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Report on

GROUND WATER HYDROLOGY & HYDRAULICS

by

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FOREWARD

This report is the result of a special study of current literature and practices.

No attempt has been made to contribute original concepts; rather to provide a compendium of the available information. In some cases, the material is taken verbatim from the references listed in the Bibliography.

Of the references listed, in our estimation, Nos. 4, 6, 7, 9, 10, 13, 15 and 17 are particularly useful to the student interested in various aspects of ground water.

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Introduction

Modern society's heavy demand upon the water resources of Nature has forced the recognition of the need for the conservation of these resources. The sound development, wise use, and protection of the available water resources requires technical knowledge of the current use of water, and the effect of this use on the supply.

Of all water resources, probably the least is known about those which are underground. The tremendous increase in the use of underground water has focused attention upon the problems involved in its use.

In the United States, the estimated total use of water is 200 billion gallons per day and of this total, possibly 15% is derived from ground water sources and there is a marked upward trend in the use of ground water for public water supplies, and more especially, for irrigation. This increasing use requires increased knowledge of ground water potentialities and limitations.

Some recent reports have indicated that, in some areas, the water levels in wells have been dropping for several years. This has given rise to alarming statements that ground water resources are being generally depleted, and may be exhausted in this generation. It is true that in some areas, water is pumped from wells faster than it is replaced by nature. Actually, the total of such areas is less than 5% of the country. A much larger proportion of the country has no progressive decline in ground water level, and in some areas, ground water storage has increased appreciably.

Most serious problems of ground water shortage are in areas where significant quantities of water are removed from wells, but ground water storage has also been changed by other of man's activities.

The Problems

I. Problems related to pumping. The problems that are created by the pumping of ground water, generally fall into one of three classes.

(a) Reservoir problems: are those where the water is pumped out faster than the entire reservoir is replenished. These problems usually arise in arid regions where precipitation is generally inadequate. In humid regions, however, it is possible to have the reservoir sealed off by clay or other impervious material so that no replenishment is possible. The water which is present may have taken years or even centuries to accumulate.

Another type of reservoir problem is that in which salty or other unusable water flows into the reservoir as good water is removed.

Corrective measures that may be taken for these problems are:

(1) Prevention of waste.

(2) Prorated reduction of pumping.

(3) Prohibition of further development.

(4) Reclaiming of used water

(5) Artificial ground water replenisnment by surface water

(6) Importation of water.

(b) Pipeline problems: are those due to the slow movement of water tnrougn earth. If water is unable to move toward a well as fast as water is pumped out, water level in the vicinity of the well decreases, and the well may be pumped dry This is not necessarily indicative of an insufficient quantity of water in the entire reservoir.

Remedial measures which have been undertaken are:

(1) Reducing the draft.

(2) Increasing the inflow to the developed area.

(c) Watercourse problems: are those which result from pumping wells along rivers or streams. Thev arise either because the pumped water is not readily replaced from the stream, or because the stream water is needed downstream or is unsuitable for use.

II. Problems arising from modification in storage other than pumping.

(a) Ground water storage increased by irrigation, in some cases causing waterlogging of land.

(b) Ground water storage decreased by drainage to the detriment of some land or resulting in decreased availability for municipal use.

(c) Quantities of ground water changed by structures for storage of surface water, or flood protection.

(d) Storage of ground water changed by navigation channel improvements.

(e) City development reducing infiltration capacity and tending to lower ground water level.

(f) Contaminated water discharged into ground or into streams supplying ground water.

(g) Puncturing of protective layers may allow sea water, etc., to enter ground water reservoirs.

(h) Change in vegetal cover may affect ground water storage.

The solution of these problems to allow full utilization of the ground water available depends upon a recognition of the natural laws governing ground water. Intelligent development requires a knowledge of the hydrology of gound water, and the hydraulics of ground water movement through the earth and into wells; and a framework of law should be developed that is based upon sound hydrologic principles, and will keep pace with our increasing knowledge of the mechanics of ground water movement.

Origin of Ground Water

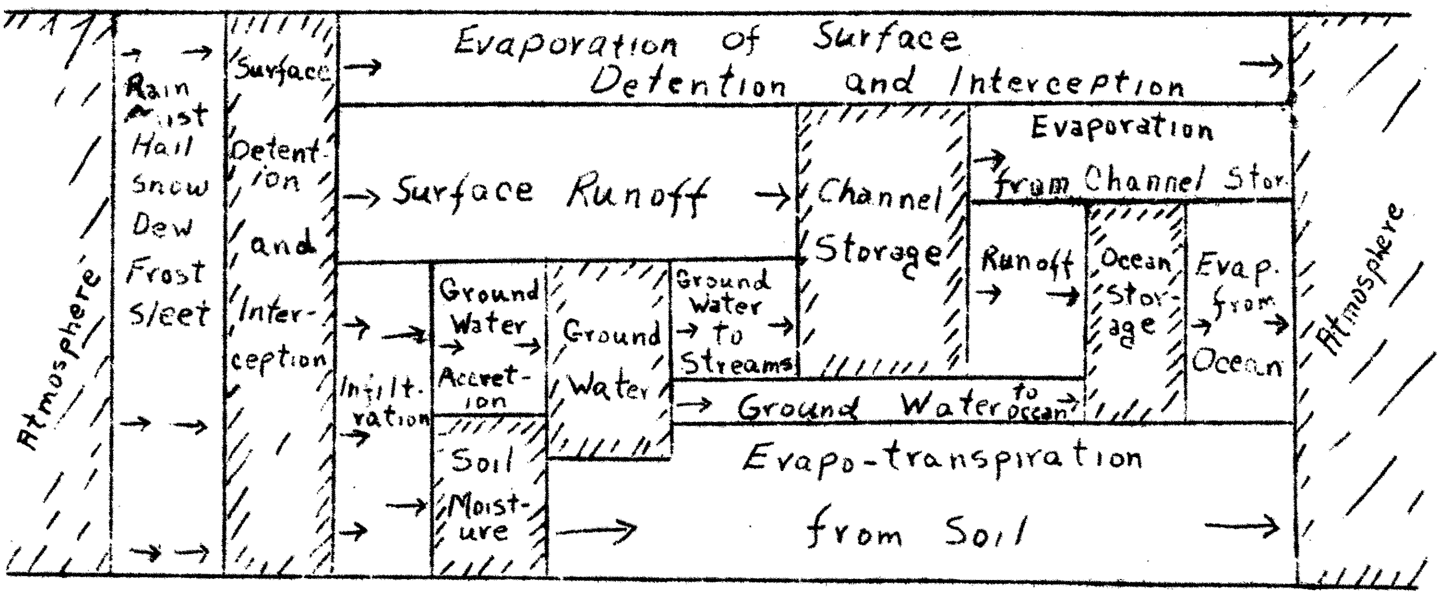
It is generally accepted that the greatest problem portion of ground water is meteoric in origin - that is, it originated in the atmosphere and reached the ground water reservoir through infiltration and percolation. Other types of water which are found are:

(1) Connate water, which was incorporated in the pores of rocks at the time of their deposition on ocean floors. This water is usually in small amounts and is often salty and unusable.

(2) Magmatic water, which has been brought close to the earth's surface by upward intrusion of ingneous rocks.

Since by far the largest portion of the available ground water supply is meteoric in origin, a study of ground water to be complete, requires a study of the hydrologic cycle, with particular emphasis on the parts of that cycle which directly apply to that portion of the water reaching the ground water reservoir.

The following diagram is a representation of the hydrologic cycle. It should be remembered that the cycle is continuous and that the diagram is repeated indefinitely. The arrows indicate movement of water; the shaded areas represent storage. The relative size of areas on the diagram has no meaning - it is simply a representation of the circulation.



-After Wisler and Brater

Infiltration:

Definition: the passage of water through the soil surface into the soil. This is to be distinguished from percolation, which is water movement within the soil. A study of the hydrologic cycle will show that water which is in the ground water reservoir must have reached there after infiltration. It should be noted also, that not all the water which infiltrates through the ground surface reaches the ground water reservoir. Water is first used up in replenishing the soil moisture deficiency, after which any access moves downward and becomes ground water.

Infiltration was first discussed by Dr. R. E. Horton in 1933.

Forces affecting infiltration. When water falls on the surface of the earth, gravity tends to pull it down through the interstitial channels leading from the surface to the water table. If the surface is dry, the molecular attraction of the soil grains is added to the force of gravity, increasing the infiltration. This increase is only temporary. After each soil grain has attracted its limiting thickness of water film, this molecular force is eliminated.

Other factors which affect the infiltration rate are:

(a) Moisture content of the soil. Soil is capable of holding water against the pull of gravity. The consequent reduction of channel area reduces the ability of water to travel through the soil. This causes moist soil to have less infiltration capacity than dry soil.

(b) Shrinking and swelling of colloids. Finely textured soils with particles of colloidal size have great reduction in infiltration capacity during early stages of storms due to the swelling of the colloids choking the channels.

(c). Effect of rain. Compaction of soil and closing of channels by beating action of raindrops reduces infiltration. Vegetal cover reduces this effect.

(d) Changes in macrostructure.

(1) Boring animals
(2) Decay of vegetal roots
(3) Cultivation
(4) Compaction

(e) Condition of vegetal cover. A protected soil may have many times the infiltration capacity of a barren soil. On the other hand, evapotranspiration from vegetation may more than offset this increase.

(f) <u>Effect of entrapped air</u>. Compression of air trapped below the ground surface tends to lower the infiltration rate.

(g) <u>Effect of temperature</u>. Viscosity of water increases as temperature decreases, lessening the infiltration rate.

<u>Infiltration Capacity</u>. The maximum rate at which a soil, in any given condition, is capable of absorbing water is called the <u>infiltration capacity</u>.

Note that infiltration capacity, f, is equal to the infiltration rate only if the rainfall intensity, p, equals or exceeds f. In any storm, f starts at a maximum, f_o decreases rapidly and then approaches a constant rate f_c. The ratio f_o/f_c depends greatly on the permeability of the soil. For example, the ratio is high for clays, low for sands.

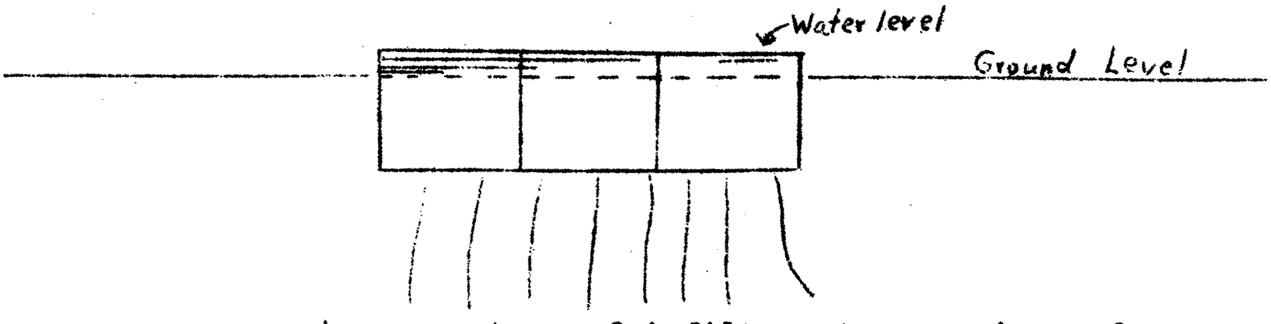
Horton has drawn attention to the fact that the curve of infiltration capacity against time for any storm is the exhaustion type, that it approaches a constant value after 1 to 3 hours and that it may be represented by an equation of the form

$$f = f_c + (f_o - f_c)\, e^{-kt}$$

where
- f = infiltration capacity at any time
- t = time of rainfall
- e = Naperian base
- f_o = infiltration capacity at beginning of rainfall
- f_c = constant value of infiltration capacity after some time has elapsed
- k = constant for any given curve.

<u>Methods of determining infiltration capacity</u>.

1. <u>Infiltrometers</u>



A common type of infiltrometer consists of two concentric rings of sheet metal, shown in section above, placed with their lower edges below the ground surface. Water is applied in both rings and is kept at the same level in both. The function of the outer ring is to prevent the water from the inner ring from spreading out over a larger area after penetrating the ground. The infiltration capacity is measured directly by measuring the amount of water which must be added to the inner ring to keep a constant level.

A variation of this is a single ring which is buried deeply enough in the ground to prevent spreading of the water after penetrating the surface.

Defects.

(a) The beating effect of raindrops is not simulated.

(b) The effect of compression of entrapped air is absent because it can escape laterally.

(c) It is impossible to place the rings without some disturbance of the soil, which will change the infiltration capacity.

2. Rain simulators. To avoid the objections above, as much as possible, various methods of determining infiltration capacity have been devised, in which water is applied by sprinkling at a uniform rate that is greater than the infiltration capacity. The surface runoff is collected and measured, and the difference between applied water and runoff gives the infiltration. Area covered is usually from 1 sq. ft. to 1/10 acre.

Horton was one of the first to use this type of infiltrometer.

Defects.

(a) Difficult to obtain sufficient results to determine average value of f for a large basin.

(b) Entrapped air can escape.

(c) Infiltration capacity determined for a small area cannot safely be used to predict for a larger area.

3. Hydrograph analysis. Accurate records of the varying intensities of rainfall during any storm period, together with a continuous record of the resulting runoff, provide a basis for the determination of infiltration capacity.

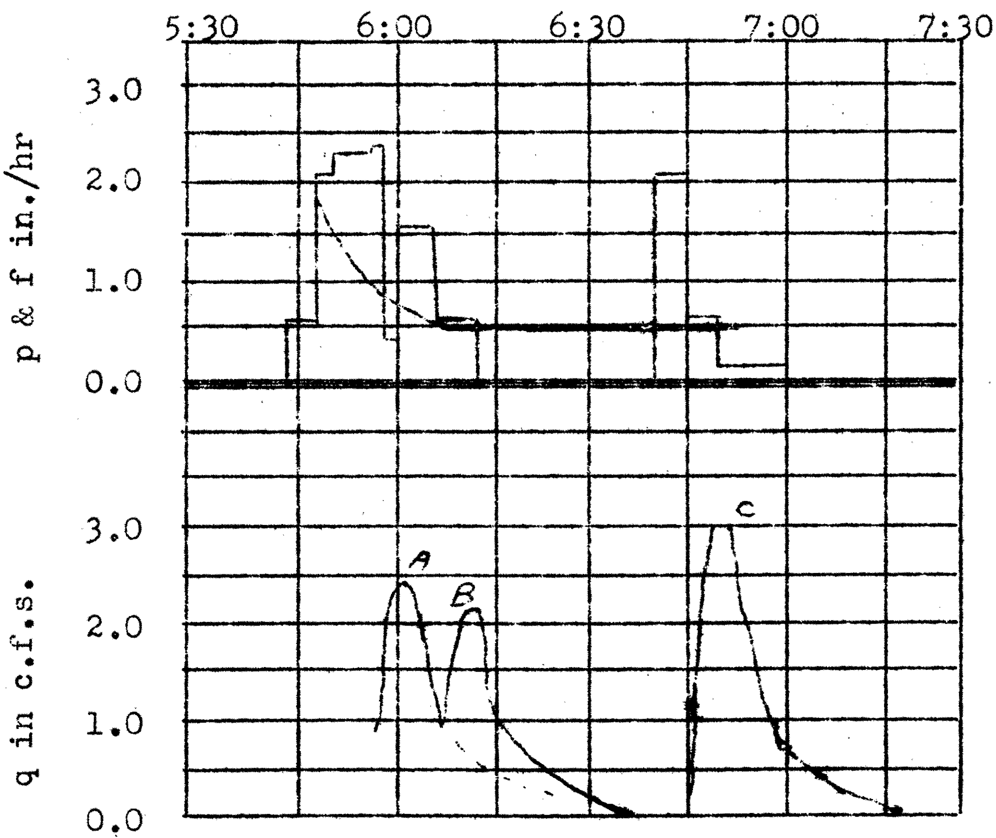
Unlike the infiltration capacities determined by infiltrometers, these results may be used to determine the infiltration capacity resulting from any given storm occurring on the same basin under similar conditions.

For small basins, where the hydrograph is quick to respond to varying intensities of rainfall, the actual manner in which f varies throughout a storm can be quite accurately determined, but for larger basins it is possible to determine only the average infiltration capacity, f_a.

(1) Small basins: Method suggested by Horner and Lloyd (Trans. Am. Geophys. Union, 1940, pp 522-541) as modified by Wisler and Brater ("Hydrology").

This method is best explained by reference to an actual example.

The diagram shows graphs of rainfall intensity and the resulting hydrograph of surface runoff, on a small basin 2.7 acres in area, for a period of two hours, during which three storms occurred.



The following steps are followed:

(a) Complete the recession curve of A by drawing dotted portion parallel to recession of B.

(b) Areas beneath A, B, C, represent depths of runoff 0.10 in, 0.09 in, 0.18 in, respectively (areas under hydrograph curves). Depths of rain producing these runoffs are 0.34 in, 0.22 in, 0.29 in (areas under p curves).

(c) For each of the periods of intense rain, the total infiltration is (difference between precipitation and runoff.) (0.34-0.10) = 0.24 in, (0.22-0.09) = 0.13 in and (0.29-0.18) = 0.11 in, respectively. Note that these values of total infiltration, F, include interception and depression storage.

(d) To reduce these to capacity rates, f, divide each by the average length of time during which infiltration was occurring at capacity rate over the entire basin.

(e) These periods begin when excess rainfall starts and continue until some time after it ends. Note: When excess rainfall ends, infiltration occurs at capacity rate over the whole basin, but this area begins shrinking from the outer boundaries toward stream channels. Any segment of this area is approximately a triangle with vertex at channel, base at divide (Horton). Therefore, assume that the equivalent period during which the same volume of residual infiltration would occur on the entire basin = 1/3 of the period from end of excess rainfall until end of overland flow. Overland flow ends at point of inflection (approx.) at recession side of hydrograph (i.e. at beginning of exhaustion curve).

(f) Therefore, in example t_1 = 11 min., t_2 = 13 min. and t_3 = 13 min.

Therefore, for first storm

$$f = 0.24 \times \frac{60}{11} = 1.31 \text{ in/hr.}$$

for second storm

$$f = 0.13 \times \frac{60}{11} = 0.60 \text{ in/hr.}$$

for third storm

$$f = 0.11 \times \frac{60}{13} = 0.51 \text{ in/hr.}$$

(g) Plot each of these values at $\frac{t}{2}$ from beginning of excess rainfall to give f curve.

(h) Note that this is really an f_a curve; i.e. it represents average infiltration capacity for each of the several periods of high storm intensity. However, (Wisler & Brater) "the divergence of the f_a curve from the true f curve is probably of minor significance."

(2) Large basins: For drainage basins larger than those for which the rain intensity may be considered as being uniform over the entire area, the average infiltration capacity f_a, that existed during any given storm may be determined by a method proposed by R. E. Horton. (Trans. Am. Geophys. Union, 1937, pp. 371-385) as reported by Wisler and Brater.

(a) Rainfall records must be available to represent rainfall variation satisfactorily. At least one of the records must be from an automatic recording gauge.

(b) Assumptions:

1. In great general storms rainfall intensity patterns at adjacent stations are very similar.

2. Surface runoff is equal to the difference between rainfall and infiltration occurring during rainfall excess. i.e. Rain which falls during and immediately after rainfall excess period, but infiltrates during the subsequent period of overland flow is ignored.

3. Rainfall periods (not necessarily rainfall excess periods) are approximately the same at all stations, though not necessarily simultaneous.

(c) Average infiltration capacity is approximated by subtracting total surface runoff from total rainfall at base station (station of recording gauge) and dividing by the period of rainfall at base station.

(d) Percentages of total rainfall occurring during each hour at the base station are computed.

(e) Then, assuming a total rainfall of some even number of inches, about equal to the average depth for the given storm, the depth of rain falling each hour at the base station is determined by multiplying this assumed depth by the various percents determined in (d).

(f) Then calculate the depths of rainfall excess during each hour of this same storm, for infiltration capacities of different amounts e.g. 0.1, 0.2, 0.3, 0.5, 0.6 in. per hour) Plot these total rainfall excesses against storm rainfall assumed in (e).

(g) Repeat (e) and (f) for assumed storms of different depths of rainfall greater and less than the assumed value of (e). This will give a set of curves showing rainfall excess vs. total rainfall, for various infiltration capacities.

(h) For each of the substations (stations where only total daily rainfall is observed), the values of rainfall excess may be obtained from curves of (g) for the total depths of rainfall at these stations, for different infiltration capacities.

(i) The average of all the stations' rainfall excesses for each infiltration capacity is plotted against infiltration capacity.

(j) This curve then will give average infiltration capacity on this basin for known values of rainfall excess.

Note: Values obtained by this method may be applied to any design storm on the same basin, but should not be used for studies on another basin.

These methods of determining infiltration rates were developed mainly for the purpose of estimating runoff from precipitation. Actually, looking at the problem from the point of view of ground water, it should be obvious that the infiltration rate obtained by these methods does not indicate the actual amount of water reaching the ground water reservoir. For one thing, interception and depression storages are included in infiltration rates calculated by hodrograph analyses. This is not too far out of line, however, as a fair portion of depression storage does eventually infiltrate.

Also, even after the water has penetrated the ground surface, it may not reach the ground water level, as there are varying amounts held in the upper levels of soil as soil water, and suspended water.

However, a comparison of infiltration capacities on different areas will at least give qualitative indication of the relative amounts of ground water accretion to be expected, and future research may provide methods of greater quantitative value than exist at present.

Ground Water

Once water has passed through the earth's surface, it becomes a potential source of ground water. It is assisted in its movement to the ground water level by the force of gravity, and is deterred from its free downward movement by the force of molecular attraction, which tends to hold a film of water to each particle of soil; by the frictional resistance to flow through the soil interstices; and by the force of capillarity which can hold a considerable height of water against the force of gravity. For a discussion of these forces, see Chapter 12, "Soil Physics" in the book "Applied Hydrology" by Linsley, Kohler, and Paulhus.

It should be realized that the rocks comprising the earth's crust are seldom, if ever, solid throughout. Their numerous openings, called interstices vary from caverns in some rocks to extremely small spaces between soil particles. Generally they are very small.

Usually, the interconnection of interstices permits movement of the percolating waters, but they may be isolated. Obviously, then, the occurrence of ground water is largely dependent on the geology of the area under consideration.

Definitions:

Aquifer. A formation which transmits water in sufficient quantity to support wells or springs.

Aquiclude. A formation which contains water, but does not permit its movement at rates sufficiently large to permit development by pumping.

Porosity. In any soil or rock, the percentage that the volume of interstices is of the total volume.

Specific Yield. The volume of water yielded by gravity drainage from saturated water bearing material, expressed as a percentage of total volume of material.

Specific Retention. The volume of water retained by a saturated material against the pull of gravity, expressed as a percentage of total volume of material.

Note: Specific retention + specific yield = porosity.

The Ground Water Reservoir

The classification of the earth's crust with reference to its properties as a reservoir, and the subdivision of this reservoir into its component parts is shown in the following diagram, taken from "Hydrology" by Wisler and Brater.

<table>
<tr><th>Structure</th><th colspan="4">Water Occurrence</th></tr>
<tr><td rowspan="4">Zone of Rock Fracture</td><td rowspan="4">Interstitial Water</td><td rowspan="3">Zone of Aeration</td><td rowspan="3">Suspended water</td><td>Soil Water</td></tr>
<tr><td>Vadose Water</td></tr>
<tr><td>Capillary Fringe</td></tr>
<tr><td>Zone of Saturation</td><td colspan="2">Ground Water</td></tr>
<tr><td>Zone of Rock Flowage</td><td colspan="4">Internal Water</td></tr>
</table>

← Water Table

The zone of rock flowage has no interstices, and the internal water is not considered here because it is not part of the hydrologic cycle.

The zone of rock fracture may have interstices and the water in this zone is called interstitial water.

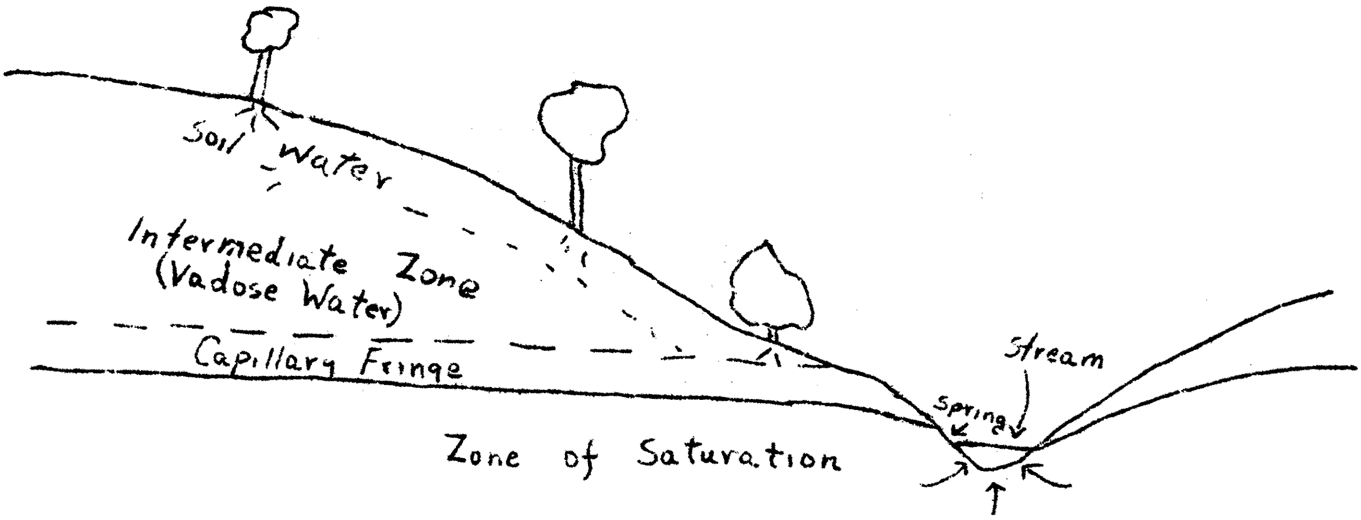
In the zone of aeration, the water is not under hydrostatic pressure, and the interstices are largely filled with gas. Water here is held by molecular attraction and is called <u>vadose</u>, <u>pellicular</u>, or <u>suspended water</u>.

