A MATHEMATICAL MODEL STUDY OF POOL 4 IN THE UPPER MISSISSIPPI AND LOWER CHIPPEWA RIVERS

Prepared for

United States Department of Interior
Fish and Wildlife Service
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Civil Engineering Department
Engineering Research Center
Colorado State University
Fort Collins, Colorado

D.B. Simons
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Chapter 1

INTRODUCTION

1.1 General

The Upper Mississippi River is part of the main riverine artery of the United States. Its exploitation both commercially and recreationally is an important aspect of the national economy. Therefore, the river must be protected and its efficiency maintained if it is to continue to be of major economic importance.

The primary purpose of this study is to construct a one-dimensional mathematical model of Pool 4 in the Upper Mississippi River below Lake Pepin and Chippewa River below Durand. An index map of the study reach is shown on Figure 1.

The mathematical model was developed by formulating the unsteady flow of sediment-laden water with the one-dimensional partial differential equations representing the conservation of mass for sediment, and the conservation of mass and momentum for sediment-laden water. The effects of locks and dams and the interactions between the Mississippi River and its main tributaries on the geomorphology of rivers and adjacent lands were considered in the modeling. The model can be used to study the impacts of the effects of different operational schemes for the locks and dams, the effects of the pools on the behavior and form of the tributary rivers, the impact of changes in the delivery of sediment and water to the study reach on the morphology of the river and adjacent lands, and the impacts of dredging and dredged material disposal on the hydraulic response and sedimentation patterns in the main channel.

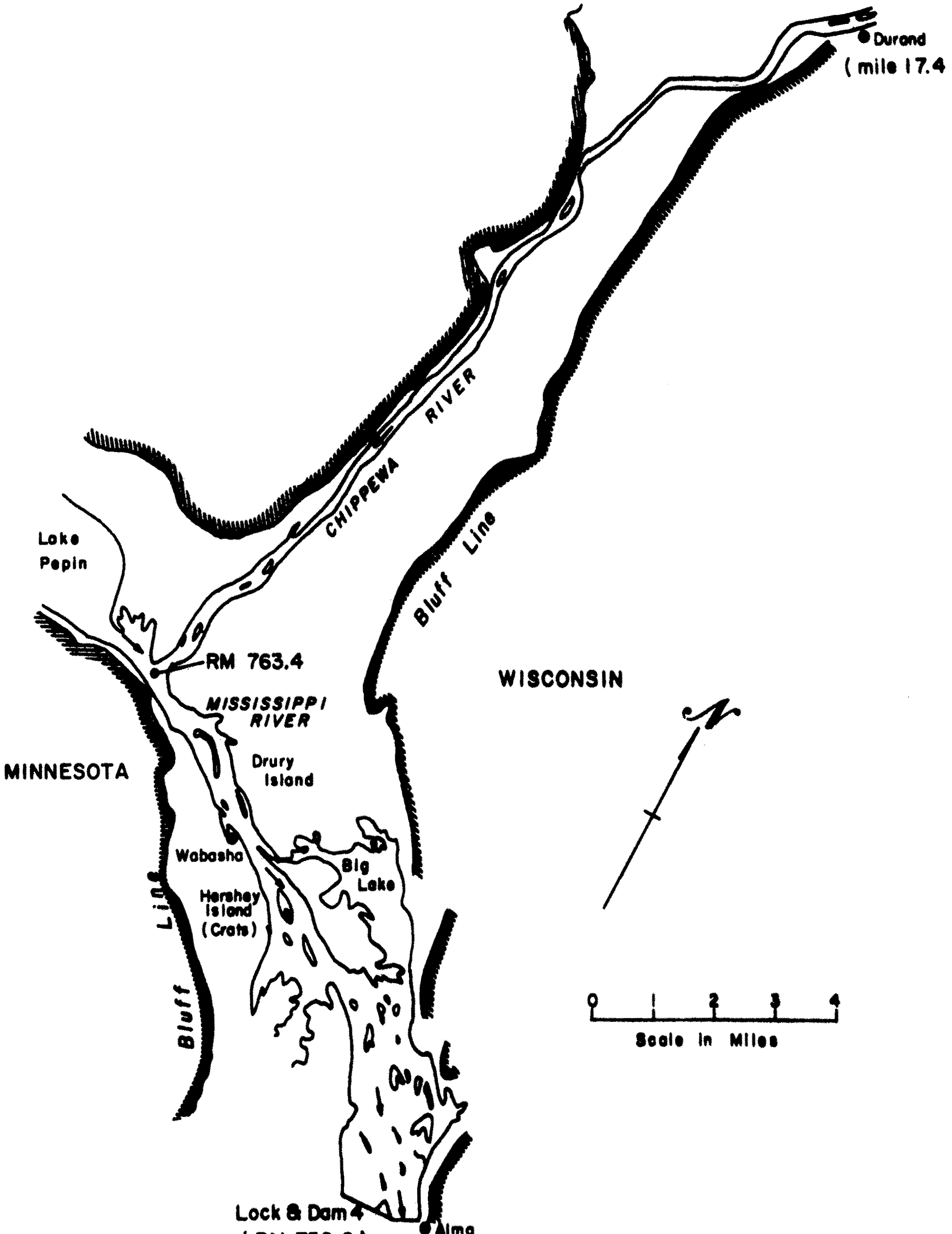


Figure 1. Index map of the study river reach

1.2 Organization of Report

In Chapter 2 the theoretical background of the mathematical model is described and the governing partial differential equations are formulated. The numerical analysis of these equations by a linear implicit method is outlined in Chapter 3. The calibration of the mathematical model and its operations are presented in Chapters 4 and 5 respectively. The results of this study are summarized in Chapter 6. The limitations of the model and its possible improvement are discussed in Chapter 7.

Chapter 2

THEORETICAL ANALYSIS

2.1 Basic equations

The one-dimensional differential equations of gradually varied unsteady flow in natural alluvial channels can be derived based on the following assumptions:

1. The channel is sufficiently straight and uniform in the reach so that the flow characteristics may be physically represented by a one-dimensional model.
2. Hydrostatic pressure prevails at every point in the channel.
3. The water surface slope is small.
4. The density of the sediment-laden water is constant over the cross section.
5. The resistance coefficient is assumed to be the same as that for steady flow in alluvial channels and can be approximated from resistance equations applicable to alluvial channels or from field data.

The three basic equations derived (Chen, 1973) are:

the sediment continuity equation

$$\frac{\partial Q_s}{\partial x} + p\,\frac{\partial A_d}{\partial t} + \frac{\partial AC_s}{\partial t} - q_s = 0 \tag{1}$$

the flow continuity equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + \frac{\partial A_d}{\partial t} - q_\ell = 0 \tag{2}$$

and the flow momentum equation

$$\frac{\partial \rho Q}{\partial t} + \frac{\partial \beta \rho Q V}{\partial x} + gA\,\frac{\partial \rho y}{\partial x} = \rho g A\,(S_o - S_f + D_\ell)$$

or

$$\frac{\partial \rho Q}{\partial t} + V \frac{\partial \beta \rho Q}{\partial x} + \beta \rho V \frac{\partial Q}{\partial x} - \beta \rho V^2 T \frac{\partial y}{\partial x} + gA \frac{\partial \rho y}{\partial x}$$

$$= \rho g A \ (S_o - S_f + D_\ell) + \beta \rho V^2 A_x^y \qquad (3)$$

where

x = horizontal distance along the channel

t = time

Q_s = sediment discharge

p = volume of sediment in a unit volume of bed layer given by ρ_b/ρ_s

ρ_b = bulk density of sediment forming the bed

ρ_s = density of sediment

A_d = volume of sediment deposited on channel bed per unit of length of channel, the value of which is negative when bed erosion occurs

A = water cross-sectional area

C_s = mean sediment concentration on a volume basis given by Q_s/Q

Q = flow discharge

q_s = lateral sediment flow per unit length of channel, a positive quantity indicates inflow and a negative value denotes outflow

q_w = lateral water flow per unit length of channel, a positive quantity indicates inflow and a negative value denotes outflow

q_ℓ = lateral flow per unit length of channel, given by $q_s + q_w$

ρ = density of sediment-laden water given by $\rho_w + C_s \ (\rho_s - \rho_w)$

ρ_w = density of water

β = momentum coefficient

V = mean flow velocity

T = $\partial A/\partial y$

y = flow depth

g = acceleration of gravity

S_o = bed slope

S_f = friction slope

D_ℓ = dynamic contribution of lateral discharge given by $q_\ell V_\ell/Ag$

V_ℓ = velocity component of lateral inflow in the main flow direction

A_x^y = departure from a prismatic channel given by $(\partial A/\partial x)_y$

h = water surface elevation

z = riverbed elevation

Δz = change in riverbed elevation

B = top width

Figure 2 is a definition sketch of an alluvial channel.

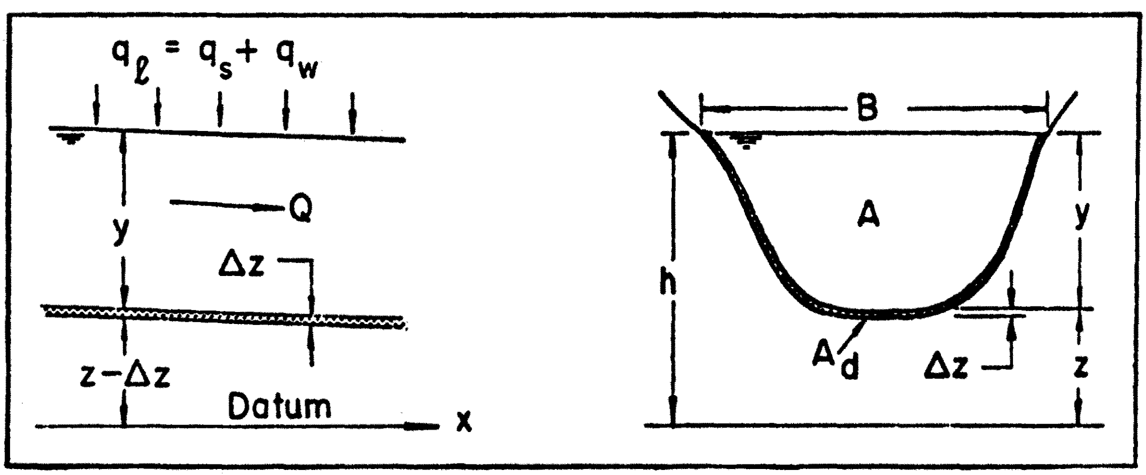


Figure 2. Definition sketch of an alluvial channel

The three equations contain three basic unknowns Q, y, and A_d. The other variables in the equations must be expressed as a function of the three unknowns in order to obtain a solution. These functions are given by the following supplementary equations which describe the physical properties of the prototype.

1. The geometric properties of cross sections are expressed as a function of stage from the known channel geometry.

2. The mean bed slope

$$S_o = -\partial z/\partial x \tag{4}$$

in which the initial bed elevation is known and its change is related to the variable A_d.

3. The friction slope S_f is a function of flow and channel characteristics. The resistance functions such as Manning's or Chezy's equations can be employed to relate S_f to the basic unknowns.

4. The lateral inflow q_ℓ consists of two components, $q_{\ell 1}$ and $q_{\ell 2}$, induced by natural and manmade activities, respectively. For overbank flow, the natural-induced lateral inflow is related to the change of water surface elevation Δh over a time period Δt

$$q_{\ell 1} = - \frac{A_f}{\Delta x \Delta t} \Delta h \tag{5}$$

where A_f = the surface area of the floodplain and Δx = length of the floodplain along the main channel. Equation 5 is formulated from the assumption that the transverse water surface (normal to the main flow direction) is horizontal and the amount of infiltration and evaporation is negligible. The lateral sediment inflow q_s has its natural and man-induced components, q_{s1} and q_{s2}, in which

$$q_{s1} = q_{\ell 1} C_b \tag{6}$$

and C_b = sediment concentration at or near the river bank.

5. The sediment discharge can be estimated from field surveys and/or from the available theories.

2.2 Lock and Dam Equations

To account for the effects of locks and dams, the following equations are utilized to simulate sediment-laden water flowing through the locks and dams:

$$Q_{s_{NL}} = Q_{s_{NL+1}} \tag{7}$$

$$Q_{NL} = Q_{NL+1} \tag{8}$$

$$Q = CW\,a\,\sqrt{2g(h_{NL} - h_{NL+1})} \tag{9}$$

where

C = gate discharge coefficient

a = the height of the gate opening

W = the width of the gate

h = the stage (water surface elevation)

NL and NL+1 = the sections immediately above and below the lock and dam respectively.

2.3 Confluence Equations

The interaction between the Upper Mississippi River and its tributaries can be simulated by the following continuity and energy equations:

$$Q_{NC+1} = Q_{NC} + Q_N \tag{10}$$

$$Q_{s_{NC+1}} = Q_{s_{NC}} + Q_{s_N} \tag{11}$$

$$z_{NC} + y_{NC} + \frac{V^2_{NC}}{2g} = z_{NC+1} + y_{NC+1} + \alpha_{NC}\frac{V^2_{NC+1}}{2g} + h_{f_{NC}} \tag{12}$$

$$z_N + y_N + \frac{V^2_N}{2g} = z_{NC+1} + y_{NC+1} + \alpha_N\frac{V^2_{NC+1}}{2g} + h_{f_N} \tag{13}$$

where

α = the correction factor for energy loss, the value of which is assumed equal to 1 in the model.

h_f = the energy head loss given by S_f x

NC, NC+1 and N = the sections in the Mississippi River immediately above and below the confluence and the section at the mouth of its tributary, respectively.

Equations 1 through 13 govern the flow and sediment movement in the study reach. Changes in flow and channel characteristics can be assessed from the solution of these equations. Because of the non-linearity of these equations, the only feasible method of solution is by numerical methods.

Chapter 3

NUMERICAL ANALYSIS

3.1 Finite-Difference Approximations

Equations 1 through 3 and 7 through 13 can be solved by a linear-implicit method using a digital computer. The finite-difference approximations employed to express the values and the partial derivatives of a function f within a four-point grid (Fig. 3) formed by the intersections of the spacelines x_i and x_{i+1} with the time lines t^j and t^{j+1} are given by

$$f \approx \frac{1}{2}(f_i^j + f_{i+1}^j) \tag{14}$$

$$\frac{\partial f}{\partial x} \approx \frac{1}{\Delta x}(f_{i+1}^{j+1} - f_i^{j+1}) \tag{15}$$

and

$$\frac{\partial f}{\partial t} \approx \frac{1}{2\Delta t}[(f_i^{j+1} - f_i^j) + (f_{i+1}^{j+1} - f_{i+1}^j)] \tag{16}$$

in which f represents Q, A, y, etc. This finite-difference scheme (Chen, 1973) was obtained by linearlizing and modifying the four-point implicit method of Amein and Fang (1970), which was found to be the most efficient method for flood routing problems (Price, 1974). In this adopted numerical scheme, the spatial derivative was approximated by the fully implicit equation 15. This formulation achieved better stability of computation than a central-implicit scheme. The central time difference defined by Eq. 16 usually gave more accurate solutions than a forward or backward finite-difference approximation. A more detailed discussion of various numerical methods is given by Chen (1973), Price (1974), and Liggett and Cunge (1975).

