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Power Spectrum, Structure Function, Vertical Wind Shear, and Turbulence in Troposphere and Stratosphere

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With 1 Figure

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Summary

It has been shown that a power law exists between vertical velocity differences $\overline{\Delta v}$ measured over a layer thickness Δh, of the form $\overline{\Delta v} = a_0 (\Delta h)^{a_1}$. This expression may be related directly to a structure function and to a power-spectrum function. Statistical derivations of the exponent a_1 from detailed vertical wind profile measurements show resemblance with the spectral functions obtained for conditions of clear air turbulence.

Zusammenfassung

Potenzspektrum, Strukturfunktion, vertikale Windscherung und Turbulenz in Troposphäre und Stratosphäre

In früheren Arbeiten wurde gezeigt, daß zwischen den Windgeschwindigkeitsdifferenzen $\overline{\Delta v}$, gemessen über einer Schichtdicke Δh, ein funktioneller Zusammenhang der Art $\overline{\Delta v} = a_0 (\Delta h)^{a_1}$ besteht. Dieser Ausdruck kann mit einer Strukturfunktion und einer Spektrum-Funktion in Einklang gebracht werden. Statistische Herleitungen des Exponenten a_1 aus detaillierten Messungen vertikaler Windprofile zeigen gute Übereinstimmung mit Spektrum-Funktionen, die unter Clear-Air-Turbulenz-Bedingungen erhalten wurden.

1. Introduction

Many investigations of clear air turbulence conducted in recent years have used power-spectrum analysis (see e. g. REITER and BURNS [14, 15]) since TUKEY [17] introduced this important mathematical tool. The utilization of structure functions has been advocated by TATARSKI [16]. They have, however, rarely been applied to turbulence problems in the free atmosphere. Not much attention has been paid, either, to the structure of wind shears, although close association between wind shears and turbulence exists. It will be shown in this article that knowledge of the structure of wind shears will provide a better understanding of turbulence spectra.

2. Statistical Wind Shear Relationships

ESSENWANGER [5] (see also [7]) demonstrated with the aid of detailed wind soundings that the vertical wind shear is related to the shear interval by a power law as follows:

$$\overline{\Delta v} = a_0 (\Delta h)^{a_1} \tag{1}$$

with

$$\sigma_{\Delta v} = c (\Delta h)^{a_1} + A_0 \tag{2}$$

and

$$\sigma_{\Delta v} = A_0 + A_1 \overline{\Delta v}. \tag{3}$$

In these equations $\overline{\Delta v}$ denotes the mean total vector shear, expressed in m/sec, Δh the shear increment, and $\sigma_{\Delta v}$ the standard deviation. a_0, c, A_0, and A_1 are coefficients or constants, and a_1 is the exponent of the relationship in question.

As *described* by ESSENWANGER [5], the exponent a_1 proved to be 0.5 for *mean* Δv, and 1/3 for *mean extreme* shears. The exponent of 0.5 has also been confirmed by ARMENDARIZ and RIDER [1], and by BELMONT and SHEN [2].

In a subsequent investigation ESSENWANGER [6] showed that the exponent is related to the vertical structure of the wind profile, and that for the mean vector wind shear the vertical persistence (i. e. the meso-structure) causes the exponent to be 0.5. Extreme values of shear appear independent of the bivariate distribution law for zonal and meridional wind shear, and thus the exponent becomes 1/3. Therefore, the conditions for these extreme values may closely resemble the structure of isotropic turbulence.

To gain a better understanding of the relationship between vertical wind structure and the exponent, the following experiment was performed. The wind profile as function of altitude was separated into stationary and non-stationary parts (see ESSENWANGER and BILLIONS [8]), and both parts were subjected individually to a wind shear analysis. The resulting distributions are shown in Fig. 1.