Water in the zone in which plant root systems are found is called <u>soil water</u>.

The belt overlying the zone of saturation contains interstices, some or all of which are filled with water in connection with, and a continuation of, the zone of saturation. Its thickness depends on particle size. It may be a few inches in sand, to many feet in fine grained soils. This belt is called the <u>capillary fringe</u>.

Below the capillary fringe lies the zone of saturation in which all interstices are filled with water. This is the most important zone from the point of view of water supply. Water in this zone is called <u>ground water</u>, and the upper surface of the zone is called the <u>water table</u>.

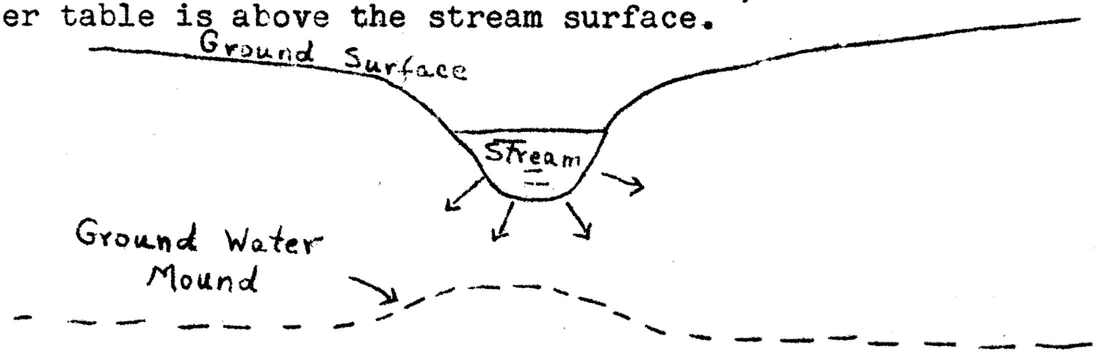
The following diagram represents the cross section of a drainage basin, to show the relation of the various zones mentioned.



(From "Hydrology" - Wisler & Brater)

Note: That the intermediate zone does not always exist. If the capillary fringe reaches the ground surface, then water is lost from the zone of saturation by evaporation aided by capillarity. If the water table reaches the surface, then a spring is formed.

The stream shown is an effluent stream because water flows from the zone of saturation to the stream; this is because the water table is above the stream surface.

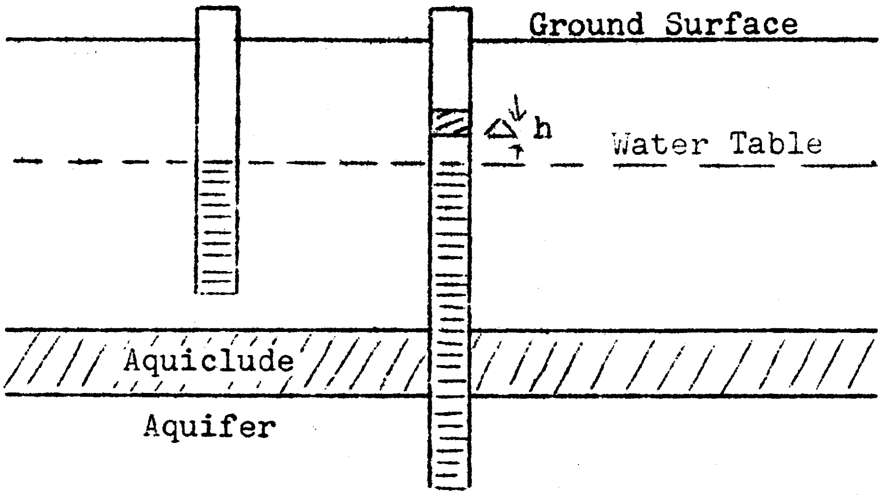


The reverse condition is shown in the above diagram, where the water table is below the stream surface. This is an influent stream. If the water table is below the stream bed, then a ground water mound or ridge is formed under the stream.

Artesian Aquifers

If an aquifer is overlain by a confining bed of relatively impervious material, and if the water level in a well penetrating the aquifer, rises above the level of the bottom of the confining bed, the aquifer is called artesian. The surface formed by connecting the heights of water level in well, tapping the aquifer is called the piezometric surface.

Elastic properties of Artesian Aquifers. It is common in some artesian wells for water levels to change during periods of large fluctuations in barometric pressure. The explanation of this is as follows.



Idealized Section after "Hydrology" - Wisler and Brater.

Consider the shallow well which is not artesian. An increase in barometric pressure, tending to force the water down in the well is balanced by an equal pressure exerted on the shallow water table. Therefore, no change in water level occurs.

If the same barometric increase occurs with respect to the deeper artesian well, the aquiclude does not transmit the full increase to the underlying water, as it has some resistance to the pressure. Consequently, the barometric increase on the water level in the well is not balanced by an equal pressure in the water in the aquifer. As a result, the water level decreases by an amount Δh.

The ratio of the water level change to the barometric pressure change (in equivalent units) is called the barometric efficiency of the well, or the aquifer.

For the same reason, loading due to tides (or other superimposed loadings) may cause fluctuations in well elevation. The ratio of rise in water level in the well to the increase in level of the tide (corrected for different densities of water) is called the tidal efficiency of the aquifer.

See Page 33 for a discussion by Jacob.

It should be noted that although the overlying aquiclude which confines an artesian aquifer is generally considered to be impervious, such is not always the case, and it is possible for large quantities of water to be lost due to the permeability of confining aquicludes. Also, many artesian aquifers would long ago have been pumped dry were it not for the fact that the permeability of the confining aquiclude allows water to enter the aquifer in areas where the piezometric surface is below the lower surface of the aquiclude.

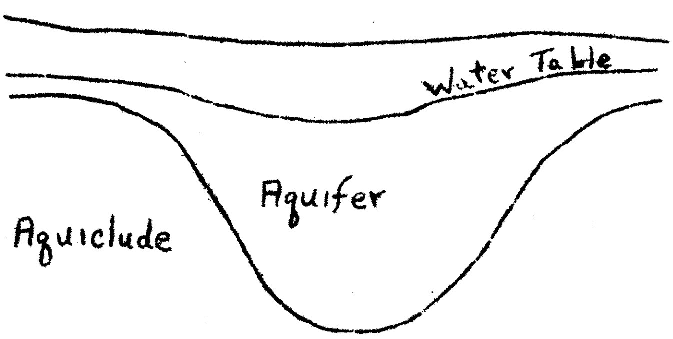
Other methods of recharge of artesian aquifers are:

(a) Infiltration on outcrop areas

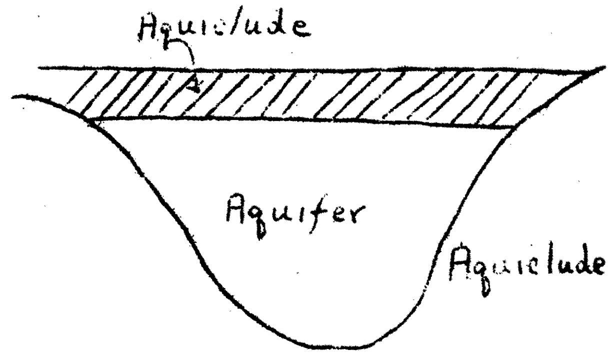
(b) Percolation through breaks in the confining aquiclude

Underflow: is a term used to describe the movement of ground water through an underground channel, such as may be found in sedimentary rocks or in alluvial soil, where a channel defined by impermeable aquicludes if filled with a permeable aquifer.

The term underflow channel is used where a water table condition exists, and underflow conduit where the flow occurs under artesian conditions. Many stream channels in dry seasons, when the surface appears dry, may carry appreciable quantities of water as underflow.



Underflow Channel



Underflow Conduit

Conclusion

It should be noted that an essential character of ground water reservoirs is movement of water through them. Even without wells, the ground water phase of the hydrologic cycle is one of movement from the place where water enters the aquifer - the recharge area - to the place where the water is discharged from the reservoir, either by evapotranspiration, by springs, or by seepage to streams or lakes or the ocean.

Thus, as a rule, usable ground water does not remain at rest under a piece of land until used, but is continually moving to some point of discharge. Thus, except in a very few places, ground water is a transient resource which cannot be conserved for the future merely by not using it.

A wise use of our ground water resources, therefore, requires careful investigation of the manner in which ground water moves and the laws governing that movement.

Experimental Basis of Darcy's Law

Flow of viscous liquids thru porous materials which they saturate follow a law first discovered experimentally by Henri Darcy, a French hydraulic engineer, in 1856.

Consider: "Uniform Laminar Flow thru Sand".

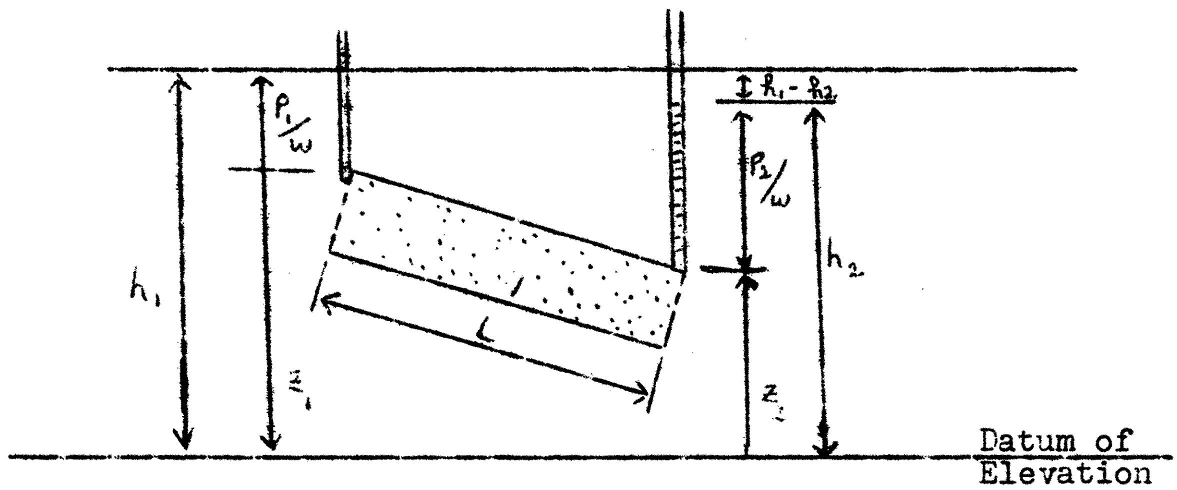


FIGURE I

Then from Darcy's exp. we have given that

$$Q = KA \frac{h_1 - h_2}{L} \quad \text{or} \quad Q = KAi \qquad (1)$$

Terms occurring in the equation are defined in the above sketch.

h_1 = inflow head

h_2 = outflow head

L = length of flow

A = cross-sectional area

K = coefficient of permeability

Piezometric head is defined as h

and $h = z + \frac{p}{w}$

Each of the terms making up "h" has the dimensions of energy per unit weight.

h_1 and h_2 may be regarded as potential energies per unit weight of water entering and leaving the sand respectively. Kinetic energy in the Darcy Exp. was equal to a constant and was also very small. In most ground water problems kinetic energy is negligible even though variable. The difference $h_1 - h_2$ = energy loss per unit weight of fluid. This is also the flow energy that is converted into heat as water flows from points of high to points of low piezometric head. To be more nearly correct one should write

$$h = z + \int_{p_0}^{p} \frac{dp}{w}$$

where w is considered some function of pressure between some standard p_0 and the prevailing pressure at the point of elevation, z. In general it will be permissible to neglect variation of specific weight in space on the other hand variation of specific weight should be considered when it varies with time. Ground water flows from points of high piezometric head to points of lower piezometric head.

<u>Range of Validity of Darcy's Law.</u> Darcy's law states that velocity of flow is proportional to the first power of the hydraulic gradient. Since velocity in laminar flow is also proportional to the first power of the gradient it can be inferred that flow must be laminar for Darcy's law to hold. Reynolds number defined as follows:

$$R = \frac{v_d}{\nu}$$

is the general index used to determine if flow is laminar or turbulent.

v = velocity of flow

d = ave. grain diameter with respect to pore space.

ν = kinematic viscosity

also

$$R = \frac{v d \rho}{\mu}$$

where

$$\nu = \frac{\mu}{\rho}$$

μ = dynamic viscosity

ρ = density

Jacob states that flow thru uniform spherical grains shows a departure from the laminar condition at values of R between 1 and 10 depending upon range of grain sizes and shapes. Departure from laminar flow can be expected in the immediate vicinity of a typical discharging well, it is here that ground water velocities reach a maximum. Large departures from laminar flow introduce error and will be considered as a separate problem later. On the other hand there is no lower limit of applicability of Darcy's law for even the smallest pores.

Permeability Coefficient

Solving equation (1) for K,

$$K = \frac{Q/A}{\frac{h_1 - h_2}{L}}$$

Common units for K are ft/sec. This coefficient was called the transmission constant by Slichter. It has been more recently called the Coefficient of Permeability, though quite inappropriately so because K depends not only on the permeability of the soil but also upon significant properties of the fluid flowing thru the sand. Fluid properties affecting flow are: 1. Viscosity, and 2. Specific weight. The fluid viscosity, μ, is a measure of the resistive force per unit area per unit transverse velocity gradient within the pores of the sand. Specific weight, w, may be considered as a driving force per unit volume per unit hydraulic gradient. The relationship between K, μ, w, and d can be determined directly by dimensional analysis as follows:

$$K = f(\mu, \; w, \; d)$$

$$\frac{L}{T} = \left(\frac{FT}{L^2}\right)^a \left(\frac{F}{L^3}\right)^b (L)^c$$

L	$1 = -2a + c - 3b$	Solving these equations
T	$-1 = a$	simultaneously it is
F	$0 = a + b$	found that:

$$a = -1$$

$$b = +1$$

$$c = +2$$

Then

$$K = \frac{Cd^2w}{\mu} \qquad (2)$$

Cd^2 depends on the character of the sand alone and hence is rightly called the Coefficient of Permeability or simply the permeability. C depends on the porosity of the sand, the range and distribution of grain sizes, the shape of the grains, and their orientation and arrangement, all of which are characterized by dimensionless length ratios or angles.

$$K = k\frac{w}{\mu}$$

$$k = Cd^2 \text{ has the dimensions of } (L^2)$$

The ratio of specific weight to viscosity varies from liquid to liquid and temperature to temperature. However, in many ground water problems the temperature does not vary appreciably and K may be regarded as a constant in the flow equation.

Units:

$$1 \text{ Centipoise} = 0.01 \text{ Poise} = 0.01 \frac{\text{dyne sec}}{\text{cm}^2}$$

$$\text{Darcy} = \frac{\text{centipoise cm}^3 \text{ sec}^{-1}/\text{cm}^2}{\text{atmosphere}/\text{cm}}$$

Conversions

$$\frac{478 \text{ Poise}}{\frac{\text{lb.sec.}}{\text{ft}^2}} \qquad \frac{929 \text{ Stokes}}{\text{ft}^2/\text{sec}}$$

The unit of Permeability used in the petroleum industry is the darcy as defined above.

Example:

Given that water with a kinematic viscosity $\nu = 1.21 \times 10^{-5}$ ft^2/sec flows thru a sand having a permeability of 7.8 darcys, find the transmission constant.

$$K = \frac{kw}{\mu} = \frac{gk}{\nu} = \frac{32.2}{(1.21 \times 10^{-5})} (7.8)\ (1.062 \times 10^{-11})$$

$$K = 2.20 \times 10^{-4} \text{ ft/sec.} = 19.0 \text{ ft/day} = 142 \text{ gpd/ft}^2$$

Notes on Permeability From Thesis by Fred. W. Kiefer at Colorado A & M College

Did not consider material containing cohesive or collodial particles.

Basic Formula:

$$V = K \frac{h_f}{L}$$

Disagreement exists on definition of "K" (See Dimensional Analysis, Page 19). Hazen suggests that "K" is a function of d and proposed using the 10% size, others have used d_{50} and d_{34}.

Also used is: $d = \frac{\sum nds}{n}$

n = no. of particles

d_s = sieve size assuming spherical shape

Hatch defined d as follows:

$$\frac{1}{d} = \frac{s}{100} \sum \frac{p}{d}$$

s = shape factor = 6 for spheres

Dimensional Analysis

$$\phi\,(\rho, \mu, h, L, V, g, d_m, (\sigma d, a, \sigma_c, r_m, \sigma m,) = 0$$

C is a function of these

Completing the dimensional analysis gives

$$\frac{h}{L} = \frac{C}{g} \; V^n \; d_m^{\,n-3} \; \left(\frac{\mu}{\rho}\right)^{2-n} \qquad (1_a)$$

If $n = 1$ the Hagen Poiseullie equation results.

If $n = 2$ the equation is in the Darcy form. ($\frac{h}{} = f \frac{L}{D} \frac{V^2}{2g}$)

Next Considered Porosity

$R = \frac{A}{p}$ = Hyd radius

Developed a "R" term for Soils

$$\frac{A}{p} \frac{L}{L} = \frac{\text{Volume of Voids}}{\text{Surface Area}}$$

Then substitute: Volume of Voids = $\frac{\alpha}{1 - \alpha}$

and $\frac{A}{p} \frac{L}{L} = \frac{\frac{\alpha}{1-\alpha}}{\text{Surface Area}}$ (Vol of Solids)

for spheres

$$\frac{\pi d^3}{6 \pi d^2} \; \frac{\alpha}{1 - \alpha} = \frac{d}{6} \; \frac{\alpha}{1 - \alpha}$$

σ = shape factor for spheres for sand we have a shape factor = 7.7.

Substitution in Eq. 1_a gives

$$\frac{h}{L} = \frac{c}{g} V^n \left(d \frac{\alpha}{1-\alpha}\right)^{n-3} \left(\frac{\mu}{\rho}\right)^{2-n}$$

Divide V by α to correct for velocity thru the pores.

$$\frac{h}{L} = \frac{c}{g} \left(\frac{V}{\alpha}\right)^n \left(\frac{d\,\alpha}{1-\alpha}\right)^{n-3} \left(\frac{\mu}{\rho}\right)^{2-n}$$

If n = 1

$$\frac{h}{L} = \frac{c}{g} \frac{V}{\alpha d^2} \left(\frac{1-\alpha}{\alpha^3}\right)^2 \frac{\mu}{\rho} \quad \text{Laminar flow eq.}$$

If n = 2

$$\frac{h}{L} = \frac{c}{g} \frac{V^2}{d} \left(\frac{1-\alpha}{\alpha^3}\right) \quad \text{For turbulent or Laminar flow.}$$

If flow is laminar

$$C = \psi R, \qquad \text{(same C)}$$

or C is some function of Reynolds number.

From $V = K \frac{h}{L}$ the following equation is obtained

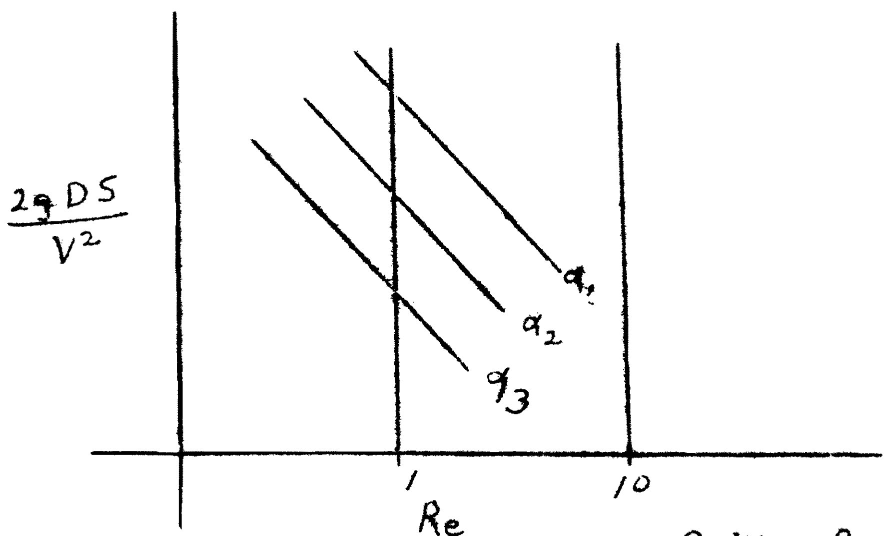
$$\frac{h}{L} = \frac{V}{K}$$

Now equate this to the previous equation for $\frac{h}{L}$ and solve for K

$$K = 2g \frac{\rho}{\mu} \frac{d^2}{c} \frac{\alpha^3}{(1-\alpha)^2}$$

a two (2) was introduced into the equation to make it consistent with other literature.

Represent the above by means of a graph (Like the one of Nickuradse)



$Re\,\Psi = $ Constant

Considering a graded material

$$K = 2g\left(\frac{\rho}{\mu}\right)\frac{d^2}{C}\frac{\alpha^3}{(1-\alpha)^2} = \text{Transmission Coef.}$$

$$k = \frac{\alpha^3}{C(1-\alpha)^2}\,2gd^2 = \text{Coef. of permeability}$$

and

$$\frac{\alpha^3}{C(1-\alpha)^2} = C_s$$

As stated previously the critical range for N_R (R_e) is 1-10, an attempt is being made to narrow these limits down.

Values of C_s determined are:

C_s	= 320 - 350	20-30 Ottawa Sand
C_s	= 650 - 670	Poudre River Sand
C_s	= 320 - 350	Uniform Lead Shot
C_s	= 450 - 458	Steel Bearings

The lead shot varied in size from 0.01" to 0.3", one size being checked at a time. The value of C_s remained constant.

The steel bearings were 1/4" dia. arranged to give 40% pore space. By arrangement of the bearings the % pore space can be varied from 26% to 40%. In the case of the lead shot the pores seem to offer less resistance to flow, being more streamlined.

From above

$$C_s = f\,(C_m\,,\ \sigma d\,,\ r\,)$$

c_m = shape factor

σd = standard deviation for size

r = roughness

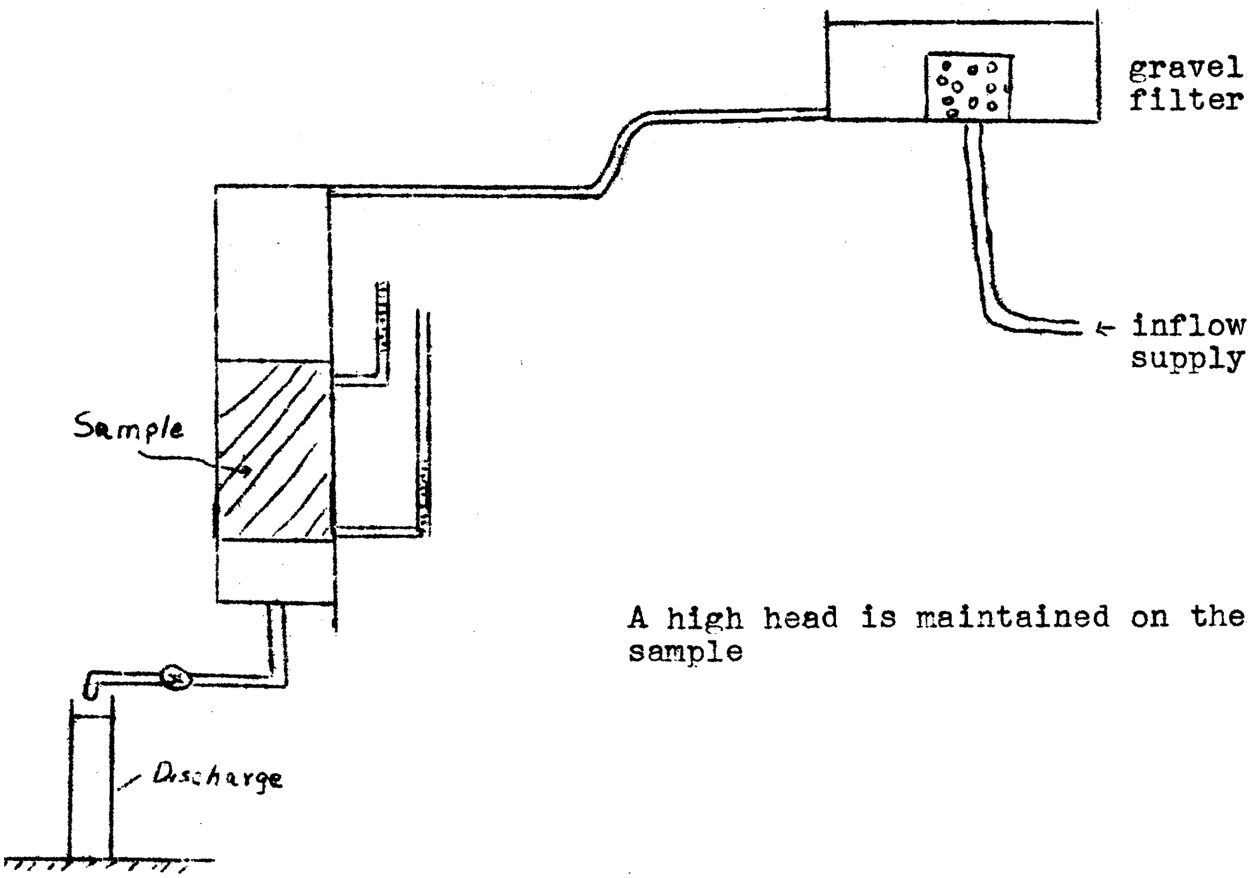
$$C_s = \left(\frac{1-\alpha^2}{\alpha^3}\right) C$$

$\frac{\alpha^3}{(1-\alpha)^2}$ changes 1.5 times for the following range in porosity (26% - 40%).

$$\text{Porosity} = \frac{\text{Vol Voids}}{\text{Vol Solids}}$$

Permeability is inversely proportional to C_s.

Sketch of Apparatus



The soil sample tested ranged in height from 6" to 12".

For more complete sketch see Mr. Kiefer's thesis. (Not completed at present, July, 1952.)

