UNITED STATES DEPARTMENT OF THE INTERIOR

EXPLORATORY LABORATORY STUDY OF LATERAL TURBULENT DIFFUSION AT THE SURFACE OF AN ALLUVIAL CHANNEL

by

W. W. Sayre and A. R. Chamberlain

Open File Report

GEOLOGICAL SURVEY
WATER RESOURCES DIVISION

Fort Collins, Colorado

CER61ARC66

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SELECTED LIST OF SYMBOLS

Symbol	Definition	Units
K_z	Coefficient of lateral turbulent diffusion	cm^2/sec
L_t	Lagrangian integral time scale of turbulence in the lateral direction $$L_t = \int_0^{\infty} R_{w'}(\alpha)\, d\alpha$$	sec
$R_{w'}(\alpha)$	Lagrangian correlation coefficient which correlates the instantaneous lateral turbulent velocity components, w', of a fluid particle, at the times t and $t + \alpha$	
t	Dispersion time	sec
Δt	Time interval between release of particles	sec
$\overline{U}$	Mean velocity of flow in the longitudinal direction	cm/sec
$\overline{U}_s$	Mean velocity of flow in the longitudinal direction at the water surface	cm/sec
x	Longitudinal distance downstream from the source	cm
X_o	The x intercept of the extension of the straight portion of the $\sigma_z^2(x)$ curve.	cm
$\overline{w'^2}$	The mean of the squared instantaneous lateral turbulent velocity components, w'	cm^2/sec^2
z	Lateral distance from the source which is located at $x = 0$, $z = 0$	cm

Symbol	Definition	Units
α	Delay time associated with the Lagrangian correlation coefficient $R_{w'}(\alpha)$	sec
δ_c	Lateral deviation of the center of concentration of a group of dispersed particles from $z = 0$	cm
σ_z^2	Variance of the lateral distribution of a group of fluid particles which have passed through the point $x = 0$, $z = 0$. In the experiments, the variance of the lateral particle-concentration-distribution curve was taken as an estimate of σ_z^2	cm^2
$\sigma_z^2(t)$	σ_z^2 as a function of t	cm^2
$\sigma_z^2(x)$	σ_z^2 as a function of x	cm^2
$\overline{\sigma_z^2}$	The mean variance of concentration distribution curves obtained for several values of Δt , other conditions remaining the same	cm^2
$\overline{\sigma_z}$	The mean standard deviation of concentration distribution curves obtained for several values of Δt, other conditions remaining the same	cm

EXPLORATORY LABORATORY STUDY OF LATERAL TURBULENT DIFFUSION AT THE SURFACE OF AN ALLUVIAL CHANNEL

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ABSTRACT

In natural streams turbulent diffusion is one of the principal mechanisms by which liquid and suspended-particulate contaminants are dispersed in the flow. A knowledge of turbulence characteristics is therefore essential in predicting the dispersal rates of contaminants in streams.

In this study the theory of diffusion by continuous movements for homogeneous turbulence is applied to lateral diffusion at the surface of an open channel in which there is uniform flow. An exploratory laboratory investigation was conducted in which the lateral dispersion at the water surface of a sand-bed flume was studied by measuring the lateral spread from a point source of small floating polyethylene particles. The experiment was restricted to a single set of flow and channel geometry conditions.

The results of the study indicate that with certain restrictions lateral dispersion in alluvial channels may be successfully described by the theory of diffusion by continuous movements. The experiment demonstrates a means for evaluating the lateral diffusion coefficient, and also methods for quantitatively estimating fundamental turbulence properties such as the intensity and the Lagrangian integral scale of turbulence in an alluvial channel.

The experimental results show that with increasing distance from the source the coefficient of lateral turbulent diffusion increases initially, but tends toward a constant limiting value. This result is in accordance with turbulent diffusion theory. Indications are that the distance downstream from the source required for the diffusion coefficient to reach its limiting value is actually quite small when compared to the length scale of most diffusion phenomena in natural streams which are of practical interest.

INTRODUCTION

One of the many important functions of natural streams and rivers is to transport industrial and domestic wastes. Considering the increasing quantities of wastes which must be disposed of together with the increasing demand on available water resources for many competing uses, it is obviously imperative that strict controls be imposed on the rates at which pollutants are released into streams. The establishment of realistic control measures requires that adequate criteria be available for predicting the rates at which pollutants are dispersed by natural mechanisms once they have been discharged into streams. At present adequate criteria are not available. The need for more knowledge concerning dispersion in open-channel flows is especially acute in relation to the disposal of low-level liquid radioactive wastes. This is due at least in part to the fact that tolerance levels for radioactive contaminants are generally several orders of magnitude lower than tolerance levels for most other contaminants. Furthermore, many radioactive contaminants cannot be removed from the water by conventional water-treatment practices.

The U. S. Geological Survey, in cooperation with the Division of Reactor Development of the Atomic Energy Commission, is engaged in several research projects which are concerned with the feasibility of using the natural environment for the disposal of certain kinds of radioactive wastes. One area of investigation is concerned with developing criteria for predicting the rate of dispersal of low-level liquid radioactive wastes in natural waterways.

In streams and rivers the initial dispersion of liquid and suspended-particulate contaminants is accomplished principally by turbulence and convection. Turbulent diffusion occurs in all turbulent flows. Common examples of convective dispersion in natural streams are longitudinal dispersion caused by mean velocity gradients and the associated mass transfer, and dispersion induced by flow around bends.

This paper is restricted to a discussion of turbulent diffusion in a broad open channel under conditions of steady, uniform flow. Experimental results are presented for one set of flow conditions in a laboratory-scale alluvial channel. The experiment was an exploratory one, conducted for the purpose of determining whether certain experimental techniques which have been successfully applied to the study of turbulence and diffusion in rigid-boundary open channels could be similarly applied to alluvial-channel studies.

TURBULENT DIFFUSION IN OPEN CHANNELS

According to Hinze (1959), satisfactory solutions to transport problems in turbulent flows depend to a large extent on the adequacy with which Lagrangian statistical functions describing turbulent motion are determined. It follows that turbulent diffusion can be more effectively investigated if the Lagrangian turbulence characteristics are also investigated.

In turbulent flow, the relative positions of a group of neighboring fluid particles change in such a way that the particles tend to spread out or disperse with the passage of time. Dispersion which is due to the random turbulent motion of the fluid particles, as opposed to molecular diffusion or convective dispersion, is called turbulent diffusion. The dispersion of dyes or small suspended particles which have inertial properties identical to those of the fluid particles are visible manifestations of turbulent diffusion.

The rate of dispersion due to turbulent diffusion depends on the turbulent motions of the fluid particles. In a wide, straight channel, with uniform flow, the turbulence characteristics depend on the geometrical characteristics of the channel (i.e., cross-sectional dimensions, boundary roughness and slope) and the flow discharge.

Diffusion theories which are based on the idealized concept of homogeneous turbulence may be applied to some extent in open channels because in channels such as described in the preceding paragraph, the turbulence is approximately homogeneous in planes which are parallel to the water surface, except close to the channel boundaries. Homogeneity means here that the statistical properties of the turbulence are independent of position in a given plane. Due to the variation of mean shear stress with depth, however, the statistical properties of the turbulence do vary in the direction normal to the planes.

Theory

The following discussion is bases on Taylor's (1921) theory of diffusion by continuous movements. In the discussion it is assumed that the turbulence field is at rest with respect to its coordinate system, and that the statistical properties of the turbulence do not vary with time. Although the discussion is restricted to a consideration of turbulent diffusion in the lateral or z direction, the concepts also apply to turbulent diffusion in the longitudinal or x direction.

The state of dispersion of a group of fluid particles at any given time may be described statistically as the variance, $\sigma^2(t)$, of their distribution about the center of concentration. In a homogeneous turbulence field, the fundamental equation of turbulent diffusion derived by Taylor (1921) and expressed in terms of lateral diffusion in the z direction is:

$$\sigma_z^2(t) = 2\,\overline{w'^2} \int_0^t \int_0^\alpha R_{w'}(\alpha_1)\, d\alpha_1\, d\alpha \tag{1}$$

Kampe de Feriet (1939) transformed equation 1 to a simpler form through integration by parts and obtained

$$\sigma_z^2(t) = 2\,\overline{w'^2} \int_0^t (t-\alpha)\, R_{w'}(\alpha)\, d\alpha \tag{1a}$$

in which

$\sigma_z^2(t)$ is the variance at time t of the lateral distribution of a group of fluid particles which were located at $z = 0$ at time $t = 0$

$\overline{w'^2}$ is the mean of the squared instantaneous turbulent velocity components in the z direction

t is the dispersion time

$$R_{w'}(\alpha) = \frac{\overline{w'(t)\;\; w'(t+\alpha)}}{\sqrt{\overline{[w'(t)]^2}}\cdot\sqrt{\overline{[w'(t+\alpha)]^2}}}$$

is the Lagrangian correlation coefficient which correlates values of w' for a fluid particle at the times t and $t + \alpha$

$L_t = \int_0^\infty R_{w'}(\alpha)\, d\alpha$ is the Lagrangian integral time scale of turbulence

Exact solutions of equation 1 are possible only when the Lagrangian correlation function, $R_{w'}(\alpha)$, is known; usually the form of $R_{w'}(\alpha)$ is not known. However, even when the form of $R_{w'}(\alpha)$ is not known, there are some approximate solutions of equation 1 which apply over limited ranges of dispersion time. For example, when the relative dispersion time, t/L_t, is very small, the correlation coefficient, $R_{w'}(\alpha)$, is nearly one and equation 1 reduces to

$$\sigma_z^2(t) \cong \overline{w'^2}\, t^2 \tag{2}$$

When t/L_t is large, equation 1 reduces to

$$\sigma_z^2(t) \cong 2\,\overline{w'^2}\, L_t\, t - 2\,\overline{w'^2} \int_0^\infty \alpha\, R_{w'}(\alpha)\, d\alpha \tag{3}$$

In a turbulent flow with a given $\overline{w'^2}$ and a finite Lagrangian correlation function, the last term in equation 3 is a constant. When t/L_t is very large, this constant is small relative to the other terms in the equation, and equation 1 reduces further to

$$\sigma_z^2(t) \cong 2\,\overline{w'^2}\, L_t\, t \tag{4}$$

At intermediate relative dispersion times, the variance, $\sigma_z^2(t)$, depends on the nature of the Lagrangian correlation function. The general functional form of equation 1 is indicated on figure 1; there may be some variation, depending on the form of the Lagrangian correlation function.