All the variables are known at all nodes of the network on the time line t^j. The unknown values of the variables on the time line t^{j+1} can be found by solving the system of linear algebraic equations formulated by substitution of the finite-difference approximations 14, 15, and 16 into Eqs. 1 through 3 and 7 through 13. The schematic diagram shown in Figure 4 explains the solution scheme.

3.2 Formulation of Finite-Difference Equations

As Eqs. 14, 15, and 16 are substituted into Eqs. 1, 2, and 3, or 7, 8, and 9 by assuming $\partial A_d/\partial z = \partial A/\partial y = T$ and employing the first order Taylor series expansion (e.g., $Q_s^{j+1} \simeq Q_s^j + (\partial Q_s/\partial Q)^j (Q^{j+1} - Q^j) + (\partial Q_s/\partial y)^j (y^{j+1} - y^j) + (\partial Q_s/\partial z)^j (z^{j+1} - z^j)$), three linear algebraic equations are formed. They can be written as

$$K_{k1} Q_i + K_{k2} y_i + K_{k3} z_i + K_{k4} Q_{i+1} + K_{k5} y_{i+1} + K_{k6} z_{i+1} = E_k \qquad (17)$$

$$K_{m1} Q_i + K_{m2} y_i + K_{m3} z_i + K_{m4} Q_{i+1} + K_{m5} y_{i+1} + K_{m6} z_{i+1} = E_m \qquad (18)$$

$$K_{n1} Q_i + K_{n2} y_i + K_{n3} z_i + K_{n4} Q_{i+1} + K_{n5} y_{i+1} + K_{n6} z_{i+1} = E_n \qquad (19)$$

where $k = 3i$, $m = 3i+1$, and $n = 3i+2$ when applied to the grid formed by sections i and $i+1$. The coefficients, K and E, are functions of variables evaluated at the time step t^j and therefore are known. To ensure the stability of the scheme, the friction slope S_f is taken on the time line t^{j+1} by employing a first order Taylor series expansion.

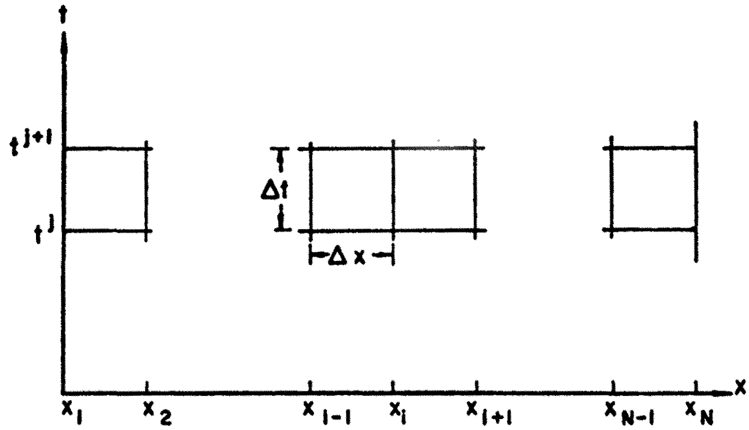


Figure 3. Network for the implicit method

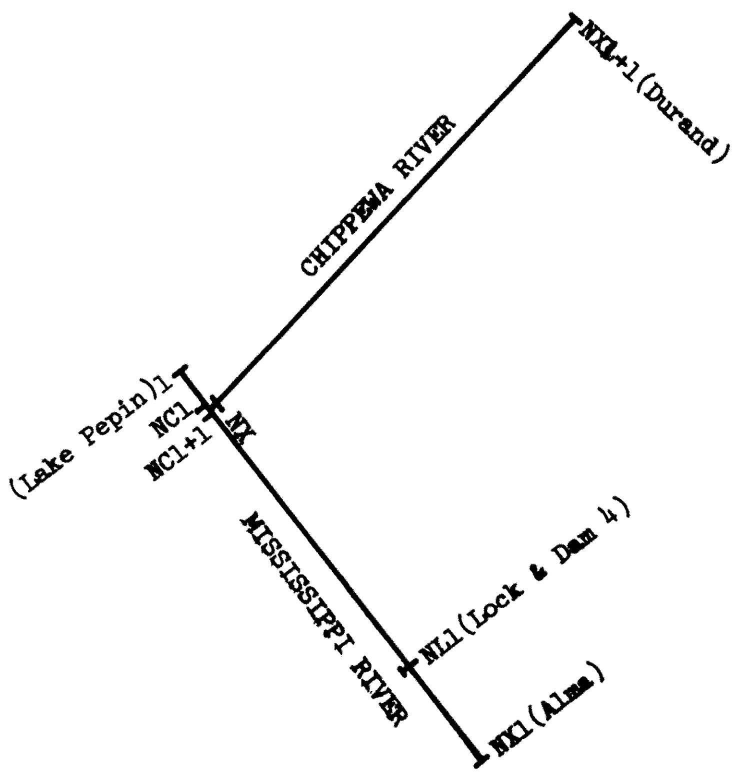


Figure 4. Schematic diagram of the study river reach

There are six unknowns in Eqs. 17, 18, and 19 at the time step t^{j+1} which cause the system to be indeterminate. However three unknowns are common for any two neighboring rectangular grids. Consequantly, there are

$$[(NC1-1) + (NX1-NC1-1) + (NX-NX1-1)$$

or

$$(NX - 3)$$

sets of three equations containing 3(NX) unknowns. Nine additional equations supplied by four upstream boundary conditions (one sediment and one flow discharge hydrograph at the upstream section of the Mississippi River and the Chippewa River, respectively), one downstream boundary condition (a stage-discharge relationship at Sec. NX1), and four confluence equations for the confluence of the Mississippi and Chippewa Rivers make this system of equations mathematically determinable. All nine equations can be expressed in the form of the following linear algebraic equations.

1. The flow discharge

   $$Q_i = f(t)$$

   can be written as

   $$K_{m4} Q_i + K_{m5} y_i + K_{m6} z_i = E_m \tag{20}$$

   in which $K_{m4} = 1$, $K_{m5} = K_{m6} = 0$, $E_m = f(t)$. The subscript i denotes the upstream boundary sections 1 or (NX1+1), and $m = 3(i-1)+1$. Figure 4 shows the schematic locations of these sections.

2. The sediment discharge hydrograph

   $$Q_{si} = f_2(t)$$

   can be approximated by

$$f_2(t^{j+1}) = f_2(t^j) + (\frac{\partial Q_s}{\partial Q})^j_i \quad (Q_i^{j+1} - Q_i^j) + (\frac{\partial Q_s}{\partial y})^j_i \quad (y_i^{j+1} - y_i^j)$$

$$+ (\frac{\partial Q_s}{\partial z})^j_i \quad (z_i^{j+1} - z_i^j)$$

or rearranged as

$$K_{n4}\ Q_i + K_{n5}\ y_i + K_{n6}\ z_i = E_n \tag{21}$$

in which the subscript $i = 1$ or $(NX1+1)$, and $n = 3(i-1)+2$.

3. The rating curve

$$Q_{NX1} = f_3(h_{NX1})$$

can be approximated by

$$Q_{NX1}^{j+1} = Q_{NX1}^j + (\frac{\partial f_3}{\partial y})^j_{NX1}(y_{NX1}^{j+1} - y_{NX1}^j) + (\frac{\partial f_3}{\partial z})^j_{NX1}(z_{NX1}^{j+1} - z_{NX1}^j)$$

or rearranged as

$$K_{3(NX1),1}Q_{NX1} + K_{3(NX1),2}y_{NX1} + K_{3(NX1),3}z_{NX1}$$

$$= \quad E_{3(NX1)} \tag{22}$$

4. The confluence equations 10 to 13 can be linearized as

$$P_{28}\ Q_{NC1+1} + P_{29}\ Q_{NC1} + P_{30}\ Q_{NX} = P_1 \tag{23}$$

$$P_{31}\ Q_{NC1+1} + P_{32}\ y_{NC1+1} + P_{33}\ z_{NC1+1} + P_{34}\ Q_{NC1} + P_{35}\ y_{NC1}$$

$$+ P_{36}\ z_{NC1} + P_{37}\ Q_{NX} + P_{38}\ y_{NX} + P_{39}\ z_{NX} = P_2 \tag{24}$$

$$P_{14}\ Q_{NC1} + P_{15}\ y_{NC1} + P_{16}\ z_{NC1} + P_{17}\ Q_{NC1+1} + P_{18}\ y_{NC1+1}$$

$$+ P_{19}\ z_{NC1+1} = P_{20} \tag{25}$$

$$P_{21}\ Q_{NX} + P_{22}\ y_{NX} + P_{23}\ z_{NX} + P_{24}\ Q_{NC1+1} + P_{25}\ y_{NC1+1}$$

$$+ P_{26}\ z_{NC1+1} = P_{27} \tag{26}$$

in which the coefficient P is a function of known variables.

3.3 Solution of Finite-Difference Equations

Equations 17 through 26 constitute a system of 3(NX) linear algebraic equations in 3(NX) unknowns. Any of the standard methods, such as the Gaussian elimination method or the matrix inversion method, can be used for its solution. A double-sweep method is applied here for solving this system of linear equations (Chen, 1973). This method offers two advantages. First, the computations do not involve any of the many zero elements in the coefficient matrix, which saves considerable computing time. Second, the required computer core storage is reduced significantly from that required from a 3(NX) x 3(NX) matrix to that required for a 3(NX) x 6 matrix, a desirable feature of the matrix solution technique when the matrix is large and the computer storage capacity is limited.

The principles of the double-sweep method can be explained by the following example. Consider a river reach being divided into three sections and the linear equations derived are

$$K_{1,4}Q_1 + K_{1,5}y_1 + K_{1,6}z_1 = E_1 \tag{27}$$

$$K_{2,4}Q_1 + K_{2,5}y_1 + K_{2,6}z_1 = E_2 \tag{28}$$

$$K_{3,1}Q_1 + K_{3,2}y_1 + K_{3,3}z_1 + K_{3,4}Q_2 + K_{3,5}y_2 + K_{3,6}z_2 = E_3 \tag{29}$$

$$K_{4,1}Q_1 + K_{4,2}y_1 + K_{4,3}z_1 + K_{4,4}Q_2 + K_{4,5}y_2 + K_{4,6}z_2 = E_4 \tag{30}$$

$$K_{5,1}Q_1 + K_{5,2}y_1 + K_{5,3}z_1 + K_{5,4}Q_2 + K_{5,5}y_2 + K_{5,6}z_2 = E_5 \tag{31}$$

$$K_{6,1}Q_2 + K_{6,2}y_2 + K_{6,3}z_2 + K_{6,4}Q_3 + K_{6,5}y_3 + K_{6,6}z_3 = E_6 \tag{32}$$

$$K_{7,1}Q_2 + K_{7,2}y_2 + K_{7,3}z_2 + K_{7,4}Q_3 + K_{7,5}y_3 + K_{7,6}z_3 = E_7 \quad (33)$$

$$K_{8,1}Q_2 + K_{8,2}y_2 + K_{8,3}z_2 + K_{8,4}Q_3 + K_{8,5}y_3 + K_{8,6}z_3 = E_8 \quad (34)$$

$$K_{9,1}Q_3 + K_{9,2}y_3 + K_{9,3}z_3 = E_9 \quad (35)$$

Equations 27 and 28, Eqs. 29 to 34, and Eq. 35 have the form of the upstream boundary equations 20 and 21, of the interior equations 17 to 19, and of the downstream boundary equation 22, respectively.

Equations 27 and 28 with 3 unknowns can be reduced to

$$Q_1 = L_{1,2} + L_{1,3}\, z_1 \quad (36)$$

and

$$y_1 = L_{2,2} + L_{2,3}z_1 \quad (37)$$

where the coefficient L is a function of K and E. Substituting Eqs. 36 and 37 into the first three interior equations, 29 to 31, yields

$$L_{3,3}z_1 + L_{3,4}Q_2 + L_{3,5}y_2 + L_{3,6}z_2 = M_3 \quad (38)$$

$$Q_2 = L_{4,2} + L_{4,3}z_2 \quad (39)$$

and

$$y_2 = L_{5,2} + L_{5,3}z_2 \quad (40)$$

Equations 27 to 31 are reduced to Eqs. 36 to 40. The same procedure can be repeated to reduce the next three interior equations, 32 to 34, by substituting Eqs. 39 and 40 into them yielding

$$L_{6,3}z_2 + L_{6,4}Q_3 + L_{6,5}y_3 + L_{6,6}z_3 = M_6 \quad (41)$$

$$Q_3 = L_{7,2} + L_{7,3}z_3 \quad (42)$$

and

$$y_3 = L_{8,2} + L_{8,3}z_3 \tag{43}$$

The coefficients L and M in Eqs. 36 to 43 can be computed by recurrence equations and therefore can be easily programmed. The procedure of using the recurrence equations to compute the values of the coefficients in Eqs. 36 to 43 is called the "forward-sweep." √

Equations 42 and 43 derived from the forward-sweep can be combined with Eq. 35 to form a set of 3 equations in 3 unknowns. The values of Q_3, y_3, and z_3 can be easily determined. Thereafter, the values of z_2, Q_2, y_2, z_1, Q_1, and y_1 can be determined backward from Eqs. 41 to 36. The recurrence equations can be easily formulated for programming. This procedure of using the recurrence equations to compute the values of unknowns is called the "backward-sweep." The whole procedure is designated as the "double-sweep" method. The method can be extended to solve a set of linear equations formulated for any number of sections in a channel reach.

The double-sweep method is used to solve the set of linear equations formulated in the study river reach. The forward-sweep is started from Section 1 to NC1 and from Section (NX1+1) to NX (Fig. 4). This results in four equations in the form of Eqs. 42 and 43. With the aid of the four confluence equations 23 through 26, a set of eight linear equations in nine unknowns is formed at the confluence. Two equations in the form of Eqs. 42 and 43 can then be derived for Section (NC1+1). The forward-sweep can be extended across the confluence to the Chippewa River to Section (NC1+1) and then continued to reach the downstream boundary section at NX1. By solving the resulting two equations (containing unknowns only at Section NX1) from the forward-sweep and the downstream boundary equation, the unknowns Q_{NX1}^{j+1}, y_{NX1}^{j+1}, and z_{NX1}^{j+1} can be computed. The unknown variables at the other sections can then be solved by the backward-sweep. After the flow condition at each node section on the time line t^{j+1} is computed, the computation is moved to the next time step. A flow chart is given in Figure 5 to show the principal programming steps.

The change of sediment area over a time step from t^j to t^{j+1} is given by

$$\Delta A_d = T(z^{j+1} - z^j) \tag{44}$$

where ΔA_d is assumed to be uniformly distributed over the channel width when formulating the finite-difference equations 17 through 26. However, to directly solve for A_d, the finite-difference equations may be derived in terms of Q, y, and A_d. A certain distribution of ΔA_d can then be assumed from theoretical or empirical information.

