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DISCUSSION OF

"NUMERICAL ANALYSIS OF CONTINUOUS UNSTEADY FLOW IN OPEN CHANNELS"

by PIN-NAM LIN

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J. C. Schönfeld (Central Research Division, Directorate General of the Rijkswaterstaat, The Hague, The Netherlands)--The paper presents a useful contribution to the application of the theory of characteristics to the propagation of long waves under influence of frictional resistance, a subject to which the present writer's thesis [Propagation of tides and similar waves, Staatsdrukkerij, The Hague, 232 pp., 1951] was likewise devoted. It may be interesting to compare the author's new method to other methods, and to define more exactly the field of application of the new method. We first give a brief description of the methods to which we shall compare the author's new method.

In the first place we have the method of Craya and the first method of the present writer [see Thesis, chap. 8] with continuous characteristics and, if necessary, operated by trial-and-error (or rather iterative) processes. In the present writer's first method, which is essentially the same as Craya's method, although differing in various details, the solution is constructed for the characteristic variables $f^+ = \sqrt{gy} + v/2$ and $f^- = \sqrt{gy} - v/2$ (y = depth; v = velocity of flow). Along lines

$$dx/dt = c^+ = v + \sqrt{gy} \quad \text{or} \quad dx/dt = c^- = v - \sqrt{gy}$$

(characteristics), we have

$$df^+/dt = - a^+ = - (g|v|v)/(2\,C^2\, y) \text{ respectively } df^-/dt = - a^- = (g|v|v)/(2\,C^2\, y)$$

Pairs of values of f^+ and f^- are found each time by constructing two characteristics. The construction consists of drawing one projection of the characteristic in a tx-diagram (slope c^+ or c^-), and another projection in a diagram of tf^+ or tf^- (slope a^+ or a^-). The quantities c^+ and a^+ or c^- and a^- are defined from estimated averages of f^+ and f^-; instead of being computed, they may be found by means of a nomogram [see Thesis, chap. 14, par. 21]. The estimations for f^+ and f^- should be checked after the execution of the two constructions, and, if necessary, the constructions are repeated with better estimations (iteration; trial and error).

A second method was developed by the present writer [see Thesis, chap. 12, par. 23] for application in irregularly shaped channels. The channel is schematized by a division in rather short sections. Distinction is made between the width of the stream bed, b_s, and the width of the water surface, b, which are both considered as functions of the height of the water level above datum h. Across a transition of one section to another section with a different area of cross section A, the velocity of flow v and the water level h vary suddenly. The discharge Q, on the other hand, varies continuously across the transition, and the energy head above datum H does likewise, or very nearly so. Therefore, the characteristics are constructed in a diagram of HQ, together with a tx-diagram, in a way similar to Craya's method.

The construction of a characteristic is based on estimated average values $\overline{Q}$ and $\overline{h}$ of Q and h. The subcharacteristics (projections of the characteristics in the tx-plane) are defined by

$$dx/dt = c^+ = \gamma\,\overline{Q} + c_o \quad \text{and} \quad dx/dt = c^- = \gamma\,\overline{Q} - c_o$$

where $\gamma = (b + b_s)/(2bA)$ and $c_o = \sqrt{g\,A/b}$ are treated as given functions of the average water level h. Although the estimation of the averages requires a check, the construction of the subcharacteristics is seldom obstructed by trial and error.

The construction of the contra-subcharacteristic (projection in the HQ-plane) follows the equations

$$\Delta H = - W|Q_1|Q_1 - Z\Delta Q \text{ respectively } \Delta H = W|Q_1|Q_1 + Z\Delta Q$$

where Q_1 is the (known) value of Q at the beginning of the characteristic, and

$$Z = Z_o - \zeta \overline{Q} + W|\overline{Q}| \text{ respectively } Z = Z_o + \zeta \overline{Q} + W|\overline{Q}|$$

whereas

$$Z_o = 1/\sqrt{gAb},\ \zeta = (b - b_s)/2gb\,A^2, \text{ and } W = \Delta x/(C^2A^3/b_s)$$

(Δx = length of channel section) are treated as given functions of $\overline{h}$. In these formulas the resistance has been reduced in such a way that the construction becomes very insensitive to errors in the estimation of $\overline{Q}$. This eliminates trial-and-error processes nearly entirely.

Although not quite essential, the method is preferably executed in fixed sections, and the characteristics are defined in such a way that their intersections correspond to the transitions between the sections. This means that the characteristics are not continuous; in this aspect the method agrees with the author's new method. The difference between the two methods lies in the fact that the author operates with constant Δt, while the present writer operates with fixed Δx.

The discontinuities in the characteristics are overcome by inter- or extrapolations in Ht- and Qt-diagrams; these diagrams are useful recapitulations of the calculation, moreover.