Derivation of the Fundamental Equations

Generalization of Darcy's Law. In the unidirectional case of fig (1) the head loss $h_1 - h_2$ may be considered uniformly distributed along the direction of flow. On the scale of pore size, of course, the head is unevenly distributed, for along any typical stream line the gradient of the head changes continuously. However, if the head is averaged at different points over small volumes containing a great number of pores the gradient of this one head will be sensibly uniform. Considering flow per unit area in a similar fashion it will also be uniform. Considering the physical meaning implied we can write

$$v = - k \frac{\partial h}{\partial s}$$

v and h are regarded as continuous functions of s , the distance along the average direction of flow. If we have an anisotropic soil as regards permeability, the components of velocity in the three rectangular coordinate directions may be assumed to be

$$\left.\begin{aligned} v_x &= - K_x \frac{\partial h}{\partial x} \\ v_y &= - K_y \frac{\partial h}{\partial y} \\ v_z &= - K_z \frac{\partial h}{\partial z} \end{aligned}\right\} \quad \text{Anistropic Soil} \qquad (3)$$

where K_x , K_y , and K_z are transmission coefficient for flow in the x , y , and z directions, respectively, each being the product of $\frac{w}{\mu}$ and the respective permeability k_x , k_y , or k_z . The negative signs are required because the K terms are positive constants and water moves in the direction of decreasing head. The velocity at any point in a sand would be equal to the vector sum of the three components of equation 3. Present considerations will be restricted to systems that are isotropic as regard permeability for which

$$\left.\begin{aligned} v_x &= - K \frac{\partial h}{\partial s} \\ v_y &= - K \frac{\partial h}{\partial y} \\ v_z &= - K \frac{\partial h}{\partial z} \end{aligned}\right\} \quad \text{Isotropic Soil} \qquad (4)$$

Significance of Ground Water Velocity Potential. According to equa. 4 the component of velocity in any direction in an isotropic material is the product of transmission **coefficient** and negative rate of change of head in that direction.

Total velocity vector = (K) (max. rate of decrease of head in space)

= (K) (negative gradient of scolar h)

For convenience ϕ = velocity potential = Kh.

The negative gradient of this potential is considered equal to the velocity.

The existance of a velocity potential implies irrotational motion. This first off might seem to contradict two facts:

1. In Ground Water flow loss of energy is thru viscous resistance.

2. Motion of viscous liquids is invariably rotational.

It is true that motion within individual pores is rotational because of viscous resistance occurring in the stream tube. However, when a volume of material is considered, containing many pores, the rotations occurring within the individual tubes cancel and we have irrotational motion.

Continuity and Compressibility

Consider the parallelepiped.

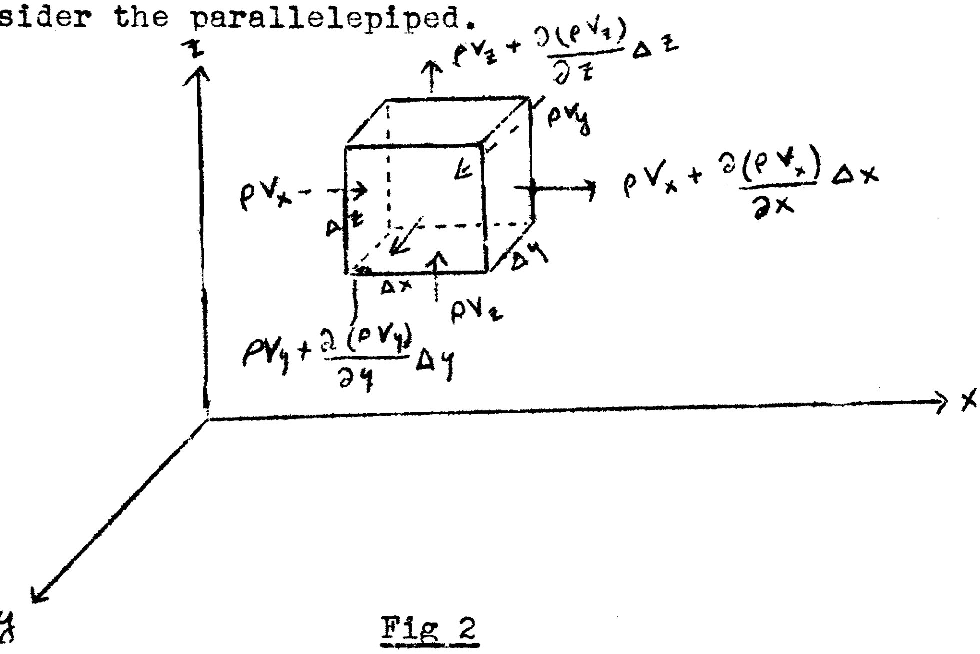


Fig 2

Water flowing thru the faces as shown. By the law of conservation of matter net inward flux must equal the rate at which water is accumulating within that volume.

Net inward flux =

$$(\rho V_x \Delta y \Delta z + \rho V_y \Delta x \Delta z + \rho V_z \Delta x \Delta y) - (\rho V_x \Delta y \Delta z + \frac{\partial(\rho V_x)}{\partial x} \Delta x \Delta y \Delta z + \rho V_y \Delta x \Delta z + \dots\dots + \frac{\partial(eV_y)}{\partial y} \Delta y \Delta x \Delta z + \rho V_x \Delta x \Delta y + \frac{\partial(eV_z)}{\partial z} \Delta z \Delta x \Delta y$$

on passing to Differential and factoring equation (5) results.

$$\text{Net inward flux} = -\left[\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z}\right] dx \; dy \; dz \tag{5}$$

The mass of water contained in the elemental volume is

$$\Delta M = \rho\theta \; \Delta x \; \Delta y \; \Delta z$$

θ = porosity of soil or sand

Sand as well as H_2O is assumed compressible

then θ, ρ, and Δz will be variable and

Δx, & Δy will remain constant relatively speaking

Rate of change of Mass of Water is

$$\frac{\partial(\Delta M)}{\partial t} = \Delta x \, \Delta y \; \frac{\partial}{\partial t}(\rho, \theta, \Delta z)$$

$$= \Delta x \, \Delta y \left[\frac{\partial p}{\partial t}\theta\Delta z + \frac{\partial\theta}{\partial t}\rho\Delta z + \frac{\partial(\Delta z)}{\partial t}\theta\rho\right] \tag{6}$$

The height of the elemental volume varies with the vertical component of compressive stress

σ_z = compressive stress

α = vertical compressibility of sand

$$E_\alpha = -\frac{d(\sigma_z)}{\frac{d(\Delta z)}{\Delta z}}$$

$$d(\Delta z) = \frac{1}{E_\alpha}\Delta z \; d(\sigma_z)$$

$$d(\Delta z) = -\alpha\Delta z \; d(\sigma_z)$$

$$d(\Delta z) = -\alpha\Delta z \; d(\sigma_z)$$

or

$$\frac{\partial(\Delta z)}{\partial t} = -\alpha\,\Delta z \;\frac{\partial\sigma_z}{\partial t}$$

Consider that the volume of solid material in the parallelepiped remains constant since compressibility of individual sand grains is small compared to compressibility of water and change in porosity.

Then

$$\Delta V_s = (1 - \theta)\,\Delta x \Delta y \Delta z = \text{Constant}$$

or

$$d\,(\Delta V_s) = \left[(1 - \theta)(d(\Delta z)) - \Delta z\,(d\theta)\right] \Delta x / \Delta y = 0$$

Whence

$$\frac{d\,\theta}{1 - \theta} = \frac{d\,(\Delta z)}{z}$$

And

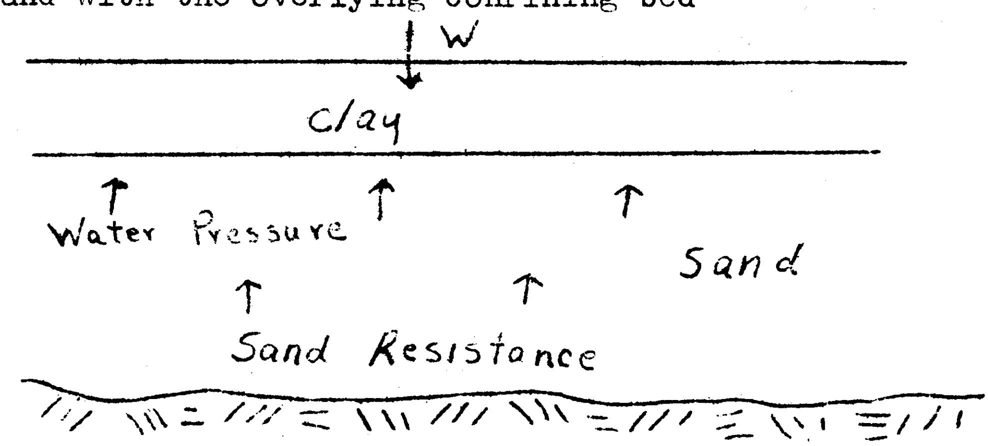
$$\frac{\partial \theta}{\partial t} = \frac{1 - \theta}{\Delta z} \frac{\partial (\Delta z)}{\partial t} \qquad \text{Substitute } \frac{\partial (\Delta z)}{\partial t} = -\alpha \Delta z \frac{\partial \sigma z}{\partial t}$$

In the above to give

$$\frac{\partial \theta}{\partial t} = \frac{1 - \theta}{\Delta z} \left(-\alpha \Delta z \frac{\partial \sigma z}{\partial t}\right) = -\alpha(1 - \theta) \frac{\partial (\sigma z)}{\partial t}$$

The compressibility of sand is relatively important only when the sand is completely saturated with water which is confined by an impermeable layer of clay or similar material overlying the sand. For the unconfined case all that results from compression is a vertical displacement of the water table.

In the confined flow system the vertical component of compressive stress, σz, and water pressure, P, are in equilibrium with downward acting forces on the plane of contact of the sand with the overlying confining bed



Rock or Similar Material

Then: (1) Water pressure acts thru the entire Volume.

(2) Compressive stress is assumed to act uniformly over whole plane of contact.

For the above

$$P + \sigma_z = \text{Vertical Component of stress on the plane of contact.}$$

If arching action can be neglected

$$P + \sigma_z = \text{dead wt/unit area at plane of contact.}$$

The problem is then a <u>Vertical Strain Problem</u>. Include in the dead wt. the wt. of the atmosphere.

$$P + \sigma_z = C$$

or

$$dP = d\,\sigma_z = 0$$

or

$$dP = -\,d\,\sigma_z$$

We see then that increasing the water pressure decreases the solid compressive stress. Assume that water pressure increases at a rate $= \frac{\partial P}{\partial t}$ find the rate of increase in the density of the water.

$$\text{Bulk Modulus} = E_o = \frac{dP}{\frac{d\rho}{\rho}}$$

Solving for $d\rho$

$$d\rho = \frac{\rho\, dP}{E}$$

$$\frac{1}{E} = \text{the compressibility}$$

$$d = \beta\,\rho\, dP$$

or for variation with respect to time

$$\frac{\partial \rho}{\partial t} = \beta \rho_o \frac{\partial P}{\partial t}$$ This is the rate density of water increases as pressure increases. (9)

ρ_o = reference density usually taken as atmospheric.

<u>Thickness of sand increases at rate.</u>

$$\frac{\partial(\Delta z)}{\partial t} = \alpha\,\Delta z\,\frac{\partial P}{\partial t} \tag{10}$$

<u>Porosity of sand increases at rate.</u>

$$\frac{\partial \theta}{\partial t} = \alpha(1 - \theta) \quad \frac{\partial P}{\partial t} \tag{11}$$

Substitute equations (9) , (10) and (11) in equation 6.

$$\partial\left(\frac{\Delta M}{\partial t}\right) = \left[\theta \beta \rho_0 \frac{\partial P}{\partial t} \Delta z + \rho \alpha (1 - \theta) \frac{\partial P}{\partial t} \Delta z + \rho \theta \alpha \frac{\partial P}{\partial t} \Delta z\right] \Delta x \Delta y$$

$$= \left[\theta \beta \rho_0 + \rho \alpha (1 - \theta) + \rho \theta \alpha\right] \frac{\partial P}{\partial t} \Delta x \quad \Delta y \quad \Delta z$$

$$= \left[\theta \beta \rho_0 + \rho \alpha - \rho \alpha \theta + \rho \alpha \theta\right] \quad \frac{\partial P}{\partial t} \Delta x \quad \Delta y \quad \Delta z$$

$$= \left[\theta \beta \rho_0 + \alpha \rho\right] \frac{\partial P}{\partial t} \Delta x \quad \Delta y \quad \Delta z$$

Pass to differentials and

Equate to the net inward flux

$$(\theta \beta \rho_0 + \alpha \rho) \frac{\partial P}{\partial t} dx \ dy \ dz = -\left[\frac{\partial(\rho V x)}{\partial x} + \frac{\partial(\rho V y)}{\partial y} + \frac{\partial(\rho V z)}{\partial z}\right] dx \ dy \ dz$$

Simplifying the resultant equation is

$$-\frac{\partial(\rho V x)}{\partial x} - \frac{\partial(\rho V y)}{\partial y} - \frac{\partial(\rho V z)}{\partial z} = \theta \rho \left(\beta + \frac{\alpha}{\theta}\right) \frac{\partial P}{\partial t} \tag{12}$$

The very small differences between ρ and ρ_0 have been neglected.

<u>Example</u>

An extensive confined sand has a thickness = 200'.0.

θ = 35%.

Area is overlain by an extensive body of tide water.

Tide depth is nearly uniform - (Assume Uniform.)

The water level in a tightly cased well tapping the sand fluctuates with the tide. The amplitude of fluctuation is 42% of tide fluctuation.

Find:

(1) Compressibility of sand under this stress variation.

Neglect the small volume of water flowing into and out of the well.

One Dimensional Prob. since flexual rigidity of thin confining beds may be neglected

$$d\sigma_z + dP = wdH$$

$$H = \text{stage of tide}$$

The relative amplitude of tidal fluctuations in the well is

$$\frac{\frac{dP}{w}}{dH} = \frac{d\,P}{dP + d\partial_z}$$

This is shown as follows: Starting with original equation

$$dP \times d\sigma_z = wdH$$

Multiply both sides by dP

$$dP^2 + d\,(\sigma_z)\,dP = wdHdP$$

Divide thru by wdH

$$\frac{(dP)^2}{wdH} + \frac{d\sigma z\ dP}{wdH} = dP$$

factor

$$\frac{d^P/w}{dH}$$

$$\frac{d^P/w}{dH}\ (dP + d\,\sigma_z) = dP$$

or

$$\frac{d^P/w}{dH} = \frac{dP}{dP + d\sigma z} = \frac{d^P/d\,O\,z}{1 + \frac{dP}{d\sigma z}} \qquad (1)$$

The volume of water in the sand is $V_w = \theta\, V$ where V = total volume of sand. As the sand and water are compressed the change in overall volume = change of water volume if sand grains are assumed incompressible.

Then

$$d\,\forall_w = d\forall$$

And

$$\frac{d\,\forall_w}{\forall_w} = \frac{d\forall}{\theta\forall} \qquad \theta\forall = \text{Volume occupied by water}$$

Then

$$E = -\frac{dP}{\frac{d\forall_w}{\forall_w}} \quad \text{or} \quad \frac{d\forall_w}{\forall_w} = -\frac{1}{E}\,dP = -\beta dP$$

And

$$\frac{d\forall_w}{\forall_w} = -\beta\,dP \tag{2}$$

Also

$$E = -\frac{d\sigma z}{\frac{d\forall}{\forall}} \quad \text{or} \quad \frac{d\forall}{\forall} = -\frac{1}{E}\,d\sigma_z$$

Divide both sides by θ

$$\frac{d\forall}{\theta\forall} = -\frac{\alpha d\,\sigma z}{\theta} \tag{3}$$

Eq. (2) and Eq. (3) are equal so equate and

$$-\beta dP = -\frac{\alpha\, d\,\sigma z}{\theta}$$

Then

$$\frac{dP}{d\,\sigma z} = \frac{\alpha}{\theta\beta} \tag{4}$$

sub eq. (4) in eq.(1)

and

$$\frac{dP/w}{dH} = \frac{\frac{X}{\theta\beta}}{1 + \frac{\alpha}{\theta\beta}} = 0.42$$

or

$$\frac{\frac{\alpha}{\theta\beta}}{1 + \frac{\alpha}{\theta\beta}} = 0.42$$

solve for α

$$\theta = 0.35$$

$$\beta = 3.3 \times 10^{-6} \quad in^2/lb$$

$$\alpha = 0.84 \times 10^{-6} \; in^2/lbs$$

Example III The sand of the previous example is subject to uniform variation of atmospheric pressure. Find the relation between the fluctuations of atmospheric pressure and simultaneous fluctuation of water level in a tightly cased well tapping the sand neglecting again the volume of water flowing into and out of the well.

$$P + \sigma_z = P_a$$

$$dP + d\sigma_p = dP_a$$

P_a = Atmospheric Press

The change of water pressure in the sand is given by

$$dP = dP_a + wdh$$ where h is the water

surface elevation in the well. As before we can put the equation above in the following form

$$\frac{dh}{\frac{dPa}{w}} = \frac{dP - dPa}{dPa} = \frac{-d\sigma z}{dP + d\sigma z} = \frac{-1}{1 + dP/d\sigma_z}$$

and

$$\frac{dP}{d\sigma_z} = \frac{\alpha}{\theta\beta}$$

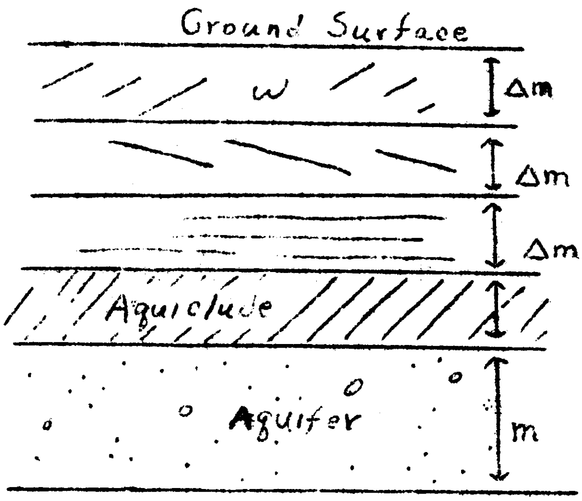
or

$$\frac{dh}{dPa/w} = \frac{-1}{1 + \frac{\alpha}{\theta\beta}} = \frac{-1}{1 + \frac{0.82 \times 10^{-6}}{(0.35)(3.3 \times 10^{-6})}} = -.58$$

In other words the water level in the well is depressed 58% as much as the atmospheric head rises. Water level fluctuations in the well reproduces the record of a water barometer although inverted and reduced to 58% of normal amptitude.

Barometric Efficiency and Tidal Efficiency. From an Article by J. E. Jacob in Trans. Am. Geophys. Union, 1940, page 582.

Consider an artesian aquifer overlain by several layers of different thicknesses.



Let w = weight of unit volume of material in stratum thickness Δm.
P_o atmospheric pressure.
b = proportion of plane of contact between aquifer and aquiclude on which hydrostatic pressure is effective.
Note: $b = 1.0$ for uncemented granular material
b = porosity for solid aquifer with tubular channels.
P = pressure of water within aquifer
σ_z = unit stress in aquifer

Then $P_o + \Sigma w_1 \Delta m_1 = (\sigma_z + b P)$

If $\Sigma w_1 \Delta m_1$ is constant, and the barometric pressure changes, then

$$\Delta P_o = \Delta\sigma_z + b\Delta P$$

If B = barometric efficiency of a well in this aquifer

$$\Delta P = \Delta P_o = B\Delta P_o \quad \text{by definition}$$

$$= (1 - B)\Delta P_o$$

therefore, $\Delta P = (1-B)(\Delta\sigma_z + b\Delta P)$

Then $$\frac{\Delta P}{\Delta\sigma_z} = \frac{1-B}{1-b(1-B)}$$

If the aquifer and the contained water are compressed by extensive uniform load, the decrease in volume of water is equal to the decrease in volume of aquifer.

$$\Delta V_w = \Delta V$$

Therefore $\dfrac{\Delta V_w}{V_w} = \dfrac{\Delta V}{\Theta V}$, where Θ = porosity of aquifer

(assuming the actual particles of aquifer are incompressible and no leakage occurs)

Also $\dfrac{\Delta V_w}{V_w} = -\dfrac{\Delta P}{E_\beta}$ and $\dfrac{\Delta V}{V} = -\dfrac{\Delta\sigma z}{E_s}$

where E_β and E_s are moduli of elasticity of water and aquifer respectively

therefore $\dfrac{\Delta P}{E_\beta} = \dfrac{\Delta\sigma z}{E_s}$ and $\dfrac{\Delta P}{\Delta\sigma_z} = \dfrac{E_\beta}{\Theta E_s}$

So that $\dfrac{E_\beta}{\Theta E_s} = \dfrac{1-B}{1-b(1-B)}$

$$\text{Therefore} \quad B = 1 - \frac{\frac{E\beta}{\theta E_s}}{1 + \frac{b}{\theta}\frac{E\beta}{E_s}}$$

A similar analysis may be made considering that P_o is constant and that the total load on the aquifer increases due to a tide level change on the overlying area.

This gives a value for tidal efficiency = C

$$C = \frac{\frac{E\beta}{\theta E_s}}{1 + \frac{b\ E\beta}{\theta\ E_s}}$$

Note from value of B

$$C + B = 1.0$$

For a granular aquifer, $\beta = 1.0$

$$\text{and} \quad B = 1 - \frac{\frac{E\beta}{\theta E_s}}{1 + \frac{E\beta}{\theta E_s}}$$

$$= \frac{1}{1 + \frac{E\beta}{\theta E_s}}$$

Now, from equation on page 31 the storage coefficient S for a confined aquifer is

$$S = \theta\ w\ m\ (\beta + \frac{\alpha}{\theta})$$

$$= \theta\ w\ m\ (\frac{1}{E\beta} = \frac{1}{\theta E_s})$$

$$= \frac{\theta\ w\ m}{E\beta}\ (1 - \frac{E\beta}{\theta E_s})$$

$$S = \frac{\theta\ w\ m}{E\beta}\ (\frac{1}{B})$$

$$\text{or } S = \frac{\theta\ w\ m}{E\beta}\ (\frac{1}{1 - C})$$

<u>Example</u>: The tidal efficiency of a well in a confined aquifer is observed to be 44%. The aquifer is 85 feet thick and its porosity is 0.35. Assuming E = 300,000 psi, what is the storage coefficient for this reservoir?

$$S = \frac{0.35 \times 62.4 \times 85}{300.00 \times 144} \left(\frac{1}{1 - 0.44}\right)$$

$$= 7.65 \times 10^{-5}$$

Note: the value of storage coefficient obtained in this manner assumes that no leakage of water occurs through overlying or underlying aquicludes.

Differential Equation for Steady Flow. Consider Eq. 12

$$-\frac{\partial(\rho V_x)}{\partial x} - \frac{\partial(\rho V_y)}{\partial y} - \frac{\partial(\rho V_z)}{\partial z} = \theta \rho \left(\beta + \frac{\alpha}{\theta}\right) \frac{\partial P}{\partial t} \qquad (12)$$

For steady flow pressure distribution does not vary with time, hence Eq. 12 reduces to

$$\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z} = 0 \qquad \text{Continuity Equation for steady flow.}$$

Performing the operation indicated

$$\rho \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}\right) + V_x \frac{\partial \rho}{\partial x} + V_y \frac{\partial \rho}{\partial y} + V_z \frac{\partial \rho}{\partial z} = 0 \qquad (13)$$

Velocity components are expressed in terms of components of the hydraulic gradient as follows

$$\begin{aligned} V_x &= -K \frac{\partial h}{\partial x} \\ V_y &= -K \frac{\partial h}{\partial y} \qquad (4) \\ V_z &= -K \frac{\partial h}{\partial z} \end{aligned}$$

Also the components of the density gradient may be expressed as follows:

$$\left.\begin{aligned} \frac{\partial \rho}{\partial x} &= \rho_o \beta \frac{\partial P}{\partial x} = \rho_o^2 \beta g \frac{\partial h}{\partial x} \\ \frac{\partial \rho}{\partial y} &= \rho_o \beta \frac{\partial P}{\partial y} = \rho_o^2 \beta g \frac{\partial h}{\partial y} \\ \frac{\partial P}{\partial z} &= \rho_o^2 \beta g \left(\frac{\partial h}{\partial z} - 1\right) \end{aligned}\right\} \qquad (14)$$

Substitute equations 4 and 14 in (13) to get (15)

<u>From 4</u> we have

$$V_x = K \frac{\partial h}{\partial x}$$

$$\frac{\partial V_x}{\partial x} = - K \frac{\partial^2 h}{\partial x^2}$$

Eq. for V_y and V_z take same form.

$$K \rho \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right) + K \rho_o^2 \beta g \left[\left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2 + \left(\frac{\partial h}{\partial z}\right)^2 - \frac{\partial h}{\partial z} \right] = 0 \quad (15)$$

2nd Order 0

Giving

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (16)$$

Eq. 16 is used in steady conduction of electricity & heat, in steady diffusion, and the theory of elastic membranes.