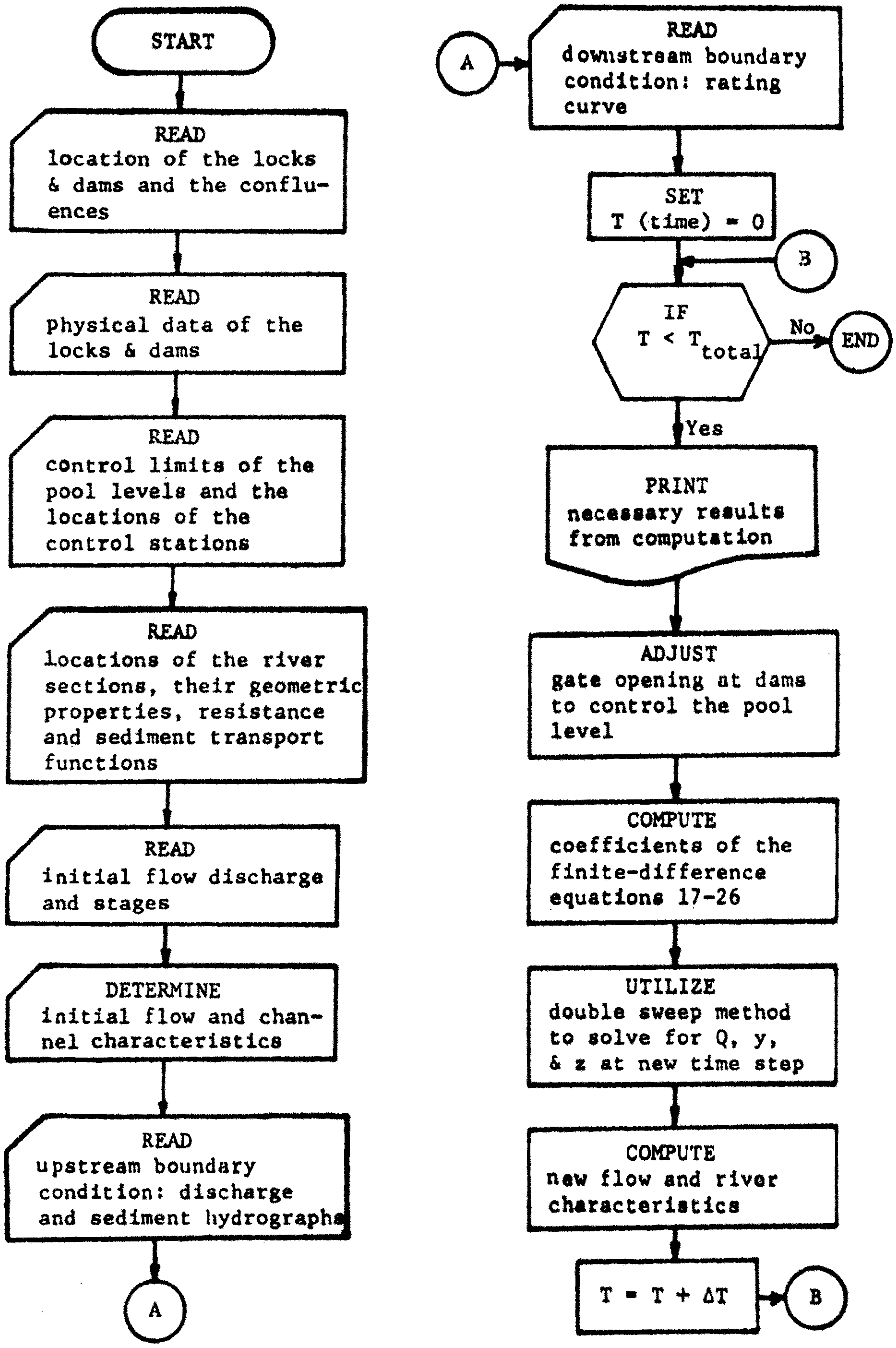


Figure 5. Flow chart of the mathematical model

Chapter 4

CONSTRUCTION AND CALIBRATION OF THE MATHEMATICAL MODEL

4.1 General

The construction of a mathematical model involves evaluating supplementary relations to basic equations (including relations on geometric properties, riverbed and friction slopes, lateral flows, and sediment discharges) from the field data and/or theories. The mathematical model is then calibrated by modifying these supplementary relations (excluding the geometric relations) such that the mathematical model would reproduce the historical response of the modeled river system. This is similar to the construction and calibration of a physical model.

Hydrographic maps of the modeled river reach, hydrographs of stage, flow, and sediment discharge data, and geological and physical properties of the bed and bed material are needed to perform the construction and calibration of the mathematical model. From this, the geometric properties of the river reach and the relations for S_f, Q_s, q_ℓ, and V_ℓ can be evaluated. If part of the data is not available, relations based on experimental, empirical, or theoretical approaches can be used. The resistance function for S_f and the sediment transport function for Q_s must be tested and modified to accomplish the model calibration until the historical data along the river reach can be reproduced by the mathematical model.

4.2 Data Availability and Model Construction

The mathematical model of the river system being studied includes the Upper Mississippi River from below Lake Pepin (RM 764.57)* to below Lock and Dam 4 (RM 748.5) and the Lower Chippewa River from Durand (Mile 17.4) to the mouth of the Chippewa River (Fig. 1). The river reach was divided into 71 sections with space increments ranging from 0.04 miles in the Chippewa River near the mouth to 2.0 miles below Lock and Dam 4. A list of the river sections is given in Table 1. The available field data that were used to construct and calibrate the mathematical model include:

1. The 1929 to 1973 discharge hydrographs of the Mississippi River at Prescott, Wisconsin (RM 811.4) and of the Chippewa River at Durand, Wisconsin (Mile 17.4); the 1965, 1966, 1971, 1974, and 1975 discharge hydrographs of the Mississippi River at Lock and Dam 4 (RM 753.8); and the 1965, 1966, and 1971 discharge hydrographs of the Cannon River at Welch, Minnesota (U.S. Geological Survey 1929-1960, 1961-1973; U.S. Army Engineer District, St. Paul, 1965, 1966, 1971, 1974, and 1975).

2. The 1965, 1966, 1971, 1974, and 1975 stage hydrographs of the Mississippi River at Wabasha, Minnesota (RM 760.4), at above Lock and Dam 4 (RM 752.8) and at Alma, Wisconsin (RM 748.5); and of the Chippewa River at Durand (U.S. Army Engineer District, St. Paul, 1965, 1966, 1971, 1974, 1975).

3. The 1975 hydrographic survey maps of the modeled river reach (U.S. Corps of Engineers, St. Paul, 1975; U.S. Fish and Wildlife Surface, Twin Cities, 1975).

4. The 1965 and 1966 dredging survey maps of the Chippewa River near the mouth (U.S. Corps of Engineers, St. Paul, 1965, 1966).

5. The 1974 and 1975 dredging survey maps of the Mississippi River near Reads Landing (RM 762.7), above Crats Island (RM 759.0) and above Teepeeota Point (RM 757.5) (U.S. Corps of Engineers, St. Paul, 1974, 1975).

*River miles of the Mississippi River above the mouth of the Ohio River.

Table 1

River Sections in the Mathematical Model

Section	Location (river miles)	Remarks*
MI1	764.57	Upstream boundary (MI)
MI2	764.11	
MI3	763.68	Chippewa River enters at RM 763.4
MI4	763.06	
MI5	762.90	
MI6	762.82	
MI7	762.74	
MI8	762.66	
MI9	762.56	
MI10	762.29	
MI11	761.51	
MI12	761.00	
MI13	760.42	Pool 4 control point
MI14	759.92	
MI15	759.59	
MI16	759.44	
MI17	759.32	
MI18	759.23	
MI19	759.14	
MI20	759.05	
MI21	758.96	
MI22	758.66	
MI23	758.22	
MI24	757.90	
MI25	757.63	
MI26	757.43	
MI27	757.14	
MI28	756.77	
MI29	756.43	
MI30	756.02	
MI31	755.50	
MI32	755.28	
MI33	755.00	
MI34	754.72	
MI35	754.23	

*In Fig. 4, NL1 = 37 and NC1 = 3. MI denotes the river section in the Mississippi River.

Table 1 (continued)

Section	Location (river miles)	Remarks**
MI36	753.67	
MI37	753.04	Lock and Dam 4 (upper)
MI38	752.50	Lock and Dam 4 (low)
MI39	750.50	
MI40	748.50	Downstream boundary (MI)
CH41	17.40	Upper boundary at Durand (CH)
CH42	16.50	
CH43	15.60	
CH44	14.60	
CH45	14.00	
CH46	13.40	
CH47	12.40	
CH48	11.40	
CH49	10.40	
CH50	9.90	
CH51	9.20	
CH52	8.60	
CH53	7.90	
CH54	7.40	
CH55	6.30	
CH56	5.60	
CH57	4.90	
CH58	4.40	
CH59	3.88	
CH60	3.37	
CH61	2.76	
CH62	2.12	
CH63	1.79	
CH64	1.48	
CH65	1.06	
CH66	0.71	
CH67	0.67	
CH68	0.62	
CH69	0.47	
CH70	0.37	
CH71	0.14	Downstream boundary (CH)

**CH denotes the river sections in the Chippewa River.

6. The sediment data in the Upper Mississippi River and in the Chippewa River (Lane, 1938; Upper Mississippi River Basin Coordinating Committee, 1972; U.S. Army Engineer District, St. Paul, 1974).

7. The physical data and the regulation method of Dam 4 is given in Table 2 (U.S. Army Engineer District, St. Paul, 1969).

Table 2

Physical Data and Operation Method of Dam 4

General	Dam 4
Location--river mile	752.8
Normal upper pool elevation (ft, above 1912 adjusted mean sea level)	667.0
Normal lower pool elevation	660.0
Dam	
Length of movable section (clear opening, in ft)	1,130
Tainter gates	22 @ 35 x 15 ft
Roller gates	6 @ 60 x 20 ft
Elevation of gate sills (ft):	
Tainter gates	652.0
Roller gates	647.0
Operation	
Control point (river mile)	Wabasha (760.4)
Control elevation (ft)	666.5 - 667.0
Flow at beginning of drawdown (cfs)	19,000
Flow at open river (cfs)	89,000

Using this information, the following supplementary relations were evaluated at all 71 sections in the modeled river reach:

1. The geometric properties of the river sections including cross-sectional areas, top widths, bed elevation, and floodplain surface area were estimated from the hydrographic

maps. The cross-sectional area and the top width were expressed as a function of stage to obtain geometric similiarity between the model and the prototype. The floodplain surface area was assumed not to vary with stage.

2. The Manning equation was employed to relate the friction slope to the flow and channel characteristics. The Manning roughness coefficients were determined from the steady water surface profiles for given discharges, where the stage-discharge relationships were assessed from the stage and the discharge hydrographs measured or computed at the gaging stations in the river reach being studied. The Manning roughness coefficients were expressed as functions of stage. These functions were modified during the model calibration to obtain kinematic and dynamic similarities. In general, the Manning coefficients decrease with increase in stage. Their values vary from 0.040 with low flow to 0.015 with high flow. A typical relation is shown on Fig. 6.

3. Sediment discharge is related to the flow and channel characteristics by a sediment transport dunction. By fitting the available data and applying the Toffalleti's method (Toffalleti, 1969) for bed material of 0.65 mm, the following relations were established:

$$C_s = K\ V^{3.2} D^{-1} \tag{45}$$

$$Q_W = 3.5 \times 10^{-13} Q^{2.8} \tag{46}$$

where

C_s = mean concentration of bed-material load on a volume basis, obtained by fitting the values of velocities versus the calculated bed material discharges from Toffalleti's method (e.g., Fig. 7),

K = empirical coefficient varied from 0.000001 to 0.000008,

V = flow velocity in fps,

D = hydraulic depth in ft, and

Q_W = discharge of wash load in cfs obtained from an empirical equation as shown in Fig. 8.

Equation 45 obtained an estimate of about 500,000 tons per year of sediment load in the Lower Chippewa River. This agrees with that stated in the Environmental Impact Statement (U.S. Army Engineer District, St. Paul, 1974). U.S. Geological Survey is collecting sediment data in the Chippewa River. These new data will be used to test Yang's Unit Stream Power Equation (Yang, 1973, and Yang and Stall, 1976) and to update this mathematical model. If Yang's method is proven to be satisfactory, his method will be adopted in the mathematical model.

4. For overbank flow, the natural-induced lateral flow $q_{\ell 1}$ was assessed from Eq. 5. On the rising limb of the hydrograph, the water carries sediment to the floodplain, depositing its