Summarizing, we have three types of method: (1) methods with continuous characteristics (Craya's method, present writer's first method); (2) method with constant Δt (author's new method); and (3) method with fixed Δx (present writer's second method).

The fields of application of the above methods can be separated by considering two criteria. In the first place we may distinguish between regularly and irregularly shaped channels, and in the second place between rapidly varying motions (relatively short waves) where the inertia is predominant, and slowly varying motions (relatively long waves) where the resistance is predominant.

In opposition to the methods with constant Δt and with fixed Δx, the methods with continuous characteristics can avoid inter- or extrapolations. This means less work, and the elimination of a source of inaccuracies. Moreover, the operation is very simple. In the present writer's first method all computations are reduced to one adjustment in one nomogram.

It is therefore clear that the methods with continuous characteristics are preferable as long as they are well applicable.

According to the present writer's experience, trial and error usually becomes obstructive when the resistance dominates the inertia. In case of a dominating inertia (and not too long steps along the characteristics) a skilled computer can at once estimate the required averages with sufficient accuracy.

Most natural conduits like rivers and estuaries, and some artificial canals as well, are of such an irregular shape that a schematization by a prism is rather crude. The schematization of such an irregular channel by rather short sections, distinction between b_s and b, and treatment of b_s and b as functions of the height of the water level, has proved itself justified in a long experience in Holland in connection with harmonic, iterative (power series) and characteristic methods.

Hence we conclude that the methods with continuous characteristics (Craya's method, the present writer's first method) are preferable in case of regularly shaped canals, and rapidly varying motions (predominant inertia).

The author's new method with constant Δt will become superior when, in regularly shaped canals, the resistance becomes predominant (slowly varying motion). A competitive method in this case may perhaps be obtained by a modification of Craya's method in such a way that the resistance is treated in a manner similar to the present writer's second method.

In irregularly shaped channels and channel networks the present writer's second method should be applied.

Pin-Nam Lin (Department of Civil Engineering, Colorado A. and M. College, Fort Collins, Colorado, Author's closure)--The method proposed by the writer consists mainly in the use of a constant time interval and an independent set of characteristics in each step of computation. As previously indicated on page 228 of the paper, the reason for using constant Δt is to eliminate the trial process arising from the presence of the resistance terms. When the resistance is assumed to be negligible, one has a relatively simple case that can be solved in a straightforward manner, and hence does not need special treatment. The use of constant Δt for this case is therefore basically unnecessary. The use of an independent set of characteristics in each step, that is, the use of "discontinuous" characteristics, provides greater solvency in planning the computation, in order that computed points can be so distributed as to best define the shape of the integral surface. To follow the procedure of keeping Δt constant, one must work with "discontinuous" characteristics. The converse, however, is not true. In this connection, the reader may find an example of the use of "discontinuous" characteristics (prepared by the writer in 1950) of interest [see ROUSE, H., Engineering hydraulics, John Wiley and Sons, pp. 703-707, 1950].

In so far as the very purpose of performing numerical computations is to obtain useful results for practical applications, the writer cannot agree with Schönfeld that "methods with continuous characteristics can avoid inter- or extrapolations," and that "this means less work and the elimination of a source of inaccuracies." The writer, however, is not concerned with extrapolation since he has not used it in his method. With regard to the matter of interpolation, the following comments may be offered.

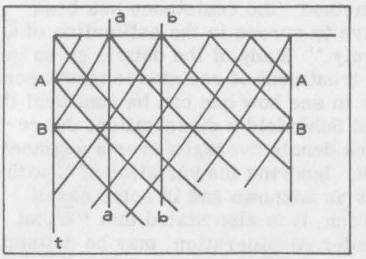

Fig. 11--Flow data as a function of time

There are in general two types of information that are of practical value, namely, the flow data at a given section as a function of time (for instance, data at points on aa, bb of Fig. 11), and those at a certain instant as a function of distance along the channel (for instance, data at points on AA, BB, Fig. 11). In the methods of "continuous" characteristics, flow data are computed only at intersections of characteristics. As a result, such information as just mentioned cannot, in general, be provided without resorting to interpolation. Using the method of "continuous" characteristics, the information would be even more questionable because, for practical reasons, it is likely that interpolation will be based on only two or three adjacent points, whereas, using the method proposed by the writer, results of a step computation are plotted, and the smooth curve giving best fit to the computed points is used in subsequent computation. Thus one can see that in order to provide useful data none of the methods can really avoid interpolation. It will be necessary sooner or later.

In general, the writer does not believe that the error resulting from curve fitting is of significant consequence in open-channel calculations. It is rather the distribution of the computed points that could seriously affect the accuracy of the computations. Thus if salient features such as a peak in the integral surfaces were missed, the resulting error could be unacceptable. Using methods with continuous characteristics, one is by no means sure that the strategic points in the integral surface are included in all stages of computations, as the general location of each characteristic is predetermined by previous computations. If additional characteristics are required to improve the accuracy of computations, then either interpolation has to be used or new characteristics have to be started from the very beginning. This means either the use of a procedure that the methods of "continuous" characteristics are said to avoid or considerable amount of additional work. According to the writer's procedure of using "discontinuous" characteristics, one is free to plan the computations in each step. Consequently, the distribution of the computed points in each step of computation should be the best in the sense that the integral surface could be most accurately defined with minimum number of computed points.

