUBCATCHMENT AND DETENTION RESERVOIR DATA

Sub-basin No. (i)	[1] SFMP No.	S_{max} [2] Alternate C (10^6 ft^3)	Q_{max} [3] (cfs)	Drainage Area (acres)	$\frac{Q_{max}}{S_{max}}$ [i] (ALTC)	Reservoir Routing Constant, K (hrs)	[5] Dry Weather Flow (cfs)	Overflow Capacity (cfs)	Q^i_{max} cfs
1	16-6	.66	530	456	691	.1239	4.6		
	5	1.09	240	748	686	.1586	7.5		
	4	.24	260	168	700	.0752	1.7		
	3	.16	295	112	700	.0614	1.1		
	8	.30	226	204	680	.0828	2.0		
	2	.13	370	88	676	.0544	0.9		
TOTALS		$2.58=S^1_{max}$		1776			17.8	416	370
2	1	$.22=S^2_{max}$	18	90	409	.0550	0.9		18
3	7	$.18=S^3_{max}$	25	124	688	.0646	1.2	54	25
4	14-1	.14	63	60	428	.0449	.6		
	2	.79	119	541	684	.1349	5.4		
TOTALS		$.93=S^4_{max}$		601			6		119
5	13-11	.57	155	387	678	.1141	3.9		
	10	.23	190	153	665	.0717	1.5		
	9	.25	140	165	660	.0745	1.6		
	8	.21	250	145	690	.0698	1.5		
	7	.15	95	101	673	.0583	1.0		
	6	.23	185	154	669	.0720	1.5		
	5	1.45	419	1012	697	.1845	10.1		
TOTALS		$3.09=S^5_{max}$		2117			21.1	960	419
6	4	.18	110	126	700	.0651	1.3		
	3	.18	200	122	677	.0641	1.2		
	2	.27	85	186	688	.0791	1.9		
TOTALS		$.63=S^6_{max}$		434			4.4	230	85
7	1	$1.13=S^7_{max}$	151	770	681	.1609	7.7		151
8	12-3	.19	82	129	678	.0659	1.3		
	5	.40	165	276	695	.0964	2.8		
	4	.32	170	222	693	.0864	2.2		
	2	.95	253	655	689	.1484	6.6		
TOTALS		$1.86=S^8_{max}$		1282			12.9	960	253
9	1	$.54=S^9_{max}$	73	370	685	.1116	3.7		73
10	2	.27	380	182	674	.0782	1.8		
	5	.36	432	246	683	.0910	2.5		
	1	.24	107	165	687	.0745	1.6		
TOTALS		$.87=S^{10}_{max}$		593			5.9		107

NOTES:

[1] Corresponding numbering of component detention basins in the San Francisco master plan.

[2] S_{max} storage capacity of associated detention basin S^i_{max} total storage capacity of subbasin i.

[3] Q_{max} maximum flow capacity into collector sewer from associated detention basin Q^i_{max} maximum flow capacity into interceptor from subbasin.

[4] Ratio between maximum release and detention basin capacity illustrates a design consistency tending to justify aggregation techniques.

[5] Inflow forecast models used dry weather flows as minimum values.

basins as well as the total storage available within each encompassing subbasin. The capacities of overflow bypass facilities have been included where such facilities exist for subbasin locations. These limits were modeled explicitly in the LP formulations, but not in the DP as used. The distribution of necessary overflows in time and space is, however, influenced by the weighting coefficients on overflows. These coefficients can be easily modified within the optimization to reflect onto the decision framework the exceedance of various thresholds (i.e., capacity of a bypass facility). The criterion function which considers the squared overflows will, in addition, tend to distribute overflows through time and space and will have much the same effect as constraining limits on overflows at particular locations.

The maximum subbasin release was assumed to be the maximum detention basin release from the detention basin located at the most downstream location in the subbasin. This is consistent with the development of the aggregated dynamic programming algorithm, presented earlier, as well as the method chosen for interaction between it and the detailed subbasin problem. These developments assumed that the control policy for the aggregated subbasin was identical to the control policy for the last or lowest (i.e., furthest downstream) detention basin. In situations where this may not be the case (e.g., two or more detention basins which contribute to a sewer below the lowest detention basin in the subbasin), the maximum subbasin release would be either the capacity of the connecting sewer, or the combined maximum release of both (or all) contributing downstream detention basins, whichever is less. In such a case the detailed subbasin problem would recognize the planned subbasin release as a constraint on the summation of these downstream detention basin releases. The discretizations used within the subbasin problems, as was noted in Chapter IV were based upon these maximum releases.

C. PROCEDURE FOR TOTAL ALGORITHM DEMONSTRATION

The simulation of this adaptive approach for on-line use was carried out on the noninteractive batch mode computer facility at Colorado State University. The simulation proceeded as follows:

1. Ten variants of the inflows from a historic storm were generated by multiplying each inflow value by a random factor between 0.5 and 1.5. These synthetic storms were fed through the parameter identification procedure to establish the base line forecast model parameters.
2. Based upon this same historic inflow sequence, which actually caused significant overflows in the existent system, one additional inflow sequence was established in the same manner and is referred to as the *real* event to be considered.
3. Data representing the *real* inflows for the first control interval were supplied to the forecast model for forecasting subsequent inflows. For this first time period, then, it was assumed that actual data describing the *real* inflows were available for that time period. Inflows for the first time period could have been based upon an average historic storm for initiating the forecast. In the context of the demonstration as conducted, however, there would have been little difference between these average initial storm values and the *real* values which were used. For control periods subsequent to the first, the forecast model was supplied with data

describing the *real* inflows experienced up to but not including, that control period.

4. The *actual* states of the subbasins were generated to simulate real data from the measuring devices located at the subbasins. For the first time increment the initial storage levels were assumed to be zero. For subsequent time intervals the *actual* states were determined by applying the continuity relationships for subbasin storage using the selected controls and the *real* inflow data (not the forecasted inflow data).

5. The total control algorithm (master problem and subproblems) then established the optimal $\underline{Q}^*_{max}$ vector, given the initial states of the subbasins, the treatment plant capacity assigned to the Richwood Sunset area Q_{Tmax}, and the control period assumed to be imminent.

6. The subbasin controls derived for the first of the remaining control intervals for each iteration of the entire control algorithms were assumed operative. That is, they would be used as the actual controls for the furthest downstream control point of each subbasin. The detailed subbasin problems were not, consequently, necessary for the demonstration of the adaptive aspects of the proposed procedure.

7. This process was repeated by returning to step three until the entire *real* storm event had passed and a complete set of operative optimal controls had been derived for the subbasins.

This general procedure is illustrated in Figure VI-1, as the total algorithm demonstration flowchart. It is worth repeating that although controls were derived for all remaining time increments, the total

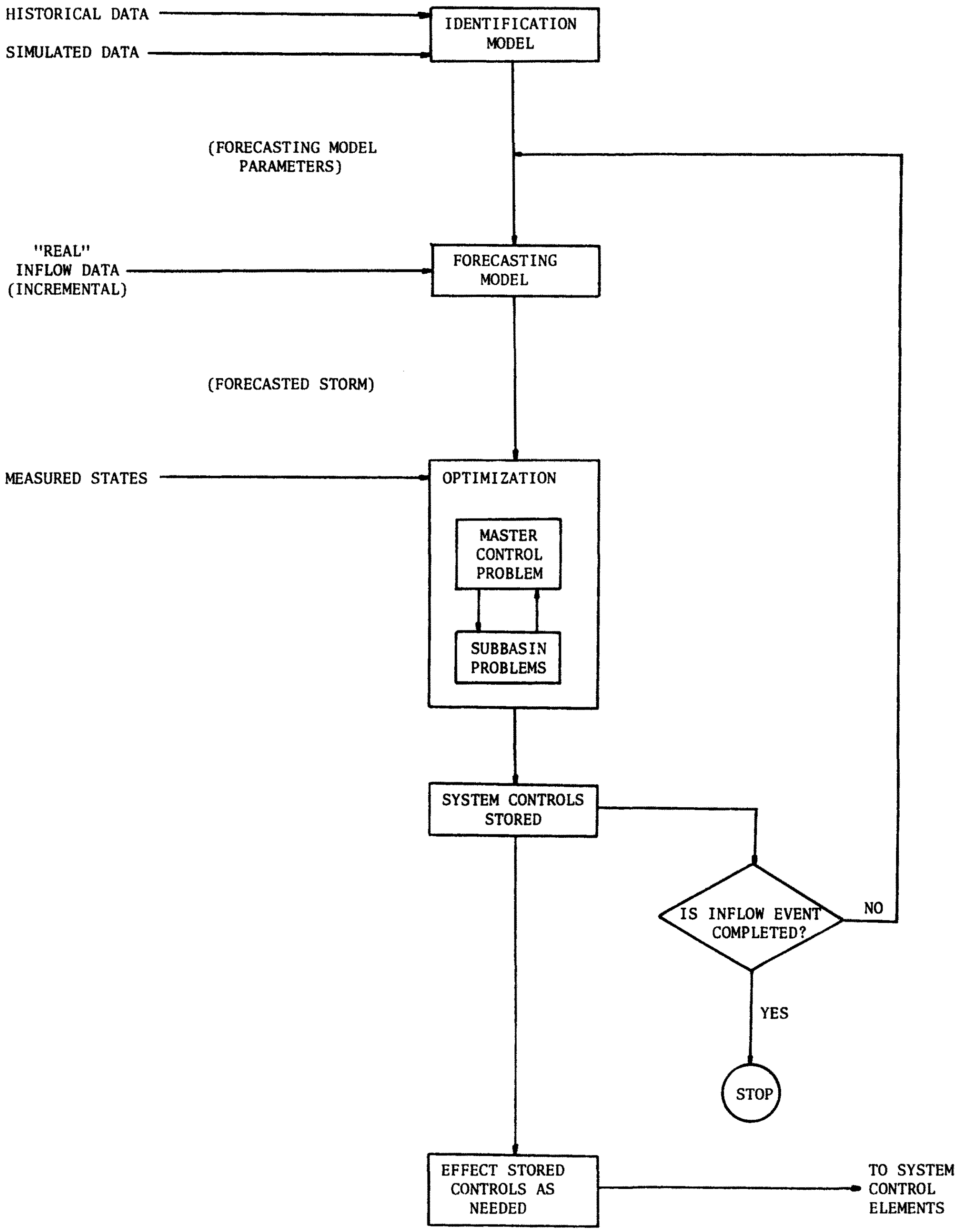


Figure VI-1. Algorithm Demonstration Flow Chart control algorithm was exercised every time increment. This was a conservative decision. In the actual operational system, it is envisioned that the control algorithm will be exercised only on demand. That is, if actual and forecasted inflows diverge beyond a tolerable threshold and/or measured states depart significantly from anticipated states, then it is assumed that the previously derived control strategy is suboptimal for the conditions being experienced. In such a case an updated control strategy is necessary which is based on updated forecasts and relevant system measurements.