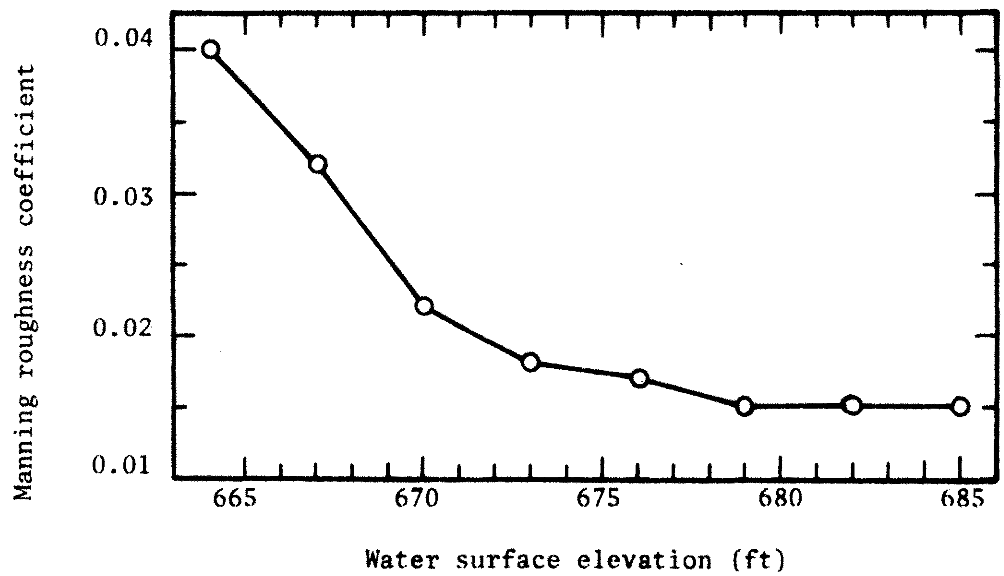


Figure 6. A typical relation between Manning roughness coefficient and stage

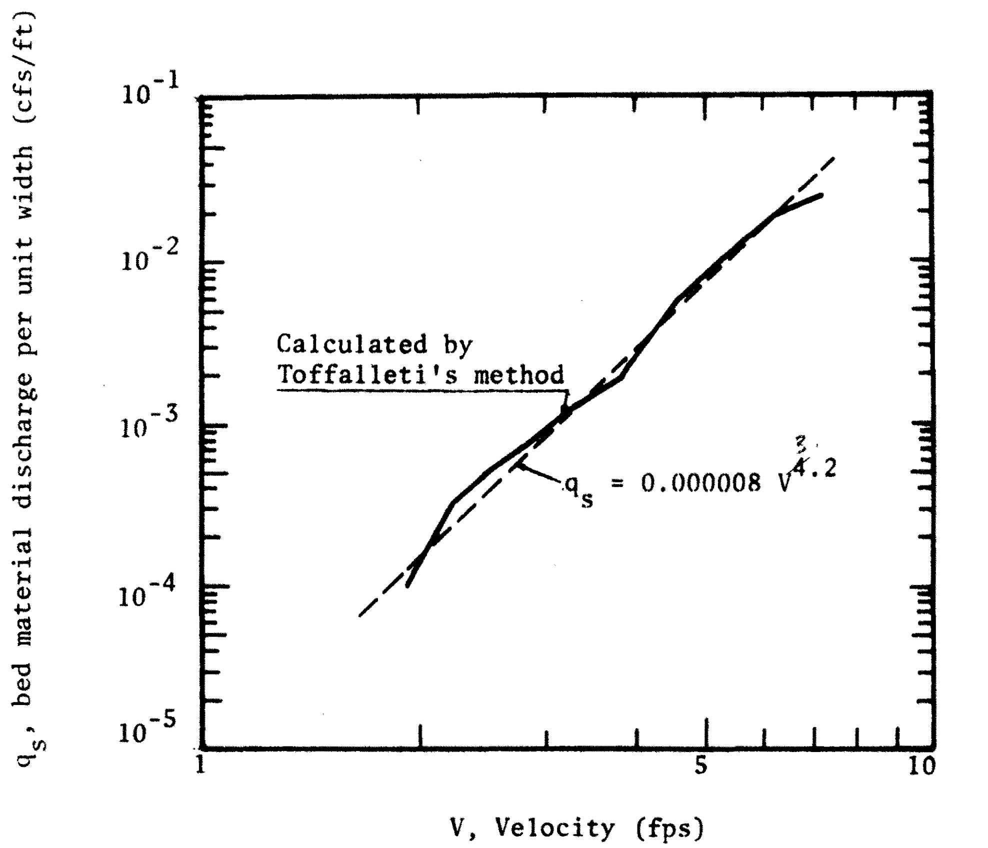


Figure 7. A typical relation between bed-material discharge and flow velocity

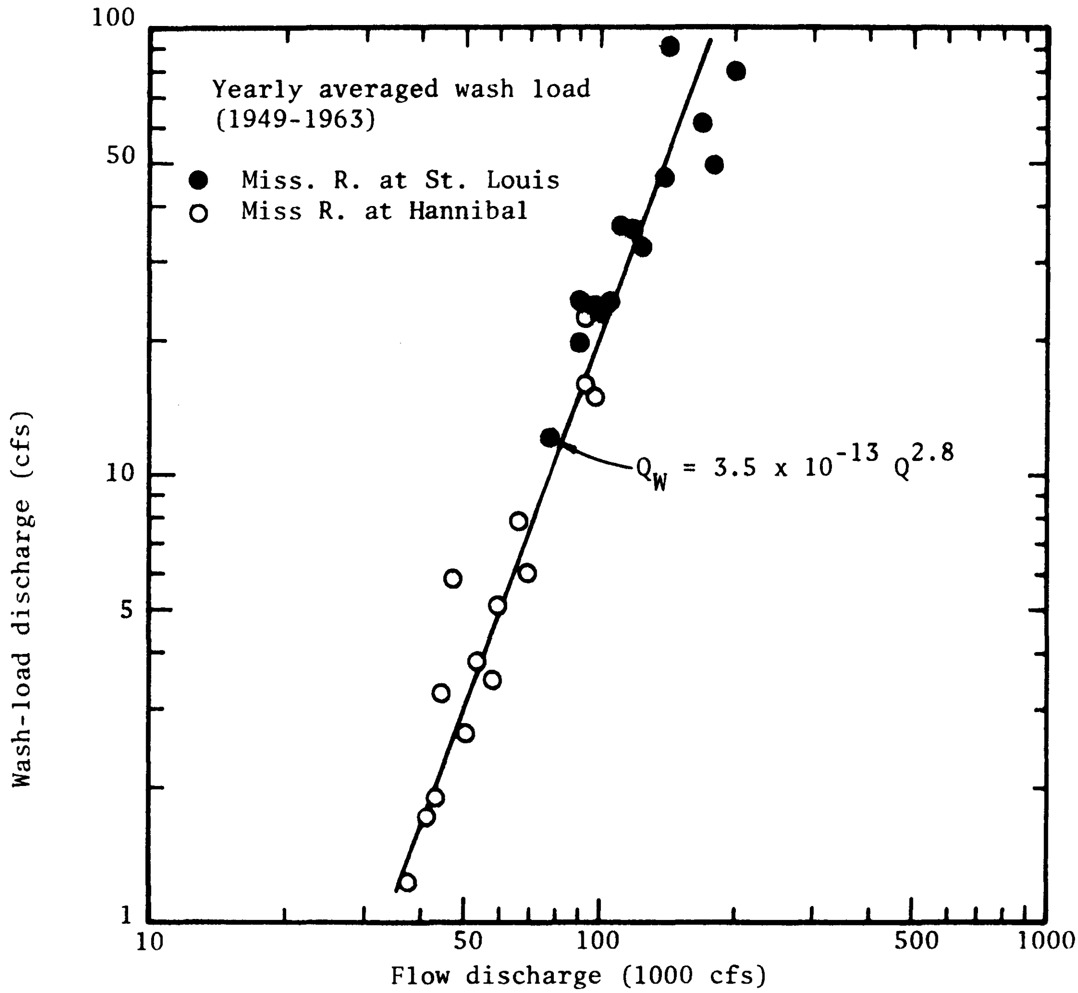


Figure 8. Relation between wash-load discharge and flow discharge (data obtained from "Summary and Analysis of Sediment Records in Relation to St. Louis Harbor Sediment Problem," by P. R. Jordan, U.S.G.S., Open-File Report, 1968)

coarse material along the river bank to raise the heights of natural levees. A triangular shape of natural levee was assumed with bottom angles 36 (face to the main channel) and 2 degrees. In this case, the quantity $q_{\ell 1}$ was negative and the lateral sand flow q_{s1} was determined from Eq. 6. The sediment concentration at or near the river bank was assumed equal to the mean concentration calculated from Eq. 45. During the falling limb, the water returns to the main channel carrying a negligible amount of sand, $q_{s1} \simeq 0$. Thus, the increase in the height of the natural levee over a period of time, $\Sigma\Delta t$, is

$$\Delta z_f = \left\{ \frac{-2\Sigma(q_{s1}\ \Delta t)}{p(\cot 2^\circ + \cot 36^\circ)} \right\}^{1/2} \tag{47}$$

Landward from the nautral levees, the deposition of sediment (mainly silt and clay) on the floodplain was assessed by

$$\Delta z_w = - \frac{(\Sigma q_{\ell 1}\ C_w\ \Delta t)\Delta x}{p\ A_f} \tag{48}$$

where C_w was calculated from Eq. 46 by Q_w/Q for $q_{\ell 1} < 0$, and was assumed to equal zero when $q_{\ell 1} > 0$.

5. The gate discharge coefficient C in Eq. 9 was evaluated based on the design charts for tainter gates prepared by the U.S. Army Engineers Waterways Experiment Station (U.S. Army Corps of Engineers, 1959). The relation for determining C is

$$C = C_t(h_s/a) \tag{49}$$

where h_s is the tailwater depth over gate sill in ft and C_t is defined by

$$C_t = k_1(h_s/a)^{k_2} \tag{50}$$

in which

a. For $1.5 \leq h_s/a$, $k_1 = 0.90$ and $k_2 = -1.11$.

b. For $1.0 \leq h_s/a < 1.5$, $k_1 = 1.50$ and $k_2 = -2.39$.

c. For $1.0 > h_s/a$, $k_1 = 1.50$ and $k_2 = 0$.

4.3 Calibration of Model

It was desired to reproduce the flow characteristics and geomorphic changes in the study reach to insure existance of similarity between the mathematical model and the modeled river reach such that the model can be used to predict future changes. The important features to be simulated include:

1. The water discharges and water surface profiles.
2. The cross-sectional area and bed elevation changes.
3. The sediment transport rates.

Two flow discharge hydrographs were used as upstream boundary conditions for the modeled reaches of the Mississippi and Chippewa Rivers. They are the discharge hydrograph recorded at Durand, Wisconsin, on the Chippewa River; and the discharge hydrograph synthesized by adding the discharge data at Prescott, Wisconsin, and at Welch, Minnesota, on the Cannon River, or synthesized by taking the discharge at Lock and Dam 4 substracting the discharge at Durand. The corresponding sediment discharges delivered to the study reach were assumed equal to the sediment transport capacities of the upstream boundary channel sections calculated from Eq. 45.

When these discharge hydrographs were routed through the modeled river reach, the flow discharge, velocity, water surface and riverbed elevations, sediment discharge, bankfull cross-sectional area (cross-sectional area at bankfull stage), open height of gates, and deposition on the floodplain at each section were calculated for each time step. The size of the time step varied from 6 hours to 7 days depending on the rate of change in flow discharge. A larger time step was used when the rate of change was small.

The calculated flow discharges, water surface and riverbed elevations, and bankfull cross-sectional areas were compared with measured data. Calibration continued through a large number of trials.

Extensive efforts were made to modify the Manning roughness coefficients and the empirical coefficients in the sediment transport equation at each section until the known historical changes were reproduced.

4.3.1 Calibration Results on Flow Characteristics

The calculated 1965 and 1971 water surface hydrographs and profiles are compared with the measured stages in Figures 9 through 12. These figures show an agreement between the measured and calculated values. Some differences were caused by the neglection of ice jam effects and by the differences in the regulated pool elevations. The effects of ice jams on the flow characteristics were neglected because these effects on geomorphic changes in the study reach were generally small. The pool elevation at the low and medium flow was regulated automatically in the model according to the authorized regulation method. The regulated pool elevations in the model might not be exactly equal to the measured values which in turn affected the water surface computation.

4.3.2 Calibration Results on River Geomorphology

To simulate the geomorphic changes in the Chippewa River and the sediment transport from the Chippewa to the Mississippi, the model was calibrated to reproduce the filling process of the 1965 dredged cut made in the Chippewa River near the mouth. This dredged cut served as a sediment trap to temporarily reduce the Chippewa sediment inflow to the Mississippi and thereby to reduce sedimentation problems. Figure 13 shows a good agreement between the measured and calculated bankfull cross-sectional areas as well as the mean riverbed elevations (average of the riverbed elevations in the deepest 400-ft width of

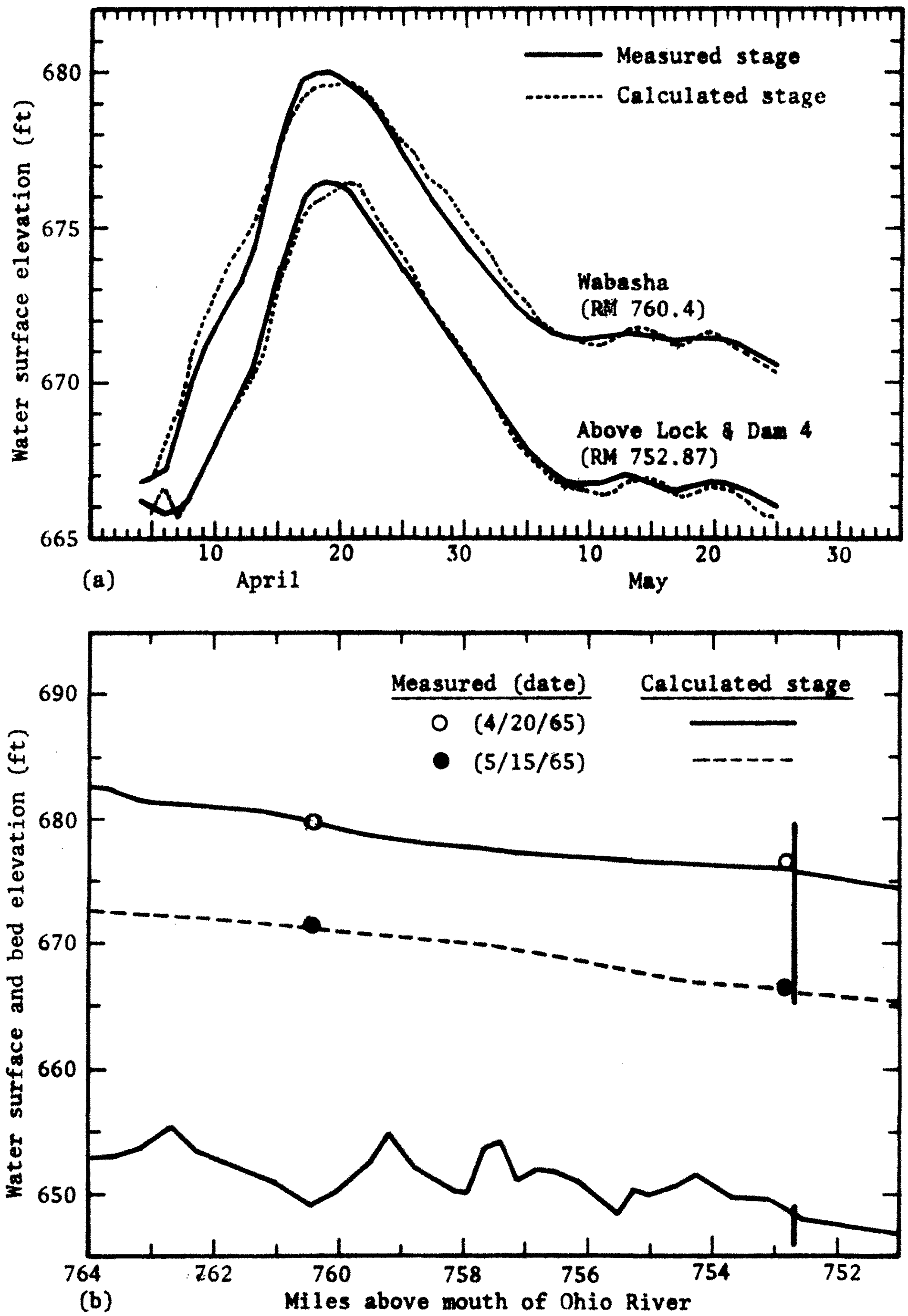


Figure 9. Mathematical model reproduction of 1965 water surface hydrograph and profile in Pool 4 of the Upper Mississippi River

