On the Nature if Clean-Air Turbulence (CAT)

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Problems associated with clear-air turbulence (CAT) observation, such as the resonance action of the aircraft itself are reviewed. A hypothesis is outlined, whereby CAT is considered an effect of internal waves on stably stratified interfaces. Such wave formation may be brought to light by detailed cloud studies.

CLEAR-AIR TURBULENCE still is a phenomenon, which at times is as puzzling to aviators as it is to meteorologists. Not only because it may hit an aircraft rather unexpectedly, but also since it may appear with a force, violent enough to constitute one of the major hazards of present-day aviation. It may occur at all levels used by aviation, even in the stratosphere and ozonosphere.

"CAT" is used synonymously to "bumpiness in flight through clear air," which actually would be a better definition of the phenomenon, because of the following two facts:

(a) Many of our CAT data obtained from VGH recordings are difficult to pin-point in space and time. Some of these gusts interpreted as CAT may actually have occurred in thin cirrus or haze layers and thus might not be truly representative for clear-air conditions.

(b) The atmospheric flow, giving rise to CAT, may be completely "laminar" in a hydrodynamic sense. On the other hand, truly "turbulent" flow will not be recorded as CAT, whenever the scale of turbulence elements is below or above certain limiting values.

For convenience sake, we will retain the terminology of "CAT," avoiding a lengthy paraphrase. We shall, however, keep statements (a) and (b) in mind when we do so.

If we consider the effect of turbulence on aircraft structure and passenger comfort, statement (a) becomes irrelevant. For gust-load and stress computations it is of no consequence how the resulting accelerations were brought about.

As shall be pointed out later, bumpiness in flight seems to be most frequently associated with stable and baroclinic layers in the atmosphere. It is only in this respect that statement (a) bears some significance, since the experienced pilot will expect some turbulence in cloud or haze layers, thin as they may be.

A good number of turbulence reports stems from stratospheric levels. As a matter of fact, there does not seem to be any level at all within the atmospheric layer so far used by winged aircraft that were completely void of CAT, not even at operational heights

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of U-2 and X-15. For tropospheric flow conditions we may visualize part of the CAT to be due to convective and hydrodynamically turbulent motions within adiabatic or even slightly superadiabatic layers. For the thermodynamically stable stratosphere this source of turbulence will be difficult to realize. There is, however, an increased possibility of CAT wherever the stratosphere shows baroclinicity, according to a modified version of Richardson's criterion,[13, 15] which shall be explained further.

Aircraft in Turbulent Flow

The most readily available reports on high-level turbulence come from aircraft observations. All we can tell with confidence, however, either from pilot reports or from instrument recordings, is that the aircraft experienced a certain amount of accelerations, occurring at certain frequencies. As soon as we try to deduce the magnitude and direction of atmospheric gust velocities responsible for these accelerations, we will have to make certain assumptions on the shape of this gust, which are very difficult to verify.[10,15] The most simple version, which, however, is completely suitable to illustrate our problem, assumes a "sharp-edged" gust—i.e., a sharply bounded gust of uniform up- or downdraft, and of infinite extent to one side of the bounding edge. The relationship for the derived vertical gust velocity W_{de} is given in Ref. 12.

$$W_{de} = (2b\ \Delta n/\rho K a V_i) \tag{1}$$

ρ is the air density, b the wing load (lb/ft^2), Δn the acceleration increment over gravity, in units of gravity, K is a gust alleviation factor (dimensionless), V_i the horizontal true air speed and $a = (dC_A/d\alpha)$, the change of the lift coefficient C_A with the angle of attack α which depends on the type of aircraft.

As may be seen from Eq. (1), W_{de} strongly depends on a, and therewith on the specific type of aircraft, which carries out the observations. Generally speaking, the design of slow-flying aircraft offers a larger value of a, than the one of fast-flying jet planes. Thus, with all other conditions assumed equal, the former would experience larger vertical accelerations from the same sharp-edged gusts, than the latter. The fact, however, that the air speed V_i appears in the denominator of Eq. (1) more than compensates for this effect. Thus, it is usually fast-flying planes, which have more difficulties with CAT.

The fact that fuel burn-off during flight decreases the wing load b continuously, makes the assessment of atmospheric gusts from acceleration records even more difficult.