In the actual operational system, a forecast of the number of control intervals necessary for the occurring event will need to be made. This forecast may also need to be updated as the event progresses. For the demonstration described here it was assumed that ten control intervals would be sufficient. Each control interval represented 20 minutes. The selection of 20 minute time intervals was based upon the availability of 20 minute inflow data for an overflow-causing event. The ten control intervals were then adequate to exceed the two hour duration of the considered event. It is anticipated that the control intervals for an actual system would be much shorter, possibly in the range of five minutes, and that the number of control intervals capable of being included in the optimizations would, hence, need to be greater. The speed of the algorithms will influence the choice of control interval size, and hence, the number of control intervals needed within the optimization. Experience with a prototype system will be needed to ascertain the proper balance between control interval size and the number necessary for desired system sensitivity.

It is noted, however, that despite an increased number of time intervals and the need for greater computer time which accompanies a shorter control period, the algorithm is capable of on-line real-time optimization for an event of any length. This is possible because of the forward moving time horizon or lead time possible within the optimization. That is, as real time progresses, the lead time need not shorten, thereby modeling a limited duration event, but may remain constant.

In the real-time on-line system this demonstration is attempting to illustrate, the sensors in the detention basins will be reporting the actual storage levels to the control center. These actual states of the subbasins will then be used as the initial conditions for the next iteration of the forecasting model and master control procedures.

These actual levels are a result of the actual inflows encountered and the previously effected controls. Therefore, in the simulation, the values which represent the measured states are calculated from the inflow data modeled as the *real* event and the derived releases which were based on the forecasted event. In this way each iteration of the master control procedure starts from *correct* initial conditions rather than derived states based on the forecasted storm inflows and calculated releases.

The inflow value which has been selected to be the *real* inflow for the demonstration is represented as $R^{ai}(k)$ for location i and time period k. The simple continuity relationship used to derive the *actual* state level subsequent to the selection of a control which is henceforth assumed operative is simply written as

$$S^i(k_r + 1) = S^i(k_r) + R^{ai}(k_r) - \hat{Q}(k_r) - \hat{O}^i(k_r)$$

where k_r indicates the current real time period for which the derived optimal control is assumed to have operated. This continuity relationship, it is recalled, differs from the one used within the optimization. In the optimization, the control decision must be made prior to the actualization of the real inflow. Only forecasted inflow values are, consequently, available for estimating the resultant state.

It is obvious that since the release from each aggregated subbasin flows directly to the interceptor, there is no situation in which the routing of upstream releases, into a downstream subbasin is needed. Further, routing effects between the actual detention basins which constitute a particular subbasin have also been ignored within this phase of the total algorithm. A justification for this is based upon the net effect of routing. Routing, in general, delays and attenuates flow profiles. The aggregated problem which lumps together all inflows assumes instant availability of flows for subbasin control decisions. The actual detailed control of the subbasins which is derived subsequent to the solution of the aggregated problem may be incapable of delivering the planned subbasin release if routing is included in the model. In the first time periods this may prove to be a built-in margin of safety. In subsequent time periods, however, the advantage may be reversed, resulting in a greater threat of overflow. Further experimentation with this aspect of the problem will need to be conducted.

Flow routing effects have a somewhat similar effect within the interceptor. Routing in the interceptor will modify the phasing of the downstream arrival of release patterns for the various subbasins. This will alter the total flows in any particular time period.

These altered total flows, which are then compared to the upper limit on flows to the treatment plant Q_{Tmax} will hence influence the allocation by the master problem of treatment capacity to the various subbasins. Within the algorithms, as developed, routing can easily be applied to the interceptor flows prior to their summation. Each time the constraint is checked tentative releases from all subbasins and time periods are available. Therefore, with the physical characteristics of the various interceptor reaches known and all inflows available, interceptor routing can be as accurate (hence complicated) as time permits. In this demonstration, however, since all calculations were performed sequentially by one central computer, routing was deleted. It is noted, however, that the parallel computations possible within the anticipated hierarchy of mini-computers or microprocessors should compensate sufficiently for their slower speeds so that simple routing models may be incorporated into the total on-line algorithm without necessitating longer control intervals.

D. IDENTIFICATION OF FORECASTING MODEL

D.1 Selection of Synthetic and Historic Inflows

The demonstration of the identification procedure required a degree of compromise with reality due to the limitations of available data. The number of intense storms which resulted in significant overflows from the existent system, and for which complete inflow records were available, was limited. To statistically simulate additional inflow sequences based on this limited sample and then attempt to identify parameters for a forecast model from the augmented sample would bias the forecast directly in the direction of the limited sample. In order to create a sample of storms large enough for the identification of the forecasting model parameters without this inherent but unknown bias, it was decided to artificially preordain a particular bias by constructing a set of synthetic, apparently diverse storms which maintained pre-selected characteristics. If the identification model detected this planned bias, and it was reflected in the forecasted events, the identification technique would be verified to some degree. The bias introduced was in the form of a constant phasing of the inflow hydrograph peaks. The difference in times of concentration within a subbasin, as well as the effects of the spatial and temporal distribution of the causal rainfall, results in different times to peak for the inflow hydrographs. A series of artificial storms were created which maintained, to a rough extent, this phasing. The simulated inflow sequences were produced by taking each inflow value of the original historic overflow producing event and multiplying it by a random number between 0.5 and 1.5. This resulted in a sample of storms with a planned bias. This bias would then, hopefully, be identified and reflected in the subsequently forecasted inflows. The identification process would ideally filter out the random component, leaving a base line model which is based, essentially, on the original event.

A series of autoregressive transfer function models were then derived, one for each subbasin, using the subbasin inflows from the augmented sample of inflow events. It is noted that the historical event upon which these simulated inflow events were based caused considerable street flooding and overflows within the existing system. The inflow at a particular point was related to the two previous inflows at the other ten locations as explained in Chapter V. Table VI-2 presents the historic detention basin inflows as reported by

TABLE VI-2

HISTORIC INFLOW DATA

Subbasin No. (i)	Detention Basin No. (j)	$R_j(k)$ [2] k=1 [3]	2	3	4	5	6	7	8	9	10	SFMP[1] No.
	1	5.	5.	125.	249.	247.	26.	6.	5.	5.	5.	16-6
	2	8.	8.	200.	402.	381.	64.	14.	8.	8.	8.	5
	3	2.	2.	65.	135.	51.	3.	2.	2.	2.	2.	4
	4	1.	1.	50.	101.	16.	1.	1.	1.	1.	1.	3
	5	2.	2.	85.	169.	54.	4.	2.	2.	2.	2.	8
	6	1.	1.	40.	81.	8.	1.	1.	1.	1.	1.	2
1	TOTAL	19.	19.	565.	1137.	757.	99.	26.	19.	19.	19.	
	7	1.	1.	40.	81.	17.	1.	1.	1.	1.	1.	16-1
2	TOTAL	1.	1.	40.	81.	17.	1.	1.	1.	1.	1.	
	8	1.	1.	20.	111.	10.	1.	1.	1.	1.	1.	16-7
3	TOTAL	1.	1.	20.	111.	10.	1.	1.	1.	1.	1.	
	9	1.	10.	32.	36.	1.	1.	1.	1.	1.	1.	14-1
	10	5.	100.	210.	356.	51.	9.	6.	5.	5.	5.	2
4	TOTAL	6.	100.	242.	392.	52.	10.	7.	6.	6.	6.	
	11	4.	65.	145.	268.	30.	5.	4.	4.	4.	4.	13-11
	12	2.	50.	117.	55.	2.	2.	2.	2.	2.	2.	10
	13	2.	55.	128.	56.	3.	2.	2.	2.	2.	2.	9
	14	2.	35.	74.	40.	2.	2.	2.	2.	2.	2.	8
	15	1.	35.	74.	40.	1.	1.	1.	1.	1.	1.	7
	16	2.	60.	133.	37.	2.	2.	2.	2.	2.	2.	6
	17	10.	350.	701.	341.	64.	18.	11.	10.	10.	10.	5
5	TOTAL	23.	650.	1372.	837.	104.	32.	24.	23.	23.	23.	
	18	1.	40.	83.	59.	2.	1.	1.	1.	1.	1.	13-4
	19	1.	50.	108.	27.	1.	1.	1.	1.	1.	1.	3
	20	10.	27.	164.	22.	2.	2.	2.	2.	2.	2.	2
6	TOTAL	12.	117.	355.	108.	5.	4.	4.	4.	4.	4.	
	21	8.	250.	517.	300.	47.	13.	8.	8.	8.	8.	13-1
7	TOTAL	8.	250.	517.	300.	47.	13.	8.	8.	8.	8.	
	22	20.	41.	100.	7.	1.	1.	1.	1.	1.	1.	12-3
	23	3.	10.	170.	135.	10.	3.	3.	3.	3.	3.	5
	24	2.	17.	190.	46.	3.	2.	2.	2.	2.	2.	4
	25	150.	307.	378.	60.	12.	7.	7.	7.	7.	7.	2
8	TOTAL	175.	366.	838.	248.	26.	13.	13.	13.	13.	13.	
	26	75.	147.	249.	27.	5.	4.	4.	4.	4.	4.	12-1
9	TOTAL	75.	147.	249.	27.	5.	4.	4.	4.	4.	4.	
	29	41.	83.	119.	8.	2.	2.	2.	2.	2.	2.	11-2
	30	40.	79.	186.	16.	3.	3.	3.	3.	3.	3.	5
	31	40.	79.	105.	5.	2.	2.	2.	2.	2.	2.	1
10	TOTAL	121.	241.	410.	29.	7.	7.	7.	7.	7.	7.	