Figure 1.--Variance of the lateral distribution, $\sigma_z^2(t)$, as a function of relative dispersion time, t/L_t

Batchelor (1949) has shown that turbulent diffusion can be described by a simple Fickian diffusion equation provided that the one-dimensional concentration distribution of a contaminant released from a point source conforms to the normal distribution law. According to Hinze (1959) the experimental evidence of many investigators shows that for diffusion in homogeneous turbulence the concentration distribution is normal for both long and short dispersion times. It is reasonable to assume that this result applies for intermediate dispersion times also.

The one-dimensional Fickian diffusion equation for lateral dispersion is

$$\frac{\partial C}{\partial t} = K_z \frac{\partial^2 C}{\partial z^2} \tag{5}$$

in which

K_z is the coefficient of lateral turbulent diffusion

C = $C(z,t)$ is the amount of contaminant per unit width ~~at time t~~, as a function of z and t.

Assuming that K_z is a constant, the solution of equation 5 for the case of a quantity of contaminant, q , released from an instantaneous point source at time $t = 0$ into a homogeneous turbulence field is

$$C(z,t) = \frac{q}{\sqrt{4 \pi K_z t}} e^{-\frac{1}{2} \frac{z^2}{(2 K_z t)}}$$

which is a normal distribution function with variance

$$\sigma_z^2 = 2 K_z t$$

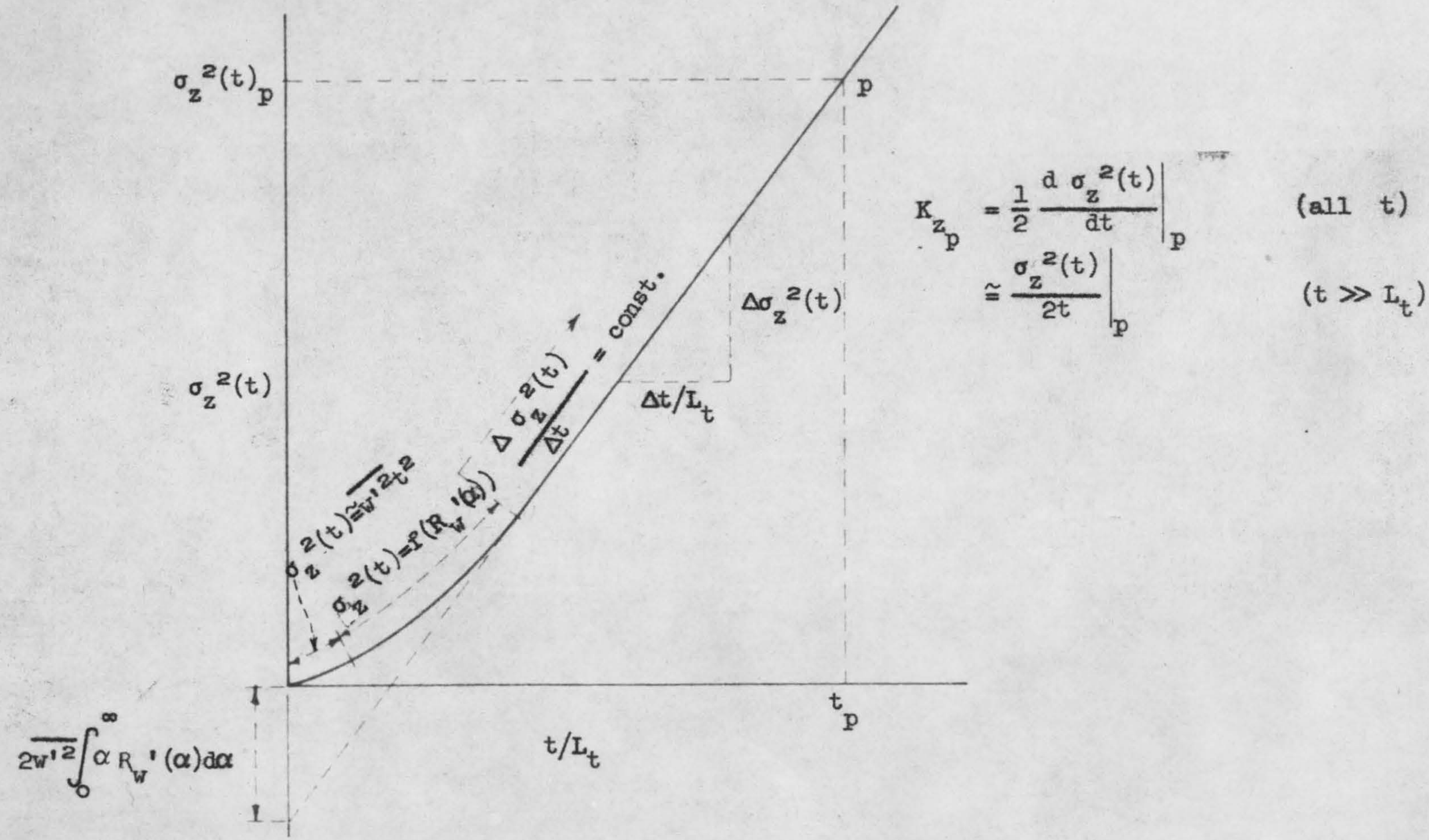


Figure 1.--Variance of the lateral distribution, $\sigma_z^2(t)$, as a function of relative dispersion time, t/L_t.

Equating this variance to equation 4 results in

$$K_z = \frac{\sigma_z^2(t)}{2t} = \overline{w'^2}\, L_t$$

This relationship which assumes that the diffusion coefficient is a constant and states that it is equal to the product of the Lagrangian velocity variance and the Lagrangian integral time scale of turbulence has been found by many investigators to be approximately true when $t/L_t \gg 1$. For short and intermediate dispersion times the diffusion coefficient, in keeping with the theory of diffusion by continuous movements, is usually defined as

$$K_z = \frac{1}{2} \frac{d\,\sigma_z^2(t)}{d\,t}$$

and is in general a function of dispersion time.

Solutions of equation 5 for this and various other initial and boundary conditions are available in the literature (e.g. Frenkiel, 1953; Hinze, 1959; and Scull and Mickelsen, 1957).

The foregoing discussion of turbulent diffusion pertains to a turbulence field which is stationary with respect to its coordinate system. However, with the transformation

$$x = \overline{U}\, t$$

the theory applies also to a homogeneous turbulence field which is being transported bodily at a constant velocity, $\overline{U}$, in the x direction. The discussion has also been simplified to the extent that complicating factors such as confining boundaries and velocity gradients, which influence the diffusion process in open channels, have not been considered. In spite of these simplifications, the concepts presented constitute a theoretical framework for studying turbulent diffusion in open channels.

Turbulence Measurements

In experimental fluid mechanics, studies of turbulence in water have lagged considerably behind corresponding studies in air. The lag is at least partially due to the fact that instruments such as the hot-wire anemometer, which has proven satisfactory for turbulence measurements in air, have not been developed to the level of adequacy required for turbulence measurements in water. Without such instruments, measurements of dispersion afford one of the most promising means by which fundamental turbulence characteristics can be quantitatively estimated. Specifically, the intensity and the integral scale of turbulence can be determined, and the Lagrangian correlation coefficient can be estimated on the basis of a statistical analysis of dispersion data. Baldwin and Mickelsen (1962) have demonstrated that these Lagrangian properties can be related empirically to the corresponding Eulerian properties which are usually obtained from anemometer measurements.

Considering again, the lateral diffusion in the xz plane of a contaminant released from a point source in a uniform flow having a uniform mean velocity, $\overline{U} = x/t$, the intensity of turbulence is given by:

$$\frac{\sqrt{\overline{w'^2}}}{\overline{U}} = \lim_{x \to 0} \left(\frac{\sigma_z(t)}{x} \right) \tag{6}$$

The Lagrangian integral time sca!~ of turbulence is given by

$$L_t = \frac{\overline{U}}{2\,\overline{w'^2}} \lim_{x \to \infty} \left(\frac{\sigma_z^{\,2}(t)}{x} \right) \tag{7}$$

Once $\overline{w'^2}$ is known, the Lagrangian correlation coefficient can be estimated graphically by successive differentiations of a $\sigma_z^{\,2}(t)$ curve, since by applying Leibnitz' rule to equation 1 and differentiating twice,

$$R_{w'}(t) = \frac{1}{2\,\overline{w'^2}} \frac{d^2 \sigma_z^{\,2}(t)}{dt^2} \tag{8}$$

If the form of the Lagrangian correlation function is known or assumed in advance, and is integrable, the intensity and scale of turbulence can be determined from diffusion data obtained at intermediate values of t/L_t, say $(0.1 < t/L_t < 10)$. This technique is useful when, as is often the case, it is difficult or impossible to obtain diffusion data at the limiting conditions specified by equations 6 and 7.