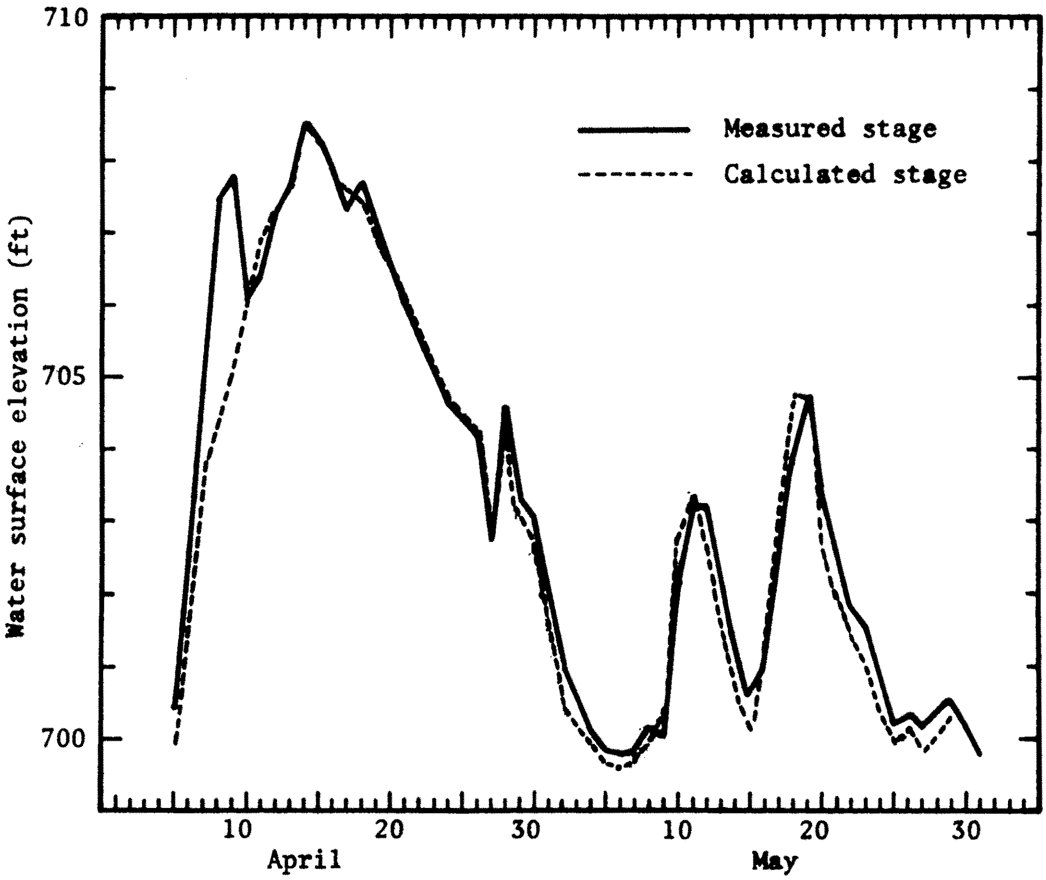


Figure 10. Mathematical model reproduction of 1965 stage hydrograph in the Chippewa River at Durand

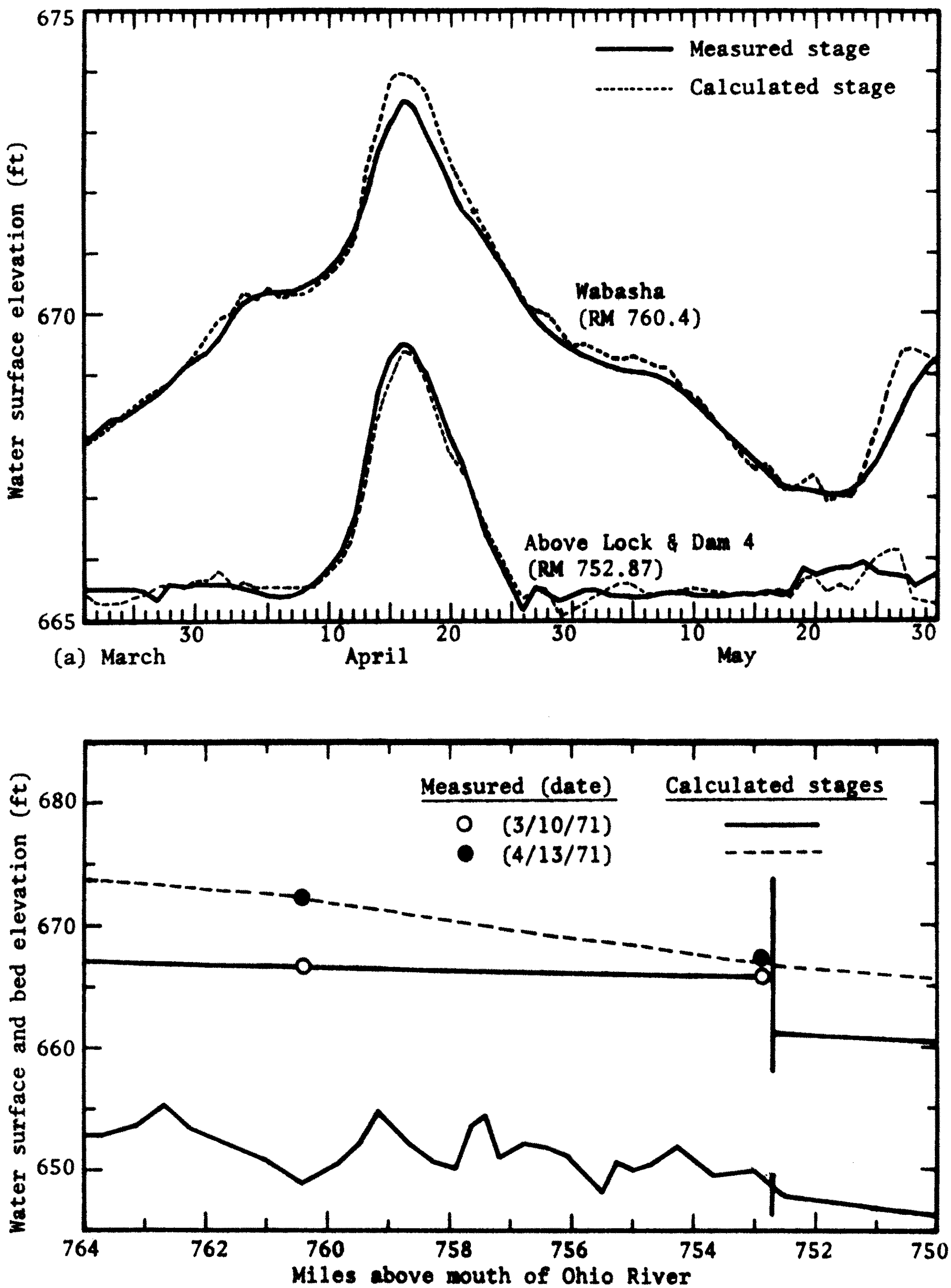


Figure 11. Mathematical model reproduction of 1971 water surface hydrograph and profile in Pool 4 of the Upper Mississippi River

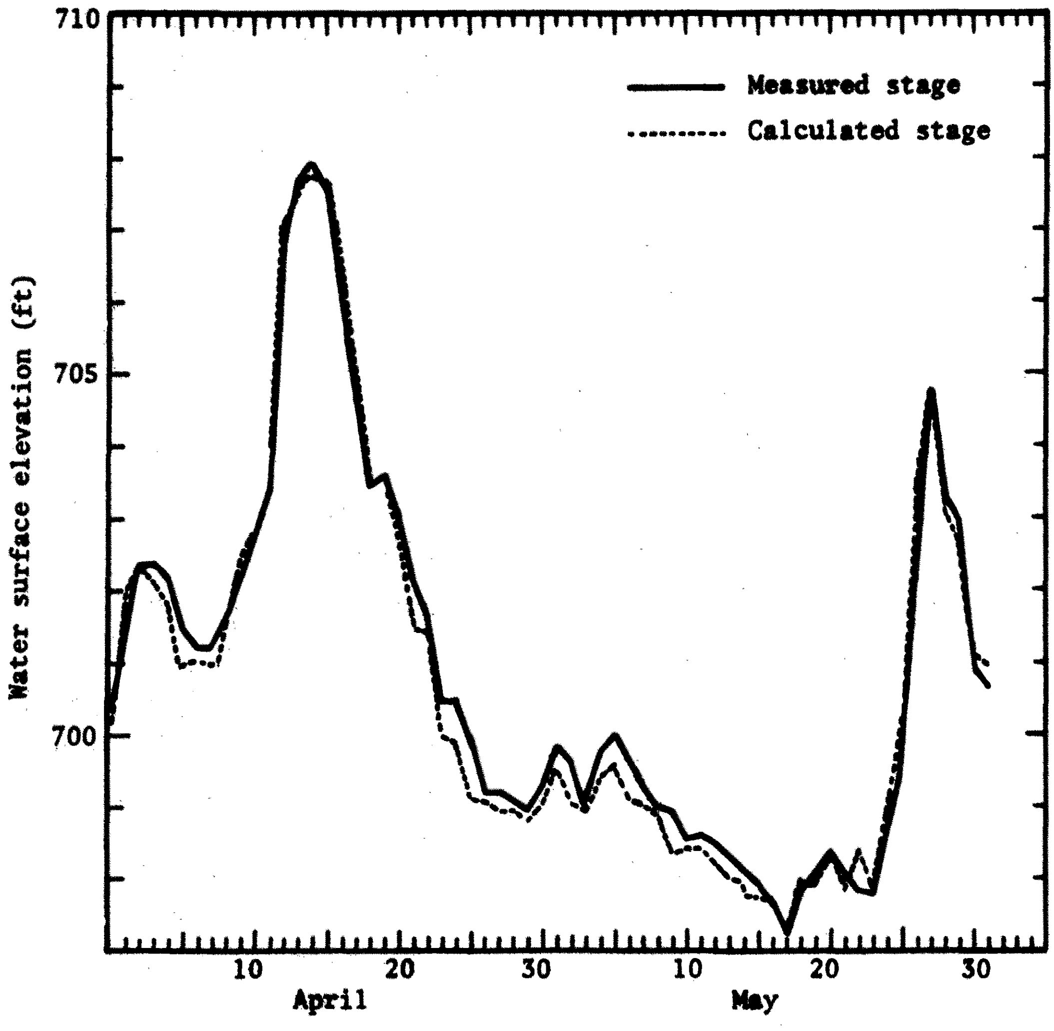


Figure 12. Mathematical model reproduction of 1971 stage hydrograph in the Chippewa River at Durand

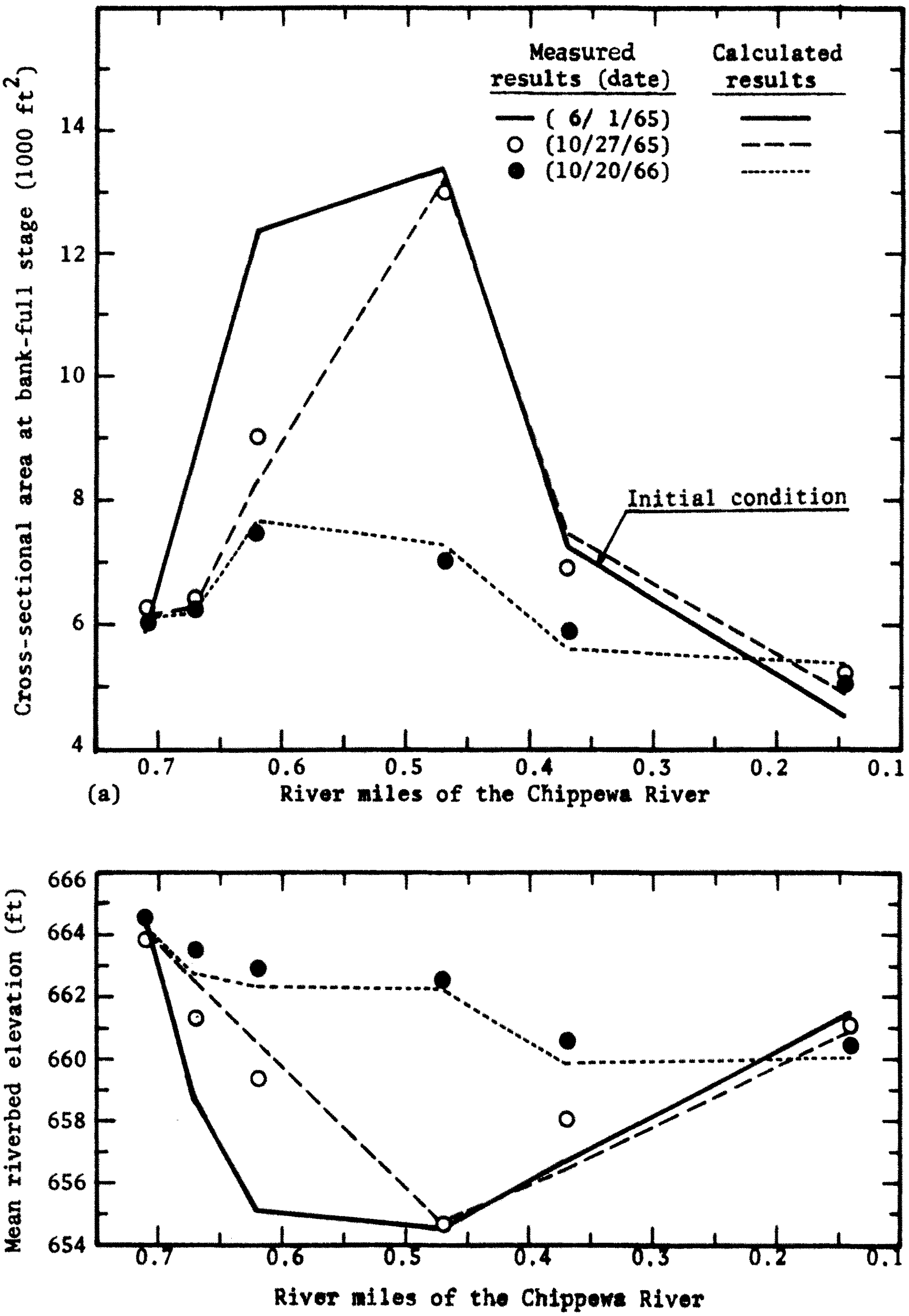


Figure 13. Mathematical model reproduction of the filling process of the 1965 dredged cut in the Lower Chippewa River

river channel). This indicates that the modeled Chippewa River can carry adequate sediment load into the Mississippi, and the mathematical model can thus be used to study the effect of changes in the Chippewa sedimentation on the Mississippi River. However, since the quantitative model calibration only covered the Lower Chippewa River near the mouth, the geomorphology of the rest of the modeled reach of the Chippewa River can only be studied qualitatively. A better prediction of the river's geomorphic changes can be performed if additional geomorphic data are available.

In the Pool 4 reach of the Mississippi River, the calculated changes in bankfull cross-sectional areas and in average riverbed elevation (average of the riverbed elevations in the deepest 700-ft width of river channel) near Reads Landing, above Crats Island and above Teepeeota Point from 1974 to 1975 are compared with the measured changes in Figures 14 through 16. These three locations are problem areas which require extensive dredgings. The agreements between the measured and calculated values are fairly good. The simulation of average riverbed elevation changes was less satisfactory than that of bankfull cross-sectional area mainly due to the assumption of a uniform distribution of sediment over the width. A better agreement could be obtained if a sediment distribution function is developed.