Whereas the smoothed (stationary) part rendered an exponent of 4/5 (Essenwanger and Billions [8]), the exponent was zero for the non-stationary part. According to expression (1) this would mean

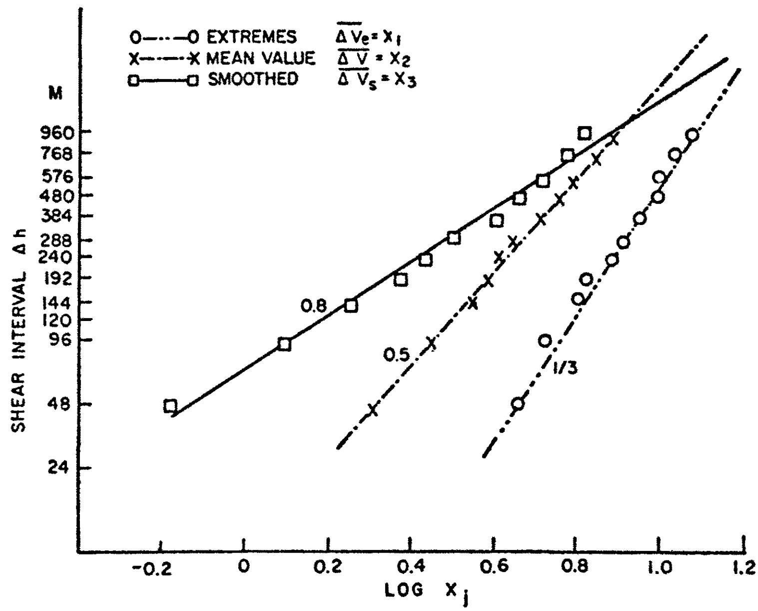

Fig. 1. Relationship Between the Shear Interval Δh and the Mean Shear Δv

that the non-stationary wind fluctuations were completely independent of the thickness of the shearing layer, and scattered about a constant mean value

$$\overline{\Delta v} = a_0. \tag{4}$$

3. Structure Function and Power Spectrum

Tatarski [16] defines the structure function for locally homogeneous and isotropic random fields by

$$D_f(r) = \overline{[f(\vec{r}+\vec{r}_1) - f(\vec{r}_1)]^2} \tag{5}$$

with

$$D_f(r) = \int_{-\infty}^{\infty} (1 - \cos k r)\, V_{(k)}\, dk \tag{6}$$

where $\vec{r}$ and $\vec{r}_1$ are position vectors, and $V_{(k)}$ is a one-dimensional spectral density function. If we consider the structure function

$$D_f(r) = c^2 r^p \text{ for } (0 < p < 2) \tag{7}$$

then the corresponding one-dimensional spectral density function results in

$$V_{(k)} = \frac{\Gamma(p+1)}{2\pi} \sin \frac{\pi p}{2} c^2 k^b \tag{8}$$

where

$$b = -(p+1) \tag{9}$$

and Γ represents the gamma function.

For $f \equiv v$, $\vec{r}_1 \equiv h$, and $\vec{r} \equiv \Delta h$ expression (5) assumes the role of a structure function of the velocity field, the latter given in terms of vertical wind profiles.

4. Structure Function, Turbulence, and Wind Shear

It has also been shown by TATARSKI [16] that a locally isotropic random distribution of the wind vector $\vec{v}(r)$ yields nine structure functions of the form

$$D_{ik}(\vec{r}) = \overline{[v_i(\vec{r}+\vec{r}_1) - v_i(\vec{r}_1)][v_k(\vec{r}+\vec{r}_1) - v_k(\vec{r}_1)]} \tag{10}$$

With local isotropy it follows that

$$D_{ik}(\vec{r}) = [D_{rr}(r) - D_{tt}(r)]\, n_i n_k + D_{tt}\, \delta_{ik} \tag{11}$$

where the δ_{ik} are Kronecker deltas ($\delta_{ik} = 1$ for $i = k$; $\delta_{ik} = 0$ for $i \neq k$); n_i are components of the unit vector along r_1 and

$$\begin{aligned} D_{rr}(\vec{r}) &= \overline{[v_r(\vec{r}+\vec{r}_1) - v_r(\vec{r}_1)]^2} \\ D_{tt}(\vec{r}) &= \overline{[v_t(\vec{r}+\vec{r}_1) - v_t(\vec{r}_1)]^2} \end{aligned} \tag{12}$$

are longitudinal and transverse structure functions. v_r are the velocity components parallel to the vector r_1, v_t the components normal to $\vec{r}_1$.