There is experimental evidence, (Ippen and Raichlen, 1957) that at large Reynolds numbers the correlation coefficient may be approximated reasonably well by an equation of the form

$$R_{w'}(\alpha) = e^{-\frac{|\alpha|}{L_t}} \tag{9}$$

By converting equation 1 to the dimensionless form

$$\frac{\sigma_z^2(t)}{\overline{w'^2}\, L_t^2} = 2 \int_0^{t/L_t} (t/L_t - \alpha/L_t)\, R_{w'}(\alpha)\, d\left(\frac{\alpha}{L_t}\right) \tag{10}$$

and by making the substitution indicated by equation 9, Frenkiel (1953) obtained the following approximate solutions to equation 10.

$$0 < t/L_t \leq 0.030 \qquad \frac{\sigma_z^2(t)}{\overline{w'^2}\, L_t^2} = \left(\frac{t}{L_t}\right)^2 \tag{10a}$$

$$0.030 < t/L_t \leq 3.63 \qquad \frac{\sigma_z^2(t)}{\overline{w'^2}\, L_t^2} = 2\left(e^{-t/L_t} + t/L_t - 1\right) \tag{10b}$$

$$3.63 < t/L_t \leq 101 \qquad \frac{\sigma_z^2(t)}{\overline{w'^2}\, L_t^2} = 2\left(t/L_t - 1\right) \tag{10c}$$

$$101 < t/L_t \qquad \frac{\sigma_z^2(t)}{\overline{w'^2}\, L_t^2} = 2\, t/L_t \tag{10d}$$

The intensity and scale of turbulence can now be determined graphically by superimposing a $\sigma_z(t)$ curve, based on experimental data and represented on logarithmic coordinates, on the theoretical $\frac{\sigma_z}{\sqrt{\overline{w'^2}}\ L_t}\left(\frac{t}{L_t}\right)$ curve as determined from equation 10 a-d, so that the curves match. The ratios of the respective ordinates and abscissas then define the ratios $t/t/L_t$ and

$$\frac{\sigma_z(t)}{\sigma_z(t)/\sqrt{\overline{w'^2}}\ L_t}$$ from which L_t and $\sqrt{\overline{w'^2}}$ may be evaluated.

The corresponding parameters describing the longitudinal turbulence characteristics may be obtained in an analogous manner by observing the longitudinal diffusion in the xz plane of a contaminant released instantaneously from a point source, or a line source which is parallel to the z axis.

Application of these and similar techniques to the determination of turbulence characteristics in open channels has been pioneered by Kalinske and Pien (1944) and Orlob (1959). Kalinske and Pien (1944) obtained measurements of the lateral diffusion of a hydrochloric acid-alcohol mixture (sp. gr. = 1) in a 3-dimensional turbulence field in a laboratory flume. Orlob (1959) studied the lateral diffusion of small polyethylene particles (sp. gr. = 0.975) in a 2-dimensional turbulence field defined by the water surface in a laboratory flume.

Diffusion at the Water Surface

The water surface (or a plane parallel thereto) of a wide channel in which there is uniform flow constitutes a 2-dimensional, nondecaying, turbulence field in which the turbulence may be assumed to be statistically homogeneous except in regions very close to the sidewalls. These conditions conform approximately to the restrictions by which diffusion theory for homogeneous turbulence is limited. Therefore, the water surface of a channel such as the one defined above is a good medium for the quantitative determination of turbulence characteristics from dispersion observations. Furthermore, measurements of dispersion on the water surface of a laboratory flume may be easily and accurately obtained, as Orlob (1959) has demonstrated.

The extent to which the diffusion patterns at the water surface reflect the diffusion process and turbulence characteristics beneath the surface is questionable, particularly in alluvial channels. The experimental results of Kalinske and Pien (1944), however, tend to confirm the Reynolds analogy that the local diffusion coefficient, $K(y)$, and the kinematic eddy viscosity, ϵ, in the equation

$$\tau = \epsilon \rho \frac{du}{dy}$$

are essentially equivalent. In the above equation

$\tau \cong \tau_o (1 - \frac{y}{D})$ is the mean intensity of the local shear, where τ_o is the mean intensity of shear at the bed and D is the mean depth of flow.

ρ is the mass density of the fluid.

u is the mean velocity at distance, y, above the bed.

If the Reynolds analogy is indeed a reasonable approximation, measured velocity profiles could be used to predict $K(y)$, and perhaps other turbulence characteristics, as functions of y.

Alluvial Channels

In general, diffusion phenomena in open alluvial channels should be similar to the corresponding phenomena in rigid-boundary open channels. Owing to the differences in the mechanics of flow associated with the two types of channels, however, certain differences are to be expected. For example, in alluvial channels there is a mutual interaction between the boundary form and both the turbulence characteristics and the velocity distribution (the flow characteristics determine the bed form and vice versa); in rigid-boundary channels, the form and roughness of the boundary influence the turbulence characteristics and the velocity distribution, but there is no reciprocal effect. Another important difference is that the individual ripples and dunes, which usually make up the dominant roughness elements in alluvial channels, migrate in the direction of flow and undergo continuous transformation in shape and size, although a certain statistical constancy prevails. The extent to which factors such as these affect the diffusion process and the turbulence characteristics in open alluvial channels has not been systematically investigated.

DESCRIPTION OF EXPERIMENT

An exploratory experiment was conducted in an 8-ft wide, 150-ft long, laboratory flume. The bed of the flume was covered with sand to a depth of about 8 inches. A schematic diagram of the flume circulation system is shown in figure 2.

Figure 2.--Schematic diagram of the flume

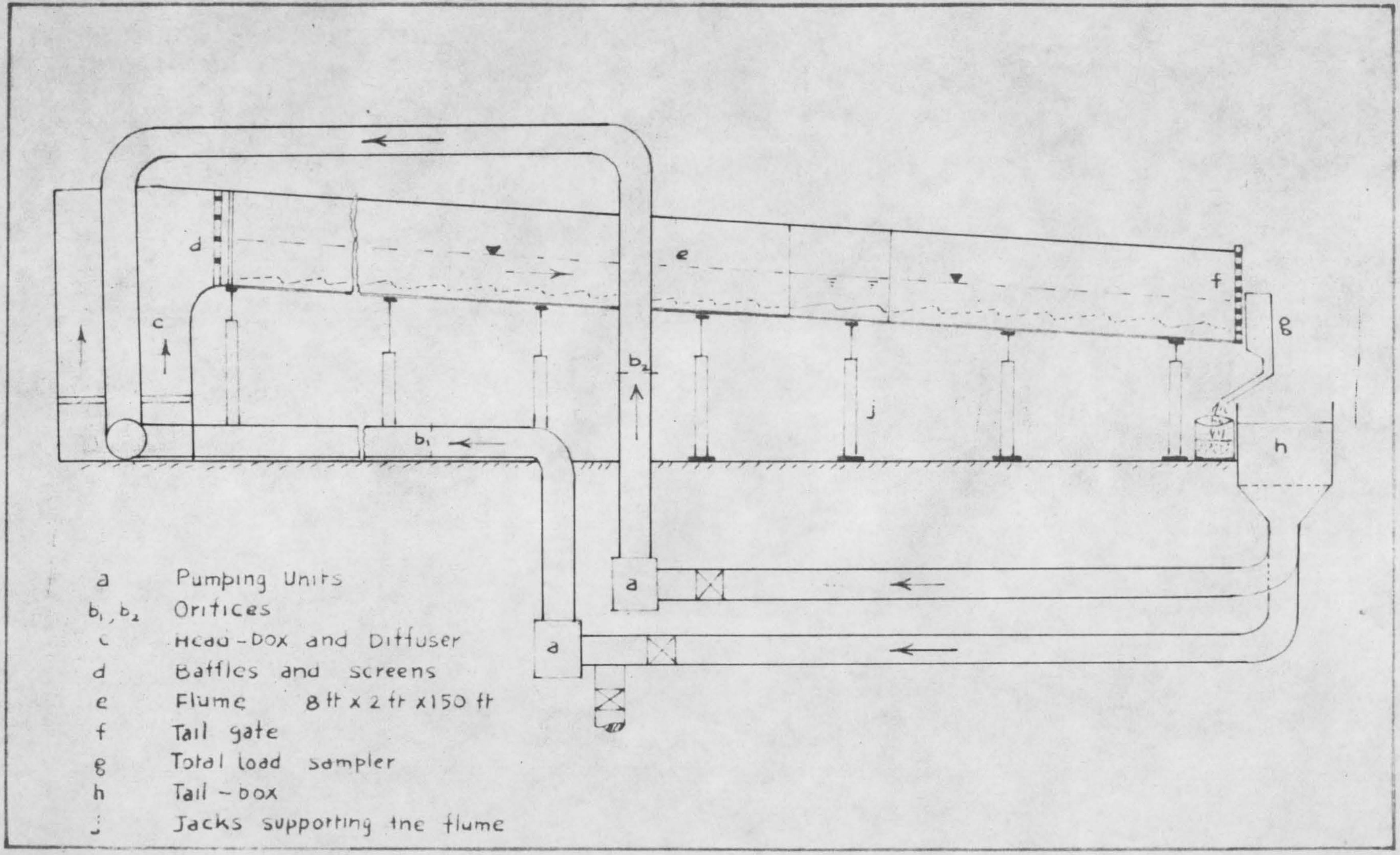


Figure 2.--Schematic diagram of the flume.

Diffusion data were obtained for one set of flow conditions. Data describing the flow conditions are as follows:

Discharge - 7.41 cubic feet per second

Average depth - 0.58 feet

Average water-surface and bed slope - 0.00071

Water temperature - 17.4 degrees centigrade

Total sediment concentration - 63 parts per million

Bed configuration - small dunes

Average dune length - 2.8 feet

Average dune height - 0.029 feet

Average dune velocity - 0.067 feet per minute

Bed material

Median fall diameter - 0.93 millimeters

Gradation coefficient $- \frac{1}{2}\left(\frac{d_{50}}{d_{15.9}} + \frac{d_{84.1}}{d_{50}}\right) = 1.53$

The equipment and procedures employed in obtaining the above data are described by Simons, ~~and~~ Richardson ~~(1961).~~ and Albertson (1961).

