DISSERTATION

BIOECONOMIC MODELING OF LIVESTOCK PRODUCTION, RANGELAND MANAGEMENT AND FORAGE SYSTEMS IN A DYNAMIC CONTEXT

Submitted by

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In partial fulfillment of the requirements

For the Degree of Doctor of Philosophy

Colorado State University

Fort Collins, Colorado

Summer 2008

COLORADO STATE UNIVERSITY

June 18, 2008

WE HEREBY RECOMMEND THAT THE DISSERTATION
PREPARED UNDER OUR SUPERVISION BY JOHN RITTEN ENTITLED
BIOECONOMIC MODELING OF LIVESTOCK PRODUCTION, RANGELAND MANAGEMENT AND FORAGE SYSTEMS IN A DYNAMIC CONTEXT
BE ACCEPTED AS FULFILLING IN PART REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY.

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ABSTRACT OF DISSERTATION

BIOECONOMIC MODELING OF LIVESTOCK PRODUCTION, RANGELAND MANAGEMENT AND FORAGE SYSTEMS IN A DYNAMIC CONTEXT

This work focuses on optimal livestock management in a dynamic framework. The first essay utilizes linear programming to analyze proper herd management during periods of drought. It also examines the use of summer hay as an option to alleviate the negative impacts of drought on cattle producers. Findings show that while financial returns are greatly impacted by varying cattle prices, optimal management decisions are driven more by weather changes than price changes. Further analysis shows that although allowing summer feed improves long term returns to producers, the main benefit of such a strategy is the ability to carry increased inventories though drought, with the increased returns coming post-drought.

The second essay utilizes dynamic programming to determine proper stocking rates when future forage production is related to current use of rangelands. The model maximizes the Bellman Equation using a Chebychev interpolation process. Results show that profit maximizing producers will leave just over half of total production as standing forage. Further analysis shows that while returns are impacted by both cattle and corn prices, optimal management decisions do not change with changes in either of these. Stocking decisions are mainly driven by animal efficiency and land productivity.

The third essay adds the element of stochastic weather to the model utilized in the second essay. Specific attention is given to how producers make stocking decisions in the face of random weather events. Again, producers leave just over half of carrying capacity as standing forage when acting optimally. However, if growing season precipitation is unknown at the time the stocking decision is made actual standing forage may vary from this desired outcome, resulting in a decrease in future stocking rates. It is shown that a producer with knowledge of growing season precipitation will be more profitable than a producer without this knowledge on average by 21%. Again, stocking decisions are mainly driven by land productivity and animal efficiency as well as whether or not a producer has knowledge of current year precipitation.

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ACKNOWLEDGEMENTS

First I would like to thank my major advisor, Dr. W. Marshall Frasier, for all of his help and guidance through this process. Without his insight and motivation, I would not have completed this task; I am indeed indebted to him. I would also like to thank the other members of my committee, Dr. Chris Bastian, Dr. James Pritchett, and Dr. Larry Rittenhouse, as all of them have had major contributions to this body of work. Dr. Pritchett was very instructive in his help to extend much of this work to an extension setting. Dr. Rittenhouse's knowledge of range applications was of great use. I would like to especially thank Dr. Bastian who invited me to participate in this exciting project, as well as allowing access to additional funding from the University of Wyoming though an AES competitive grant. I would also like to thank Dr. Steven Koontz, who helped guide the econometric portions of this work. I would like to thank all of my friends, family and fellow students who helped me in this process. I am lucky enough to have too numerous of these people to mention all by name, but I appreciate you all. Finally, I would like to thank my lovely wife Chian for her motivation and support. She always seemed to be able to prod or distract me when I needed it most. It is to her I dedicate this work.

TABLE OF CONTENTS

INTRODUCTION

Cattle producers are involved in complex decisions based not only on animal management, but must also include grazing and feeding strategies. Both livestock performance (herd and individual animal) and forage systems are dynamic in nature, requiring constant attention from cattle producers. These producers need an adaptive approach to management. What strategy works for one situation most likely will not translate to success in another situation. Just as the situations faced by cattle producers are rarely identical, the producers themselves are not homogenous entities. Producers range from mere hobbyists and part time ranchers to very large operations. Even for a given size of producer, objectives are likely to vary across long run or current year profit maximization, to guaranteeing a minimum yearly cash flow, or even just the enjoyment of the lifestyle associated with raising animals. Just as there are many sizes of producers, there are also many stages of producers in the cattle production chain including cow/calf operations, stocker operations, and feedlots.

Strategies for managing similar situations will most likely be different according to the size and type of producer. For example, for a given weather and price outcome faced by producers, cow/calf producers must decide whether to market heifer calves, or retain them for breeding stock. If heifer calves are retained for breeding stock there is a time lag before production of calves occurs, as opposed to marketing of these animals, which results in instant returns. Yet stocker operations must decide how many animals to stock given current range productivity and prices. Overstocking can lead to degraded range health, while under stocking may leave some of the range underutilized. Regardless of the type of operation, attention must be given to the dynamic aspect of both forage production and cattle prices.

Just as cattle management entails a wide range of producers and situations, research involving cattle management has been very diverse in scope. For example studies have focused on cattle use of and impacts on riparian and wetland areas, (see for example Parsons, et al. (2003), Stillings, et al. (2003), and Kirby, et al. (2002)). Other research has examined the interaction of cattle and large game species (see for example Shwiff and Merrell (2004), and Bastian, et al. (1991)). Bonham (1987) focuses on the removal of forage on rangelands from herbivores other than cattle. Lecain, et al. (2000) give attention to carbon cycling on grazing pastures.

However, much research has been conducted on the relationship between cattle and rangelands. Grings, et al. (1996) focus on cattle efficiency across differing rangelands, while Phillips, et al. (2003) examine animal performance given differing herd management strategies for a given range. Others have focused attention on the interaction of the animals and the range more specifically either by analyzing the stability of such systems (see for example Smith and Slatkin (1973) and Loehle (1985)), or by examining the long-run tradeoffs of grazing decisions (see for example Torrell, et al. (1991)). May, et al. (2002) examine the impacts of differing grazing leases on management decisions regarding these intertemporal tradeoffs.

While research in the field of grazing management has provided some answers for cattle producers, response to drought has been a major concern in recent years (see for example Nagler, et al. (2006)). Weather can greatly impact annual forage production as well as potential feed supplies. This decline in forage and available feed translates into lower production possibilities for cattle producers, requiring adaptations in management strategies. Clark and Annexstad (1988) analyzed this problem by focusing research on feeding strategies during drought while Parsch, et al. (1997) give attention to strategies for grazing animals during periods of drought.

Cow/calf producers must decide if it is beneficial to destock during drought, with the implications of restocking costs post-drought, or whether it is better to carry larger inventories through drought conditions in order to continue to market calves over this period. Current cattle prices, as well as expectations of future prices, will play a role in this decision. Liquidating during low prices with expectations of restocking during high prices should not be a viable option for cow/calf producers. Stocker operations face a similar problem during drought conditions, but the inventory effect is not as exaggerated as a cow/calf producer. Yet, stocker producers must weigh the benefit of carrying animals during periods of poor forage production against the cost of degrading the rangeland. While changes in cattle prices across years is less important on decisions for stocker producers, seasonal differences during the year and across weight classes need to be considered when making these types of stocking decisions.

This work will address decisions by both cow/calf (Chapter 1) and stocker producers (Chapters 2 and 3). In all three essays, cattle production is modeled for Central Wyoming, which has recently experienced a severe drought. The first chapter addresses strategies of a profit maximizing cow/calf producer in the face of varying weather and cattle prices over time. The second chapter addresses a stocker operation focusing on the evolution of rangelands by modeling forage systems and animal behavior simultaneously.

Attention is given to stocking strategies that returns to land. The third chapter expands on the results in the second chapter by adding the element of varying forage production due to variations in annual precipitation.

CHAPTER 1

Introduction

Recent droughts have had a major impact on cattle producers in the intermountain west. Although ranchers need not graze all available standing forage in any given year, their grazing decisions are limited to what standing forage will allow. Drought negatively affects forage production which is utilized by cattle producers as feed for animals. This decrease in forage production can alter management decisions by forcing ranchers to carry smaller herds or increase feed purchased or acreage grazed. Ranchers must make decisions regarding herd size and make up, as well as how to meet the nutritional requirements through stocking and feeding decisions. However ranchers also respond to market conditions. High prices can give producers the incentive to carry larger herds resulting in the ability to sell more calves. It is rancher response to these combinations of forces that is examined in this paper.

Problem Statement

If both the weather and market are favorable, producers will not need to alter herd management strategies much, if at all. However, what should a producer do if the weather is favorable but the market is down, or conversely if prices are up but poor precipitation has negatively affected range condition? The impact of both variable weather and cattle prices affect management decisions and ultimately profitability. Proper producer responses to these varying weather and market conditions are examined. Holecheck (1994), in a drought study of New Mexico cattle producers, has shown that producers who respond to high prices without regard for poor forage production can extremely overgraze pastures, a result that can require pastures be completely destocked for recovery.

It is obvious that producers must consider both weather and market forces when making grazing decisions. Yet often, these exogenous forces do not move together, so producers must consider the impacts both of these factors when making their decisions. However, some management strategies may be able to alleviate some of the limiting impacts of poor forage production during drought, allowing producers the ability to take advantage of high prices. Therefore, special attention is given to how altering management decisions may be able to improve the negative financial impact caused by adverse weather conditions.

While the problem of proper herd management during drought situations is common for many cattle producers in the west, the focus of this paper will be Central Wyoming. Wyoming has over 5,800 cattle producers carrying over 1 million head of cattle, with 2,800 of these having at least 100 head of cattle. (Wyoming Ag Statistics, 2007) Recently experiencing an extended drought, ranchers have faced reduced range and cattle productivity resulting in lower ranch incomes and reduced owner's equity (Nagler et al., 2006).

Much work has been done analyzing the management implications of grazing in Wyoming covering topics ranging from cattle and game interactions (Bastian, et al., 1991), costs of predation (Shwiff and Merrell, 2004), how cattle respond to different grazing systems (Hepworth et al., 1991), costs of invasive species (Etchepare, 1985), and how grazing impacts carbon exchange rates (Lecain et al., 2000). While there has also been recent work regarding optimal herd management decisions in the face of weather and forage uncertainty, (see Parsch et al. (1997), Carande et al. (1995), Rodriguez and Taylor (1988), Garoian et al. (1990), Ogden (1987), and Clark and Annexstad, (1988)), there has yet to be much research directed specifically at cattle management in Wyoming in the face of drought situations.

Objective

The objective of this paper is to address the impacts of both price cycle and weather fluctuations, focusing specifically on interactions of weather and price on optimal management decisions over long planning horizons. Focus is given to how profit maximizing producers respond to fluctuations in both market forces as well as growing season precipitation. The paper also examines the possibility of counteracting poor forage production during drought by allowing substitute feed during summer months. Comparisons of present value of ranching incomes across an entire planning horizon are conducted. Specific attention is also given to specific periods of drought in order to determine if management decision can alleviate some of the negative consequences during these periods of poor forage production.

Data and Methods

The research employs a mathematical programming model aimed at modeling producer behavior in the face of fluctuating forage production and market prices. This model represents a ranching operation where herd decisions are based on an explicit objective of profit maximization. As stated previously, poor forage production can limit grazing opportunities for cattle producers. When faced with drought situations, these physical constraints, specifically forage availability, are generally more binding than during normal weather conditions. The mathematical model incorporates the profit maximizing objective of producers while limiting decisions to those that fit the limited resources available to the producer. The model will also incorporate fluctuating cattle prices. Due to the constrained maximization nature of the problem, linear programming will be utilized.

The linear programming model is parameterized for comparison of outcomes over differing management strategies. The production systems and strategies analyzed are obtained from survey results of Wyoming cattle producers conducted by Nagler, et al. (2006), which include partial and full liquidation, as well as supplemental feeding. The strategy referred to as supplemental feeding is not feeding of supplements specifically to address nutrition deficits; rather it is a strategy which hay is feed during summer months as a substitute for forage. The feeding of hay is seen to supplement the lacking forage production of range land. The model is used to evaluate management decisions in response to the negative affects of drought on forage productivity and consequently ranching incomes.

The linear programming model follows previous modeling work done by Torell, et al. (2001). The model is formulated to maximize the present value rancher income over a given planning horizon in an iterative and sequential approach over time, subject to both physical and financial constraints. In order to account for variability in prices, the model is solved using GAMS numerous times over the entire planning horizon beginning at different starting points in the price cycle. Choice variables include amount and type of land to graze, herd size/characteristics, as well as amount of additional feed purchased. Mathematically model is as follows:

$$MaximizePV = \sum_{t=0}^{T}(1+r)^{-t}(NetIncome_t) \tag{1.1}$$

Where:

$$NetIncome_t = (Gross_t - LandCost_t - HerdCost_t - LoanCost_t) \tag{1.2}$$

$$\begin{aligned} Gross_t = & \sum_{AnimalClass}(AnimalsSold_{t,AnimalClass} * Animal\Pr ice_{t,AnimalClass}) \\ & + \sum_{CropClass}(CropSold_{t,CropClass} * Crop\Pr ice_{t,CropClass}) \end{aligned} \tag{1.3}$$

$$LandCost_t = \sum_{Season}\sum_{LandType}(LandUsed_{t,Season,LandType} * LCost_{t,Season,LandType}) \tag{1.4}$$

$$\begin{aligned} HerdCost_t = & \sum_{AnimalClass}(AnimalsRaised_{t,AnimalClass} * AnimalCost_{t,AnimalClass}) \\ & + \sum_{AnimalClass}(AnimalsPurchased_{t,AnimalClass} * PurchaseWeight_{AnimalClass} \\ & * Purchase\Pr ice_{t,AnimalClass}) \end{aligned} \tag{1.5}$$

$$LoanCost_t = (1 + LoanRate) * ShortTermBorrowing_{t-1} \tag{1.6}$$

$$\begin{aligned} AnimalsRaised_{AnimalClass,t+1} = & AnimalsRasied_{AnimalClass,t} \\ & + AnimalsPurchased_{AnimalClass,t+1} - AnimalsSold_{AnimalClass,t} \\ & + AnimalsBorn_{AnimalClass,t+1} - DeathLoss_{AnimalClass,t+1} \end{aligned} \tag{1.7}$$

$$CropRaised_{CropType,t} = \sum_{Season} LandType_{Season,CropType,t} * CropYield_{CropType} \quad (1.8)$$

$$AUM_t = \sum_{AnimalClass} AnimalsRasied_{AnimalClass} * AnimalUnitEquivalent_{AnimalClass} * GrazingTime \quad (1.9)$$

$$Savings_{t+1} = Savings_t * (1 + SavingRate) + NetIncome_t + OffRanchIn\ come_t - FamilyExpenses_t - FixedExpenses_t + ShortTermBorrowing_t \quad (1.10)$$

Subject to:

$$AUM_t \leq \sum_{CropType} CropRaised_{CropType,t} * CropConversion_{CropTupe} \quad (1.11)$$

$$CropsSold_{CropType,t} \leq CropRaised_{CropType,t} - CropsGrazed_{CropType,t} \quad (1.12)$$

$$AnimalsSold_{AnimalClass,t} \leq AnimalsRasied_{AnimalClass,t} \quad (1.13)$$

$$LandUsed_{LandType,t} \leq \overline{L}_{Landtype,t} \quad (1.14)$$

$$Savings_t \geq 500 \quad (1.15)$$

$$ShortTermBorrowing_T = 0 \quad (1.16)$$

Where:

- *PV* is the present value of ranch income over planning horizon
- *NetIncome* is yearly net income
- *Gross* is gross revenues from animal and crop sales
- *LandCost* is cost associated with land use, including cost for land use as well as cost of forage purchased
- *HerdCost* is the cost of managing the entire herd

- *LoanCost* is cost of repaying loans
- *AnimalsSold* is number of animals per class sold
- *AnimalPrice* is price received per animal per class
- *CropSold* is amount of crop sold
- *CropPrice* is price received per unit of crop sold
- *LandUsed* is amount of land used for grazing or crop production per type of land
- *LCost* is cost of using land types, including grazing fees
- *AnimalCost* is the cost of carrying a specific class of animal
- *AnimalsPurchased* is the number a animals purchased for a given class
- *PurchaseWeight* is the weight of each class when purchased
- *PurchasePrice* is the per weight price of a given class of animals
- *LoanRate* is the interest owed on short term loans
- *ShortTermBorrowing* is the amount of short term loans
- *AnimalsBorn* is amount of newborn calves
- *DeathLoss* is the number of animals dead due to normal death loss
- *LandType* differentiate different types of land, as well as public and private land
- *CropYield* is amount of crop produced according to land type
- *AUM* is Animal Unit Months required for the herd, based both amounts and class of animals raised
- *FixedExpenses* include taxes, depreciation, insurance, and machinery costs
- *CropConversion* converts crops into AUM equivalents
- $\overline{L}$ represents total land in each land class available to producer

The model is parameterized for an operation typical of Fremont County, WY. Fremont County producers alone carry over 100,000 head of cattle per year (Wyoming Ag Statistics, 2007). The model assumes an annual off-ranch income of $24,000, which is offset by a family allowance of $24,000, so returns represent only those from the ranching operations. Based on these parameters, the model is designed to maximize total net present value of profits over the specified planning horizon. Terminal value for the ranch after the planning horizon is set at $1.

Herd Management

Producers are free to set herd size following equation 1.7. Parameters are set to keep herd characteristics (e.g. birth rates, minimum replacement rates, maximum percentage of heifers kept, and bull ratios) in line with observations from enterprise budgets, but the producer is free to buy or sell animals as long as they are inline with these parameters. Also under management control is the amount of land to be grazed and amount of feed to be purchased, again restricted by actual amount of land/feed available in the area (equation 1.8 and constraint 1.14). Land can be used for grazing or raising hay crops. Any unused crops can be sold. Land available to producers simulates a representative ranch for Fremont County, and includes privately owned land as well as the option to utilize public and private grazing leases. Producers are able to graze and/or feed as long as animal requirements, based on all animals reaching a required weight by, are met, but there is no minimum amount of feed required in any given year. Nutritional requirements are accounted for within each season as well as across years. Producers do not determine weight gain of each animal, only how to get each animal to the required weight for their class of animal according to equation 1.9.

Forage Production

The model accounts for yearly variability in forage production due to weather impacts. This fluctuation was modeled altering the forage production possibly of each land type each year. Therefore in years experiencing poor precipitation, each land class produces lower amounts of forage and/or hay crops. The variable affecting forage production was estimated using a regression from Smith (2005). Smith ascertains that spring precipitation, specifically precipitation from March 5 through May 25, is a good predictor of yearly forage production for this region of Wyoming grasslands.

$$\text{Predicted Forage (kg/ha)} = 241.972 + 54.3073*X \qquad \text{(r squared = .32)} \qquad (1.17)$$

Where X is total precipitation (in inches) occurring from March 5 through May 25

Since the model has been parameterized for Fremont County Wyoming, weather data from the Riverton Weather Station obtained from the National Climatic Data Center (NCDC, NOAA 6/6/2007) were used to feed the forage prediction regression equation. Data was available and used from 1921 through 2006 (86 years). These weather data were used to estimate forage production over this same time horizon for the study area. This predicted forage is used to generate yearly forage production as a percentage of mean production. The yearly predicted amount of forage as a percentage of mean predicted forage over this time horizon is used as a scalar to adjust the annual forage production in the model, specifically setting differing values for the *CropYield* parameter across years in equation 1.8 for grazing land. Hay availability for purchase, however, was not adjusted as most hay production in the area is irrigated, and is therefore not highly impacted through drought years. In fact dry land hay production in the region has reached a maximum of only 1.27% of total annual hay production since 1959 (Wyoming Ag Statistics, 5/30/07).

The model optimizes over an 86 year planning horizon. To evaluate how initial forage production impacts herd decisions, the model has also been parameterized featuring three different starting points along the forage production path as estimated using the precipitation data. The weather data were looped so that regardless of starting point the entire 86 year data set could be utilized. These three distinct beginning points along the forage production path were chosen by looking at 10 year moving averages of forage production, and represent the beginning of relatively good (Wet Start), poor (Dry Start), or average (Average Start) period of forage production.

Cattle Prices

Prices received for sold livestock, *AnimalPrices* in equation 1.3, are modeled to fluctuate over time as well. Actual prices paid at the Torrington, WY auction over the period of 1968 through 2006 formed the basis for the price parameters in the model. The majority of data was obtained from the Livestock Marketing Information Center (LMIC), (unpublished data supplied by Jim Robb, LMIC, Lakewood, Co., June 22, 2007), however some data was not available there. Bred cow prices were obtained from Cattle-Fax (unpublished data, from Cattle-Fax Inc., Centennial, Co., accessed August 21, 2007), and were thus based on West-wide, not Wyoming specific, prices. Even with the supplemental Cattle-Fax data, some months had missing observations. The series with missing values were Bull, Cull and Bred cow prices. As cattle prices tended to move together, the missing values were calculated based on the existing data available from the Torrington auction. Complete data sets for Steer calf, Heifer calf prices and Yearling prices were used to estimate the missing values for Bull, Cull and Bred cow prices. OLS was used to estimate the missing data. The estimation results are shown here (t stats in parenthesis):

$$Bullprice = 13.5388 + \underset{(1.66)}{0.183325^{*}}\ SteerCalfprice - \underset{(-1.141)}{0.112003^{*}}\ HeiferCalfprice + \underset{(4.007)}{0.380734^{*}}\ Yearlingprice \qquad \text{(Adjusted R-Squared = .55471)} \tag{1.18}$$

$$Cullprice = 9.07837 - \underset{(-.204)}{0.261142^{*}}\ Steerprice + \underset{(.150)}{0.019977^{*}}\ HeiferCalfprice + \underset{(2.482)}{0.415433^{*}}\ Yearlingprice \qquad \text{(Adjusted R-Squared = .32798)} \tag{1.19}$$

$$BredCowprice = -126.51 + \underset{(8.14)}{3.3585^{*}}\ SteerCalfprice + \underset{(1.74)}{0.55196^{*}}\ HeiferCalfprice + \underset{(15.97)}{6.01246^{*}}\ Yearlingprice \qquad \text{(Adjusted R-Squared = .745)} \tag{1.20}$$

Since all the existing costs in the model were based on 1997 prices, all monthly price data were converted into 1997 prices using the PPI index. The model has a sale date of November 1, so average prices of October 1 through November 30 were used in the model. With all missing data calculated, the average November 1 prices were truncated to a data set of 1980-2006 (27 years). As price cycle is expected to perpetuate in a similar manner, these 27 years were then looped over time. The model was then reconstructed to have 27 iterations per weather start, each starting at a different year of the price cycle. The model then has 3 runs (Average, Wet and Dry starts) of 86 years, each with 27 iterations (different starting points on the price cycle). The result is a data set including optimal decisions and resulting financial returns for all possible combinations of market and weather states for all three weather starting point runs, a total of 6,966 yearly observations.

Results

It is expected that droughts negatively affect ranch income; however management decisions may be able to alleviate some of this impact. Depending on both length and severity of drought, various management decisions should be able to reduce the negative impacts of extended periods of drought. Proper herd management decisions in the face of drought situations following both traditional and non-traditional management practices are examined.

For comparison, the model was initially solved in the absence of weather impacts, or constant forage production in all years, (*NO DROUGHT*). This scenario uses the average forage production over the 86 year planning horizon for every year. The model was also solved under a baseline (*BASE* scenario) condition under existing, traditional management practices, with variable weather impacting yearly forage production. Specific attention was given to modifying herd size in response to these weather fluctuations, as local producers have stated they have engaged in partial and full liquidation during periods of drought (Nagler et al., 2006). As stated above, this scenario was solved utilizing three distinct starting points along the weather cycle. Therefore output was generated for *BASE* – Average Start Weather, *BASE* – Dry Start Weather, and *BASE* – Wet Start Weather scenarios.

As stated above, an objective of this paper was to determine if allowing supplemental feed could help alleviate some of the negative impacts associated with drought conditions. Therefore, the model was also solved after altering some constraints on producers. These additional solutions included (1) allowing supplemental feed (*FEED* scenario) during summer months as well as (2) requiring a minimum herd size (*FLOOR* scenario) throughout the horizon. The first was chosen as a strategy to potentially offset some of the reduced forage production caused by a drought situation, and the latter was chosen to show what additional cost would be imposed for producers to keep herd genetics intact over the drought period. The *FLOOR* scenario placed a lower limit of 500 Animal Unit Years (AUY) so that a producer could quickly replenish his/her herd from their existing stock after forage production recovers. The *FLOOR* scenario, however, was not drastically different from the *FEED* scenario as the minimum herd size observed with the *FEED* scenario was 483 AUY. Output is also shown for the FEED scenario across the three different weather starting points, *FEED* – Average Start Weather, *FEED* – Dry Start Weather, and *FEED* – Wet Start Weather iterations.

Financial Outcomes

As expected, results reveal that drought unfavorably affects ranch income. Figure 1.1 shows the comparison of total discounted profits over the 86 year planning horizon over the *NO DROUGHT* case, as well as the *BASE* and *FEED* scenarios for the three different starting points. The case with average forage production in all years (*NO DROUGHT*) outperforms any of the scenarios that impose forage production based on precipitation. If forage production is impacted by weather, for all three starting points the

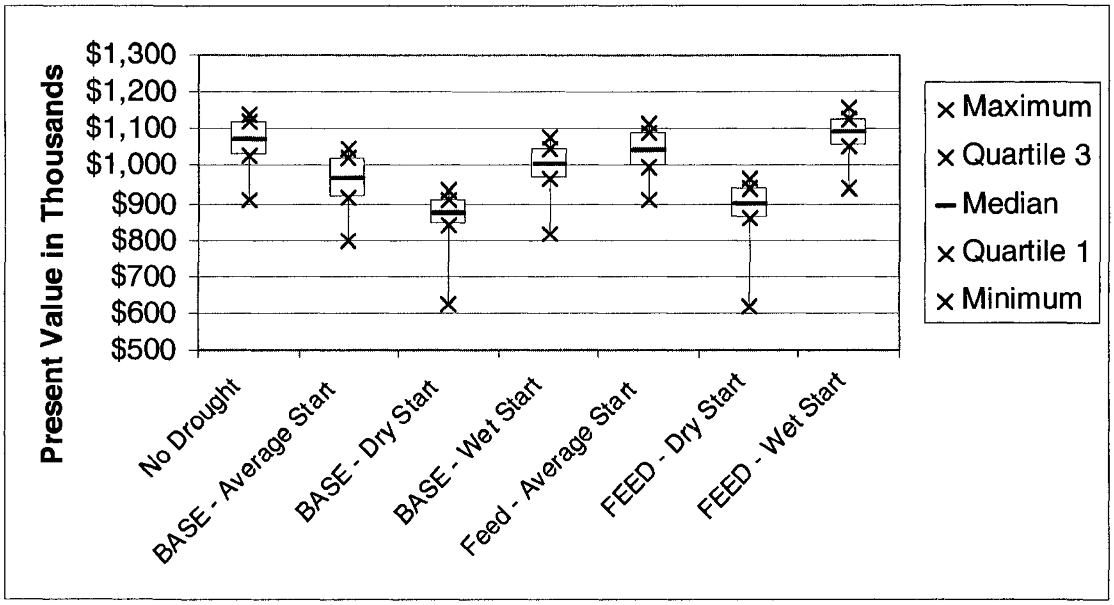

Figure 1.1 – Distribution of Net Present Value of Ranch Income across Scenarios and Weather Starting Points

ability to supplemental feed increased total discounted ranch income over the 86 year planning horizon when compared to the base model. The ability to feed had the greatest impact in the average and wet starting point runs.

Decision Variables

Results of the baseline drought model (*BASE*) show that it is in fact optimal to partially liquidate in the face of drought conditions, as seen in Figure 1.2. However, the model never resulted in a fully liquidated herd. In the *FEED* scenario, allowing producers the ability to purchase supplemental feed during these periods can help alleviate the negative impacts of drought over the entire planning horizon. As seen in Figure 1.3 for the average start scenario, average yearly summer feed allowed is greatest during dry years (graphs for the dry and wet start scenarios can be located in Appendix 1). Producers who adjust both herd size and allow supplemental forage will generally have better long term financial results when faced with fluctuating forage production.

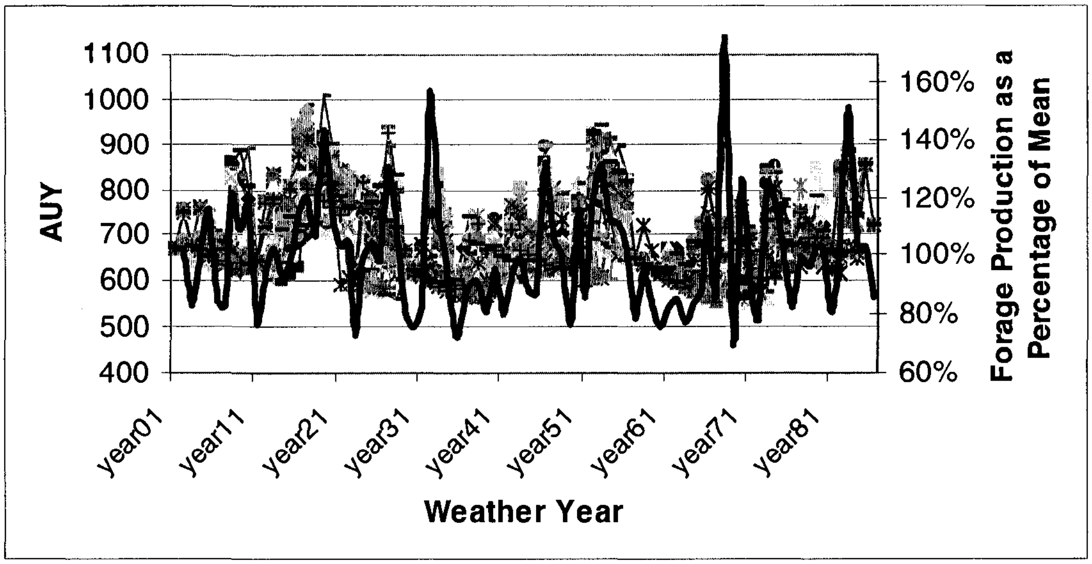

Figure 1.2 – Comparison of AUY and Forage Production as a Percentage of Mean Production across Weather Years for *Base* Scenario, Average Start (Black line represents forage production, other lines represent AUY for different price cycle iterations)

However, the size of herd and amount of additional feed purchased also depends on where the producer is in the price cycle. Therefore, there is no single "right" decision for a manager when faced with drought situations.

Whether or not supplemental feed is allowed, producers generally graze most of the land they have access to. However, when supplemental feed is allowed, the distribution of acres utilized is shifted upward. This implies allowing producers the ability to supplemental feed results in more thorough utilization of rangeland. It is interesting to note that as supplemental feeding is allowed, on average, producers will feed slightly less in years of favorable forage production, selling more than under the *BASE* scenario. However, during periods of poor forage production they tend to feed significantly more than they would have under the *BASE* scenario. Although average herd sizes are very similar whether or not supplemental feed is allowed, the ability to supplemental feed tends to slightly increase herd size.

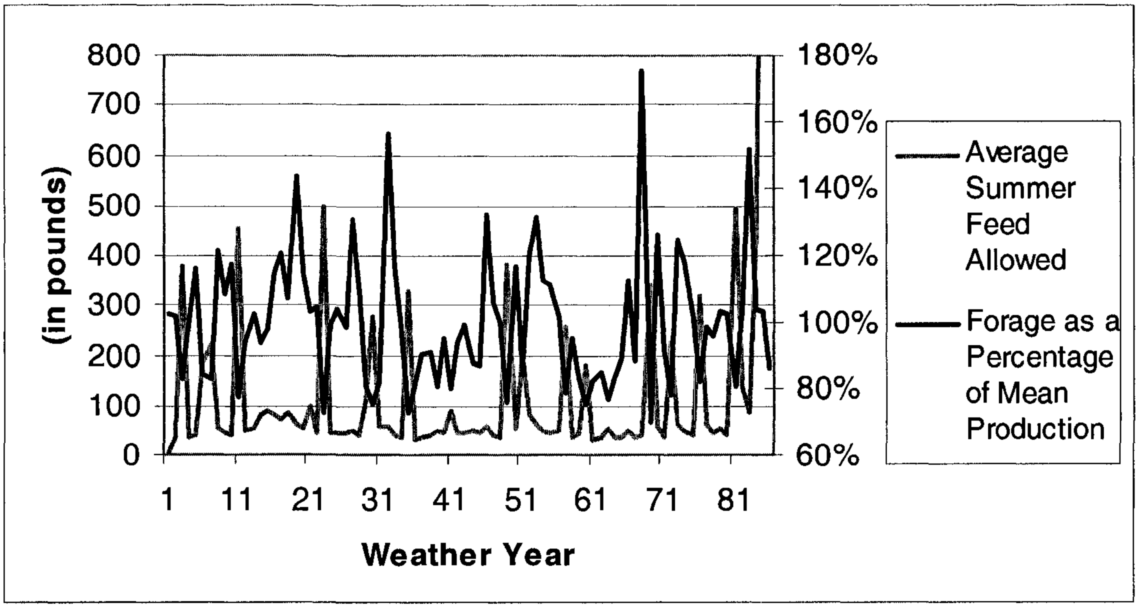

Figure 1.3 – Average Summer Feed Allowed Compared to the Forage Production as a Percentage of Mean Production across Weather Years for *Feed* Scenario, Average Start

Whether or not supplemental feed is allowed, producers generally graze most of the land they have access to. However, when supplemental feed is allowed, the distribution of acres utilized is shifted upward. This implies that allowing producers the ability to supplemental feed results in more thorough utilization of rangeland. It is interesting to note that as supplemental feeding is allowed, on average, producers will feed slightly less in years of favorable forage production, selling more than under the *BASE* scenario. However, during periods of poor forage production they tend to feed significantly more than they would have under the *BASE* scenario. Although average herd sizes are very similar whether or not supplemental feed is allowed, the ability to supplemental feed tends to slightly increase herd size.

Driving Factors

When analyzing herd management decisions, timing of decisions are important. It seems many decision variables, such as herd sizes, follow, at least somewhat, weather patterns. However, many of the financial outcomes, such as yearly returns, seem to follow market years more closely. In Figure 1.4, the time paths represent total acres grazed using different market starting points for the *BASE* scenario. Each line depicts the time path of total acres grazed for given weather years, starting at a different year in the price cycle. Figure 1.5 shows the same data aligned across market years, with differing starting weather years. Figures 1.6 and 1.7 show the same comparison across weather and market years for yearly returns. As can be seen, total acres grazed seems to follow a pattern more closely aligned to weather year as opposed to market year, while the opposite is true for yearly returns

As all combinations of weather and price realizations were modeled, data were available to determine how each of these factors affected management decisions and financial returns. The relationship between management decisions, as well as financial returns, and the variation in weather and market prices were of interest. With the large amount of data generated, an approach was formulated to most precisely estimate the impacts of these exogenous variables on management outcomes.