NOTES:

[1] Corresponding numbering of component detention basin in the San Francisco master plan.

[2] All flows are in cfs.

[3] Time intervals are 20 minutes.

Bradford [1] and the *lumped* subbasin inflows used in the base-line parameter identification procedure.

D.2 Results of Identification Algorithm

A total of ten inflow events were generated with this planned bias. Since each inflow event was described by the inflow into each of the ten locations and ten periods of time, there were thus 80 pieces of data available for each of the ten models generated. This was due to the autoregressive aspect of the model in which each inflow value was related to the previous two vectors of inflows which supplied the needed autoregressive and transfer function terms. Convergence to the base-line parameters was achieved within seven seconds on the CDC 6400 for many of the models. The parameter convergence for a typical transfer function term which relates the inflows at one location to those at another is illustrated in Figure VI-2. The convergence trends of the autoregressive parameters were generally of the same nature as the transfer function terms. Initial erratic behavior which yielded to the establishment of a trend after five or six simulated inflow events seemed to be the most frequent convergence pattern.

D.3 Results of Forecasting Model

Table VI-3 presents the inflow data for the storm event that was assumed to be the *real* event experienced. Although derived from historic inflow sequence it exceeded the historic inflow in total inflow volume by two percent.

The relationship between the various forecasts and the *real* inflow sequence for two subbasins that ultimately experienced the largest portion of overflows is illustrated in Figure VI-3. The

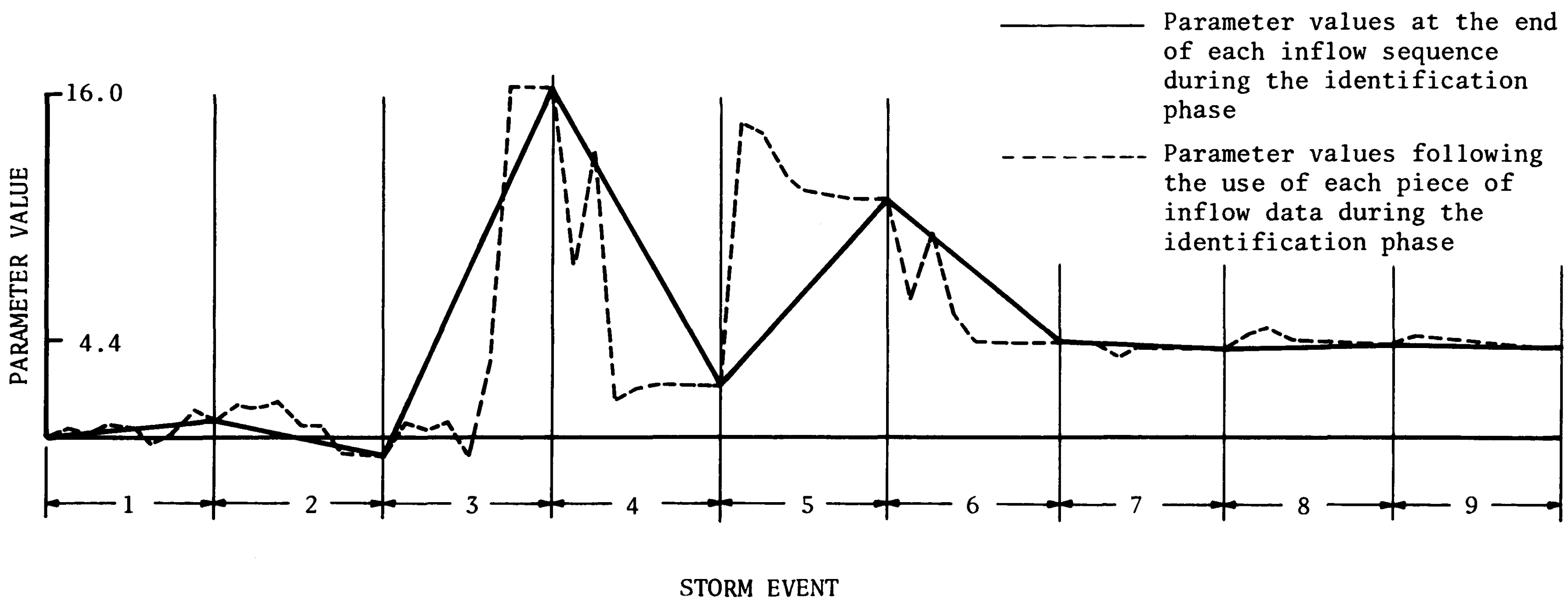


Figure VI-2. Parameter Convergence Trends

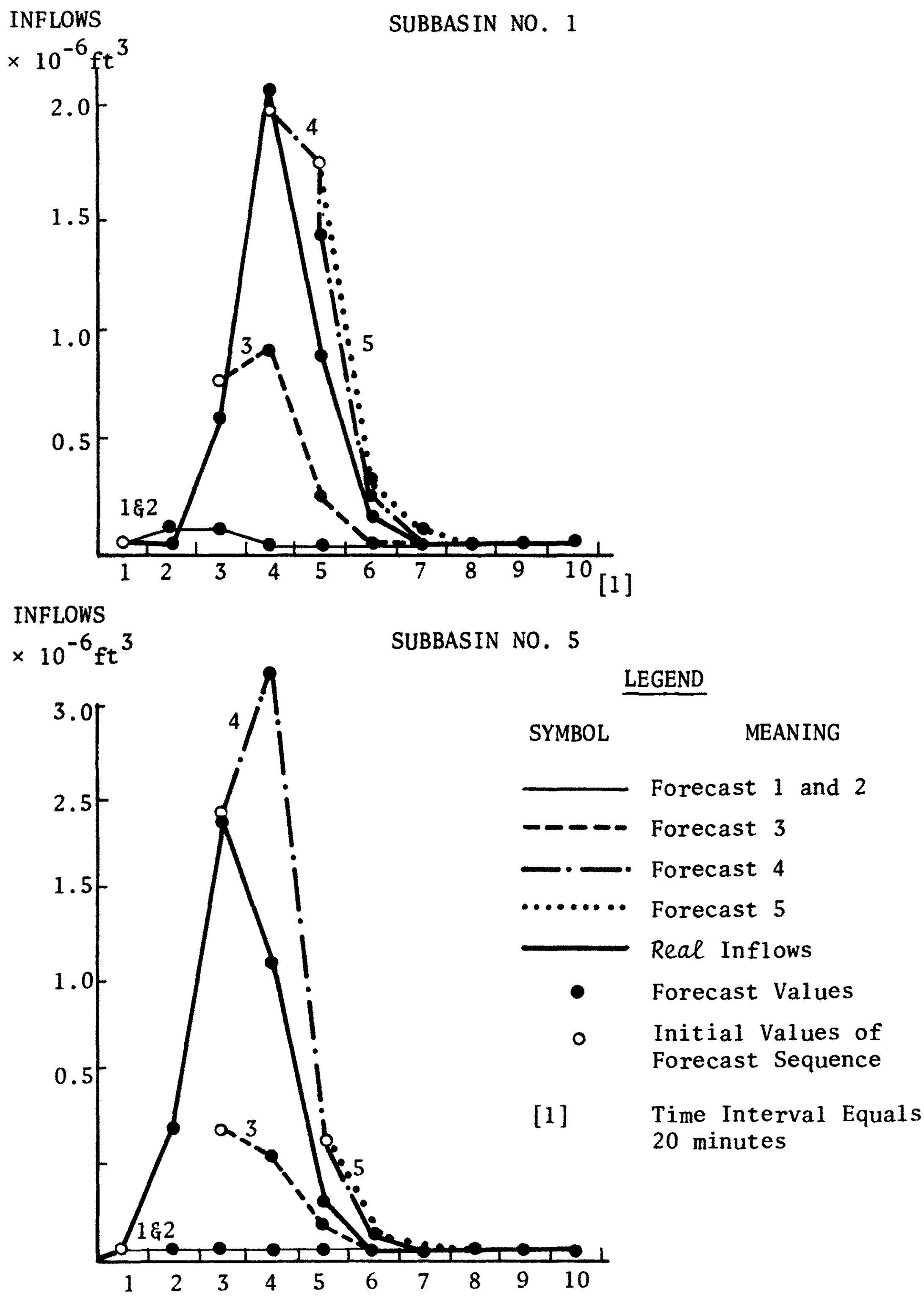


Figure VI-3. Comparison of *Real* Inflows and Forecasts for Subbasin One and Five error trends in these two series of forecasts were similar to those of the other eight subbasins. In all cases the initial forecasts were lower than the *real* event. This was due in part to the nature of the model. It was derived by assuming that the inflows were related to the previous *two* measurements at all locations. For the demonstration, the first two forecasts were based entirely upon the first set of inflow measurements representing only one period of time. Half of the model was, consequently, unable to contribute to the forecast, since the measurements previous to the first were assumed zero.