4.3.3 Remarks

The mathematical model has been calibrated to reproduce two historical flood events and some one-year geomorphic changes in the Lower Chippewa River near the mouth and in the Upper Mississippi River at Reads Landing, Crats Island and Teepeeota Point. It was found

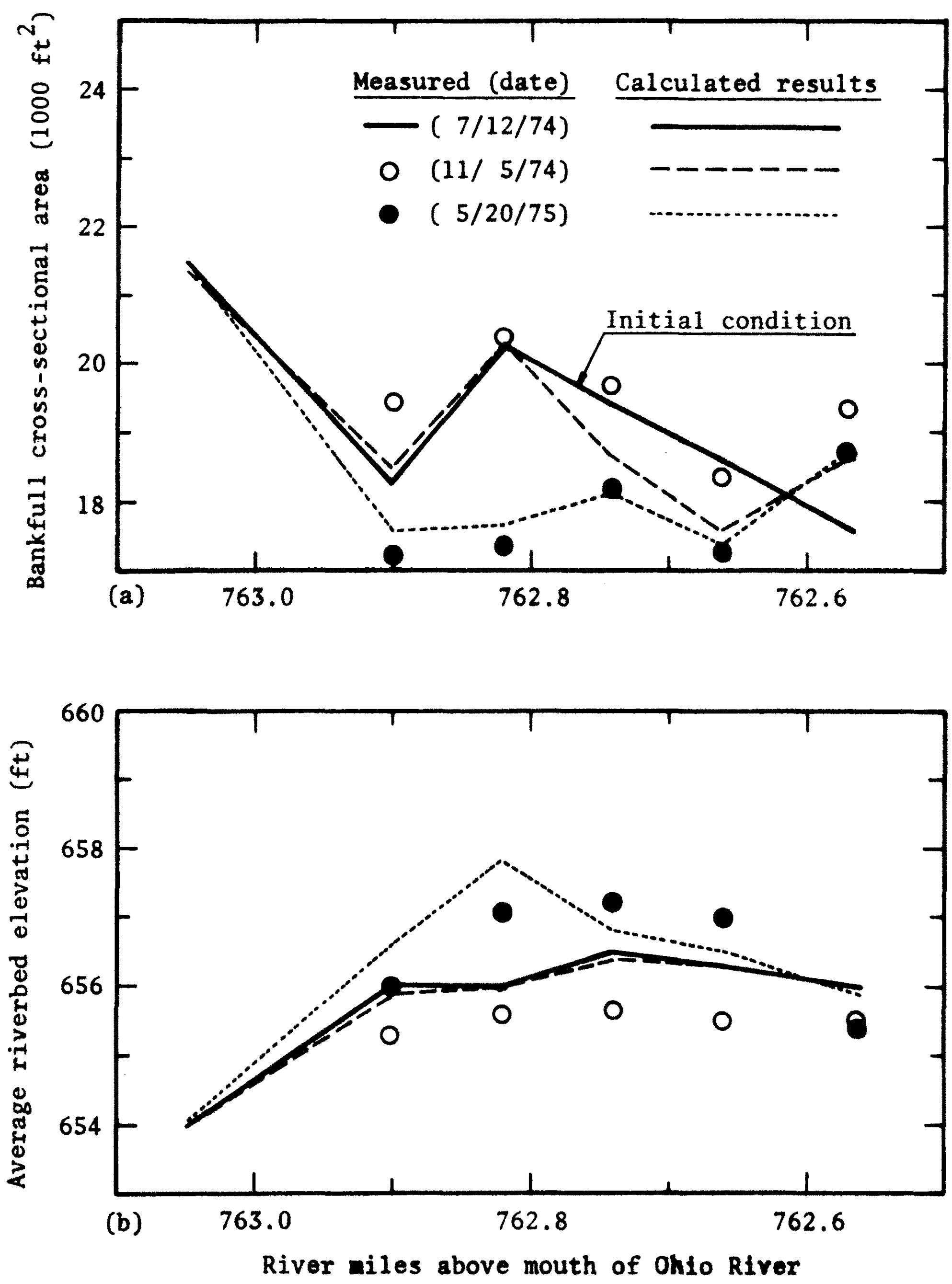


Figure 14. Mathematical model reproduction of the geomorphic changes in the Mississippi River near Readings Landing from 1974 to 1975

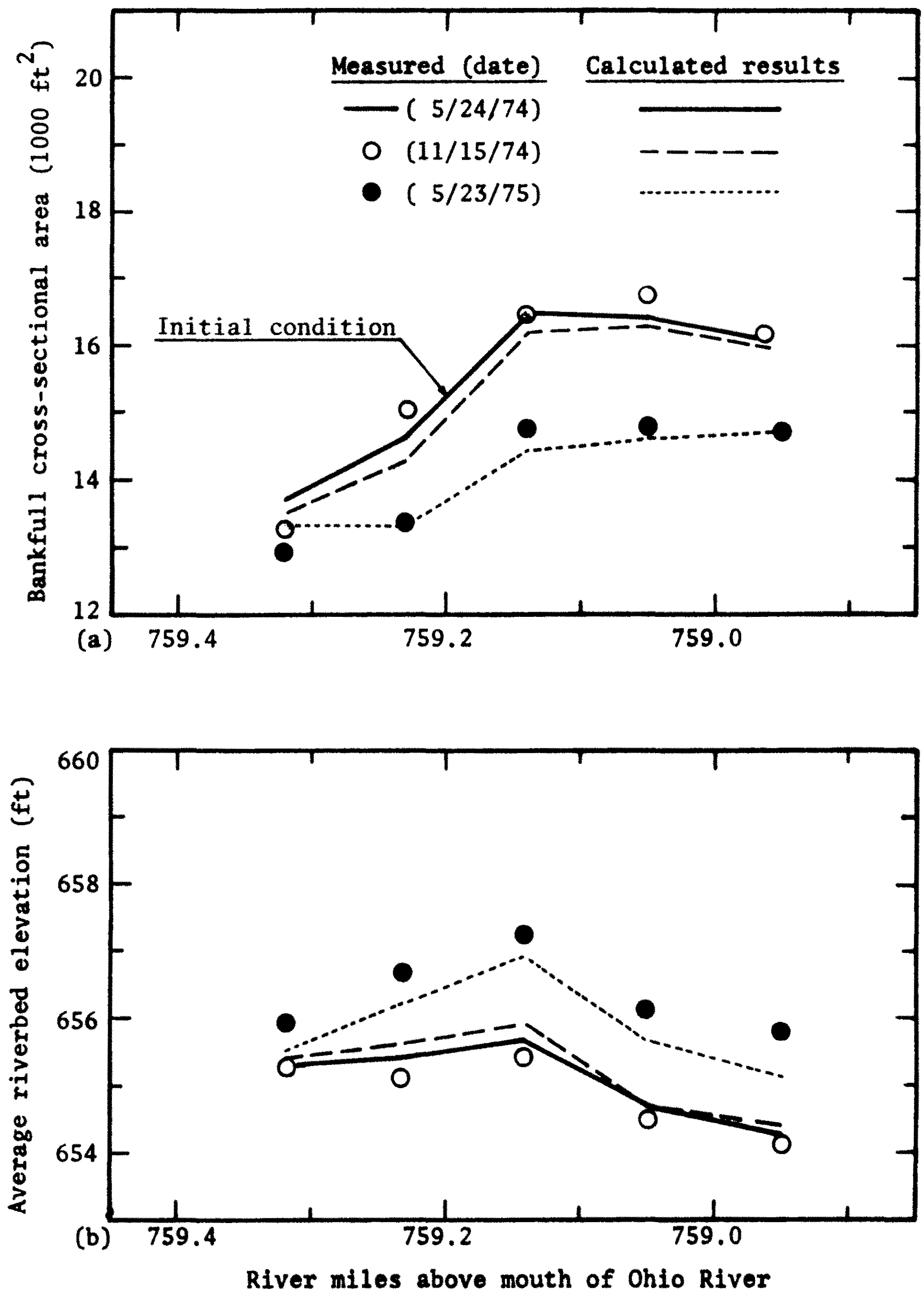
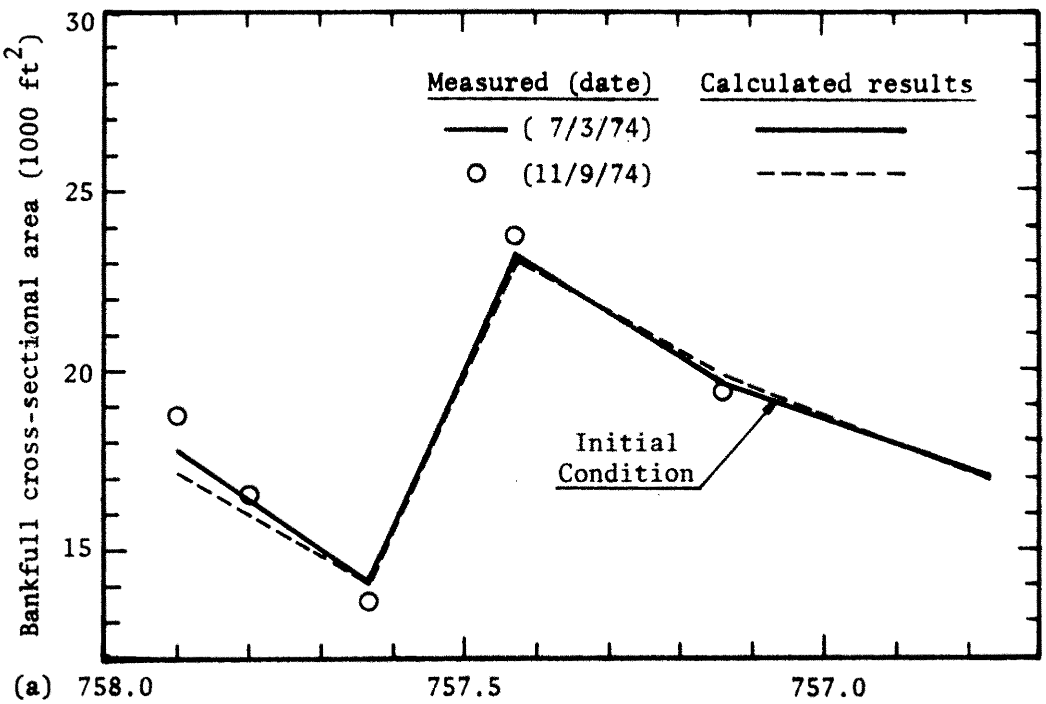


Figure 15. Mathematical model reproduction of the geomorphic changes in the Mississippi River above Crats Island from 1974 to 1975



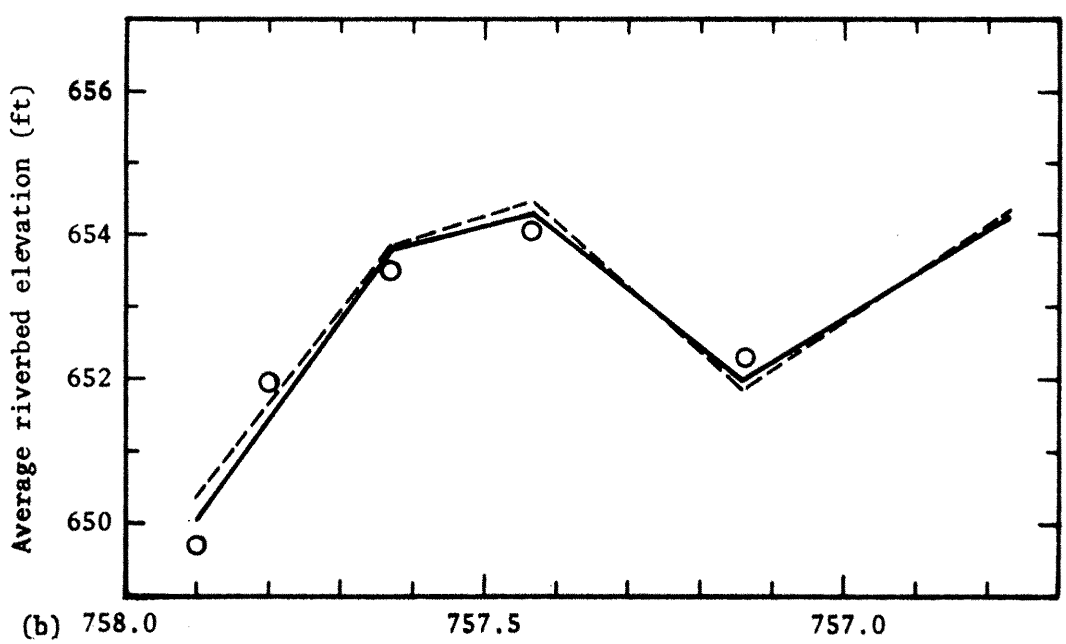


Figure 16. Mathematical model reproduction of the geomorphic changes in the Mississippi River above Teepeeota Point in 1974

that the available geomorphic data were not sufficient to calibrate the entire river reach. Very little reliable sedimentation data are available for the Chippewa River. Fortunately, the 1965-1966 dredging survey data in the Lower Chippewa River served a good basis to evaluate the sediment supply from the Chippewa into the Mississippi River.

In summary, since the calculated flow characteristics and trends of geomorphic changes agree with the measured values, it was concluded that the mathematical model as calibrated was as good as the available field data and could be employed to study the river's response to future development quanitatively to some extent. The predicability of the model should be good at least near the river confluence, and above Crats Island and Teepeeota Point a few years into the future. To improve the model predicability, the model should be verified and updated whenever additional data are available. It is desired to have hydrographic surveys conducted twice a year in the next few years in both the Mississippi River and the Chippewa River.

Chapter 5

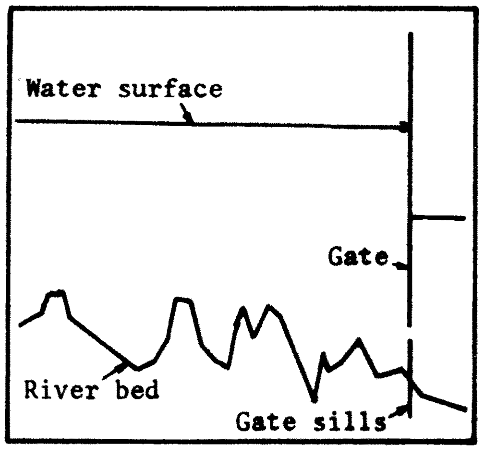
OPERATION OF THE MATHEMATICAL MODEL

5.1 General Model Operation

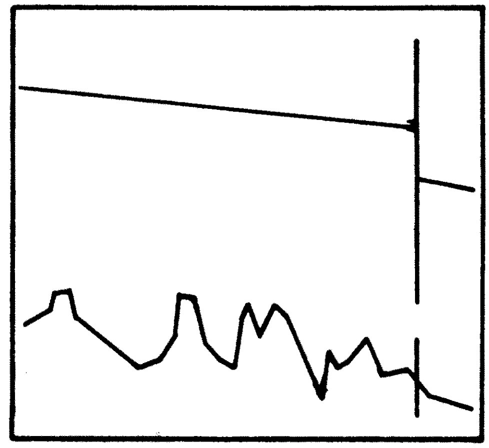
The calibrated mathematical model can be employed to assess the impacts of factors considered in this study. Typical results of routing a one-year hydrograph through the modeled river reach are given in Figures 17, 18 and 19. The water-surface profile in the Upper Mississippi River for $Q = 12{,}700$ cfs at 76 days is shown in Figure 17a. To maintain the normal pool levels, the control gates are lowered close to the gate sills. As the inflow increases, the pool stage is lowered at the dam by gradually opening the gates to maintain the level at the control stations within the prescribed control limits (Table 2) as shown in Figure 17b. As inflow continues to increase, the gates are opened further to increase the outflow until the gates are out of water and the river becomes an open river (Fig. 17c). After flood recedes the gates are then partially lowered into the water as required to restore the pool as shown in Figure 17d.