<u>Differential Equation for Unsteady Flow.</u> Start with Eq. (12)

$$- \frac{\partial(\rho V_x)}{\partial x} \quad \frac{\partial(\rho V_y)}{\partial y} - \frac{\partial(\rho V_z)}{\partial z} = \theta \rho \left(\beta + \frac{\alpha}{\theta}\right) \frac{\partial P}{\partial t} \quad (12)$$

Differentiation of $\rho (V_x + V_y + V_z)$

Gives

$$\rho \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}\right) + V_x \frac{\partial \rho}{\partial x} + V_y \frac{\partial \rho}{\partial y} = V_z \frac{\partial \rho}{\partial z} = \theta \rho \left(\beta + \frac{\alpha}{\theta}\right) \frac{\partial P}{\partial t}$$

Velocity Components as before are

$$\left.\begin{aligned} V_x &= - K \frac{\partial h}{\partial x} \\ V_y &= - K \frac{\partial h}{\partial y} \\ V_x &= - K \frac{\partial h}{\partial z} \end{aligned}\right\} \quad (4)$$

And the components of density gradient may be expressed as follows. (See Eq. 18)

$$\left.\begin{aligned}
\frac{\partial \rho}{\partial x} &= \rho_o \ \beta \frac{\partial P}{\partial x} = \rho_o^2 \ \beta g \ \frac{\partial h}{\partial x} \\
\frac{\partial \rho}{\partial y} &= \rho_o \ \beta \frac{\partial P}{\partial y} = \rho_o^2 \ \beta g \ \frac{\partial h}{\partial y} \\
\frac{\partial P}{\partial z} &= \rho_o^2 \ \beta \ g \ \left(\frac{\partial h}{\partial z} - 1\right)
\end{aligned}\right\} \qquad (14)$$

Substitute Eq. 4 and 14 in the above, first noting the following

From Eq. 4

$$\frac{\partial V_x}{\partial x} = - K \frac{\partial^2 h}{x^2}$$

$$\frac{\partial V_y}{\partial y} = - K \frac{\partial^2 h}{\partial y^2}$$

$$\frac{\partial V_z}{\partial z} = - K \frac{\partial^2 h}{\partial z^2}$$

Then we obtain

$$K \rho \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right) + K \rho_o^2 \beta g \overbrace{\left[\left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2 + \left(\frac{\partial h}{\partial z}\right)^2 - \frac{\partial h}{\partial z}\right]}^{0} =$$

$$= \theta \rho \left(\beta + \frac{\alpha}{\theta}\right) \frac{\partial P}{\partial t}$$

Second Part, left hand side is very small $\rightarrow$ 0 leaving

$$K \rho \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right) = \theta \rho \left(\beta + \frac{\alpha}{\theta}\right) \frac{\partial P}{\partial t}$$

or

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{\theta}{K} \left(\beta + \frac{\alpha}{\theta}\right) \frac{\partial P}{\partial t}$$

Then

$$P = wh$$

And

$$\frac{\partial P}{\partial t} = w \frac{\partial h}{\partial t}$$

Substituting

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{\theta w}{K} \left(\beta + \frac{\alpha}{\theta}\right) \frac{\partial h}{\partial t} \tag{17}$$

This is the approximate partial differential equation governing the unsteady flow of water in compressible sands.

Equations of the same form occur in the theories of unsteady flow of heat and electricity.

<u>For the Special Case</u>

For flow in a uniformly thick and essentially horizontal bed of thickness, b , Eq. 17 becomes

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{\theta wb}{kb} \left(\beta + \frac{\alpha}{\theta}\right) \frac{\partial h}{\partial t} \tag{17}$$

Since $\frac{\partial^2 h}{\partial z^2} \rightarrow 0$

Then:

If the coefficient of storage is defined as

$$S = \theta wb \left(\beta + \frac{\alpha}{\theta}\right)$$

the volume of water that a unit decline in head releases from storage in a vertical column of the aquifer of unit x-section

and

the product $Kb = T$

T = transmissivity

(since bed is transmissive while the water is transmissible we call it transmissivity coefficient)

We then have for Eq. 18

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T} \frac{\partial h}{\partial t} \tag{19}$$

T is a function of permeability, thickness of bed, specific weight and viscosity of water. The latter two vary with temperature.

Dimensions of T are $\left[L^2/T \right]$

The coefficients S and T may be regarded as emperical values to be determined principally by pumping test techniques. (Will discuss later). A rough estimate of S however can be made.

<u>Given</u>:

$$w = 62.4$$
$$b = 200$$
$$\theta = 0.35$$
$$\beta = 3.3 \times 10^{-6} \ \ in^2/\#$$
$$\alpha = 2.0 \times 10^{-6} \ \ in^2/\#$$

Then

$$S = \theta wb \left(\beta + \frac{\alpha}{\theta} \right)$$

Substituting

$$S = (0.35)(\overset{62.4}{w})(200) \left[\frac{3.3}{10^6} + \frac{2.0}{(10)^6(0.35)} \right] \frac{1}{144} = 0.000270$$

b , θ , and β are generally known are or determinable.

α is not easily evaluated.

For an area = 1000' x 1000' the water released from storage = 270 ft^3 for a head decline = 1.0'

<u>Example</u>

From a pumping test

$$S = 0.00028$$

<u>Given</u>:

$$b = 200'$$
$$\theta = 0.35$$

Determine the compressibility of the sand under the stress variation accompanying the lowering of head caused by the pumping. Find the fraction of storage attributed to expansibility of the water and that attributed to compressibility of the sand.

$$S = \theta wb \left(\beta + \frac{\alpha}{\theta} \right)$$

Can write as

$$S = \theta wb \beta \left(1 + \frac{\alpha}{\theta \beta} \right)$$

$\theta wb \beta$ = storage coefficient sand would have if incompressible

One (1) in parenthesis stands for that part of H_2O derived due to expansion of water.

$\frac{\alpha}{\theta\beta}$ = part derived from compression of sand

$$\theta w b \beta = (0.35)(62.4)(200)\frac{(3.3 \times 10^{-6})}{144} = 1.0 \times 10^{-4}$$

Compressibility of sand = α

$$S = \theta w b \beta \left(1 + \frac{\alpha}{\theta\beta}\right)$$

Substitute

$$0.00028 = 1 \times 10^{-4} \left(1 + \frac{\alpha}{\theta\beta}\right)$$

$$2.8 = 1 + \frac{\alpha}{\theta\beta}$$

$$\frac{\alpha}{\theta\beta} = 1.8$$

$$\alpha = 1.8\ \theta\beta$$

$$= (1.8)(0.35)(3.3 \times 10^{-6}) = \underline{2.1 \times 10^{-6} \qquad \alpha}$$

comp. of sand

Note

$$\frac{\alpha}{\theta\beta} = 1.8$$

Fraction derived from expansion of water

$$= \frac{1}{2.8} = 36\%$$

Fraction derived from compression of sand

$$= \frac{1.8}{2.8} = 64\%$$

Boundary Conditions.

General solutions of Equations 16, 17, 18, and 19 may be obtained by the techniques of partial differential equations. Solutions of practical interest will be those particular solutions that satisfy certain boundary conditions regarding distribution and variation of head on one or more boundaries or the flow across them. In general there are two boundary conditions for each independent coordinate. In unsteady flow there will be one additional specification regarding distribution of head with time. The foregoing partial differential equations are linear in h. Particular solutions may be added to different boundary conditions or initial conditions or different combinations thereof may be combined linearly to obtain useful solutions to practical problems, Furthermore, a particular solution that satisfies a certain set of boundary conditions is unique, it is the only independent solution for that problem. (Reference: "Uniqueness Theorem", any text on potential theory.) So far we have idealized the ground water system as a continuous one, it is expedient to also idealize conditions on the boundary of the system in order that they too can be expressed mathematically.

There are several kinds of ground water boundaries in nature:

1. Underlying and overlying rock beds

2. Contiguous rock masses (as faults)(as along a wall of a buried rock valley)

3. Dikes.

Across such boundaries

$$V_n = - K \frac{\partial h}{\partial h} = 0$$

n = direction of a line normal to the bounding surface.

Boundary is a stream line in 2-dimensional flow and a stream surface in 3-dimensional flow.

Permeable or semi-permeable boundaries are formed by:

1. bottoms of rivers
2. bottoms of canals
3. bottoms of lakes
4. bottoms of other bodies of surface H_2O

These may be treated as equipotential surfaces if the body of surface water is large in volume, so its level is uniform to independent of changes in ground water flow.

The uniform head on a boundary of this kind could, however, vary with time.

We know that flow lines are perpendicular to lines of equal potential. Accordingly flow lines entering or leaving bounding surfaces of uniform head in ground water systems do so at right angles to those surfaces if the material is isotropic.

For "V" perpendicular to a bounding surface, its component along the surface = 0.

$$V_n = - K \text{ or } \frac{\partial h}{\partial h} = 0$$

where n is the direction of some line drawn normal to the velocity vector.

The expression $\frac{\partial h}{\partial n} = 0$ says h = c over the boundry. In some cases flow across a permeable boundry is specified, like pumping a well where total flow thru the bounding surface may be held constant. Then

$$V = - K \frac{\partial h}{\partial s} = \text{constant and } \frac{\partial v}{\partial t} = 0$$

There are permeable or semi permeable boundaries formed by the surface of the ground along the banks of a body of surface water (so called seepage faces) from which ground water emerges either to evaporate or trickle down the boundary face. Modified seepage faces occur above the water surface in shallow wells that have screens at that level. Such faces may be regarded as surfaces of uniform atmospheric pressure or $P = 0$ and

$h = z$ from $h = z + \frac{P}{w}$. In most problems of steady unconfined

flow where downward percolation of surface water to the water table may be disregarded the water table is taken as the upper bounding surface of flow so

$$P = 0 \quad \text{and} \quad h = z$$

Water table is defined by height of rise in piezometric tubes and wells just topping the zone of saturation. The water table under these conditions is a stream surface and in the special case of two dimensional flow in a vertical plane is a stream line, or

$$\frac{\partial h}{\partial n} = 0$$

If rate of infiltration is not negligible, the stream lines instead of lying in the water surface cross it at angles.

$$P = 0$$

$$h = z \qquad \text{very difficult to solve}$$

$$\text{but } \frac{\partial h}{\partial n} = 0$$

More will be said later about special simplifications necessary to satisfy certain types of boundary conditions.

Example 5. The rate of infiltration downward to the water table is assumed to have a constant and uniform magnitude w

of dimensions $(\frac{L}{T})$. Find the relation between the angle of

inclination of the water table to the horizontal and the refraction of the flow lines as they cross the water table.

α = angle between refracted flow lines and the vertical.

w = rate of infiltration downward to the water table.

$\frac{\Delta x}{w}$ = distance between neighboring flow lines of unsaturated flow.

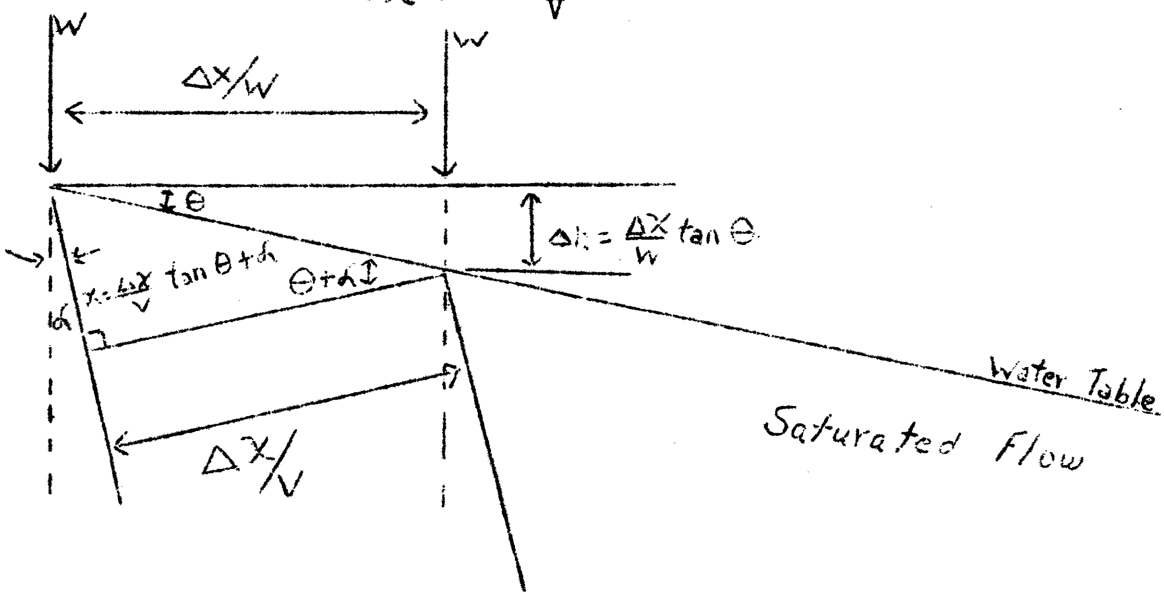
$\Delta h = \frac{\Delta x}{w} \tan \theta$ = difference of head between two points where these above flow lines reach the water table.

$\frac{\Delta x}{V}$ = distance between same two flow lines below the water table.

V = velocity of unsaturated **flow.**

The distance traveled on the higher stream line (or the lift in the sketch) from its intersection with the water table in losing a unit head Δh is

$$\Delta x \tan \frac{\theta + \alpha}{V}$$



$$\text{Tan } \theta = \frac{\Delta h}{\frac{\Delta x}{W}} \qquad \text{or } \Delta h = \frac{\Delta x}{W} \tan \theta$$

and

$$\text{Tan } \theta + \alpha = \frac{x}{\frac{\Delta x}{V}}$$

or

$$x = \frac{\Delta x}{V} \text{ Tan } \theta + \alpha$$

Can now calculate the hydraulic gradient

$$V = K \frac{\partial h}{\partial s}$$

$$\frac{V}{K} = \frac{\partial h}{\partial s} = \frac{\Delta h}{\frac{\Delta x}{V}} \tan (\theta + \alpha)$$

Sub value of Δh

$$= \frac{\frac{\Delta X}{w} \tan \theta}{\frac{\Delta X}{V} \tan (\theta + \alpha)} = \frac{V \tan \theta}{w \tan \theta + \alpha}$$

or

$$\frac{V}{K} = \frac{V \tan \theta}{w \tan (\theta + \alpha)}$$

$$\frac{W}{K} = \frac{\tan \theta}{\tan \theta + \alpha}$$

$$\tan \theta + \alpha = \frac{K}{W} \tan \theta$$

$$\theta + \alpha = \tan^{-1} \left(\frac{K}{W} \tan \theta\right)$$

$$\alpha = \tan^{-1} \left(\frac{K}{W} \tan \theta\right) - \theta$$

Example

If $W = 0.001\ K$ and $\theta = 1^{\circ}$

then $\alpha = 85^{\circ} - 43'$

If in the limit $W \rightarrow 0$

then $\alpha = 90^{\circ} - 0'$

and stream line lies in the water surface.

Steady Confined Flow.

Flow in Sand of Uniform Thickness.

Many problems of this type can be reduced to two dimensional form or flow in a horizontal plane.

For:
1. Vertical lateral boundaries for the sand.
2. Condition of vertical lateral boundary uniform.
3. Slope of sand is small.

Then vertical component of flow = 0 and

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad (20)$$

Solutions for equation (20) can be assumed to be of the form

$$h = h_o\, e^{\alpha x + \beta y}$$

and then finding the restrictions on the coefficient α and β.

$$\frac{\partial h}{\partial x} = \alpha ho\, e^{\alpha x + \beta y}$$

$$\frac{\partial^2 h}{\partial x^2} = \alpha^2 h_o\, e^{\alpha x + \beta y}$$

$$\frac{\partial h}{\partial y} = \beta\, h_o\, e^{\alpha x + \beta y}$$

$$\frac{\partial^2 h}{\partial y^2} = \beta^2 h_o\, e^{\alpha x + \beta y}$$

Substitute value of $h = h_o\, e^{\alpha x + \beta y}$

$$\frac{\partial^2 h}{\partial x^2} = \alpha^2 h$$

$$\frac{\partial^2 h}{\partial y^2} = \beta^2 h$$

or

$$\alpha^2 h + \beta^2 h = 0$$

$$\alpha^2 + \beta^2 = 0$$

or

$$\beta = \pm\, i\, \alpha \qquad \text{where } i = \sqrt{-1}$$

Having evaluated β and α in terms of one another the following are possible solutions:

$$h = h_o\, e^{\alpha x}\, e^{i \alpha y}$$

$$h = h_o\, e^{\alpha x}\, e^{-i \alpha y}$$

$$h = h_o e^{-\alpha x}\, e^{i \alpha y}$$

$$h = h_o e^{-\alpha x}\, e^{-i \alpha y}$$

Linear combinations of these will also be solutions of equation 18 in the plane of rectangular coordinates

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = \frac{\theta \alpha_o b}{Kb} \left(\beta + \frac{\alpha}{\theta} \right) \frac{\partial h}{\partial t} \qquad (18)$$

The method of combining solutions of this form to satisfy non uniform boundary conditions is by the method of Fourier. (See any text on Partial Differential Equations)

Consider the flow of ground water in one direction in a sand of uniform thickness. Eq 20 reduces to

$$\frac{d^2h}{d_x{}^2} = 0$$

Integration gives as a general solution

$$h = C_1 x + C_2$$

If $h = 0$ when $x = 0$, then $C_2 = 0$ and $\frac{-Vo}{K} = \frac{dh}{dx} = \frac{-qo}{Kb}$

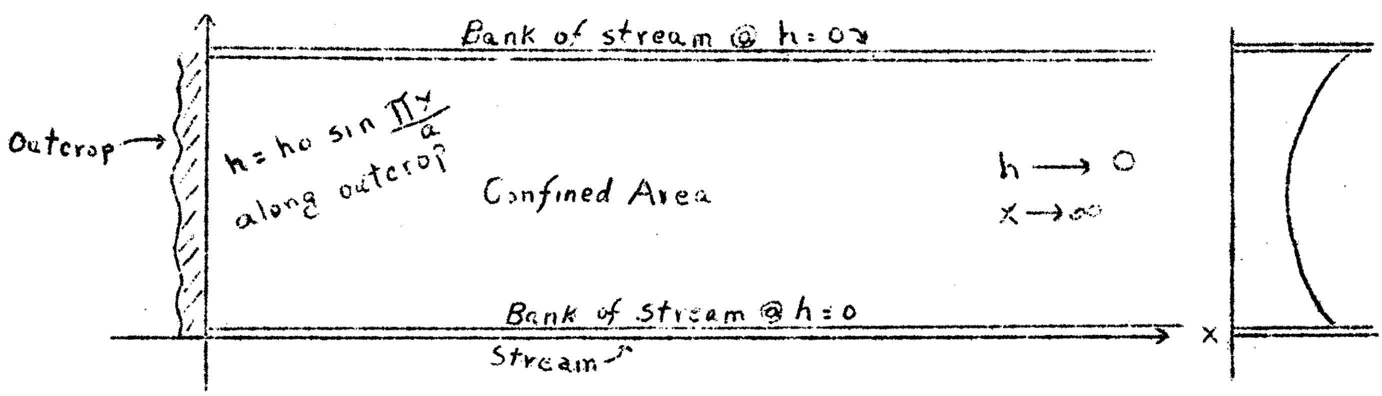
Where qo = discharge per unit of width measured normal to the direction of flow and b is the thickness of the sand, then

$$h = \frac{-qo^x}{Kb} = \frac{-qo^x}{T}$$

If flow is in the negative (x) direction the head will increase uniformly with x at the rate q^o/T.

Example 6. Confined sand extends indefinitely in one direction between parallel streams of vanishing low grade that cut completely thru it.

Other data is indicated on the sketch.



1. Find the distribution of head throughout the confined area.

2. Determine total confined flow from the outcrop to the two streams.

Take last two equations of (21). Add them, divide by i. The following solution results

$$h = h_o e^{-\alpha x} \sin \alpha y$$

This satisfies equation at $y = 0$, $y = a$, and $x = \infty$ and can be made to satisfy 4th boundary condition by putting $\alpha = \frac{\pi}{a}$ or

$$h = h_o e^{\frac{-\pi x}{a}} \sin \frac{\pi y}{a}$$ is the desired solution.

To determine total flow

Take $\frac{\partial h}{\partial x}$

$$\frac{\partial h}{\partial x} = h_o \left(-\frac{\pi}{a}\right) e^{\frac{-\pi x}{a}} \sin \frac{\pi y}{a}$$

as $x \longrightarrow 0$

$$\frac{\partial h}{\partial x} = \frac{-\pi}{a} h_o \sin \frac{\pi y}{a}$$

Then

$$dQ = - K\, dA \frac{\partial h}{\partial x}$$

$$dA = b dy$$

$$Q = \int_o^a - K\, b dy \left(-\frac{\pi}{a} h_o \sin \frac{\pi y}{a}\right)$$

$$T = Kb$$

$$Q = T \int_o^a \frac{\pi}{a} h_o \sin \frac{\pi y}{a} dy$$

$$= - T \left[h_o \operatorname{Cos} \frac{\pi y}{a} \right]_o^a$$

$$= - T h_o \left[\operatorname{Cos} \pi \frac{a}{a} - \operatorname{Cos} \frac{(\pi)(o)}{a} \right]$$

$$= - T h_o \left[(-1)-(+1) \right] = 2Th_o$$

Flow in a Sand of Nonuniform Thickness. The steady flow of ground water in a sand of nonuniform thickness is described by equ. 16

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (16)$$

If confining layers are impermeable

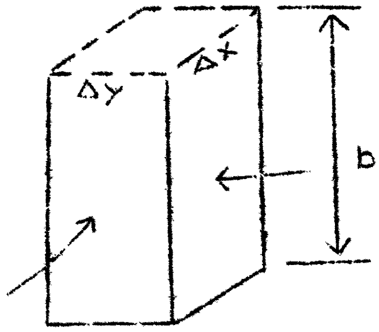
$\frac{\partial h}{\partial n} = 0$ for upper and lower surfaces of the sand.

Usually difficult to find an exact solution to equation (16) satisfying all boundary conditions.

If the depth of sand is small compared to lateral extent Eq (16) may be replaced by an equation that is easier to solve.

Assume $b = f(x,y)$

The principal of continuity is then applied to an elemental prism of the aquifer of height b and area = $(\Delta x \; \Delta y)$



Net inward flow thru the vertical faces normal to the x direction is

$$-K \, \Delta y \left(\frac{\partial (bV_x)}{\partial y}\right) \Delta x$$

and that normal to the faces in the y direction is

$$-K \, \Delta x \left(\frac{\partial (bV_y)}{\partial y}\right) \Delta y$$

by the continuity principle.

In steady flow the net inward flow = 0 and

$$\frac{\partial (bV_x)}{\partial x} + \frac{\partial (bV_y)}{\partial y} = 0 \qquad (22)$$

We are in effect assuming flow is horizontal but recognizing changes in total thickness of flow. If b can be expressed as a function of x, y equation 22 can be integrated. To illustrate, assume the sand is maintained at constant head h_o along the bounding line $x = 0$ and that it thins out in the x direction in accordance with

$$b = b_o e^{-ax}$$

then there is no flow in the y direction and equ. 22 reduces to

$$\frac{d(bV_x)}{dx} = b\frac{dV_x}{dx} + V_x \frac{db}{dx} = - Kb \frac{d^2h}{d_x2} - K \frac{dh}{dx}\frac{db}{dx} = 0$$

dividing thru by -Kb and substituting for b

$$\frac{d^2h}{d_x2} - a \frac{dh}{dx} = 0$$

It may be verified that the solution of this differential equation is

$$h = h_o \epsilon^{ax}$$

Satisfying the boundary conditions $h = h_o$ at $x = 0$

$$\frac{dh}{dx} = a\, h_o \epsilon^{ax}$$

and

$$b\, V_x = b_o \epsilon^{-ax} (-K)\, ah_o \epsilon^{ax} = K\, a\, b_o\, h_o$$

which is independent of x or in other words

$$\frac{d(bV_x)}{dx} = 0$$

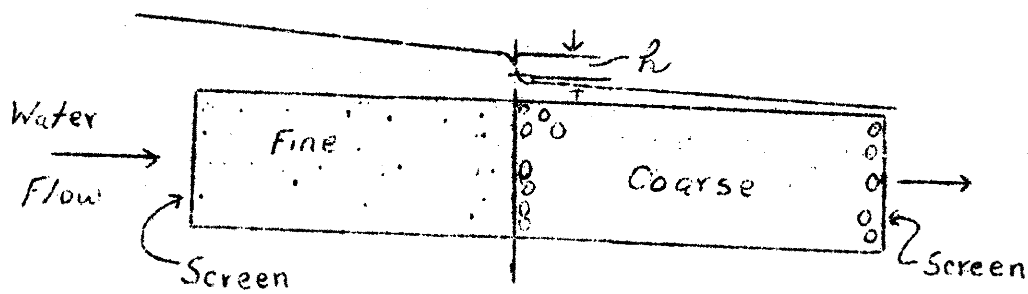
The solution for h is independent of z. It defines a single piezometric surface for all stream lines. Say that a well at a given point is virtually independent of the depth to which the well penetrates the sand providing flow is gradually varied.

Head Loss at an Interface

Experiments have been performed at Colorado A & M College to determine the head loss as water passes through an interface between an aquifer consisting of fine particles and one consisting of large particles.

This study is particularly applicable to the interface between a natural aquifer and the gravel packing around a well casing, but applies equally well to any similar condition.

Apparatus consisted of a horizontal conduit filled with sand and gravel size material as shown.



The head loss was taken as the vertical difference between tangents to the piezometric surface before and after passing through interface.

By Dimensional Analysis

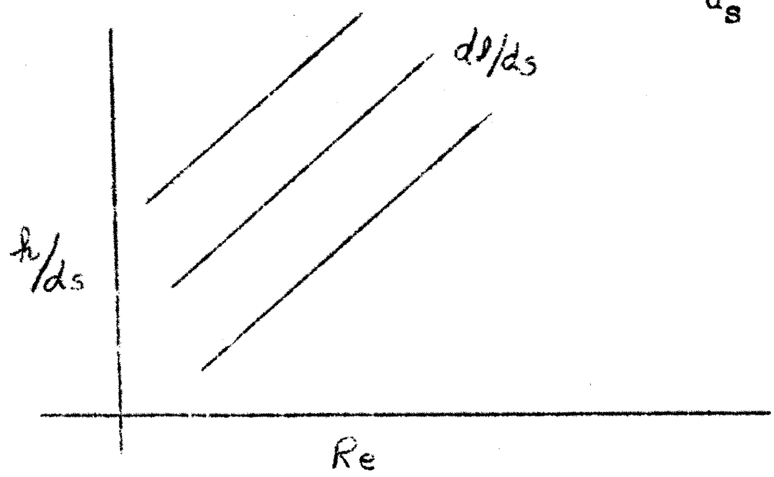
$$\phi = (h,\ d_\ell,\ d_s,\ \sigma_\ell,\ \sigma_s,\ \mu,\ \rho,\ V_b,\ \alpha,\ \text{etc.})$$

By assuming some of these values constant and using uniform sizes of material in the fine and coarse portions respectively, this reduced to

$$\phi \left(R_e \, , \frac{h}{d_s} \, , \frac{d_\ell}{d_s} \right) = 0$$

Where Re = Reynold's Number
h = head loss
d_s = diameter of fine material
d_ℓ = diameter of coarse material
$R_e = \frac{V d_s \rho}{\mu}$

Plotting the results of tests in the form $\frac{h}{d_s}$ against R_e, gave a series of uniform curves for various values of $\frac{d_\ell}{d_s}$.