Forecasts subsequent to the first two showed a significant improvement. The full magnitude of the peak was, however, not reasonably forecasted until the control interval previous to it was reached. At that time the total remaining inflows were, however, generally predicted to be greater than the *real* event.

Table VI-3. Inflows from Simulated *Real* Storm $R^i(k)$

Subbasin No.	Time (k) = 1	2	3	4	5	6	7	8	9	10
1	19.00	30.00	400.00	1400.00	600.00	120.00	50.00	19.00	19.00	19.00
2	1.00	5.00	35.00	70.00	25.00	10.00	1.00	1.00	1.00	1.00
3	1.00	1.00	30.00	90.00	20.00	1.00	1.00	1.00	1.00	1.00
4	6.00	50.00	300.00	250.00	100.00	20.00	10.00	6.00	6.00	6.00
5	23.00	450.00	1500.00	1000.00	204.00	50.00	24.00	23.00	23.00	23.00
6	12.00	150.00	300.00	120.00	5.00	4.00	4.00	4.00	4.00	4.00
7	8.00	200.00	570.00	250.00	100.00	50.00	8.00	8.00	8.00	8.00
8	150.00	400.00	800.00	300.00	30.00	13.00	13.00	13.00	13.00	13.00
9	50.00	175.00	250.00	25.00	8.00	4.00	4.00	4.00	4.00	4.00
10	90.00	270.00	370.00	60.00	20.00	7.00	7.00	7.00	7.00	7.00

Figure VI-4 illustrates the error series between the assumed *real* inflows and the final forecast available. The final forecast available is the inflow forecasted in each immediately following control period for each value of real-time. The control decisions for these subsequent time periods are assumed to be implemented in the system since no further opportunity exists to refine the forecast prior to the application of the needed controls. For control algorithms based upon a simple one step ahead forecast this error series is of primary importance. For control algorithms which recognize the interdependence in time of the control decisions the importance and impact of this error series is shared by the error series between *real* inflows and forecasted inflows for *all* subsequent time periods. This forecast error is illustrated in Figure VI-5 which compares the total remaining forecasted inflows to the total remaining assumed *real* inflows for subbasins one and five. It is noted that for subbasin five, initial underestimations of individual subsequent inflows (Figure VI-4) is accompanied by a moderate underestimation of total inflows (Figure VI-5). For subbasin one, however, an initial overestimation of individual subsequent flows is accompanied by a general underestimation of total subsequent flows.

E. CONVERGENCE PROPERTIES OF MASTER CONTROL PROBLEM

The master control problem allocates the assigned interceptor capacity, Q_{Tmax}, to the subbasins. The capacity is assigned increment by increment. The increments are based upon the discritization used in the particular subbasin considered. The subbasin which results in the best improvement in the total objective function without causing a

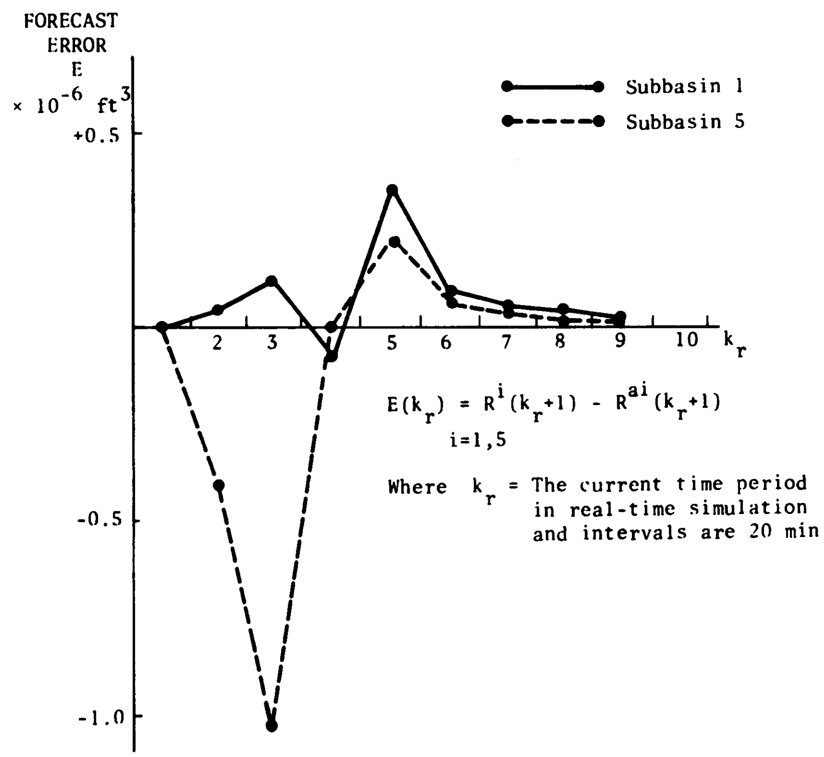


Figure VI-4. Error Series Between *Actual* Inflows and Forecasted Inflows for Each Subsequent Time Penalty

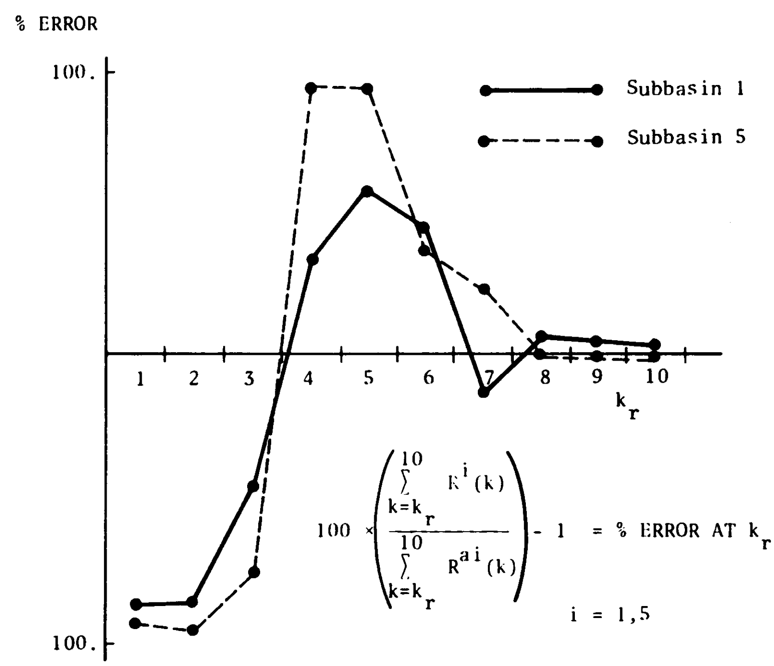
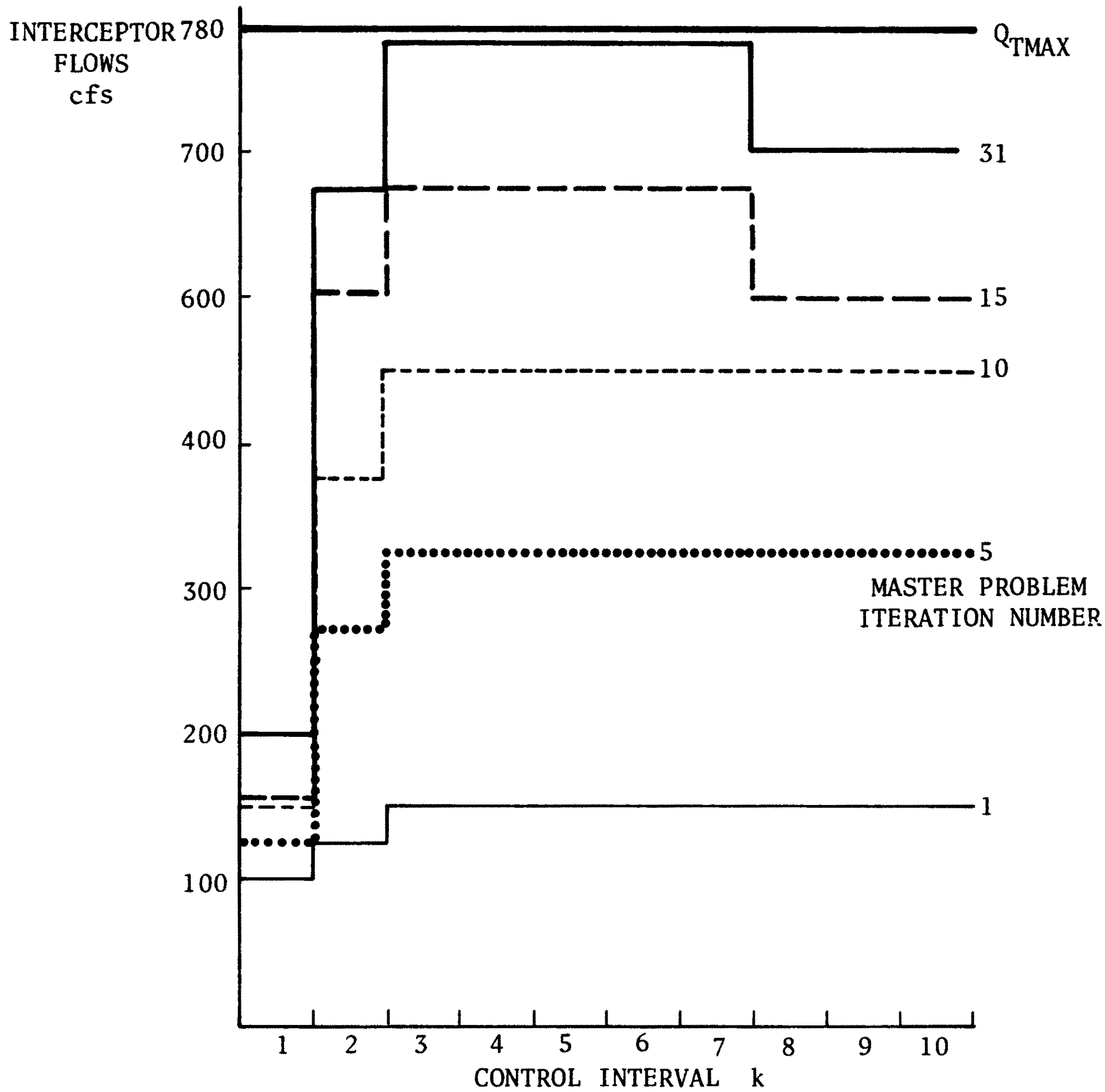
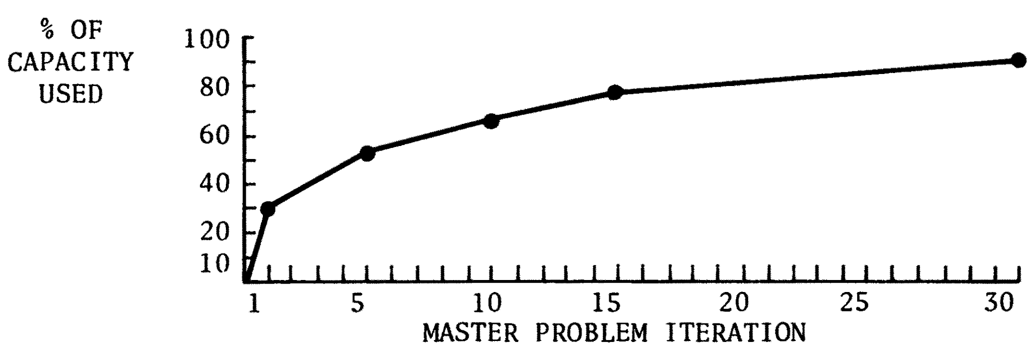