Furthermore, horizontal and vertical gusts may have similar acceleration effects upon the aircraft, only that the latter usually are four times as powerful than the former, assuming equal gust velocities.[9, 15]

The problem becomes even more complex if one considers the elastic properties of the wings, which may by some resonance action exaggerate the effect of atmospheric gusts in certain frequency ranges. This is characterized by the relationship

$$\phi_0(\omega) = [T(\omega)]^2\ \phi_i(\omega) \tag{2}$$

$\phi_i(\omega)$ and $\phi_0(\omega)$ are the frequency spectra of input—e.g., atmospheric gust accelerations, and of output—e.g., aircraft accelerations, and $T(\omega)$ is the frequency response function which depends on the elastic properties of the aircraft.[10] Types of airplanes with large and relatively elastic wings, as for instance the U-2, will be particularly sensitive to this kind of exaggerated or distorted "turbulence." If we were concerned with CAT forecasting for one particular type of aircraft only, the foregoing would not concern us much. There is, however, not only a large number of aircraft and missile types to give advice to, but CAT research should also consider future type air vehicles, which may be sensitive to different portions of the perturbation spectrum of the atmosphere. Therefore, it is of paramount importance to: (1) formulate useful working hypotheses on the formation of atmospheric perturbations which may lead to CAT, and (2) collect data on this perturbation spectrum by direct measurements, which do not involve an aircraft. Such measurements can only be carried out by tracking more or less inertia-free bodies, like cloud matter, chaff, or balloons.

A Working Hypothesis of CAT Formation

It has been mentioned that CAT may be brought about by hydrodynamically turbulent flow, or by