The experimental techniques for studying lateral diffusion were similar to those employed by Orlob (1959). Polyethylene particles were released one at a time from a point source located on the water surface at a point on the center line of the flume, 30 feet downstream from the head box. As the particles floated down the flume, their paths, which tended to diverge, defined a lateral dispersion pattern. The dispersion pattern was determined by intercepting the particles at definite distances downstream from the source and noting their lateral positions.

The polyethylene particles were disc-shaped, having a diameter of 1/8-inch and a thickness of 1/16-inch. The specific gravity of the particles was approximately 0.96. Therefore, there was little density effect in the inertial response of the particles to turbulent impulses.

The particles were sufficiently large to eliminate any appreciable effect due to molecular diffusion, but probably not too large to trace the motions of the larger turbulent eddies at the water surface. The effect of particle size was not investigated in this study, however, recent and as yet unpublished data obtained by the senior author in the same flume and at about the same Reynolds number indicate that there is little difference in the lateral dispersion patterns of 1/8" polyethylene particles dispersing on the surface and Rhodamine - B dye dispersing beneath the surface. This would indicate that the particles were not too large.

The particles were released into the flow through a funnel which was mounted on a point-gage assembly. The 3/16-inch diameter tip of the funfel was located just above the water surface. The particles were ejected sequentially in lots of 100 into the funnel at a constant rate from a manually-operated dispenser. In order to detect any effect due to periodicity in the turbulence, the experiment was repeated for several lots at different rates of release, by varying the time interval, Δt , between releases from 1 to 5 seconds. No periodicity effect was detected. The particle-introduction method simulated continuous release from a point source in a 2-dimensional system, or if diffusion in the y direction is neglected, a vertical line source in a 3-dimensional system.

The particles were caught in a compartmented sieve which was inserted in the flow, normal to the flow direction. The collector extended across the entire width of the flume. The collector was divided into 1-centimeter compartments as shown in figure 3, and each compartment was numbered.

Figure 3.--Section of compartmented-sieve particle collector

Because only the bottom 1/2 inch, or less, of the collector was immersed in the flow, disturbance to the flow was assumed to be negligible. The lateral distribution of particle paths for each 100-particle lot was determined by

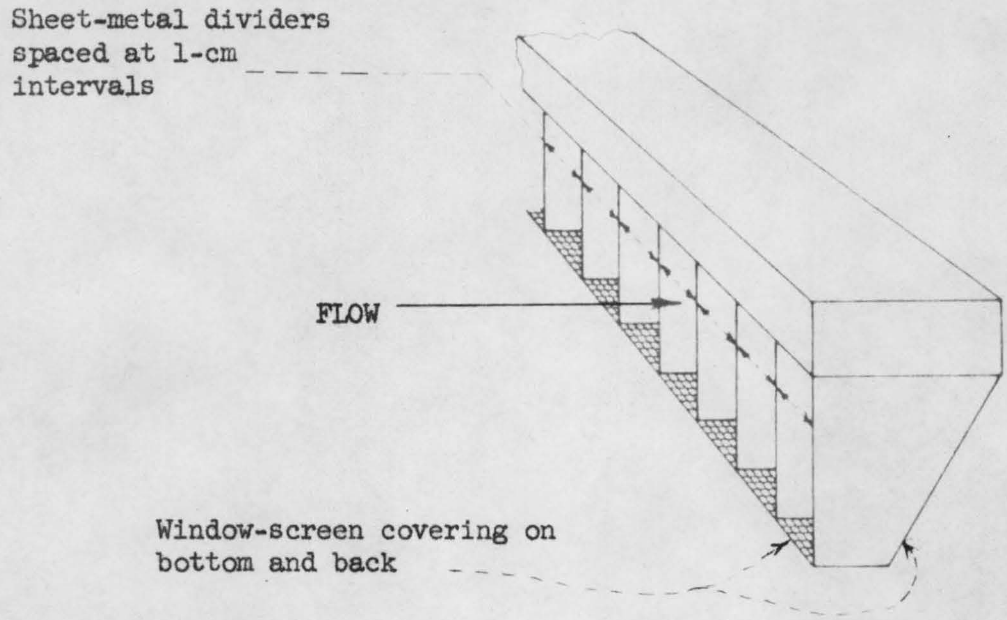


Figure 3.--Section of compartmented sieve particle collector.

counting and recording the number of particles trapped in each compartment. Lateral distributions of 3 to 5 lots of 100 particles were determined in this manner for source-to-collector distances of 15, 30, 48, 60, 120, 240, 300, 450, 600, 900, 1500, 1950, 2400, 2850, and 3300 centimeters.

The diffusion theory discussed in the previous section is based on the Lagrangian reference system in which attention is focused on the displacement histories of individual particles with respect to time. If the data had been obtained in strict accordance with the theory, the lateral position of each particle at a specified dispersion time, t, following release would have been determined, and the extent of dispersion would be described as the variance, $\sigma_z^2(t)$, of the lateral distribution of particles about the mean lateral position at time t. Actually the lateral position of each particle was determined at a specified distance, x, downstream from the source, rather than at a specified t, and the extent of dispersion was described as $\sigma_z^2(x)$. Because the flow was uniform ($x = \overline{U}\,t$), and x was large compared to the lateral deviations from the mean path, z, it seemed reasonable to assume that $\sigma_z^2(t) = \sigma_z^2(x)$. In the analysis which follows, no distinction between $\sigma_z^2(t)$ and $\sigma_z^2(x)$ is made, and σ_z^2 is substituted for $\sigma_z^2(x)$.

The times of transit over various known distances was recorded for 27 particles. The mean surface velocity computed from these data was $\overline{U}_s = 62.5$ centimeters per second. Extrapolation of three measured velocity profiles to the water surface indicated a mean surface velocity of 65.8 centimeters per second. The velocity as determined by the particle-timing method was used in analyzing the data.

Data on longitudinal diffusion were also collected at distances from the source of 240, 600 and 900 centimeters. Particles were released from the source at 10-second intervals and times of transit for each longitudinal distance were determined indirectly by measuring the times between arrival of successive particles at the end station. This procedure was repeated with 100 or more particles for each of the three longitudinal distances. This method, although theoretically sound, did not prove to be sufficiently accurate in practice, because operator errors were of the same order of magnitude as the spread about the mean transit time. The results were therefore, inconclusive.

RESULTS

The lateral diffusion data were plotted on arithmetic probability paper. A typical set of data, for a source-to-collector distance of 300 centimeters, is shown in figure 4. These results indicate that the distribution of particle paths along the z axis followed the normal probability law.

Figure 4.--Cumulative distribution plots for $x = 300$ centimeters

This was typical of all runs. It was also typical that no systematic variation of distribution due to variation of the time interval, Δt, between particle releases could be detected. The center of concentration of the particles deviated from the center line of the flume by an amount δ_c. In nearly all runs this deviation was to the left. The standard deviation, σ_z, of the particle distributions was determined from the cumulative distribution plots according to the relationship

$$\sigma_z = \frac{z_{84.1} - z_{15.9}}{2}$$

where $z_{84.1}$ and $z_{15.9}$ are the positions along the z axis where the 84.1 and 15.9 percentile lines intersect the distribution curves. The procedure for determining σ_z is illustrated in figure 4. The parameters determined from the cumulative distribution plots and other experimental data are listed in table 1.

In figure 5, the variance of lateral spread, σ_z^2, is shown as a function

Figure 5.--Variance of lateral distribution as a function of distance downstream from source

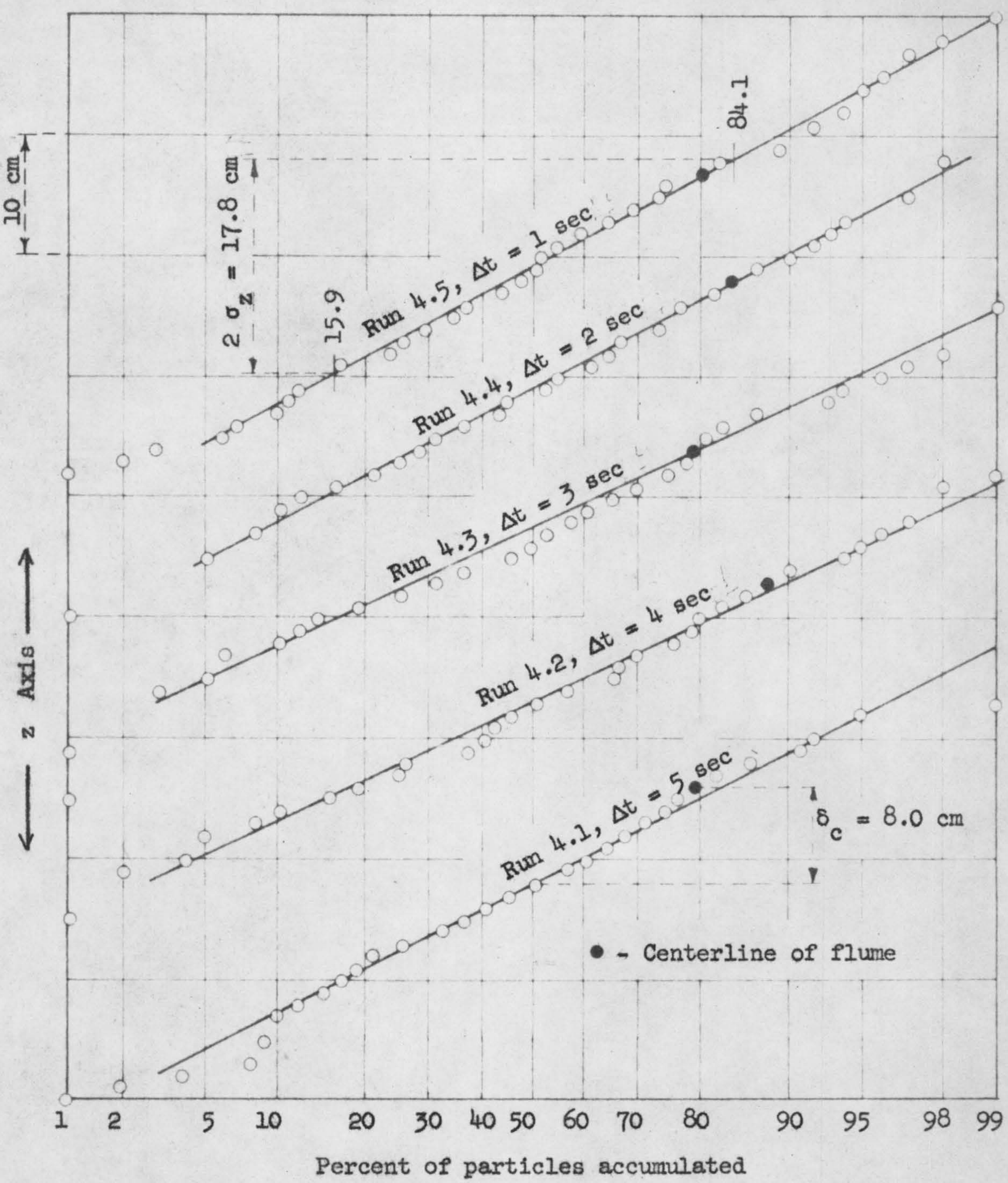


Figure 4.--Cumulative distribution plots for x = 300 centimeters.