Linear regression was utilized to estimate the impacts of both weather and price variability on decision and financial variables. As the regression was performed on mean values for all variables, the estimation output can be used as a measure of elasticity. These elasticities can be viewed as a measure of sensitivity of the optimal management

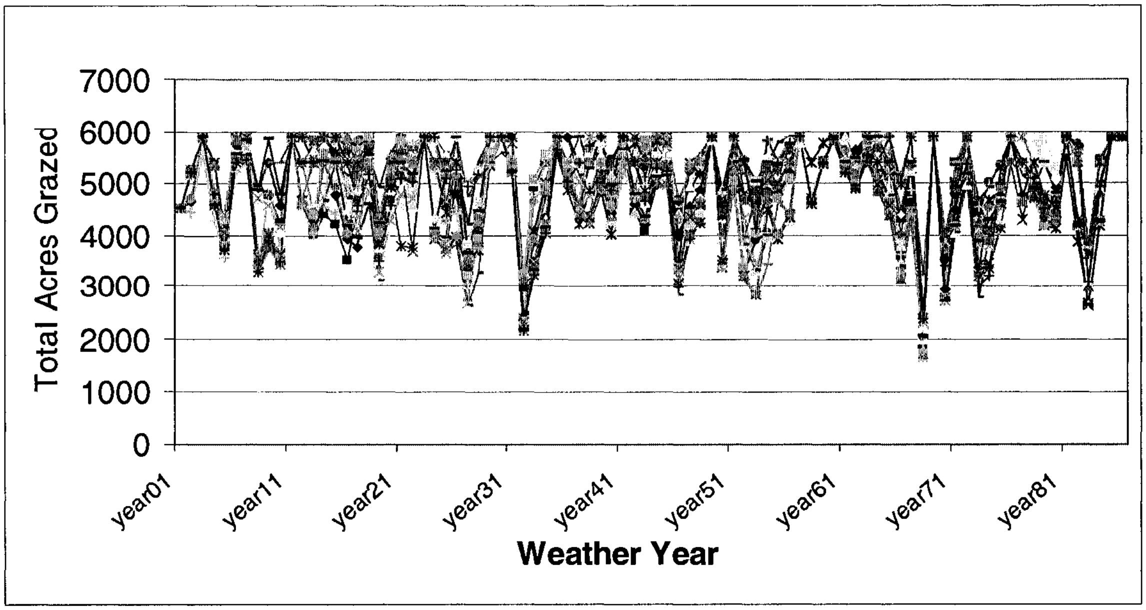

Figure 1.4 – Time paths of Total Acres Grazed across Weather Years given Differing Starting Market Years (Lines represent acres grazed for different price cycle iterations)

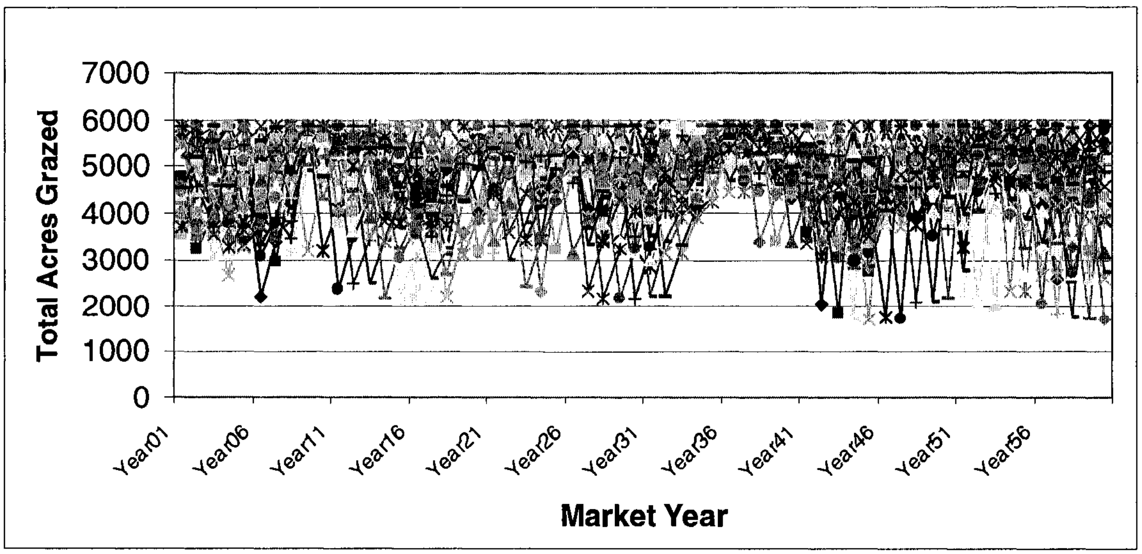

Figure 1.5 – Time paths of Total Acres Grazed across Market Years given Differing Starting Weather Years (Lines represent acres grazed for a given market year over different iterations of weather years)

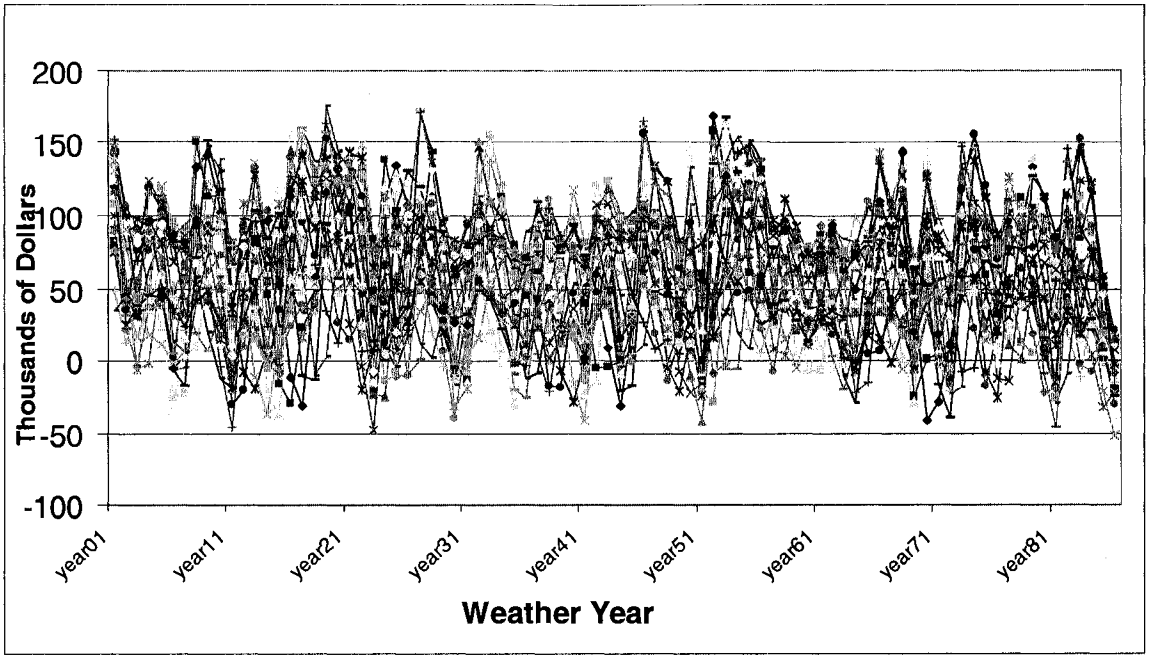

Figure 1.6 – Time paths of Yearly Returns across Weather Years given Differing Starting Market Years (Lines represent yearly returns for different price cycle iterations)

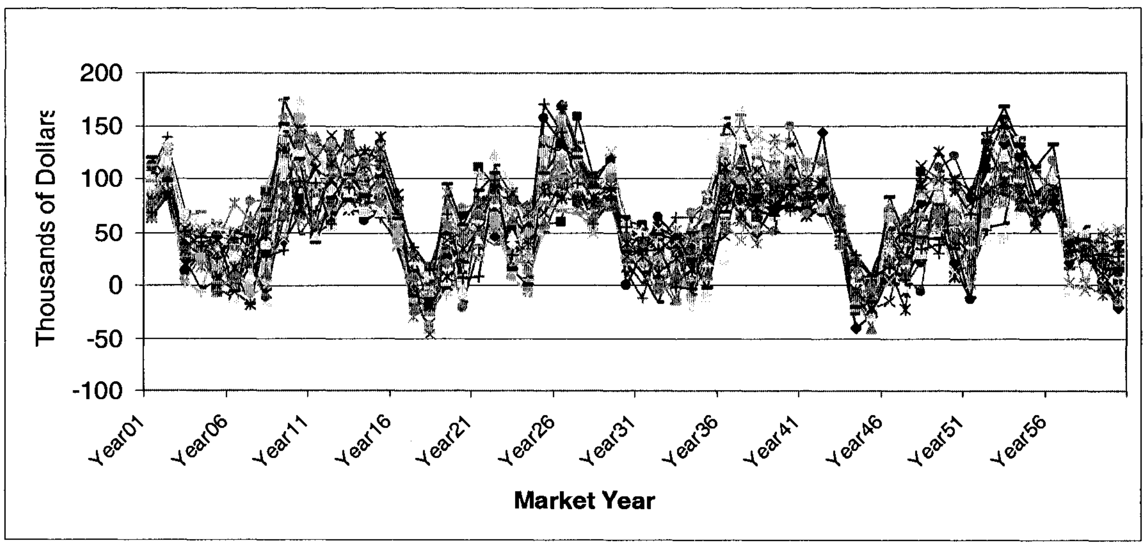

Figure 1.7 – Time paths of Yearly Returns across Market Years given Differing Starting Weather Years (Lines represent yearly returns for a given market year over different iterations of weather years)

and the financial outcomes to changes in exogenous variables representing fluctuations in market prices and forage production due to changes in precipitation. The following regression was estimated for the financial returns and management decision variables:

$$Y_t = \beta_0 + \beta_1 * Market_t + \beta_2 * Weather_t + \varepsilon_t \tag{1.21}$$

Where Y_t is an outcome of the model in year t, $Market_t$ is steer calf price in year t, and $Weather_t$ is growing season precipitation in year t, and ε is an error term. All of these variables are presented as a percent of their associated means. Weather data was already in terms of percentage of mean, so price data was converted likewise. Steer calf price was used as a proxy for market fluctuations and were likewise converted to a percentage of their mean. The decision and financial variables from the model were converted to a percentage of their mean as well. The coefficients therefore are an estimation of elasticity that the independent variables (price and precipitation) have on the dependent variable of interest; *Net Yearly Returns*, *AUY* (Herd Size), *Cull* (both total number and as a percent of herd size), *Acres Grazed* (both total acres and acres per AUY), and *Feed* (both total feed and feed per AUY).

Elasticities were estimated for all weather runs combined (the entire 6,966 year data set), as well as independently for each of the three starting points of weather years (2,322 yearly observations each). However since the independently calculated elasticities follow the same trends as the combined data, only the combined results are shown. Table 1.1 displays the results of these regressions for the *BASE* scenario.

The coefficients are displayed in the "Market" and "Weather" columns, with associated t-statistics reported directly under the coefficients. These cells show corresponding elasticities of these variables on the related variable in the given row. "Comparison t-statistics" are based on a null hypothesis that the two elasticities across a row are equal. Therefore t-stats greater than 1.96 indicate significant difference in the coefficients. Highlighted cells show where there is a statistical difference between elasticities at the 95% level. The highlighted cell is the elasticity which has a greater absolute value. For example, under the *BASE* scenario, a 1% increase in "Market," or Steer Calf Price, will encourage producers to increase acres grazed 0.3%, however a 1% increase in "Weather," or growing season precipitation, will encourage producer to decrease total acres grazed by 0.674%.

These elasticities show that yearly returns are heavily impacted by market variations, much more so than weather variation. In the *BASE* run, as expected, as producers face a better market year, they respond by increasing herd size, acres grazed and total feed. They also increase total culling activities, but culling percentage actually drops. However, in most cases decision variables are more responsive to weather than market fluctuations. If producers face a better weather year, they respond by increasing herd size and total feed. However producers are able to reduce both total acres grazed and acres per animal as the land is more productive during these wetter years. In fact, both total acres and acres per animal are more responsive to weather than market changes. With respect to an increase in weather, producers increase total feed, but less is offered to each animal. However, managers can partially liquidate a herd as a strategy to overcome unfavorable weather; yet it was never optimal to fully liquidate. Once the herd

Table 1.1 Market and Weather Elasticities of Production and Financial Returns for *BASE* Scenario

	BASE			
	Market	Weather	R Squared	Comparison t Statistic
Yearly Returns	4.018 103.113	1.009 36.226	0.632	-51.213
AUY	0.254 25.707	0.326 46.133	0.286	1.225
Cull	0.202 21.784	0.210 31.648	0.175	0.133
Acres Grazed	0.300 31.445	-0.674 -98.684	0.606	-6.357
Total Feed	0.233 22.493	0.326 44.157	0.261	1.598
Feed/AUY	-0.022 -20.572	-0.002 -2.273	0.058	0.349
Acres/AUY	0.069 16.602	-0.997 -334.394	0.942	-15.791
Cull/AUY	-0.049 -11.000	-0.102 -32.067	0.142	-0.904

Values represent elasticities estimated using OLS. Values under coefficients are associate t statsagainst the null hypothesis that the coefficients are equal to zero. Also reported are associated R Square Values, as well as comparison t stats against the null hypothesis that the coefficients are equal.

size is reduced, managers need less total feed, however culling will occur at a higher rate than when faced with favorable weather. Producers can also alleviate some of the effects of poor forage production by increasing total acres grazed, but more specifically increasing the allowable acreage per animal, accomplished by partial liquidation.

Table 1.2 reports the elasticities for the scenario where supplemental feed is allowed in traditionally "off-season" months. Again, proper management in regards to weather fluctuations allows producers to take advantage of favorable market conditions. The main differences are that when examining market impacts more total feed and more feed per animal are allowed in response to favorable market years. Only the coefficient representing elasticity of "Market" on Acres/AUY in the *FEED* scenario was not statistically significant at the 95% confidence level.

The major differences when looking at weather impacts are that the sign on total feed has switched, indicating producers will actually allow less total feed during good weather years. When producers are allowed to supplemental feed, the weather year is also the dominant driver in total herd size and both feed variables (total and per animal). This implies that as producers allow supplemental feeding management decisions are able to respond more to weather impacts than they do market movements, even though their yearly returns are in fact more heavily impacted by market prices than precipitation changes under this scenario. Also, under the *FEED* scenario, elasticities associated with herd size, total feed, feed/AUY with respect to weather impacts become statistically different (and greater in absolute value) than those with respect to market impacts. Allowing supplemental feed during summer months can place producers in a position to benefit greater from beneficial markets than relying solely on herd liquidation.

More Detailed Examination of Yearly Outcomes within Period of Drought

The model aims to maximize the net present value of all yearly incomes, and allowing supplemental feed during drought years will tend to increase this stream of discounted incomes. Analysis to this point has focused on total outcomes over the entire planning horizon, however a producer will be interested in how the decision to allow supplemental feed impacts the yearly returns during the period of drought specifically. Can proper management alleviate adverse weather conditions allowing producers to take advantage of favorable markets? The output was examined over a five-year drought in the beginning stages of the planning horizon to specifically analyze differences in yearly decisions and outcomes during a drought across the scenarios. The following analysis

Table 1.2 Market and Weather Elasticities of Production and Financial Returns for *FEED* Scenario

	Feed			
	Market	Weather	R Squared	Comparison T Stat
Yearly Returns	4.453 40.378	1.690 21.441	0.231	-47.019
AUY	0.235 15.534	0.405 37.346	0.190	2.882
Cull	0.198 14.031	0.271 26.863	0.117	1.244
Acres Grazed	0.224 27.840	-0.559 -97.149	0.595	-5.701
Total Feed	0.339 8.882	-0.608 -22.272	0.076	-4.578
Feed/AUY	0.139 6.764	-1.060 -72.115	0.430	-15.672
Acres/AUY	0.004 0.461	-0.917 -148.073	0.759	-15.543
Cull/AUY	-0.037 -8.319	-0.107 -33.713	0.148	-1.195

Values represent elasticities estimated using OLS. Values under coefficients are associate t statsagainst the null hypothesis that the coefficients are equal to zero. Also reported are associated R Square Values, as well as comparison t stats against the null hypothesis that the coefficients are equal.

shows how yearly returns are impacted during a drought when supplemental feed is allowed. In the "average start" scenario, a five-year drought occurred in the eleventh year of the planning horizon. Table 1.3 and Figure 1.8 show the distribution of returns for the different market iterations over this five-year span across the *BASE* and *FEED* scenarios.

It appears as though the ability to supplemental feed during a drought has little, if any, impact on average yearly returns. It would be difficult to show any producer the benefit of such a plan based on these figures. However, during this window, a producer that allows supplemental feed is able to carry a larger herd, which will allow them the benefit of selling more animals immediately post drought, while a producer that does not

Table 1.3 Comparison of Distribution of Net Returns during 5 Year Drought Across *BASE* and *FEED* Scenarios (Both Individual Net Yearly Returns as well as Net Yearly Returns Summed over 5 Drought)

	BASE	FEED
	Yearly Returns	
Minimum	-$45,068	-$49,863
Average	$46,651	$46,932
Maximum	$136,012	$134,562
Standard Deviation	$42,447	$43,265
	Sum of Yearly Returns Over Drought	
Minimum	$14,059	-$7,123
Average	$233,255	$234,659
Maximum	$431,373	$425,593
Standard Deviation	$128,365	$121,617
	Difference between Feed and Base	
Minimum	-$25,693	
Average	$1,404	
Maximum	$27,902	

allow supplemental feed must rebuild their herd after more severe liquidation. This inventory effect of allowing summer feed is the main benefit of such a strategy. The producer that allows supplemental feed has higher costs throughout the drought; however these costs are at least partly off-set by having a more constant stream of calves to sell. A producer that does not allow supplemental feeding more aggressively liquidates their herd, reducing the costs of carrying animals during the drought while also benefiting in the short term by increasing sales through liquidation. However, immediately following a drought when conditions are again favorable, these producers must spend time rebuilding the herd in order to produce a similar number of calves, while producers that allow supplemental feed still have a larger herd intact. Table 1.4 and Figure 1.9 show the difference in returns over the 3 years immediately following the five year drought.

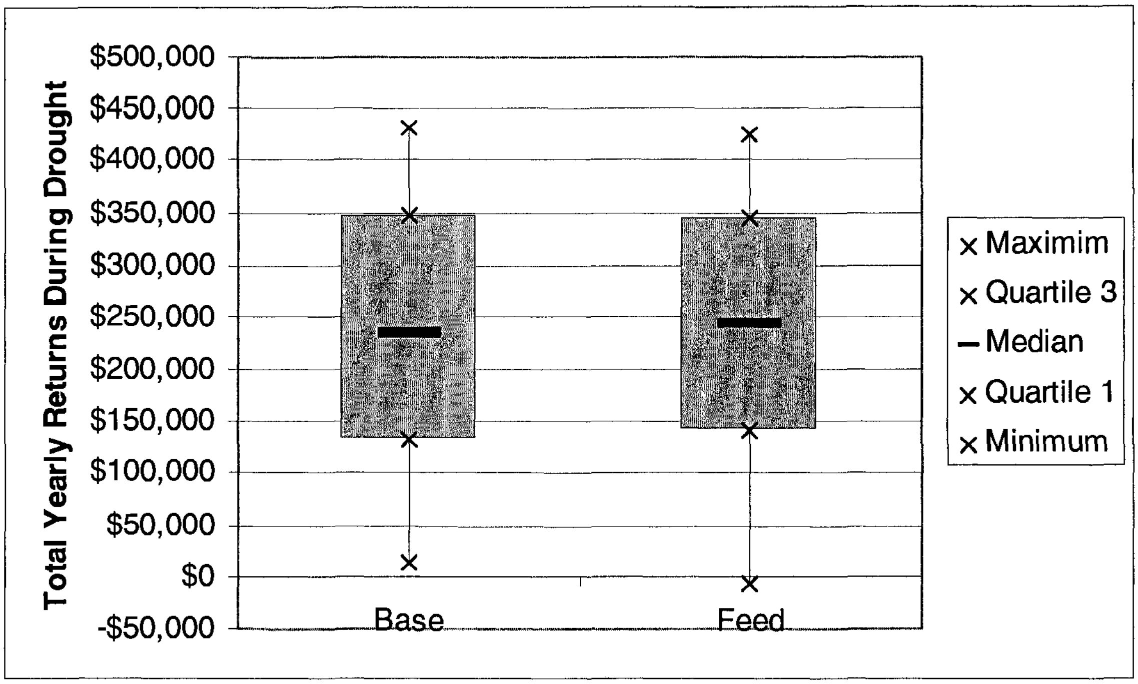

Figure 1.8 Graphical of Distribution of Net Yearly Returns Summed over 5 Year Drought across *BASE* and *FEED* Scenarios

Table 1.4 Comparison of Distribution of Net Returns over 3 Years Immediately Following 5 Year Drought Across *BASE* and *FEED* Scenarios (Both Individual Net Yearly Returns as well as Net Yearly Returns Summed over 3 Years Immediately Post-Drought)

	Base	Feed
	Yearly Returns	
Minimum	-$30,095	-$11,856
Average	$83,612	$94,648
Maximum	$161,212	$177,057
Standard Deviation	$48,937	$47,844
	Sum of Yearly Returns Post-Drought	
Minimum	$16,321	$58,776
Average	$250,837	$283,944
Maximum	$417,947	$444,627
Standard Deviation	$127,556	$121,702
	Difference between Feed and Base	
Minimum	-$17,637	
Average	$33,107	
Maximum	$89,304	

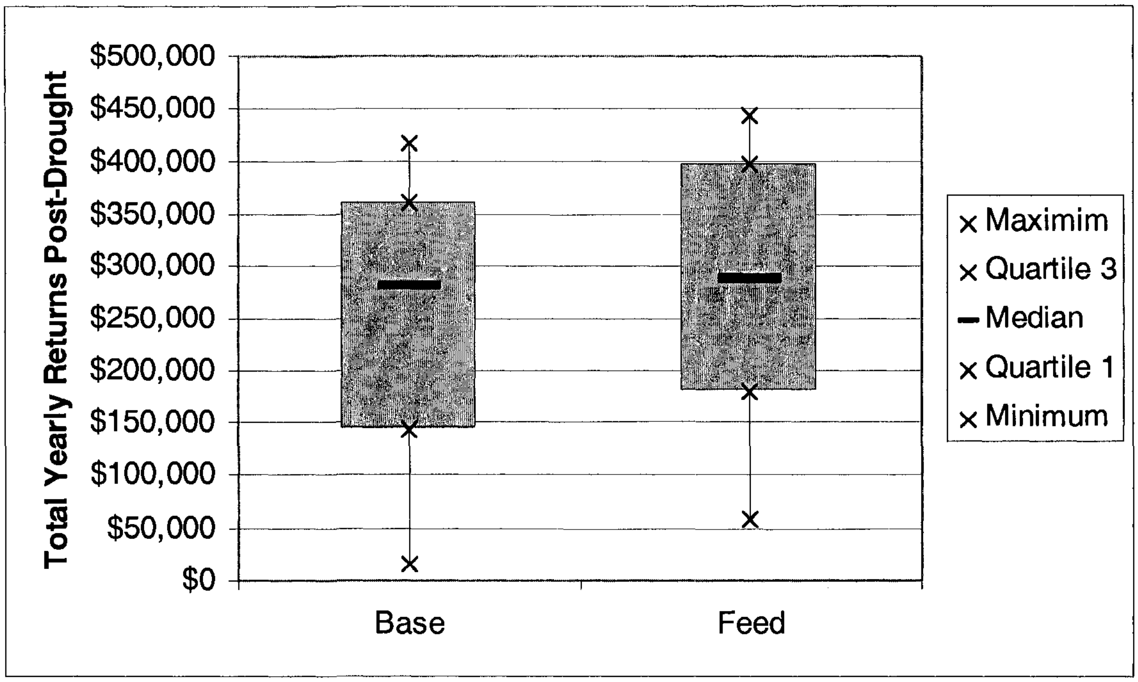

Figure 1.9 Graphical Comparison of Distribution of Summed Net Returns 3 Years Immediately Following 5 Year Drought across *BASE* and *FEED* Scenarios

As stated above, the ability to supplemental feed can help a producer's financial standing, however the benefit usually comes post-drought. This is due to the fact that a producer must increase costs in order to allow additional feed, but the benefit is realized after the drought as they were able to carry a larger herd throughout the drought, resulting in more sales immediately following the drought. This inventory effect drives the difference in outcomes observed when summer feeding is allowed. Table 1.5 and Figure 1.10 show the impact supplemental feeding can have on the time frame including the drought and planning horizon immediately following the drought. So, although supplemental feeding does impact producers' financial standing over the long run, when looking at individual drought occurrences, the true benefit of supplemental feed during a drought is realized after the event by having larger inventories intact instead of having to retain additional animals.

Table 1.5 Comparison of Distribution of Net Returns over 5 Year Drought and 3 Years Immediately Following Across *BASE* and *FEED* Scenarios (Both Individual Net Yearly Returns as well as Net Yearly Returns Summed over 5 Year Drought and 3 Years Immediately Following Drought

	BASE	FEED
	Yearly Returns	
Minimum	$305,576	$325,074
Average	$484,092	$518,603
Maximum	$709,791	$737,590

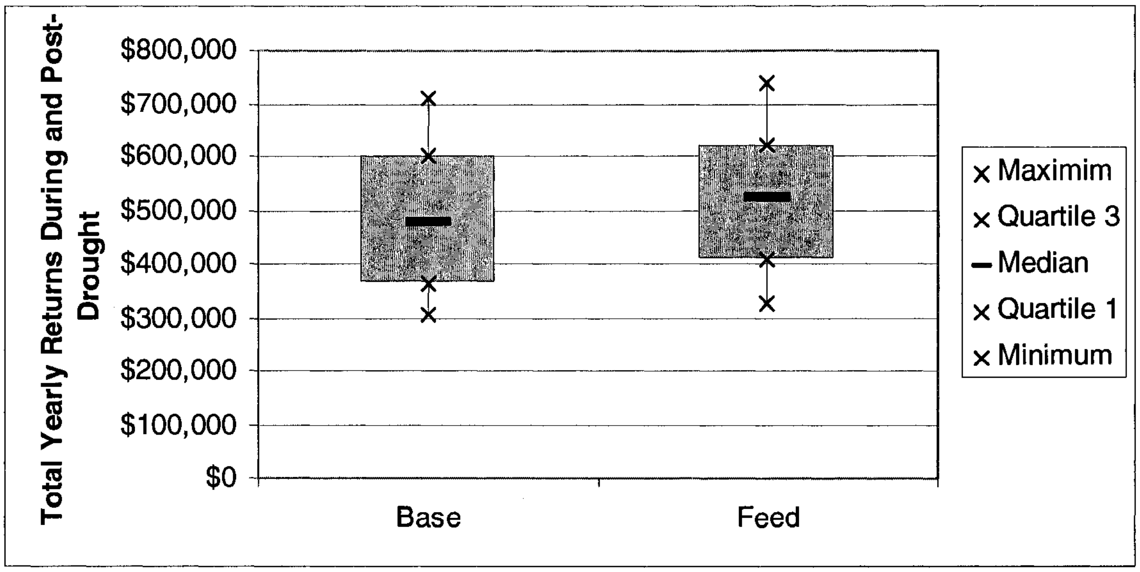

Figure 1.10 Graphical Comparison of Distribution of Total Summed Net Yearly Returns over 5 Year Drought and 3 Years Immediately Following Drought across *BASE* and *FEED* Scenarios

Conclusion

Recent droughts have greatly impacted cattle producers through decreases in yearly forage production. Cattle prices also have a role in ranching outcomes. The objective of this paper has been to address the impacts that variations in price cycle and weather conditions have on ranching outcomes, giving attention to the possibility of alleviating some negative impacts of drought by allowing supplemental feeding.

As producers are expected to be driven by the motive of profit maximization, and are constrained both in terms of financial and physical resources, a linear programming model maximizing present value of ranch income over a predetermined planning horizon was utilized to determine these impacts. Analysis was focused on financial outcomes as well as management decisions compared across a drought scenario with traditional management decisions including herd liquidation as well as a scenario which allows the ability to allow supplemental feed in traditionally off-season months. The model was solved iteratively starting at each of 27 potential years over a loop of market prices. An 86 year loop of weather data was used to estimate forage production, with the model being solved over three distinct starting weather patterns for each of the potential 27 market iterations.

Results show that financial outcomes and management decisions are in fact influenced by both the current state of the weather and the market. An interesting finding is that, independent of the state of the market, most management decisions are driven by growing season precipitation, and therefore forage production. In order to reduce the negative forage impacts of drought situations, ranchers, when faced with reduced forage production, should partially liquidate their herd, increase acreage (both in total and per animal), and increase feed (both total and per animal). The results also show that allowing the ability to supplemental feed during summer months will ultimately help ranchers' financial standing in the long run.

The ability to supplemental feed will have a larger impact on financial status post drought than during drought, as the ability to carry larger inventories through the drought also requires additional costs as compared to a producer with a more aggressive liquidation strategy. However, the additional costs also allow more animals to be sold during drought as compared to a more drastically liquidated herd, offsetting some of the higher costs. More importantly the strategy of allowing summer feeding allows more animals to be sold post-drought as opposed to requiring a period of herd build up associated with a producer that culled more aggressively.

Ultimately there is no single right decision for ranchers when faced with fluctuating forage production. Proper strategies for producers must incorporate the current status of both weather and the market. The results show how movements in these two exogenous variables affect cattle management decisions. The modeling efforts of this paper focus mainly on how allowing supplemental feeding can alleviate some of the pressures of reduced forage production in the face of unfavorable weather conditions. Caution must be used however, as this model does not fully account for forage dynamics across years. The next step in this modeling process will be to link forage production across years based on prior use of rangelands. No effort has been made in this paper to address how much pressure should be allowed on grazing lands. There is no consequence modeled for overgrazing, it is modeled such that forage production is independent of prior range use. It is expected, however, that current cattle management decisions can affect future forage production, which implies even more caution needs to be used when stocking decisions are made. Therefore a dynamic model incorporating the impacts of current grazing on future forage production will be developed in chapter 2.

CHAPTER 2

Introduction

While chapter 1 was useful in examining how producers respond to fluctuating forage production due to stochastic weather, inter-year dynamics of rangeland productivity was ignored. Proper range management should account for long term effects of decisions concerning stocking rates. Vegetation evolves over time due to both weather impacts as well as prior use of the land. The viability of the rangeland at any point in time depends directly on prior management decisions. While chapter 1 focused on maximization of yearly returns of herd decisions over a fixed planning horizon, this chapter evaluates maximizing the discounted sum of returns to land from grazing operations across an infinite horizon. The physical relationships utilized in this essay follow a predator-prey model of rangeland evolution, specifically as analyzed by Noy-Meir (1976). His analysis focused on the stability of such grazing systems. This research extends his idea by showing that a producer interested in maximizing the value of the land, not just short-term profits, will ensure the long-term productivity of the rangeland.

Problem Statement

Producers can degrade long term range productivity while trying to take advantage of short term opportunities. Manley, et al. (1997) have shown that individual producers, when acting to maximize profit, will stock at rates that can reduce the condition of the range over time. He has shown this to be especially true when cattle prices are high. Likewise, in a dynamic framework, Pope and McBryde (1984) demonstrated that individual producers often stock public rangeland at a rate higher than socially optimal. This essay similarly focuses on management over time, however with special attention giving to dynamics of the rangeland due to management decisions.

Dynamic analysis is not new to the study of cattle production. Chavas, et al. (1985), Meyer and Newett (1970), Glen (1980), and Apland (1985) studied optimal feeding strategies over time. These studies however focused mainly on how and what to feed cattle for finishing, rather than looking at rangeland viability over time. Tess and Kolstad (2000) modeled how forage quality affects animal performance, yet did not specifically address how the rangeland evolves over time, ignoring the impact grazing has on forage production. Vetter (2005) examined rangelands with a focus on the idea of equilibrium, but not necessarily optimal stocking decisions. Rodriguez and Roath (1987) analyzed optimal stocking decisions over time. They analyzed stocking decisions over the short-term focusing specifically on seasonal response, and concluded that stocking rates should decline over the grazing season as forage declines. However, they based their argument entirely on the decline in animal performance as forage production decreases rather than the impact an over utilized rangeland will have on future periods.

Torell, et al. (1991) investigated interactions of stocking rates and range condition. Using a dynamic model calibrated for eastern Colorado, they found that ranchers do not have any incentive to continuously overgraze rangelands. They addressed random weather by treating each year as an averagely productive year in terms of weather impacts on forage grazing. They statde that current performance drives the economic decisions and impacts on future productivity are not as important as current implications of stocking rates. They based this conclusion on a fixed (40 year) planning horizon. Although they did notice that optimal stocking rates are decreased slightly when future forage productivity is accounted for, the fixed planning with a fixed terminal value horizon is expected to have a different outcome than with an infinite planning horizon.

Noy-Meir's study (1976) focused on stability of grazing systems. He modeled the grazing mechanism as a predator-prey relationship, with cattle being the predators of forage. His main assumptions (which are utilized in this essay) are as follows: (a) a single plant, or set of plant species, which have identical growth functions and are equally grazed, (b) a single herbivore species, (c) grazing is on green vegetation in the growing season, with constant plant growth, (d) herbivore requirements/reactions are constant over time. He also initially forced constant herd sizes in order to determine the stability of steady states in the grazing system. By using simple equations of motion, he is able to identify numerous outcomes based on plant growth functions and stocking density. He then went on to compare his results to actual grazing system data. It is assumed that if producers fully incorporate the long term costs of grazing decisions, they will desire a grazing system that ensures long term rangeland health and the associated value of the grazing of such as system. However, Noy-Meir's focus on simulation showed results of given actions and potential outcomes, but does not incorporate manager's decision behavior.

Objective

The objective of this essay is to analyze the long-term trade offs of grazing management decisions. As stated above, much research has been done along similar lines with mixed results. Here, specific attention is given to management decisions given the knowledge that current grazing decision will impact future forage production. This essay generates an economic model that integrates the biophysical relationships outlined by Noy-Meir (1976). Noy-Meir was interested in the stability of such grazing systems, being able to determine what stocking rates were able to keep a tract of range in a stable condition, yet his study lacked economic focus. This essay expands the physical model analyzed by Noy-Meir in order to evaluate grazing decisions that maximize land values over time.

Model Development

In order to account for the dynamic nature of rangeland production, dynamic programming will be employed. A major benefit of dynamic programming is that it allows decisions to change over time. As the rangelands evolve over time, dynamic programming offers a convenient tool for analysis. Unlike some previous work in range economics, this paper recognizes that current forage production is influenced by past management decisions, as reflected in the ending state of the forage in the previous period. It is therefore clear that current decisions will directly impact future forage production and future profitability. Dynamic programming allows for the consideration of these effects when modeling management decisions.

The model presented here is based on the physical relationships presented by Noy-Meir in his analysis of the predator-prey relationship associated with rangelands. Emphasis is placed on equilibrium outcomes of both animal and rangeland performance, and the resulting economic consequences associated with those steady state values. The model encompasses not just animal performance over time, but also how stocking decisions affect the evolving condition of rangelands. As the model incorporates expectations of future forage production into current period stocking decisions, it is expected that stocking rates will be lower in current periods so as to not reduce future productivity. As long as property rights are assured, this forward looking view of rangeland health should better align individual producers' incentives with that of the socially desired outcome, especially where it concerns public land management.

Huffaker and Wilen (1991) use a very similar model. They first evaluate the impact of forage deterioration on a single season model, ultimately adding the dimension of multiple seasons. Their objective is to compare season long stocking rates versus intensive early stocking rates. However they also model the decision as a decision to either graze animals on pasture or feed via dry lot. This is to "shift the attention away from the market dynamics of the overall animal investment decision and allows the model to focus on the complexities of optimal stocking under deteriorating forage nutrients (1991, pg 1214)."

The model presented here however does not focus on whether to graze or not, rather it assumes a land's best use is grazing, and aims to determine the proper stocking rate when considering the impacts of such decisions on future forage growth. Producers are expected to maximize the value of land, an objective consistent with one who holds title to the land, or is ensured continued transferable grazing rights to the land. The model infers that the producer is aware that future forage production is tied to the ending state of the range in the current period. The model will also assume that producers expect that animal consumption is tied to the current state of standing forage. Therefore, relationships are needed to tie future forage production to the current state of the range, as well as animal consumption to current forage production. Noy-Meir's model will be utilized to formulate these relationships. Following Noy-Meir's assumptions grazing is only allowed on this tract of land during the growing season. The general Noy-Meir model is specified as follows. Forage growth (*G*) is of the logistic growth form, specifically:

$$G(V) = \gamma V(1 - \frac{V}{V_m}) \tag{2.1}$$

Where γ is maximum growth rate per unit of time, V is vegetation density per unit of land (standing pounds of forage/acre in the current study), and V_M is the maximum plant biomass for a unit of land (carrying capacity).

The logistic equation to model population dynamics can be traced back to work by Verhulst (1838), and Lotka (1928) and Volterra (as translated in Chapman 1931). Lotka and Volterra also tackled the idea of predator-prey relationships, leading to Holling (1959, 1966) to develop his "disk" theory of predation, which is in fact a Michaelis-Menten type equation. (See Berrymman, 1992, for a more complete history of the evolution of predator-prey modeling) The use of the sigmoid equation to model forage growth in a grazing setting is not limited to the Noy-Meir study, however. Cacho (1993) argues that the sigmoid curve accurately reflects pasture growth in a grazing setting, and his argument applies directly to studies involving a steady state in that he shows that an area "maintained at a steady state by continuous grazing exhibits a constant mass (pg. 388)." Cooper and Huffaker (1997), likewise, utilize the logistic growth equation to model pasture growth in a pasture setting.

Consumption of plant biomass is of the Michaelis-Menten form, with:

$$C = cS = c_m\{(V - V_R)/[(V - V_R) + V_K]\}S \tag{2.2}$$

Where C is total consumption per unit of land, c_m is the level of daily consumption associated with satiation, c is consumption per animal per unit of land, S is stocking density per unit of land, V_r is any ungrazeable residual or mandatory carryover biomass, and V_k is the plant biomass at which consumption equals half of satiation, also known as the "Michaelis Constant." This function has the properties over the relevant range of $C'(V)>0$, and $C''(V)<0$. The Michaelis-Menten equation was first utilized by Michaelis and Menten (1913) in the research of kinetic enzymes. The equation has since been used in many different applications partly due to "definition of the growth rate as a function of the growth rate limiting substrate is very convenient in practice, as it is a continuous function with the properties $\mu(0)=0$ and $\mu(s)\rightarrow\mu_m$ when $s\rightarrow\infty$ (Holmberg, 1981, pg. 24)."

Allden and Whittaker (1970) studied consumption patterns by sheep and showed consumption per animal is closely related to herbage availability. Their study showed a relationship of the consumption and herbage available very similar to the Michaelis-Menten form as seen in Figure 2.1. Although Allden and Whittaker did not specifically address this functional form in their study, the relationship did lead others such as Noy-Meir to follow utilize such an equation.

The Michaelis Constant in a grazing setting can be interpreted a few ways. Cooper and Huffaker (1997) explain it as inversely related to the efficiency of a grazing animal, so a lower number translates into an animal that is able to achieve desired performance with less forage. However, in this model, it can also represent a measure of forage quality. Allden and Whittaker in fact show that the consumption relationship to herbage allowance can be shifted due to density of pastures. Although the overall relationship of increased consumption until satiation as herbage allowance is increased, the curvature of the relationship is affected by herbage density. Although not measured in the Allden and Whittaker study, it is assumed the same relationship would hold true for pasture of differing forage quality. As the forage in this model is assumed to be homogenous, a system with a lower Michaelis Constant is analogous to a system which has higher quality forage, in other words two identical animals will perform differently on pastures with different associated Michaelis Constants. Whichever interpretation is taken, both of these can be under a producer's control, either through altering herd genetics to get more efficient grazers, or by improving the quality of the pasture.

By combining these two biological functions, plant biomass evolution over time can be formulated. The equation of motion for plant biomass becomes:

$$\dot{V} = G(V) - C(V,S) = G(V) - c(V)S \tag{2.3}$$

With vegetation biomass growth accounted for, an equation relating animal performance to consumption of biomass is needed. Huffaker and Wilen (1991) utilized a forage conversion coefficient to convert animal consumption to animal gain of 0.096. So total gain per animal is (0.096 * Consumption) over the grazing season.

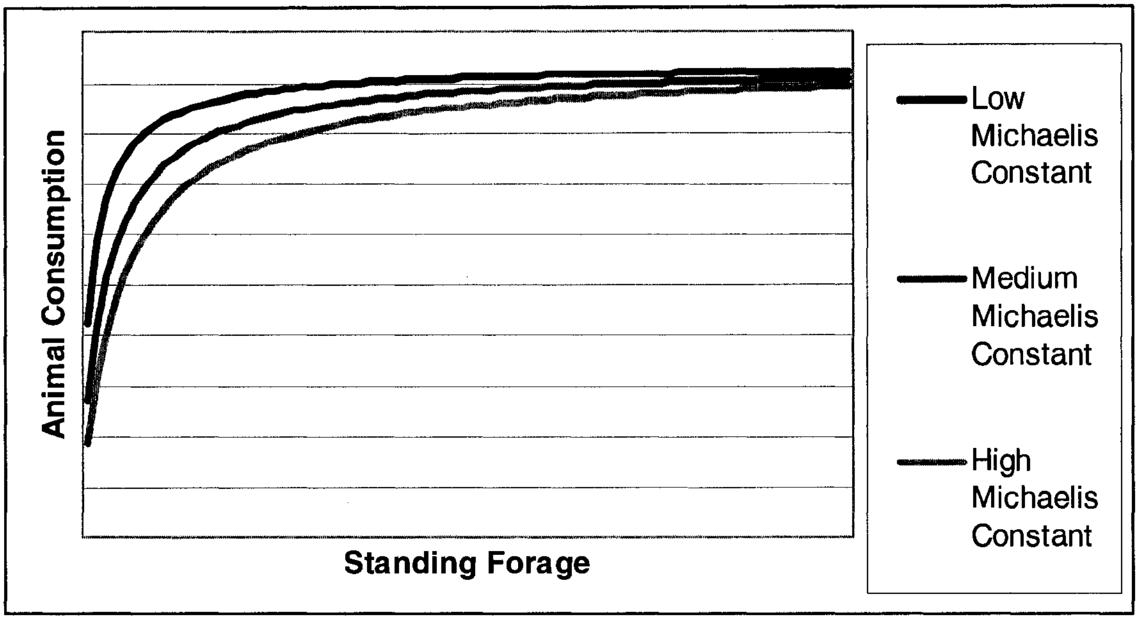

Figure 2.1 Relationship of Consumption to Standing Forage using the Michaelis-Menten Equation

Gain per animal then becomes:

$$Gain(V) = 0.096 * Consumption = 0.096 * (c_m\{(V - V_R)/[(V - V_R) + V_K]\}) \tag{2.4}$$

Animal weight at the end of the grazing season becomes:

$$W_{end} = W_{init} + Gain(V) \tag{2.5}$$

Where W_{end} is the weight of an animal coming off range and W_{init} is the weight of the animal when put on the range.