According to the writer's experience, it is the trial process that is the most time-consuming. Such mechanical or routine work as plotting and fitting a curve to the computed points at the conclusion of each step of computation certainly takes little additional time. By using "discontinuous" characteristics and keeping Δt constant as proposed by the writer, trial solution on the U-V plane is eliminated in most cases. Time saved in this manner far exceeds the time required in straightforward plotting and curve fitting. Plotting of results of computations as each step proceeds also offers the opportunity of visual check against gross mistakes.

Both of Schönfeld's methods require trial processes in the solutions on the t-f and H-Q planes. In the first method, the trial process is aided by a nomogram, which is prepared on the basis of the following

$$\begin{cases} dx/dt = c^{+} = V + \sqrt{gy} \\ (d/dt)\ (V/2 + \sqrt{gy}) = df^{+}/dt = -\ (g/2)\ S_f = -\ g\,|V|\,V/2\ C^2\ y \end{cases}$$

and

$$\begin{cases} dx/dt = c^{-} = V - \sqrt{gy} \\ (d/dt)\ (V/2 - \sqrt{gy}) = df^{-}/dt = -\ (g/2)\ S_f = -\ g\,|V|\,V/2\ C^2\ y \end{cases}$$

where the Chezy coefficient C is defined by a formula in the form of

$$C = \alpha(1 - \beta/\sqrt{gy})$$

A nomogram so constructed can only be used for the calculations of points in the channel. For points along the boundaries, further aid has to be devised.

Schönfeld refers to his second method as a method of fixed Δx. The method as explained in Chapter 12 of his dissertation, however, does not use constant Δx and there is no hint of using "discontinuous characteristics. Schönfeld states that in this method "the resistance has been treated in such a way that construction becomes very insensitive to errors in the estimation of $\overline{Q}$," and that "this eliminates trial-and-error process nearly entirely." Study of the details given in his dissertation impresses the present writer that Schönfeld's treatment of resistance terms seems to be rather arbitrary, and it is difficult for the present writer to see how one can be confident that the treatment will be good in general. According to page 186 of Schönfeld's dissertation, the resistance head loss is expressed as $H_r = \overline{W}\,\overline{|Q|Q}$, where the bars denote averages over a segment of a characteristic, Q stands for discharge, and $W = \Delta x/C^2A^2R$. Ignoring the variation of C with depth of flow, one can see that W still varies with A^2R which is an unknown and in some cases could be quite sensitive to the change in depth. In the dissertation, it is also stated that "$\overline{W}$, an average value of the coefficient for resistance of the section under consideration, may be defined from $\overline{H}$." This means that the computer would have to estimate the average value of W and then check it later. The quantity $\overline{|Q|Q}$ is arbitrarily written as $\overline{|Q|Q} = \overline{S} = S_m + Q_m\ (Q_2 - Q_1)$. Rules are then given to compute Q_m and S_m. When Q_1 and Q_2 have the same sign, $S_m = |Q_1|Q_1$ and $Q_m = |(2/3)Q_1 + (1/3)Q_2|$. When Q_1 and Q_2 are opposite in sign

$$S_m = -\,\mu\,|Q_2 - Q_1|\ (Q_2 - Q_1) \quad \text{and} \quad Q_m = \lambda\ |Q_2 - Q_1|$$

where $\mu = 1/4 + r^2 - (4/3)\ r^3$ and $\lambda = (1/4) + (r/2) + r^2 - (2/3)\ r^3$, in which

$$r = (Q_1 + Q_2)/2\ (Q_2 - Q_1)$$

The writer wonders how many would prefer to follow all of the foregoing rules without assurance regarding how much and how often the trial process would be reduced.

When the idea of "discontinuous" characteristics is applied, Δx can be kept constant, and the result is the method of fixed Δx proposed by Schönfeld. In this case, however, the solution of the equations of "contra-subcharacteristics" is not direct because there are two distinct values of z existing at every intersection of characteristics. The advantages gained in this procedure will then be principally those associated with the use of "discontinuous" characteristics. When the non-uniform channel can be approximated by fairly long reaches of uniform channels, it appears that the method of constant Δt can be used to advantage. When the reaches are very short, the method of fixed Δx seems very promising.

Although there are points on which the present writer takes a different view, the discussion by Schönfeld is very informative and stimulating. The writer should like to express his sincere thanks to Schönfeld for the fine discussion as well as for the courtesy of lending a copy of his dissertation.