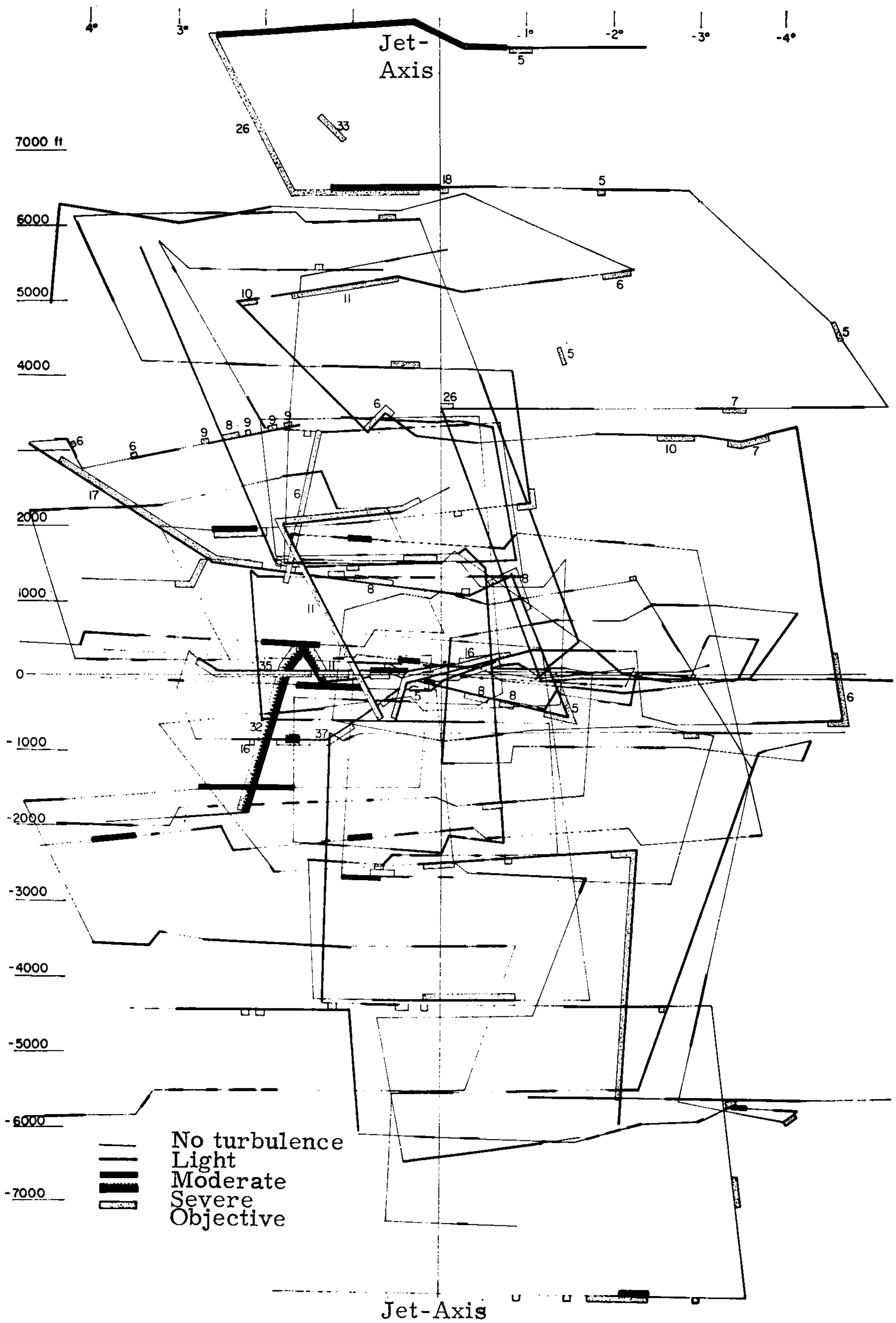

Fig. 1. CAT distribution in 12 Project Jet Stream research flights (Nos. 1, 5, 11, 12, 13, 17, 18, 19, 20, 27, 29, and 30), in a coordinate system based upon the vertical jet axis. Subjective CAT observations: thin lines: no turbulence; medium-weight lines: light turbulence; heavy lines: moderate turbulence; heavy lines with dots: severe turbulence. Objective turbulence measurements: shaded areas; numbers indicate vertical gust velocities in fps.

laminar flow as well. Let us define the first as having closed, or almost closed, streamlines in a coordinate system fixed to the earth, and in horizontal and/or vertical planes intersecting the current, indicative of vortex motions within turbulent eddies. The eddies should contain either horizontal or vertical accelerations of a magnitude, which would qualify them as CAT. Among these considerations, convective patterns in the form of Bénard cells would qualify. (Up-drafts of a magnitude as experienced in Cb—clouds, while satisfying our definition of "turbulence," usually are associated with condensation processes, and do not qualify, therefore, as CAT. The same holds for turbulence observed in orographically produced rotor clouds.) While such situations may occur away from jet streams, where sometimes light turbulence is observed, indeed, it is believed that they would produce only isolated bumps, rather than prolonged "cobblestone" turbulence.

In the vicinity of jet streams, shears usually are too large to permit Bénard cells to form. They will be distorted into helical vortices whose stream lines describe a wave pattern in a coordinate system fixed to the earth. However, if we use the definition given above, this brings us back to "laminar" flow conditions. Assuming perturbations with magnitudes of about 10 percent of the mean flow (= basic current), we realize, that under jet stream conditions they would, at the most, cause fluctuations in the u, v, and w components, but never would they lead to a reversal of flow as would have to be the case in real turbulence.

We will, therefore, be quite safe to treat the CAT problem as a wave phenomenon in a current with vertical and/or horizontal gradients of wind speed and/or of potential temperature. A good number of theoretical treatments have been forwarded on this subject.[4, 15, 17] It will suffice here to outline only a few basic considerations.

Richardson's Criterion

One of the most widely misused criteria in CAT research is the one by Richardson.[22]

$$Ri = \frac{(g/T)[(\partial T/\partial z) + \Gamma]}{[(\partial u/\partial z)]^2 + [(\partial v/\partial z)]^2} = \frac{(g/\theta)(\partial\theta/\partial z)}{[(\partial V/\partial z)]^2} \quad (3)$$

g = acceleration of gravity, T = temperature, θ = potential temperature, Γ = dry adiabatic lapse rate, u, v = the components of the wind vector, whose magnitude is V, z = vertical coordinate.