As Noy-Meir focused on stability of grazing systems, his study lacked economic focus. In this paper, producers are assumed to be concerned with profit maximization when they make their stocking rate decision. Therefore, the producers will optimally make decisions based on profitability of stocking rate decisions, not just stability of the grazing system. The single season herd return to land equation will then be a function of forage, stocking rate and prices.

$$\pi(V,S,P_i,P_e,C)=[P_e*W_{end}-P_i*W_{init}-CC]*S-FC \tag{2.6}$$

Where P_e is price per pound of the animal at W_{end}, and P_i is weight per pound at the W_{init}, CC is carrying costs per animal, and FC is the fixed cost associated with grazing a tract of land.

Output Prices

A shortcoming of some previous dynamic modeling in this area take constant price per weight over differing weight classes when evaluating optimal decisions. Prices per hundred weight (cwt) however do not exhibit constant prices. Producers are faced with declining prices per cwt as weight per animal is increased. Cooper and Huffaker (1997) acknowledged this price slide effect, and modeled a system where animals were purchased at 272.16 kg at $1.74/kg, and sold at the end of the season for only $1.43/kg. In order to account for the price slide effect in the current model, an equation forecasting prices was generated from data available for the Torrington, WY auction. This allowed for a continuous slide over the relevant range of potential weight gain. The data was received from the Livestock Marketing Information Center (LMIC), (unpublished data supplied by Jim Robb, LMIC, Lakewood, Co., June 22, 2007). Weekly prices were available from 1992 through 2006. The prices were deflated to base year 1982 prices using PPI. It was hypothesized that grain prices would affect the prices slide so corn prices were obtained from LMIC and were likewise deflated using Producer Price Index.

Ordinary Least Squares was conducted to estimate price per cwt as a function of weight and corn price. The resulting equation is:

$$P(W_{end}, P_{corn}) = \beta_0 + \beta_1 * W_{end} + \beta_2 * W_{end}^{\ 2} + \beta_3 * W_{end}^{\ 3} + \beta_4 * P_{corn} + \beta_5 * W_{end} * P_{corn} \tag{2.7}$$

Table 2.1 Price Slide Regression Output

Variable Name	*Coefficients* *(t statistics)*
Intercept	293.9348 1.619
End Weight	-0.7396 -0.929
End Weight Squared	0.0010 0.829
End Weight Cubed	0.0000 -0.801
Corn Price	-32.2459 -2.872
Corn Price * Weight	0.0227 1.437

(R Square 0.4717)

As expected, the output shows a declining price per weight as weight is increased. This decline is less drastic when corn prices are high. When the corn price is relatively low, feedlots prefer to purchase lightweight animals and add weight themselves. However, as corn prices rise, the cost of gain for feedlots also rises, so they are less likely to pay a premium for lighter animals. Regardless of corn prices, marginal value per pound received by cattle producers is maximized between 650-750 pound animals as seen in Figure 2.2. Figure 2.2 shows how price per hundred weight decreases as animal weight is increased, while Figure 2.3 shows the marginal value per pound as animal weight is increased. As seen in Figure 2.3, even as price per weight decreases as animal increase weight, initially the marginal value per pound increases.

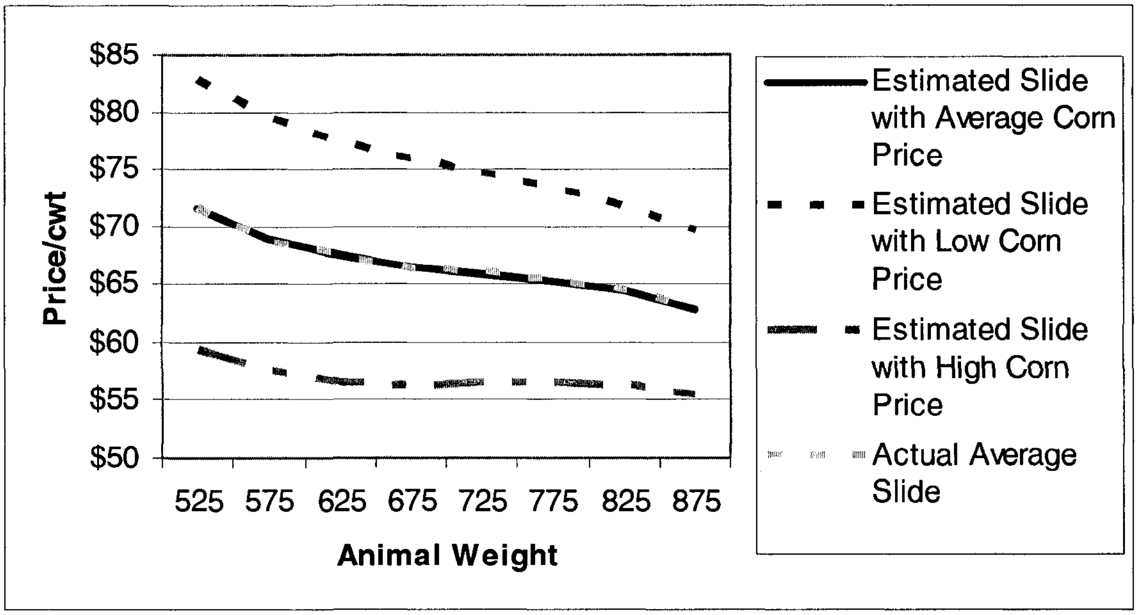

Figure 2.2 Estimated Cattle Prices as a Function of Animal Weight and Corn Prices

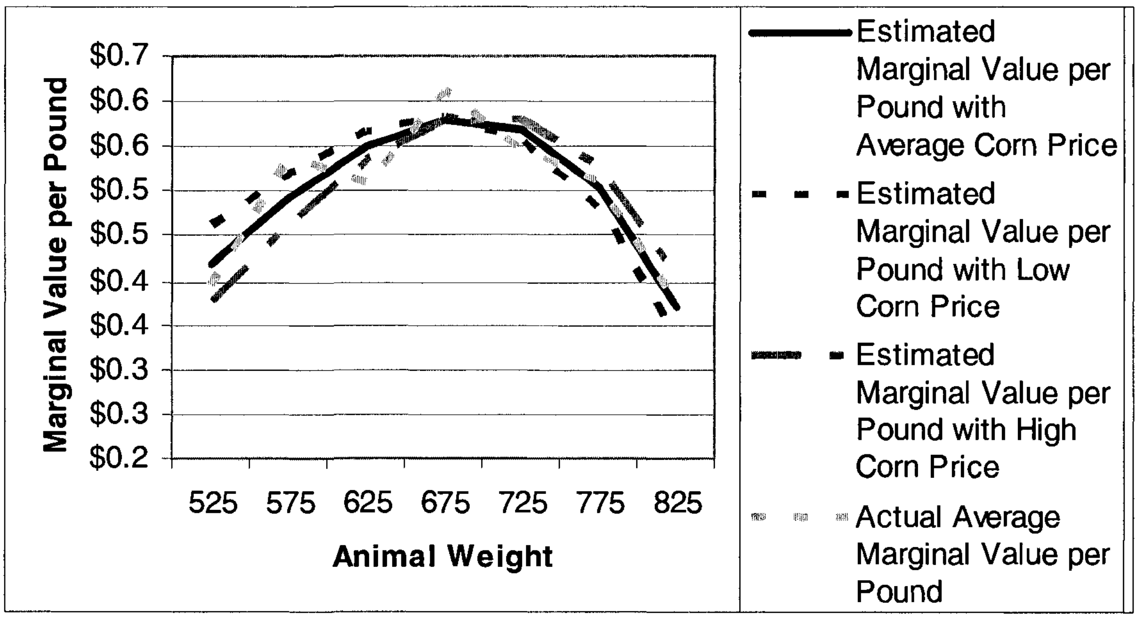

Figure 2.3 Marginal Value of Animal Weight Realized by Cattle Producers

Model parameterization

The model used in this process is:

$$\underset{S}{Max} \int_0^T \beta^t * \{[P_e * W_{end} - P_i * W_{init} - CC] * S * (1 - Deathloss)\} dt \tag{2.8}$$

$$s.t. \dot{V} = G(V) - C(V,S) = G(V) - c(V)S \tag{2.9}$$

With:

$$G(V) = \gamma V (1 - \frac{V}{V_m}) \tag{2.10}$$

$$c(V) = c_m \frac{(V - V_R)}{(V - V_R) + V_K} \tag{2.11}$$

$$Gain(V) = 0.096 * c(V) \tag{2.12}$$

$$W_{end} = W_{init} + Gain(V) \tag{2.13}$$

$$P(W_{end}, P_{corn}) = \beta_0 + \beta_1 * W_{end} + \beta_2 * W_{end}^2 + \beta_3 * W_{end}^3 + \beta_4 * P_{corn} + \beta_5 * W_{end} * P_{corn} \tag{2.14}$$

The model is representative of a stocker operation in central Wyoming. Producers determine their stocking rate in early summer, and sell all animals in the fall. Estimated parameters are for an acre of land, and are given in Table 2.2. The growth rate of forage parameter (γ) used (0.1) is from Noy-Meir (1976) and represents a rangeland of high productivity. As Noy-Meir relates this rate to that of "highly productive" rangeland, so the model was also solved for growth rates of 0.5, 0.4, and 0.3 for sensitivity analysis. Following the work of Torell, et al. (1991), weather is not explicitly stochastic, and the growth parameter is used to represent average productivity each year. This will allow a true steady state to emerge, and initial sensitivity analysis to be performed. This assumption is relaxed in the following chapter. The parameter representing the maximum plant biomass for this area (V_m) is based on Bastian et al. (2005) estimate of 0.39 AUM per acre productivity for Fremont County Wyoming with an AUM representing 800 pounds of grazeable forage. Huffaker and Wilen (1991) represent daily animal consumption of 15.6 pounds of dry matter per day over a grazing season taking an animal from 587 to 770 pounds. Over a 120 grazing season, this translated into 1872 pounds of dry forage consumption per animal. Huffaker and Wilen (1991), based on Noy-Meir (1976), also utilizes a 20% of carrying capacity for the Michaelis constant for consumption, translating here into 62.4. Without a better estimate for the Michaelis Constant, the model was also solved with values of 30 and 100 to determine how sensitive the outcomes are to this parameter.

Producers are generally aware of market prices when making stocking decisions, so the model was solved for differing market conditions. Cattle prices (both initial and final) are based on equation 2.33. However, the model was solved with cattle prices (both initial and final) increased, and likewise decreased, by 20% for comparison to the baseline outcome. The price of corn is based on mean values ($1.65/bu) from the LMIC data over the time period used in estimating the price function. The model was also solved for differing corn prices, specifically over the maximum ($2.25/bu) and minimum ($1.65/bu) prices observed in the LMIC data. Initial Weight (550 lbs.), and days on pasture (120), are in line with a study done for the Wyoming Red Desert by Bastian et al. (1991). Van Tassell et al. (1997) calculate animal costs per AUM in a study including Wyoming. The sum of association fees, veterinary, moving, herding, miscellaneous labor and mileage, salt and feed, water, horse and improvement maintenance costs from his study are \$9.08 per AUM. Deflating these animal costs to 1982 dollars result in animal costs of \$7.69 per AUM. This translates to animal carrying costs of \$21.67 per head (\$7.69*4 months*0.7 AU equivalent) based on average animal weights over the season in this study. The discount rate used initial was 10%, but results were generated for discount rates of 0.1%, 5%, and 20% for comparison.

Table 2.2 Parameters used in Dynamic Model

Parameter	Value (Alternative Vales Used in Sensitivity Analysis)
γ (Relative Growth Rate of Forage)	.1 day^{-1} (.05, .04, .03)
V_m (Maximum Standing Vegetation)	312 lb/acre
C_m (Maximum Daily Consumption)	15.6 lb/ animal/ day
V_r (Mandatory Forage Residual)	0 lb/ acre
V_k (Michaelis Constant)	62.4 lb/ acre (30, 100)
W_{init} (Initial Weight)	550 lb.
P_c (Price of Corn)	1.65 (1.10,2.25)
β (Discount Factor)	0.909091 (.833333,.952381,.990099)
FC (Fixed Acre Cost)	\$10.18/ acre
CC (Carrying Cost per Animal)	\$21.62/ animal
Days on Pasture	120
Death loss	0.02

Mechanism for Solving the Model

Optimization Technique – Bellman Approach

The model is designed to be dynamic in nature, so the producer will not make decisions strictly to maximize single season returns. Producers will account for forage dynamics when making stocking rate decisions, so the intertemporal profit function is represented as:

$$\underset{S}{Max} \int_0^T \beta^t * \{[P_e * W_{end} - P_i * W_{init} - CC] * S * (1 - Deathloss)\} dt \qquad (2.15)$$

$$s.t. \dot{V} = G(V) - C(V) = G(V) - c(V)S \qquad (2.16)$$

Where β is the discount factor.

In order to solve this maximization problem, the model utilizes the Bellman equation. The equation becomes:

$$U_t(V) = \max_{s \in S(v)} \{ f(v,s) + \beta * U_{t+1}(g(v,s)) \}, v \in V, t = 1,2,....T \qquad (2.17)$$

Where U_t is value function, namely the maximum of current and future returns, $f(v,s)$ is the per season return function, $g(v,s)$ is the forage dynamic equation. This function then represents the trade-offs of current returns, $f(v,s)$ and all future returns U_{t+1}. It is expected that a private producer will aim to maximize the present value of their ranch, including the terminal value, or that of a public land manager who aims to ensure land values across generations, therefore the model assumes T=∞. This outlook will place a value on the tract of land used in cattle production, rather than just the value of current livestock production. Therefore, by maximizing the Bellman equation, overall value of the land (as long as cattle production is the best use for the tract of land) will be maximized. Rational producers are expected to find the most profitably long run steady state, as well as the best way to reach that state in the short run.

Since this is a continuous state problem, finding an optimal policy rule at each state is impossible. The fact that there are infinite possible states adds much complexity to the issue. Often this is overcome by discretizing the state space; however this approach results in less precision. Mathematical techniques are now available to overcome the hurdle of continuous state problems by allowing the optimal policy function to be approximated. The approximation technique allows for an optimal policy rule for any possible state, while leaving little error from the true unknown policy function. When the time horizon is infinite, meaning the problem is time-separable, and the problem is autonomous, Judd (1998) defines the value functions (using variable definitions as used in the current model) as:

$$U(v) = \sup_{\Psi(v)} \sum_{t=0}^{\infty} \beta^t \pi(v_t, s_t) \mid v_0 = v \tag{2.18}$$

Where $\Psi(v)$ is the set of possible actions given a stock v, and this value function then satisfies the Bellman equation:

$$U(v) = \sup_{v \in D(v)} \pi(v,s) + \beta\{U(v^+) \mid v, s\} \equiv (TV)(v) \tag{2.19}$$

This will translate into an optimal policy function which solves:

$$S(v) \in \arg\max_{s \in D(v)} \pi(v,s) + \beta\{U(v^+) \mid v, s\} \tag{2.20}$$

Where v^+ is defined as the state in the subsequent period resulting from current period actions. Judd's theorem 12.1.1 (pg 402) states that if S is compact, $\beta<1$, and π is bounded above and below, the map TV is monotone in U, is a contraction mapping with modulus β in the space of bounded functions, and has a fixed point. This existence of a fixed point ensures both the existence and uniqueness of the value function, while also suggesting how to find the optimal decision rule with associated optimal value.

Approximation of Value Function – Chebychev Approach

It is impossible to calculate the value function for every possible state given a continuous state function. However, many mathematical techniques have been devised for approximating the value function. "[T]he Bellman equation that characterizes the solution of an infinite horizon dynamic optimization model is a function fixed-point equation (Miranda and Fackler, 2002, pg 115)." According to Miranda and Fackler (2002), we can approximate this value function through contraction mapping, basically finding a map T that satisfies $V=TV$. Howitt et al. (2002) have formulated a numerical approximation to this technique for infinite time horizon problems. The idea of mapping, namely $V=TV$, can be accomplished by the following two propositions; (1) any function can be approximated by a polynomial of sufficient order, and (2) such a function can be found within some finite number of iterations (pg. 4). With these holding, they show an approach to approximate a function which in fact maximizes the total value function as stated above. They prescribe the use of Chebychev polynomials for their orthogonal nature, as seen in Figure 2.4. This orthogonality is due to the terms of the Chebychev polynomials being sinusoidal in nature, where:

$$\Theta_n(\hat{x}) = \cos(n * \cos^{-1}(\hat{x})) \tag{2.21}$$

Which numerically becomes:

$$\Theta_1(\hat{x}) = 1$$

$$\Theta_2(\hat{x}) = \hat{x}$$

$$\Theta_3(\hat{x}) = 2 * \Theta_2(\hat{x}) - \Theta_1(\hat{x})$$

$$\vdots$$

$$\Theta n(\hat{x}) = 2 * \Theta_{n-1}(\hat{x}) - \Theta_{n-2}(\hat{x})$$

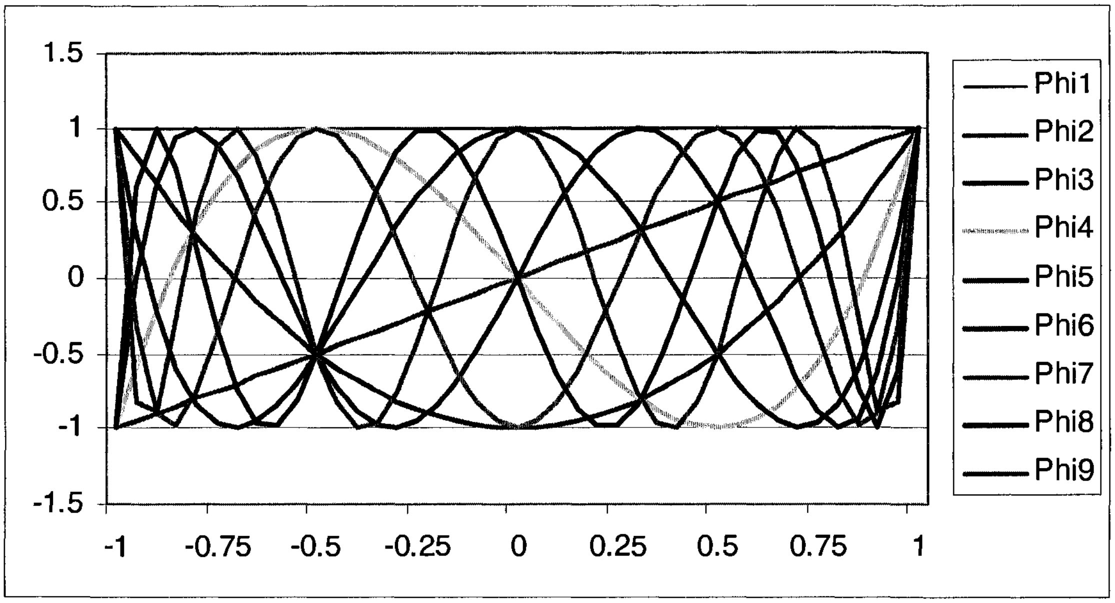

Figure 2.4 Chebychev Polynomials over the [-1,1] Interval

Approximating the value function utilizes the interpolation process. Miranda and Fackler (2002) approximate this intractable real-valued function f with the tractable function $\hat{f}$. They suggest an interpolation scheme utilizing a function form as a linear combination of n linearly independent basis functions Φ_1, Φ_2, ... Φ_n,

$$\hat{f}(x) = \sum_{j=1}^{n} c_j \Theta_j(x) \tag{2.22}$$

With basis coefficient c_1, c_2, ...c_n to be determined throughout the interpolation process. The approximation technique utilizes these polynomials in Chebychev regression over the interval $[a,b]$ at n number of interpolation nodes, x_1, x_2, ...x_n. With n nodes and n basis functions the solution of the coefficients reduces to solving a linear equation. In order to solve for the coefficients by solving the interpolation conditions:

$$\sum_{j=1}^{n} c_j \Theta_j(x_j) = f(x_j), \forall i = 1,2,...,n \tag{2.23}$$

Using linear algebra, this can be written in matrix form as:

$$\Phi c = y \tag{2.24}$$

Letting $y_i = f(x_i)$ be the function value at the ith node, then

$$\Phi_{ij} = \Theta_j(x_i) \tag{2.25}$$

is the typical element of the interpolation matrix representing the jth basis function evaluated at the ith node. We then can choose the coefficients that minimize the sum of squared errors, basically least-squares approximation, namely:

$$e_i = f(x_i) - \sum_{j=1}^{n} c_j \Theta_j(x_i) \tag{2.26}$$

With estimation of the coefficients of:

$$c = (\Phi'\Phi)^{-1}\Phi' y \tag{2.27}$$

When choosing the n nodes over the $[a,b]$ interval, one may be tempted to choose them at even spaces. However, often the errors are greatest towards the ends of the interval. Chebychev nodes correct for this by placing nodes more closely spaced towards the endpoints of the interval, leaving more space between nodes in the center of the interval. "Chebychev-node polynomial interpolants are very nearly optimal polynomial approximants. Specifically, the approximation error associated with the nth-degree

Chebychev-node polynomial interpolant cannot be larger than 2 π log(n)+2 times the lowest error attainable with any other polynomial approximant of the same order (Miranda and Fackler, 2002, pg. 119-120)." These nodes are chosen in the following fashion:

$$x_i = \frac{a+b}{2} + \frac{b-a}{2}\cos\left(\frac{n-i+0.5}{n}\pi\right), \forall i = 1,2,....,n \tag{2.28}$$

In order for better coverage of the state space, it is recommended to fit the state variable to the interval [-1,1], by defining

$$z = 2\frac{x-a}{b-a} - 1 \tag{2.29}$$

Where z is the map of the Chebychev coefficients over the [-1,1] interval.

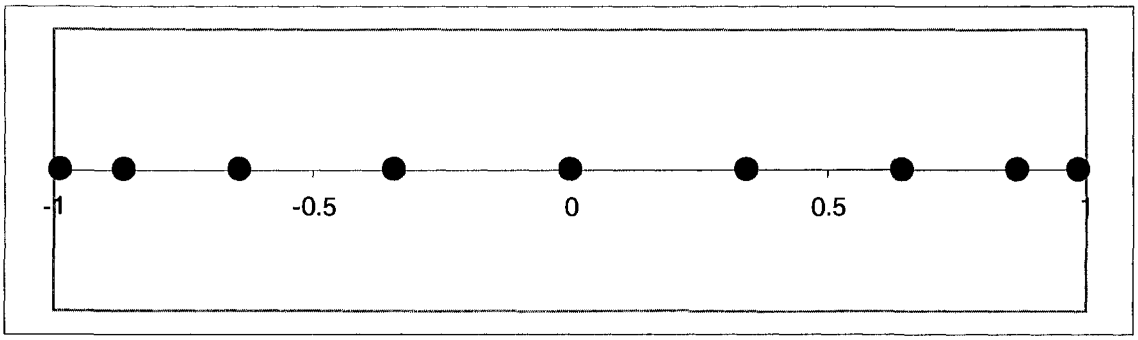

Figure 2.5 Spacing of Chebychev Nodes over the [-1,1] Interval

Howitt et al. (2002) describe this iterative process by using the following regression to update the polynomial coefficients for the kth iteration as (using current notation):

$$c_i^{(k)} = \frac{\sum_{j=1}^{n} U_k(x_j)\Theta_i(\hat{x_j})}{\sum_{j=1}^{n} \Theta_i(\hat{x_j})\Theta_i(\hat{x_j})} \tag{2.30}$$

Then updating the value function using the coefficients:

$$U^{(k)}(x) = \sum_i c_i(k)\Theta_i(2\frac{x-a}{b-a}-1) \tag{2.31}$$

Again, this process is continued until convergence, as Howitt et al. (2002) define until the error sum of squares $(c^k\text{-}c^{k-1})^2$ is less than a predetermined tolerance. Once convergence is obtained, we have the approximation of the value function at all possible states. The approach used in this analysis is that of policy iteration. Of interest is the best stocking rate to apply given any stock forage level. Miranda and Fackler (2002) use Newton's method to approximate this function. They begin by defining the rewards f (in this case the return equation), transition probabilities P (unneeded in this case as all info is deterministic), discount factor δ, and some initial value guess v. Here is where the policy iteration begins. First is to update x (the policy) given initial guess for v:

$$x \leftarrow \arg\max_x \{f(x) + \delta v\} \tag{2.32}$$

Then the value is updated in the following fashion:

$$v \leftarrow [I - \delta]^{-1} f(x) \tag{2.33}$$

The new v is compared to the initial guess for v. If there is a significant difference, the new value for v is used, and the process is repeated. This process will find an improvement in the value function for at least one state until there is no change in v. Once this occurs, the optimal policy has been found. Because this is evaluated at the finite number of Chebychev nodes, as long as there is a finite number of admissible policies the policy iteration will find an optimal in a finite number of iterations (Miranda and Fackler, 2002). When looking at optimal decisions for broiler production, Kennedy et al. (1976) state policy iteration "is preferred because convergence is rapid and the exact solution is obtained (pg. 25)."

Results

The model was solved using GAMS utilizing the Chebychev collocation process maximizing the Bellman equation. The model was initially solved using the baseline parameters. For an acre of land given the initial parameters, the policy function converges to the long run equilibrium of 173.9 pounds of forage per acre with an associated stocking rate of 0.67 per acre, or 1.5 acres per head. The policy function will show optimal convergence to this amount of forage from starting points either above or below this amount. The model was also solved for the other parameter values for growth rate, discount rate, corn price, and Michaelis Constant as shown in Table 2.2. The model was also solved across differing values for cattle prices. Table 2.3 compares outcomes across these differing parameter values.

With respect to all parameters except the Michaelis Constant and output price, ending forage values are around 174 pounds of standing vegetation. This implies it is optimal for producers to leave just over half of standing vegetation when considering future forage impacts due to current grazing. This would imply a lack of motivation for producers to overgraze rangelands as long as previous assumptions about maximizing land value and guaranteed property rights exist. The outcome is line with the traditional view of land manager to "take half, leave half" as a general rule for range management. Even with a lower Michaelis Constant, it is not optimal to leave less than half of potential standing forage at seasons end. Again, if the Michaelis Constant is interpreted as

Table 2.3 Results for Steady State Values from Dynamic Optimization

		Growth Rate	Discount Rate	*Steady State Values* State Value	Stocking Rate	End Weight	Returns per acre
		0.1	0.01	175	0.6734	682	$24.37
			0.05	175	0.6742	681	$24.37
			0.1	174	0.6753	681	$24.37
			0.2	173	0.6773	681	$24.37
		0.03	0.1	172	0.2040	681	$7.31
		0.06		173	0.4061	681	$14.62
Corn Price	1.10	0.1	0.1	173	0.6774	681	$25.60
	1.65			174	0.6753	681	$24.37
	2.25			175	0.6726	682	$23.04
Michaelis Constant	30	0.1	0.1	164	0.5896	702	$28.20
	64.2			174	0.6753	681	$24.37
	100			185	0.7448	667	$20.76
Ouput Prices	Decreased 20%	0.1	0.1	179	0.6644	682	$16.61
	Base			174	0.6753	681	$24.37
	Increased 20%			171	0.6807	681	$32.17

(Unless otherwise noted, Michaelis constant 64.2, Corn Price 1.65, growth rate .1, discount rate .1)

previously mentioned, a producer can improve returns to land significantly through either carrying more efficient grazers, or improving the quality of the forage on the range.

When the forage growth parameter is 0.1, optimal long run stocking rates are around 0.67 head/acre, or equivalently 1.5 acre/head. With a growth parameter of 0.1, this number fluctuates only with changes to the Michaelis Constant, and varies between 0.59 and 0.75 head/acre, or 1.25 and 1.75 acre/head for the parameters used. This is due to the fact that per animal consumption is determined by standing forage, and altering the Michaelis Constant ultimately alters the consumption per animal. Once consumption per animal is determined the only way to remain at a steady state is to find the stocking rate that equates total consumption to total growth.

If the forage growth parameter used is 0.06, the stocking rate drops to 0.4 head/acre or 3 acre/head, and if the growth parameter is only 0.03, this falls to 0.2 head/acre or 5 acre/head. Therefore stocking rate is very dependent upon potential forage production, and optimal stocking rates should be aligned with this forage growth parameter.

End weight for cattle in all cases except varying Michaelis Constant are around 680 lbs. at sale date. However, due to declining prices for higher sale weights, this lighter weight animal optimizes returns over heavier animals. Even with different Michaelis Constants, optimal sale weight does not exceed 702 pound animals going to auction. Again, this is due to the fact that stocking rate is determined by setting total consumption equal to total growth, and per animal consumption is in part determined by the Michaelis Constant. Therefore, since the Michaelis Constant alters per animal consumption and stocking rates at the steady state, it is not surprising that ending weights will also very with the Michaelis Constant.

Returns per acre are most responsive to changes in the forage growth parameter. Land with more forage production potential can carry more animals over the season to the same ending weights resulting in much higher returns. The Michaelis Constant also has a large impact in return per acre as well. Again, a producer with more efficient grazers or higher quality forage can produce more weight gain per acre of land, resulting in higher returns to the land base. This is not only due to the ability to produce more gain per area of land, but the ability to do so with a lower stocking rate, resulting in lower variable costs per acre.

Producers are also often aware of fluctuating prices. Indeed, cattle prices have a large impact on returns per acre; however management decisions vary little across different cattle price levels. Similarly, corn prices have an impact, although less so than cattle prices, on financial returns per acre, but optimal decisions vary little in respect to either cattle or corn price levels. Unfortunately, this implies there is little a producer can do by means of herd management to alleviate the impact of either low cattle or high corn prices.

These results of the model are consistent with Noy-Meir's (1976) analysis. Ignoring economic consequences, he says the "safe carrying capacity (pg. 93)" is defined as

$$H_s = \frac{\gamma V_k}{c_m} \tag{2.34}$$

which is .4 with the given base parameters.

He says that the maximum carrying capacity can be defined as

$$H_x = H_s + \frac{\gamma}{4c_m V_m}(V_m - V_k)^2 \tag{2.35}$$

which is .72 given above base parameters. Given the above parameters, the maximum carrying capacity is approached but never realized, consistent with Noy-Meir's statement that a stocking rate just below this maximum capacity "may be a reasonable choice of 'normal' stocking in a commercial pasture (1976, pg. 9)."

Optimal Stocking Rate

A convenient outcome available with dynamic programming is the ability to determine the optimal policy function, which prescribes stocking rate in this case. In the beginning of the grazing season, a producer must make stocking decisions. Given that initial standing forage is observable, the approach utilized allows a producer to make the stocking decision that will maximize total returns to land over the infinite horizon based solely on that standing forage level. Figure 2.6 shows what stocking rate should be set for various levels of initial standing forage across the different scenarios. As seen in Figures 2.6, 2.7 and 2.8 , discount rate, corn price levels, nor output price levels alter the optimal stocking rate for a given standing forage level. However, as seen in Figures 2.9 and 2.10 growth rate, or plant productivity, has a major impact on optimal stocking rate, and the Michaelis Constant also alters optimal stocking patterns greatly for a given standing forage level. This implies that regardless of a producer's personal discount rate, the price level of corn, or the output price level, the stocking rate is determined predominantly on standing forage for given biological response parameters. The difference in optimal stocking rate in terms of varying growth rate or Michaelis Constant is greatest in the middle of the state space, and less drastic towards either carrying capacity or extremely overstock range.

Obviously, a producer with more productive rangeland should set a higher stocking rate. It is interesting, however, that as a producer has a situation relating to a higher Michaelis Constant, whether through less productive grazers or lower quality forage, they should in fact stock at a higher rate and end with lower weight animals. Producers who are faced with a situation that relates to a lower Michaelis Constant, whether having more efficient grazers or higher quality forage, should stock at a lower rate and end up putting more weight on their animals. Again, this ultimately is due to the

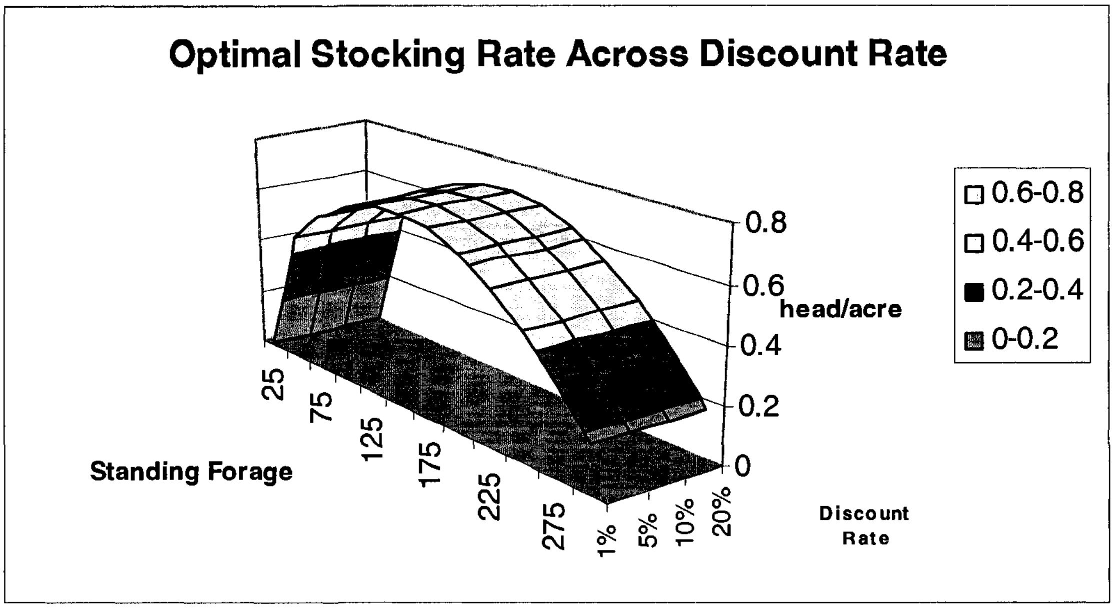

Figure 2.6 Optimal Stocking Rate for given Standing Forage Levels across Discount Rates

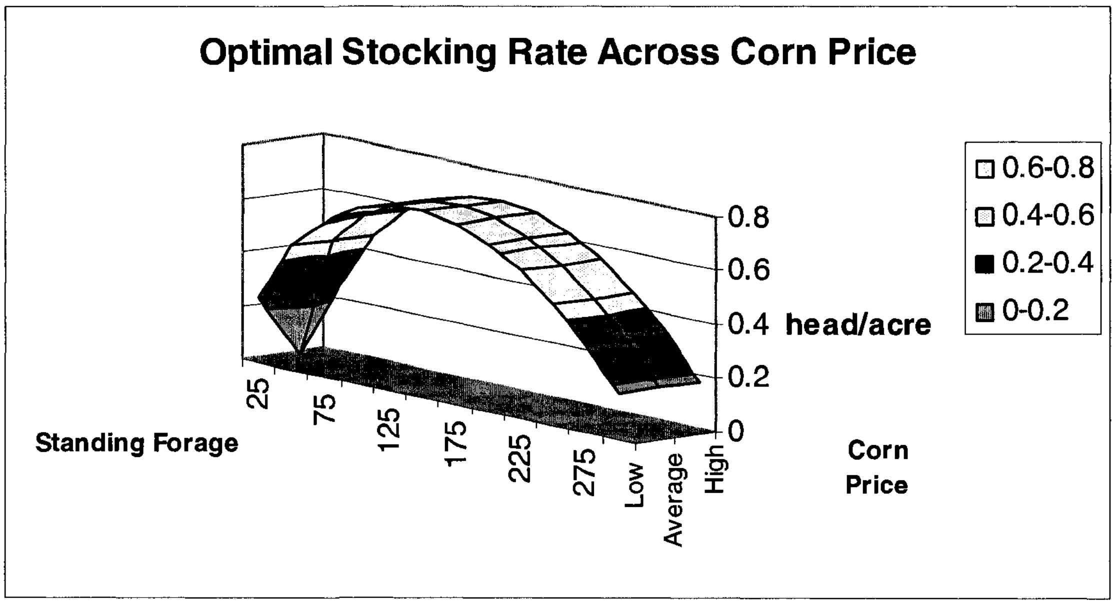

Figure 2.7 Optimal Stocking Rate for given Standing Forage Levels across Corn Prices

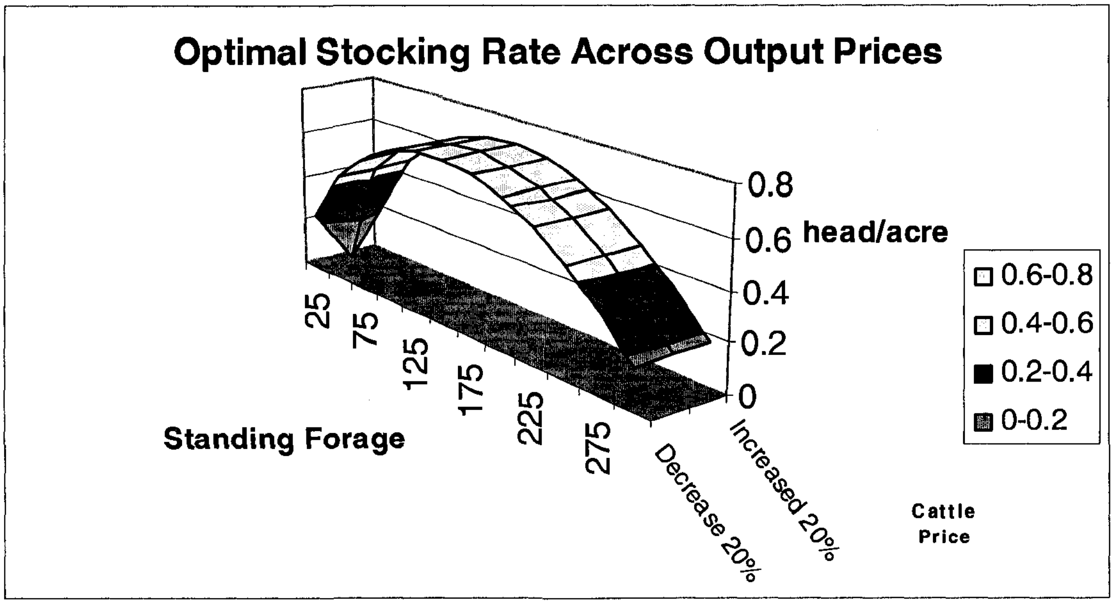

Figure 2.8 Optimal Stocking Rate for given Standing Forage Levels across Output Prices

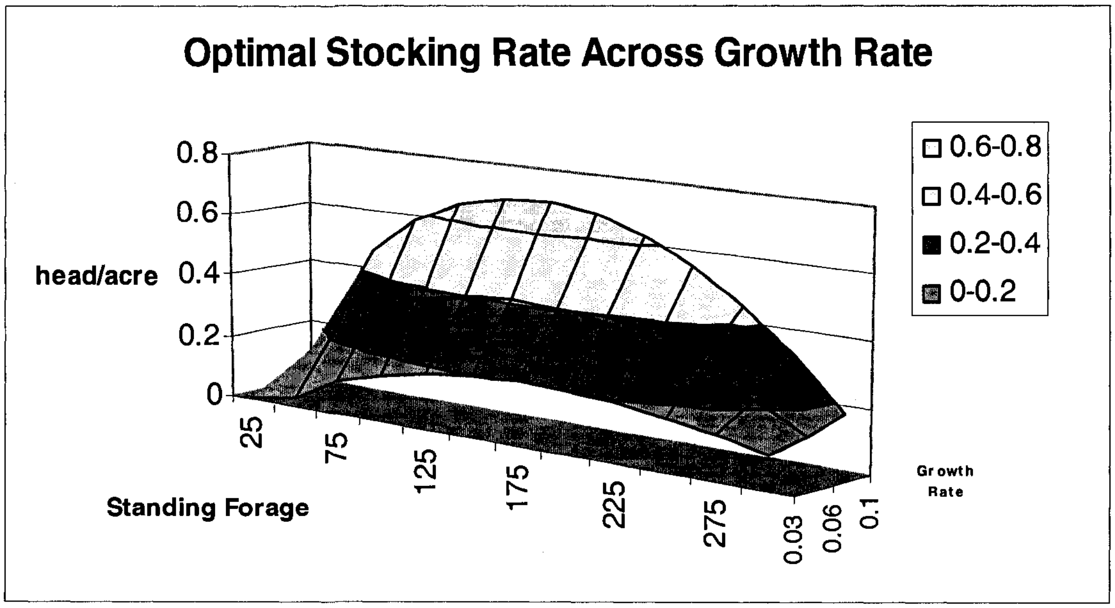

Figure 2.9 Optimal Stocking Rate for given Standing Forage Levels across Forage Growth Rates

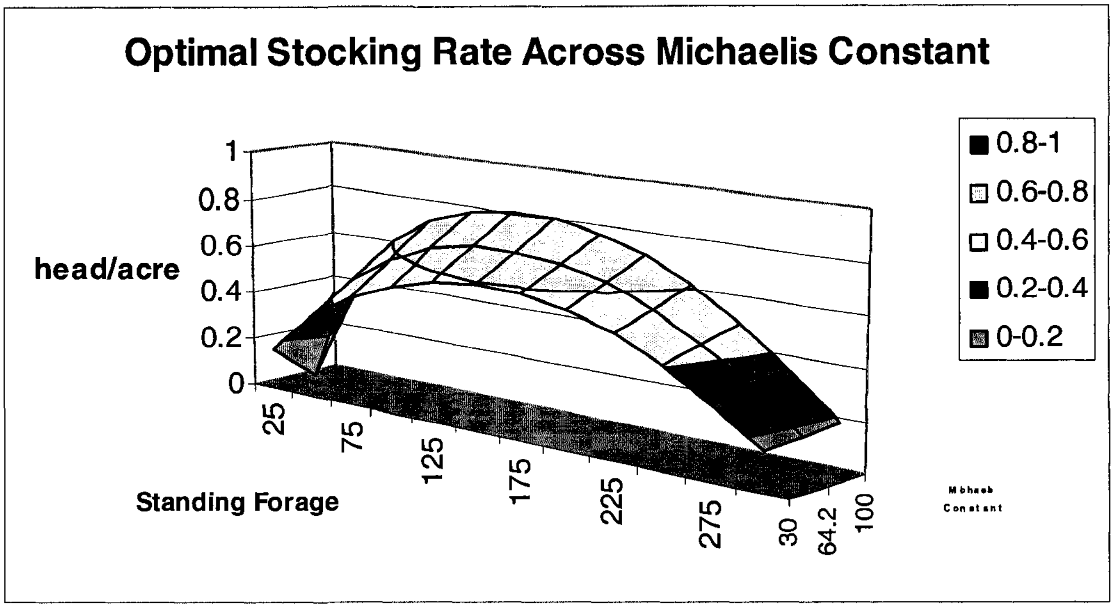

Figure 2.10 Optimal Stocking Rate for given Standing Forage Levels across Michaelis Constant

ability to allow more gain per animal while maintaining lower carrying costs associated with lower animal numbers.