During the same flood routing, the water surface profiles in the Lower Chippewa River were determined and some results are shown in Figure 18. Because of the regulation of Pool 4, the stage in the Chippewa River near the confluence is raised, reducing the flow velocity and causing changes in river morphology.

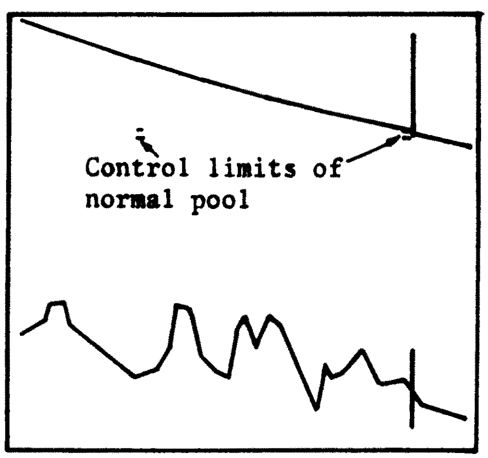
Some changes in riverbed elevation in the Upper Mississippi River during the same flood routing are shown in Figure 19. The difference between the solid and the dashed line indicates the changes in the



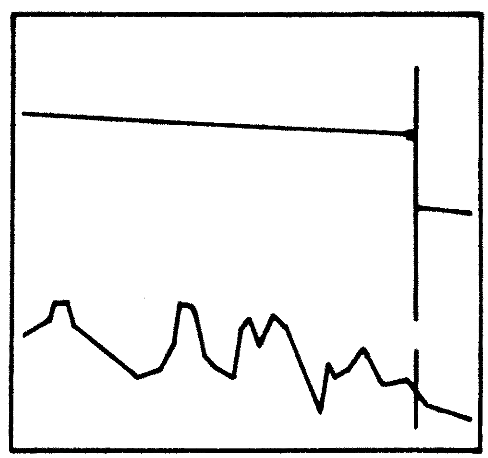
(a) Normal pool stages with control gates lowered close to gate sills
Q = 12,700 cfs at Dam 4
76 days



(b) Pool regulation shifted to secondary control at dam gates gradually opened
Q = 38,700 cfs at Dam 4
94 days

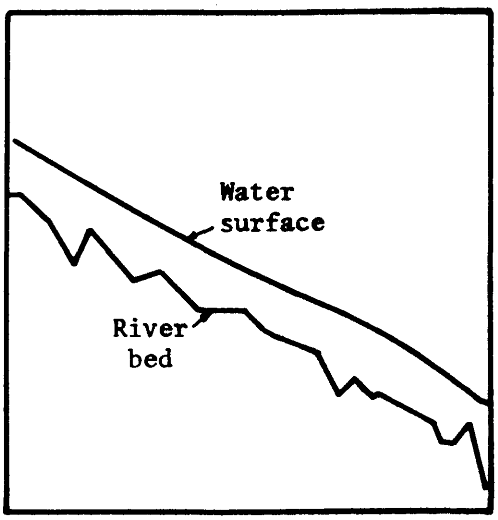


(c) Gates entirely out of water at flood crest
Q = 101,000 cfs at Dam 4
101 days

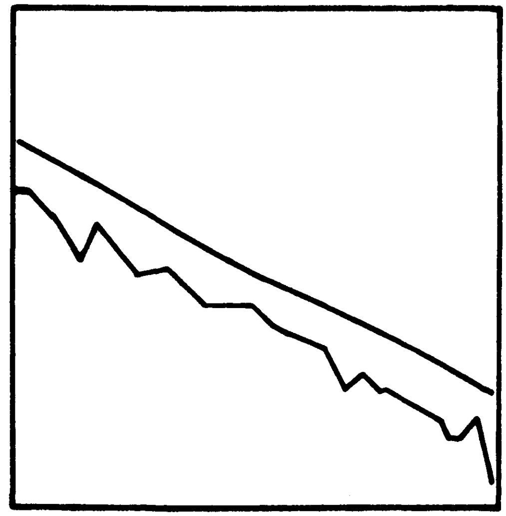


(d) Gates partially lowered to restore pool after flood recedes
Q = 29,000 cfs at Dam 4
144 days

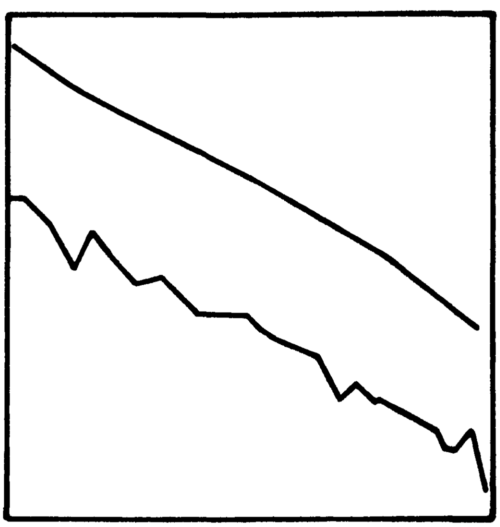
Figure 17. Water surface profiles in Pool 4 of the Upper Mississippi River during a flood routing



(a) Q = 5,300 cfs in the Chippewa
Q = 12,700 cfs in the
Mississippi, 76 days



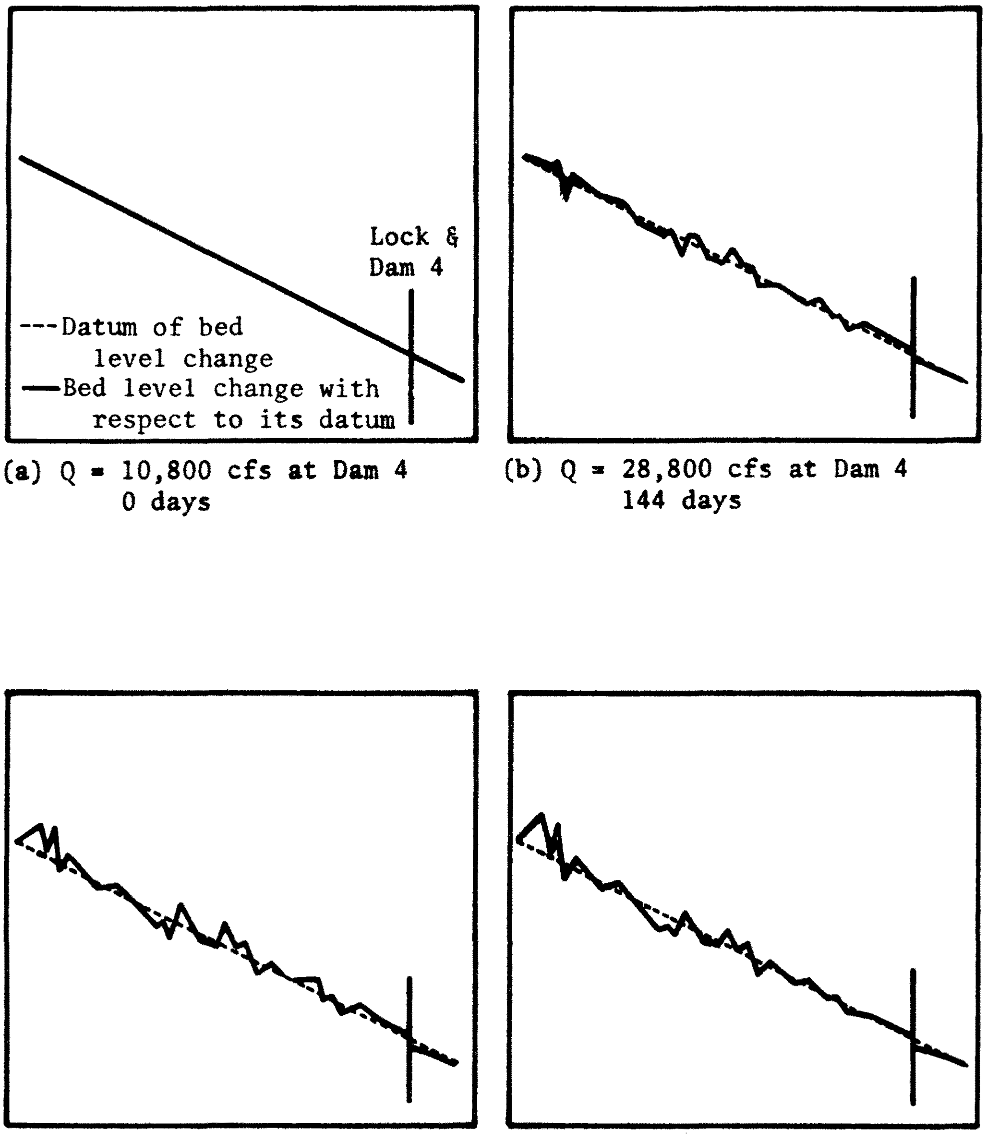
(b) Q = 3,600 cfs in the Chippewa
Q = 38,700 cfs in the
Mississippi, 94 days



(c) Q = 54,000 cfs in the Chippewa
Q = 101,000 cfs in the
Mississippi, 101 days

(d) Q = 7,400 cfs in the Chippewa
Q = 29,000 cfs in the
Mississippi, 144 days

Figure 18. Water surface profiles in the Lower Chippewa River during a flood routing



(a) Q = 10,800 cfs at Dam 4
0 days

(b) Q = 28,800 cfs at Dam 4
144 days

(c) Q = 36,000 cfs at Dam 4
174 days

(d) Q = 9,100 cfs at Dam 4
279 days

Figure 19. Bed elevation change in the Pool 4 reach of the Upper Mississippi River during a flood routing

riverbed elevation. It can be seen that the river bed does not continuously aggrade or degrade but fluctuates with a trend of aggradation or degradation.

5.2 Mathematical Model Prediction

What will Pool 4 in the Upper Mississippi and Lower Chippewa Rivers look like in the future? The answer may be obtained from the operation of the calibrated mathematical model of the present river system. Three 10-year flow simulations were conducted using the mathematical model. In this active river reach, a 10-year prediction would be more meaningful than a longer period prediction. An identical series of input flow rates was used for each simulation. This input series was developed from the peak discharges and flow volume frequency curves for the period 1929 to 1973 as follows:

1. The peak discharge and the flow volume frequency curves for the Mississippi River at Prescott, Minnesota and the Chippewa River at Durand, Wisconsin were constructed from the 1929 to 1973 flow data.

2. The peak discharges and the flow volumes for return periods of 1, 2, 5, 20, and 50 years were determined from the peak discharge and the flow volume frequency curves.

3. After examining the 1929 to 1973 flow data, the yearly flow having both the peak discharge and the flow volume closest to those determined from the frequency curves was selected to be the typical hydrograph for the specific return period. The yearly flows obtained are given in Table 3.

4. The typical 1, 2, 5, 20, and 50-year floods were combined in random sequence into a 50-year series of flows. Only the first 10-year series of flow was routed through the model. This 10-year series contained two 5-year, six 2-year, and two 1-year floods. The input series of flow to the outlet of Lake Pepin was assumed equal to the flow at Prescott multiplied by a factor of 1.05 to count the inflow from the Cannon River and other miscellaneous inflows.

From the study of the historical records of river flows, it was concluded that these flow series were adequate to represent the future river flow in the next 10 years. There were some occasions when the flood

Table 3

Typical Hydrographs

Return Period in years	Terminology used	Year Duration Curve	
		Prescott	Durand
1	1-yr annual hydrograph	1934	1933
2	2-yr annual hydrograph	1956	1956
5	5-yr annual hydrograph	1971	1965
20	20-yr annual hydrograph	1952	1954
50	50-yr annual hydrograph	1965	1938

discharge through Lake Pepin was much larger than from the Chippewa. Under these circumstances the sedimentation problems in the Lower Pool 4 reach would be negligible. The clear water flow discharged from Lake Pepin would be sufficient to transport away the sediment supplied from the Chippewa. However the reverse was also true. It was anticipated that these extremes would be rare and their effect would be counter-balanced. A large flood occurring in the Chippewa accompanied by a small flow in the Mississippi would cause severe sedimentation problems.

The three major 10-year simulations conducted to assess future geomorphic changes were: 1) present scheme of operation, 2) holding the pool level 1 ft above the normal pool, and 3) reducing the sediment inflow into the Chippewa River at Durand by 50 percent.

Simulations (1) and (2) were conducted to assess the effects of different operation schemes for Lock and Dam 4 on the geomorphology of the study reach in the next 10 years. The effects of other alternative operational schemes can be determined in a similar way. Simulation (3) was performed to estimate the effects caused by the reduction in the delivery of sediment to the study reach. The effects of Pool 4 on the behavior and form of the Chippewa River were also estimated during the simulations.

Simulation (1) was performed by simply routing the 10-year series of flow through the model. To conduct the latter two simulations some minor modifications of the control statements of the mathematical model were made. For Simulation (2) the control limits of the pool levels were raised 1 foot. For Simulation (3) the sediment discharge entering the Chippewa River at Durand was set to be 50 percent of the sediment transport capacity. Other alternative development programs in the modeled river reach can be studied in a similar manner. The results of model predictions are presented in the following sections.

5.2.1 With Present-Day Operations

5.2.1.1 Riverbed Changes

The mathematical model of the present river system was operated to assess future geomorphic changes that would result if the present scheme of operations to maintain the 9-ft channel were continued for 10 years. The hydrographs used in the model were synthesized from the 1929 to 1973 peak discharge and flow volume frequency curves as described previously. The sediment supply rates employed were those obtained in the calibration of the model.

The anticipated riverbed elevation changes in the study reaches in the next 10 years are given in Tables 4 and 5.

Table 4

Future Riverbed Elevation Changes in the Upper Mississippi River

Location	Riverbed Elevation Change after 1975,* ft	
Pool 4:	1980	1985
Lower one-third (RM 752.8-756.3)	0.3	+0.7
Middle one-third (RM 756.3-759.8)	0.1	+0.2
Upper one-third (RM 759.8-763.3)	+0.7	+1.3

*Positive and negative changes signify aggradation and degradation respectively.

Table 5

Future Riverbed Elevation Changes in the Lower Chippewa River

Location	River Elevation Change after 1975,* ft	
	1980	1985
Lower one-third (Mile 0-5.8)	+0.1	+0.1
Middle one-third (Mile 5.8-11.6)	-0.6	-0.7
Upper one-third (Mile 11.6-17.4)	+0.6	+0.5

*Positive and negative changes signify aggradation and degradation respectively.