For incompressible fluids ($v \ll c$, where c is the velocity of sound) TATARSKI [16] derives

$$D_{tt}(\vec{r}) = \frac{1}{2r} \frac{d}{dr} (r^2 D_{rr}). \tag{13}$$

According to KOLMOGOROV [11] and OBUKHOV [12] turbulence in the (isotropic) inertial subrange follows the "two-thirds law"

$$D_{rr}(\vec{r}) = C(\varepsilon r)^{2/3} \text{ for } l_0 << r << L \tag{14}$$

where C is a dimensionless constant of the order of unity, ε is the rate of dissipation of kinetic energy, and l_0 and L are the inner and outer scales of turbulence. It follows from (13) that

$$D_{tt}(\vec{r}) = \frac{4}{3} C(\varepsilon r)^{2/3} \text{ for } l_0 << r << L. \tag{15}$$

A comparison between expression (1) and (12) shows that the vertical shear of the horizontal wind vector relates to the transverse structure function as

$$\overline{\Delta v} = \sqrt{D_{tt\,(\Delta h)}} \tag{16}$$

or, for the inertial subrange of turbulence

$$a_0 (\Delta h)^{a_1} = \sqrt{\frac{4}{3} C} \cdot \varepsilon^{1/3} (\Delta h)^{1/3}. \tag{17}$$

From this it follows that

$$a_1 = {}^1/_3 \text{ and } 2a_1 = p = {}^2/_3$$

which ESSENWANGER [5] found to hold for *mean extreme shears*. a_0 turns out to be a function of the rate of dissipation, ε. At the same time, because of (7) and (8), velocity spectra follow the well-known "$-5/3$ law"

$$V(k) \propto k^{-5/3}. \tag{18}$$

The exponent for mean shears was reported by ESSENWANGER to be $a_1 = 1/2$. From (16) and (7) it follows that $p = 1$, and from (9) that the "spectrum slope" should be $b = -2$. It has been mentioned earlier that these *mean shears* contain turbulence as well as meso-structure. It has been shown by HINES [10] and by WEINSTEIN et al. [14] that the meso-structure present in vertical wind profiles above the tropopause may be explained by the effect of internal gravity waves (inertial waves) having horizontal wavelengths of 10^2 to

10^3 km. PHILLIPS [13] considered the spectra of internal waves and arrived at the striking result that

$$V(k, \Delta h) \propto k^{-2} \tag{19}$$

for $K \cdot \Delta h \ll 1$, where Δh characterizes the thickness of the layer with strong stratification. This condition is certainly met with internal waves described by HINES [10]. Thus, PHILLIPS' theoretical derivations are in excellent agreement with data obtained from detailed vertical wind profiles.

The non-stationary wind profiles, found by ESSENWANGER and BILLIONS [9] to yield an exponent $a_1 = 0$ [see Eq. (4)], lead to $p = 0$ and hence a "white noise" spectrum with a slope $b = -1$.

The smoothed (stationary) part of vertical wind profiles, according to ESSENWANGER and BILLIONS [8] which contained the *persistent meso-scale* of atmospheric structure, yielded $a_1 = 4/5$. This results in $p = 8/5$ for the structure function, and $b = -13/5$ for the spectrum slope. This value lies between $b = -11/5$ derived by BOLGIANO [3, 4] for the "buoyant subrange", and $b = -3$ found by VINNICHENKO et al. [18] to hold for spectra measured in *horizontal* flight by aircraft at wave length along the flight path of $\lambda_x > 600$ to 800 m.

A *linear trend* in wind shears may be expressed by Eq. (1), with $a_1 = 1$. Such a trend will yield a spectrum slope of $b = -3$, if one extended the condition imposed upon Eq. (7) to $p = 2$. Hence, one may speculate to what extent VINNICHENKO's et al. [18] measurements were influenced by a linear trend.

5. Conclusion

Even though the comparisons between vertical wind shears, structure functions, and power spectra which have been presented in the foregoing discussion have been made for transverse velocity fluctuations $(\overline{\Delta v} \perp \overline{\Delta h})$ similar reasoning may be applied to longitudinal fluctuations $(\overline{\Delta v} \| \overline{\Delta x})$ as they may be obtained, for instance, from aircraft records [see Eq. (12) through (15)]. The same exponential relationship should hold as derived above, only constant coefficients [a_0 in Eq. (1)] should differ by a factor of 3/4 [see Eq. (15)]. One may, thus, avoid the computation of spectra from the Fourier transformation of autocorrelation functions by resorting to a simple treatment of vertical and/or horizontal wind shears as outlined by Eq. (1). The relationship between wind shears and power spectra, demonstrated by TATARSKI's [16] structure function, also simplifies the interpretation of turbulence data.

From the foregoing it appears that the relation between Δv and Δh is limited by "white noise" conditions ($a_1 = 0$) on the one hand, and by a linear trend without turbulence or mesostructure ($a_1 = 1$) on the other hand.

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