These had a uniform slope of 45°, but the sequence of curves did not follow the sequence of values of $\frac{d_\ell}{d_s}$.

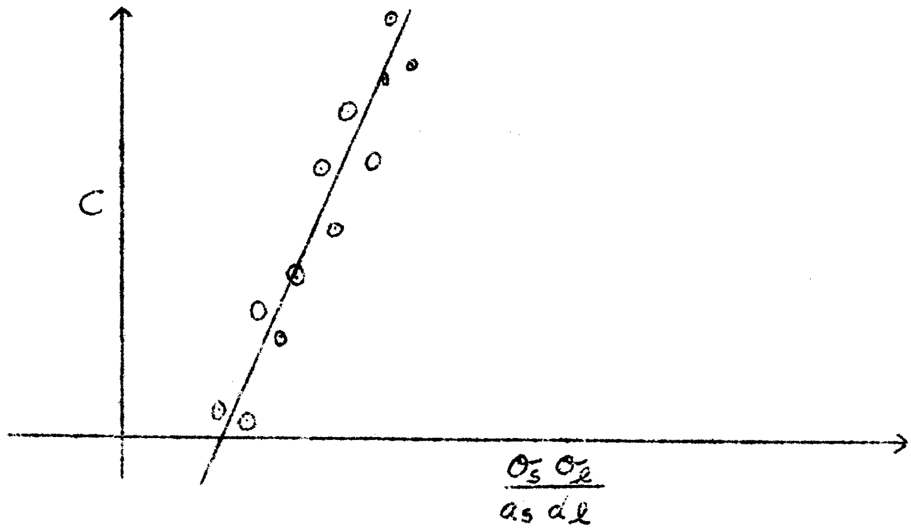
Replotting these with $\frac{\sigma_s}{d_s} \times \frac{\sigma_\ell}{d_\ell}$ as the third parameter (i.e. the product of the uniformity coefficients of the two materials) gave the correct sequence of curves. σ_ℓ and σ_s are standard deviations from mean particle size of large- and small-sized material, respectively. The curves obtained have the equation:

$$\frac{h}{d_s} = C \, R_e$$

or $h = C \, R_e \, d_s$ = head loss at interface where C is a function of

$$\left(\frac{\sigma_s}{d_s}\right) \times \left(\frac{\sigma_\ell}{d_\ell}\right)$$

After making tests on various uniform materials, and one test on non-uniform material, the values of C were plotted against $\log\left(\frac{\sigma_s}{d_s}\frac{\sigma_\ell}{d_\ell}\right)$ to obtain a graph of this form:



The equation of the resulting straight line was

$$C = C\left(\frac{\sigma_s}{d_s}\frac{\sigma_\ell}{d_\ell}\right)^n$$

and $n = 4.6$

$C_\alpha = 1.06 \times 10^{+8}$

Therefore, the head loss at an interface is

$$h = R_e\, d_s\, C_\alpha \left(\frac{\sigma_s}{d_s}\frac{\sigma_\ell}{d_\ell}\right)^n$$

Equilibrium Flow to Wells[1]

Basic Equations for Uniform Steady Flow in Isotropic Media.

Darcy - $$V = k \frac{h}{\ell} \qquad (1a)$$

$$V = k \text{ grad } h = K \nabla h \qquad (1b)$$

$$V = k \frac{\partial h}{\partial s}$$

Continuity - $$Q = AV \qquad (2a)$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \qquad (2b)$$

$$\text{div } V = 0$$

Laplaces - $$\nabla^2 h = 0 \qquad (3)$$

(rect. coordinates) $$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (4)$$

(cylindrical coordinates) $$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{1}{r^2}\frac{\partial^2 h}{\partial \theta^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (5)$$

where r , θ and z are the cylindrical coordinates.

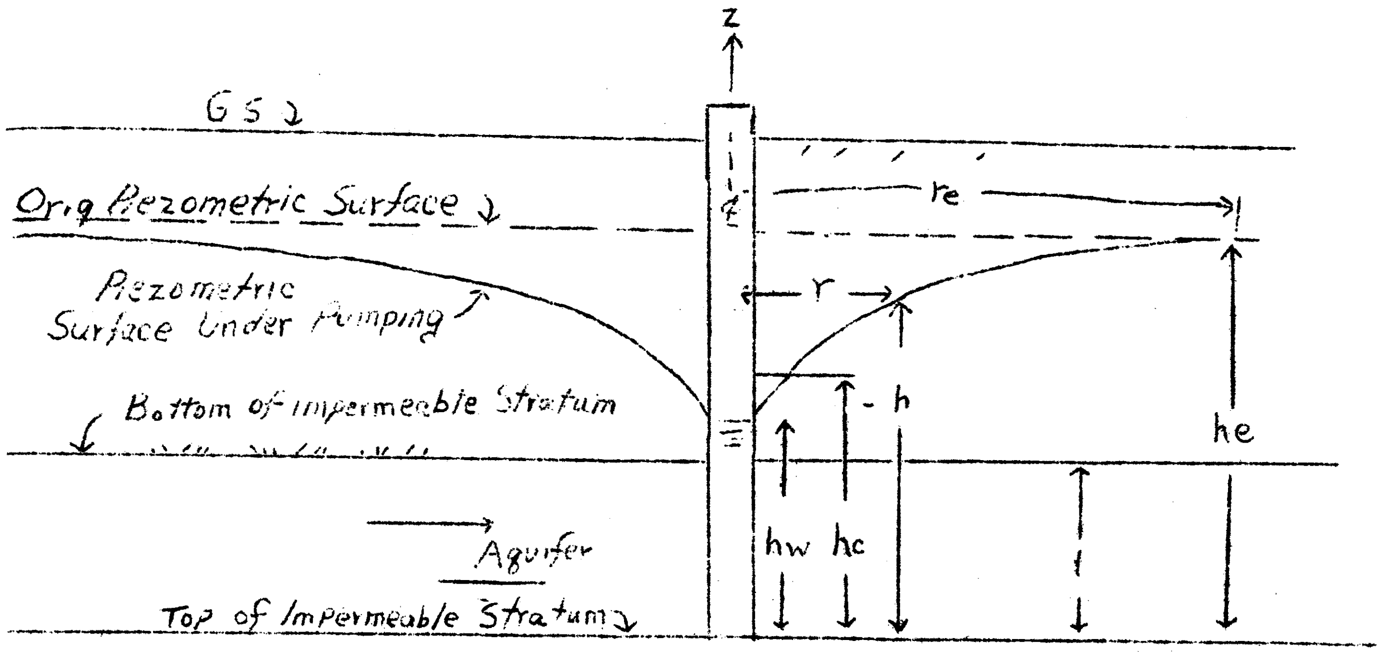
The cylindrical components of velocity are given by:

$$V_r = k \frac{\partial h}{\partial r}, \quad V_\theta = \frac{k}{r}\frac{\partial h}{\partial \theta} \quad \text{and} \quad V_z = k \frac{\partial h}{\partial z} \qquad (6)$$

Confined Flow Systems. For confined flow system with well extending to bottom of permeable stratum, if all flow is assumed horizontal, radial and symetrical, there is no component of velocity in either the direction of θ or z , hence $\frac{\partial h}{\partial z}$ and $\frac{\partial h}{\partial \theta}$ and all higher derivitives of h with respect to z and θ are equal to zero, and

$$\frac{d^2 h}{d_r^2} + \frac{1}{r}\frac{dh}{dr} = 0 \qquad (7)$$

[1] A review of "Hydraulics of Wells", D. F. Peterson, Jr., O. W. Israelsen and V. E. Hansen, Bul. No. 351, Utah Agric. Exp. Sta., March, 1952.



with boundary conditions:

$$\text{at} \quad r = r_w \, , \; h = h_w$$

$$\text{and at} \quad r = r_e \, , \; h = h_e$$

Integrating (7) and substituting boundary values:

$$\frac{h - h_w}{\ln r/r_w} = \frac{h_e - h_w}{\ln r_e/r_w} \qquad (8)$$

and

$$h = h_w + \frac{h_e - h_w}{\ln r_e/r_w} \ln \frac{r}{r_w} \qquad (9)$$

At a distance, r , from the well, combining the equation of continuity in the form $Q = AV$ with that of Darcy yields:

$$Q = 2\pi r \, t \, k \frac{dh}{dr} \qquad (10a)$$

Differentiating equation (9) with respect to r , and substituting in equation 10a, the value thus obtained for $\frac{dh}{dr}$, gives

$$Q = 2\pi t k \frac{h_e - h_w}{\ln r_e/r_w} \qquad (10b)$$

The same equation can be obtained by integrating equation (10a) between the limits r_e and r_w , and h_e and h_w .

Eliminating $(h_e - h_w)$ from equations 9 and 10b and solving for $(h - h_w)$ gives

$$h - h_w = \frac{Q}{2\pi tK} \ln \frac{r}{r_w} \qquad (11)$$

Likewise, this same equation can be obtained by integrating equation (10a) between the limits r and r_w, and h and h_w.

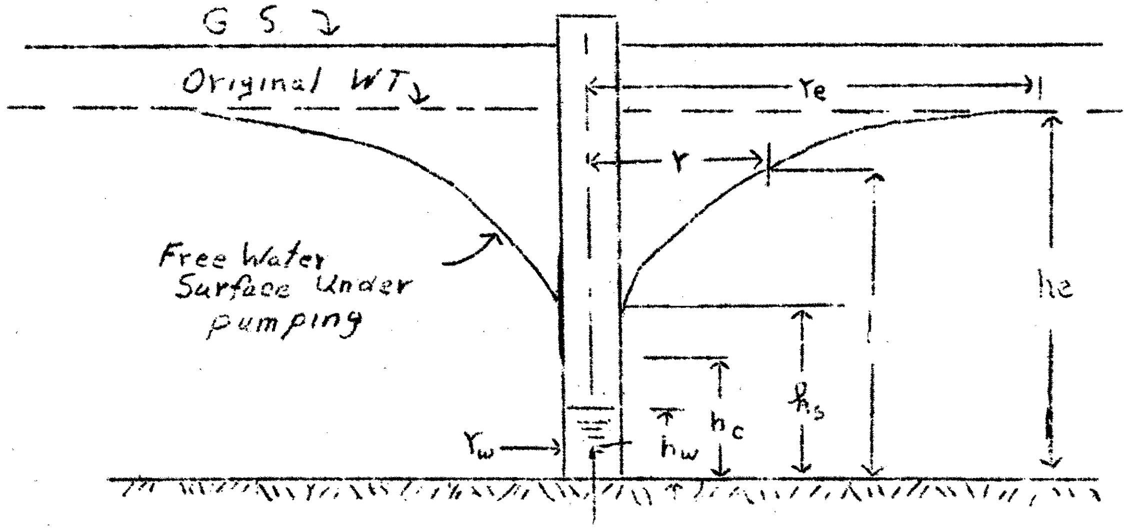
The total drawdown, D_w, which equals $(h_e - h_w)$ as obtained from equation (10b) is:

$$D_w = \frac{Q}{2\pi tk} \ln \frac{r_e}{r_w} \qquad (12)$$

Equation 12 presents some technical difficulties . . . assumes piezometric surface level at elevation h_e, implying no flow toward the well beyond r_e, a condition which can not exist. Actually the concept of a radius of influence is fallacious.

Unconfined Flow Systems and the Seepage Surface. If there is no impermeable stratum overlying the permeable water-bearing aquifer, or if the drawdown is greater than the depth from the static water level to the bottom of the confining stratum, the flow system may be classified as unconfined or partially confined.

The problem of developing a rational equation to find the drawdown is more complicated (than the confined system) because the position of the boundary of the region of flow is unknown. In addition, the top of the surface of the flow region intersects the well at an elevation somewhat greater than the elevation of the water in the well. A real seepage surface always exists around a well in an unconfined system while pumping. (The existance and length of this seepage surface is very important to the functioning of a well for drainage purposes.)



Duouit Solution:

Based on assumption of horizontal flow through any concentric cylinder:

$$Q = 2\pi r h k \frac{dh}{dr}$$

integrating to, $\frac{Q}{\pi k} \ell n \, r = h^2 + c$

introducing boundary conditions at r_w

$$\frac{Q}{\pi k} n \frac{r}{r_w} = h^2 - h_w^2 \qquad (13)$$

or substituting $h_e = h$ when $r = r_e$

$$\frac{Q}{\pi k} \ell n \frac{r_e}{r_w} = h_e^2 = h_w^2 \qquad (14)$$

Equation 14 while not rigorously correct has been shown by experiment (Wyckoff, 1932) to give correct values of discharge.

The equation for the free surface

$$h^2 = \frac{h_e^2 - h_w^2}{\ell n \, r_e/r_w} \ell n \frac{r}{r_w} + h_w^2 \qquad (15)$$

is obtained by eliminating Q from equations 13 and 14.

Dupuits solution for the position of the free surface gives quite accurate results at fairly large distances from the well but is incorrect in the neighborhood of the well where the assumptions made in developing equation 13 are less applicable.

<u>Recent Solutions and the Discharge Number</u>:

Babbitt and Caldwell (1948) plotted the percent of drawdown of the free surface at any distance, r, to the drawdown of the free surface at the well, $100 (h_e - h)/(h_e - h_s)$ against the ratio r/r_e and found the shape of the free-surface curve to be independent of the physical dimensions of the system. The following equation was proposed for the position of the free surface:

$$h_e - h = \frac{2.30 \, C_x}{\pi k \, h_e} \log \frac{r_e}{0.1 \, h_e} \qquad (16)$$

where C_x is the ratio of the drawdown of the free surface at any distance r, to the maximum possible drawdown when the well is discharging at maximum.

Hansen (1949) proposed the emperical equation,

$$C_x = - 0.3 \log \frac{r}{r_e} \qquad (17)$$

for values of r/r_e greater than 0.05. Substituting equation 17 in equation 16 gives

$$(h_e - h) = \frac{0.69 \, Q}{k \, h_e} \log \frac{r_e}{0.1 \, h_e} \log \frac{r_e}{r} \qquad (18)$$

which indicates that the elevation of the free surface is a linear function of log r for any value of Q. (Only in the vicinity of the well.)

Hansen points out the desirability of expressing the constant of integration in terms of the radius of the well, instead of the radius of influence, and developed the relation

$$\frac{Q}{k\ r_w^2} = F\left(\frac{h_s}{r_w}, \frac{h_w}{r_w}\right) \tag{19}$$

for which the fractional parameters are dimensionless.

The term $\frac{Q}{k\ r_w^2}$ is designated "Discharge Number" and from experimental data, a family of curves were plotted for this Discharge Number, against the ratio $\frac{h_s}{r_w}$ for various values of $\frac{h_w}{r_w}$ (Figure 4 F the **Reference**).

<u>The Seepage Surface for Unconfined Systems</u>. The Dupuit equation is valid for calculation of discharge and can be written in terms of the Discharge Number as

$$\frac{Q}{k\ r_w^2} = \frac{\pi\ (h_e^2 - h_w^2)}{r_w^2\ \ell n\ {}^{r_e}/_{r_w}} \tag{20}$$

which makes possible calculations of the Discharge Number from the theoretical computations of Yang (1949) and provides basis for extending the experimental curves of Hansen over a wide range of Discharge Numbers.

<u>Unconfined Flow Replenished by Vertical Percolation</u>:

In the case of drainage wells in a valley fill, a large part of the discharge of the well comes from water moving horizontally from within the cone of influence.

From Dupuit assumptions, the flow Q_r through any concentric cylinder at radius r, is

$$Q_r = Q - \pi q_v\ (r^2\ r_w^2) \tag{21}$$

where q is the rate of replenishment vertically per unit horizontal area.

Since r is large in relation to r_w the equation may be written

$$Q_r = q - \pi q_v\ r^2 \tag{22}$$

From 22 and Darcy's law

$$Q = \pi q_v r^2 = 2\pi r h k \frac{dh}{dr}$$

which may be integrated to give

$$\frac{Q}{\pi k} \ell n\ r = \frac{q_v}{2k} r^2 = h^2 + c \qquad (23)$$

Evaluating C when $r = r_w$ and $h = h_w$, gives

$$\frac{Q}{\pi k} \ell n \frac{r}{r_w} - q_v \frac{r^2 - r_w^2}{2k} = h^2 - h_w^2 \qquad (24)$$

and at the radius of influence

$$\frac{Q}{\pi k} \ell n \frac{r_e}{r_w} - q_v \frac{r_e^2 - r_w^2}{2k} = (h_e^2 - h_w^2) \qquad (25)$$

By the approximation

$$r_e^2 - r_w^2 = r_e^2$$

and noting by definition

$$n = \frac{\pi r_e^2 q_v}{Q} \qquad (26)$$

n is the ratio of discharge of water from vertical percolation to the total discharge of the well.

Equation 25 may be written in forms.

$$Q = \frac{\pi k (h_e^2 - h_w^2)}{2.303 \log r_e/r_w - (n/2)} \qquad (27)$$

and

$$q_v = \frac{\dfrac{nk (h_e^2 - h_w^2)}{r_e^2}}{2.303 \log r_e/r_w - (n/2)} \qquad (28)$$

From equations 24 and 26, and ignoring the term r_w/r_e because of its negligible effect, the equation of the draw down curve will be given by the relation

$$Q = \frac{\pi k (h^2 - h_w^2)}{2.303 \log r/r_w - n/2\ (r/r_e)^2} \qquad (29)$$

and substituting for Q from equation 27

$$h^2 = (h_e^2 - h_w^2) \frac{2.303 \log r/r_w - n/2\ (r/r_e)^2}{2.303 \log r_e/r_w - n/2} + h_w^2 \qquad (30)$$

Validity of Equations: No data are available for checking equations for unconfined flow replenished by vertical percolation. The seepage surface which exists at the well has been neglected in the theoretical development. Also Dupuit's assumption of horizontal flow through concentric cylinders is not entirely valid. Equation 30 can not be expected to give reliable results for positions of the free water surface near the well; however, equation 27 should give reliable results for discharge. Equation 29 may be expected to give reliable results if the measurements of piezometric head are made at points where the assumption of horizontal radial flow (through vertical potential surfaces) applies. This is true along the impermeable boundary and at greater elevations as the radius increases.

Determination of Seepage Surface:

Equation 27 may be rewritten

$$(Q = \frac{\pi k\ (h_e^2 - h_w^2)}{2.303 \log \frac{r_e}{e^{n/2}\ (r_w)}} \tag{31}$$

and substituting an effective $r_w^1 = r_w\ e^{\ n/2}$

Equation 31 reduces to the form of equation 14 with the resulting geometry of the transformed well the same as for the Dupuit case having the same discharge, but for a well of radius equal to $e^{\ n/2}\ r_w$. For $n = 1$ (the well system is replenished entirely by vertical percolation) $r_w^1 = 1.643\ r_w$. The experimental curves may then be used for finding the height $(h_s - h_w)$ of the seepage surface above the water surface in the well.

Effect of Replenishment. A condition of steady flow implies that the total replenishment of the influence cone equals the discharge of the well.

$$F\ (D_w,\ r_w,\ k,\ t,\ q,\ Q) = 0 \tag{32}$$

involves only dimensions of L and T choosing r_w and k as repeating variables.

$$F_1\ (\frac{D_w}{r_w},\ \frac{t}{r_w},\ \frac{q}{k},\ \frac{Q}{k\ r_w^2}) = 0 \tag{33}$$

or

$$\frac{Q}{r_w^2} = F_2\ (\frac{D_w}{r_w},\ \frac{t}{r_w},\ \frac{q}{r}) \tag{34}$$

$Q/k\ r_w^2$ is dimensionless and depends only on the geometry of the well $(r_w,\ D_w)$, and hydrology of the ground water system (k, t, q). $Q/k\ r_w^2$ is dependent only on the well and its environment and may be considered a measure of discharge in dimensionless terms. Hansen demonstrated it to be the ratio of the Froude number to the Reynolds number, indicating the ratio of viscous to gravity forces. It has been designated as the discharge number, denoted as N.

Consideration of replenishment gives some reality to the measuring of the radius of influence. It is a characteristic length describing the area within which the total replenishment equals the discharge,

Unconfined System Replenished by Horizontal Flow: Designating the discharge number for the unconfined case as N_{uh}, and substituting $D_w = h_e - h_w$, equation 34 may be written:

$$N_{uh} = F_3 \left(\frac{h_e}{r_w}, \frac{h_w}{r_w}, \frac{q}{k}\right) \tag{35}$$

Equation 20 may be written

$$N_{uh} = \frac{\pi \left[\left(\frac{h_e}{r_w}\right)^2 - \left(\frac{h_w}{r_w}\right)^2 \right]}{\ln r_e/r_w} \tag{36}$$

The right side of equation 36 contains only linear dimensions; thus the numerical value of N_{uh} defines the shape of the influence region. q/k may be taken as the natural slope, i_n, of the water table in the region, if no water comes into the region of influence by vertical percolation.

Under conditions of steady flow

$$Q = 2\, r_e\, k\, h_e\, i_n \tag{37}$$

Solving equation 37 for r_e and substituting in equation 36 yields

$$N_{uh} = \frac{\pi \left[\left(\frac{h_e}{r_w}\right)^2 - \left(\frac{h_w}{r_w}\right)^2 \right]}{2.303 \log \left(\frac{N_{uh}\, r_w}{2\, i_n\, h_e}\right)} \tag{38}$$

N_{uh} depends on the radius of the well, initial and final depths of water in the well and natural slope of the water table.

Large values of N_{uh} indicate deep, narrow drawdown cones of influence; while small values indicate broad shallow cones. Both flows into the zones of influence and low permeability cause cones to become narrow and steep; whereas, slow replenishment and high permeability cause them to be broad and shallow.

A design chart of N_{uh} vs. the replenishment factor $\frac{i_n}{h_e/r_w}$ is given by Fig. 5 of the Reference.

Confined Systems: The discharge number for the confined system will be designated by N_c. From equation 10,

$$N_c = \frac{Q}{k\ r_w^2} = \frac{2\pi(D_w/r_w)(t/r_w)}{\ln r_e/r_w} \qquad (39)$$

and

$$Q = 2\ r_e\ k\ t\ i_n \qquad (40)$$

Solving 40 for r_e and substituting in 39 gives

$$N_c = \frac{2\pi(D_w t/r_w^2)}{2.303 \log\left(\frac{N_c\ r_w}{2\ i_n t}\right)} \qquad (41)$$

Equation 41 satisfies the functional relationship equation 34 and makes possible computation of values of $i_n/D_w/r_w$ for various values of $N_c/D_w t/r_w 2$ to form the curve of Figure 6 of the Reference.

Unconfined System Totally Replenished by Vertical Percolation:

Let $N_{uv} = Q/k\ r_w^2$. Combining equations 26 and 27 for n = 1 yields,

$$N_{uv} = \frac{\pi\left[\left(\frac{h_e}{r_w}\right)^2 - \left(\frac{h_w}{r_w}\right)^2\right]}{2.303 \log\left(\frac{h_e}{r_w} - \sqrt{\frac{Q}{qv}}\right) - \frac{1}{2}} \qquad (42)$$

Substituting $Q = N_{uv}\ k\ r_w^2$ gives

$$N_{uv} = \frac{\pi\left[\left(\frac{h_e}{r_w}\right)^2 - \left(\frac{h_w}{r_w}\right)^2\right]}{1.151 \log - \sqrt{\frac{(N_{uv}k)}{q_v \pi}} - \frac{1}{2}} \qquad (43)$$

Which satisfies the relationship required by equation 34.

Equ. 43 may be rewritten:

$$\frac{N_{uv}}{\left(\frac{h_e}{r_w}\right)^2} = \frac{\pi\left[1 - \left(\frac{h_w}{h_e}\right)^2\right]}{1.151 \log\left[\frac{N_{uv}}{\left(\frac{h_e}{r_w}\right)^2} - \log \frac{(\pi q_v)/k}{\left(\frac{h_e}{r_w}\right)^2}\right] - \frac{1}{2}}$$

The ratio h_e/r_w depends on dimensions of the well, while q_v/k is the ratio of the unit replenishment to permeability.

From equation 44, $N_{uv}/(h_e/h_w)^2$ can be computed for values of $(q_v/k)/(h_e/r_w)^2$ for various drawdown ratios, H_w/h_e. Graphs of the results are presented in Figures 7 and 7a of the Reference.

Significance of the Discharge Number. If the value of $Q/k\ r_w^2$ is known, the effective radius of influence may be established by consideration of the hydrologic factors and the well radius.

$$N_c = F\left(\frac{r_e}{r_w}, \frac{t}{r_w}, i_n\right) \tag{45}$$

$$N_{uh} = F\left(\frac{r_e}{r_w}, \frac{h_e}{r_w}, i_n\right) \tag{46}$$

$$N_{uv} = F\left(\frac{r_e}{r_w}, q/k\right) \tag{47}$$

For the confined case, equation 11 can be written

$$\frac{N_c\ r_w}{t} = \frac{2\pi\ \frac{h-h_w}{r_w}}{2.303\ \log \frac{r}{r_w}} \tag{48}$$

This relationship is shown by Fig. 8 of the Reference.

Effectiveness of Wells. Defined by Wenzel (1942) as

$$E_w = \frac{100\ (h_e - h_c)}{(h_s - h_w)} \tag{49}$$

Because of head loss through casing, E_w is less than 100%.

Confined System: For a confined system E_w has quality of well efficiency because $(h_e - h_c)$ is the power per unit weight of discharge delivered by the well to the fluid outside; whereas $(h_e - h_w)$ is the power per unit weight of discharge imported by the pumps to water inside the well. The difference represents the power loss per unit weight of discharge through the boundary of the well. $(h_e - h_c)$ can not be obtained directly by measurement; however the head loss for the confined case of radial flow is a linear function of the logarithm of the radius (Equ. 11), and h_c may be obtained by projection of a plot of h vs. $\log\ r$, to the well casing (See Fig. 9 of the Reference).