In the Lower Pool 4 reach of the Mississippi River, the riverbed aggrades in the next 10 years because Lock and Dam 4 raises the water surface level which in turn reduced the flow velocity and river ability to transport the sediment inflowed from the Chippewa. This results in a 0.7 ft, 0.2 ft and 1.3 ft aggradation respectively on the lower, middle, and upper one-third of the Pool 4 reach in the next ten years. These predicted values are reasonable when compared to what actually occurred between 1929 and 1975 (Simons et al., 1976). During this time period, the riverbed has aggraded 1.3 ft and 1.0 ft on the lower one-third and middle one-third of the Pool 4 reach. It is anticipated the riverbed will aggrade another 0.7 and 0.2 ft on these reaches. Nevertheless, between 1929 and 1975 the riverbed in the upper one-third of the Pool 4 reach has degraded 2.7 ft from 1929 to 1975. It is also noted that the river has narrowed 150 ft in this time period. The net result was a slight reduction of the bank-full cross-sectional area. This indicates a deposition occurred in this upper reach which is the same as indicated by the calculated results. Therefore, if the river width remains unchanged in the next ten years, aggradation in this upper reach is expected.

In the Chippewa River, some small net aggradation is expected in the lower one-third reach as shown in Table 5. Actually, the deposition is relatively large during the low flow season because of the backwater effect of Pool 4. However, during high flow this deposited sediment is flushed out to the crossing areas in the Mississippi River, causing navigation problems in the main channel and sedimentation problems on the floodplain and backwater areas. It is then clear that the riverbed generally fluctuates with time as the sandbar moves downstream. The crossing areas accumulate sediment easier than the other portions of the river reach. Therefore, at crossings the bed elevation fluctuates with a trend toward aggradation. The predicted bed elevation changes in the middle and upper one-third reaches of the Chippewa River are not reliable because the model is not well calibrated in these reaches.

The predicted riverbed elevation changes in the study reach are produced by a common 10-year flow series. If an extremely large flood occurs above Lake Pepin in the next ten years, the clear water flowing down to the river reach may erode out the deposited sandbars. Conversely, an unusually large flood occurring in the Chippewa will transport tremendous amounts of sand into the Middissippi causing severe sedimentation problems.

5.2.1.2 Floodplain Deposits

The natural levees along the Upper Mississippi and Lower Chippewa River bank lines continue to grow in the ten years simulated. It is estimated that the natural levees are raised 0.5 ft in ten years.

Away from the natural levees, the deposition of sediment (mainly silt and clays) on the floodplain is not large. In the Pool 4 reach approximately 0.5 in. of silt and clay is deposited in ten years.

5.2.2 One Foot Above Normal Pool

The geomorphic changes in the study reach caused by holding the pool level 1 ft above the normal pool level for ten years are not significantly different from operation at normal pool level. The geomorphic changes of these two systems are similar. However, increasing the pool level reduces the sediment transport capability of the river reach. The reach aggrades more and degrades less when the pool is held 1 ft higher than the normal pool. The maximum difference is on the order of 1.0 ft in the aggradation reach immediately below the mouth of the Chippewa on the Mississippi River.

There is a 10 percent increase in floodplain deposits of silts and clay resulting from holding the normal pool level 1 ft higher, but as these floodplain deposits are small, the increase is of little significance. The natural levee heights are not increased significantly either.

5.2.3 Reduction of Sediment Inflow

Suppose it were possible to reduce the transport of sediment into the Chippewa River at Durand by 50 percent, then the river system in the Chippewa and Pool 4 would degrade. These amounts of degradation have been calculated assuming present-day operations for the next ten years. The maximum degradation occurring below Durand is estimated to be about 3 ft. The effects of this reduction in sediment supply reaches Pool 4 after some time exceeding 10 years.

The reduction of sediment supply to the Chippewa River may be accomplished by bank stabilization above Durand. However, a pure reduction of sediment inflow may cause degradation sufficient to undercut the Chippewa banks in the degrading reach. Therefore, a good improvement plan may require a combination of bank stabilization and a treatment in the Lower Chippewa to reduce its sediment transport ability. Further research is definitely needed to identify and evaluate improvement programs if the sedimentation problems in the Mississippi River in the St. Paul District is to be relieved.

5.3 Dredging

Dredging is important in the maintenance, extension, and improvement of the navigable waterway in the Upper Mississippi River. The problem of dredging, dredged material disposal and sedimentation in the channel and on the adjacent floodplain has been studied by employing the mathematical model of the river system. The effects of dredging on the hydraulics of the study reach have been estimated and some dredging guidelines have been developed.

In Pool 4 the crossing near Reads Landing that has required extensive dredging was modeled. In the model a simulated dredge cut 1 ft deep, 1000 ft wide, and 1300 ft long (from River Mile 762.90 to 762.66) was made in the crossing area. The cut was made at the beginning of the low-water season and the riverbed level changes in the modeled reach were computed during the next year for a 2-year annual hydrograph and a 5-year hydrograph. These riverbed levels were compared with those that would occur during the same year if no dredge cut were made.

As shown in Figure 20, a larger flood would produce more severe sedimentation problems in Pool 4 than a smaller flood. This agrees with the statement given in the Environmental Impact Statement (U.S. Army Engineer District, St. Paul). On the Upper Mississippi River below the confluence, a 2-year annual hydrograph would produce a deposition of approximately 0.5 ft compared to a 1.5 ft deposition produced by a 5-year annual hydrograph.

If the 1 ft deep dredged cut was made in this reach, the dredged cut would not have survived after passing of a 5-year annual hydrograph as shown in Figure 21. The resulting bed elevation would be essentially the same as on the natural channel without dredging. The filling of the dredged cut occurred mainly during the high flow. As a 2-year

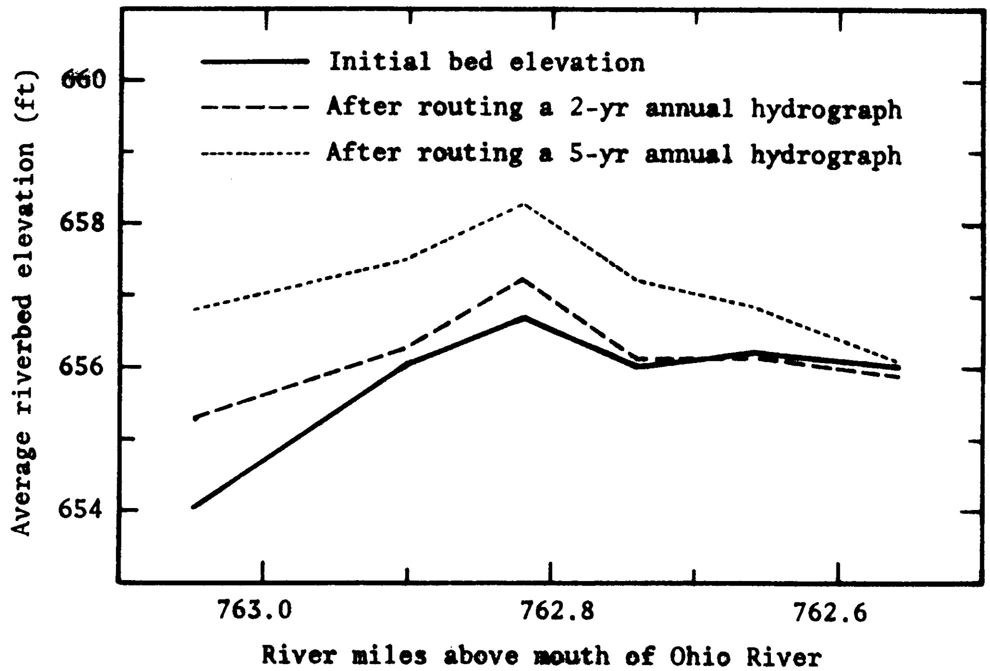


Figure 20. Riverbed elevation changes near Reads Landing after routing an annual hydrograph (without dredging)

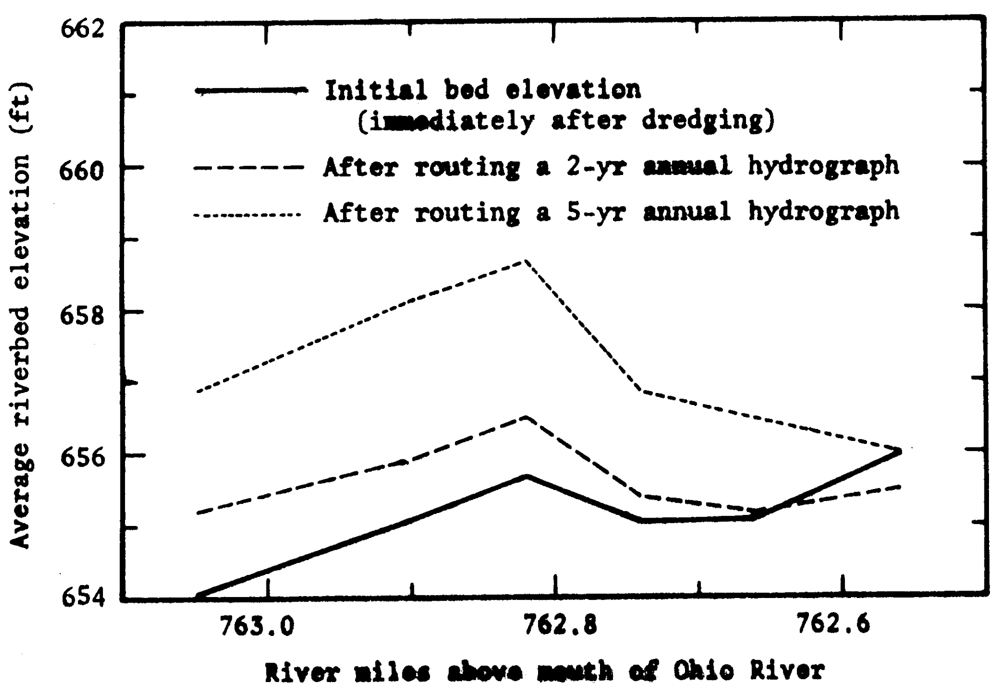


Figure 21. Riverbed elevation changes near Reads Landing after routing an annual hydrograph (with dredging)

annual hydrograph is passed through the reach, the size of the dredged cut would be reduced but might last for another year.

A sequential effect of a dredged cut was also observed in the model. With dredging near Reads Landing a larger amount of sediment was deposited on the dredged cut. This reduced the sediment deposition on the riverbed upstream of Crats Island (3 miles downstream of the dredged cut) by 0.1 ft after passing of a 5-year annual hydrograph. The implication is that a dredged cut can serve as a sediment trap to reduce the sedimentation problem down the river.

To verify this idea, a dredged cut of 4 ft deep, 600 ft wide, and 1600 ft long was made on the Chippewa bed near the mouth (Mile 0.67 to 0.37). At the end of a 5-year annual hydrograph the dredged cut was almost filled up. This dredged cut intercepted a large portion of the Chippewa sediment before it entered the Mississippi River and thus reduced the deposition amount near Reads Landing by 50 percent.

Other alternative maintenance or development programs can be studied by changing the control statements in the model and operating the model under suitable boundary conditions. The model results can be used to evaluate different programs (e.g., to determine a minimum dredging requirement on a crossing area in a dry year or in a wet year). This simple and economic methodology of mathematical modeling is very valuable for planning and decision making.

Chapter 6

SUMMARY

A mathematical model of Pool 4 in the Upper Mississippi and Lower Chippewa Rivers has been constructed, calibrated and applied to study responses of the river system to man-induced activities. The major findings are summarized.

6.1 Construction and Calibration of Model

1. The mathematical model has been constructed by evaluating the supplemental relations to basic flow equations (including relations on geometric properties, riverbed and friction slopes, lateral flows, and sediment discharges) from the field data and/or theories, and then solving the equations by using numerical methods and a digital computer.

2. The mathematical model has been calibrated by modifying the supplemental relations to reproduce two historical flood events and several one-year geomorphic changes in the Lower Chippewa River near the mouth and in the Upper Mississippi River at Reads Landing, Crats Island and Teepeeota Points. However, the available field data are not sufficient to calibrate the entire river reach. Having a good agreement between the simulated and the measured flow characteristics and trends of geomorphic changes, it is concluded that the mathematical model as calibrated is as good as the available field data and could be employed to study the river's response to future development quantitatively to some extent. The predicted results would be more reliable for the river reaches near the calibrated regions. Because of the dynamic behavior of the river system, a long-term prediction of using the present mathematical model is not recommended until additional data are available to verify and update the model to cover the entire modeled reach.

6.2 Mathematical Model Prediction

Future geomorphic changes that may occur in Pool 4 in the Upper Mississippi and Lower Chippewa Rivers due to present and anticipated future developments have been assessed. The responses expected are as follows.

1. If the pools are operated in the present-day manner for the next 10 years and if the sediment load to the study reach remains essentially unchanged, the riverbed in Pool 4 would have aggraded approximately 0.7 ft overall. The lower one-third of the Chippewa River would have aggraded 0.1 ft.

2. Under the present-day manner of operation and with normal sediment loads, the natural levees along the riverbanks and on the islands would grow on the average approximately 0.5 ft in height in the next 10 years.

3. Under the present-day manner of operation, on the average approximately 0.5 inch of silts and clays would be deposited on the unprotected floodplains along the study reaches in the next 10 years.

4. The geomorphic changes caused by operating with the pool one ft above normal pool for 10 years are not significantly different from operation at normal pool level. Increasing the pool level causes aggrading reaches to aggrade more and degrading reaches to degrade less.

5. Holding the pool one ft above normal for 10 years causes increased deposits on the natural levees and on the floodplains but these increases are not significant.

6. If the sediment inflow to Durand in the Chippewa River is reduced by 50 percent the river would degrade in the Chippewa River below Durand but there would be little effect on riverbed elevations in Pool 4 in the next 10 years.

7. A larger flood would produce severer sedimentation problems in Pool 4 than a smaller flood.

8. A one-ft-deep dredged cut near Reads Landing would not have survived after passing of a five-year annual hydrograph but may last through a two-year annual hydrograph.

9. A dredged cut may serve as a sediment trap to reduce the sedimentation problem down the river. A large dredged cut made in the Lower Chippewa River would reduce the deposition rate in the Mississippi below the confluence.

Other alternative maintenance or development programs can be studied by changing the control statements in the model and operating the model under suitable boundary conditions. Operation of the mathematical model is simple and economic and is very valuable for planning and decision making.

Chapter 7

LIMITATION OF THE MATHEMATICAL MODEL AND RECOMMENDATION FOR IMPROVEMENT

The principal limitation of the mathematical model constructed herein is its assumption of one-dimensional flow. Only the general pattern of the river geomorphology can be considered. To perform a detailed study, either a two-dimensional model should be developed or a modification of the present model can be made by using a compound stream approach (Dass, 1975). Since there was no width predictor included in the mathematical model, the changes in channel width with time should be accepted as a known quantity or should be evaluated using qualitative geomorphic concepts.

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