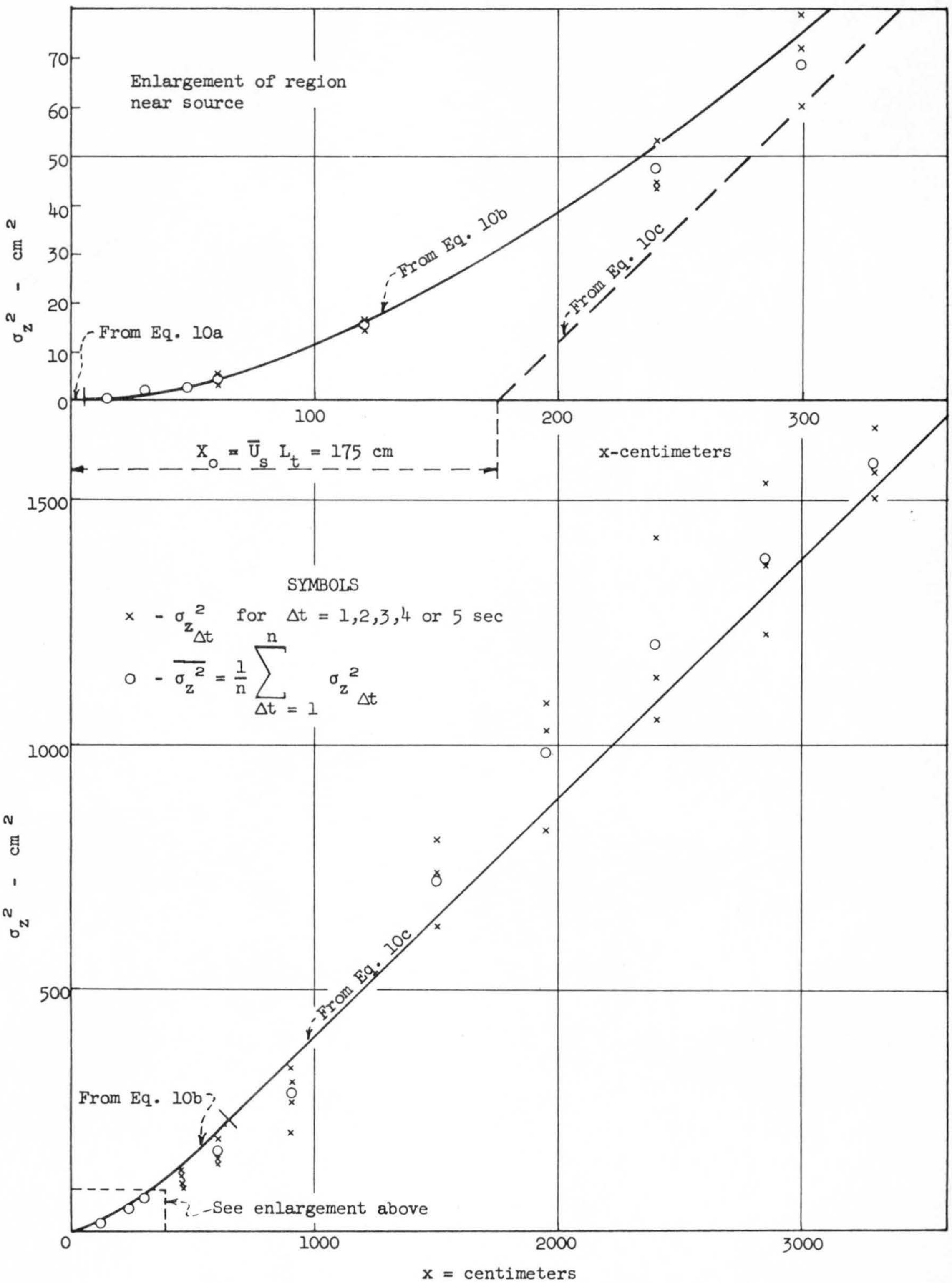


Figure 5.--Variance of lateral distribution as a function of distance downstream from source.

TABLE 1

LATERAL DIFFUSION DATA AND COMPUTED PARAMETERS

Run No.	x cm	Δt sec	$t = x/\overline{U}_s$ sec	σ_z cm	σ_z^2 cm^2	$\overline{\sigma_z}$ cm	$\overline{\sigma_z^2}$ cm^2	$\frac{\overline{\sigma_z^2}}{x}$ cm	$\frac{\overline{\sigma_z}}{x}$	δ_c cm	$\overline{\delta_c}$ cm
15.1	15	4	0.240	0.70	0.490	0.74	0.548	.0366	.0493	1.40	1.25
15.2		3		0.77	0.593					1.45	
15.3		2		0.77	0.593					1.15	
15.4		1		0.72	0.518					1.00	
14.1	30	4	0.480	1.50	2.25	1.42	2.02	.0673	.0473	1.70	1.74
14.2		3		1.40	1.96					1.70	
14.3		2		1.35	1.82					1.90	
14.4		1		1.42	2.03					1.65	
13.1	48	4	0.769	1.70	2.89	1.71	2.92	.0608	.0356	1.95	1.84
13.2		3		1.75	3.06					2.50	
13.3		2		1.80	3.24					1.05	
13.4		1		1.58	2.50					1.85	
7.1a	60	3	0.960	2.38	5.66	2.22	4.94	.0823	.0370	2.50	2.28
7.1b		3		2.42	5.86					2.45	
7.2a		2		2.19	4.80					2.45	
7.2b		2		2.29	5.24					1.98	
7.3a		1		1.97	3.88					2.00	
7.3b		1		2.05	4.20					2.30	
6.1	120	3	1.92	4.10	16.8	3.94	15.5	.129	.0328	3.8	3.4
6.2		2		3.92	15.4					3.4	
6.3		1		3.80	14.4					3.0	

TABLE 1 - continued

LATERAL DIFFUSION DATA AND COMPUTED PARAMETERS

Run No.	x cm	Δt sec	$t=x/\overline{U}_s$ sec	σ_z cm	σ_z^2 cm^2	$\overline{\sigma_z}$ cm	$\overline{\sigma_z^2}$ cm^2	$\frac{\overline{\sigma_z^2}}{x}$ cm	$\frac{\overline{\sigma_z}}{x}$	δ_c cm	$\overline{\delta_c}$ cm
5.1	240	5	3.84	7.3	53.3	6.87	47.3	.197	.0286	7.2	7.1
5.2		3		6.6	43.6					6.6	
5.3		2		6.9	47.6					6.3	
5.4		1		6.7	44.9					8.3	
4.1	300	5	4.80	8.5	72.3	8.28	68.8	.229	.0276	8.0	8.2
4.2		4		7.8	60.8					9.8	
4.3		3		7.7	59.3					6.4	
4.4		2		8.5	72.2					8.8	
4.5		1		8.9	79.2					7.8	
2.1	450	5	7.20	10.5	110.2	10.6	112.6	.250	.0236	3.5	7.4
2.2		1		10.3	106.1					5.4	
2.3		4		11.3	127.7					7.5	
2.4		3		10.9	118.8					11.4	
2.5		2		10.0	100.0					9.0	
3.1	600	5	9.60	13.8	190.4	12.9	166.9	.278	.0215	10.0	9.0
3.2		4		13.6	185.0					8.8	
3.3		3		12.0	144.0					10.0	
3.4		2		12.7	161.3					6.9	
3.5		1		12.4	153.8					9.4	
1.1	900	5	14.4	18.4	339	17.0	289	.322	.0189	8.4	8.0
1.2		4		17.4	303					7.7	
1.3		3		16.3	266					13.0	
1.4		2		18.4	339					6.0	
1.5		1		14.2	202					4.7	