Since Richardson derived this expression under laboratory conditions, it will be of rather dubious value for the free atmosphere. It will be even more dubious when based upon parameters taken from radiosonde ascents, because these, at best, reveal the atmospheric macro-structure, while CAT is a microstructural phenomenon.[15, 17]

Nevertheless, we may adopt the position, that Richardson's criterion in essence gives the ratio between the damping forces of vertical thermal stability, and the turbulence generating forces* derived from vertical wind shear. Even though the limiting value of $Ri = 1$ derived under laboratory conditions, and separating the turbulent state of flow from the laminar state, will not hold true in the atmosphere, the physical principles underlying the criterion will still be valid. Statistical findings of Petterssen and Swinbank[11] in the free atmosphere over England suggest a limiting value of 0.65. This ties in nicely with earlier results given by Fage and Falkner who found that the ratio of the exchange coefficient of heat, A_T, to the exchange coefficient of momentum, A_M, for *free* flow is $(A_T/A_M) = 2$, while Prandlt's mixing length theory considers these two quantities to be equal, and so does Richardson's criterion. If one computes Ri under the assumption of $(A_T/A_M) = 1$ (this ratio would appear as a factor on the right-hand side of Eq. (3)), a limiting value of 0.5 which lies very close to Petterssen's value of 0.65 and which is obtained from considerations of vertical stability and shear only, would be brought back to $Ri = 1$ by applying the factor of $(A_T/A_M) = 2$ to it.

Eq. (3) would indicate increasing probability of turbulence with decreasing vertical stability. This, however, is in disagreement with the frequent observations of CAT in stratospheric levels (Fig. 1).[18] It also contradicts the findings from Project-Jet-Stream analyses, which showed an increase in CAT-frequency within stable and baroclinic layers, such as the "jet-stream fronts" above and below the jet core.[16, 19–21, 23]

The observations can be made to agree with Richardson's criterion, if the latter is transformed by substituting the vertical wind shear in the denominator of Eq. (3) by the thermal wind equation.[13, 15] This procedure is irregular insofar as the original Richardson criterion was not meant for motions on a scale that would be influenced by the Coriolis parameter, which appears in the thermal-wind relationship. Nevertheless, this modified criterion has been tested with atmospheric flow conditions, and it seems to show some correlation, at least with the amplitudes of mesoscale perturbations in the jet-stream region (Fig. 2).

With the substitution as outlined, one arrives at

$$Ri^* = \frac{f^2\theta}{g(\partial\theta/\partial z)[(\partial z/\partial n)_\theta - (\partial z/\partial n)_p - (\theta/g)(\partial \dot{V}/\partial\theta)]^2} \quad (4)$$

where the coordinate n is measured normal to the current; the indices θ or p indicate differentiation on an isentropic or isobaric surface, and $\dot{V} = (dV/dt)$. The last term in the denominator is hard to obtain from observations; it will be dropped, therefore, although it may be of considerable influence on Ri^*, especially in the jet-stream region, where there may be a sizeable vertical gradient of horizontal acceleration. If, furthermore, one neglects the slope of isobaric surfaces against the slope of isentropic surfaces, one obtains the following approximation

$$Ri^* \cong \frac{f^2\theta}{g(\partial\theta/\partial z)\,[(\partial z/\partial n)]_\theta{}^2} \tag{5}$$

Eq. (5) now states that perturbations are likely to amplify in stable and baroclinic layers. This approximate form of the modified Richardson criterion has been used in constructing (Fig. 2).

As may be seen from cross-sections through the jet stream[18] the baroclinicity in the jet-stream fronts has about the same order of magnitude above and below the jet core, only with reversed sign. Mean vertical wind profiles in the jet-stream region[14] even seem to indicate, that the wind shear $(\partial V/\partial z)$ is about 15 to 20 percent stronger on the average above the jet core than below. If we assume isothermal conditions in both, the upper and the lower "jet-stream front"—i.e., the stable and baroclinic layer above and below the jet core—this slightly higher wind shear above the maximum wind level would make the CAT probability according to Eq. (5) larger in the upper jet-stream front, than in the lower one. There seems to be a slight trend in this direction in the measurements over oceans[4] but not so much over continental areas,[23] although the tropopause, and especially the tropopause "break," in any event, harbor a large amount of CAT. Considering that the "tropopause break" actually lies in the region of the upper jet-stream front,[16] this should not be surprising.

The foregoing leaves room for speculation as to possible turbulence at still higher levels of the atmosphere. We know that the polar-night jet at about 25 mb is capable of vertical wind shears of the same magnitude as the polar front or subtropical jet streams.[5] This would imply large baroclinicities combined with strong thermal stabilities, as they are characteristic for the stratosphere. While the air density is only about 1/10 of what it is near the tropopause—thus, according to Eq. (1), reducing the vertical accelerations of the aircraft due to atmospheric gusts to the same fraction—there may still be sufficient energy in small-scale wave perturbations to make it felt as CAT. In this, we will have to consider furthermore, that we still lack knowledge of the spectrum of wavelengths at these heights. Fast-flying aircraft which will operate at these levels will respond to longer wavelengths of perturbations than our present jet aircraft.