Conclusion

A dynamic model of rangeland evolution was evaluated for optimal stocking decisions. The model is representative of a stocker operation located in central Wyoming with a goal of maximizing the value of the land. As expected, with an infinite time horizon, it is optimal for producers to incur lower returns initially in order to improve rangeland health as opposed to a producer interested in maximizing current year profits only. Although optimal levels of standing forage are reliant on growth rates of forage and consumption characteristics of animals which can be hard to quantify, the idea of "take half, leave half" is fairly consistent with optimal stocking decisions. In fact in most cases, the optimal standing forage at season end is 55% of potential production. For a private producer whose goal is to sell his land at the end of his ranching horizon, the approach of this model will ensure that the land is at its most valuable state when the end of their planning horizon is reached. Likewise, if the tract of land in question is owned by the public, the appropriate manager can evaluate the individual rancher's decisions to see if actions are in line with socially desired result, namely keeping the public land in its most valuable state. However, in order for individual producers to act in a manner consistent with this approach, the public land manager must somehow enforce the optimal decisions. The easiest way would be to guarantee the transferable right to private access on public grazing lands as long as this approach is used.

A producer must be aware of current conditions of the range in order to make optimal decisions. Although selecting the proper stocking rate is vital to maintaining long-term range health, one of the largest impacts a producer can have is to carry efficient grazers, or have high quality forage. Also, both cattle and corn price levels will have a major impact on financial returns; however a producer should not alter their management strategy based on variation in either of these price levels. Regardless of price levels, producers never had an incentive to overgraze the range in any of the scenarios evaluated.

Although the model utilized sheds some insight into how optimal stocking rates depend on both current and future productivity, the model lacks the realism of fluctuation forage production across years. A model linking future forage production to current decisions and stochastic weather realizations will be helpful in determining how producers should respond to natural fluctuations in forage production. Such a model is developed and analyzed in Chapter 3.

CHAPTER 3

Introduction

The second essay showed that if a producer accounts for the impact current decisions have on future productivity, they will not have an incentive to overgraze. In fact, they will manage towards the optimal amount of standing forage even if it means forgoing more current period rents. This essay extends the analysis to include the effects stochastic weather has on forage production as well. Relaxing the assumption that a tract of land has a constant productivity allows for analysis of how producers should make grazing decisions when faced with uncertainty in grazing season precipitation.

Problem Statement

Proper range management is crucial to long-term sustainable forage production. As seen in the previous chapter, when concerned with the long run, a producer should forgo short term profits to ensure future productivity. However, forage production is dependent not only on the current state of range viability, but also on current year growing season precipitation. While the dynamic programming model utilized in the previous section demonstrated how long term costs can outweigh short term gains of overstocking rangeland, if forage production does vary with precipitation, the previous model has limited usefulness for yearly decision making.

While Stochastic Dynamic Programming (SDP) has gained popularity in agricultural scenarios, not much attention has been given to rangeland applications.

Recent work often has focused on forest management strategies (see for example Couture (2008), and Moore and Conroy (2006)) or optimal water reservoir strategies (see for example Abolpour (2007), Mujumdar (2007), and Mousavi (2004)). Recent range studies have focused on optimal improvement strategies to preserve range condition (see Bernardo (1989), and Pope and McBryde (1984), for example) rather than optimal herd management to control range productivity. Likewise Carande, et al. (1995) understand the need for dynamic analysis, but limit themselves to analyzing optimal sale dates across a fixed number of stocking rates and precipitation outcomes. Kobayashi, et al. (2007) analyze the impacts of limited capital on sheep stocking strategies in Kazakhstan, mainly in the vein of showing a need for improving localized capital markets.

But, if in fact "(m)anagement of the forage base, as well as the livestock, is the key to improved livestock performance" (Manley, et al., 1997, pg. 644) a producer must be aware of variability of precipitation, as it will most assuredly have an impact on forage production. Indeed, as stated by Hart, et al. (1998), increases in the stocking rate may increase short-term livestock gain, a producer must understand the potential long-term decrease in range productivity, and this can only be more dramatic during times of low precipitation. Westoby, et al. (1989) liken this ongoing decision to a "continuing game, the object of which is to seize the opportunities and evade the hazards, so far as possible (pg. 266)." In fact, "(i)n grazing systems with very high climatic variability, forage availability varies to such a great degree with rainfall that herbivore population dynamics are driven by rainfall via its direct effect on forage availability in any given year…(r)ainfall and stocking rate interact, with low rainfall exacerbating the effects of high stocking rate, and high rainfall mitigating them (Vetter, 2005, pg. 324)."

In the previous chapter, forage production was assumed to be constant across years, just as Torell, et al. (1991) accounted for varying weather by using the average forage production for all years. However as Smith (2007) states, "the better years are often remembered as normal. The reality is half or more years are below average (pg. 1)." If a producer makes stocking decision based on average precipitation, and ignores the fact that many years are below "average," they will overstock the range, and ultimately drive down both forage production and the value of the land.

Similar to the approach presented in this paper, Passmore and Brown (1991) evaluate the idea of proper rangeland management in a stochastic setting, looking specifically at biomass as the best proxy for range condition, and as stocking rate being the most likely decision to impact this condition. They incorporated variable weather and analyzed optimal sheep stocking an arid rangelands in Australia. They stress that although producers often lack the technical expertise needed to fully take advantage of such models, dynamic programming in this stochastic setting is valuable in providing useful insights into the "intertemporal nature of the rangeland setting (pg. 154)" and highlighting what information is necessary to make better decisions.

Objective

The objective of this essay is to examine how producers make grazing decisions in the face of stochastic weather, with the understanding that future forage productivity will be directly impacted by the ending state of the range. Attention is given to decisions with knowledge only of the probabilities of potential weather outcomes each year as producers often must make stocking decisions before growing season precipitation can be fully observed. This essay addresses how a tract of land will evolve over time given these management decisions. Analysis of how these decisions would be altered with knowledge of current year precipitation is also examined.

Model Development

Obviously, stochastic forage production must be represented in a model aiming to describe optimal range management. Loehle (1985) analyzed the Noy-Meir in order to analyze equilibrium state in a catastrophe theory based model. He incorporated random weather effects through altering the carrying capacity variable (V_m in equation 2.1). He ascertains that the carrying capacity of the land will vary year to year based on fluctuations in forage production as driven by precipitation. However, if carrying capacity is in fact the amount of plant biomass an ecosystem can support, this variable should not change based solely on precipitation.

The model utilized in this chapter will incorporate variable forage production through equation 2.1 as well, but the parameter affected by precipitation is the relative growth rate, γ. An area is expected to be capable of sustaining a given amount of plant biomass in any given year, but precipitation may limit the biomass from reaching that level in any given year. Also, in high precipitation years, a land area cannot produce beyond carrying capacity, but that threshold may be reached quicker than in drier years. Therefore, the model utilized in chapter 2 is updated to account for variable weather by making the growth parameter, γ, stochastic. This chapter follows the work of the previous chapter, with specific attention given to the effect of stochastic weather patterns on optimal grazing management. Rangeland productivity responds to both grazing pressure and natural fluctuations in weather. A producer therefore must make grazing decisions in response to current state of the range and expectations about nature. A producer makes decisions before current year precipitation is realized based on expectations of possible weather realizations.

The model presented in this chapter is directly related to the model utilized in chapter 2 based on previous work by Noy-Meir (1976). The model maximizes the value of rangeland, subject to the physical relationship outlined by Noy-Meir. The model has been updated to account for changes in forage growth due to stochastic weather. It is assumed that the carrying capacity (V_m) of a given tract of land is fixed, yet yearly growth is also dependent upon growing season precipitation. Often, stochastic events such as this are represented in the equation of motion by simply adding a random term, often some sort of Brownian motion. However, in the instance of forage growth it may be reasonable to assume that the rate of growth of forage is directly impacted by precipitation. In order to reflect this connection, equation 2.1 remains as:

$$G(v,\omega) = \gamma(\omega)V(1-\frac{V}{V_m}) \tag{3.1}$$

However, in this chapter γ is taken to vary among years, resulting in differing actual growth across years with differing precipitation even with identical beginning standing forage. Consumption is the same as equation 2.2, and the equation of motion remains the same as equation 2.3, however the change in forage is now dependent on precipitation. So the growth equation and actual equation of motion are now:

$$C(V,S) = cS = c_m\{(V - V_R)/[(V - V_R) + V_K]\}S \tag{3.2}$$

$$\dot{V} = G(V,\omega) - C(V,S) = G(V,\omega) - c(V)S \tag{3.3}$$

With ω denoting that the growth rate, and ultimately seasonal growth, depend on random weather events. The equations for animal gain, ending weight, and return to area of land remain the same as equations 2.4, 2.5, and 2.6 respectively. Prices are still calculated according to equation 2.7. This model is also solved using the Bellman approach. The objective is identical to maximize the return to land,

$$\underset{S}{Max} \int_0^T \beta^t * \{[P_e * W_{end} - P_i * W_{init} - C] * S * (1 - DeathLoss)\} dt \tag{3.4}$$

however, equation 2.9 is altered to update the dependence on weather.

$$s.t. \dot{\mathrm{V}} = \mathrm{G}(\mathrm{V}, \omega) - \mathrm{C}(\mathrm{V}, \mathrm{S}) = \mathrm{G}(\mathrm{V}, \omega) - \mathrm{c}(\mathrm{V})\mathrm{S} \tag{3.5}$$

Optimization Technique – Bellman Approach

As with Chapter 2, the model will be solved utilizing the Bellman equation. However, equation 2.18 has been updated to account for stochastic weather events. With the implication that weather will impact the growth function, the goal now is to maximize the discounted *expected* returns, which is represented by:

$$U(v) = \sup_{\Psi(v)} E_{\omega} \left\{ \sum_{t=0}^{\infty} \beta^t \pi(v_t, s_t) \mid v_0 = v \right\} \tag{3.6}$$

This value function is such that the Bellman equation (3.7) is satisfied over the expectations and leads to the policy function which solves (3.8) (From Judd, 1998 and Miranda and Fackler, 2002)

$$U(v) = \sup_{v \in D(v)} \pi(v, s) + \beta E_{\omega} \{U(v^+) \mid v, s\} \equiv (TV)(v) \tag{3.7}$$

$$S(v) \in \underset{s \in D(v)}{\arg\max}\, \pi(v, s) + \beta E_{\omega} \{U(v^+) \mid v, s\} \tag{3.8}$$

Again, as in Chapter 2, the value function is again approximated using the Chebychev interpolation process to estimate the basis coefficients, c_j.

$$U(v) \approx \sum_{\mathrm{j=1}}^{\mathrm{n}} c_j \Theta_{\mathrm{j}}(\mathrm{x}) \tag{3.9}$$

This is done by replacing the Bellman equation with the following system of n equations and n unknowns, which allows to computation of the coefficients, based on the expectations of weather events.

$$\sum_{\mathrm{j=1}}^{\mathrm{n}} c_j \Theta_{\mathrm{j}}(\mathrm{x_j}) = \max_{s \in S(x_j)} \left\{ f(v_i, s) + \beta E_\omega \sum_{j=1}^{n} c_j \Theta_j(g(x_i, s, \omega)) \right\} \tag{3.10}$$

In this case, however, the problem is stochastic and attention must be given computing the expectations. Miranda and Fackler (2002) state "the continuous random variable (ω) in the state transition function is replaced with a discrete approximant, say, one that assumes values $\omega_1, \omega_2, \ldots, \omega_k$ with probabilities $p_1, p_2, \ldots, p_k$, respectively.(pg. 229)" This alters the collocation function for the Bellman equation thusly:

$$u_i(c) = \max_{s \in S(v_i)} \left\{ f(v_i, s) + \beta \sum_{k=1}^{n} \sum_{j=1}^{n} p_k c_j \Theta_j(g(v_i, s, \omega_k)) \right\} \tag{3.11}$$

However, here we have p_k which represents the probabilities associated with each distinct weather outcome. Therefore, the interpolation process solves for the coefficients that maximize the expectation of the value function. The producer is assumed to have a constant expected forage production for any given state across years without regard to any previous realized forage production. The model therefore has the same forage production probabilities, p_k, regardless of prior year weather outcomes.

Weather Parameterization

As stated above, in order to utilize this approach, the weather impact on growth must be in discrete form. As in chapter 2, the model is parameterized for central Wyoming. Weather data is available for Riverton, WY from NOAA from 1928. Using equation 1.7, predicted forage for Fremont County, Wyoming using Smith's "Casper Equation" was calculated.

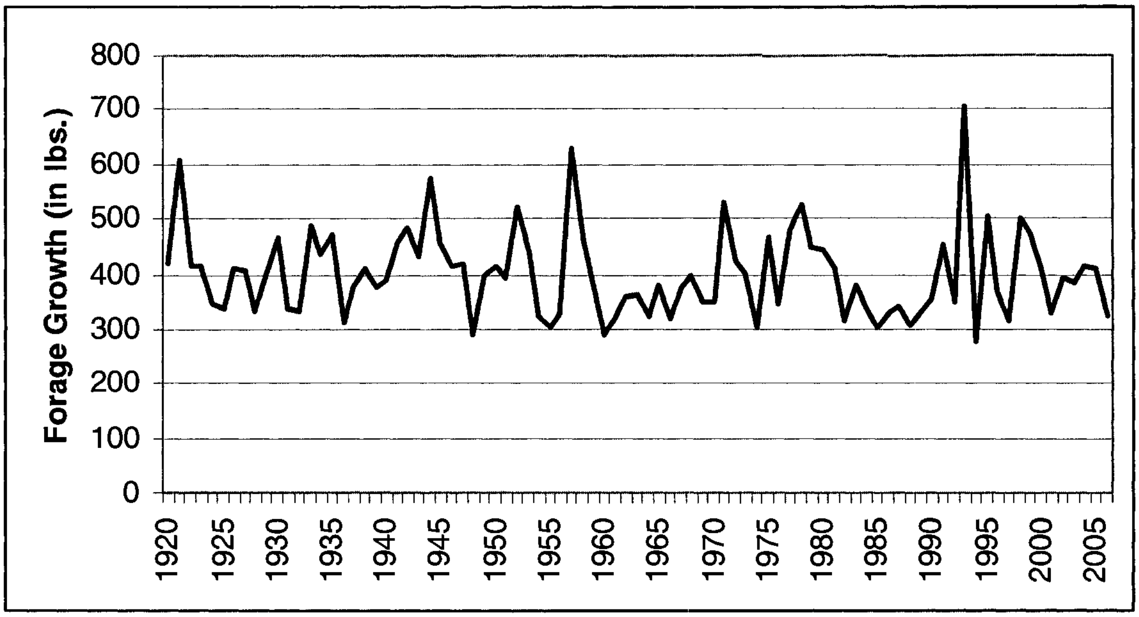

Figure 3.1 Predicted Forage Growth for Fremont County Wyoming

However, in order for the Bellman equation to optimize over the random weather, this stochastic variable needs to be discrete. The predicted forage production outcomes were discretized into 7 equal range outcomes over the relevant range, as reported in Table 3.1. As can be seen, as Smith (2007) state, more than half of the years see less than average precipitation, and less than average forage production, reinforcing the need to incorporate variable weather in any decision making process.

Table 3.1 Frequency of Forage Production (in lbs/acre)

Range	*Median Value*	*Frequency*	*Cumulative %*
[250,315)	282.43	9	10.34%
[315,380)	347.30	28	42.53%
[380,445)	412.18	28	74.71%
[445,510)	477.06	15	91.95%
[510,575)	541.93	3	95.40%
[575,640)	606.81	3	98.85%
[640,704)	671.69	1	100.00%

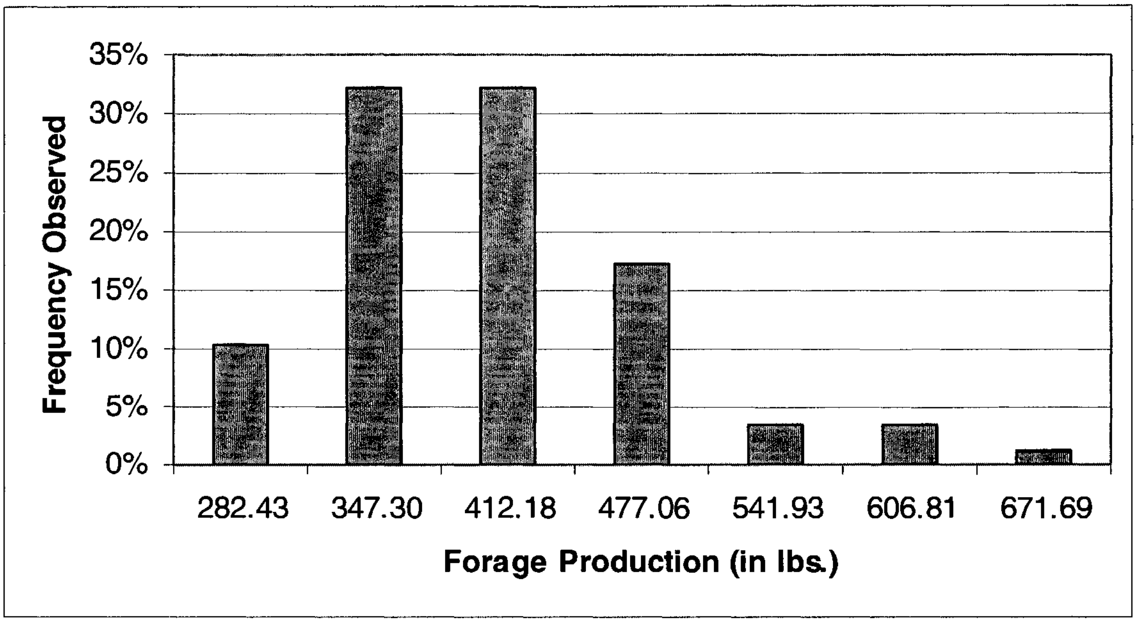

Figure 3.2 Frequency of Forage Production

In order to calibrate forage production for this area, the value of gamma from equation 3.12 was found that corresponded to forage production observed with each of the bins. This gamma was estimated assuming the observed forage production from equation 1.7 occurred when standing forage was at the state that maximized forage production, or ½ of carrying capacity. The equation was then solved for the γ that satisfied:

$$G(v,\omega) = \gamma_{\omega} V(1 - \frac{V}{V_m}) \qquad (3.12)$$

With V=156, V_m=312.

Table 3.2 shows these gammas and the associated probabilities of observation over this 86 year horizon:

Table 3.2 Calculated Growth Rates and Associated Probabilities of Occurrence

gamma	Probability
0.0302	10.34%
0.0371	32.18%
0.0440	32.18%
0.0510	17.24%
0.0579	3.45%

Table 3.3 Parameters used in Stochastic Dynamic Model

Parameter	Value (Alternative Vales Used in Sensitivity Analysis)
γ (Relative Growth Rate of Forage)	Stochastic
V_m (Maximum Standing Vegetation)	312 lb/acre
C_m (Maximum Daily Consumption)	15.6 lb/ animal/ day
V_r (Mandatory Forage Residual)	0 lb/ acre
V_k (Michaelis Constant)	62.4 lb/ acre (30, 100)
W_{init} (Initial Weight)	550 lb.
P_c (Price of Corn)	1.65 (1.10,2.25)
β (Discount Factor)	0.909091 (.833333,.952381,.990099)
CC (Carrying Cost per Animal)	$21.62/ animal
Days on Pasture	120
Death loss	0.02

Results

Model Simulation

The model was solved for using GAMS utilizing the Chebychev collocation method. This value function collocation method described above is able to determine the state value that maximizes the Bellman equation. This solution procedure determines the optimal decision rule for any given state value. However, as weather is stochastic, the forage level will not converge to a single value over time, so in order to determine optimal behavior a time path was simulated. Outcomes can be mapped over a select time path though using a simulation method. A hundred year horizon was run where the precipitation was determined using a random number generator. A simulated time path of outcomes based on the already determined decisions rules was over this predetermined time path of weather outcomes to map optimal decisions when faced with these realized weather impacts.

Given initial forage, the control is set according to the policy function in order to reach a desired expected state in the subsequent period. After the management decision is made based on the expected weather outcome the system is shocked with a realized weather impact which ultimately determines the ending state of forage. In the next period the decision is again made according to the policy function at the realized new forage state and the process is continued over the planning horizon.

The model was first simulated for a scenario of deterministic weather using the expected value of the stochastic weather in all periods. Then the model was solved using stochastic weather for the initial parameter values as state in Table 3.3. Of interest is the fact that when a producer accounts for expected forage production they will make decisions differently than that based on a deterministic view using average weather events. When producers account for variable weather, they will in fact stock at a more conservative rate. Producers will also realize lower returns to the land as a result, but again the long term cost of potentially driving down forage production outweighs the short term benefit of increased returns associated with higher stocking rates.

Although the ending state of the forage (average values reported under "Actual State" in Table 3.4) is often observed to be lower under the stochastic case, the desired ending state (Desired State in Table 3.4) is in fact higher than in the deterministic case. As producers are unaware of current year weather realizations when decisions are made, they can only make decision based on expected outcomes of weather. Producers try to leave more standing forage to ensure future productivity, yet often poor weather outcomes force the ending state below the desired state. Although, in years with favorable weather outcomes, producers leave more forage than desirable.

Table 3.4 Results from Dynamic Optimization

		Actual State	Desired State	Stocking Rate	Returns	Ending Weights
	Deterministic Outcome	173	173	0.2903	$10.44	681
Discount Rate	1%	162	176	0.2523	$8.29	675
	5%	161	176	0.2525	$8.28	675
	10%	160	175	0.2528	$8.26	675
	20%	158	173	0.2534	$8.23	674
Corn Price	low	159	174	0.2533	$8.68	674
	med	160	175	0.2528	$8.26	675
	high	161	176	0.2523	$7.81	675
Michaelis Constant	30	150	164	0.2128	$9.67	696
	64.2	160	175	0.2528	$8.26	675
	100	171	186	0.2878	$6.96	660
Cattle Prices	Decreased 20%	163	181	0.2501	$5.57	675
	Base	157	175	0.2525	$8.21	674
	Increased 20%	154	172	0.2537	$10.88	673

(Unless otherwise noted, Michaelis constant 64.2, Corn Price 1.65, discount rate .1)

The model was also solved for the other parameter values as listed in Table 3.3 for sensitivity analysis. As seen in chapter 2, the parameter with the largest impact on optimal decisions was the Michaelis Constant. Although corn and cattle price levels do impact returns to land, like chapter 2, the decisions made by a producer are not heavily influenced by price levels. An interesting difference from chapter 2 however, is that actual average ending state is less than observed in the deterministic model, although the desired state is in fact slightly higher. Another major difference from the output of chapter 2 is that returns to land are significantly lower for the model when weather deviation is observed. The outcome was also run deterministically for the average gamma (0.042837746) based on associated probabilities. When there is no deviation, returns to land are much higher every year, standing forage is consistently higher, and stocking rates are higher compared to any of the other scenarios. Again, the fact that weather is stochastic implies this will never reach a true "steady state," however frequency of outcomes can be determined by simulation.

Comparison across Discount Rates

The discount rate used had little or no impact on model results. As this is an infinite time horizon, producers will generally respond to variable weather in the same regard. Figure 3.3 shows that regardless of discount rate used, the percent of time that standing forage observed is each of the following categories is identical. Figure 3.4 shows that the discount rate did not heavily impact the amount of animals per acre. As seen in Figures 3.5 and 3.6 there is slight variation in the occurrence of different ending weights for each animal as well as yearly returns to land, but the overall pattern remains steady for different discount rates.

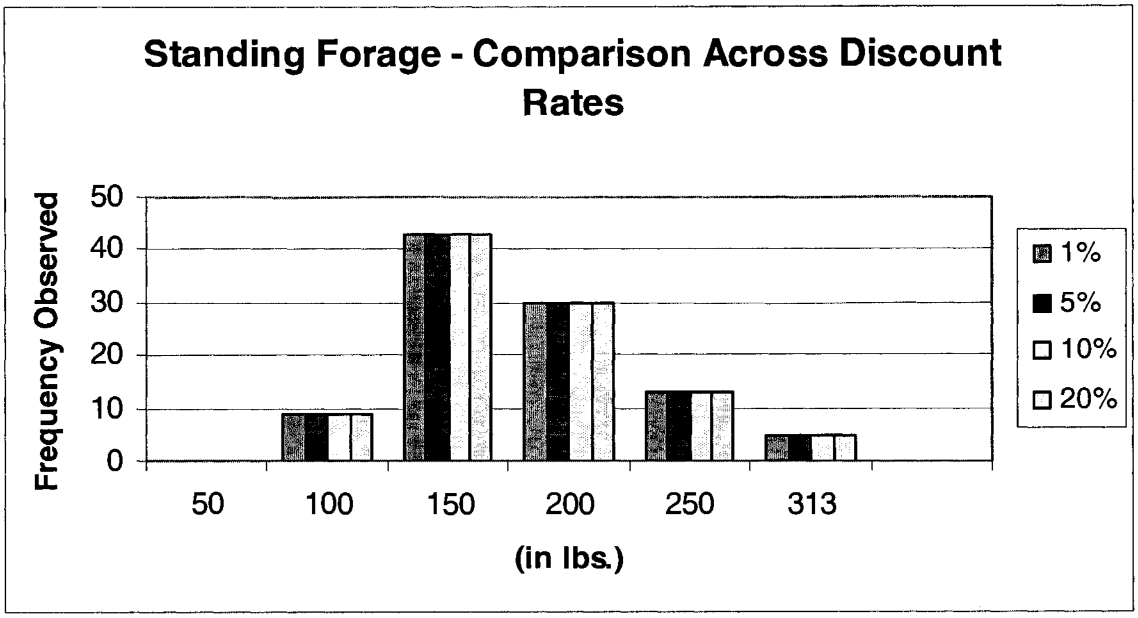

Figure 3.3 Comparisons of Standing Forage across Discount Rates

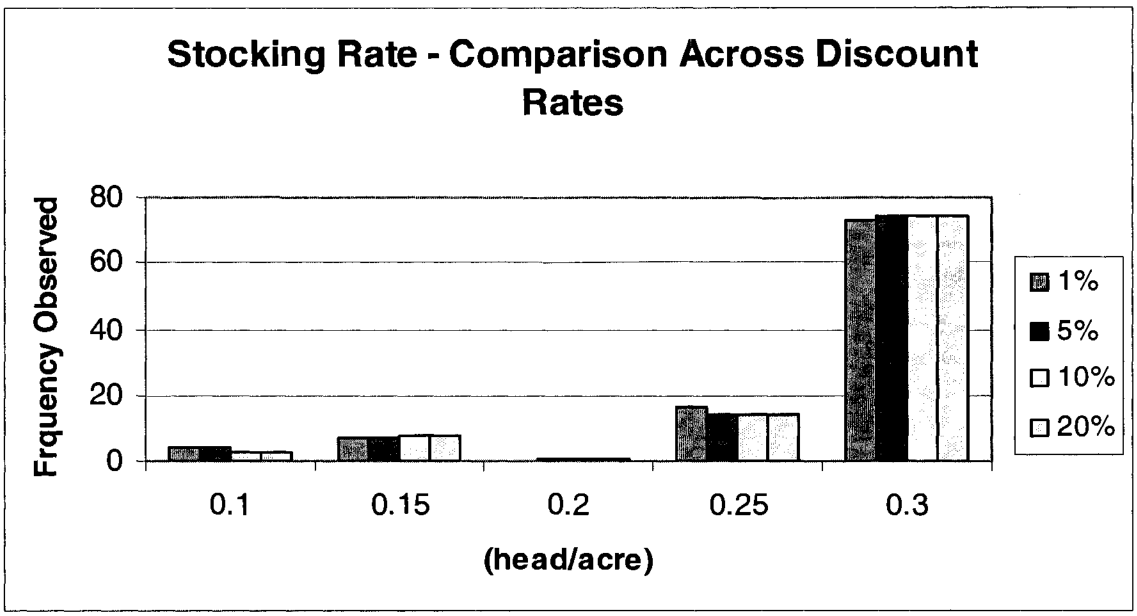

Figure 3.4 Comparisons of Stocking Rates across Discount Rates

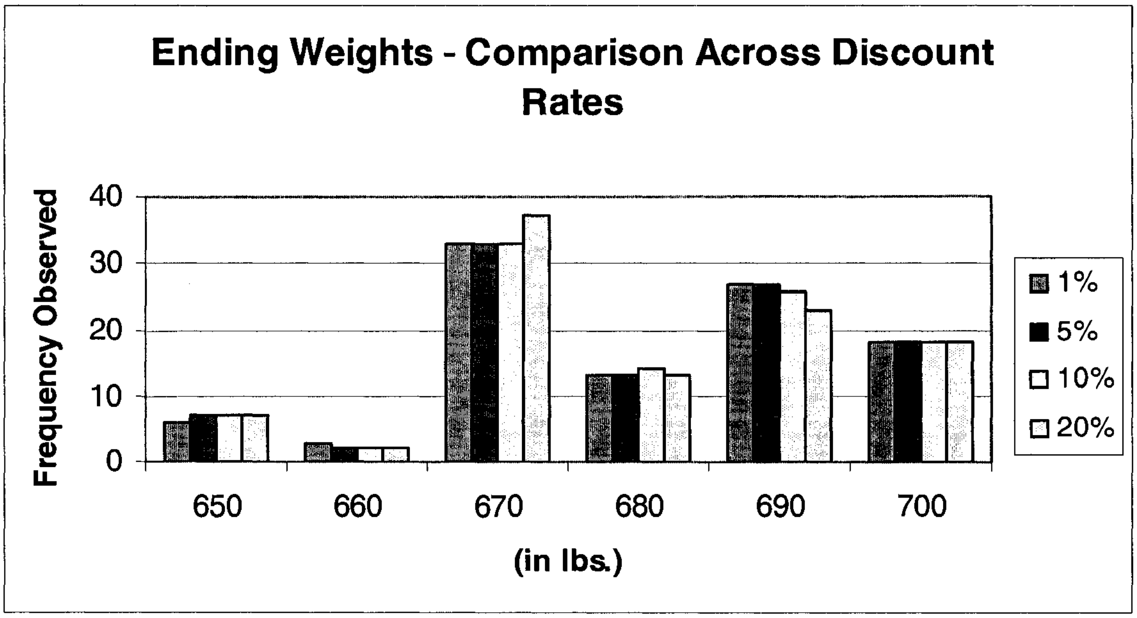

Figure 3.5 Comparisons of Ending Weights across Discount Rates

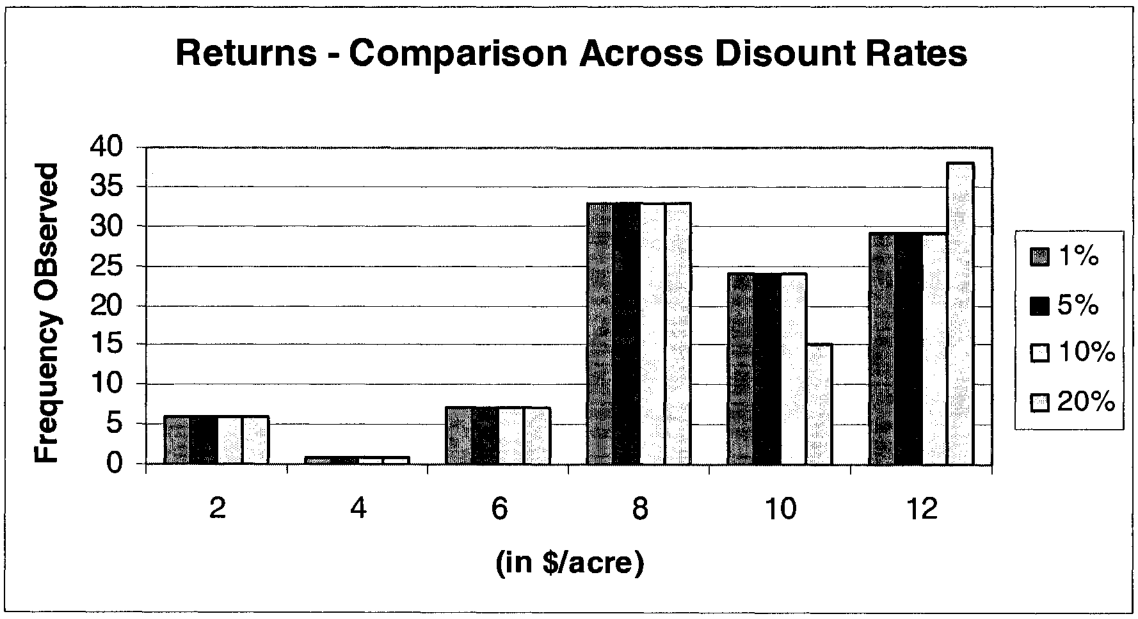

Figure 3.6 Comparisons of Yearly Returns across Discount Rates

Comparison across Corn Price Levels

As can be seen in Figures 3.7, 3.8, and 3.9, as in the comparison across discount rates, the decision variables across corn price levels are fairly consistent. The biggest difference is that lower corn prices see higher returns to land more frequently than either average or high corn prices, as can be seen in Figure 3.10. This implies that although high corn prices do negatively impact cattle producers, they should not alter decisions based solely on high corn prices.