Figure VI-5. Total Forecast Error Series violation of the interceptor capacity constraint is incremented before all others. The interceptor capacity is thereby utilized with more important flows first. The convergence to the upper limit Q_{Tmax} of total interceptor flows for each time period (k) is illustrated in Figure VI-6a. In this demonstration Q_{Tmax} was arbitrarily set equal to 780 cfs which represents approximately one-half the total available treatment capacity for the plan considered. In the demonstration to follow, a much smaller limit was used (300 cfs), and consequently, the convergence was more rapid. This demonstration, however, illustrates more dramatically the priorities of overflow minimization followed by throughflow maximization (where capacity permits), which characterizes this entire development. Total interceptor flows are shown for various intermediate steps of the master control problem. Figure VI-6b illustrates the total interceptor utilization over all time periods as a function of the number of master problem iterations. The final value of 89 percent indicates that although flows were not available to fully utilize the capacity for all time periods the final attained utilization was still high.

Figure VI-7 illustrates the corresponding convergences of the total objective function and *anticipated* overflows to their ultimate values. Anticipated overflows are those overflows which are used within the optimizations to plan the control strategy. They are based upon the control decisions being considered and the forecasted inflows. They relate to the actual overflows to the degree that the forecasted inflows are accurate. Within the optimization, however, these overflows are seen as planning tools only. Each step of the master control problem involves a subbasin evaluation. The time taken for each step

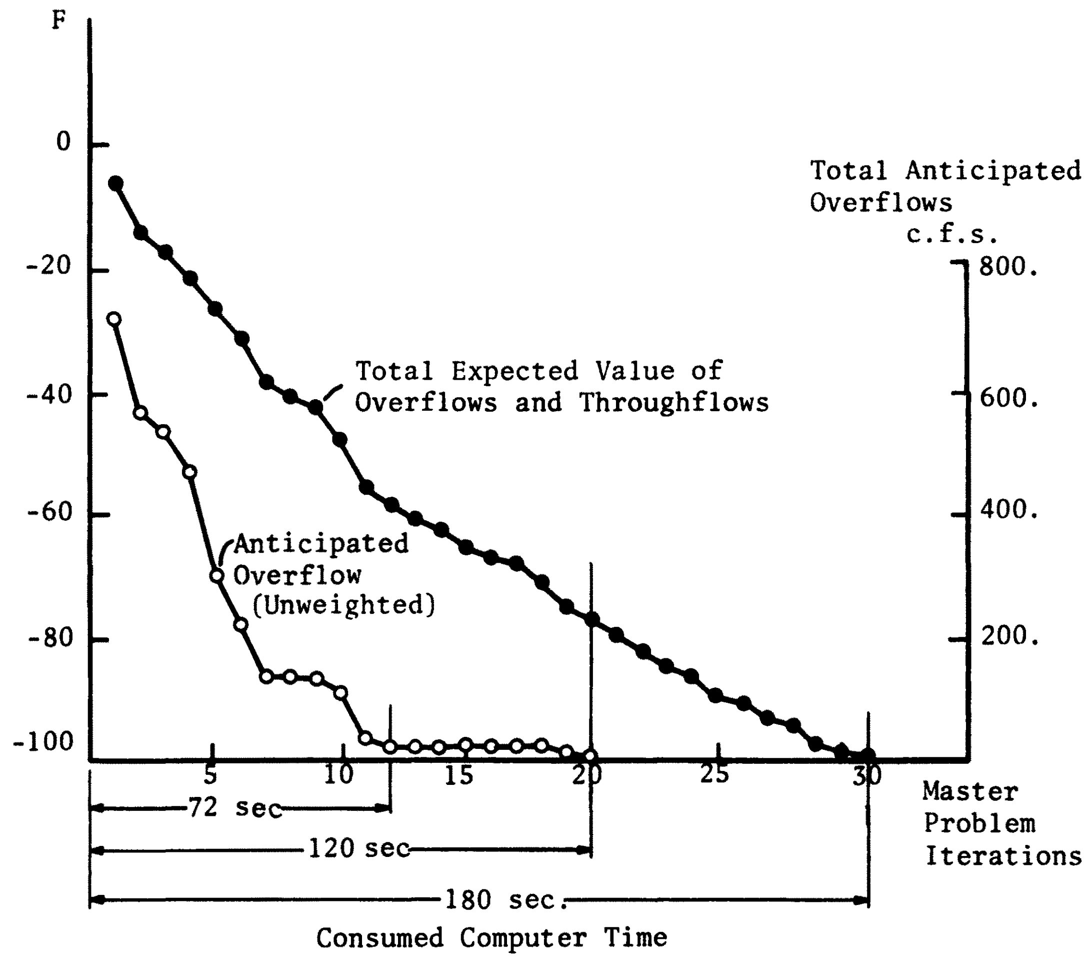


(a) Allocation of Interceptor Capacity



(b) % of Total Capacity (Q_{TMAX}) Used

Figure VI-6. Master Problem Convergence Trends



Total Anticipated Overflows $= \sum_{i=1}^{10} \sum_{k=1}^{10} O^{*i}(k)$

Total Expected Value of Overflows and Throughflows (F) $= \sum_{i=1}^{10} \tilde{O}(\underset{\sim}{Q}^{i}_{max})$

Figure VI-7. Convergence of Objective Function and Anticipated Overflow Reductions consequently depended upon the subbasin considered and its discretization. On the average, however, each step in the demonstration took approximately six seconds on a CDC 6400 computer with total convergence achieved in approximately 180 seconds or three minutes.

A control strategy which essentially eliminated anticipated overflows was achieved with 12 steps or approximately 72 seconds. (Complete elimination of overflow was obtained after 20 steps or 120 seconds.) The balance of the master control procedure resulted in a maximization of throughflows subsequent to the elimination of overflows. This is evident in Figure VI-7 by the continued decrease in the objective function after the elimination of overflows. Additional throughflow minimization after overflow elimination accounted for 23 percent of the final objective function value and required approximately 60 additional seconds.

F. ALLOCATION OF TREATMENT CAPACITY

As the Richmond-Sunset section of the system is only one of three major branches of the total system, it is assumed that another level of decision making will divide the total available treatment plant capacity among the three branches. The maximum flow to treatment Q_{Tmax} was, however, selected arbitrarily for this demonstration. A value of Q_{Tmax} was selected such that significant overflows would occur in the system if an operation rule like the one presented in Chapter IV was used for subbasin control purposes. This selection of Q_{Tmax} would then permit the comparison of system performance between control policies generated by the operating rule, using a one step ahead forecast, and the aggregated dynamic programming, using forecasted inflow for the entire remaining time horizon.

The total treatment capacity to be available for the entire city in the modeled plan is 1560 cfs. Considering the Richmond-Sunset area to represent approximately one-third of the area of the city results in an average allotment of 520 cfs of capacity. This level, however, was too high to assume significant overflows for the system when controlled either by policies derived using simple operating rules or, policies derived using aggregated dynamic programming. For the demonstration, therefore, the value of Q_{Tmax} was fixed at 300 cfs. This resulted in significant overflows for the system and hence enabled a comparison between techniques to be made.

G. RESULTS OF THE INTERACTIVE OPTIMIZATION

The master control problem and iterative forecasting model were tested using four different subbasin control algorithms for the purpose of testing and comparison. The four subbasin control strategies used were as follows:

1. The aggregated dynamic programming technique using a criterion which squared the weighted overflows and assumed that risk was associated with each forecast value.
2. The aggregated dynamic programming technique using a criterion which squared the weighted overflows and assumed that *no* risk was associated with the forecast values.
3. The aggregated dynamic programming technique using a criterion which increased linearly with the weighted overflows and assumed risk.
4. A simple operating rule, previously discussed in Chapter IV, which assumed no risk and which used a linear criterion.