Waves on Interfaces

The theory of gravity waves on interfaces has already been treated adequately by Helmholtz in Refs. 7 and 8. These wave motions may be derived by means of the perturbation method.[1–3, 6, 17] One arrives at the following expression for wave propagation speeds in a shearing current with a temperature discontinuity:

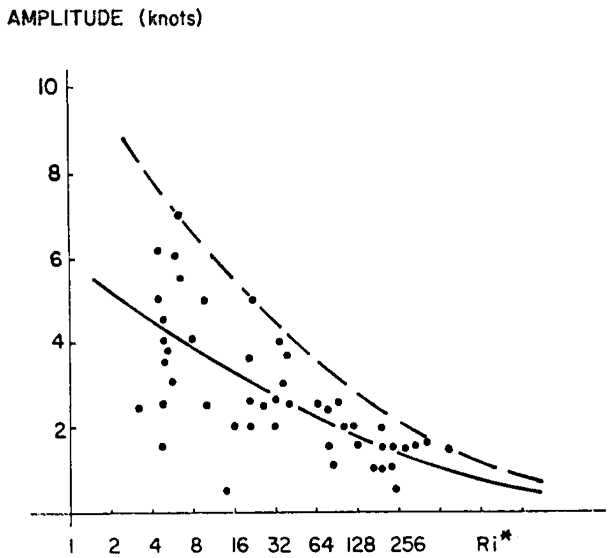

Fig. 2. Correlation of the amplitudes of the mesostructure of wind speed (ordinate) with Ri*-numbers (abscissa).

$$C = \frac{\rho_0\bar{u}_0 + \rho_1\bar{u}_1}{\rho_0 + \rho_1} \pm \sqrt{\frac{gL}{2\pi}\cdot\frac{\rho_0 - \rho_1}{\rho_0 + \rho_1} - \frac{\rho_0\rho_1}{(\rho_0 + \rho_1)^2}(\bar{u}_0 - \bar{u}_1)^2} \tag{6}$$

L being the wavelength of the disturbance. Indices 0 stand for the lower layer, 1 for the upper layer. Velocity components $\bar{u}_1$ refer to the basic current. The critical wavelength for which disturbances start to amplify is obtained by equating the expression under the square root to zero. We may also substitute temperature for air density from the equation of state and arrive at

$$L_c = (2\pi/g)[(u_0 - u_1)^2\, T_0 . T_1]/[(T_1 + T_0) \times (T_1 - T_0)] \tag{7}$$

Abbreviating $\bar{T} = (T_0 + T_1)/2$, $\Delta T = T_1 - T_0$, $\bar{u}_0 - \bar{u}_1 = \Delta\bar{u}$ we obtain $T_0 . T_1 = \bar{T}^2 - 1/4(\Delta T)^2$ and

$$L_c = \frac{\pi}{g}\,\frac{(\Delta\bar{u})}{\bar{T}}\,\frac{\bar{T}^2 - 1/4(\Delta T)^2}{\Delta T} \tag{8}$$

Since $1/4(\Delta T)^2 \ll \bar{T}^2$ we may write

$$L_c = \frac{\pi}{g}\,[(\Delta\bar{u})^2 . T/\Delta\bar{T}] \tag{9}$$

For conditions near the tropopause level with $T \cong 230°$ we obtain

$$L_c = 73.6 . [(\Delta\bar{u})^2/\Delta T] \tag{10}$$

The following table[16] gives values of ΔT and m/sec for different wavelengths.