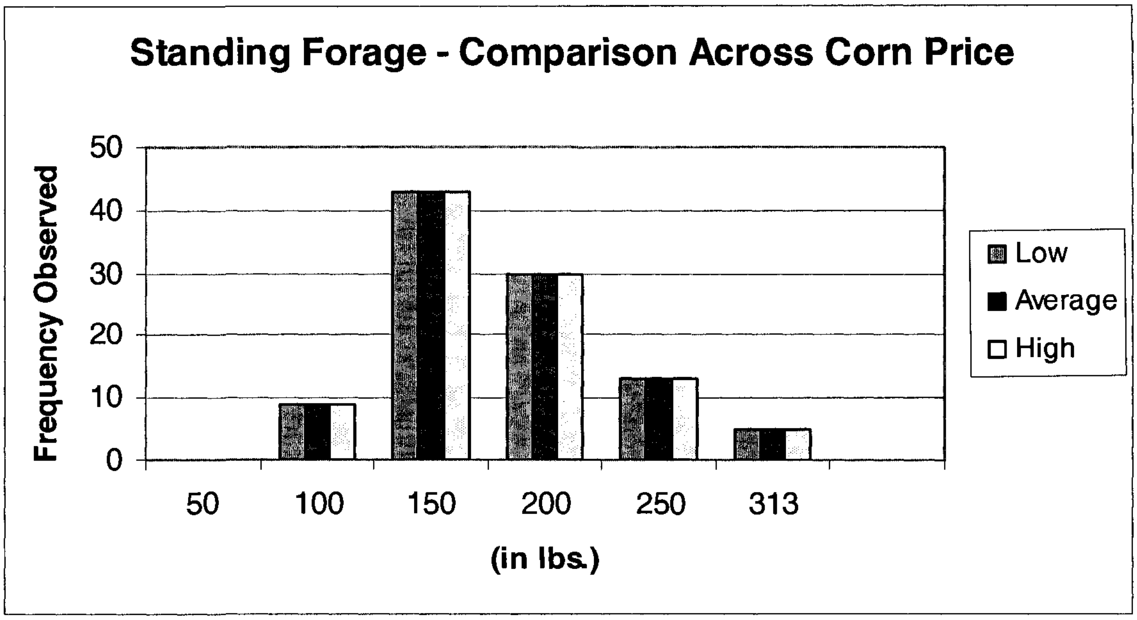

Figure 3.7 Comparisons of Standing Forage across Corn Price Levels

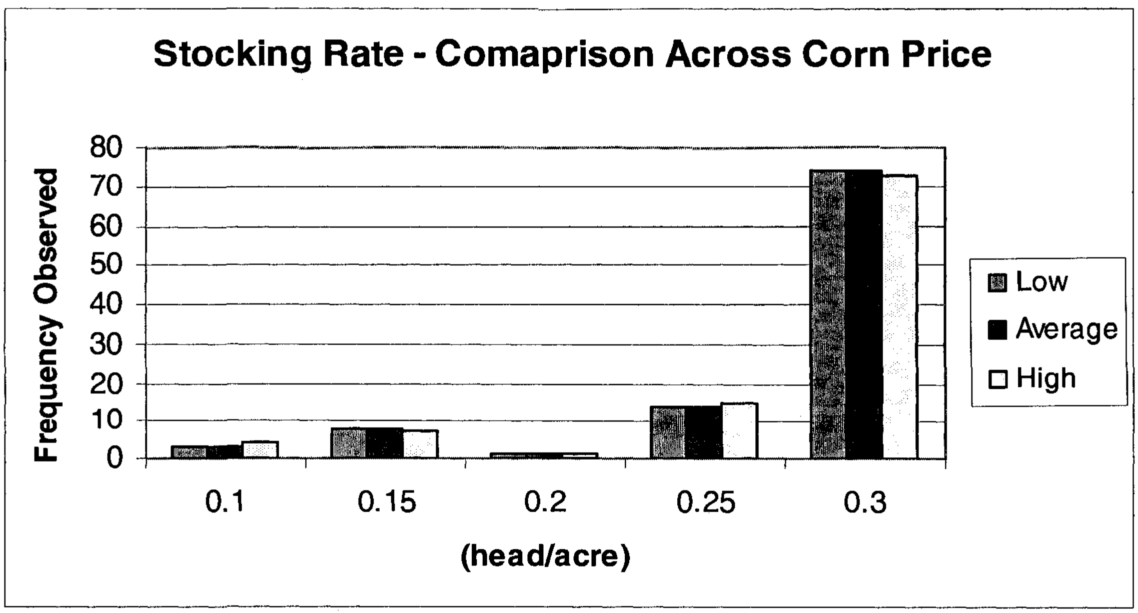

Figure 3.8 Comparisons of Stocking Rate across Corn Price Levels

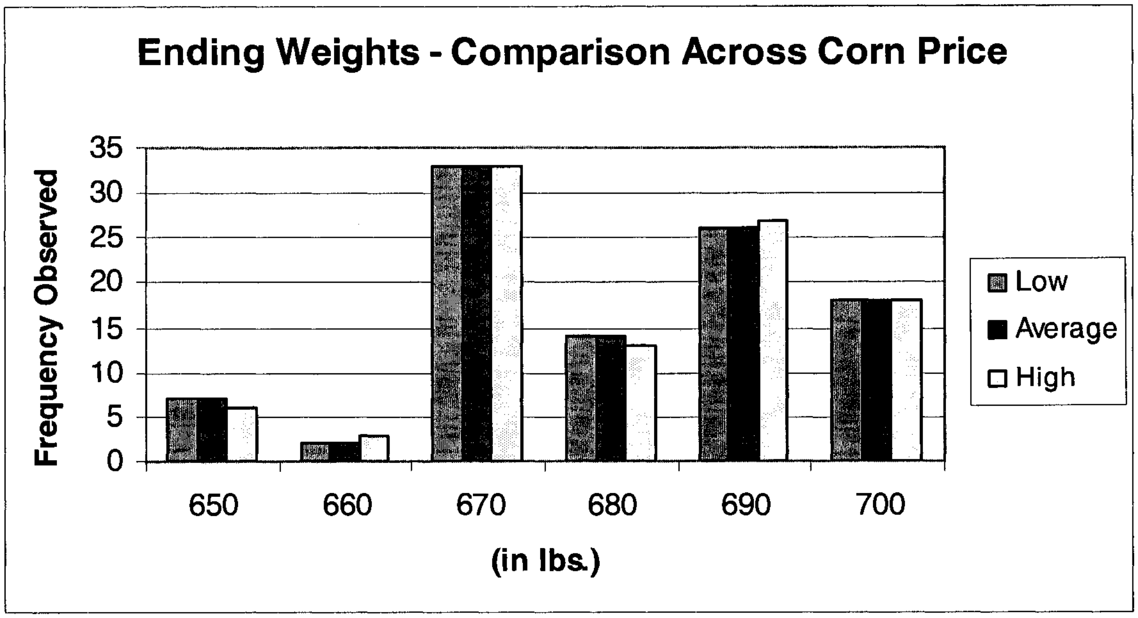

Figure 3.9 Comparisons of Ending Weights across Corn Price Levels

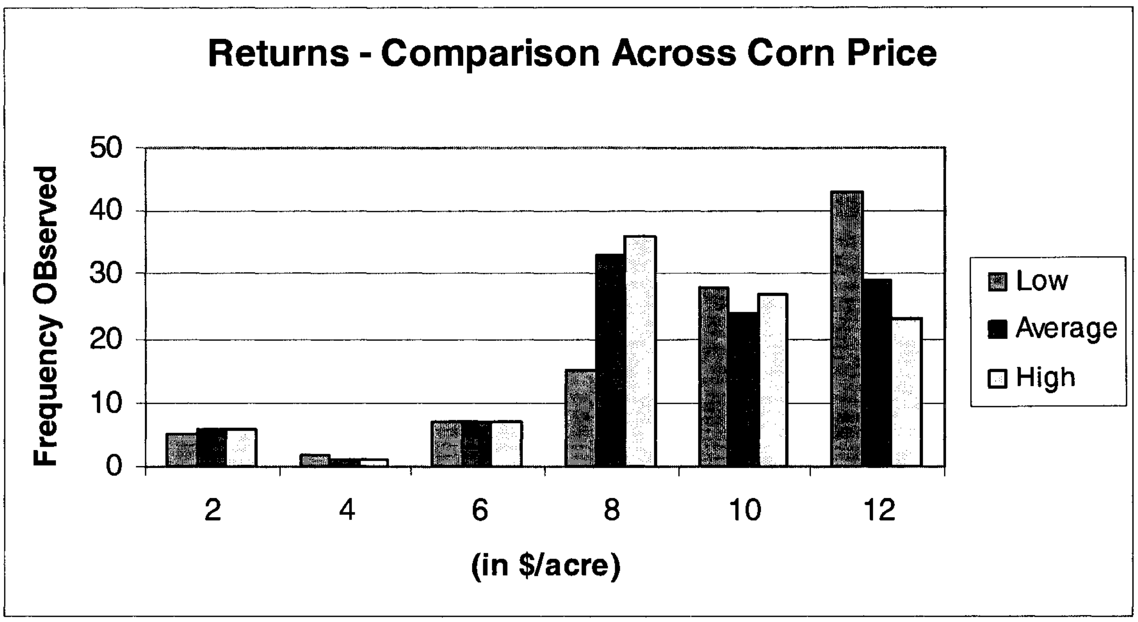

Figure 3.10 Comparisons of Yearly Returns across Corn Price Level

Comparison across Michaelis Constant

The Michaelis Constant proves to be the most important parameter when determining decisions. A lower Michaelis Constant, meaning animals are able to consume more with less standing forage. The outcome associated with cattle that are more efficient result in leaving less standing forage a seen in Figure 3.11, stocking at lower rates as seen in Figure 3.12, producing heavier cattle as seen in Figure 3.13, and having higher yearly returns as seen in Figure 3.14. In fact, the scenario with a Michaelis Constant of 30 is the only scenario in which standing forage was allowed to drop below fifty pounds, and also the only scenario in which yearly returns topped $12/acre. On the other hand, the scenario where the Michaelis Constant was set at 100, meaning more standing forage was required as the animals are less efficient, was the only scenario that saw a stocking rate over .3, and never encountered ending weights over 700 pounds. This scenario was also the most likely to see standing forage exceed 250 pounds/acre. This scenario realized the lowest returns per acre, never seeing yearly returns reach $12/acre. This result has two major implications. First, in order to properly determine optimal management decisions, this biological response parameter must be accurately determined. Secondly, if this is somehow in a producer's control, it would be very valuable to lower this parameter, whether through increasing forage quality or through better herd genetics.

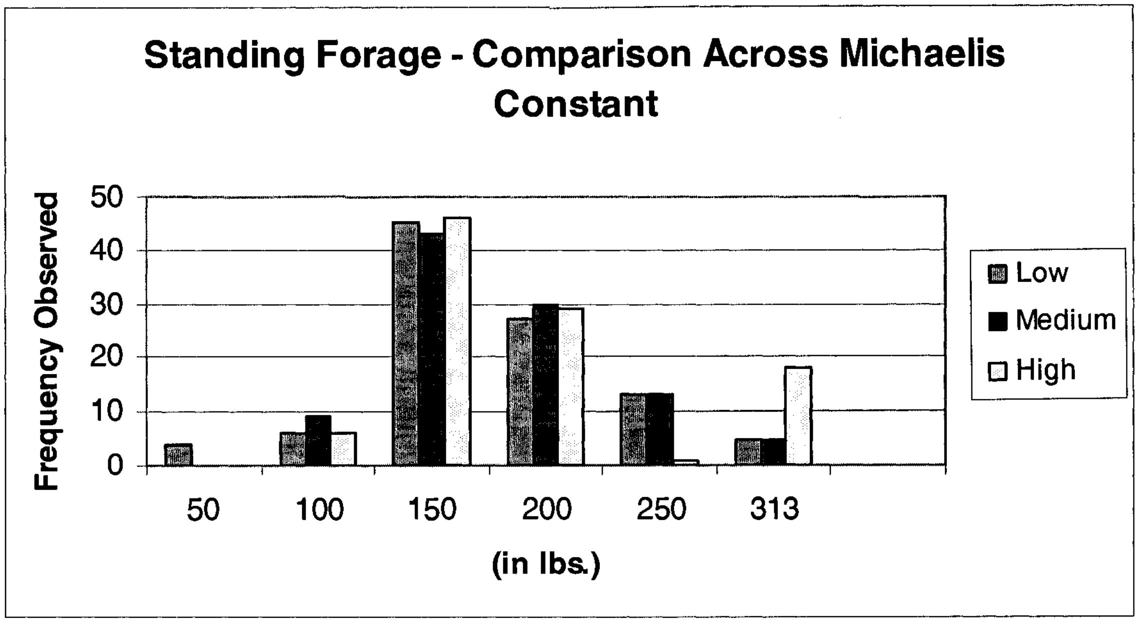

Figure 3.11 Comparisons of Standing Forage across Michaelis Constants

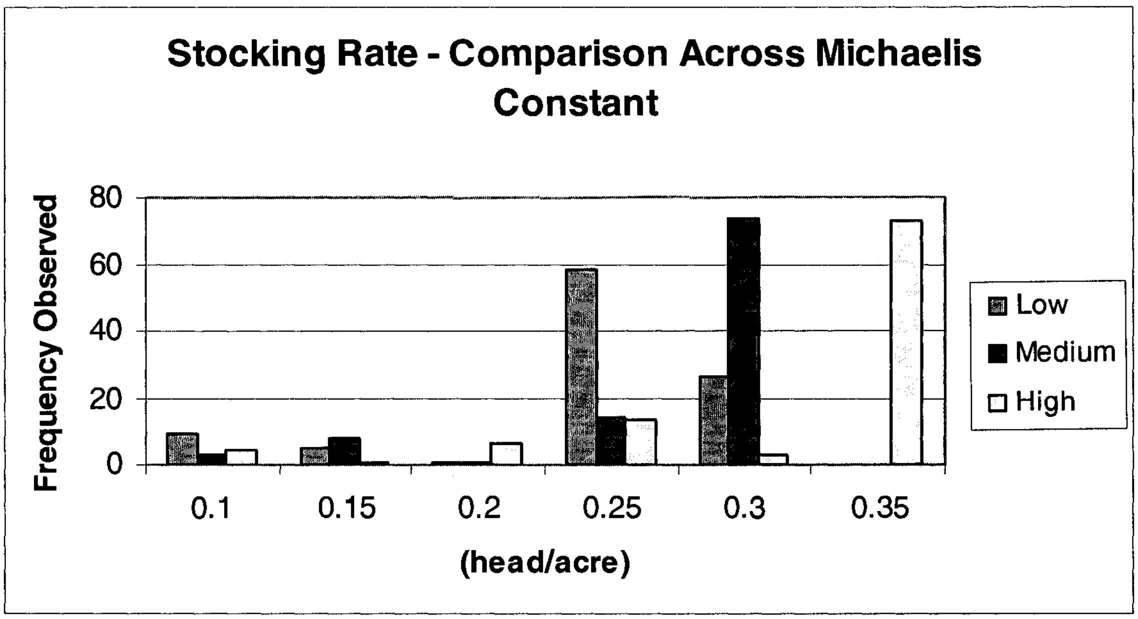

Figure 3.12 Comparisons of Stocking Rate across Michaelis Constants

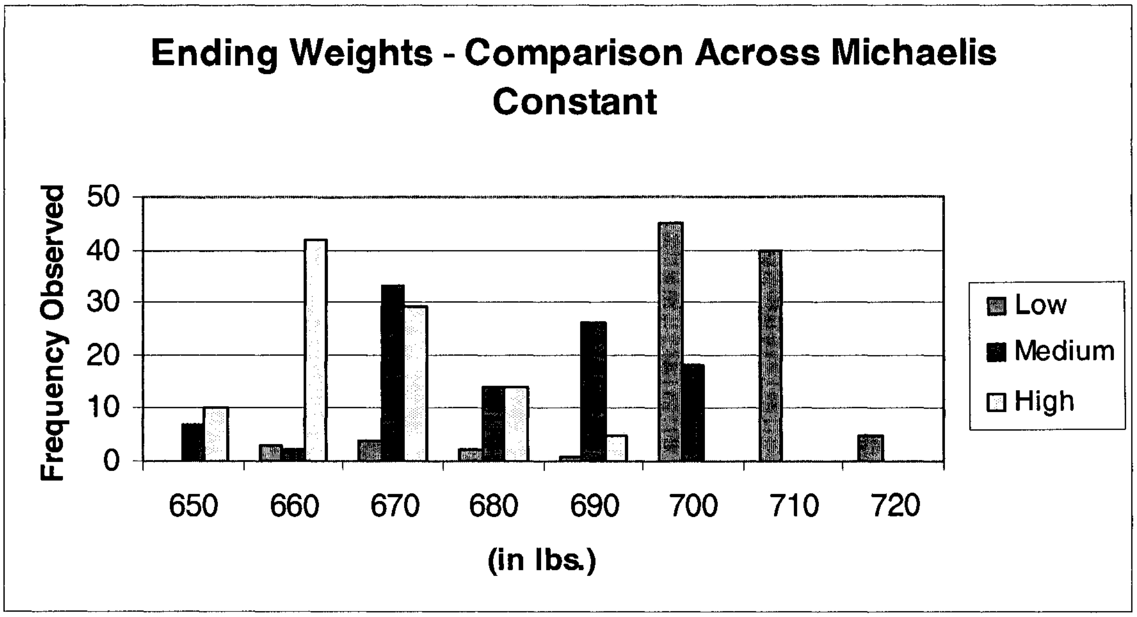

Figure 3.13 Comparisons of Ending Weights across Michaelis Constants

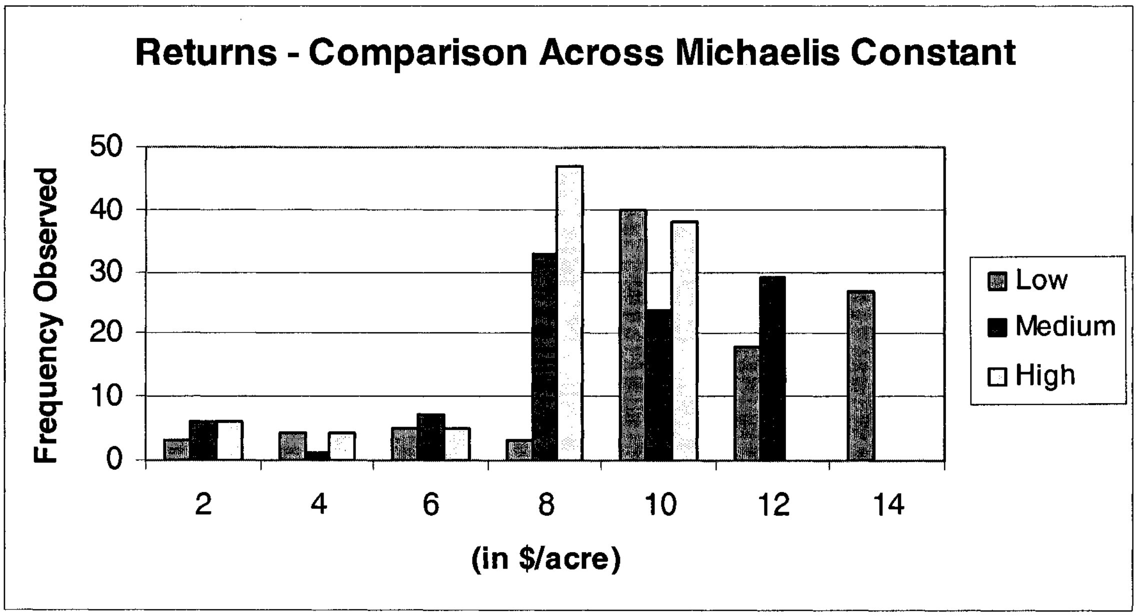

Figure 3.14 Comparisons of Yearly Returns across Michaelis Constants

Comparison across Cattle Price Levels

As in the comparison across corn price levels and discount rates, the decision variables across cattle price levels are fairly consistent. As seen in Figure 3.15, the stocking rate is fairly consistent, ranging from 0.2501 to 0.2537 head per acre from low to high cattle prices. As seen in Figures 3.15, 3.16 and 3.17, when output prices are high, producers will tend to stock slightly higher, leave slightly less forage, and end with slightly heavier cattle. However, the difference in decisions is not near as great as that observed across varying Michaelis Constants. The biggest impact varying cattle price levels have are on returns per acre, a seen in Figure 3.18. Producers will feel the impact of varying cattle price levels greatly, but should not change their decisions much due to the changes.

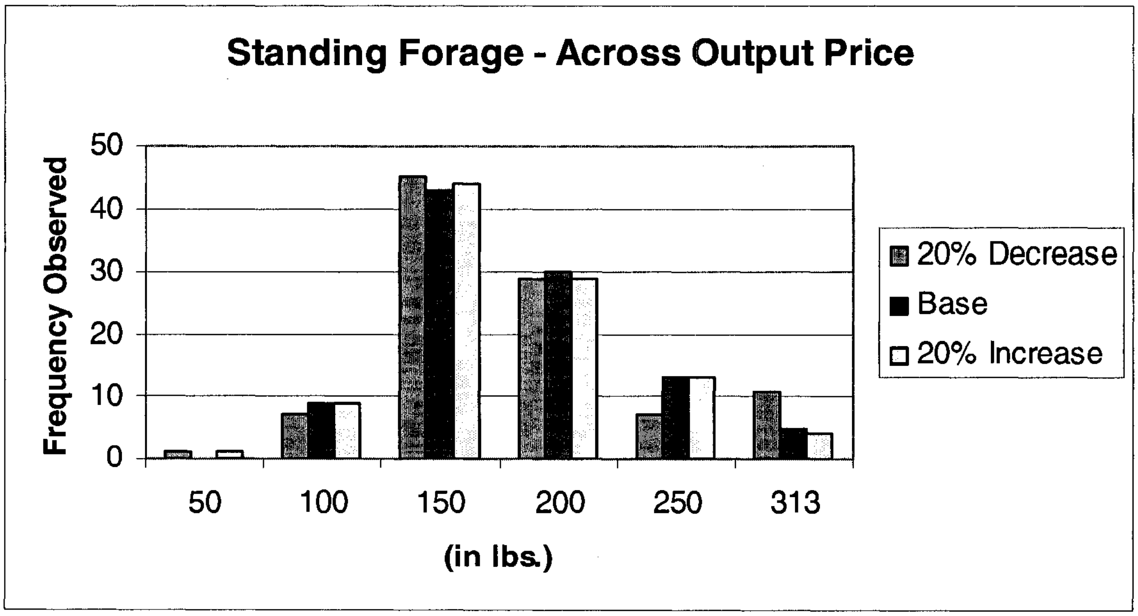

Figure 3.15 Comparisons of Standing Forage across Cattle Price Levels

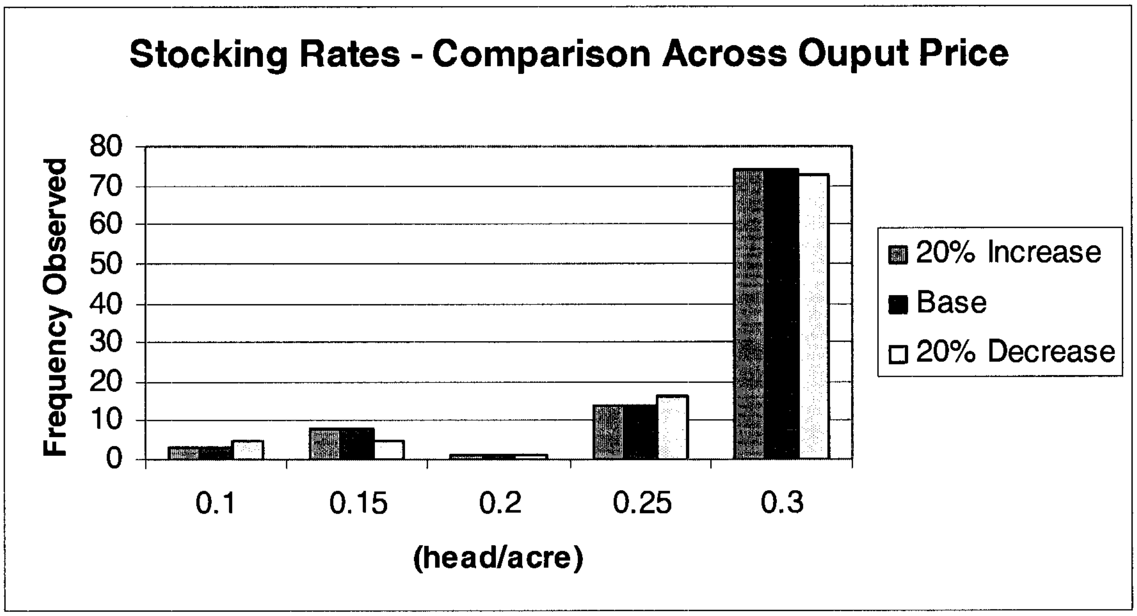

Figure 3.16 Comparisons of Stocking Rate across Cattle Price Levels

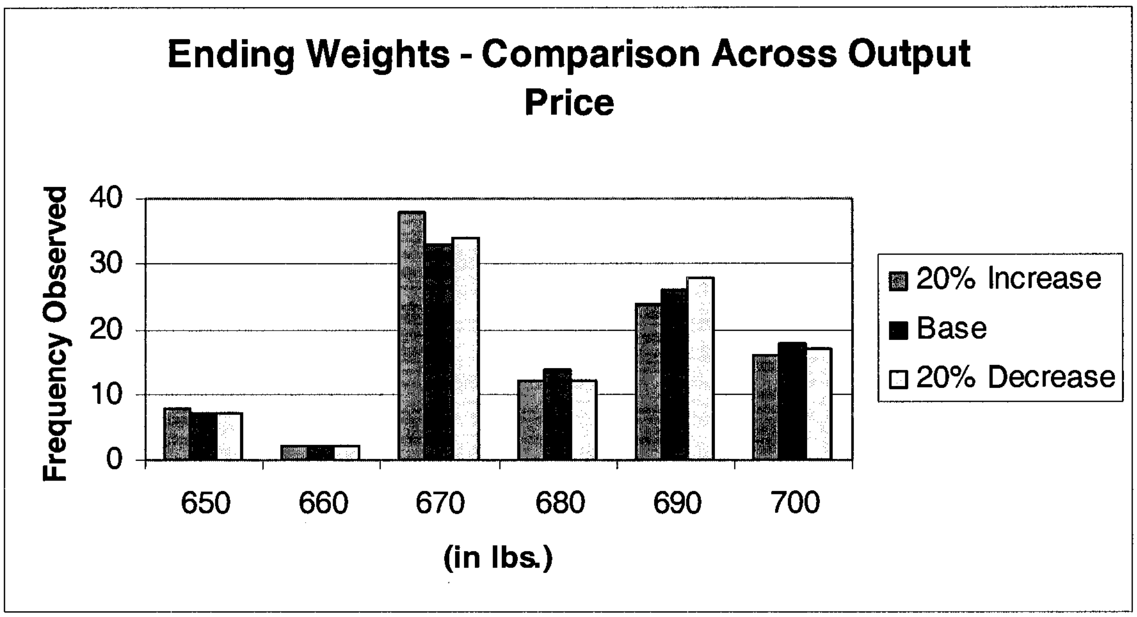

Figure 3.17 Comparisons of Ending Weights across Cattle Price Levels

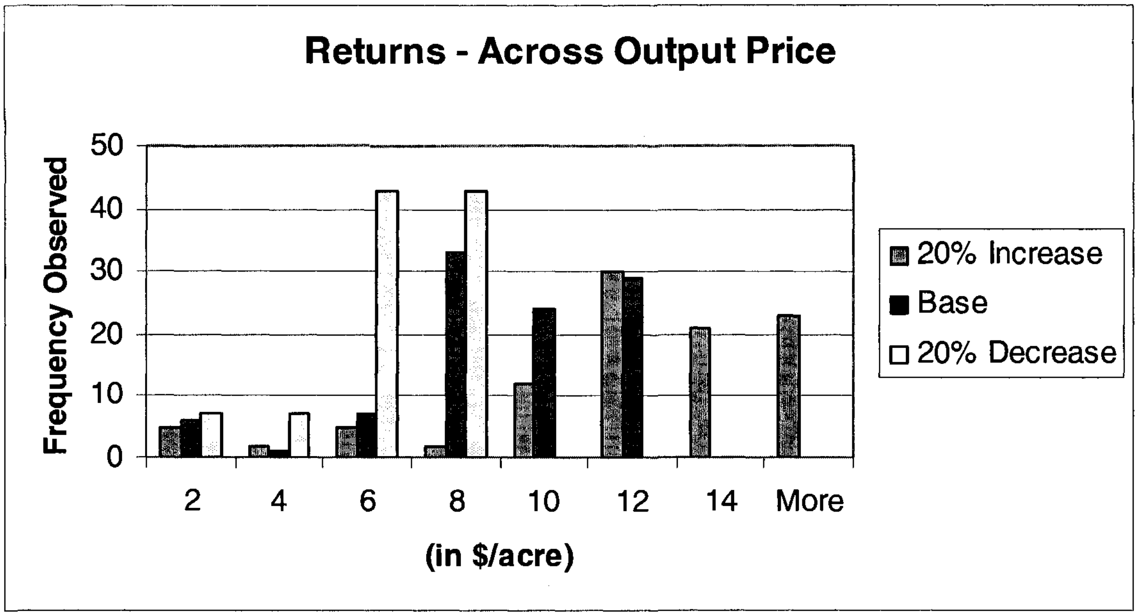

Figure 3.18 Comparisons of Yearly Returns across Cattle Price Levels

Optimal Stocking Rate

The previous outcomes were based on a producer who only has knowledge of long-term expected outcomes of random weather events. But what should a producer do if they are able to observe, or accurately predict, precipitation before the stocking rate is set? The value function is able to help determine the optimal policy function as well. Figure 3.19 shows what stocking rate should be according to observed standing forage and observed (or predicted) current year growing season weather. The graph is based on the base scenario of a 10% discount rate, average corn prices, and a Michaelis Constant of 64.2.

As expected, as precipitation increases, optimal stocking rate also increases. As observed standing forage reaches the desired standing forage level from either direction, optimal stocking rates also increase. What impact would knowledge of weather have for a producer? A simulation was run using desired standing forage levels given the

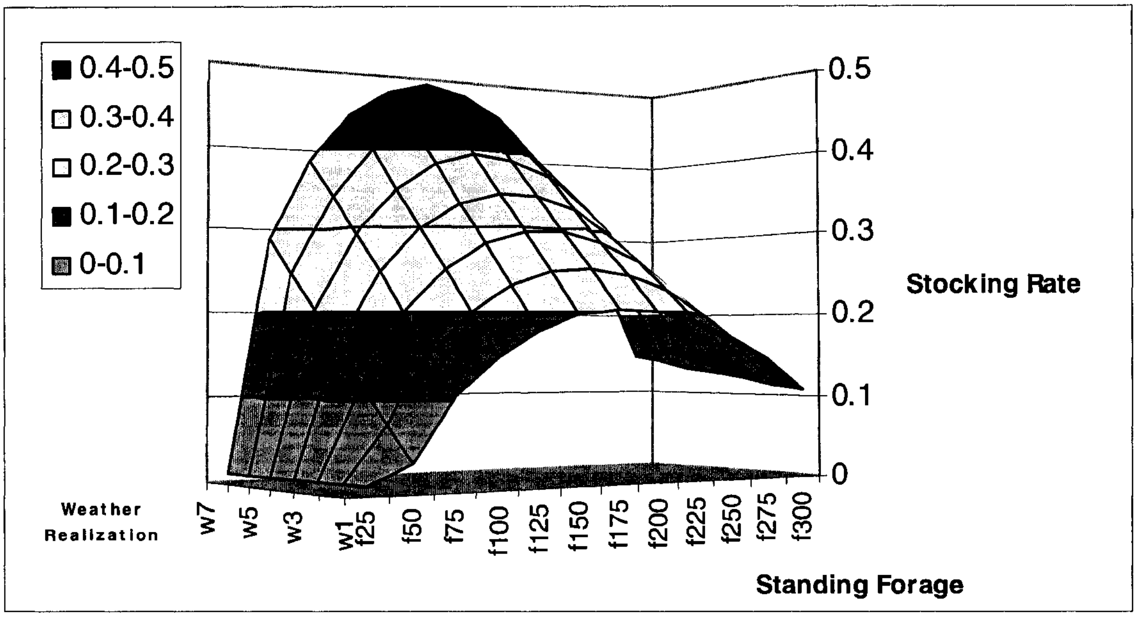

Figure 3.19 Optimal Stocking Rates across Standing Forage and Weather Realizations

stochastic dynamic programming model. The simulation was run under a scenario where a producer must make decisions without knowledge of current year growing season precipitation, and a scenario where the producer was given knowledge of current season weather before stocking rates were determined. As seen in Figure 3.20, if the producer was aware of current weather realizations, the producer sets stocking rate that tracked seasonal weather, as opposed to the scenario where decisions must be made prior to weather realization, where often stocking rates followed actual weather patterns with a one year lag. The producer who is aware of weather realizations at the time of stocking rate decisions is able to keep standing forage at the desired level, while a producer who makes decisions based solely on expectations of weather outcomes allows standing forage to fluctuate around desired levels as seen in Figure 3.21. This shows that a producer who is unaware of weather realizations allows standing forage to fluctuate, forcing them to alter future stocking rates to account for the state of the forage to be in other than the desired state. The biggest impact this will have on a producer is that yearly returns will be affected by these decisions. Figure 3.22 shows the differences in yearly returns for producers with and without knowledge of weather realizations. Table 3.5 shows that on average, a producer with knowledge of weather realizations can stock at a higher rate, leaves more standing forage, and has better average yearly returns, as well as receiving 21% total returns over a 100 year horizon.

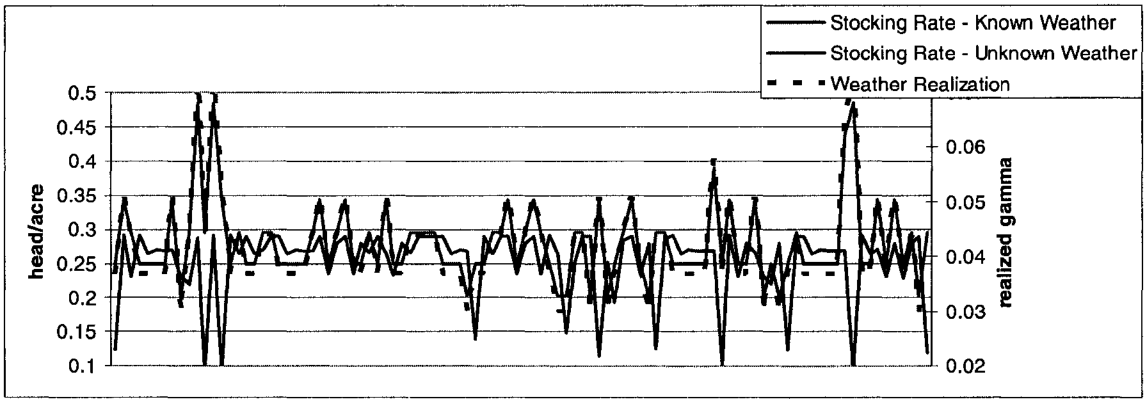

Figure 3.20 Stocking Rates over Time Compared for Known and Unknown Weather against Weather Realizations

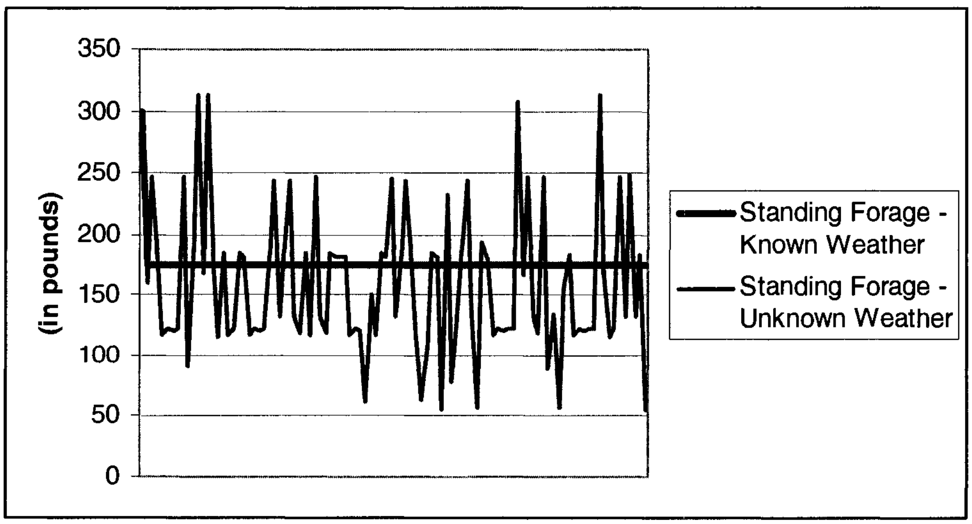

Figure 3.21 Standing Forage over Time Compared for Known and Unknown Weather Realizations

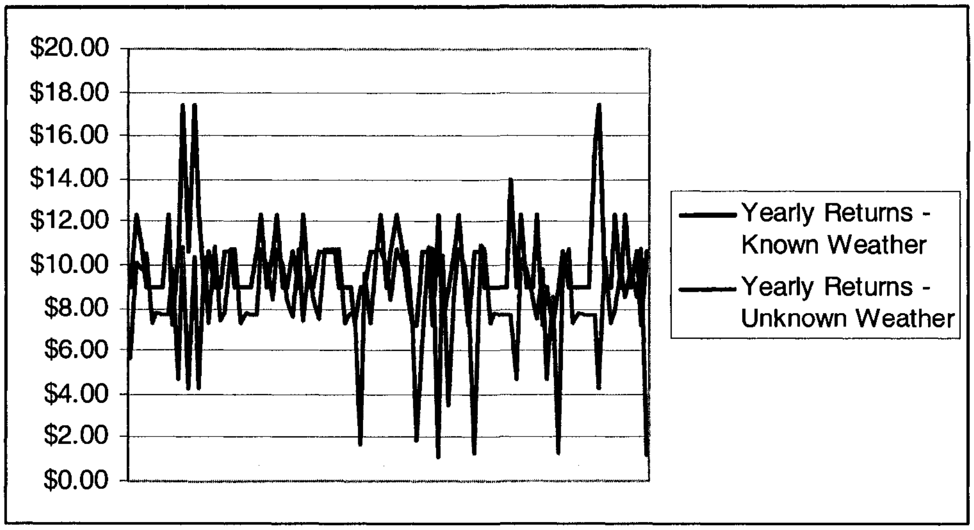

Figure 3.22 Yearly Returns over Time Compared for Known and Unknown Weather Realizations

Table 3.5 Comparison of Outcomes with Known and Unknown Weather Relizations

	Deterministic Scenario	Stochastic Scenario
Average Stocking Rate	0.2800	0.2528
Average Standing Forage	175	160
Average Yearly Returns	$10.05	$8.26
Total Yearly Returns over 100 Year Horizon	$1,005	$826

Without prior knowledge of weather, a producer can only make decisions based on expected outcomes, which leaves ending forage other than the desired state in most cases. Therefore, if a dry year occurred, the forage base was depleted due to higher than desirable stocking rates and the producer must respond by stocking at a lower rate the following year in order to rebuild the forage base. If a producer is aware of weather before the decision is made, they are able to stock the range to ensure the desired ending forage value is reached. In this scenario, over 100 year time span, the knowledge of weather increased total returns to land by $178, or an average of $1.78 per year. This follows Vetter's (2005) conclusion that basing herd numbers conservatively in the idea that maintaining a constant herd size is important is inappropriate, and that "(d)rought risks are minimized not by maintaining conservative stocking rates, but rather by allowing livestock numbers to increase in wet years (pg. 330)." Optimal management includes increasing stocking rates in response to favorable conditions and reducing stocking rates in poor years. Management decisions must account for current conditions and expected outcomes to ensure future productivity of rangeland.

Conclusion

A dynamic model maximizing land values over stochastic weather events was evaluated in terms of optimal stocking decisions. As in the previous chapter, the parameter with the largest impact on decisions was the Michaelis Constant. Producers who can procure either animals that are efficient grazers or better quality forage can increase returns to land. The individual's discount rate will not alter management decisions, nor will the price level of cattle or corn, although the latter two will impact financial returns realized.

The dynamic model optimized over expected outcomes, as compared to a model that uses average forage production, ends with usually less forage, a more conservative stocking rate, and lower financial returns. However, if a producer has knowledge of current year weather realizations, they can make more precise stocking rate determination to take advantage of increased forage in wet years without depleting the forage base in dry years. Even with this knowledge however, a producer will still stock at a lower rate on average than a producer without variable weather impacts (0.27 acre/head as compared to 0.29 acre/head). Regardless of whether a producer has knowledge of current weather or not, they must constantly monitor the state of the range and make corresponding changes to stocking rates if they are to maximize financial returns.