TABLE 1 - continued

LATERAL DIFFUSION DATA AND COMPUTED PARAMETERS

Run No.	x cm	Δt sec	$t=x/\overline{U}_s$ sec	σ_z cm	σ_z^2 cm^2	$\overline{\sigma_z}$ cm	$\overline{\sigma_z^2}$ cm^2	$\overline{\sigma_z^2}/x$ cm	$\overline{\sigma_z}/x$	δ_c cm	$\overline{\delta_c}$ cm
8.1	1500	3	24.0	28.3	801	26.8	723	.482	.0179	7.6	10.6
8.2		2		25.0	625					10.4	
8.3		1		27.1	734					13.7	
9.1	1950	3	31.2	28.7	824	31.3	981	.503	.0161	13.7	12.8
9.2		2		32.1	1030					14.3	
9.3		1		33.0	1089					10.4	
10.1	2400	3	38.4	32.4	1050	34.6	1202	.501	.0144	13.2	6.6
10.2		2		33.7	1136					10.5	
10.3		1		37.7	1421					-4.0	
12.1	2850	3	45.6	37.0	1369	37.1	1377	.483	.0130	6.0	6.7
12.2		2		39.2	1537					9.0	
12.3		1		35.0	1225					5.0	
11.1	3300	3	52.8	40.6	1648	39.6	1571	.476	.0120	8.4	9.5
11.2		2		38.8	1505					12.0	
11.3		1		39.5	1560					8.0	

of distance downstream from the source, x . The plotted experimental data follow a peculair trend in that the data fall consistently below the curve obtained from equation 10 in the range ($450 < x < 900$) and consistently above the curve in the range ($x > 1500$) with an apparent break occurring somewhere in the range ($900 < x < 1500$). It is believed that the break was due to a change in turbulence characteristics induced by a change in bed roughness. This hypothesis is supported by the photographs in figure 6 which show the condition of the flume bed after the

Figure 6.--Flume bed after the experiment

experiment. Both photographs show a path, approximately 2-feet wide, of comparatively smooth bed extending for some distance down the flume. This path, which extended from the head box to about 9 meters downstream from the source, conceivably could have caused a reduction in dispersion in the range ($0 < x < 900$) as indicated by the data in figure 5. It is not known definitely if the condition of the bed indicated by the photographs persisted throughout the entire experiment, however, previous experience with the same flume suggests the possibility. As the distance downstream from the source increases the slope of the $\sigma_z{}^2(x)$ curve becomes constant, which indicates a constant coefficient of lateral diffusion, $K_z = 15.1$ square centimeters per second.

Figure 7 illustrates the use of equations 6 and 7 in determining the

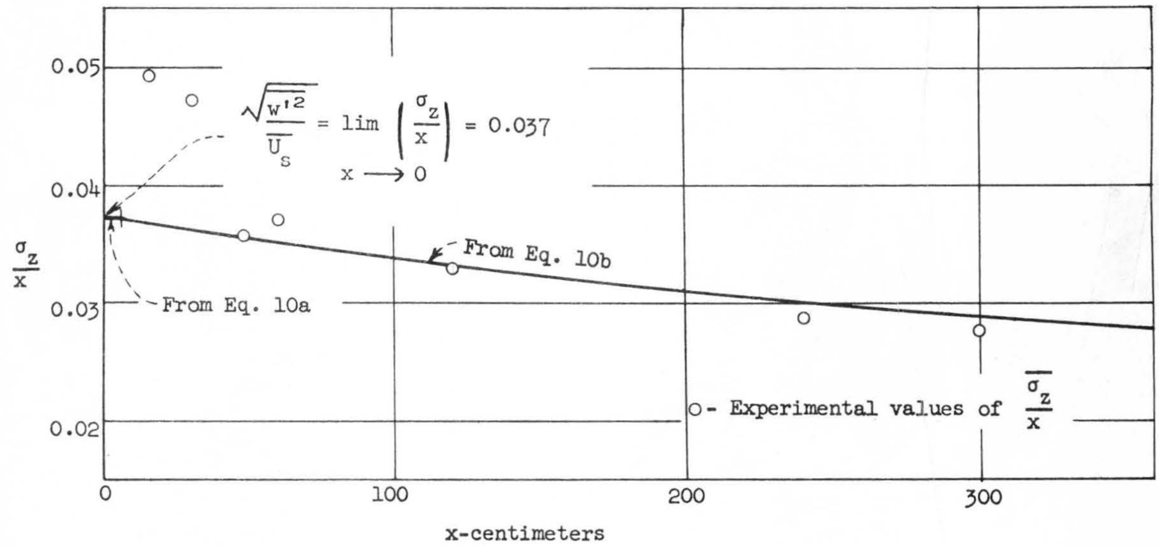
Figure 7.--Intensity and scale of turbulence as determined from limits of dispersion data

magnitudes of the intensity and integral scale of turbulence from experimental dispersion data obtained close to and far from the source. Due to the apparent shift in turbulence characteristics in the range ($900 < x < 1500$),

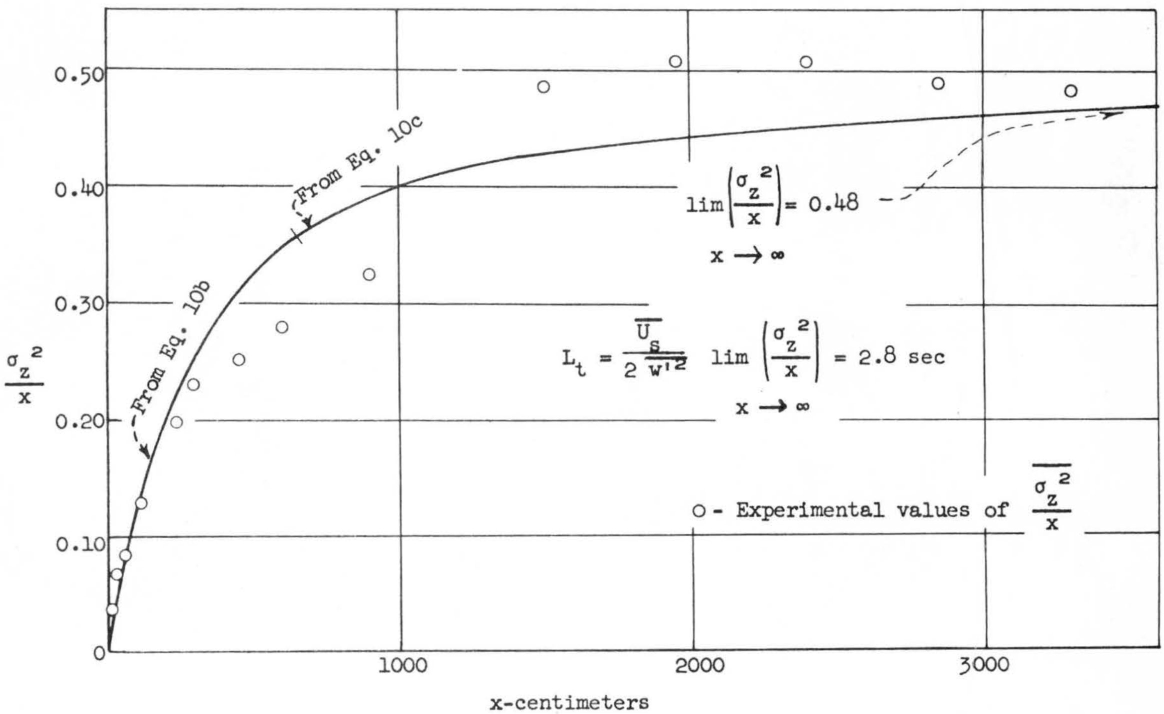
A. View of bed looking up-stream toward source.

B. View of bed looking downstream from source.

Figure 6.--Flume bed after the experiment.



A. Intensity of turbulence



B. Integral scale of turbulence

Figure 7.--Intensity and scale of turbulence as determined from limits of dispersion data.

use of experimental data obtained at the limiting distances from the source would in this case require that estimates of $\sqrt{\overline{w'^2}}/\overline{U}_s$ and L_t be based on different sets of turbulence characteristics. Also, in view of the limitations of the polyethylene-particle technique, the reliability of the dispersion measurements obtained near the source, where small scale motion is important, is questionable. Specifically, the measurements obtained at x = 15 and 30 centimeters are open to question because the diameter of the particles, the compartment spacing, σ_z , and the diameter of the tip of the funnel through which the particles were ejected, were all of the same order of magnitude.

Consequently, the method of estimating turbulence characteristics which uses an assumed Lagrangian correlation function is considered preferable for interpreting the results of this experiment. This method is illustrated by figures 8 and 9 . The correlation function