These four approaches, although not providing an exhaustive selection of alternatives, were used to gain insight into the following considerations:

1. How much advantage if any does a complete forecast of future inflows yield?
2. How much does the consideration of the inherent risk in these forecasts influence the ultimate performance?
3. How will the nature of the criterion (i.e., weighted overflows squared or not) effect the distributions in time and space of overflows?

Controls for the previously described system of subbasins were derived using each technique and the same master control problem and forecasting models (with the lead time for the simple operating policy limited to one increment).

Figure VI-8 illustrates the values of Q_{max}^{*i} for each subbasin and control interval. The Q_{max}^{*i} policies for the four techniques are also presented on the same graphs to highlight their differences while continuing to illustrate the interrelationships of particular Q_{max}^{*i} values in time and space for each individual technique. Figure VI-9 repeats the above information for subbasins one and five for which significant differences in total overflows were encountered between the four methods. This figure, in particular, illustrates the interdependence in time and space of the allocated Q_{max}^{*i} terms.

Table VI-4 presents and summarizes the overflow quantities experienced within the subbasins and time periods for which overflows were encountered. The summary table on the bottom presents the total overflows experienced and indicates their distribution in time and

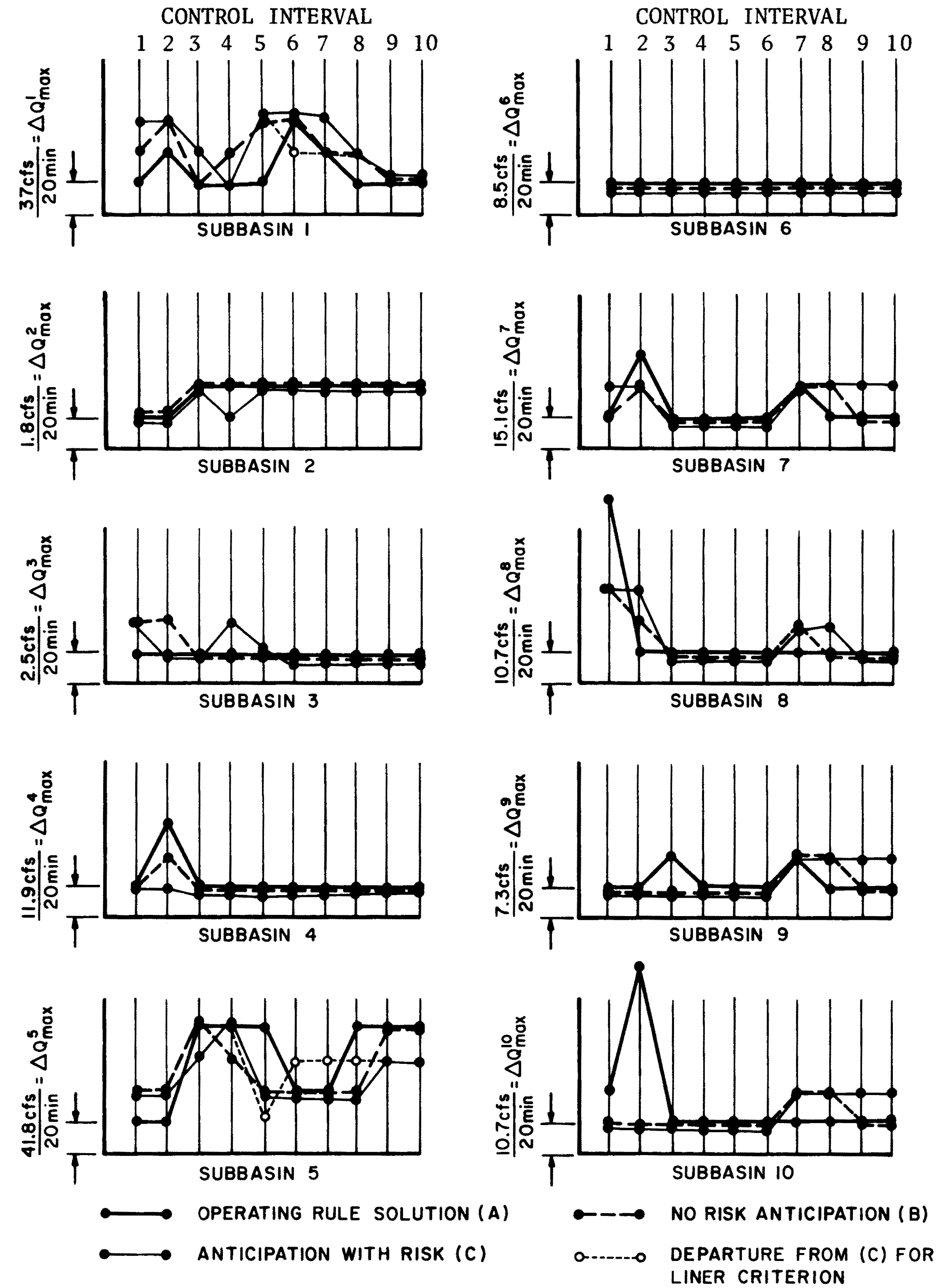


Figure VI-8. Q^{*i}_{max} Policies for All Subbasins

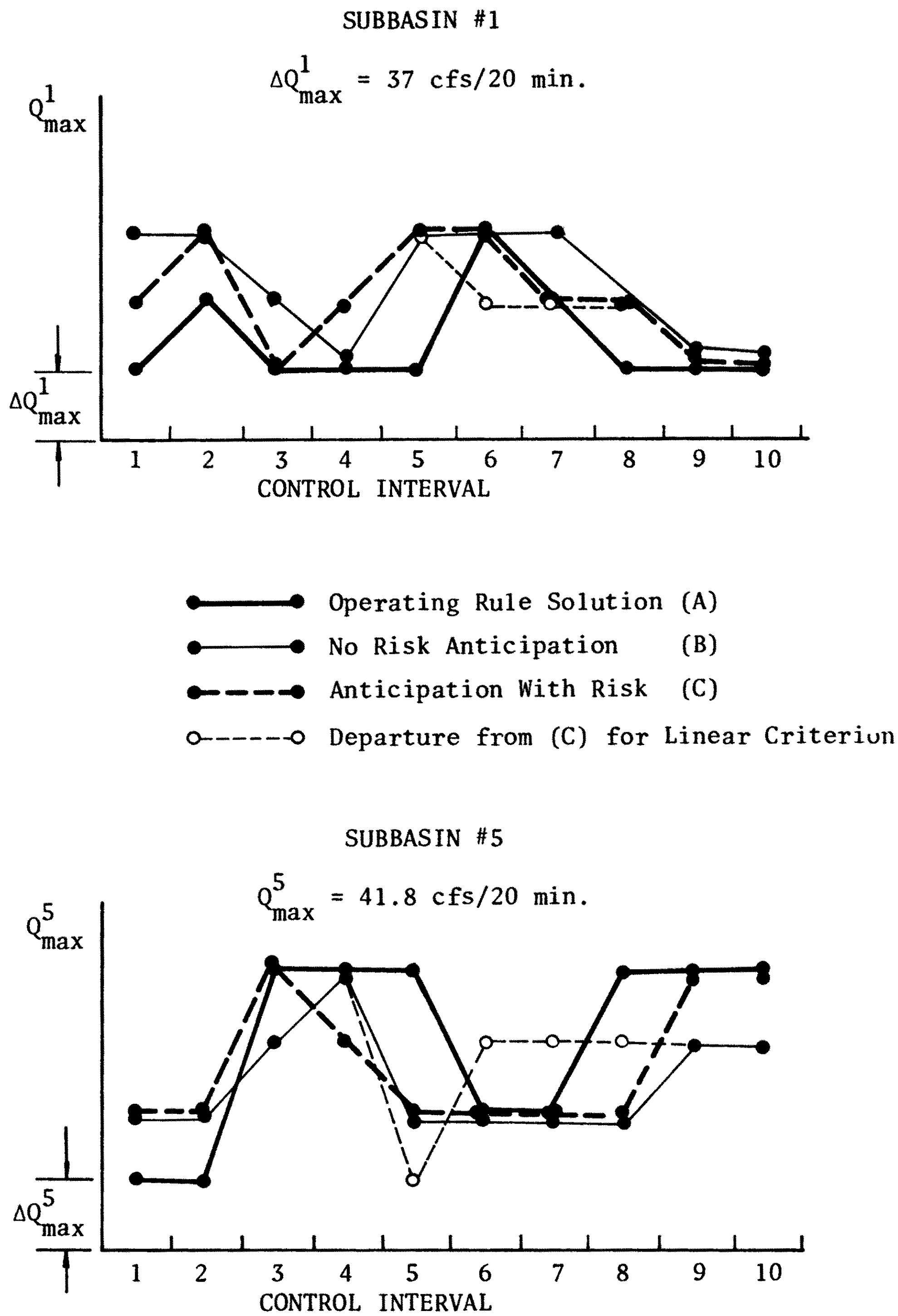


Figure VI-9. Q^{*i}_{max} Policies for Subbasin One and Five

TABLE VI-4

COMPARISON OF OVERFLOWS FOR CRITICAL SUBBASINS AND TIME PERIODS

Overflows in CFS/20 min for policies derived by:

- A - Simple Operating Rule[1]
- B - No Risk Anticipation[2]
- C - Anticipation with Risk[2]
- D - Anticipation with Risk[1]

Subbasin No.	Time Period K=3 A	B	C	D	4 A	B	C	D	5 A	B	C	D	6 A	B	C	D	7 A	B	C	D
1	0.	0.	0.	0.	148.	37.	0.	0	0.	0.	37.	0	0.	0.	0.	37.	0.	0.	0.	0.
5	0.	0.	0.	0.	0.	83.6	41.8	83.6	41.8	125.	125.	125.4	0.	0.	0.	0	0.	0.	0.	0.
7	0.	0.	0.	0.	15.1	30.2	30.2	30.2	90.6	90.6	90.6	90.6	30.2	30.2	30.2	30.2	0.	0.	0.	0.
9	0.	0.	0.	0.	0.	14.6	14.6	14.6	0.	0.	0.	0	0.	0.	0.	0.	0.	0.	0.	0.
10	0.	0.	0.	0.	0.	21.4	21.4	21.4	0	10.7	10.7	10.7	0.	0.	0.	0.	0.	0.	0.	0.
∑ Overflows All Overflow Locations	0.	0.	0.	0.	163.1	186.8	68.0	149.8	132.4	226.3	263.7	226.	30.2	30.2	30.2	67.2	0.	0.	0.	0.
∑ $(\text{Overflows})^2 \times 10^{-4}$ All Overflow Locations	0.	0.	0.	0.	2.2	1.0	.16	.9	1.0	2.4	2.5	2.4	.1	.1	.1	.2	0.	0.	0.	0.