It should be realized that the atmosphere usually shows a detailed structure, especially near jet streams, consisting of shallow (<1000 ft thick) stable and baroclinic layers interspersed with less stable layers. The (potential) temperature contrast across these stable layers frequently is of the

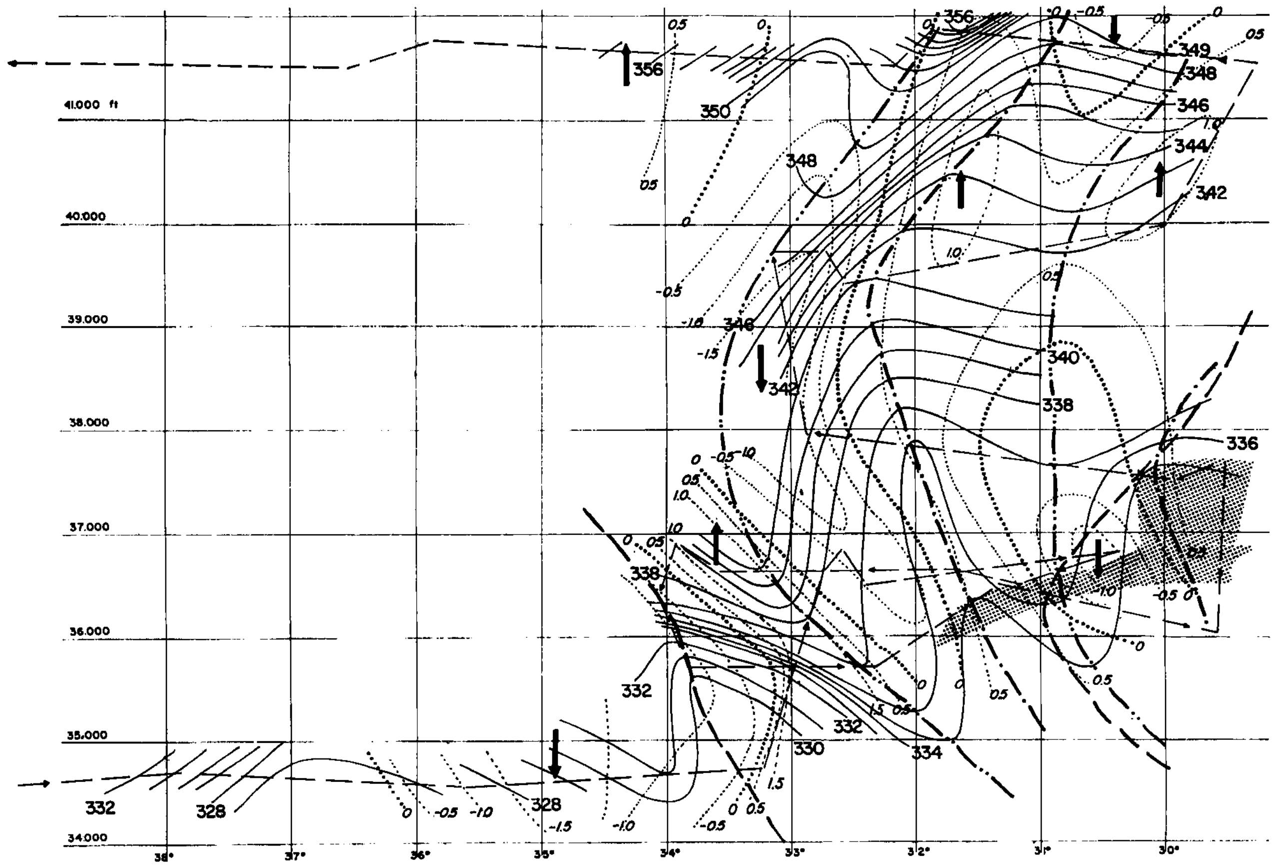

Fig. 3. Isotherms (°K) of potential temperature (full lines) and isotachs (m/sec) of vertical motion (dotted) of Project Jet Stream Flight No. 27, March 29, 1957. Flight legs are indicated by thin dashed lines, vertical jet axes by heavy dashed lines, the "isentrope hump" by a dash-dotted line, and the "isentrope trough" by a dash-double-dotted line. Cloud areas are marked by shading.

order of 2 to 4°C. The wind shear necessary to set up waves of a length which would make them felt as CAT is of the same order as meso-scale wind fluctuations so abundantly bound during Project-Jet-Stream flights.[16]

The critical wavelength is rather sensitive to small changes in shear and in temperature contrast between the two layers (Table 1). As shall be mentioned in the preliminary results of cloud photogrammetry, waves of these wavelengths, which may be observed at cirrus level under jet-stream conditions, change their appearance and configuration very rapidly. They may show up and disappear again in a matter of a few minutes. Considering the findings on atmospheric meso-structure[16] and the sensitivity of Eq. (10) to such meso-structure, this should not be surprising at all. It may also help to explain the patchiness of CAT.