$$R_{w'}(\alpha) = e^{-\frac{|\alpha|}{L_t}}$$

Figure 8.--Theoretical solution of diffusion equation with

$$R_{w'}(\alpha) = e^{-\frac{|\alpha|}{L_t}}$$

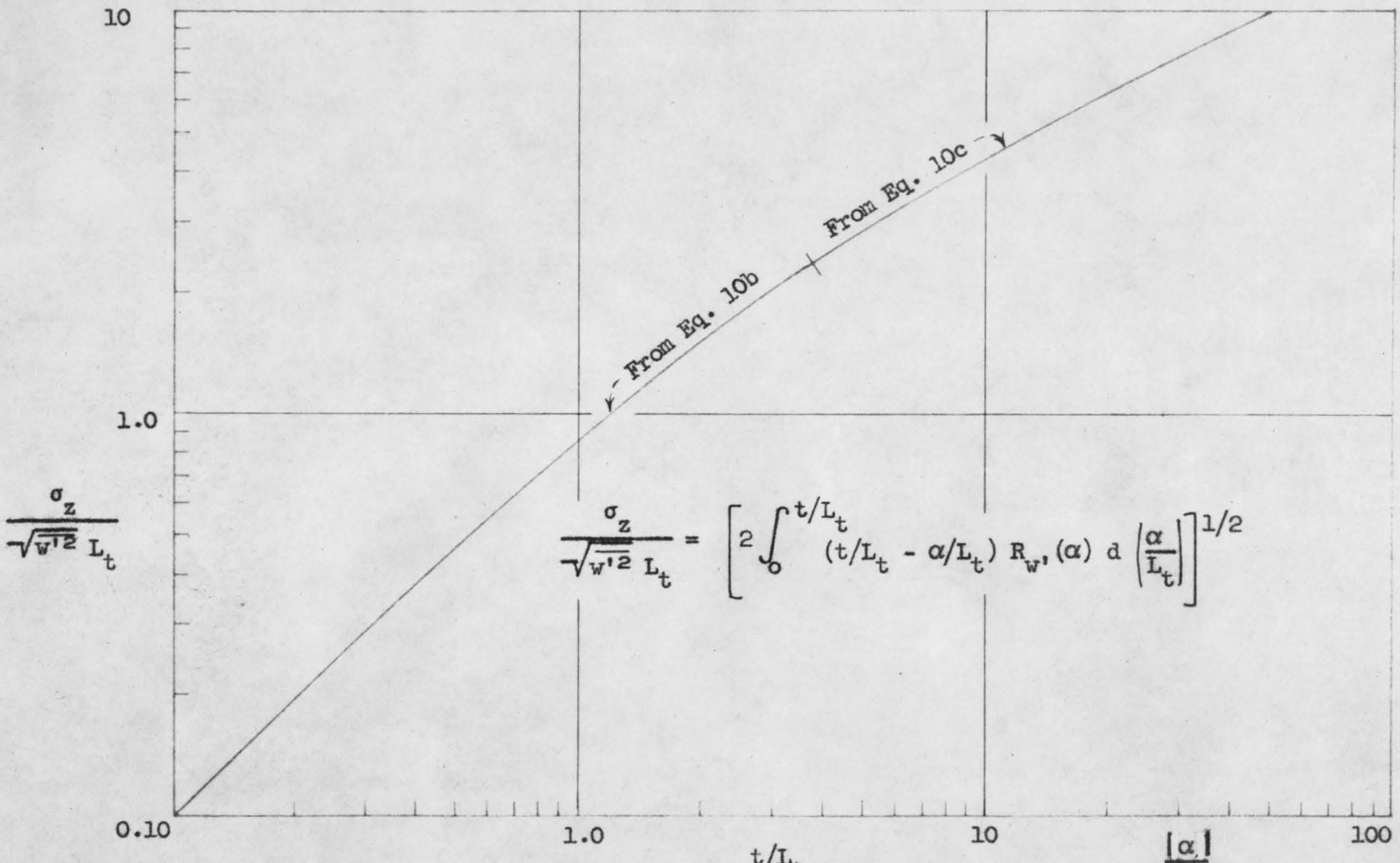


Figure 8.--Theoretical solution of diffusion equation with $R_{w'}(\alpha) = e^{-\frac{|\alpha|}{L_t}}$

Figure 9.--Standard deviation of lateral distribution as a function of dispersion time

was assumed and solutions 10b and 10c of equation 10 were used to plot the theoretical $\frac{\sigma_z}{\sqrt{\overline{w'^2}}\, L_t} \left(\frac{t}{L_t}\right)$ curve in figure 8. In figure 9, the experimental data indicating the $\sigma_z(t)$ relationship obtained by experiment were plotted. Figure 9 was superimposed on figure 8 in the position which corresponded to the best fit of the entire range of data to the theoretical curve. The ratio of the abscissas then defined the integral scale of turbulence

$$L_t = \frac{t}{t/L_t} = 2.8 \text{ seconds}$$

and the ratio of the ordinates defined the intensity of turbulence

$$\frac{\sqrt{\overline{w'^2}}}{\overline{U}_s} = \frac{1}{\overline{U}_s L_t} \frac{\sigma_z}{\sigma_z / \sqrt{\overline{w'^2}}\, L_t} = \frac{1}{2.8 \times 62.5}(6.5) = 0.037$$

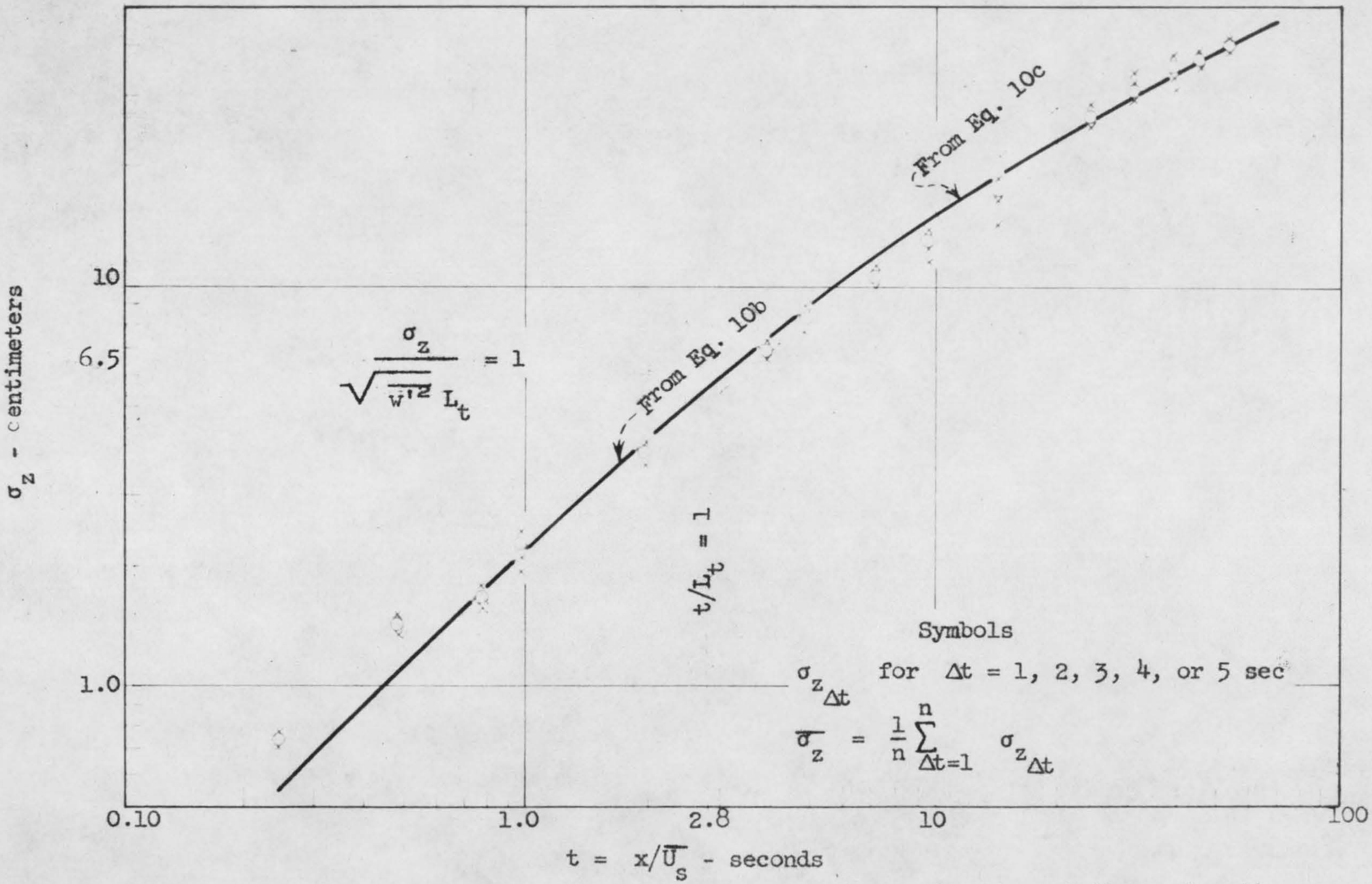


Figure 9.--Standard deviation of lateral distribution as a function of dispersion time.

The principal advantage of this method is that it involves the entire range of data, not only data obtained at limiting distances from the source.

With the values of L_t and $\sqrt{\overline{w'^2}}$ as determined in the preceding paragraph, equations 10a, 10b, and 10c were used to construct the theoretical curves shown in figures 5 and 7.

Figure 10 was plotted to indicate the manner in which the coefficient of

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Figure 10.--Lateral diffusion coefficient as a function of distance from source

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lateral diffusion, K_z, approaches a constant as the distance from the source increases. The manner of approach depends in part on how K_z is defined. The $K_z(x)$ relationship is shown in figure 10 for the two alternate definitions

$$K_z = \frac{1}{2} \frac{d\sigma_z^2}{dt} = \frac{\overline{U}_s}{2} \frac{d\sigma_z^2}{dx} \tag{11}$$

and

$$K_z = \frac{\sigma_z^2}{2t} = \frac{\overline{U}_s}{2} \frac{\sigma_z^2}{x} \tag{11a}$$

The experimental values of K_z corresponding to equation 11 were obtained by graphical differentiation of the data plotted in figure 5. Values of K_z corresponding to equation 11a were computed directly from the data. The theoretical curves were obtained from equations 10b and 10c by using the computed values of L_t = 2.8 seconds and $\overline{w'^2}$ = 5.38 square centimeters per second squared. Although the diffusion coefficient as defined in equation 11a appears frequently in the literature and the coefficients defined in equations 11 and 11a both approach the same limiting value as the distance from the source increases, the definition given in equation 11 is preferable because it is consistent with the theory of diffusion by continuous movements and because it approaches the limiting value,