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Appendix 1

Supplemental Tables and Figures Supporting Results of Chapter 1

Table A1.1 Market and Weather Elasticities of Production and Financial Returns across All Starts, Including FLOOR Scenario

	Base			*Feed*			*Floor*		
	Market	Weather	R Squared	Market	Weather	R Squared	Market	Weather	R Squared
Yearly Returns	-4.018	1.009	0.632	-4.453	1.690	0.231	-4.451	1.691	0.231
	-51.213			-47.019			46.980		
AUY	0.254	0.326	0.286	0.235	-0.405	0.190	0.234	-0.405	0.191
	1.225			2.882			-2.903		
Cull	0.202	0.210	0.175	0.198	0.271	0.117	0.197	0.271	0.117
	0.133			1.244			-1.253		
Acres Grazed	0.300	-0.674	0.606	0.224	-0.559	0.595	0.224	-0.559	0.597
	-6.357			-5.701			-5.701		
Total Feed	0.233	0.326	0.261	0.339	-0.608	0.076	0.336	-0.610	0.076
	1.598			-4.578			-4.651		
Feed/AUY	-0.022	-0.002	0.058	0.139	-1.060	0.430	0.137	-1.061	0.430
	0.349			-15.672			-15.738		
Acres/AUY	0.069	-0.997	0.942	0.004*	-0.917	0.759	-0.005	-0.908	0.859
	-15.791			-15.543			15.372		
Cull/AUY	-0.049	-0.102	0.142	-0.037	-0.107	0.148	-0.037	-0.108	0.149
	-0.904			-1.195			1.207		

Values represent elasticities estimated using OLS. Values under coefficients are comparison t stats against the null hypothesis that the coefficients are equal. Also reported are associated R Square Values.

Table A1.2 Market and Weather Elasticities of Production and Financial Returns across Average Starts, Including FLOOR Scenario

	Base Drought			Supplemental Feed			Herd Floor		
	Market	Weather	R Squared	Market	Weather	R Squared	Market	Weather	R Squared
Yearly Returns	-4.001	1.225	0.759	-4.205	2.416	0.395	-4.203	2.417	0.395
	-47.166			-30.402			30.361		
AUY	0.268	0.259	0.263	0.250	0.304	0.198	0.249	0.305	0.198
	-0.149			0.920			-0.944		
Cull	0.213	0.157	0.148	0.210	0.184	0.093	0.209	0.184	0.092
	0.056			-0.442			0.430		
Acres Grazed	0.316	-0.735	0.678	0.237	-0.590	0.642	0.237	-0.589	0.643
	-7.133			-5.982			-5.982		
Total Feed	0.244	0.257	0.233	0.358	-0.819	0.153	0.356	-0.820	0.153
	0.208			-7.827			-7.889		
Feed/AUY	-0.024	-0.003	0.066	0.140	-1.106	0.479	0.138	-1.107	0.479
	0.354			-16.400			-16.457		
Acres/AUY	0.073	-1.010	0.945	-0.003	-0.901	0.857	-0.002	-0.900	0.857
	-15.917			-15.257			15.253		
Cull/AUY	-0.051	-0.096	0.122	-0.039	-0.106	0.132	-0.038	-0.106	0.134
	-0.761			-1.139			1.152		

Values represent elasticities estimated using OLS. Values under coefficients are comparison t stats against the null hypothesis that the coefficients are equal. Also reported are associated R Square Values.

Table A1.3 Market and Weather Elasticities of Production and Financial Returns across Dry Starts, Including FLOOR Scenario

	Base Drought				Supplemental Feed				Herd Floor			
	Market	t stat	Weather	R Squared	Market	t stat	Weather	R Squared	Market	t stat	Weather	R Squared
Yearly Returns	4.096	-58.638	0.646	0.481	4.603	-63.474	0.868	0.131	4.603	63.446	0.870	0.131
AUY	0.254	2.698	0.413	0.344	0.230	5.098	0.530	0.235	0.229	-5.113	0.530	0.235
Cull	0.203	0.058	0.266	0.236	0.193	2.701	-0.352	0.176	0.192	-2.705	0.352	0.176
Acres Grazed	0.300	-5.256	-0.610	0.537	0.215	-4.967	-0.507	0.532	0.215	-4.966	-0.507	0.537
Total Feed	0.232	3.105	0.415	0.319	0.344	-2.779	-0.508	0.053	0.341	-2.872	-0.610	0.054
Feed/AUY	-0.023	0.353	-0.002	0.059	0.146	-16.107	1.094	0.414	0.143	-16.192	1.096	0.415
Acres/AUY	0.069	-15.540	-0.988	0.935	-0.010	-14.861	-0.885	0.863	-0.009	14.867	0.884	0.864
Cull/AUY	-0.049	-1.166	-0.117	0.184	-0.037	-1.411	-0.120	0.184	-0.037	1.422	-0.121	0.186

Values represent elasticities estimated using OLS. Values under coefficients are comparison t stats against the null hypothesis that the coefficients are equal. Also reported are associated R Square Values.

Table A1.4 Market and Weather Elasticities of Production and Financial Returns across Wet Starts, Including FLOOR Scenario

	Base Drought				Supplemental Feed				Herd Floor			
	Market	t stat	Weather	R Squared	Market	t stat	Weather	R Squared	Market	t stat	Weather	R Squared
Yearly Returns	3.960	-48.157	1.140	0.725	4.574	-49.249	1.691	0.281	4.672	49.204	1.691	0.281
AUY	0.241	1.115	0.306	0.261	0.226	2.607	0.379	0.158	0.225	-2.630	0.379	0.158
Cull	0.191	0.062	0.207	0.153	0.191	1.461	0.277	0.097	0.190	-1.473	0.277	0.097
Acres Grazed	0.285	-6.693	-0.677	0.608	0.220	-6.163	-0.581	0.612	0.219	-6.163	-0.580	0.612
Total Feed	0.222	1.472	0.308	0.238	0.316	-3.201	-0.503	0.050	0.313	-3.263	-0.504	0.051
Feed/AUY	-0.020	0.342	0.000	0.050	0.130	-14.494	-0.979	0.401	0.128	-14.552	-0.980	0.401
Acres/AUY	0.066	-15.921	-0.998	0.945	0.023	-16.895	-0.966	0.855	-0.002	16.009	-0.940	0.857
Cull/AUY	-0.047	-0.786	-0.093	0.124	-0.035	-1.035	-0.096	0.131	-0.035	1.048	-0.096	0.132

Values represent elasticities estimated using OLS. Values under coefficients are comparison t stats against the null hypothesis that the coefficients are equal. Also reported are associated R Square Values.

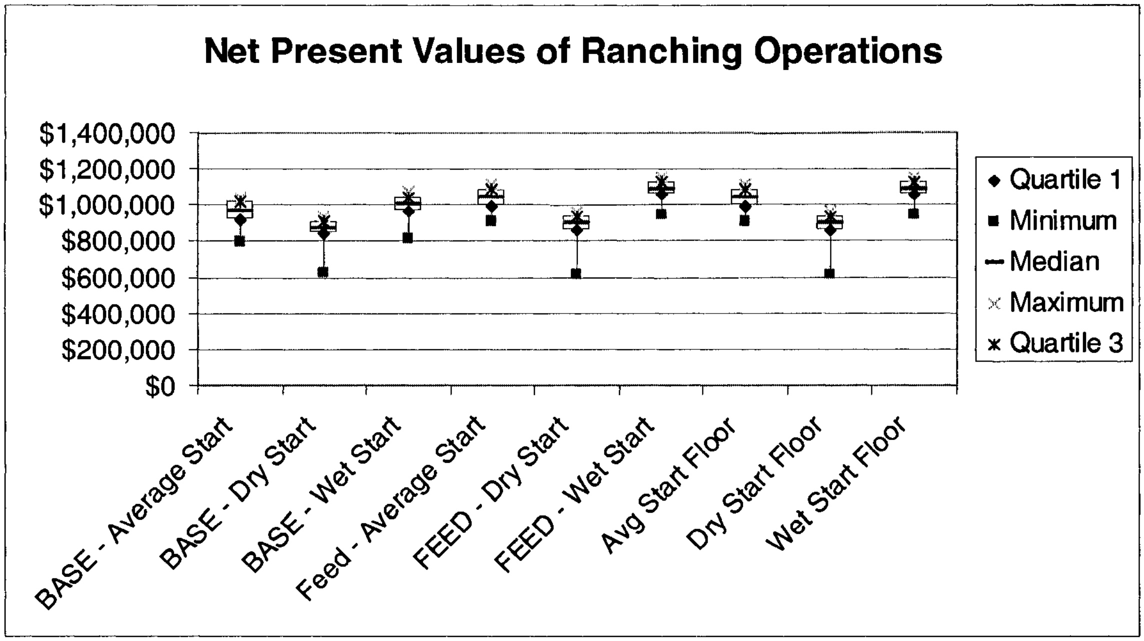

Figure A1.1 Quartile Graphs for Net Present Values across Scenarios, Including FLOOR

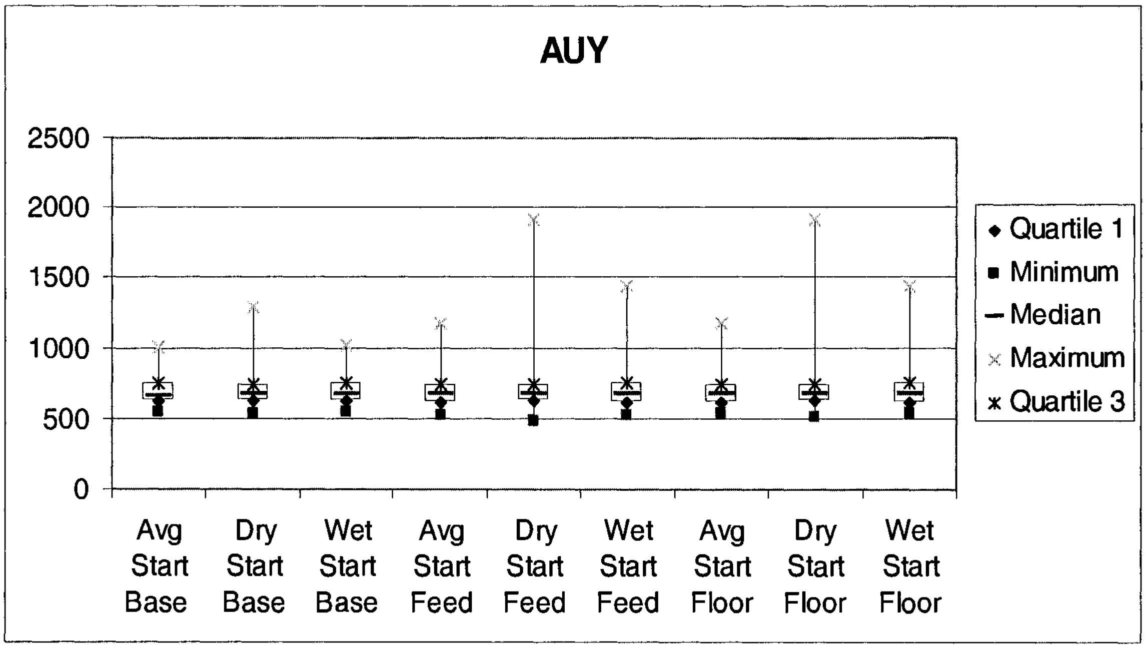

Figure A1.2 Quartile Graphs for Herd Size across Scenarios, Including FLOOR

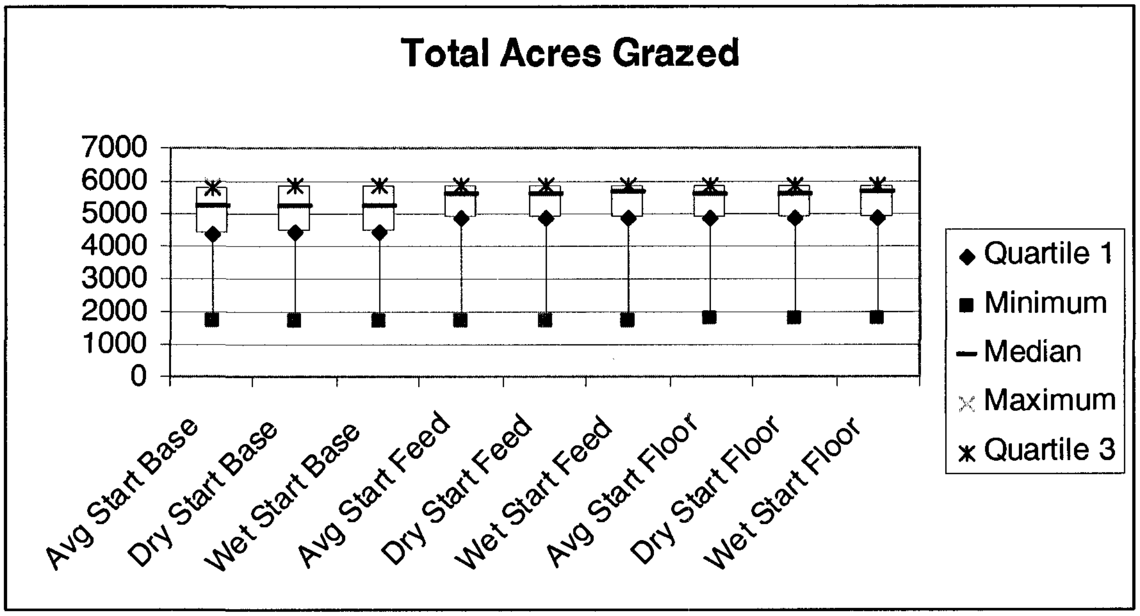

Figure A1.3 Quartile Graphs for Total Acres Grazed across Scenarios, Including FLOOR

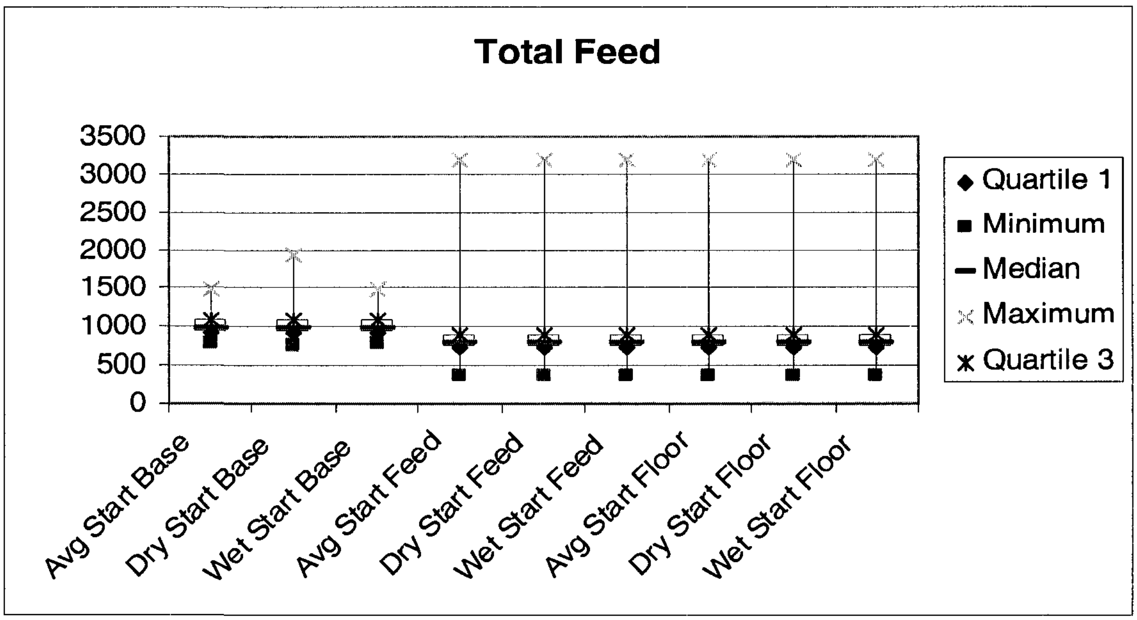

Figure A1.4 Quartile Graphs for Total Feed Allowed across Scenarios, Including FLOOR

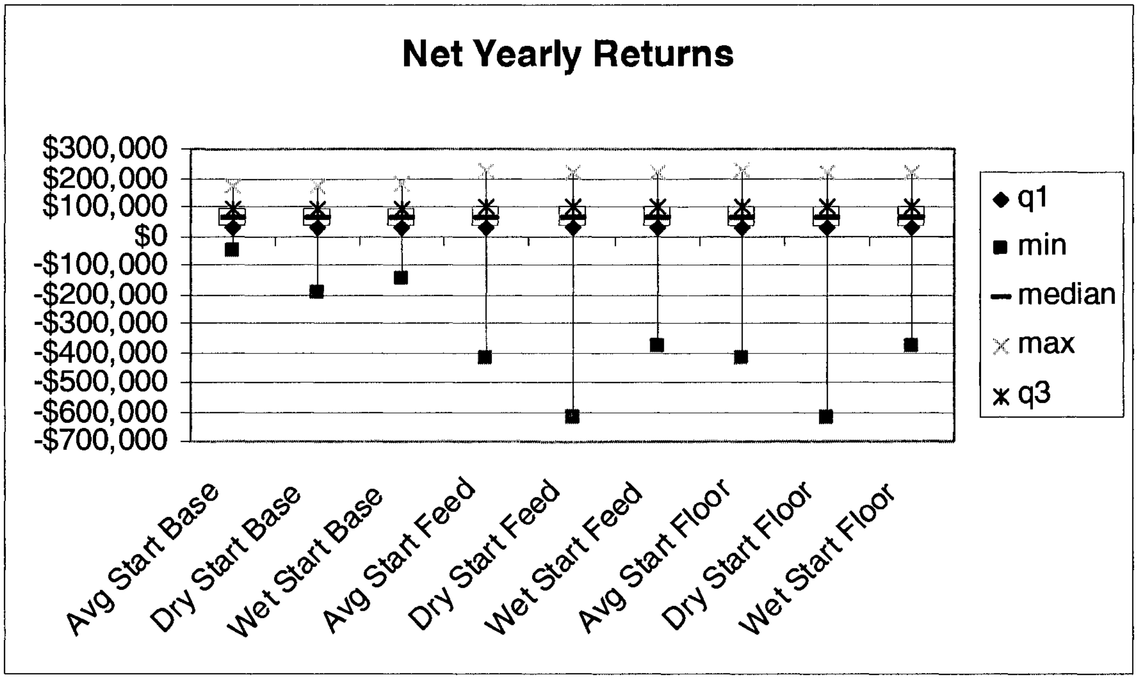

Figure A1.5 Quartile Graphs for Net Yearly Returns across Scenarios, Including FLOOR

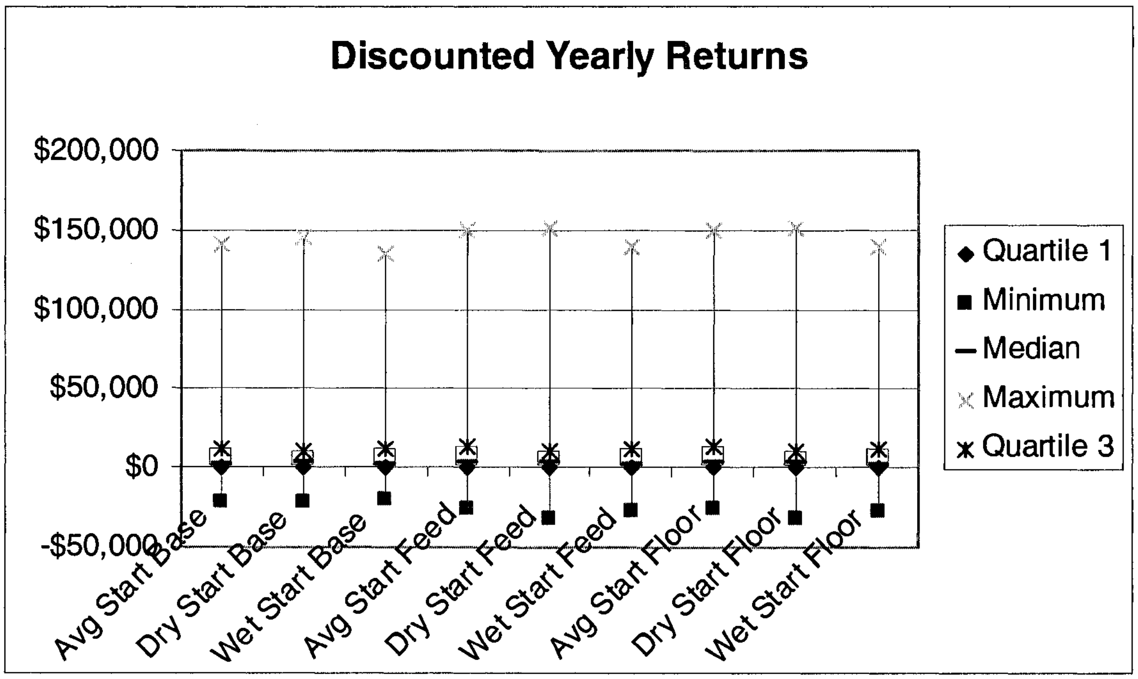

Figure A1.6 Quartile Graphs for Dicounted Yearly Returns across Scenarios, Including FLOOR