Table 1. Vertical Wind Shear $\Delta\bar{u}$(m/sec) for Different Temperature Discontinuities and Critical Wavelengths at an Interface

ΔT	L_c = 200 m	L_c = 100 m	L_c = 50 m
2°	2.3 m/sec	1.6 m/sec	1.2 m/sec
4°	3.3	2.3	1.6
6°	4.0	2.9	2.0
8°	4.7	3.3	2.3
10°	5.2	3.7	2.6

More refined treatments of wave formation, using three-layer models with a transition zone instead of a sharp discontinuity have been proposed by Sekera[25] and Sasaki. While they may serve to elucidate some detailed mechanisms of wave formation and behavior, they will not change materially the basic context of above discussion, which was aimed to point out the significance of gravity and shearing waves for CAT formation.

Results of CAT—Statistics

Fig. 1 shows no significant preference of light CAT for any particular quadrant around the jet core of the Project-Jet-Stream flights. It became evident, however, that all cases of moderate and severe CAT were located in, or very close to, the axis of a downward drop of the isentropic surfaces—the "isentrope trough."

The explanation for this phenomenon may be sought in the fact, that the "isentrope trough" is produced by sinking motion, which, in turn, tends to further stabilize existing stable layers.

Assuming adiabatic conditions of flow ($(d\theta/dt) = 0$), and no motion in the y-direction ($v = 0$). We may estimate the cause of local stability changes from the advective terms:

$$\frac{\partial}{\partial t}\left(\frac{\partial\theta}{\partial z}\right) = -\frac{\partial u}{\partial z}\frac{\partial\theta}{\partial x} - u\frac{\partial}{\partial x}\left(\frac{\partial\theta}{\partial z}\right) - \frac{\partial w}{\partial z}\left(\frac{\partial\theta}{\partial z}\right) - w\frac{\partial^2\theta}{\partial z^2} \quad (11)$$

The first term on the right-hand side of this equation contains the influence of differential temperature advection at different heights, the second term the advection of atmospheric layers with different stability. The third term indicates the effect of differential vertical motion on stability, and the fourth term the influence of vertical motion on curved vertical temperature profiles. The contributions from the first and second term will have to be estimated from future research flights which offer a better three-dimensional data distribution. Fig. 3 may serve as an illustration of the effects of the third term. Project-Jet-Stream flight no. 27 experienced severe CAT in the area between about 33° N and $32^1/_2$° N and 35,000 and 37,000 ft (Fig. 1). Derived gust velocities of up to 35 ft/sec were encountered. At 33° N and between 36,000 and 37,500 ft we obtain

$$\frac{\Delta w}{\Delta z}\cdot\frac{\Delta\theta}{\Delta z} = -\frac{2\text{m/sec}}{500\text{ m}}\cdot\frac{3°\text{C}}{500\text{ m}} = \frac{-6}{2.5\times 10^5} = -2\times 10^{-5}°\text{ C/m sec} \quad (12)$$

This would correspond to a change in lapse rate $(\partial/\partial t)(\partial\theta/\partial z)$ of about +7°C/100 m hr, if the conditions prevailing in this cross-section remained unchanged for this period of time, which would hardly be the case.

The fourth term, again, is difficult to estimate from the available measurements. The foregoing will suffice, however, to demonstrate, that conditions for CAT may change rapidly in the vicinity of jet streams due to changing stabilities, thus influencing Richardson's number (Eq. 5). The same, of course, holds for the term $((\partial z/\partial u)\theta$ in that expression. The slope of isentropic surfaces normal to the direction of flow will mainly be influenced by velocity gradients $(\partial w/\partial n)$ and $(\partial v/\partial z)$.