Unconfined System: For the unconfined case considerable care must be exercised in applying Wenzel's procedure, to clearly distinguish between h_s and h_c, h_s differs from h_c because of hydrodynamic considerations resulting from the minimum energy concept, and h_c is the effective energy head outside the well, equal to h_w plus losses thru the screen and envelope. No suitable method for determining h_c for the unconfined system has yet been developed.

It is proposed that the squares of differences between the piezometric levels near the permeable stratum and the elevation of the top of the impermeable stratum bear a **logarithmic** relationship to the radial distance from the well and may be extended on a semi-logarithmic plot of the system to intersect the well casing at h_c. Likewise a semi-logarithmic plot of the free water surface at various radii from the well when extended to the well will intersect the casing at h_c

Comparison of Unconfined and Confined Systems: Unconfined system is inherently more efficient in utilizing available specific energy than is the confined system.

The specific energy in the confined system is

$$h_e - h_w = \frac{Q \ln (r_e/r_w)}{2\pi k t}$$

and for the unconfined system is

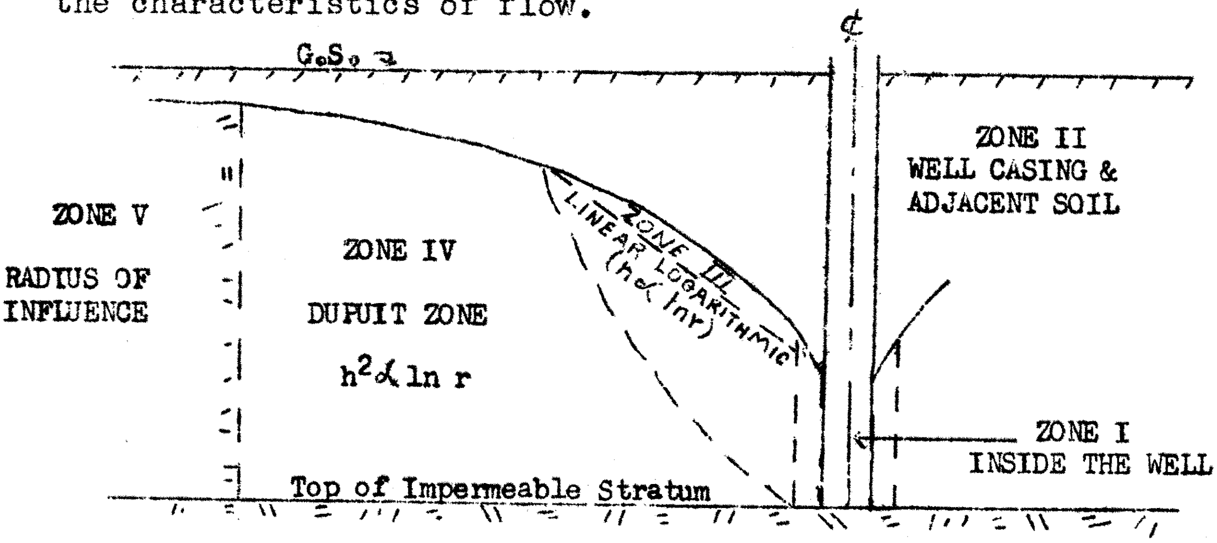
$$h_e - h_w = \frac{Q \ln (r_e/r_w)}{k\pi (h_e + h_w)}$$

for the equivalent situation

$$\frac{Q_u}{Q_c} = \frac{h_c + h_w}{2t}$$

and since h_w must be greater than t to assure confined flow, and as h_e is always greater than t. the ratio $\frac{Q_u}{Q_c}$ must be greater than unity.

Zones of Flow in Well Hydraulics. Ground water flow to wells should be considered by zones or regions according to the characteristics of flow.



Zone 1. Inside the well.

Flow parameters h_w and h_r are readily measurable and are of value in defining flow in other zones.

Zone II. The Casing and Adjacent Soil.

Zone where so-called well losses occur, difficult to obtain measurements and must be exactly located because of high and accelerating velocities. Measurements made in this zone should not be used in predicting conditions in any other region.

Zone III. Linear Logarithmic Zone.

In confined flow this zone extends from near the well to the so-called radius of influence in region V. In unconfined flow this zone applies to the free surface near the well where the flow is curilinear. In this zone the piezometric surface in confined flow and the free surface in unconfined flow is proportional to the logarithm of the radius from the well ($h \sim \ell n\ r$). In unconfined flow, measurements of piezometric head near the free surface can be used to extrapolate to the well casing to determine the height of the seepage zone, h_s.

Zone IV. Dupuit Zone for Unconfined Flow.

Applies only to those regions of unconfined flow where the velocity vector is essentially horizontal. Dupuits equation ($h^2 \sim \ell n\ r$) developed on the assumption of horizontal flow applies with increasing accuracy as the flow approaches the horizontal, particularly along the base of the aquifer. Piezometric heads measured in this region will define flow conditions if the Dupuit equation is used.

Zone V. Radius of Influence Zone.

This zone is the least accurately defined. For wells recharged horizontally an actual radius of influence does not exist, but has tangible meaning for those receiving their recharge by vertical percolation. The effects of departure from steady flow are most pronounced in the radius of influence zone. Piezometric heads measured in this region are of value in approximating the original position of the undisturbed water table, but should not be used for predicting either the discharge or permeability of the aquifer.

Non-Equilibrium Flow to Wells[1]

General Formulas. An analogy exists, to the extent the flow of ground water is governed by Darcy's Law, between the hydrologic conditions in an aquifer and the thermal conditions in a similar thermal system. Darcy's law is analogous to the flow of heat by conduction. The mathematical theory of heat-conduction developed by Fourier and subsequent writers is largely applicable to hydraulic theory.

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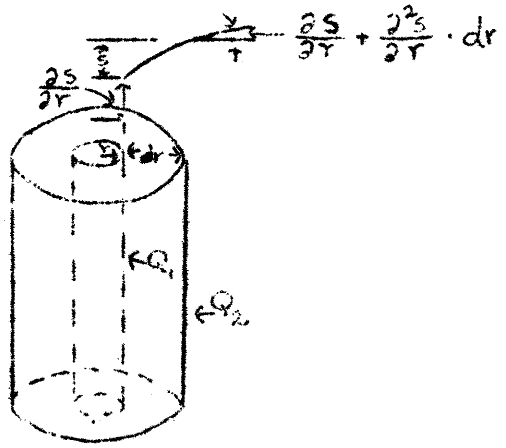
1 A review of "Relation Between the Lowering of the Ground Water Table and Discharge of a Well Using Ground Water Storage", Chas. V. Theis; Trans. AGU, Part II, 1935 (as presented in "Hydrology", Wisler & Brater, John Wiley & Sons, 1949)



T = Coefficient of Transmissibility rate of flow of water in gallons per day through a vertical strip of the aquifer 1 ft. wide and the full saturated height, under a hydraulic gradient of 100% at 60° F.

Difference in the rate of flow through the under and outer faces is the rate of reduction of storage within the cylinder.

$$Q_1 - Q_2 = \frac{dv}{dt} \qquad v = \text{volume of storage} \tag{51}$$

$$Q_1 = T\ I\ A_1 = -\ 2\pi r\ T\ \frac{\partial s}{\partial r} \tag{52}$$

$$I_2 = I_1 + \frac{\partial^2 s}{r^2}\ dr$$

$$I_2 = \frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r^2}\ dr \tag{53}$$

Then

$$Q_2 = -\ T\ I_2\ \ 2\pi(r + dr)$$

$$= -\ T\ \left(\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial r}\ dr\right)\ \ 2\pi\ (r + dr) \tag{54}$$

and the rate of change of storage:

$$\frac{dv}{dt} = (2\pi r dr)\left(\frac{\partial s}{\partial t}\ S\right) \tag{55}$$

where S = Coefficient of storage, for water table condition = specific yield; and for artesian conditions equal to the water obtained from a column of water-bearing material with 1 ft^2 base and height equal to thickness of the aquifer.

Substituting these values for Q_2 and $\frac{dv}{dt}$ into the continuity equation dividing through by $2\pi r\ t\ k\ dr$ and neglecting differentials higher than the first order,

$$\frac{\partial^2 r}{\partial r^2} + \frac{1}{r}\ \frac{\partial s}{\partial r} = \frac{S}{T}\ \frac{s}{t} \tag{56}$$

For a constant pumping rate Q ,

$$s = \frac{Q}{4\pi T} \int_{r^2 S/4Tt}^{\infty} \frac{e^{-u}}{u} \, du \tag{57}$$

When Q = gallons/min, T = gals/day, = gallons and t = elapsed time in days,

$$s = \frac{114.6Q}{T} \int_{\frac{1.87r^2S}{Tt}}^{\infty} \frac{e^{-u}}{u} \, du \tag{58}$$

$$= \frac{114.6Q}{T} \; w(u)$$

When $$u = \frac{1.87r^2S}{Tt} \tag{59}$$

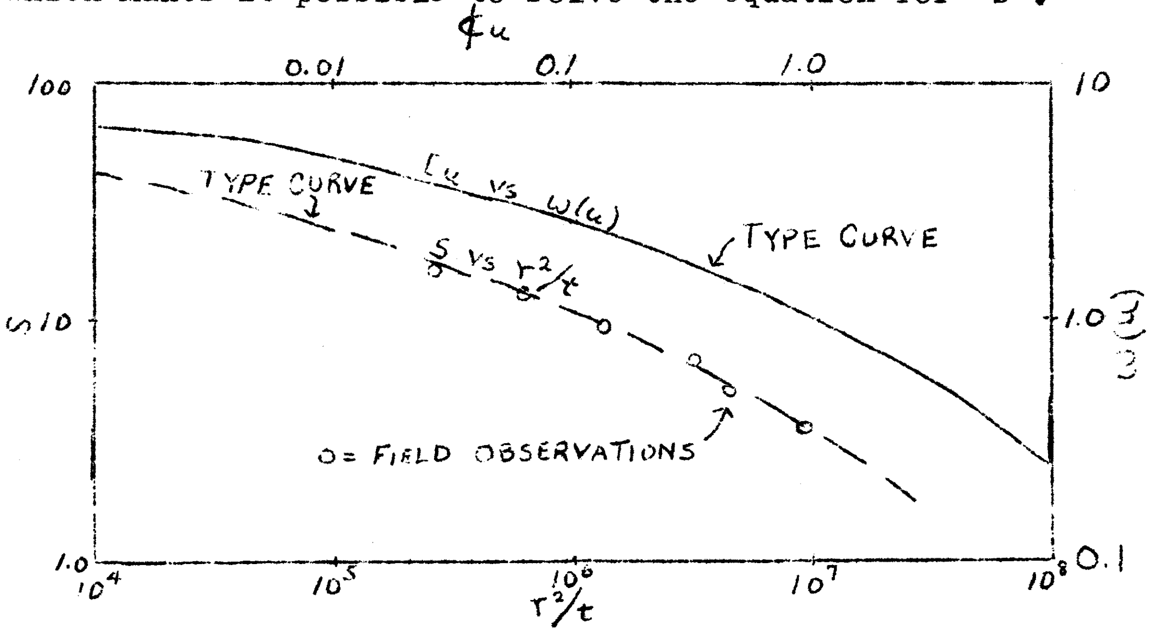
Note: Formulas 58 and 59 permit determination of T and S with one or more values of r and several values of t .

$$\int_{1.87r^2S/Tt}^{\infty} \frac{e^{-u}}{u} \, du = w\,(u) = -\,0.5777216 - \log_e u + u - \frac{u^2}{2.21} + \frac{u^3}{3.31} \tag{60}$$

w (u) is called "well function of u " and tables of its value are published: Table 32, <u>Smithsonian Physical Tables</u>, 8th revised edition, 1933.

(The values to be used are those given for E_1 (-x) with the sign changed) Values of w (u) for values of u from 10^{-15} to 9.9 as complied by Wenzel are given by Table 11, pp 236-237, of the reference.

Theis has developed a graphical method of superposition which makes it possible to solve the equation for s .



Rearranging equations 29 and 30,

$$s = \left[\frac{114.60}{T} \; Q\right] \; w\,(u) \tag{61}$$

$$\frac{r^2}{t} = \left[\frac{T}{1.875}\right] \; u \tag{62}$$

If Q is constant, the bracket portions of 61 and 62 are constant for a given pumping test and

$$\frac{r^2}{t} \; / \; s \sim \frac{u}{w(u)}$$

Consequently if values of drawdown, s , are plotted against r^2/t are plotted to the same scale as the type curve w (u) vs. u , the curves will be similar.

ith the coordinate axes parallel, the data curve is translated to a position of "best fit" on the type curve and an arbitrary match point marked on both. Noting the coordinates of the match point on both curves, and substituted into equations 61 and 62 to solve for T and S .

<u>Example</u>:

Q = 300 gpm
Total depth of well = 200 ft.
Screen setting 170-200 ft.
Well diameter 12"
Static depth to water 5' below grade

To predict performance of this well for 30 days of continuous pumping with total withdrawal from storage:

Q = 300 gpm	r = 6" = 0.5'
t = 30 days	T = 4500 gpd/ft
	S = 6.4×10^{-4}

From equ. 59

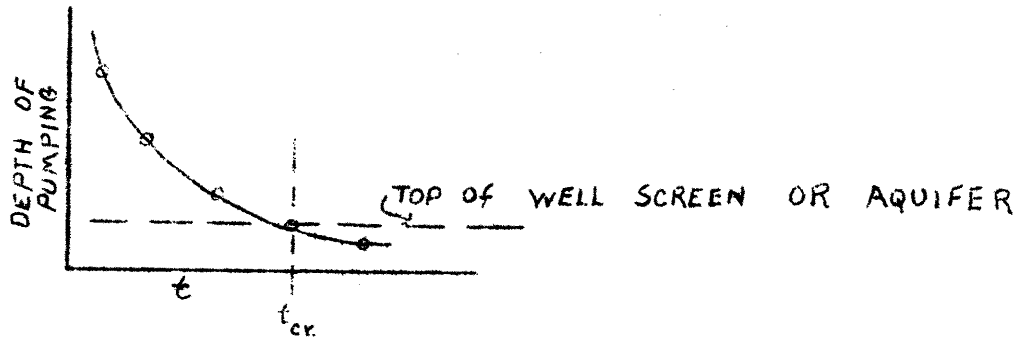
$$u = \frac{1.87\,(0.5)^2\; 6.4}{4.5 \times 10^3 \times 30 \times 10^4} = 2.2 \times 10^{-9}$$

The corresponding value of w (u) (from table 11) is 19.4

From equ. 61

$$s = \frac{114.6 \times 300 \times 19.4}{4500} = 148 \text{ ft.}$$

Pumping level at end of 30 day period = 148 + 5 (normal depth of wt) = 153 ft below grade. Further similar determinations producing a plot of pumping level vs. time, reveals time when it would reach the top of the well screen and/or aquifer.



Note: Pumping below the top of the aquifer, reducing its saturated depth and consequently, T , would accelerate the rate of fall to complete failure. In the example given a 250 gpm rate would be pumping at 28' above the top of the screen when 300 gpm would be dewatering the aquifer.

Such an example contemplates minimum performance. Normally, as the well cone increases, replenishment is induced by increased gradient causing inflow from surface streams, encomposes sufficient area to be sustained by percolation, or reduces flow to other outlets to balance the well discharge.

<u>Modified Non-Equilibrium Formulas</u>[1]: The sum of the terms in equ. 60 beyond $\log_e u$ is not appreciable when u becomes small.

From 59 it is seen that u decreases as time increases. Accordingly, for large values of t , the terms beyond $\log_e u$ in the exponential series may be neglected and equ. 61 may be written:

$$s = \frac{114.6Q}{T} \; w(u) = \frac{114.6Q}{T} \left[-0.5772 - \log_e u \right]$$

$$= \frac{114.6Q}{T} \left[\log_e \left(\frac{1}{u}\right) - 0.5772 \right] \qquad (63)$$

but $$u = \frac{1.87 r^2 S}{Tt} \quad \text{and} \quad \frac{1}{u} = \frac{Tt}{1.87 r^2 S}$$

$$s = \frac{114.6Q}{T} \left[\log_e \left(\frac{Tt}{1.87 r^2 S}\right) - 0.5772 \right] \qquad (64)$$

In applying the above equation to drawdown or recovery in a particular observation well, r will be constant and,

at time t_1 , $S_1 = \frac{114.6Q}{T} \left[\log_e \left(\frac{Tt_1}{1.87 r^2 S}\right) - 0.5772 \right]$

---

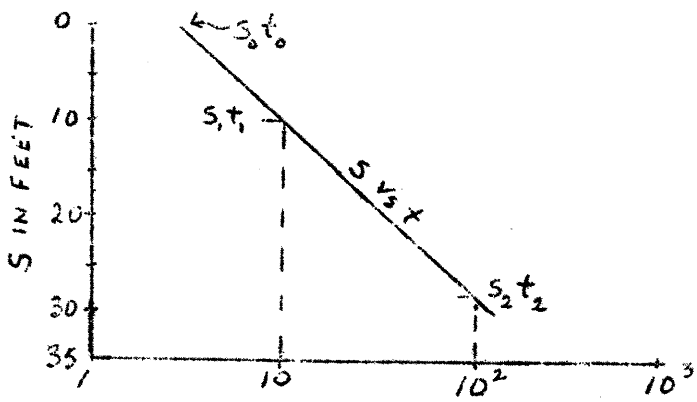
[1] A review of "Drawdown Test to Determine Effective Radius of Artesian Well", C. E. Jacob, Proc. ASCE, V. 72, No. 5 (as presented in "Hydrology", Wisler & Brater, John Wiley & Sons, 1949)

at time t_2 , $s_2 = \frac{114.6Q}{T} \left[\log_e \left(\frac{Tt_2}{1.87r_2S} \right) - 0.5772 \right]$

then $s_2 - s_1 = \frac{114.6Q}{T} \log_e \left(\frac{t_2}{t_1} \right)$ (65)

or $s_2 - s_1 = \frac{264Q}{T} \log_{10} \left(\frac{t_2}{t_1} \right)$ (66)

GRAPHICAL SOLUTION:



t = time in minutes

Observational data, s vst is plotted on semi-log paper, and an arbitrary choice made of t_1 , s_1 , and t_2 , s_2 after relationship has reached straight line proportions. If, for convenience, t_1 and t_2 are chosen one log cycle apart,

$$\log_{10} \left(\frac{t_2}{t_1} \right) = 1$$

and $$s_2 - s_1 = \Delta s = \frac{264Q}{T} \qquad (67)$$

Note: change in drawdown, per log cycle, varies directly as Q and inversely as T

or $$T = \frac{264Q}{\Delta s} \qquad (68)$$

Extrapolation of the semi-log plot to s = 0 permits computation of S , the storage coefficient.

From Equation 63

$$s = 0 = \frac{114.6}{T} \left[\log_e \left(\frac{Tt_0}{1.87r^2S} \right) - 0.5772 \right]$$

$$\log_e \left[\left(\frac{Tt_o}{1.87r^2S}\right) - 0.5772 \right] = 0$$

and $$\log_e \left(\frac{Tt_o}{1.87r^2S}\right) = 0.5772$$

or $$\frac{Tt_o}{1.87r^2S} = e^{0.5772}$$

and $$S = \frac{Tt_o}{1.87r^2 e^{0.5772}} = \frac{0.3Tt_o}{r_2} \qquad (69)$$

when t_0 = time intercept of zero drawdown axis, in days.

Adjustment of Test Data for Thin Aquifers. One of the basic assumptions of the Thiem and Theis formulas is a constant value of transmissibility. However, under water table conditions, the draw down reduces the saturated thickness of the aquifer and if appreciable, transmissibility is not constant but decreases with time (of pumping).

Jacob's method of adjustment:

$h = m - s$, where m is the original height of the water table, h is the depth of a given distance from the well when pumped and s is the drawdown.

and $h^2 = m^2 + 2\,m\,s + s^2$

and $h_2^2 = m^2 + 2\,m\,s_2 + s_2^2$

$h_1^2 = m^2 + 2\,m\,s_1 + s_1^2$

and $$h_2^2 - h_1^2 = -2\,m\,s_2 + 2\,m\,s_1 + s_2^2 - s_1^2$$
$$= (2\,m\,s_1 - s_1^2) - (2\,m\,s_2 - s_2^2)$$
$$= 2m \left[\left(s_1 - \frac{s_1^2}{2m}\right) - \left(s_2 - \frac{s_2^2}{2m}\right) \right] \qquad (70)$$

Substituting in the Dupuit equation in form,

$$\log_e \frac{r_2}{r_1} = \frac{\pi P}{Q} (h_2^2 - h_1^2)$$
$$= \frac{\pi P}{Q}\, 2\,m \left[\left(s_1 - \frac{s_1^2}{2m}\right) - \left(s_2 - \frac{s_2^2}{2m}\right) \right]$$

but $T = Pm$

$$\log_e \frac{r_2}{r_1} = \frac{2\pi T}{Q} \left[\left(s_1 - \frac{s_1^2}{2m}\right) - \left(s_2 - \frac{s_2^2}{2m}\right) \right] \qquad (71)$$

Transposing and converting to $\log_{10}$,

$$T = \frac{527.72 \log_{10} r_2/r_1}{\left[(s_1 - s_1^2/2m) - (s_2 - s_2^2/2m) \right]} \qquad (72)$$

where s_1 and s_2 are small compared with m_1

$s_1^2/_{2m}$ and $s_2^2/_{2m}$ may be omitted,

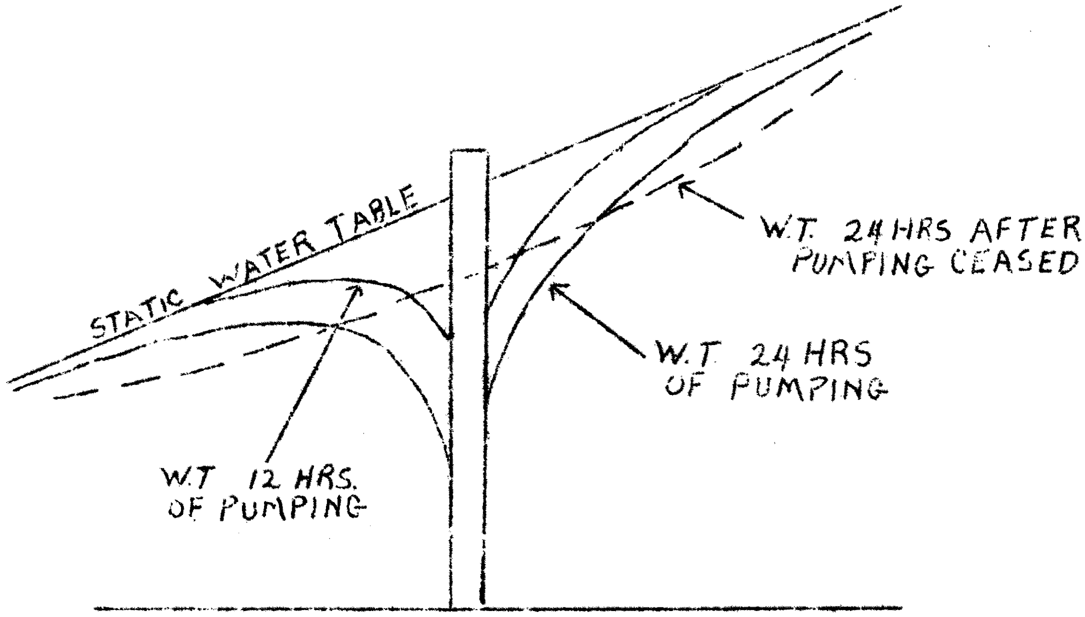
reverting to T in terms of the Thiem formula.

Affect of Slope of Normal Water Table on Flow to Wells.[1]

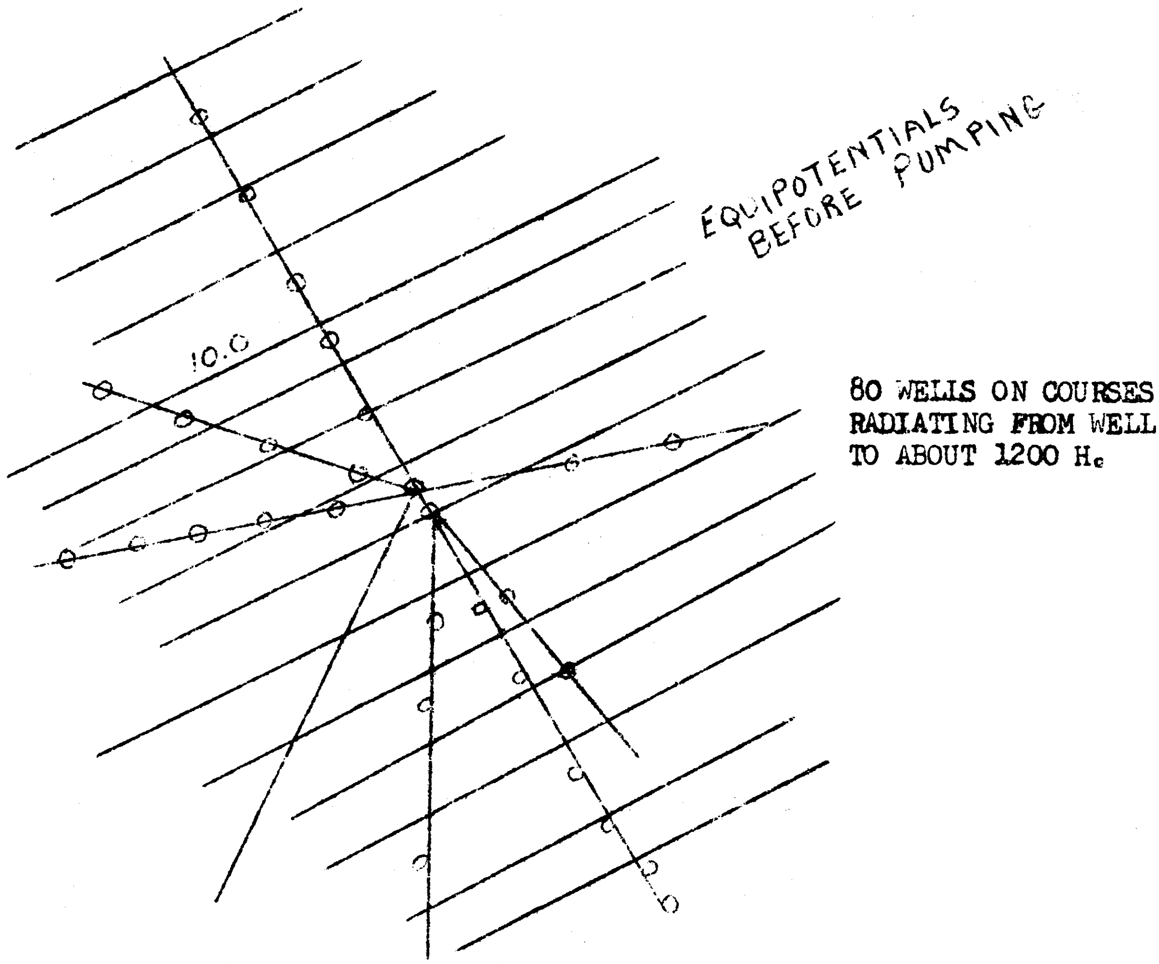
Methods for modifying well flow formulas to correct for the effects of the normal slope of the water table (or piezometric) surface, resulting in deformation of the well "cone", were developed on the basis of the extensive measurements pertaining to a well near Grand Island, Nebraska, in 1931. Formulas developed have since been verified by additional test data secured in Nebraska, Kansas, and elsewhere.