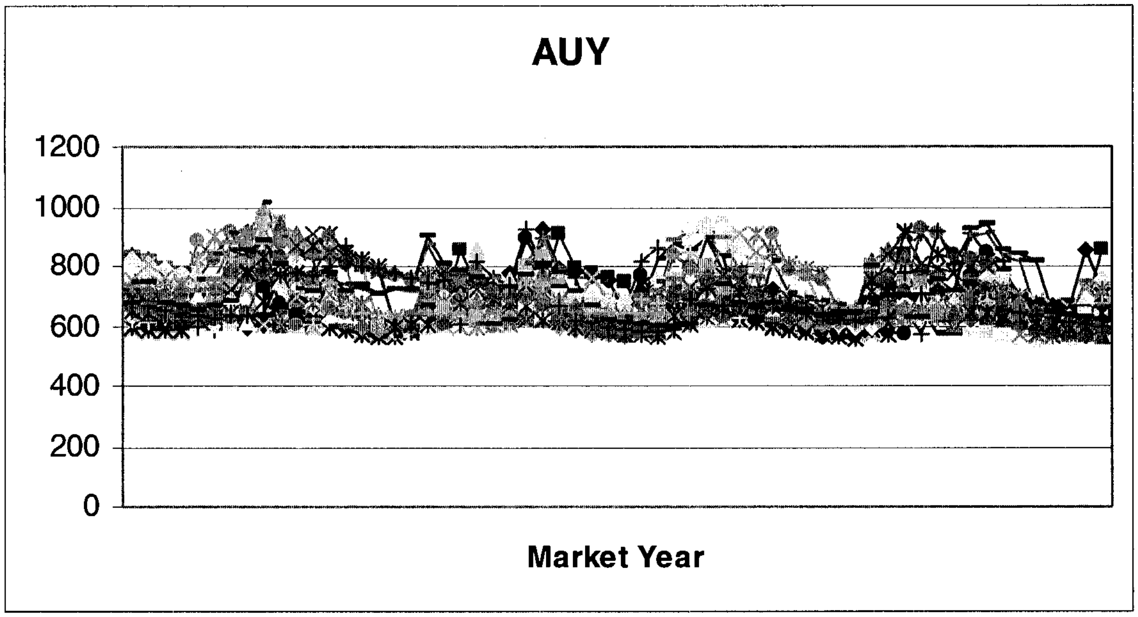

Figure A1.7 Herd Size over Market Year and Average Start

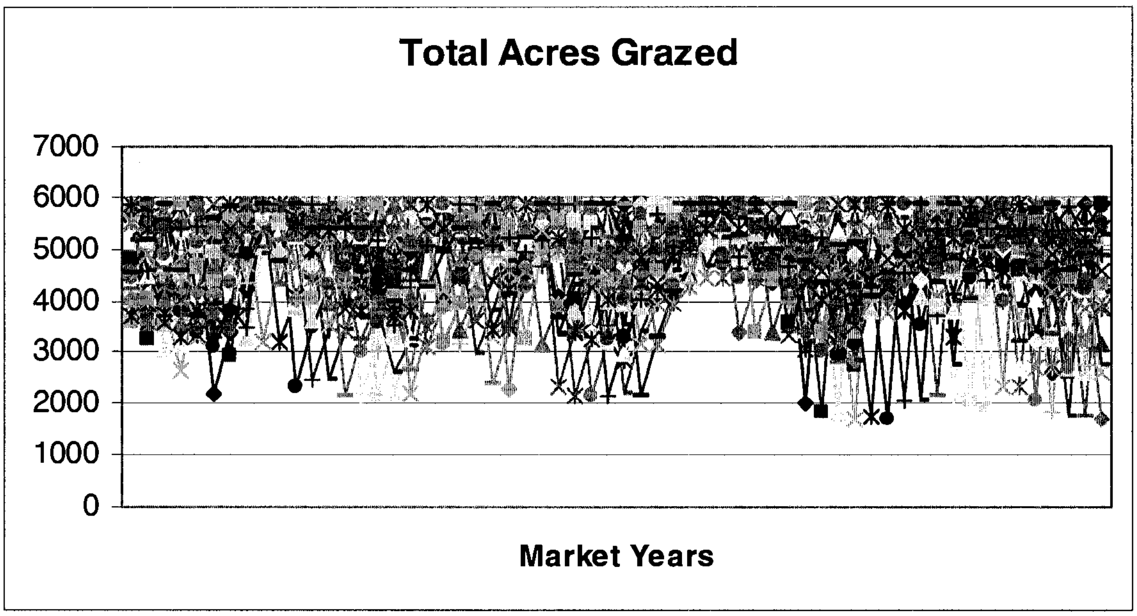

Figure A1.8 Total Acres Grazed over Market Year and Average Start

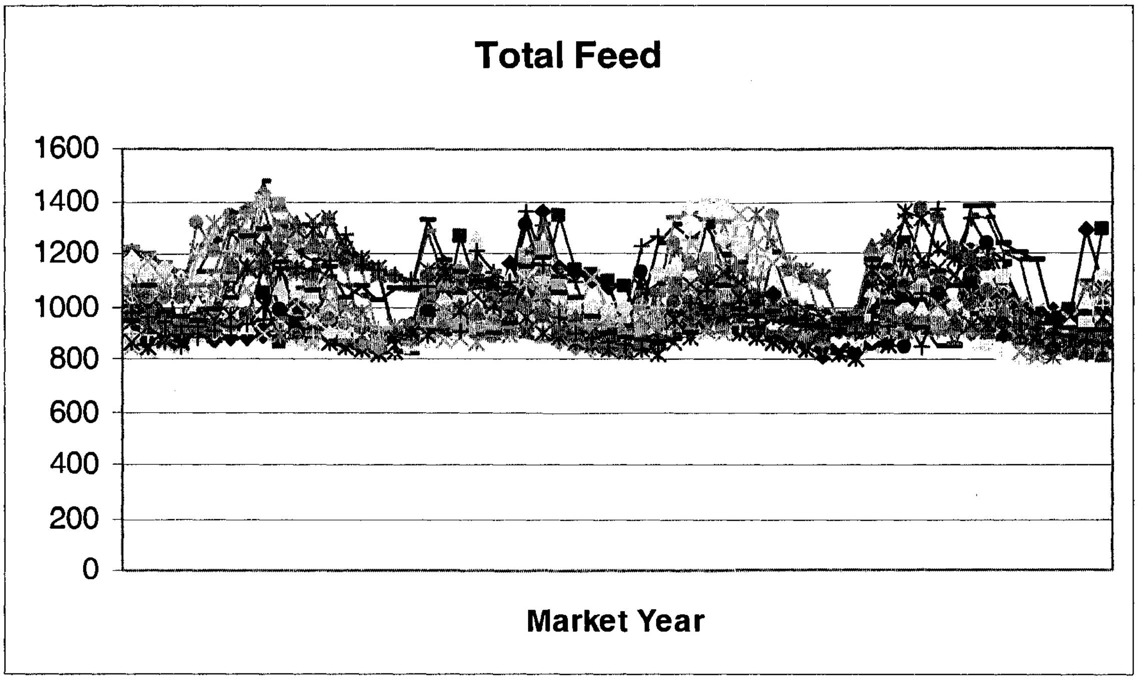

Figure A1.9 Total Feed Allowed over Market Year and Average Start

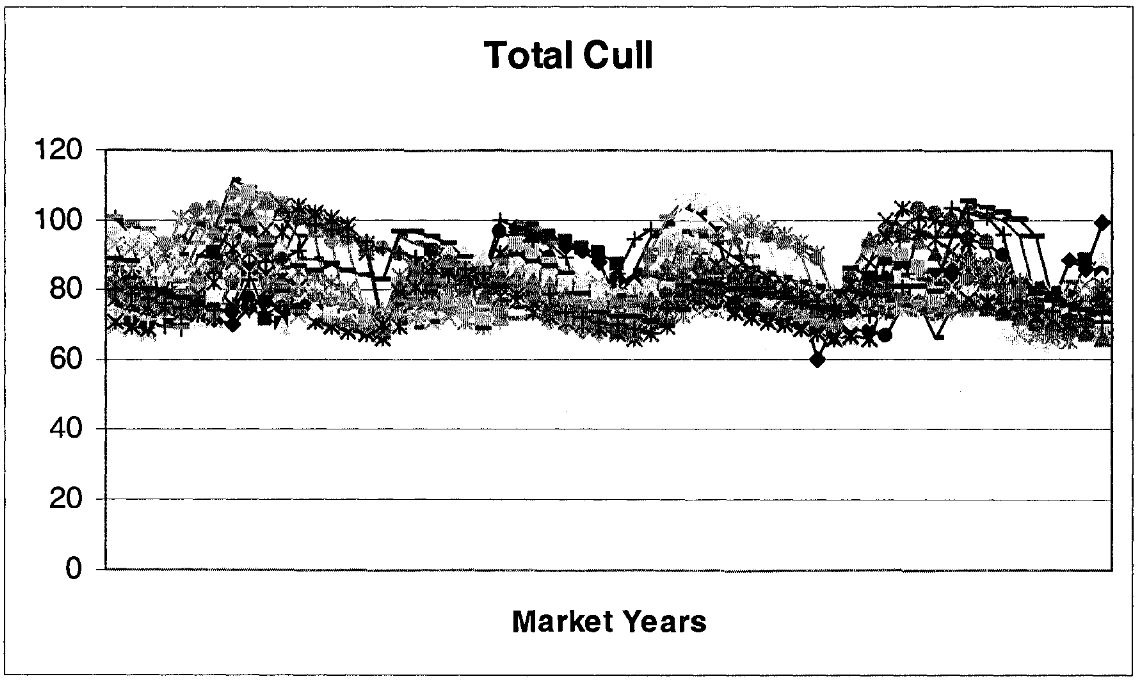

Figure A1.10 Total Culling Activities over Market Year and Average Start

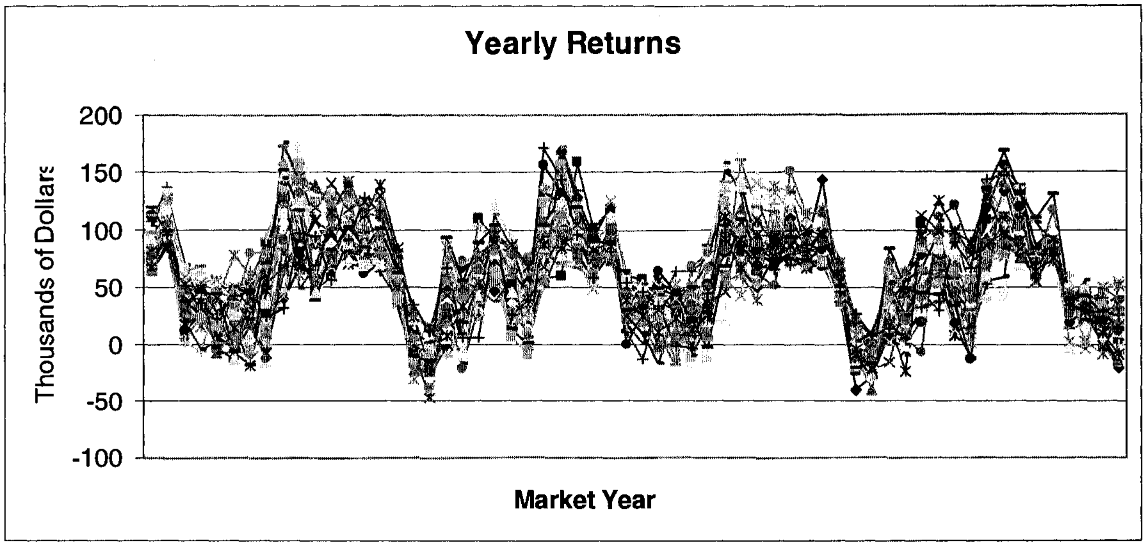

Figure A1.11 Net Yearly Returns over Market Year and Average Start

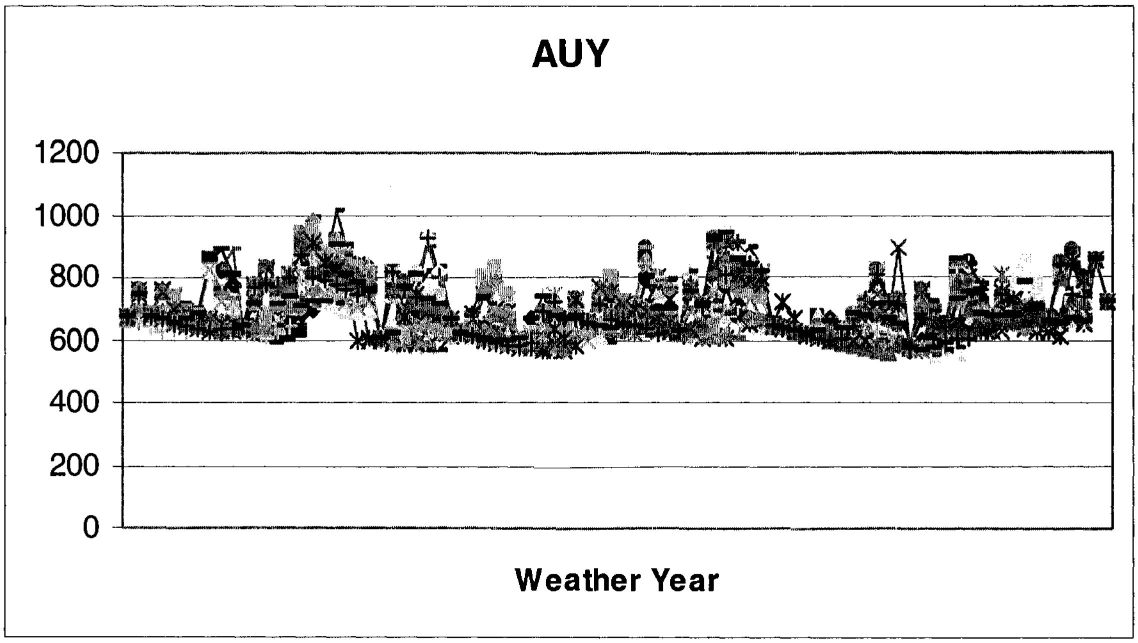

Figure A1.12 Herd Size over Weather Year and Average Start

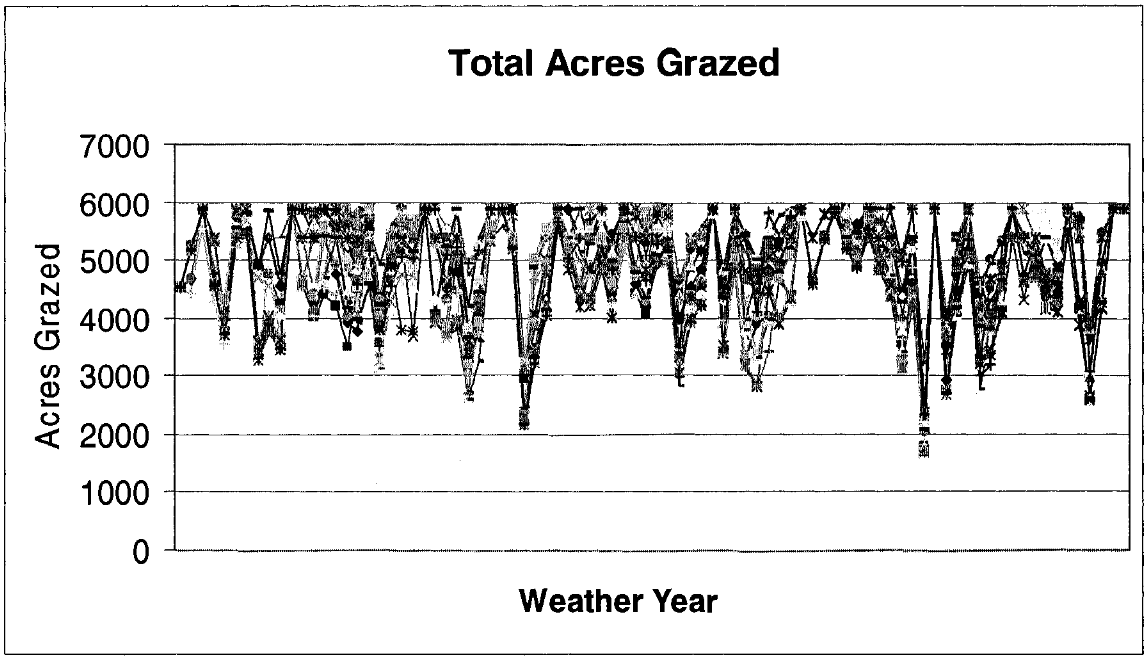

Figure A1.13 Total Acres Grazed over Weather Year and Average Start

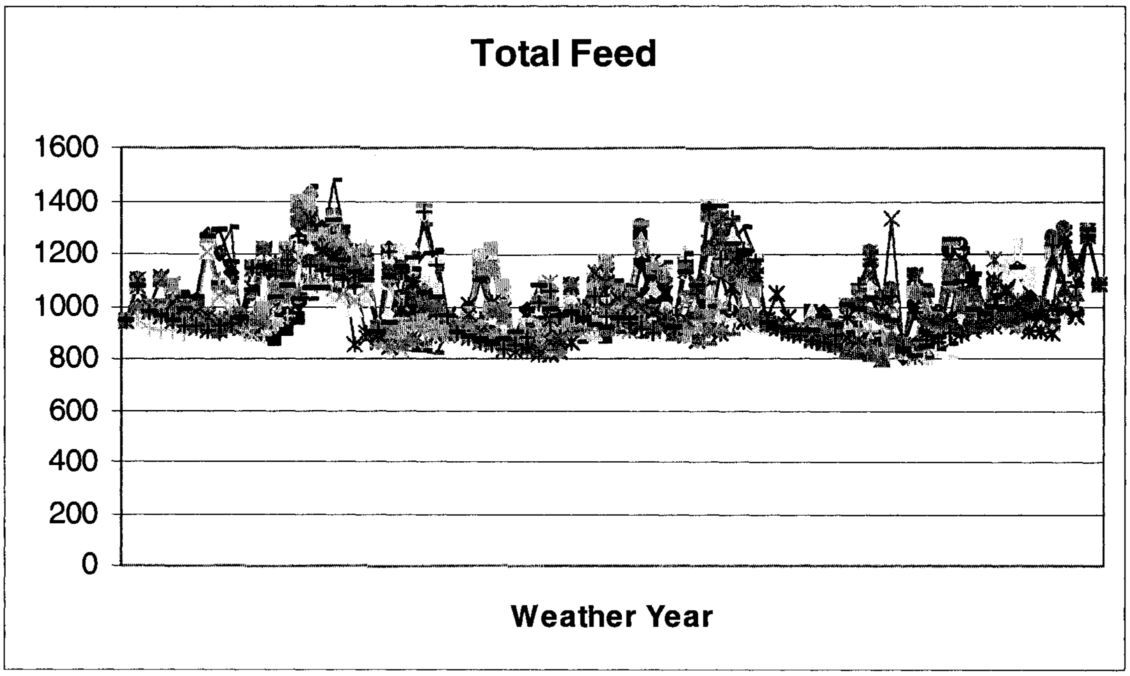

Figure A1.14 Total Feed Allowed over Weather Year and Average Start

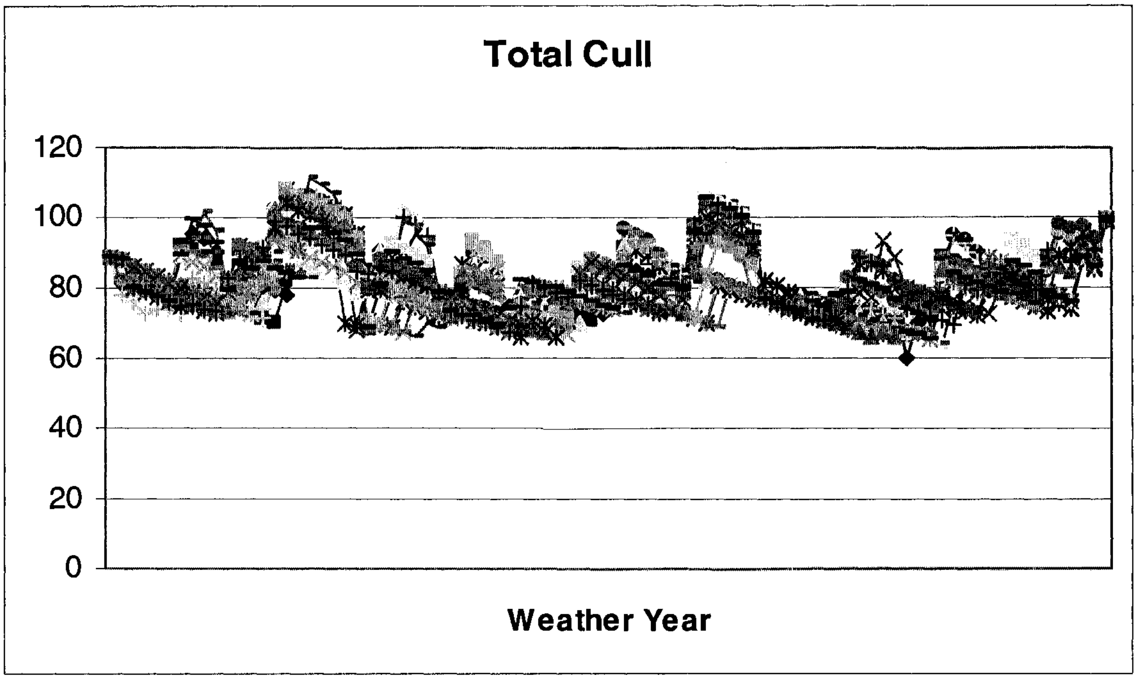

Figure A1.15 Total Culling Activities over Weather Year and Average Start

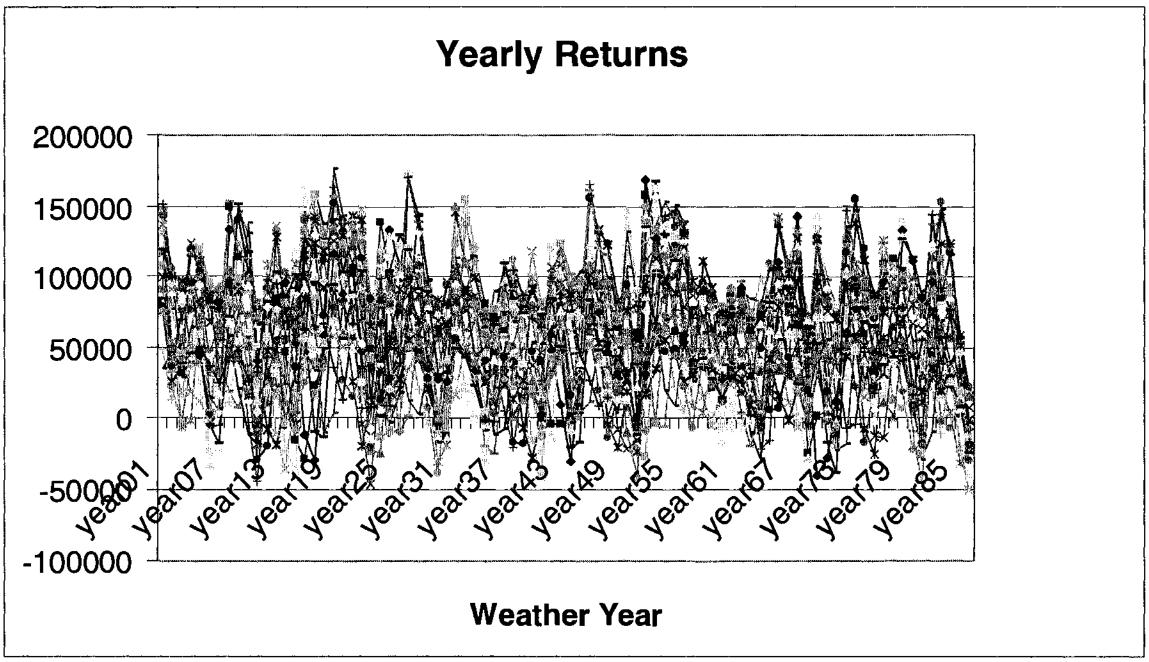

Figure A1.16 Net Yearly Returns over Weather Year and Average Start

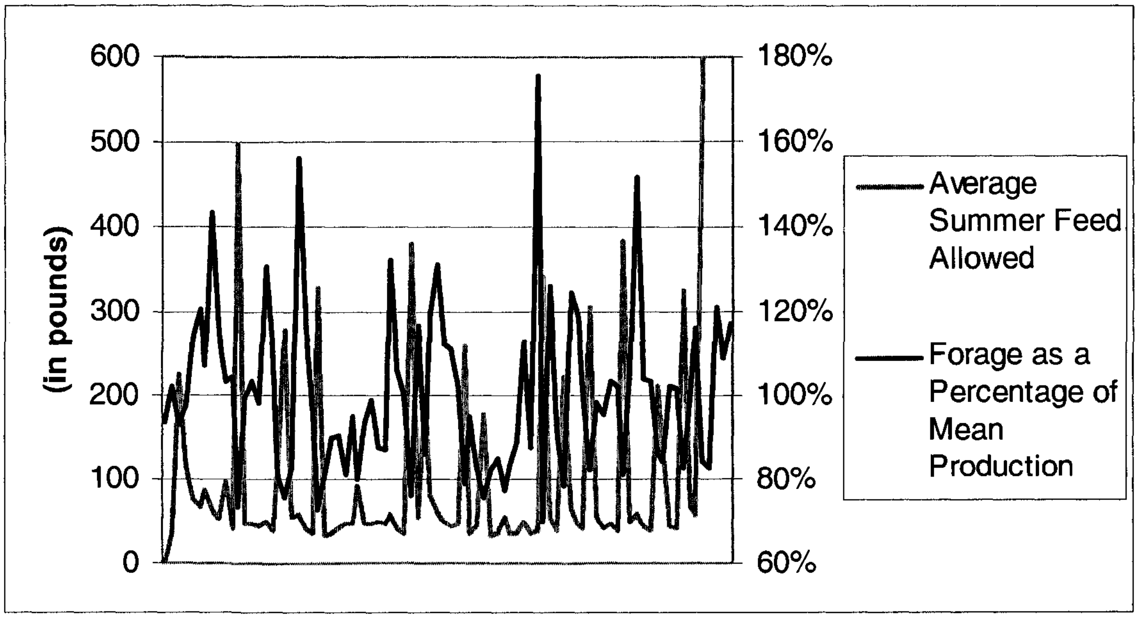

Figure A1.17 – Average Summer Feed Allowed Compared to the Forage Production as a Percentage of Mean Production across Weather Years for *Feed* Scenario, Wet Start

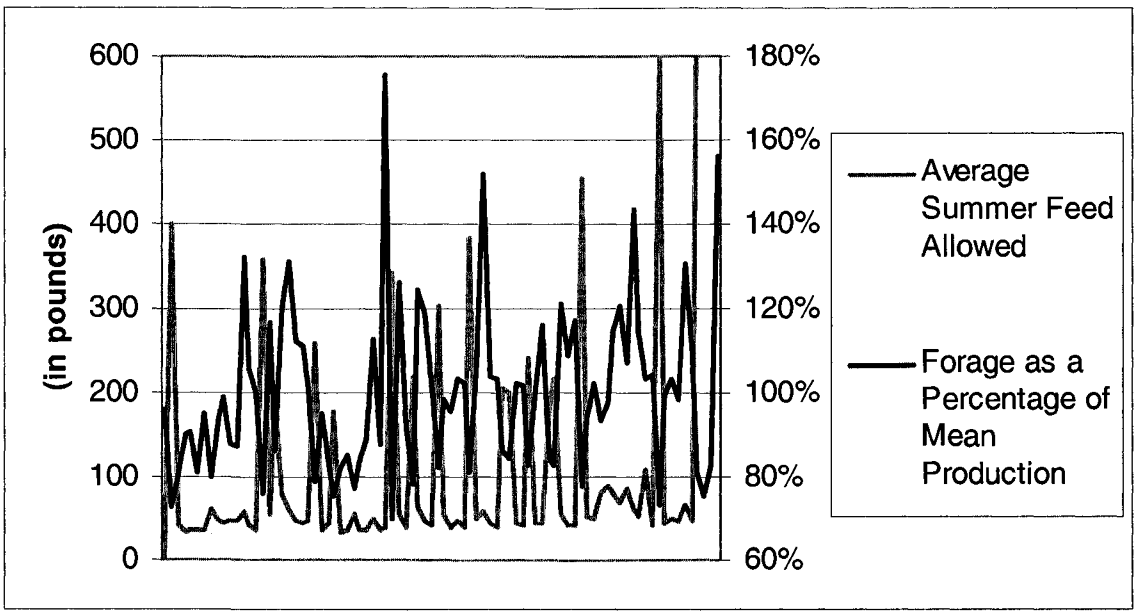

Figure A1.18 – Average Summer Feed Allowed Compared to the Forage Production as a Percentage of Mean Production across Weather Years for *Feed* Scenario, Dry Start

Appendix 2

Supplemental Tables and Figures Supporting Results of Chapter 2

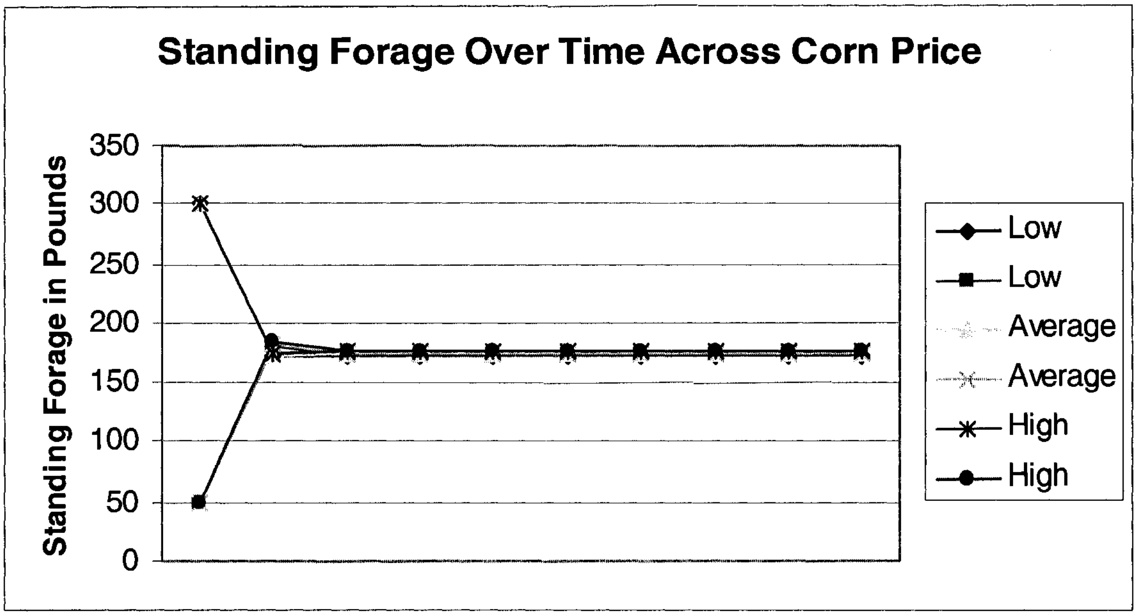

Figure A2.1 Time paths of Standing Forage across Corn Prices

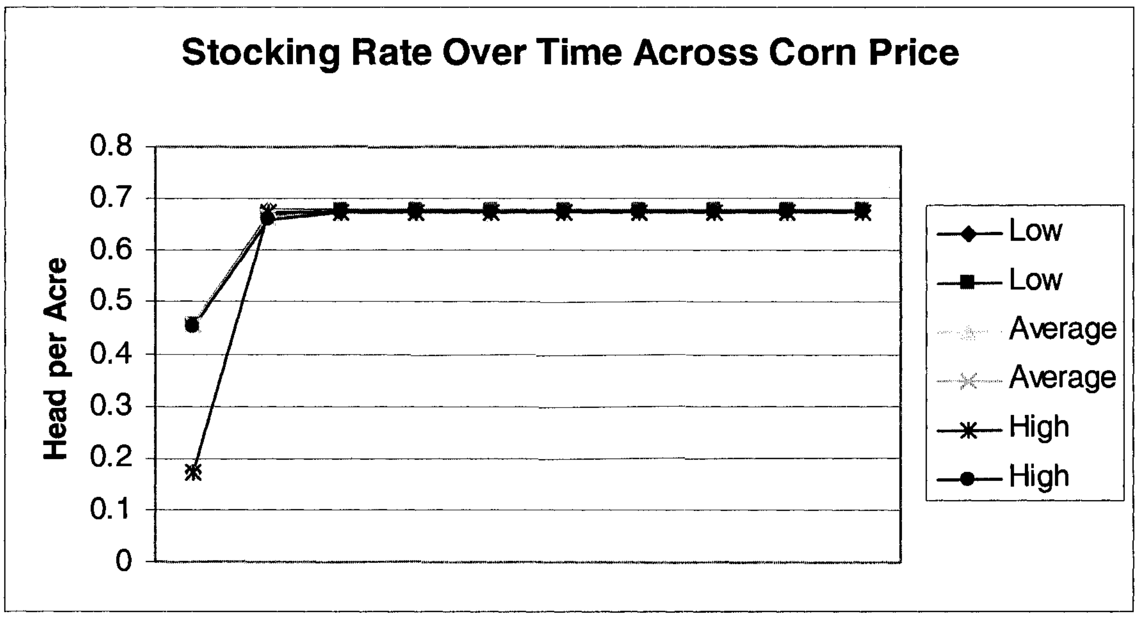

Figure A2.2 Time paths of Stocking Rate across Corn Prices

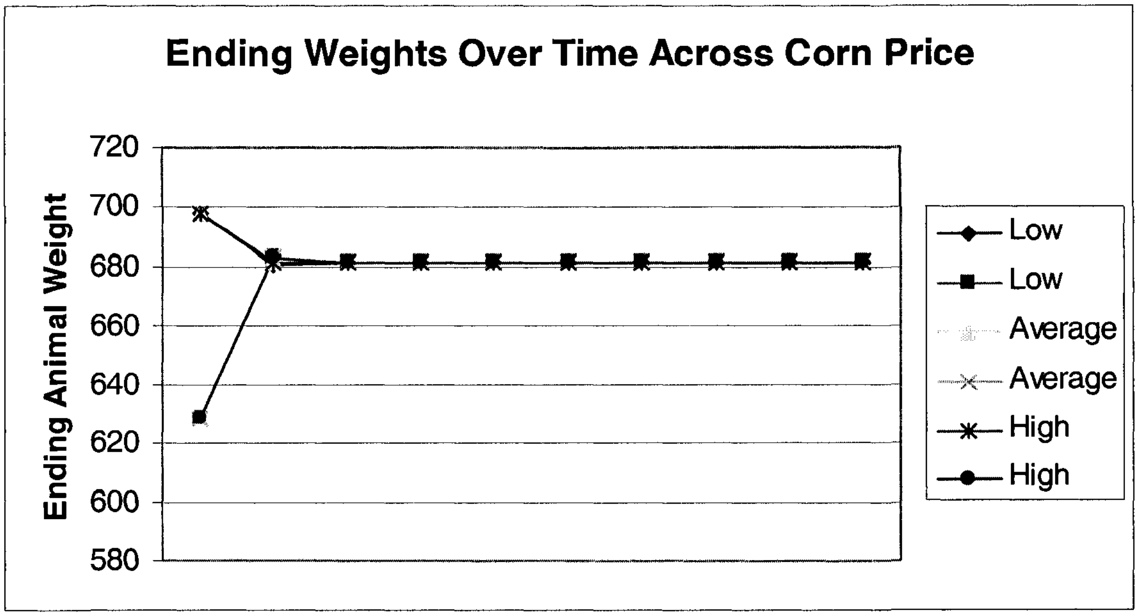

Figure A2.3 Time paths of Ending Weights across Corn Prices

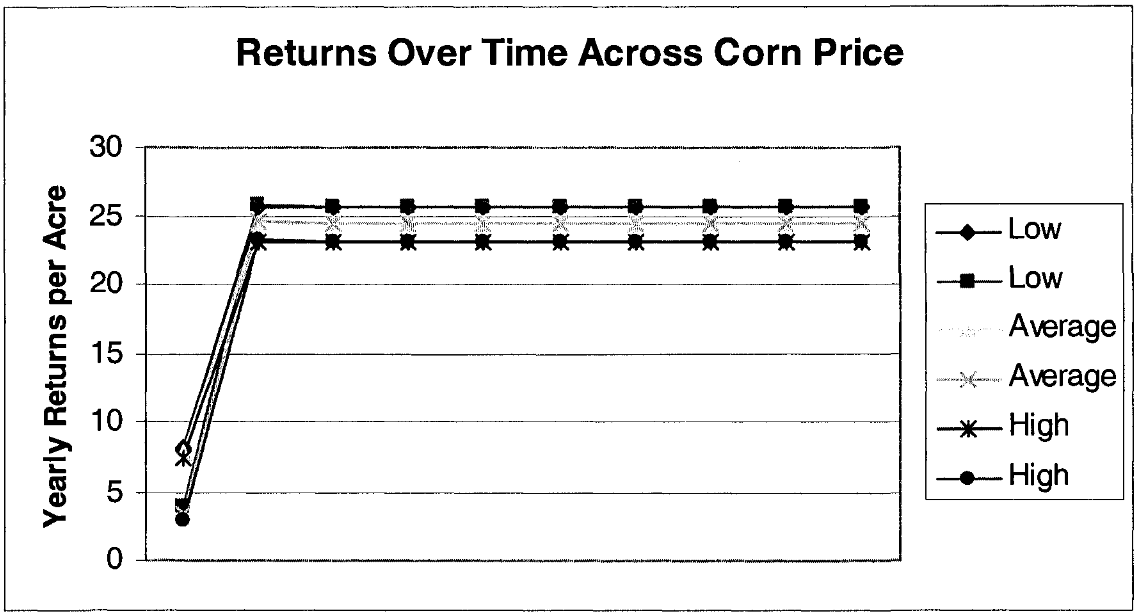

Figure A2.4 Time paths of Yearly Returns across Corn Prices

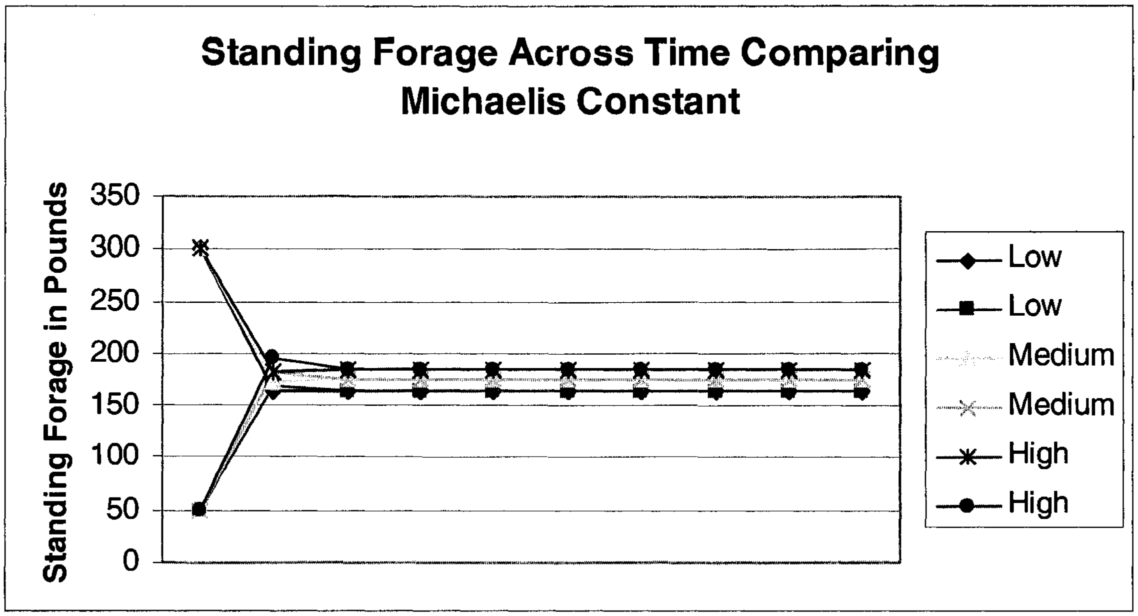

Figure A2.5 **Time paths of Standing Forage across Michaelis Constant**

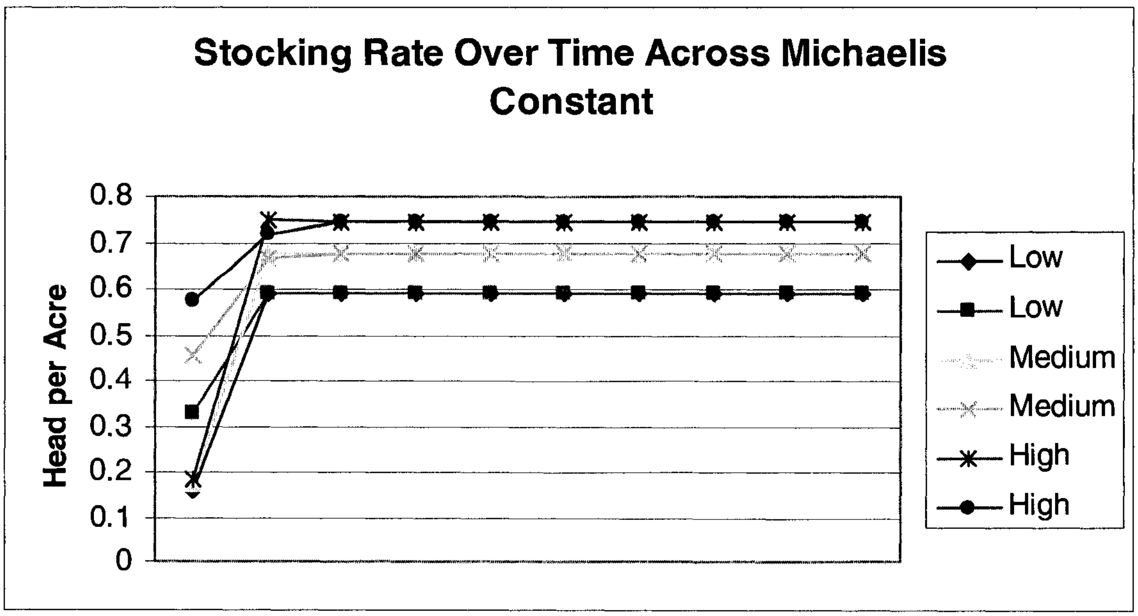

Figure A2.6 **Time paths of Stocking Rate across Michaelis Constant**

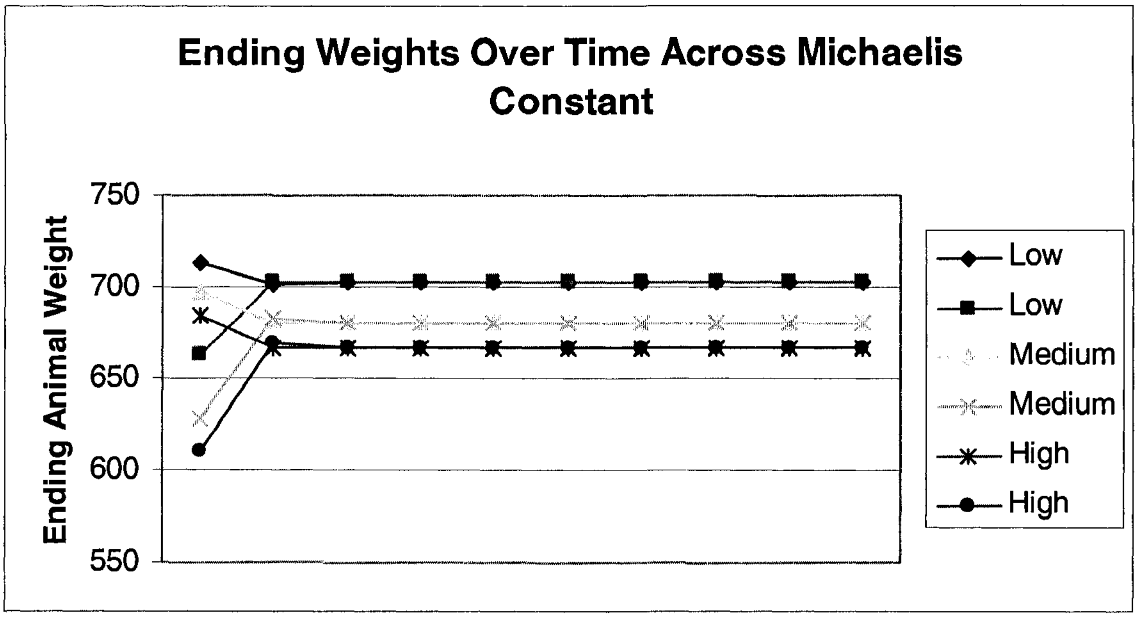

Figure A2.7 Time paths of Ending Weights across Michaelis Constant

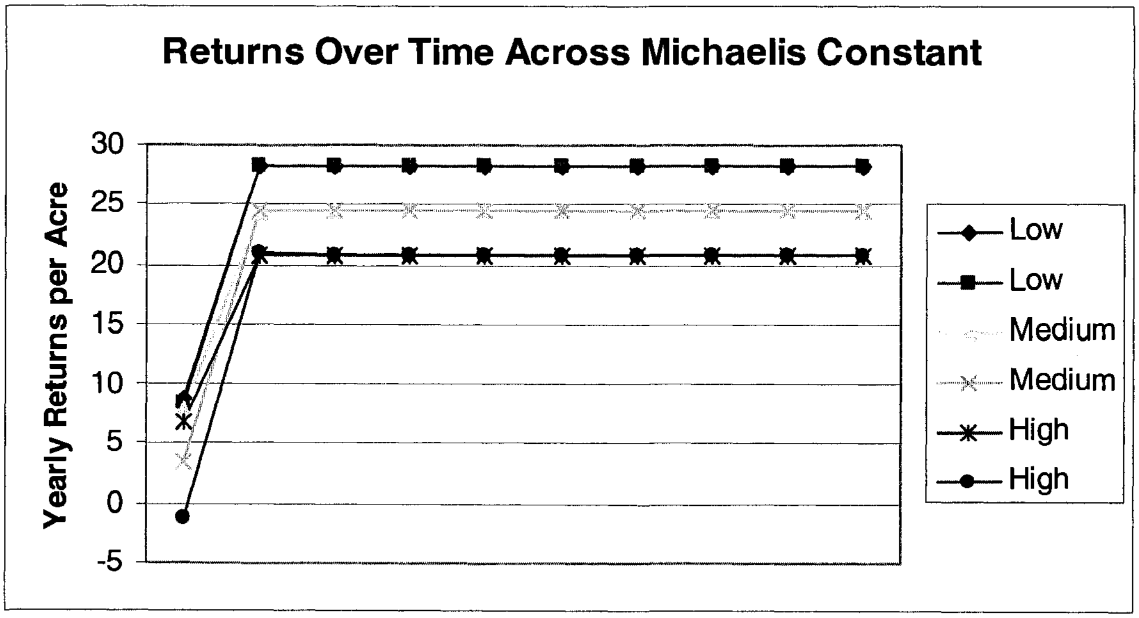

Figure A2.8 Time paths of Yearly Returns across Michaelis Constant

Table A2.1. Chebychev Coefficients

	1	2	3	4	5	6	7	8
a	2449.0474	5.5891	-11.6046	-1.9020	0.9955	-0.4305	0.0843	0.1221
b	498.8742	5.5762	-11.5894	-1.9175	1.0105	-0.4445	0.0972	0.1103
c	255.1153	5.5617	-11.5721	-1.9353	1.0277	-0.4606	0.1120	0.0968
d	133.2653	5.5373	-11.5419	-1.9666	1.0579	-0.4887	0.1378	0.0734
e	76.3459	4.9236	-3.9815	0.2456	-0.4815	0.4275	-0.2525	0.0709
f	152.9657	5.2099	-7.3442	-0.4490	-0.2116	0.5333	-0.6368	0.6071
g	267.9984	5.6792	-12.1770	-1.9069	1.0436	-0.4859	0.1350	0.0792
h	241.1199	5.4327	-10.9126	-1.9655	1.0093	-0.4320	0.0860	0.1167
i	296.2285	4.5060	-13.4448	-0.8902	0.4924	-0.2285	0.0583	0.0482
j	216.2817	6.4270	-9.6533	-2.8996	1.4557	-0.5823	0.0561	0.2502
k	173.4558	4.4498	-7.8448	-1.7750	0.9301	-0.3983	0.0711	0.1225
l	337.1645	6.6498	-15.2926	-2.0905	1.1126	-0.5063	0.1350	0.0881

	9	10	11	12	13	14	15	16
a	-0.2376	0.2932	-0.3093	0.2997	-0.2744	0.2404	-0.2027	0.1650
b	-0.2269	0.2835	-0.3005	0.2918	-0.2673	0.2341	-0.1971	0.1602
c	-0.2146	0.2724	-0.2905	0.2828	-0.2592	0.2268	-0.1908	0.1546
d	-0.1934	0.2532	-0.2731	0.2671	-0.2451	0.2142	-0.1795	0.1447
e	0.0558	-0.1094	0.1021	-0.0611	0.0141	0.0196	-0.0329	0.0292
f	-0.5040	0.3695	-0.2331	0.1136	-0.0219	-0.0382	0.0682	-0.0733
g	-0.2026	0.2652	-0.2873	0.2828	-0.2617	0.2311	-0.1963	0.1608
h	-0.2283	0.2806	-0.2941	0.2829	-0.2565	0.2220	-0.1845	0.1475
i	-0.1119	0.1469	-0.1627	0.1658	-0.1605	0.1500	-0.1363	0.1208
j	-0.4116	0.4759	-0.4755	0.4340	-0.3688	0.2929	-0.2158	0.1444
k	-0.2278	0.2743	-0.2820	0.2649	-0.2331	0.1940	-0.1530	0.1138
l	-0.2162	0.2821	-0.3070	0.3050	-0.2859	0.2570	-0.2230	0.1878

	17	18	19	20	21	22	23
a	-0.1298	0.0986	-0.0721	0.0505	-0.0334	0.0200	-0.0093
b	-0.1256	0.0951	-0.0692	0.0482	-0.0317	0.0189	-0.0088
c	-0.1208	0.0910	-0.0658	0.0455	-0.0297	0.0176	-0.0081
d	-0.1122	0.0837	-0.0598	0.0407	-0.0261	0.0152	-0.0069
e	-0.0175	0.0067	-0.0019	0.0029	-0.0062	0.0078	-0.0054
f	0.0612	-0.0400	0.0171	0.0015	-0.0124	0.0146	-0.0094
g	-0.1273	0.0974	-0.0717	0.0506	-0.0338	0.0204	-0.0095
h	-0.1134	0.0837	-0.0591	0.0397	-0.0250	0.0143	-0.0064
i	-0.1045	0.0882	-0.0722	0.0568	-0.0419	0.0276	-0.0137
j	-0.0833	0.0352	-0.0013	-0.0187	0.0261	-0.0235	0.0136
k	-0.0792	0.0505	-0.0285	0.0133	-0.0040	-0.0003	0.0010
l	-0.1535	0.1219	-0.0938	0.0694	-0.0485	0.0305	-0.0147

(a. 0.1 growth rate, 1 % discount rate, average corn price, 64.2 Michaelis constant, b. 0.1 growth rate, 5 % discount rate, average corn price, 64.2 Michaelis constant, c. 0.1 growth rate, 10 % discount rate, average corn price, 64.2 Michaelis constant, d