$$\lim_{x \to \infty} K_z = \text{constant, more rapidly.}$$

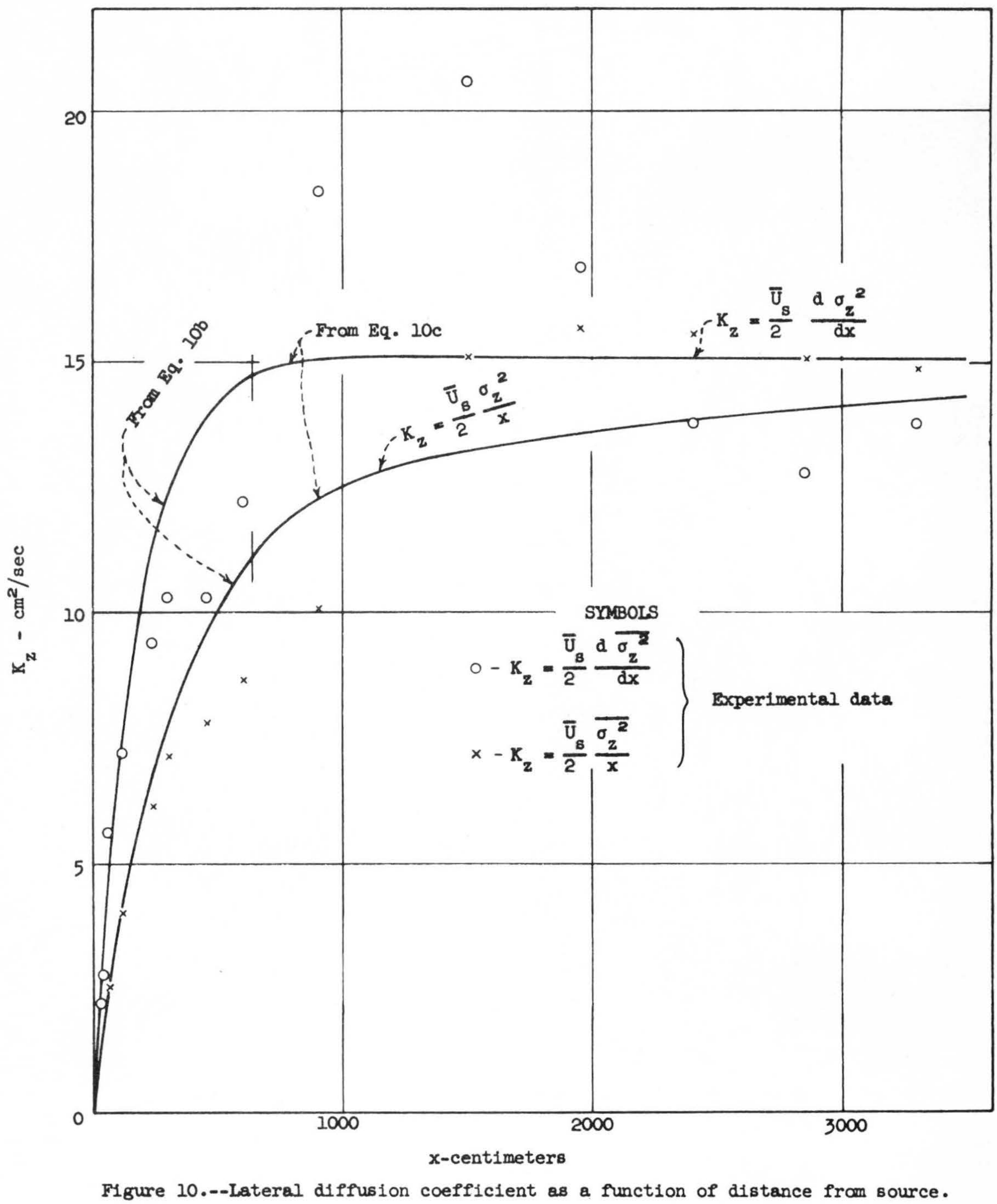

Figure 10.--Lateral diffusion coefficient as a function of distance from source.

Semi-empirical equations which relate the limiting value of K_z to mean flow parameters in a broad rigid-boundary open channel have been derived by both Orlob (1961) and Elder (1959). Orlob's equation (1961), which was based on experimental observations of lateral turbulent diffusion at the water surface of a laboratory flume with extreme bottom roughness is

$$K_z = 0.085 \; \phi(f) \, E^{1/3} \, (L_t \overline{U}_s)^{4/3} \tag{12}$$

in which

$$\phi(f) = f^{5/6} \left(1 - \frac{1}{K} \sqrt{f/8}\right)^{-1/3} \tag{13}$$

$$E = \overline{U} g S_e \tag{14}$$

In these equations

- E is the rate of energy dissipation per unit mass of fluid in a broad open channel
- f is the Darcy-Weisbach friction factor
- K is the von Karman turbulence constant and equals 0.4
- U is the mean flow velocity
- g is the acceleration of gravity
- S_e is the slope of the energy gradient
- $\overline{U}_s$ is the mean flow velocity at the water surface.

Elder's equation (1959), which was confirmed experimentally in a small flume with a smooth boundary for very small flow depths, is

$$K_z = 0.23\ D\sqrt{\tau_0/\rho} \tag{15}$$

in which

D is the mean depth of flow

$\sqrt{\tau_0/\rho} = \sqrt{gDS_e}$ is the shear velocity at the bed of the channel.

If values of the mean flow parameters determined in this experiment are used, equation 12 yields $K_z = 20.4$ square centimeters per second and equation 15 yields $K_z = 14.3$ square centimeters per second. Both of these values of K_z compare quite well with the value $K_z = 15.1$ square centimeters per second determined from the data in this experiment. Equation 12 is based on Kolmogoroff's similarity hypothesis for small-scale turbulence components whereas the extent of dispersion at large times is generally believed to be controlled mainly by the large-scale components. Due to this apparent contradiction there may be conceptual advantages to a relationship of the form of equation 15.

Application to Practical Problems

In order to predict the lateral diffusion pattern in a channel in which the mean flow conditions are known, the first and most important requirement is to estimate K_z by using a relationship such as equation 12 or equation 15. The second requirement is to estimate X_o (see figure 5), which defines the point at which an extension of the straight-line portion of the $\sigma_z^2(x)$ curve intersects the x axis. Next, by using $2K_z/\overline{U}_s$ as a slope and X_o as an intercept, σ_z^2 is established as a function of x which is valid for large distances from the source. If the Lagrangian correlation function is known, $\sigma_z^2(x)$ can be established for the range of values of x where K_z varies.

If the correlation function $R_{w'}(\alpha) = e^{-\frac{|\alpha|}{L_t}}$ is assumed then, $X_o = \sqrt{\overline{w'^2}}\, L_t$, K_z is constant for $x \gtrsim 6\ X_o$, and equations 10a and 10b may be used to predict the diffusion pattern near the source. Assuming that the numerical value of $K_z = \overline{w'^2}\ L_t$ is known, application of equations 10a and 10b requires knowledge of either L_t or $\sqrt{\overline{w'^2}}$. At present neither L_t nor $\sqrt{\overline{w'^2}}$ can be determined for alluvial channels, except by experiment. For predicting dispersion at very large distances from the source the error introduced by assuming $X_o = 0$ would generally be small.

CONCLUSIONS

A review of the theory of turbulent diffusion by continuous movements as applied to open channels coupled with the analysis of experimental diffusion data for one set of flow conditions in a laboratory alluvial channel gives rise to the following conclusions:

1. The statistical theory of diffusion by continuous movements in a homogeneous turbulence field appears to have useful application in the study of lateral turbulent diffusion in alluvial channels.

2. The scale and intensity of the lateral turbulence components at the surface of an alluvial channel with uniform flow can be estimated by analyzing observations of the lateral dispersion of small polyethylene particles floating on the surface.

3. The coefficient of lateral diffusion in an alluvial channel reaches a limiting constant value, in accordance with turbulent diffusion theory, as the distance from the source is increased. Indications are that the distance required for the diffusion coefficient to become contstant is quite small in comparison to the length scale of most diffusion phenomena in natural streams which are of practical interest.

4. The assumption that the Lagrangian velocity correlation function is exponential in form appears to be a useful first approximation in considering turbulent diffusion in an alluvial channel. This assumption permits an approximate prediction of the diffusion pattern near the source.

5. It appears that it will be possible to derive for alluvial channels a relationship between the limiting value of the lateral diffusion coefficient and mean flow, channel-geometry, and sediment parameters which is analogous to the semi-empirical relationships for rigid-boundary open channels derived by Orlob (1961) and Elder (1959).

REFERENCES

1. Baldwin, L. V. and Mickelsen, W. R., 1962, Turbulent diffusion and anemometer measurements: American Society of Civil Engineers, Proc. Paper No. 3095, Vol. 88, No. EM 2, p. 37-69.

2. Batchelor, G. K., 1949, Diffusion in a field of homogeneous turbulence, I. Eulerian analysis: Australian J. Sci. Research, Vol. 2, p. 437-450.

3. Elder, J. W., 1959, The dispersion of marked fluid in turbulent shear flow: Journal of Fluid Mechanics, Vol. 5, No. 4, p. 544-560.

4. Frenkiel, F. N., 1953, Turbulent diffusion: Mean concentration distribution in a flow field of homogeneous turbulence: Vol. III, Advances in Applied Mechanics, Academic Press, p. 61-106.

5. Hinze, J. O., 1959, Turbulence: An introduction to its mechanism and theory: McGraw-Hill Book Co., Inc., 586 pp.

6. Ippen, A. T. and Raichlen, F., 1957, Turbulence in civil engineering: Measurements in free surface streams: American Society of Civil Engineers, Proc. Paper No. 1392, Vol. 85, No. HY5, 27 pp.

7. Kalinske, A. A. and Pien, C. L., 1944, Eddy diffusion: Industrial and Engineering Chemistry, Vol. 36, p. 220-223.

8. Kampé de Fériet, J., 1939, Les fonctions aléatoires stationnaires et la théorie statistique de la turbulence homogène: Ann. Soc. Scien. de Bruxelles, Vol. 59, Series 1.

9. Orlob, G. T., 1959, Eddy diffusion in homogeneous turbulence: American Society of Civil Engineers, Proc. Paper No. 2950, Vol. 85, No. HY9, p. 75-101.

10. Orlob, G. T., 1961, Eddy diffusion in homogeneous turbulence: Discussion closure: American Society of Civil Engineers, Vol. 87, No. HY1, p. 137-143.

11. Scull, W. E. and Mickelsen, W. R., 1957, Flow and mixing processes in combustion chambers: Chapter 2 in NACA TR 1300, p. 32-82.

REFERENCES (Continued)

12. Simons, D. B., Richardson, E. V. and Albertson, ~~N.~~ M. L., 1961, Flume studies using medium sand (0.45 mm): U. S. Geological Survey Water Supply Paper 1498-A, 76 pp.

13. Taylor, G. I., 1921, Diffusion by continuous movements: Proc. London Math. Soc., A 20, p. 196-211.