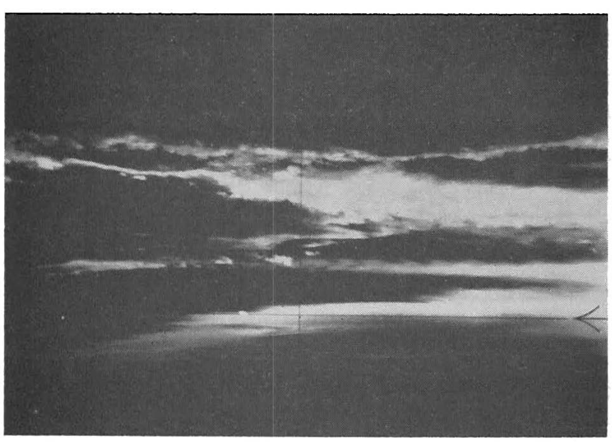

Fig. 4. Clouds seen from camera site "South" near Fort Collins, Colo., on Nov. 3, 1961, ca. 3 p.m. View towards west. Unstable waves on top of an orographically produced sheet of Ci-clouds appear above the center of the photograph. These waves are longer than the usual "CAT-waves," their unstable nature, however, may produce CAT, especially in the wave crests.

Again the accuracy of our presently available data does not permit an evaluation of these terms.

Measurement of Atmospheric Perturbations

It has been mentioned in the foregoing, that gust velocities obtained from aircraft turbulence records do not permit any far-reaching conclusions as to the perturbation state of atmospheric flow. It was decided, therefore, to investigate photogrammetrically the detailed structure of cirrus clouds, with special attention being paid to small-scale wave phenomena, which might have the same dimensions as they are present in CAT.

The considerations upon which these studies were based are the following:

(a) While at lower levels the energy from the latent heat of condensation and/or sublimation makes a large contribution to vertical circulations and to the amplification of perturbation motions, it was felt, that this contribution will be exceedingly small at the cirrus level, where dry and moist adiabats are very close to parallel. Thus, any deductions we may make, that are based on thermal stability of the atmosphere—e.g., considerations as they enter into the derivation of Richardson's number—will need only a small, even negligible, percent correction, when extrapolated to moist air conditions.

(b) Since CAT is a very patchy phenomenon even in the vicinity of a jet stream, it has to be expected, that such wave perturbations in cirrus clouds as we are looking for, will not occur too frequently. According to Ref. 23, maximum CAT frequency near jet-stream front and tropopause over the eastern United States amounts to about 50 percent of the total flying time being turbulent. On the average, we would expect about 15 percent of flying time to be bumpy near jet streams (Fig. 1). Assuming that the cirrus clouds observed during a particular jet-stream weather situation, occur at one level, we would expect about the same probability of this wave phenomenon to occur.

(c) The observation site near Fort Collins, Colorado, lies only about 40 miles east of the Continental Divide. Orographically induced perturbations may, therefore, trigger wave formations more frequently than should be expected over level terrain.

Qualitative Results of Observations

The following qualitative observations were made on this small-scale wave phenomenon:

(a) Waves of wavelengths estimated to less than 1 km, more probably in the range of 10^2 m, could be observed very clearly in sheets or bands of cirrostratus, when the jet stream is close to the area. Such waves of somewhat larger length on top of an orographically produced Ci-sheet are shown in Fig. 4.

(b) These short "CAT-waves" usually appear embedded into larger standing-lee waves.

(c) The waves seem to travel with cloud speed—i.e., very close to the speed of wind.

(d) These waves are of a highly unstable nature, while the cloud banks in which they appear may last for a long time. These short "CAT-waves" may appear in a certain portion of the main cloud in a matter of a few minutes, and dissipate equally as rapidly. They may show up again in another portion of the same cloud.

(e) In most cases, the crests of these small waves seem to be oriented at an angle >45°, or more nearly 90°, from the direction of flow. Different wave trains with slightly differing orientation and wavelengths have been observed within the same cloud sheet.

(f) On days, when field measurements were in progress on account of observations of such "CAT-waves," pilot reports from the Rocky Mountain area received over teletype indicated observations of moderate to severe CAT.

Feasibility of Detailed Cloud Studies in a Wavelength Range Corresponding to CAT

From the presently available data it follows, that the measurement of "CAT" wavelengths in cirrus clouds by photogrammetric methods is feasible, if the photographs are taken not too far away from the zenith. It will be difficult, however, to compute the speeds of these waves by tracing "corresponding" points from one pair of photographs to the next, simply because these points do not identify the true motion of cloud matter to a sufficient degree of reliability. In the author's opinion, this is a point of minor importance, however, because it will mainly be the wavelength of the wave phenomenon, that influences the vertical accelerations observed in CAT, and not so much their speed, since the latter will be very close to the speed of wind and therefore will be irrelevant in considering the motion of the aircraft relative to the air.

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