SUMMARY COMPARISON

	A	B	C	D
$\sum_k \sum_i (\text{Overflows})$ Over All Overflow Times k & Locations i	325.7	443.3	361.9	443.3
$10^{-4} \times \sum_k \sum_i (\text{Overflows})^2$ Over All Overflow Times k & Locations i	3.3	3.5	2.76	3.5

NOTE: [1]Criterion used was linear with overflows and throughflows.

[2]Criterion used was linear with throughflows but quadratic with overflows.

space by summing the square values of the individual overflows. It is noted that the simple operating rule (A) resulted in 10 percent less total overflow when compared to the seemingly second best method which used inflow anticipation risk and a quadratic criterion (C). The squared overflow, which is 24 percent larger for the rule curve, indicates, however, that this technique resulted in an inferior distribution in time and space of overflows. This can be seen by noting that A produced a maximum overflow rate (per 20 min. interval) of 148 cfs (over all subbasins and time periods), whereas the maximum rate produced by C was 125 cfs. Thus, C performed better under a criterion of minimizing the maximum rate of overflow.

H. DISCUSSION

The results presented in the preceding section provide insight into the three considerations mentioned earlier. These considerations deal with the potential advantage of: (1) long range forecasts, (2) the consideration of risk, and (3) the value of an exponent on the criterion used for influencing the distribution of overflows.

The results also yield information on the feasibility of using on-line control which is based upon the total system approach chosen in this work (i.e., adaptive optimization of a decomposed system). Figure VI-7, which demonstrated the convergence properties of the total algorithm for one real-time period, also provides data on required computer time. This computer time, it is noted, is roughly comparable to the anticipated time required for a distributed computer system to accomplish the same tasks. This is due to the countervailing impact of the larger time requirements of the individual mini-computers, and the advantage of parallel computations which are feasible when using a dendritic hierarchy of mini-computers. NOting the essential elimination of anticipated overflows within 75 seconds, and the continued improvement of throughflows for the duration of the three minutes needed for final convergence, it is seen that a wide range of potential real time control intervals can be used. Time intervals ranging from three to five minutes seem quite feasible in spite of the additional number of intervals which would consequently need to be modeled if shorter intervals were to be used.

Figure VI-6 demonstrates the utilization of treatment plant capacity. Although 80 percent utilization was achieved within approximately the same 75 seconds needed for overflow elimination, improvement thereafter was significantly slower. A full three minutes was required to achieve the ultimate utilization possible for the situation modeled. This is, again, well within the anticipated real time control interval range.

The convergence of the base-line parameter identification, illustrated in Figure VI-2, demonstrates the feasibility of using stationary parameters for the forecasting model, although, this in fact may be a weak assumption. The erratic behavior of the parameter trends within each event indicate possible nonstationarity of parameters within events. This erratic behavior seemed to dampen out, however, after five or six events.

The subsequent forecasts of storm inflows, although initially demonstrating significant error as illustrated in Figures VI-4 and VI-5, were seen to improve dramatically as the *real* event progressed. It is noted that the use of a finer discretization within the forecast model may yield an improvement of the forecasted flows which are needed for the chosen control interval used within the optimizations.

The above points appear to demonstrate the feasibility of the entire proposed on-line algorithm. The comparison of performances for the proposed system under the various subbasin considerations, discussed above, is provided in the balance of the computational results.

The advantage of long range forecasts (and, consequently, optimizations over the entire horizon) over short range forecasts (and, consequently, myopic control policies), is still uncertain. The performances of the control policies derived for each of the four subbasin assumptions have been compared by calculating the total overflow and the total of the individual overflows squared. The total squared overflow value indicates the dispersion of the overflows through time and space. Table IV-4 summarizes the performance of the various policies which were illustrated in Figure VI-9. It was noted that the full forecast model (with risk) yielded a lower maximum rate of overflow at the expense of total overflow. Total overflows were less, it is recalled, for the myopic based policies.

The performance of the full forecast model without risk was judged inferior to the same general approach with risk, and to the myopic based policies. This seems to indicate that risk is an important co-requisite to long range forecasting when used within an optimization model.

Table IV-4 also illustrates the superiority of policies derived by using a criterion function with an exponent on weighted overflows which is greater than one. Such a nonlinear criterion may also be an essential part of an optimization scheme based upon a full forecast of events. The full, or long range, forecast spans many control intervals and the nonlinear criterion accentuates the dependence in time and space of the various control decisions.

In general, it seems that the full forecast optimization model with risk and a nonlinear criterion, is a worthwhile tool. Its effectiveness is hampered, although, by the accuracy of the inflow forecast model which, this author believes, can be greatly improved upon.

Chapter VII

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

A. SUMMARY

An important source of water pollution, which has been frustrating plans for achieving a national goal of cleaning up natural waterways, is urban stormwater runoff. Achieving the national goal demands the control of urban stormwater runoff either by itself or, as it is often found, in combination with sanitary sewage, so that development of many promising alternatives has been stimulated.

The use of ambient and/or auxiliary detention storage has emerged as both technically feasible and economically advantageous when compared to other schemes for achieving the needed control. Such in-line storage would have numerous control features for which remote operation is a fairly obvious necessity. This would require the utilization of field information and a capability for rapid execution of control.

The control of complex systems of such auxiliary detention storage facilities has been approached herein by the on-line use of computerized information and control algorithms. The proposed control system would sense the current state of the system as well as the characteristics of the pollution threatening event, and telemeter this data to the computer control center. A forecasting model within the computer would then utilize the inflow data to generate a forecast of the future system inputs. These forecasts, along with the system status would be subsequently used by the optimization model to determine the controls to be exercised. The controls would then be telemetered to the system components where they would be effected. As the forecast is updated, or as the system status demands, the entire process would be repeated until the pollution threatening event passes.

The development of this adaptive on-line control algorithm for a computer controlled system of combined sewers is the problem to which this dissertation was directed. It is an adaptive system in that the event in progress is monitored and its future aspects are forecasted. The control algorithm is constantly updated with current data. It still, however, recognizes the uncertainty in the forecast and derives controls which will minimize expected weighted overflows and street flooding while efficiently utilizing the existent facilities.

A review of adaptive approaches to the problem of linked reservoir optimization was presented along with a discussion of rainfall simulation, with a view towards its application to the forecasting problem. The application of adaptive learning processes to the problem of forecasting was seen to be thoroughly justified by the literature, although the detail required in the problem addressed here had not yet been considered.

It was concluded in Chapters III and IV that a large-scale optimization technique, which reduces the excessive dimensionality of the original problem formulation, was required for the control of the large complex systems considered. This conclusion, then, formed the basis for the entire approach. Decomposition was the characteristic technique chosen for most of this work while aggregation was also applied in situations where the time required for detailed solutions was too great for iterative on-line interaction.

The large-scale decomposition approach chosen within this work relied upon a master control problem which derived needed data for independent subproblems. These subproblems were formed to correspond to physical subsections or subbasins. The use of decomposition encouraged the development of a variety of subbasin analysis techniques because the general procedure could thereby incorporate a mix of techniques for subbasin analyses. This permits the use of detailed techniques where necessary and shortcut methods where permissable.

Various detailed subbasin analysis techniques were developed under the categories of deterministic subbasin techniques and stochastic subbasin techniques. The difference between the two approaches centered around the assumptions concerning the accuracy and reliability of the inflow forecasts. In each category, routing effects were addressed and those methods capable of incorporating both the risk of the forecasted inflows and routing effects were developed. These techniques are then proposed as detailed subbasin techniques to be used subsequent to an interactive phase of optimization. The algorithms used within the interactive phase were by necessity very simple, although they still considered risk. Basically, all the subbasin techniques considered were variants of linear programming and dynamic programming.

As a separate system function, techniques were developed to analyze historic storms for the derivation of parameters for a base-line forecast model. The model could then be modified during a particular storm to forecast the future inflows of that event being considered. The techniques used, based on parameter identification theory for multiple time series analyses and forecasting were adopted for on-line use within the large scale optimization framework.

The totally on-line real-time framework for the complete computer control system was simulated on a central computer by the a-priori selection of a particular event. It was subsequently utilized sequentially as the *real* system inflows during the optimizations. As more *real* data became available to the program, simulating the passage of real-time, the entire large-scale optimization technique was repeated and new controls supplanted older controls where necessary.