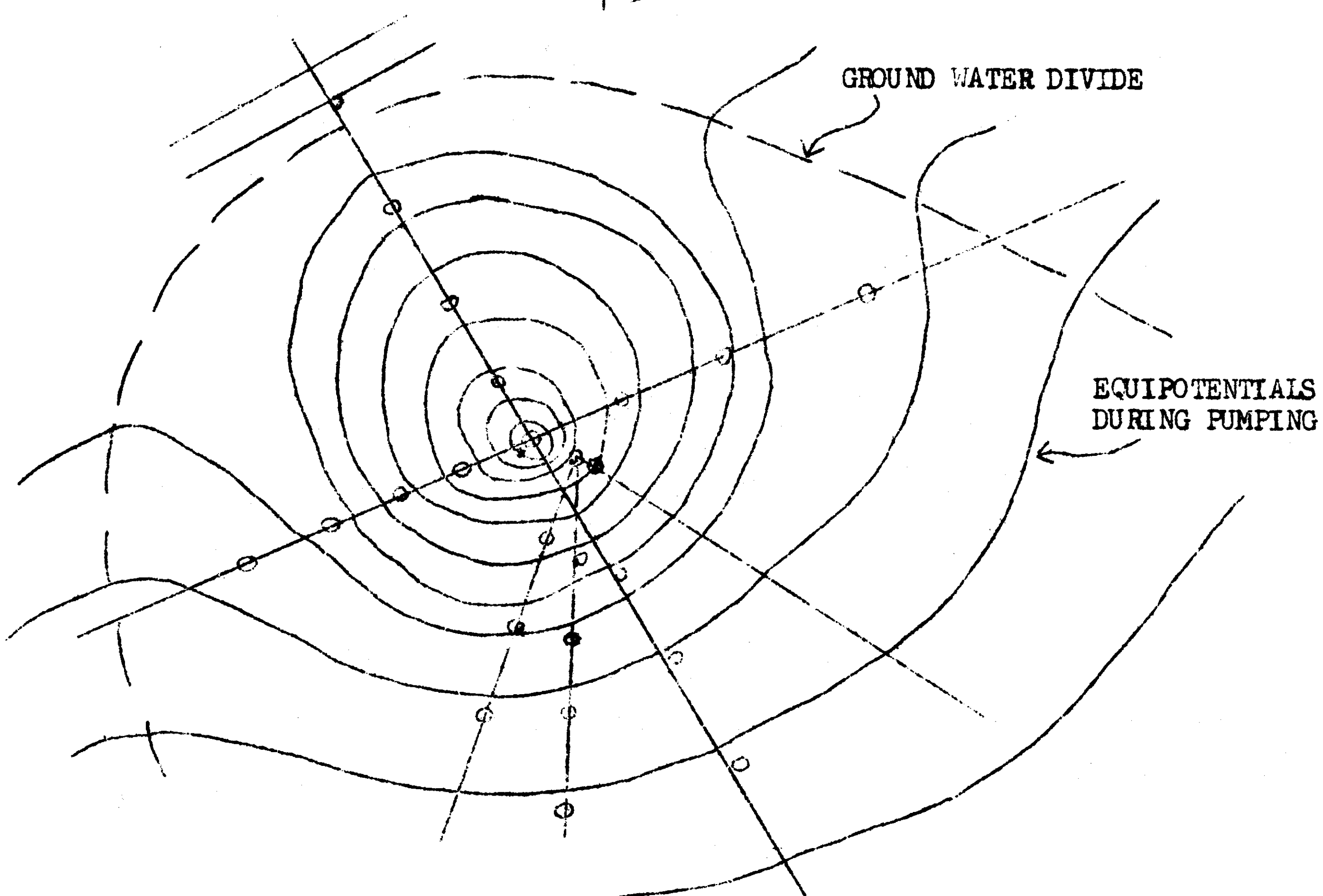
Wenzel, Water Supply Paper No. 887, USGS (As covered by Mienzer & Wenzel in "Physics of the Earth - IX, Hydrology", McGraw Hill, 1942, pp 459-476.)

The Limiting Formula:



[1] A Review of U.S. Geologic Survey Water Supply Paper 887, L. K. Wenzel, 1942 (As presented by O. E. Meinzer and L. K. Wenzel in "Physics of The Earth - IX," McGraw-Hill Book Co., 1942.


EQUIPOTENTIALS BEFORE PUMPING
10.0
80 WELLS ON COURSES RADIATING FROM WELL TO ABOUT 1200 Hc


GROUND WATER DIVIDE
EQUIPOTENTIALS DURING PUMPING

Equilibrium Conditions: Thiem formula found to apply provided s_1 and s_2 are averages of paired measurements, taken at equal distances, r_1 and r_2, respectively, up and down the normal hydraulic slope from the well. It was accordingly developed that:

q = discharge pumped in g.p.m.

P_f = Permeability in gal./day through each ft. depth of the aquifer in a width of 1 mile for each ft./mile of hydraulic gradient

$$P_f = 527.7qC \tag{73}$$

when
$$C = \frac{A}{B} \tag{74}$$

$$A = \frac{\log_{10} \frac{r_2}{r_1}}{0.25\ M} \tag{75}$$

and
$$B = \frac{(s_{1u} + s_{1d})}{2} - \frac{(s_{2u} + s_{2d})}{2} \tag{76}$$

$M = \sum m$ at the points of the 4 drawdowns. (saturated thicknesses of water bearing material)

C is obtained by plotting all possible values of A vs. B, should be of at least 3 pairs as deviation from straight line indicates variance from the basic assumptions.

For the ideal hydrologic system postulated by Dupuit

$$P = \frac{Q}{IA} \quad \frac{Q}{2\pi i\ x\ y}$$

Taking into account a normal slope of the water table

$$P = \frac{2Q}{\pi x\ (y_u + y_d)(i_u + i_d)} \tag{77}$$

Where y_u and y_d, and i_u and i_d are respectively the saturated thickness and the hydraulic slopes of k distances directly up and down the normal water table slope.

The hydraulic gradient of the point r distance from the well can be satisfactorily approximated from a profile of the cone of depression by $i_r = \frac{h_{r+b} + h_{r-b}}{2b}$. Study of data indicates b = 10 is satisfactory, from which, for water table conditions:

$$P_f = \frac{18,335q}{r\,(y_u + y_d)\,(f_{(r+10)_u} + f_{(r+10)_d}) - (f_{(r+10)_u} + f_{(r-10)_d})} \qquad (78)$$

and <u>for</u> <u>artesian</u> <u>conditions</u>:

$$P_f = \frac{9,168q}{rm\,(f_{(r+10)_u} + f_{(r+10)_d}) - (f_{(r-10)_u} - f_{(r-10)_d})} \qquad (79)$$

When y_u and y_d are respected saturated thicknesses of the aquifer of r distances, respectively, up and down the slope from the well; and $f_{(r+10)_u}$ and $f_{(r+10)_d}$ are respective altitudes of the water table (or piezometric surface) of distance $(r+10)$ up and downstream, and $f_{(r-10)_u}$ and $f_{(r-10)_d}$ are the corresponding altitudes at distances $(r-10)$ from the well in ft.

<u>Non-Equilibrium Conditions</u>: On the basis of tests, it was tentatively concluded that determinations of permeability using the non-equilibrium formula require a similar balancing of the draw-down data obtained from one or more pairs of up and down stream observation wells situated, on a line of the normal ground water (or piezometric surface) slope, through the pumped well.

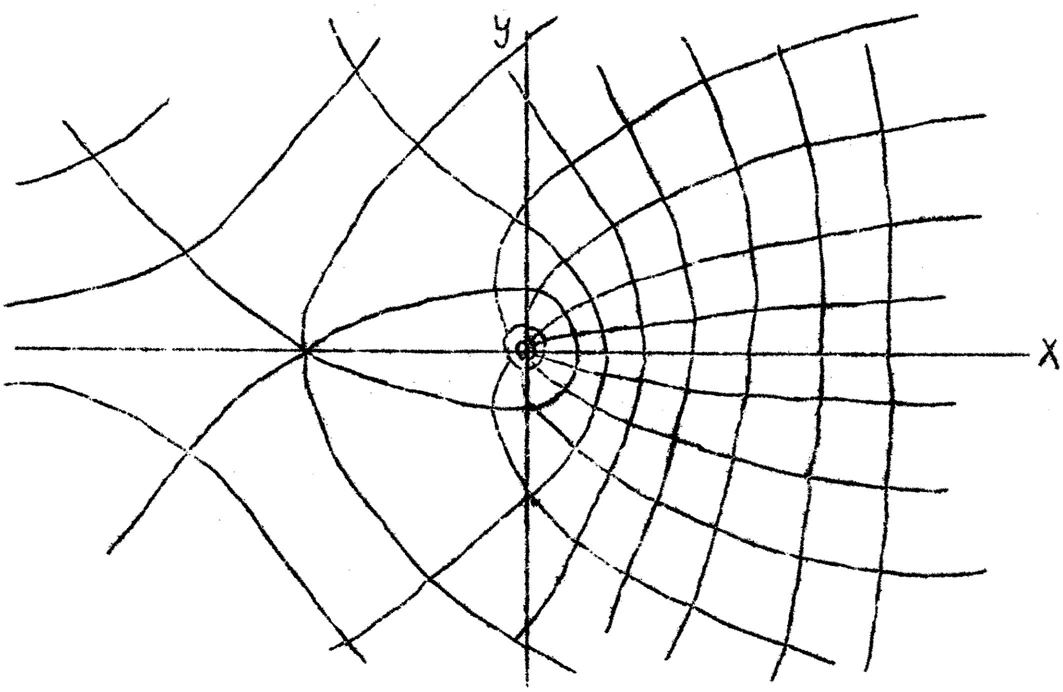
<u>Application of Equilibrium Formulas for Flow to Wells and the Extent of the Cone of Influence</u>.

Meinzer and Wenzel (Ref.9) point out that while equilibrium flow conditions may not prevail over the entire cone of influence even after very long periods of sustained pumping, that equilibrium is reached close to the well soon after pumping begins, gradually radiating to greater distances. Thus, applications of the well performance formulas, based on the assumption of steady flow, may be quite satisfactory using data of observations made fairly close to the well within a reasonable time after pumping started.

It is pointed out, however, that flow formulas based on assumed boundary conditions of the limits of the influence cone must generally be considered to be correct only to the extent that contributions to the discharge beyond such an arbitrarily assumed limit of influence can be considered to be negligible. The experience has been that equilibrium formulas have generally given satisfactory results.

While many hydraulicians have assumed the limiting radius of the cone of influence to be in the order of 500 to 1000 ft., observations have been made indicating much more extensive area of influence, up to 7 miles or more from a well.

Velocity Potential and Stream Function for a Well in Uniform Flow.



See Fig. 5 Ground Water Chapter "Hydraulics for Engineers". Edited by Rouse.

First consider flow in the negative x-direction in an effectively infinite sand of uniform thickness.

$$\phi = kh$$

$$V_x = -k \frac{\partial h}{\partial x}$$

$$\frac{\partial h}{\partial x} = \frac{V_x}{-k} \qquad V_x = C$$

$$dh = \frac{V_x \, d_x}{-k} \quad \& \quad h = \frac{V_x \,(x)}{-k} + C \qquad h = \frac{V_x \, x}{-k}$$

when $h = 0$, $V_x = 0$, & $C = 0$

$$\phi_o = \frac{V_x \,(-x) k}{-k} = V_{o_x} \, x = \text{Velocity Potential}$$

Stream function at R $\triangle$ to Velocity Potential $= V_{o_x} \, y$

Considering radial flow into the well at the origin shown in the figure

$$\phi = kh$$

$$\text{and } h = \frac{Q}{2\pi kb} \ln \frac{r}{r_w} \qquad \text{See Eq. (12)}$$

$\frac{Q}{b} = q =$ discharge per unit thickness of sand.

$$\phi = \cancel{k} \frac{q}{2\pi \cancel{k}} \ln \frac{r}{r_w} = \frac{q}{2\pi} \ln \frac{r}{r_w}$$

or $$r = \sqrt{x^2 + y^2}$$

$$1/2 \ln \left(\frac{r}{r_w}\right)^2 = \ln \frac{r}{r_w}$$

or $$\phi = \frac{q}{4\pi} \ln \frac{x^2 + y^2}{r_w^2}$$

The stream function corresponding to the velocity potential is at all times at right angles to velocity potential function

$$\chi_w = \text{stream function}$$

$$\chi_w = \frac{q}{2\pi} \theta = \frac{q}{2\pi} \tan^{-1} \frac{y}{x}$$

The combined flow in the figure is the sum of the two velocity potentials

$$\phi = \phi_o + \phi_w$$

The point of stagnation downstream from the well can be found by equating the x component of velocity to zero and solving for x

$$V_x = \frac{-d\phi}{dx}$$

$$\phi = V_{ox} + \frac{q}{4\pi} \ln \frac{x^2 + y^2}{r_w^2}$$

$$V_x = -\frac{\partial \phi}{\partial x} = -V_o \frac{-q}{2\pi} \frac{x}{x^2 + y^2} = 0 \qquad \text{with } y = 0$$

Solving for x $$-V_o \frac{-q}{2\pi} \frac{\cancel{x}}{x^{\cancel{2}}} = 0$$

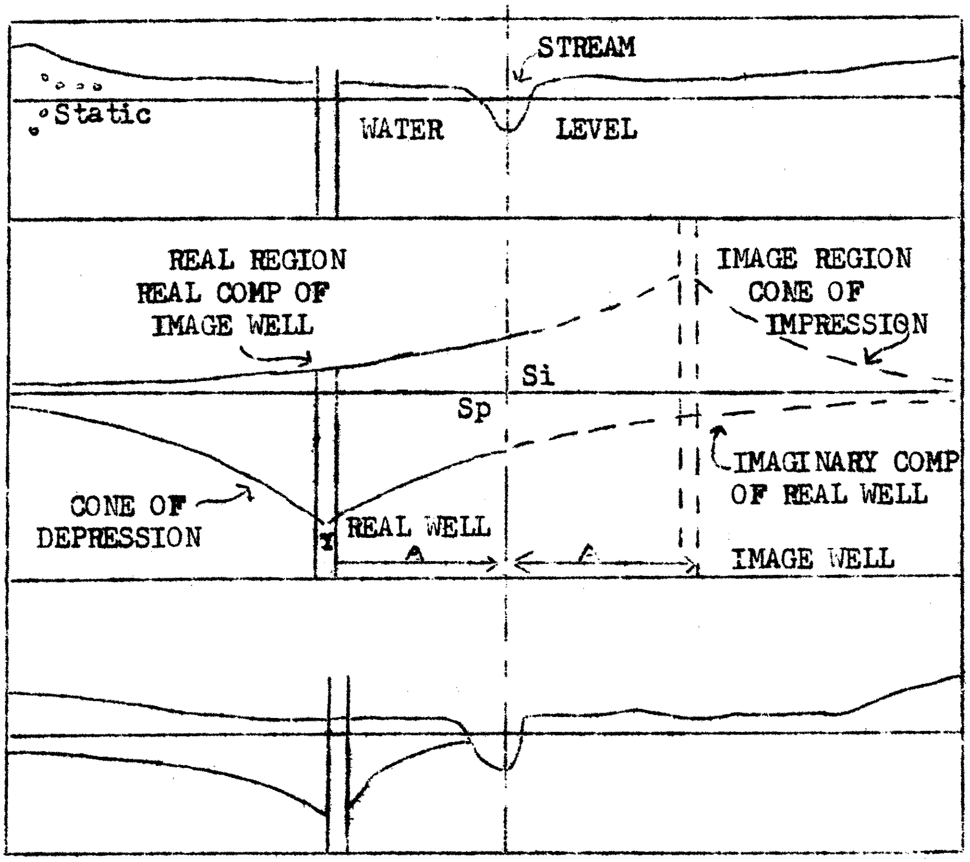
$$x = \frac{-q}{2\pi} V_o$$

<u>Method of Images</u>

The assumption that an aquifer is of infinite areal extent is necessary in the development of either the equilibrium or the nonequilibrium formula. This condition is satisfied by only a few major aquifers such as Dakota sandstone. In most areas the existence of formation boundaries or of folds and faults, or the direction by surface streams serves to limit the continuity of consolidated strata to a few miles or more. Particularly in glaciated areas the prerequisite of infinite areal extent is seldom satisfied. It is necessary to make appropriate adjustments for the effect of geologic boundaries before the above equations can be applied to problems of flow in areally limited aquifers. Impervious formations detract from the contributing area of the aquifer they bound - (called Negative Boundaries). Positive boundary is where an aquifer is intersected by a perennial stream or other body of surface water with sufficient flow to prevent development of the cone of depression beyond the surface source. See Fig. 86, Page 248, "Hydrology", Wisler and Brater, which shows several possible geological boundaries.

Most faulted structures don't occur as abrupt straight lines but rather as tapered and irregular terminals. However, in general the area covered by a well, field or pumping test site is small compared to area of such limited aquifers. Hence, it is often possible to treat a geological boundary as an abrupt discontinuity. The greater the distance the boundary is from the well site the less the error involved. Where conditions permit assuming a straight line demarkation for a geologic boundary it is possible to solve the problem by substitution of a hypothetical system that satisfies the real system.

Method of Images devised by Lord Kelvin in his work on electrostatic theory is a convenient method for solving boundary problems. Consider the following figure:



See Fig. 87 "Hydrology" by Wisler and Brater

To be effective as a boundary the stream flow must equal or exceed the withdrawal of the well since any flow below well yield, will result in drying up the stream and elimination of the boundary. Assume the stream to be of line width = line source. Assume vertical flow at stream = 0. There will then be zero drawdown at line source. Any system satisfying zero drawn down at the line source is a solution of the real problem.

<u>Steps</u> (See Above Figure)

1. Real bounded aquifer is replaced by an imaginary aquifer of infinite areal extent.

2. An imaginary well is placed on opposite side of and equidistant from the boundary.

3. Note that the image well returns H_2O to aquifer as fast as real well takes it out.

4. Image well produces a buildup of water level at boundary = drawdown of real well.

5. System results in zero drawdown at boundary and satisfies limit of real problem.

6. To secure resultant cone of depression or to evaluate drawdown at any point in real region add real components of several depression cones.

7. Drawdown is:
   a. Steepened on river side.
   b. Flattened on land side.

An aquifer bounded by an impervious strata on one side is next considered.

1. Boundary is approximated by a sharp line of demarkation.

2. Limit imposed by the barrier is that there shall be no flow across the boundary line.

3. An imaginary well is placed across the boundary at a right angle to and equidistant from the boundary.

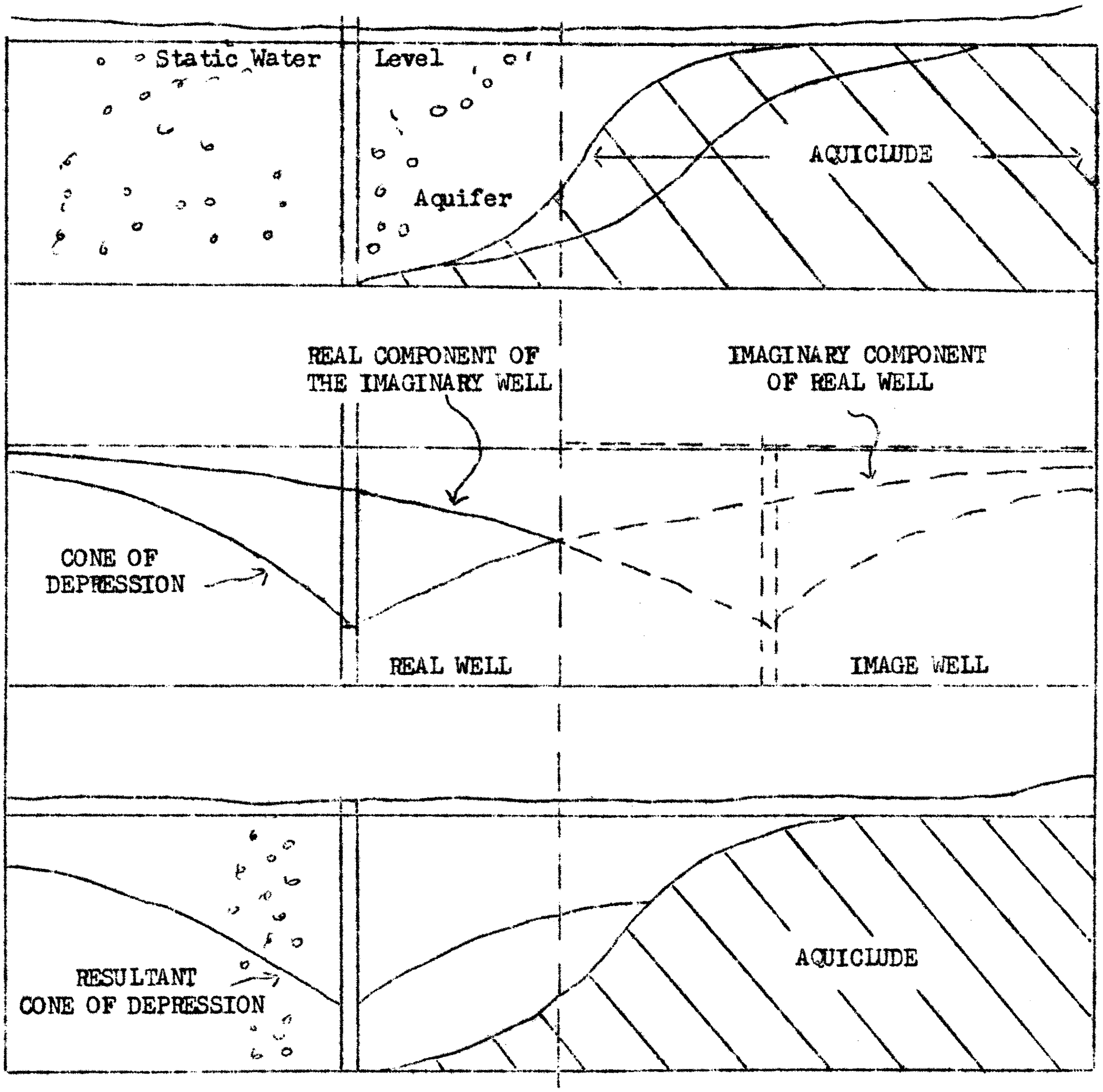
4. The symetrical drawdown cones produce a ground water divide everywhere along the boundary.

5. This image system satisfies the limit of the real problem and is therefore a solution.

6. The drawdown at any point is determined by adding the real component of each depression cone.

7. The resultant cone of depression is flattened on the side adjacent to the boundary and steepened on the opposite side of the well.

See following figure illustrating the above steps.



See Fig. 88 "Hydrology" by Wisler & Brater

Next consider an aquifer bounded by impervious strata on two sides.

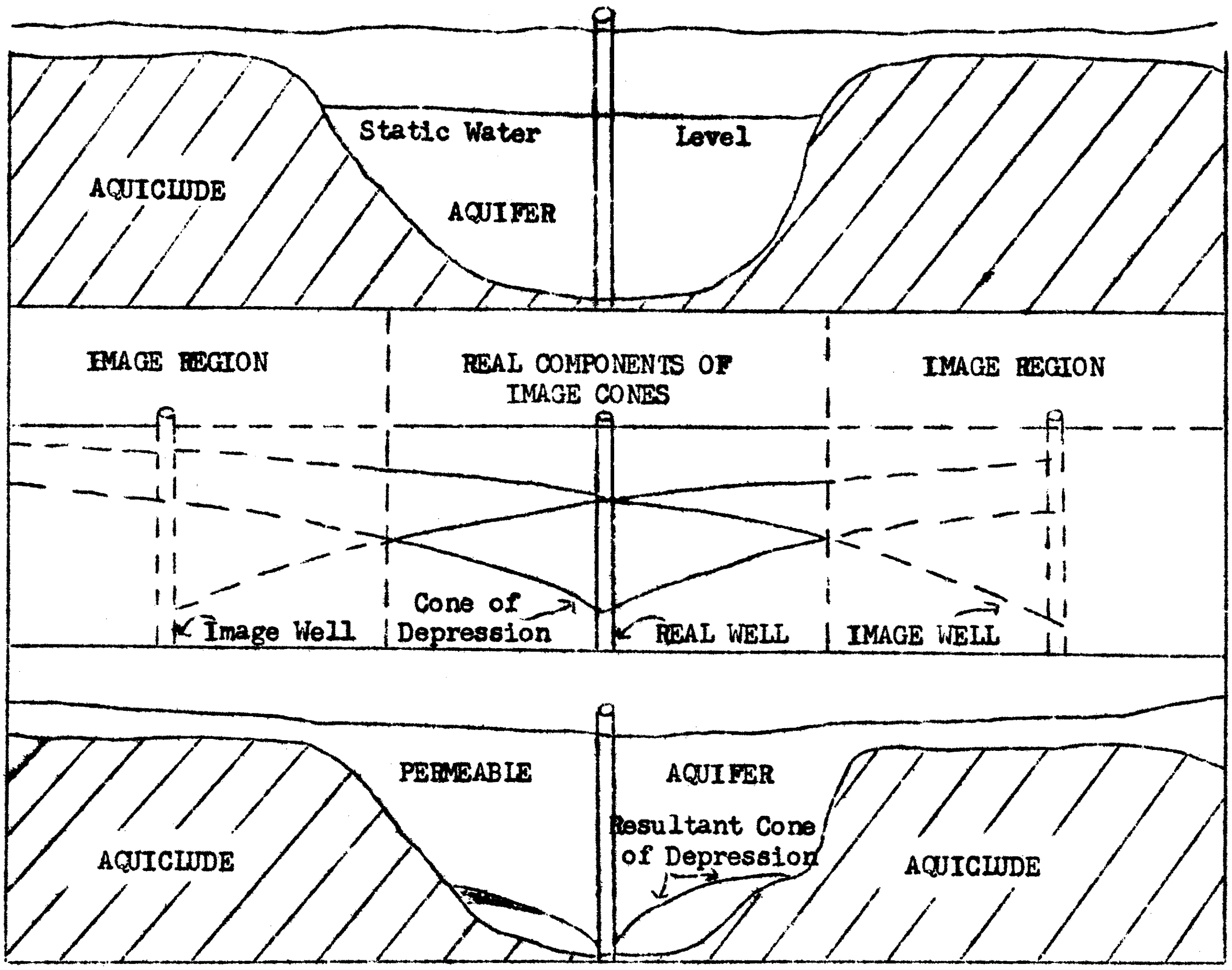


Fig. 89, "Hydrology", Wisler and Brater

The primary images balance the effect of the real well at each boundary as in the previous examples.

Although the primary images balance the effect of the real well at their respective boundaries, each image produces an unbalanced drawdown at the farther boundary. The unbalanced drawdowns at the boundaries theoretically produce a gradient and consequent flow across the boundary. It is necessary then to add a secondary set of image wells at the appropriate distances to balance the residual effect of the primary images.

Each image well in the secondary set will again distribute the balance at the farther boundary and all successive sets of images to infinity will leave residuals at the boundary. In practice add image pairs until residual effects are negligible in comparison with total effect.

The modified non-equilibrium formula is particularly advantageous for the analysis of image or boundary problems because it is easier to recognize changes of slope in a straight line than to detect changes in curvature of a log-log plot.

Consider Eq. 67

$$\Delta s = \frac{264Q}{T} \qquad \Delta s = \text{draw down difference per log cycle in ft.}$$

This equation indicates that $\Delta s/1$, the slope of the semi log graph, is dependent only on Q and T and for a given aquifer T = C and Q may be held constant.

Under above conditions the graph of drawdown versus the log of time, since the pump started, will show successive changes in slope as pumping continues. The water level will draw down at an initial rate under the influence of the real well that is nearest to the observation well. When the cone of depth of the nearer image well affects the observation well rate of drawdown will be doubled after r^2/t becomes small because total rate of withdrawal is then equal to the rate of the real well plus one image well or twice the rate of the real well. The effect of the further image well triples the slope of the original semi-log graph, after r^2/t becomes small since rate real well + rate 2 image wells equals 3 times the rate of the real well.

Jacob's method, you recall, is restricted to small values of u and u is small when r is small and t is large. Frequently image distances are large in relation to the distance from an observation well to a pumping well. For a single image the semi-log plotting of (s) vs (t) shows a two limb graph.

The transition from the first limb to second limb follows a curved path thru the region where values of u for the image well are large. When t becomes large in relation to r^2, u becomes small and the observed data follows the straight line of the second segiment of the graph.

Consider Fig. 90 "Hydrology", by Wisler and Brater. This shows effect of more than one image well.

The approximate location of image wells can be obtained by drawing tangents to observed data graph at appropriate slope values. Note the following:

1. Graph of Fig. 90 could be separated and plotted as 3 separate lines.

2. Each component has same slope and intersects zero drawdown axis at its respective value of time.

3. The time intercept on this axis permits calculation of (S) by Eq. 69 and since S = constant for a given aquifer

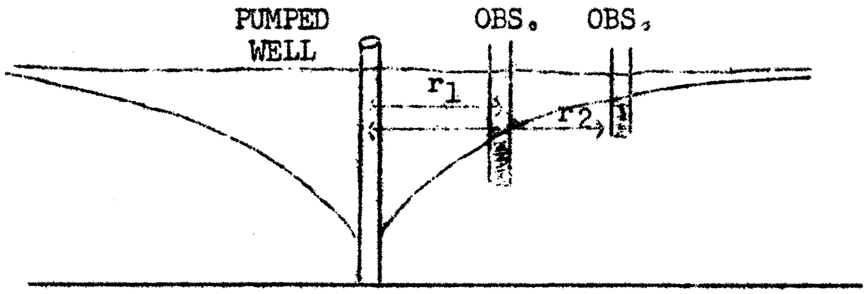
$$S = \frac{0.3Tt_1}{r_1^2} = \frac{0.3Tt_2}{r_2^2} = \frac{0.3Tt_3}{r_3^2}$$

or $$\frac{t_1}{r_1^2} = \frac{t_2}{r_2^2} = \frac{t_3}{r_3^2}$$

The time intercepts are then known for all wells real or imaginary. Now if the distance from the real well to the observation well is known, the distance to any image well can be determined from the data on the semi-log graph. This method can involve considerable error from small error in intercept locus.

The following method avoids the above.

1. Assume two observation wells at distances r_1 and r_2 from discharging well. The drawdown in each observation well can be calculated as follows.



$$S_1 = \frac{114.6Q}{T} \left[\ln \left(\frac{Tt_1}{1.87r_1^2S}\right) - 0.5772 \right]$$

$$S_2 = \frac{114.6Q}{T} \left[\ln \left(\frac{Tt_2}{1.87r_2^2S}\right) - 0.5772 \right]$$

2. Make semi-log graph for observation well (1) and for observation well (2). Then when $S_1 = S_2$

$$\frac{114.6Q}{T} \left[\ln \left(\frac{Tt_1}{1.87r_1^2S}\right) - 0.5772 \right] = \frac{114.6Q}{T} \left[\ln \left(\frac{Tt_2}{1.87r_2^2S}\right) - 0.5772 \right]$$

or upon simplifying $$\frac{t_1}{r_1^2} = \frac{t_2}{r_2^2} \qquad (81)$$

Equation 44 and 45 are identical. As a result of equations 80 and 81 we can say that for a given aquifer the times of occurrence of zero drawdown or of equal drawdown vary directly as the squares of the distance from the observation well to the discharging well and are independent of pumping rate. Equal drawdown method was used for data on Fig. 90.

The time values at the tangent intersections determine only approximate distances to image wells. The time intercepts so determined will be too small and calculated image distances will be smaller than correct distances.

The preliminary estimates of image distances may be corrected by trial in the following manner:

1. Assume locations of image wells based on values computed by first approximation.

2. Using Equation 61 and 62 we can determine the drawdown component resulting from each image well.

3. The time drawdown graph is obtained by adding these components.

4. Make successive adjustments in image distances until computed and observed graphs are in agreement.

If a pumping test is run without prior knowledge of geologic boundaries, and if semi-log graphs for all observation wells show evidence of image reflections it is possible to locate the distance to such boundaries by calculating the distance from each observation well to each image well. Then by scribing an arc from the respective observation wells we can locate the image wells at the intersection of arcs. The boundary is then located at midpoint of a line joining the real well and the image well.

PROBLEMS OF WELL CONSTRUCTION[1]

General Construction Methods and Problems

A major problem in securing construction of satisfactory irrigation wells is the employment of a competent driller. Drillers experienced only in drilling of domestic and stock water wells are not equipped to drill much larger wells required for irrigation supply. Commonly those capable of drilling irrigation wells operate in local areas. Movement of equipment over any distance to new areas of development is expensive.

Frequently drillers do not penetrate the full thickness of the aquifer, thereby losing much of the potential well capacity. It is difficult to get drillers to read and understand the available technical literature. Much good information on techniques and equipment is supplied by manufacturers of well equipment.

Unless drillers are well acquainted with an area, a test well should be put down and test pumped to secure information on nature regarding location and thickness of water bearing strata and their water producing characteristics.

Aquifers range from fine sands to cobbles and present various screen requirements, problems of development, and involve various equipment and methods of drilling. Well graded aquifers may be developed to produce their own gravel envelopes by sand removal from near the screen. Others consisting of fewer materials will require the construction of a gravel envelope to retain the sand of the aquifer and minimize entrance head losses to the well.

The requirement for construction of a gravel envelope requires a hole substantially larger than the inner casing and frequently requires the use of a temporary outer casing.

The use of the reverse rotary drill, whereby water is pumped down outside the drill stalk and returned inside results in balanced pressures against the aquifer during drilling reducing danger of caving and the need for an outer casing. The higher upward velocities enable bringing up larger drill cuttings and eliminates the necessity for special sealing materials and reduces tendency to seal the aquifer. However, it requires much larger quantities of water and is limited in depth to about 500 ft. The reverse rotary method has been slow of adoption by drillers and is not in common use.

Cable drilled wells are most common and their efficiency often depends to considerable extent on the methods and thoroughness of their development.

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[1]From lectures by W. E. Code, Irrigation Engineer, Colorado Agricultural Experiment Station.

The methods and problems of well construction are well covered by Rohwer (Ref. 12) and Johnston (4).

Problems of Well Screens and Gravel Envelopes[1]

Irrigation wells must generally produce 450 gpm or more. This involves high velocities close to the well creating problems of screening and development quite different than for domestic wells. Pumping equipment and power are available for satisfactory and economic pumping for irrigation supply. However, the design of screens and envelopes has not been well standardized and frequently imposes a limitation of satisfactory and economic pumping.

Experiments on Screens and Envelopes. Cooperative investigations are being conducted by the Soil Conservation Service, Colorado Agricultural Experiment Station and screen manufacturers on the hydraulic problems of screens and gravel envelopes. Tests are run full scale, using 12 inch diameter screen sections 2 feet long. Tests are first made without gravel packs and subsequently with varying grades of gravel.

Head loss relationship to discharge per foot of length of screen is related to the total length of the screen, longer screens producing greater losses inside as the result of turbulence. For the 2 ft test length, the loss reduced rapidly as the screen coefficient (% clear openings) up to 30-40% with little further reduction of losses for larger screen coefficients (Note: Specific relationship indicated is peculiar to the 2 ft. screen length).

Tests of gravel packs showed that for a given discharge the lowest screen loss occurred with no pack and was slightly increased for 1" diameter gravel. Smaller sizes of gravel increases the losses at the screen substantially (not losses through the pack, but at the screen as result of reducing the effective screen openings).

Observations of debris on the outside of screens following tests indicated most of the flow to be near the bottom of the screen. This was confirmed by velocity measurements within the screen. This would result in high velocities in this section and may be a cause of sand pumping. From this standpoint, it might be desirable to graduate screen opening sizes to equalize the flow over its length.

Results of the cooperative test of screens and gravel envelopes, to date are reported by Peterson, etal, (Ref. 12).

A theoretical analysis of well screen hydraulics was developed by Peterson and confirmed by his laboratory investigations (Ref. 11 and 12). The results furnished important criteria for the selection of well screens.

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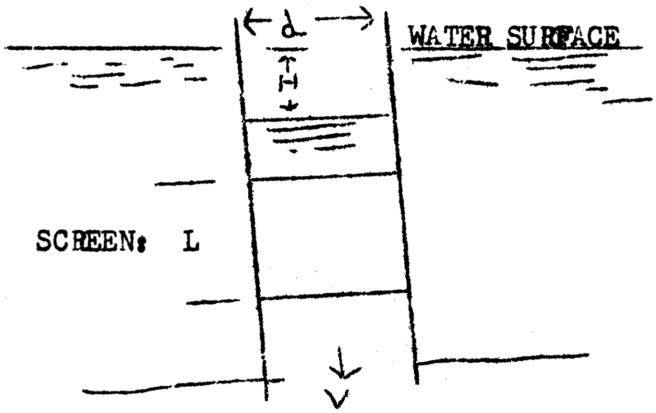
[1]From lectures by C. H. Rowher, Sr. Irrigation Engineer, U.S.D.A., Soil Conservation Service.

The problem, illustrated by the following sketch, was considered as consisting of two parts: flow through a series of orifice openings as water enters the screen and flow within a pipe manifold within the screen.



A dimensional analysis, after the elimination of the assumed insignificant variables yielded

$$\frac{\Delta h}{V^2/g} = f\ (C_c\ ,\ C_s\ ,\ \frac{L}{d}) \tag{1}$$

where L is the screen length; d the screen diameter; C_s the percent open area; C_c the contraction coefficient for the perforation; Δ h the difference in hydrostatic head between the inside and outside of the screen; V the mean velocity of the liquid in the screen. In this first analysis the effect of the surrounding gravel was omitted.

A theoretical analysis based on the principles of continuity, energy and momentum verified the selection of dimensionless parameters in Eq. (1) as well, as combining them in a more usable form. The final equations were as follows:

$$\frac{\Delta h}{Q^2/A^2 g} = \frac{\cosh CL/d + 1}{\cosh CL/d - 1} \tag{2}$$

or

$$\frac{\Delta h}{V^2/2g} = \frac{\cosh CL/d + 1}{\cosh CL/d - 1} \tag{3}$$

where C is a dimensionless number computed as equal to 11.31 C_c C_s . Reference to Eq. (1) will show that Eqs. (2) and (3) are specific combinations of the same dimensionless parameters.

Idealized screens, the values for C_c being those determined by von Mises, were used in establishing the validity of equations. A slight deviation of the experimental data from the theoretical curve resulted. However, after the effect of viscosity on the contraction coefficient was allowed for and the boundary zone conditions within the screen were allowed for by using an effective diameter equal to 96% of the true diameter, the data followed the theoretical curve very closely.

What was determined was that the loss coefficient,

$$\frac{\Delta h}{Q^2/A^2 g}$$

becomes asymptotic with 1.0 when CL/d is equal to and greater than 6. This is due, of course, to the fact that the plus or minus 1 becomes negligible to cosh 6. Therefore, it is considered that 6 is the critical value for CL/d which should be exceeded for minimum losses.

The theory was applied to the losses into and through several commercial screens and expanded to include the effects of the surrounding gravel. The coefficient K, including these factors, was substituted for C_c in C and determined for various combinations from the theoretical curve.

The following recommendations were offered:

(1) For minimum screen losses, CL/d must be greater than 6.

(2) The losses through the screen are independent of gravel size when CL/d is greater than 6, and thus the gravel can be selected on the basis of control of sand flow.

(3) The losses in a screen do decrease as the length is increased, but only until CL/d reaches 6. For values of CL/d greater than 6 the head loss depends only on the diameter, as included in the loss coefficient,

$$\frac{\Delta h}{Q/A^2 g}$$

It should be noted that increasing the diameter decreased the head loss, but at the same time decreases the values of CL/d. If CL/d is made less than 6 the length of the screen can be increased to hold losses at a minimum.

(4) The major part of the flow into a well takes place over the length of a screen (measured from the discharging end) required to obtain a CL/d value of 6. This is, therefore, the most important part of the screen.

(5) It was possible to determine the screen coefficients of only a few selected screen and gravel size combinations. These are given in Ref. 12.

WATER LAW

The main source of information for this section has been the United States Geological Survey Circular No. 117, written by C. L. McGuinness. This Circular includes a resume, by States, of the existing water laws (up to time of publication).

Legal concepts concerning water have necessarily been founded upon hydrologic knowledge at the time of their formulation, and thus reflect the incompleteness of scientific information. This situation creates difficulties, because many laws have been developed even with no knowledge of hydrologic facts and the underlying principles controlling them.

If sound development of water resources is to be made, many existing deficiencies must be corrected. A "hydrologically sound" water law should:

1. Apply the same rules to all ground water, rather than try to distinguish between supposedly different kinds.

2. Apply the same rules to surface water.

3. Be consistent from state to state where interstate water sources and problems are involved.

Principles of Water Law.

All water law may be considered as following one of two major doctrines:

1. Riparian rights doctrine, or common-law doctrine. This right is based on ownership of land overlying a water bearing formation. It does not depend on use and is not lost by non-use.

   Variations:

   (a) English Rule: (1843. Acton vs. Blundell.) "The person who owns the surface may dig therein, and apply all that is there found, to his own purposes at his free will and pleasure; and if in the exercise of such right, he intercepts or drains off the water collected from the underground springs in his neighbor's well, this inconvenience to his neighbor falls within the description of *damnum absque injuria*, which cannot become the ground for an action".

   (b) American Rule of Reasonable Use: (1862 - Bassett vs. Salisbury Manufacturing Co., New Hampshire.) A man's right to the use of percolating water is limited by the corresponding right of his neighbor, and restricts each to "a reasonable use of his own property, in view of the similar rights of others."

(c) <u>California Rule</u>: The doctrine of Correlative rights. Modification of the rule of reasonable use. (1903 - Katz vs. Walkinshaw, California) Not only must a landowner's use be reasonable, in consideration of the rights of others, but it must be correlated with the uses of others in time of shortage. "Disputes - <u>where rights are equal</u> - in cases where the supply is sufficient for all, are to be settled by giving to each a just and fair proportion." No statement was made covering cases where rights are not equal. Use is restricted to lands overlying the common supply, at times when the supply is insufficient.

2. <u>Doctrine of Prior Application</u>. Developed under arid conditions in the United States. The first in point of time to put water to a beneficial use is recognized as first in right. (U. S. Supreme Court, 1874, Atchison vs. Peterson). Originally the decision applied to mining uses. Basey vs. Gallagher, 1874, extended the doctrine to appropriations for irrigation.

<u>Desert Lands Act, 1877</u>. The right does not exceed the amount of water actually appropriated and necessarily used for irrigation. All surplus water over and above such use and appropriation remains free for appropriation and use of the public.

Note the important features of the <u>Prior Appropriation Doctrine</u>.

(a) First in time - first in right. Later appropriators must cease use in reverse order of priority in time of scarcity.

(b) Use must be a beneficial one.

<u>Problems of Water Law.</u>

A water right is a right to the use of water and is usually recognized as real property, subject to protection by law. Generally, this protection takes the form of a limitation on the rights of others, so that the water remains available to the holder of the water right.

Many problems arise both in the formulation of laws governing water rights, and in the administration and interpretation of these laws. Some of the problems which appear will be considered briefly here.

Kinds of Water. Many laws are founded on empirical classifications of water, which are unsound, in the light of present knowledge. There is widespread belief, reflected in legislation, that different kinds of water exist, to which different rules of law can be applied. Various decisions have referred to surface water in watercourses, diffused surface water, spring water, subterranean watercourses, artesian waters, underflow dependent on streams, diffused percolating waters, percolating waters tributary to springs, percolating waters supplying wells, and seepage water.

It should be obvious, from a brief study of the principles of hydrology, that to be effective a water law should recognize that water in the underground reservoir all follows the same physical laws and cannot be subdivided; and also that ground water and surface water are interconnected, and cannot be considered separately.

Principles of Law. The two doctrines - of common law and prior appropriation - are diametrically opposed in principle and cannot successfully exist side by side. The tendency has been to weaken riparian rights in favor of prior appropriation, and appropriation has been modified by ever greater stress on the words reasonable and beneficial.

Allocation of Water Among States. Problems of interstate allocation of ground water have not been serious so far, but will appear as ground water is developed from aquifers that cross state lines. There is reason to believe that where states have difficulty agreeing on compacts regarding these reservoirs, the Supreme Court will make decisions.

Protection of Means of Diversion. In the Western United States a prior appropriator of surface water is entitled to reasonable protection of his means of diversion. If he is required to make changes in his diversion works because of a later appropriation, he must do it himself if at a reasonable cost. Otherwise, the later appropriator may be required to pay the difference between reasonable and actual cost.

Applied to ground water prior appropriators may have their pumping costs increased by later appropriations. However, if the lowering of the water table is required for full development of the reservoir, earlier appropriators may be considered as maintaining a non-beneficial use in pumping from higher levels, and their "reasonable" increase in cost may be quite high relative to their former costs. But, based on a determination that under economic conditions existing in the area other users are successfully pumping water from depths as great as the new, lower, depth from which the prior appropriator must pump, a modified interpretation of the word reasonable may be made.

Excessive Early Appropriation. Earlier use of water for irrigation in many cases was insufficient and therefore many prior appropriators are normally using an amount which, by present standards, is not necessary and, in fact, harmful. Usually, in courts such rights have been reduced on the grounds that waste is not beneficial.

Aquifers of Small Perennial Yield. One of the most difficult problems for administrators to face is the problem of a reservoir which has small or no recharge. The question is, whether to limit pumping to the negligible annual amount which may safely be pumped annually, or whether to "mine" the reservoir to serve the present generation, with no regard for the future. The solution to the problem will require careful consideration. One solution may be the artificial recharging of such reservoirs.

Optimum Yield. The maximum perennial yield of an aquifer can be obtained only by depressing the water table enough to reduce natural discharge and induce additional recharge as far as practicable. Therefore, the final, safe rate of pumping is less than the rate of pumping during the reduction stage. A safe and necessary rate initially is excessive later. The problem here is whether to make the reduction at the expense of later appropriators or force all users to share the reduction.

Preferential Uses. Where water is limited, it is in the public interest that some uses of water be given preference over others. Usually domestic use is given priority regardless of prior appropriation. Under appropriation doctrine, and excluding domestic use, the principle of preferential use would be applied by granting prior appropriators some compensation if their rights were condemned in favor of a later superior use.

The recommendation of the National Reclamation Association (1946) is that a new version of the appropriation doctrine should be followed whereby the holder of the water right is entitled to its use only as long as his right is not condemned in favor of a superior use, with no compensation for the loss of the right. The order of superiority recommended is:

1. Domestic and municipal use.

2. Irrigation and stock watering.

3. Water Power.

4. Mining; industrial use.

5. All other uses.

This order might, of course, be varied in different States, and even in different parts of the same State.

Proration in Time of Scarcity. Under strict appropriative doctrine, in time of shortage of water, junior appropriators are cut off altogether and senior appropriators get their full share.

A possible modification of this would be to prorate supplies in time of scarcity to both junior and senior appropriators with the junior reimbursing the senior for reduction in his prior right. The most difficult problem here would be to fix the amount of compensation to be paid.

Salt Water Encroachment. A special problem arises where there is danger of salt water entering a ground water reservoir as good water is withdrawn. When once salt water has entered a reservoir, it is practically impossible to remove it, and the reservoir is lost to all future use, or at any rate, for many years. All important coastal aquifers should be studied and placed under any necessary control to protect them from this danger.

Air Conditioning. Recent development of air conditioning has created new problems in the use of ground water. The drain on the water supply may be reduced by reuse of the water, which may decrease the demand for new water by as much as 90 or 95% whether such devices should be required by law is a problem of economics, because when conservation devices are used, the new water is used in a consumptive way, being lost by evaporation. Therefore, for example, they are economical in New York where used water is dumped into the sea, but in Denver, where municipal water is dumped into the North Platte River and used for irrigation, the recirculation would not be economical.

Conclusion.

The general objective of water law should be to promote full development of the water resources of the country, rather than to restrict use to prevent local overdevelopment. The principle of prior appropriation will probably prove to be most capable of promoting efficient development, but the principle of priority should be modified to realize a full, yet safe development of each area's water resources. It should be accepted that water is a public resource and its use should be regulated to produce the maximum public as well as private good.

In order to have the co-operation of water users and the general public in the regulation of water use, it is absolutely essential that there be an adequate understanding of the facts A regulation whose purpose and basis is not understood is difficult and often impossible to enforce. Therefore, in addition to the necessity for a sound hydrologic basis to any law governing the use of water, there arises the necessity that the public, and indeed many technical men, be made to understand the fundamental principles of occurrence of water beneath the earth's surface.

BIBLIOGRAPHY

1. Bennison, "Ground Water - Its Uses, Development and Conservation", Published by Johnson Well Screen.

2. Jacob, C. E., "On the Flow of Water in an Elastic Artesian Aquifer", Trans. Am. Geophys. Union.

3. ____________, "Drawdown Test to Determine the Effective Radius of Artesian Wells", ASCE Trans. 112, 1947.

4. Johnston, C. M., "Irrigation Wells and Well Drilling", Cir. 404, California Agric. Exp. Sta., May, 1951.

5. Kaxmann, "River Infiltration as a Source of Ground Water Supply", Trans. ASCE Vol. 113, 1947.

6. Linsley, Kohler and Paulhaus, "Applied Hydrology", McGraw-Hill.

7. McGuiness, "Water Law With Special Reference to Ground Water", Geological Survey Circular No. 117.

8. __________, "Recharge and Depletion of Ground Water Supplies", Trans. ASCE, Vol. 112, 1947.

9. Meinzer, O. E., and Wenzel, L. K., "Physics of the Earth", Vol. IX, "Hydrology", McGraw-Hill.

10. Peterson, D. F., Israelson, O. W., and Hansen, V. E., "Hydraulics of Wells", Utah State Agric. Exp. Sta. Bul. 351.

11. Peterson, J. S., "Well Screens", Unpublished Master's Thesis, Colorado A & M College, 1951.

12. Peterson, J. S., Rohwer, C., and Albertson, M. L., "Effect of Well Screens on Flow into Wells", Mimeograph, USDA, SCS, Jan., 1952.

13. Rouse, Hunter, "Engineering Hydraulics", John Wiley and Sons.

14. Theis, C. V., "The Relation Between the Lowering of Piezometric Surface and the Duration of Discharge of a Well Using Ground Water Storage", Am. Geophys. Union Trans., 1935.

15. Thomas, H. F., "The Conservation of Ground Water," McGraw-Hill.

16. Wenzel, L. K., "Methods of Determining Permeability of Water Bearing Materials," USGS, Water Supply Paper No. 887, 1942.

17. Wisler, C. O., and Brater, E. F., "Hydrology", John Wiley & Sons, 1949.

18. Zee, Chong-Hung, "The Use of Combined Electrical Analogies to Investigate Unconfined Flow into Wells", Ph.D. Thesis, Utah State Agric. College, 1952.