The simulated on-line, real-time demonstration of the total algorithm was repeated four times using different assumptions within the subbasin problem. The assumptions dealt with:

1. The advantage of long range forecasting techniques over short range myopic techniques.
2. The advantage of considering the risk inherent in long range forecast data.
3. The advantage of recognizing the interdependence in time and space of control decision impacts.

B. CONCLUSIONS

Several of the more significant conclusions based on the results of this work are discussed below:

1. The general format of the performance criterion used throughout the various techniques is judged to be an adequate description of the actual control objectives.

The control objectives of decreasing *real* overflows and street flooding while efficiently utilizing the existent facilities was readily achieved. This was accomplished by crediting the expected value of anticipated throughflows while penalizing the expected value of anticipated overflows for releases resultant from a potential control policy and forecasted inflows.

Street flooding events which resulted from *real* inflows which exceeded the forecasted values, although not eliminated, were proven controllable to a degree. Experiments conducted prior to the final demonstrations which used a high penalty weighting factor on anticipated storage overflows influenced the decisions on planned (control based) overflows. Such penalty factors often resulted in lower subsequent *real* storage overflows. If such controlled overflows were selected in an early period, but subsequent *real* inflows were lower than forecasted inflows, it was seen that the controlled overflow previously selected was unnecessary. Without a priori knowledge concerning the error of the forecast, the weighting factors on controlled overflows and uncontrolled storage overflows (i.e., street flooding) should, in general, remain equal (unless unequal overflow impact is known to exist or can be assumed).

An exponent greater than one on anticipated overflows of either type will tend to balance both types of overflows. And, if forecast error is unbiased, overflows will in the long run be more evenly distributed. A secondary goal of achieving evenly distributed overflows is seen to be attained, in part, at the expense of total overflow reduction. The balance between these goals remains to be determined. When a quadratic criterion was used, which squared the weighted overflows, a 22 percent increase in total overflows in time and space was achieved. (That is, squared overflows were 23 percent less when the quadratic criterion was used in an otherwise identical formulation.)

The use of the expected value of the anticipated weighted overflows and throughflow as a criterion for selecting controls appears to be well justified. When risk was considered, and a stochastic optimization used to select subbasin controls, and 18 percent reduction in total *real* overflows was achieved in the demonstration. This reduction is based on a comparison of performance with an otherwise identical formulation which assumed that the forecasted inflow values each had a probability of unity.

Control policies based upon the total expected value of anticipated weighted overflows and throughflows compared well with myopic policies which were based upon a one step ahead forecast and simple operating roles. The myopic technique did, however, achieve the lowest total overflow. It was 10 percent less than the overflow achieved by the stochastic problem using a full forecast of future inflows and a quadratic objective. But, its distribution was inferior. The sum of the squared overflows was again 23 percent higher and was comparable to the full forecast problem with no risk, and to the full forecast problem with a linear criterion.

2. The forecasting of inflow values for the remaining time periods of an event is a feasible approach to the on-line real-time derivation of system controls which are dependent to each other in time and to a random inflow factor. The parameters for a set of autoregressive transfer function models (one for each location) were derived in a sequential manner amenable to on-line adaptation. The resultant models were subsequently used in a simulated real-time environment to sequentially forecast future inflows. The inflows were based upon a previously derived base line model which reflected historic trends, and current data describing the event in progress. By limiting the values of the forecasted inflows to values under the maximum expected inflows, based upon historic data, the forecasted values were

kept within the range considered in the stochastic optimization. The stochastic optimization, it is recalled, set the forecasted inflow value as the mean of a distribution ranging up to this same historic maximum. The forecasted values were seen to converge to the simulated *real* event after three or four iterations of the forecast model. This convergence however, was preceded by large errors in the forecasts of subsequent and total subsequent inflows. These errors may have influenced the control algorithms, resulting in additional overflows. The discretizations of time within the forecast models was identical to that used in the optimization models. A much finer division of time in the forecasting model may yield an improved performance, since more data will be available prior to the arrival of the *real* peak inflows which the system was most sensitive to. In particular, an underestimation of immediate inflows coupled with an overestimation of total flows (or vice versa) may be responsible for the total overflow superiority of the control policies derived by using an operating rule. Such an operating rule only reacts to the forecast of immediate inflows.

3. A large-scale problem can be reduced to manageable dimensionality by the application of decomposition techniques.

Decomposition as a characteristic approach yields four distinct advantages. (1) Decomposition permits the independent solution of low dimensional problems either sequentially or in parallel. (2) Decomposition permits the use of the appropriate subbasin technique for each particular location. (3) Decomposition permits the use of nonlinear functions within the overall system constraints; i.e., routing. (4) Decomposition lends itself well to distributed computational power, possibly in the form of a hierarchy of mini-computers, since the subproblems correspond to physical subsections of the system. The decomposed problem, which is amenable to distributed computer power, will then be capable of achieving the same total computational speed when applied to a hierarchy of slower minicomputers. The slower computational speed of the minicomputers and microprocessors proposed herein is compensated by the possibility of parallel computations, and also by the advantages of shorter computer word length. This shorter word length requires less memory and data handling for a realistic level of system precision.

4. Decomposition and aggregation techniques used in a complimentary fashion appear to be a worthwhile approach to the solution of large scale problems.

Although decomposition is the characteristic large-scale approach adopted here, the interactive subbasin problem used aggregation to further reduce the problem size. Decomposition and aggregation, when used in conjunction, enable large systems to be analyzed without the multiplicity of optimization levels needed if repeated system decomposition is used and also without the loss of solution flexibility inherent in solution via gross aggregations of components. If the proper balance is struck an efficient solution algorithm results.

C. RECOMMENDATIONS FOR FUTURE RESEARCH

The research outlined herein has generated many questions while hopefully answering a few. The work can be categorized into three major areas of concern: (1) the master control problem and the overall solution strategy, (2) the forecasting model used for the estimation of the inflow sequences, and (3) the subbasin analysis techniques with and without uncertainty and risk. Each has evolved its own questions which will be treated separately below.

Master Control and Overall Solution Strategy

1. Decomposition permits routing within subbasin problems as well as within the interceptor. The routing technique chosen must be quick and simple for iterative use but accurate enough to fully represent the attenuation and lag encounter by flows within the considered reach. Efforts to find a transfer function representation for the various reaches of a system could enable the inclusion of routing directly within the distributed system computer system while necessitating little additional software.
2. The discrete steepest descent method worked adequately for use with discretized subbasin parameters (i.e., solution via dynamic programming) but would be unnecessarily limiting for subbasin analysis techniques which can be solved with continuously arrayed values. The development of an efficient steepest descent method for use with either all continuously arrayed subbasin techniques or a mix of continuous and discrete techniques should be considered.

Inflow Forecast Models

1. Further research is desirable in efforts to discover the sufficient forecast model for the expressed purposes. Computer time, storage requirements, as well as data needs, increase with the order and degree of the model. However, diminishing improvements in forecasting ability may also attend such increases in model size. An optimal model size

should, therefore, be sought which, in conjunction with the stochastic subbasin analyses techniques, yields an efficient forecast and optimization procedure combination.

2. The analysis of historic inflow (or rainfall) data, in an effort to discover seasonal or storm type subgroups which exhibit a higher degree of stationarity, should be pursued. Such subgroups would aid in the forecast of inflows since they would result in greater model stationarity. The subgroup identifiers such as season, wind direction, storm type, etc., could be used to define which sufficient model is necessary for the forecast of the remainder of the event in progress.

3. The derivation of the *best* weighting factors for the data being used in the parameter identification procedure is a subject which would be worthwhile pursuing. The available data can be weighted according to its perceived importance both within the on-line real-time modification of the base line model, and in the off line establishment of the base line model. Older storms or storms from other logical subgroups (e.g., seasons) might be weighted lower than newer storms or storms whose patterns seem close to the emerging pattern of the storm being considered. Care must be taken, however, since to weight the newest data far greater than the historic data would yield an erratic model; while weighting the historic data too highly would yield a constrained model, unable to adapt or learn the current pattern.

4. In general, more basic work to establish the value of forecasting for the derivation of system controls (either

deterministically or stochastically) is needed. Systems affected by random disturbances must still be controlled. If tentative control policies are needed prior to the realization of an event, forecasting seems valuable. However, in view of the performance of the policies derived from a simple operating rule the value of forecasting over an entire planning horizon remains unclear.

Subbasin Analysis Techniques

1. Experiments were conducted to ascertain the advantage of using stochastically derived controls when compared to deterministically derived controls for variants of the forecasted storm inflows. These experiments indicated the superiority of stochastically derived controls for simple random variants of the forecasted storm. However, a full demonstration of this superiority would entail the consideration of all relevant cross and serial correlations in the generation of probable variants. The weighting of these variants with their probability would enable a more comprehensive evaluation of the advantage of a control strategy. Such a study would provide needed data for analyzing whether or not stochastic policies are worth the additional computer power that may be required, considering their advantages.
2. The use of upper bounding linear programming codes, either independently in the deterministic case, or built into the LPUU code in the stochastic case, would overcome the dimensionality problem encountered using simple LP approaches. Such codes would eliminate much of the computer storage

needed to explicitly represent an upper bound constraint for each variable and time period. This, in conjunction with other sparse matrix handling methods (diagonalization), could facilitate the application of LP to much larger subbasins and system subsections. This could possibly lead to the elimination of the aggregated intermediate problem used for interaction with the master problem.

3. Tradeoffs between solution optimality and speed, considering the inherent risk in the forecast, should be made. Faster subbasin analysis techniques may be needed if the control interval is to approach five minutes. The increased opportunity for updating and correcting provided by a smaller control interval, combined with greater forecast risk for such short intervals, increases the advantages of simplified solution strategies such as the myopic operating policy. The further comparison of the large scale approach outlined herein with an approach using such simplified subbasin analysis techniques is a worthwhile endeavor.

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