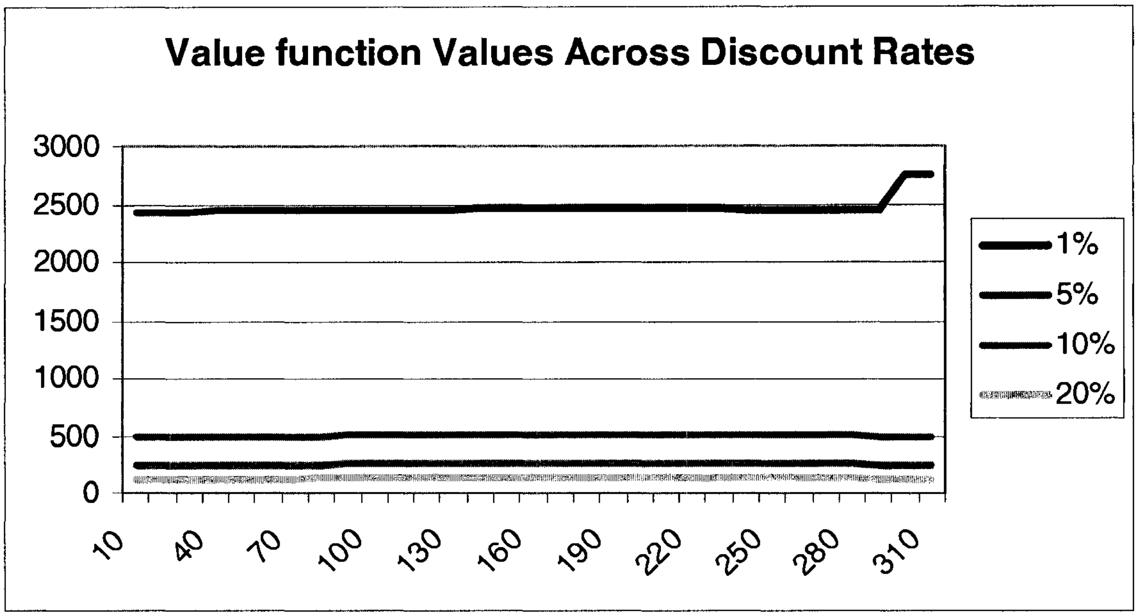

Figure A2.9 Comparison of Value Function across Discount Rates

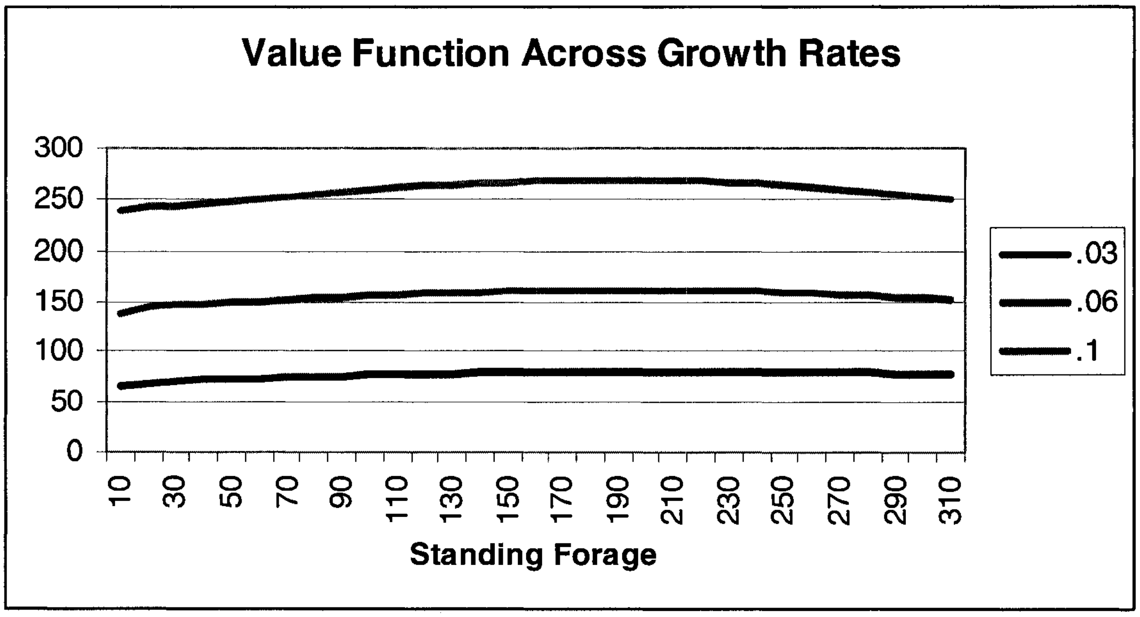

Figure A2.10 Comparison of Value Function across Forage Growth Rates

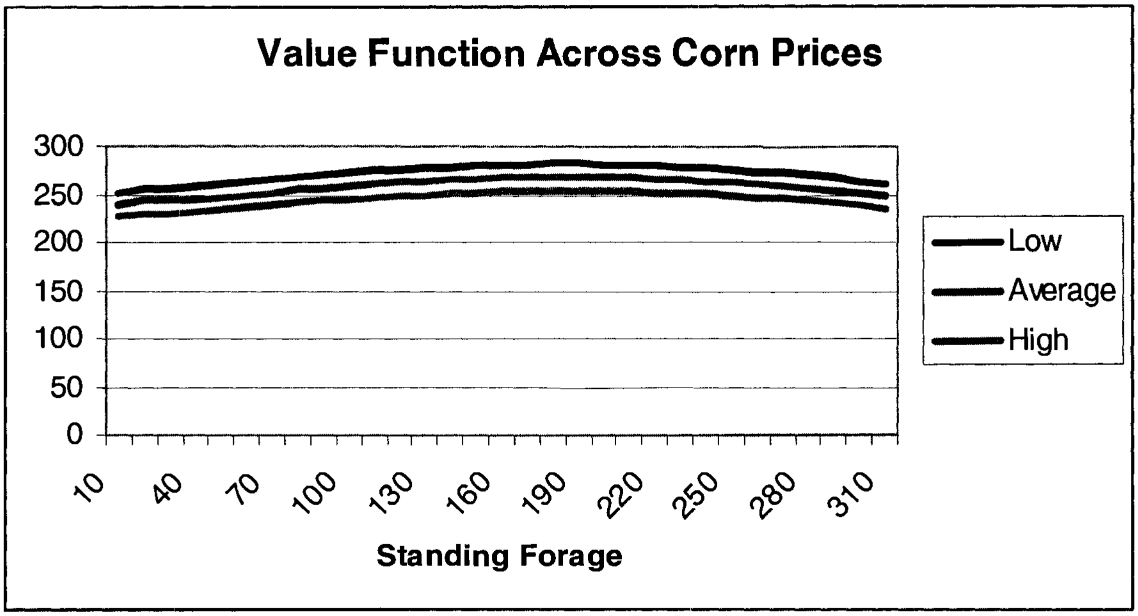

Figure A2.11 Comparison of Value Function across Corn Price Levels

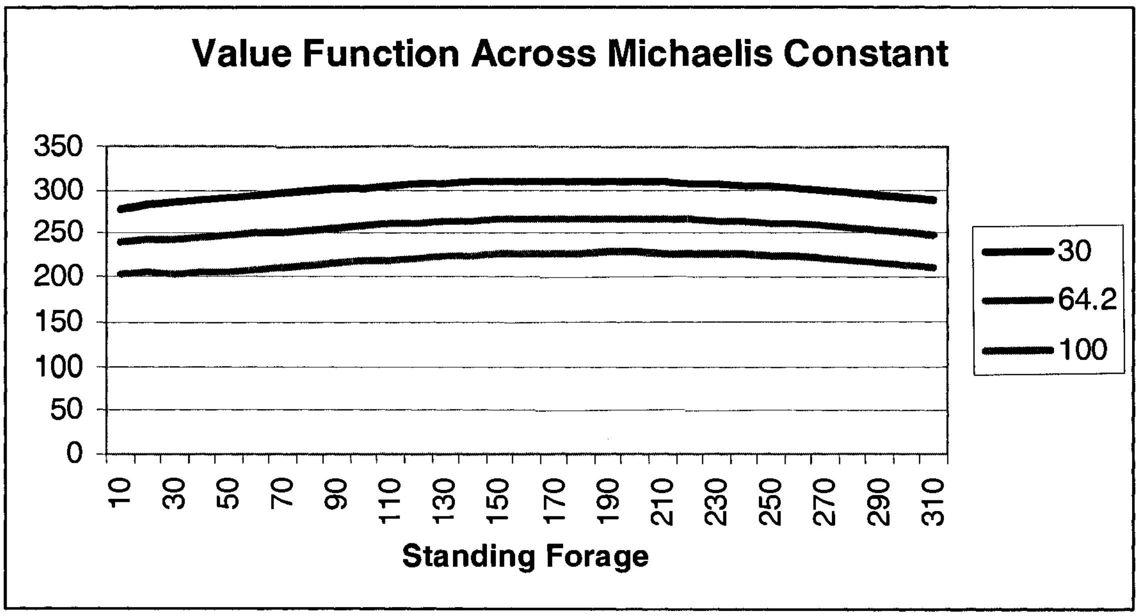

Figure A2.12 Comparison of Value Function across Michaelis Constant

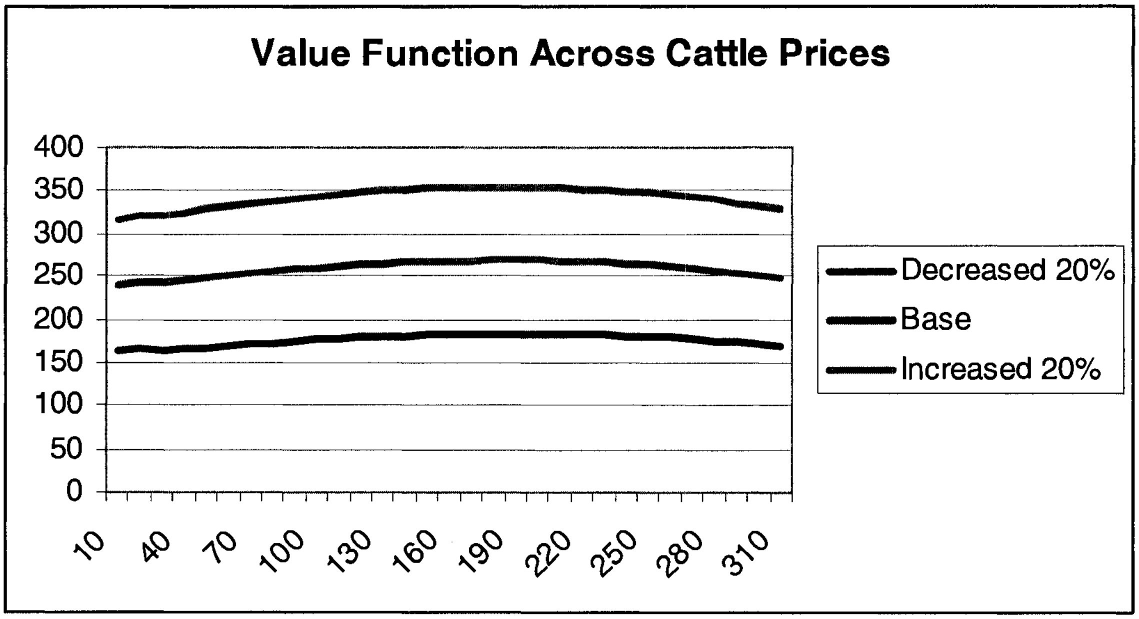

Figure A2.13 Comparison of Value Function across Output Price Levels

Appendix 3

Supplemental Tables and Figures Supporting Results of Chapter 3

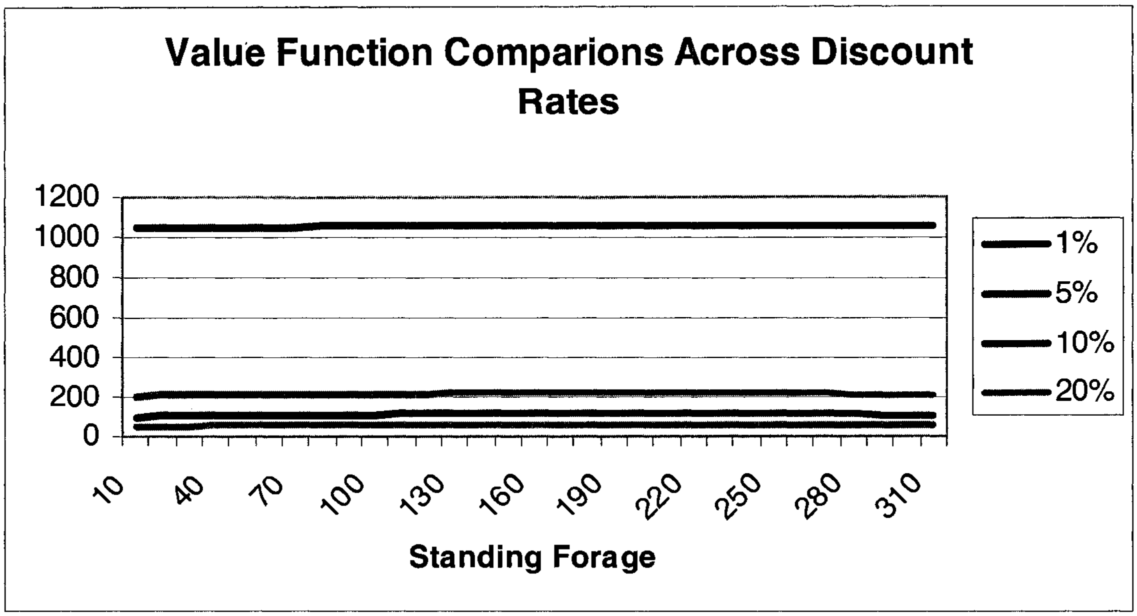

Figure A3.1 Value Function Comparisons across Discount Rates

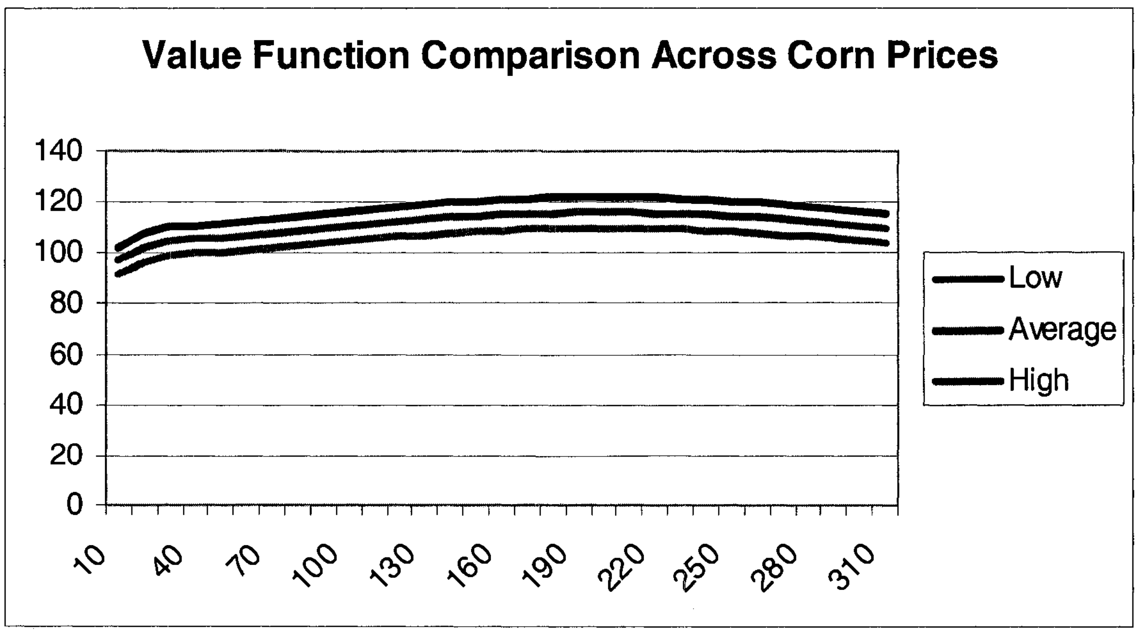

Figure A3.2 Value Function Comparisons across Corn Price Levels

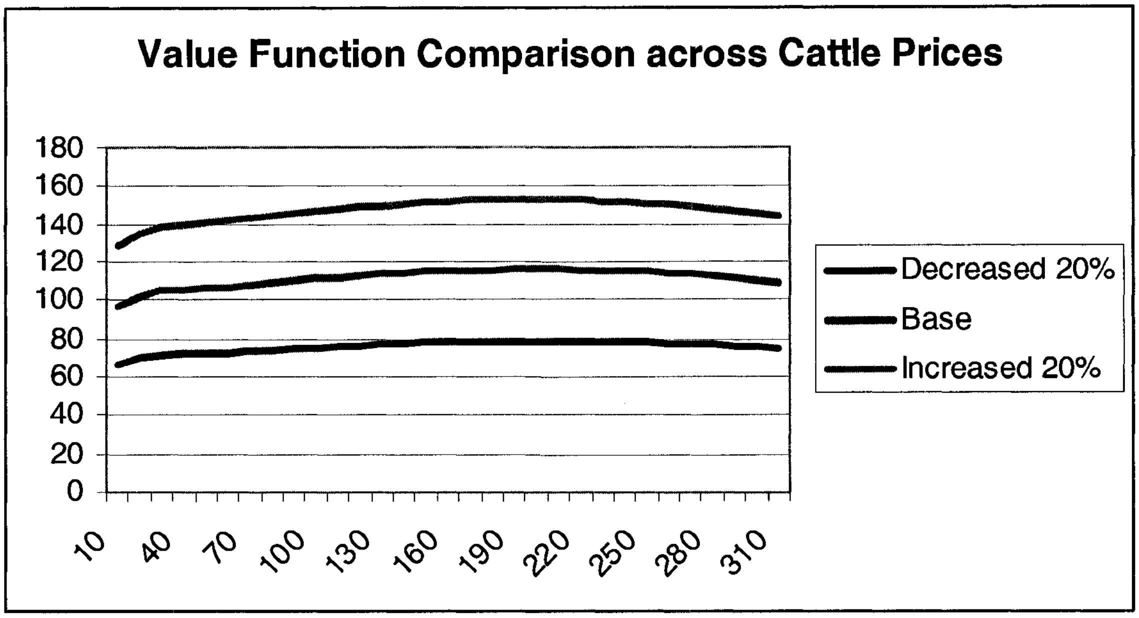

Figure A3.3 Value Function Comparisons across Output Price Levels

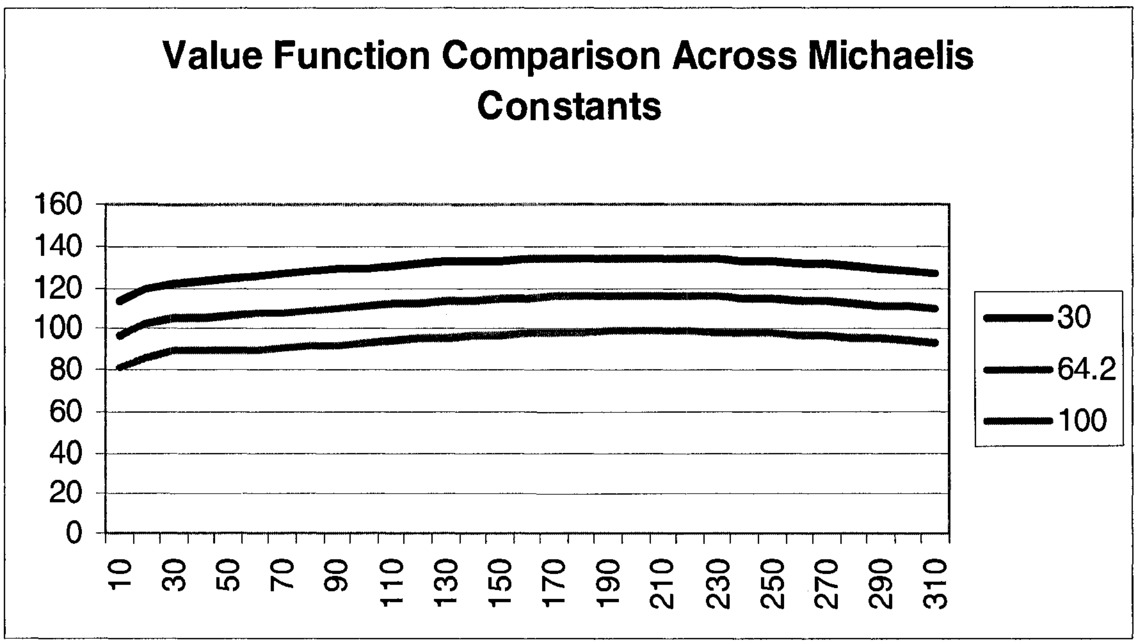

Figure A3.4 Value Function Comparisons across Michaelis Constants

Table A3.1. Chebychev Coefficients

	1	2	3	4	5	6	7
a	1054.6809	5.1155	-5.5676	0.1152	-0.5783	0.7055	-0.6351
b	214.7035	5.0553	-5.5050	0.0558	-0.5247	0.6600	-0.6000
c	109.7657	4.9903	-5.4377	-0.0058	-0.4733	0.6204	-0.5723
d	57.3415	4.8791	-5.3204	-0.1136	-0.3830	0.5510	-0.5238
e	127.4277	5.0370	-6.3060	0.2862	-0.4787	0.5236	-0.4757
f	93.0898	4.8480	-4.5771	-0.2387	-0.5104	0.7256	-0.6465
g	115.3146	5.1738	-5.7174	0.0209	-0.4835	0.6303	-0.5854
h	103.7297	4.7892	-5.1328	-0.0342	-0.4626	0.6097	-0.5578
i	109.0499	5.1196	-5.5262	0.0910	-0.5364	0.6495	-0.5718
j	74.6389	3.6243	-3.6900	-0.0922	-0.3594	0.4996	-0.4573
k	145.0392	6.3426	-7.1871	0.0890	-0.5953	0.7457	-0.6870

	9	10	11	12	13	14	15
a	-0.2781	0.1093	0.0125	-0.0796	0.0978	-0.0810	0.0464
b	-0.2677	0.1095	0.0056	-0.0714	0.0935	-0.0842	0.0580
c	-0.2592	0.1073	0.0040	-0.0680	0.0898	-0.0812	0.0560
d	-0.2444	0.1033	0.0012	-0.0620	0.0834	-0.0759	0.0527
e	-0.2635	0.1538	-0.0631	-0.0023	0.0412	-0.0568	0.0547
f	-0.2097	0.0240	0.0918	-0.1376	0.1300	-0.0920	0.0456
g	-0.2742	0.1200	-0.0050	-0.0633	0.0890	-0.0834	0.0598
h	-0.2424	0.0930	0.0140	-0.0732	0.0905	-0.0786	0.0517
i	-0.2258	0.0722	0.0327	-0.0856	0.0948	-0.0753	0.0433
j	-0.1789	0.0506	0.0364	-0.0782	0.0824	-0.0623	0.0327
k	-0.3332	0.1575	-0.0236	-0.0599	0.0964	-0.0967	0.0747

	17	18	19	20	21	22	23
a	-0.0145	0.0232	-0.0162	0.0000	0.0000	0.0000	0.0000
b	0.0025	0.0132	-0.0193	0.0181	-0.0133	0.0079	-0.0035
c	0.0025	0.0127	-0.0186	0.0176	-0.0130	0.0077	-0.0034
d	0.0026	0.0119	-0.0175	0.0166	-0.0123	0.0074	-0.0033
e	0.0228	-0.0044	-0.0102	0.0190	-0.0213	0.0178	-0.0101
f	-0.0184	0.0281	-0.0268	0.0194	-0.0099	0.0000	0.0000
g	0.0055	0.0114	-0.0190	0.0194	-0.0155	0.0100	-0.0048
h	-0.0009	0.0143	-0.0181	0.0154	-0.0100	0.0050	-0.0018
i	-0.0095	0.0190	-0.0182	0.0115	-0.0039	-0.0009	0.0018
j	-0.0118	0.0177	-0.0143	0.0063	0.0013	-0.0050	0.0040
k	0.0137	0.0088	-0.0217	0.0258	-0.0233	0.0169	-0.0088

(a. 1% discount rate, average corn price, 64.2 Michaelis constant, b. 5% discount rate, average corn price, 64.2 Michaelis constant, c. 10% discount rate, average corn price, 64.2 Michaelis constant, d. 20% discount rate, average corn price, 64.2 Michaeli

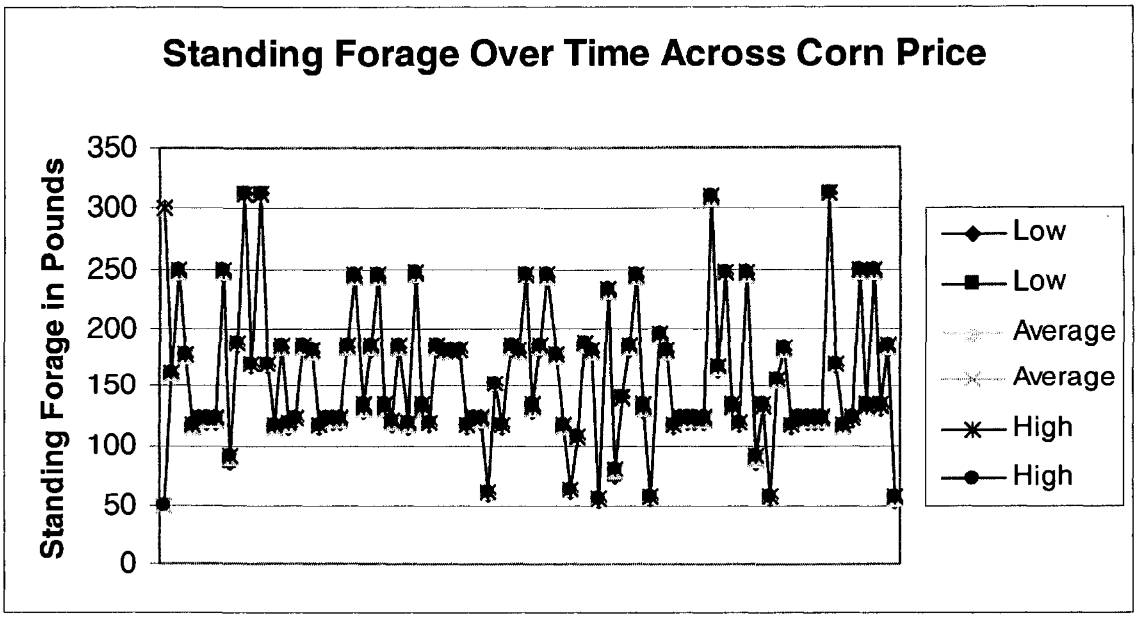

Figure A3.5 Time paths of Standing Forage comparing Corn Prices

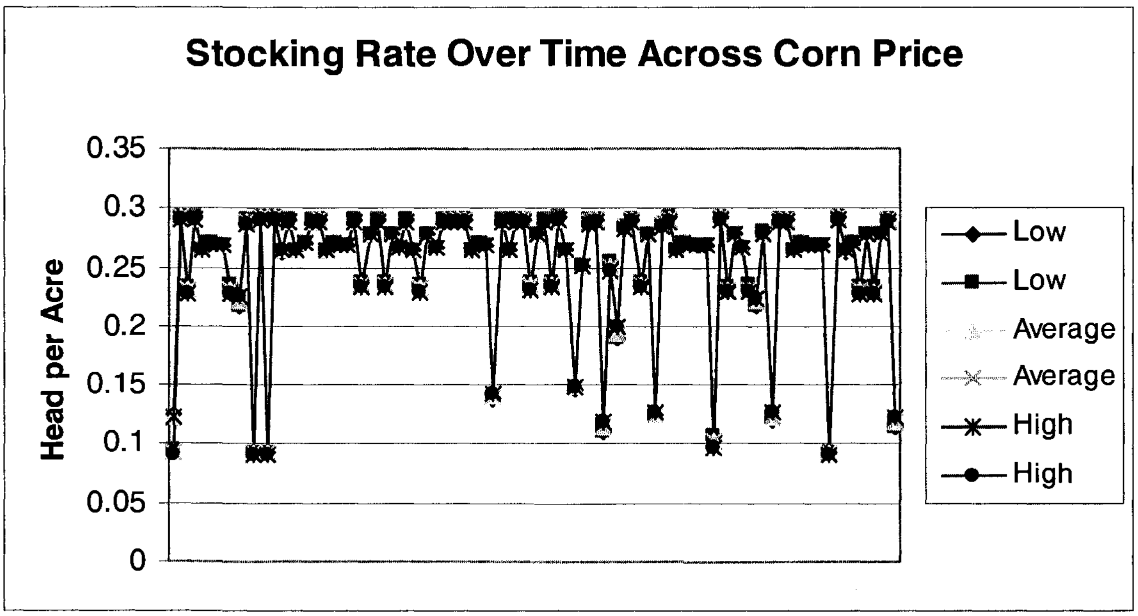

Figure A3.6 Time paths of Stocking Rate comparing Corn Prices

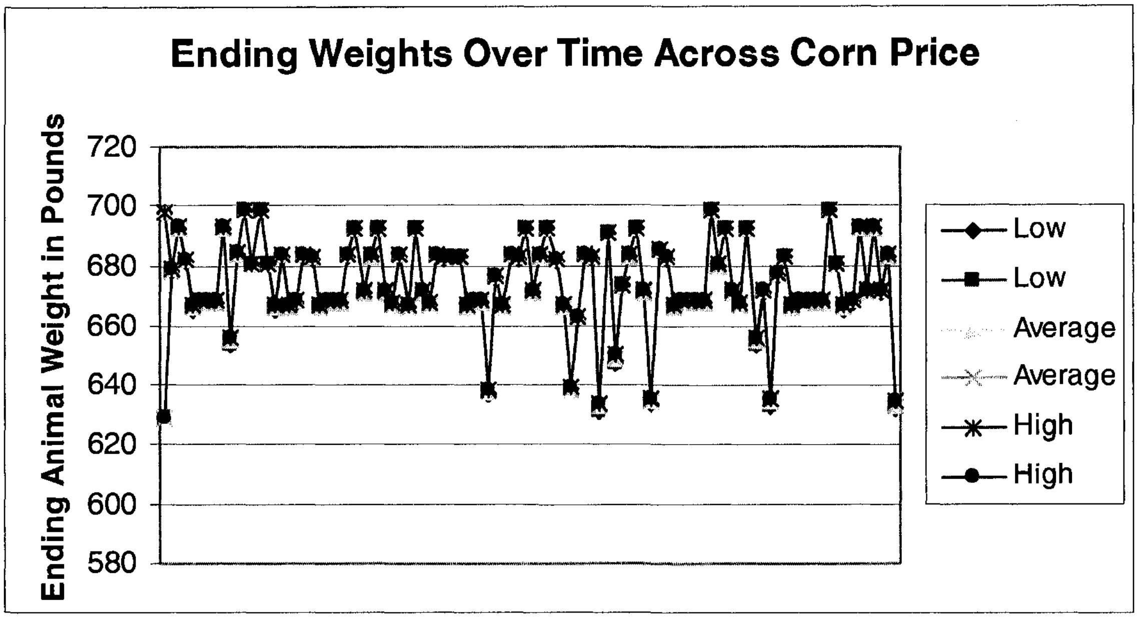

Figure A3.7 Time paths of Ending Weights comparing Corn Prices

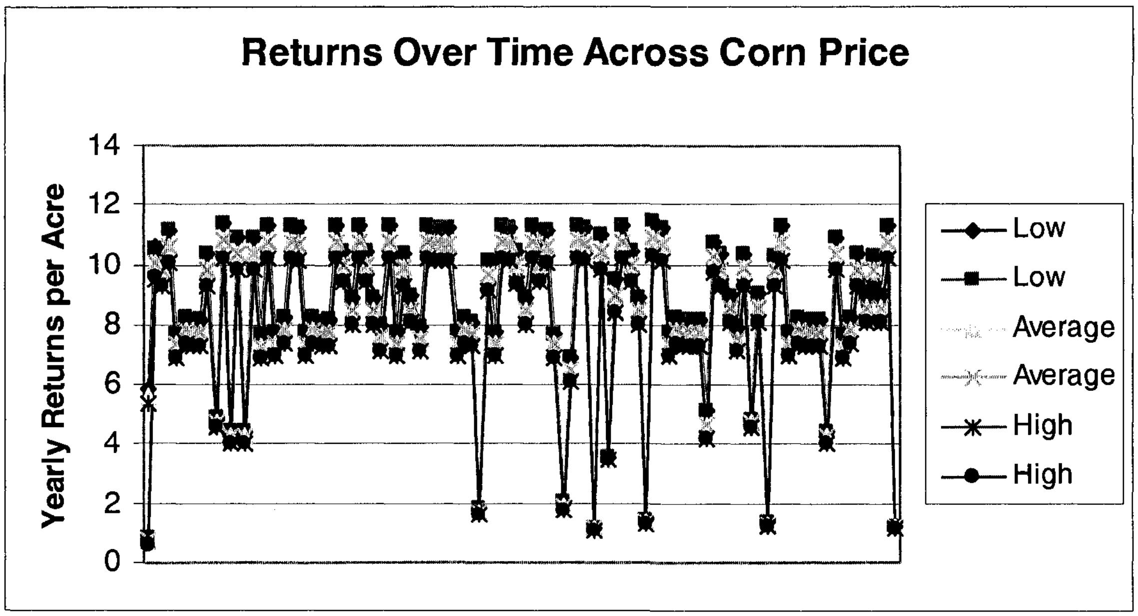

Figure A3.8 Time paths of Yearly Returns comparing Corn Prices

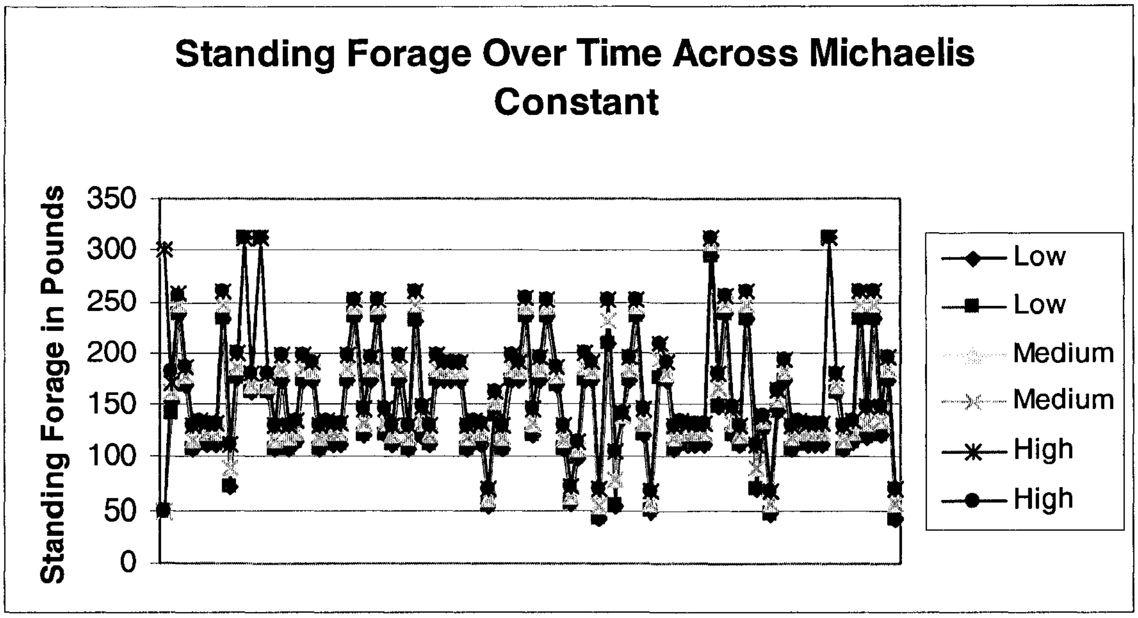

Figure A3.9 Time paths of Standing Forage across Michaelis Constant

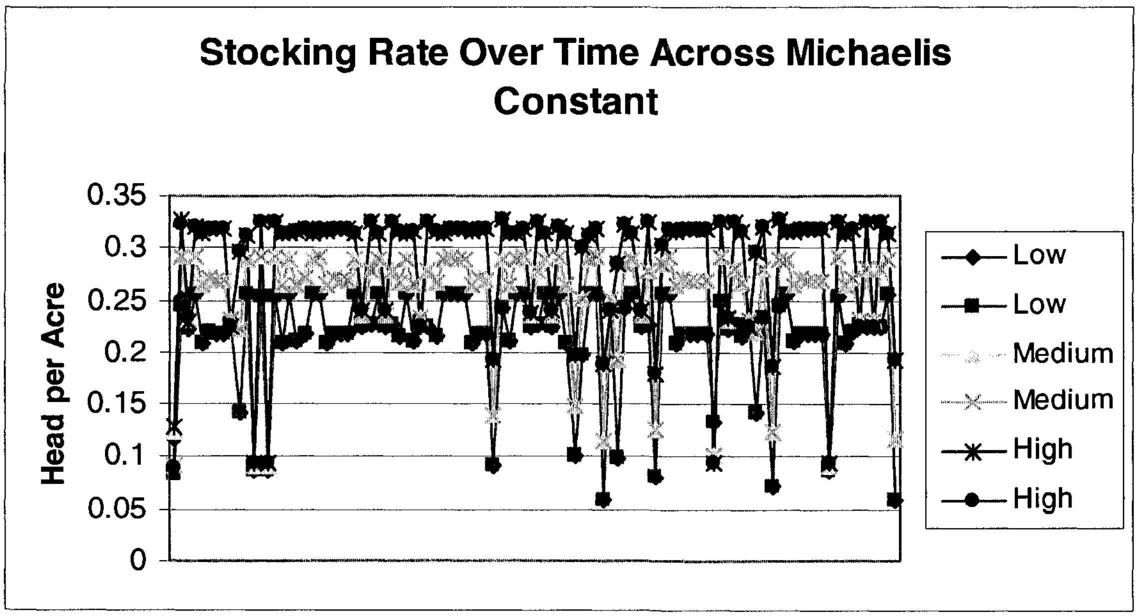

Figure A3.10 Time paths of Stocking Rate across Michaelis Constant

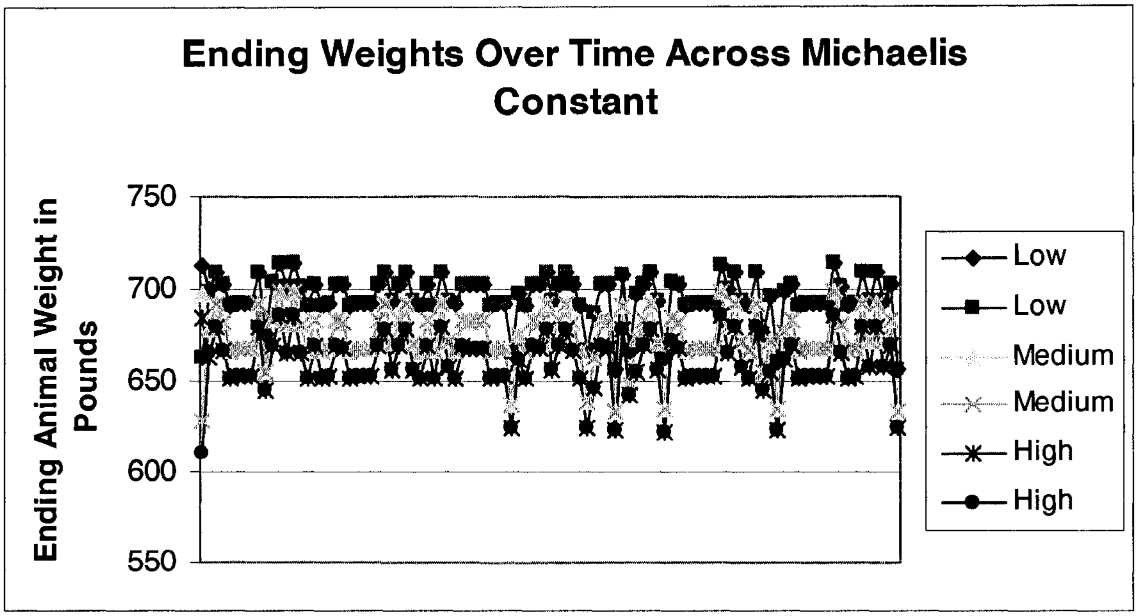

Figure A3.11 Time paths of Ending Weights across Michaelis Constant

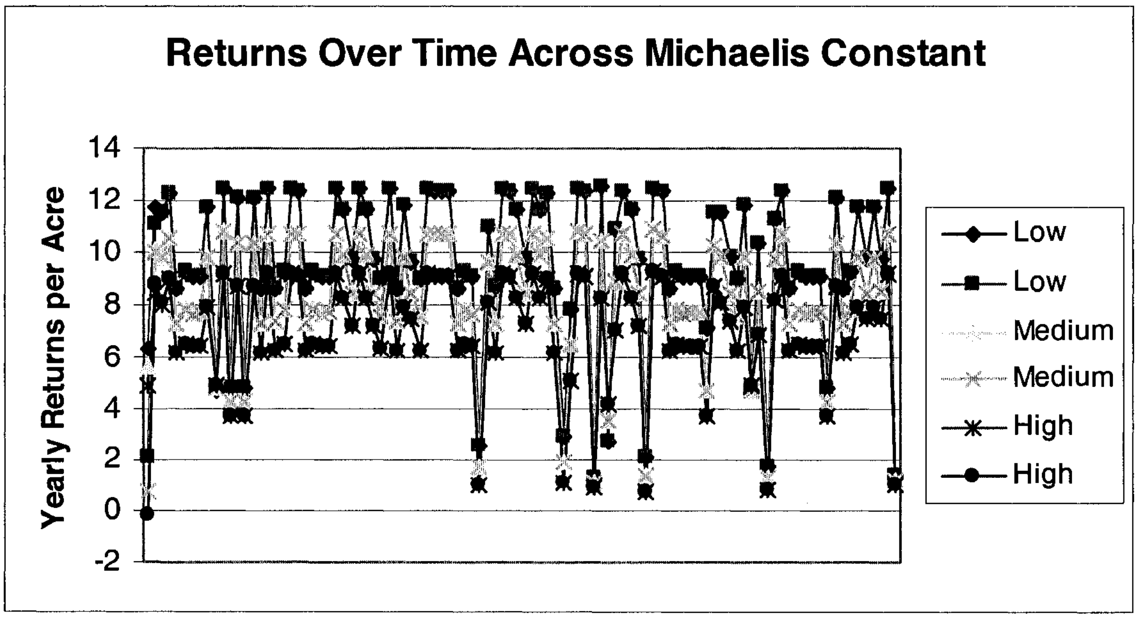

Figure A3.12 Time paths of Yearly Returns across Michaelis Constant

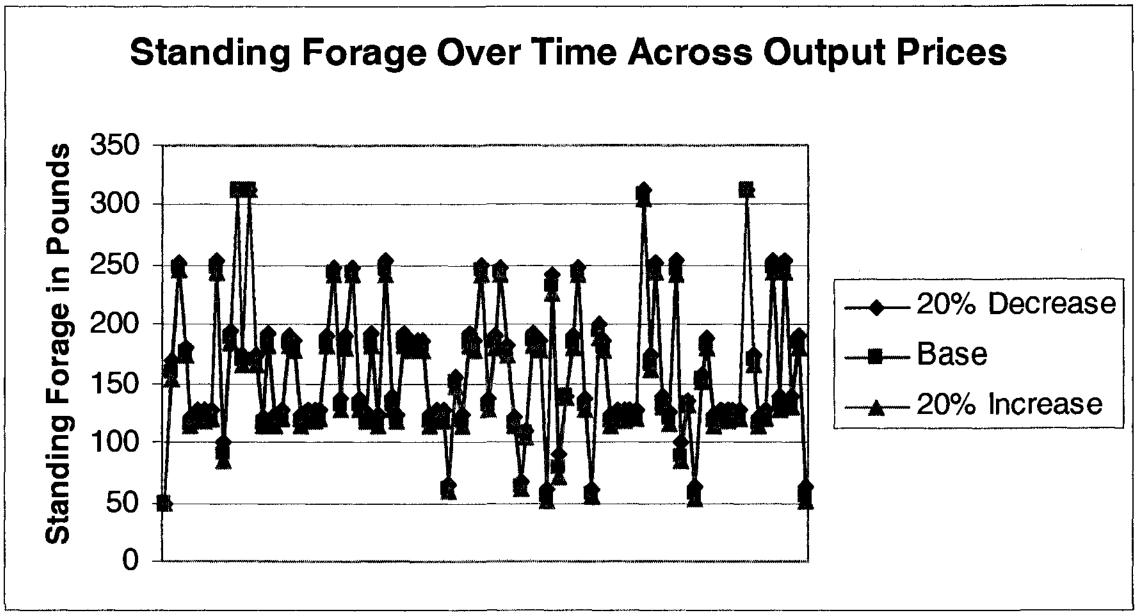

Figure A3.13 Time paths of Standing Forage across Output Prices

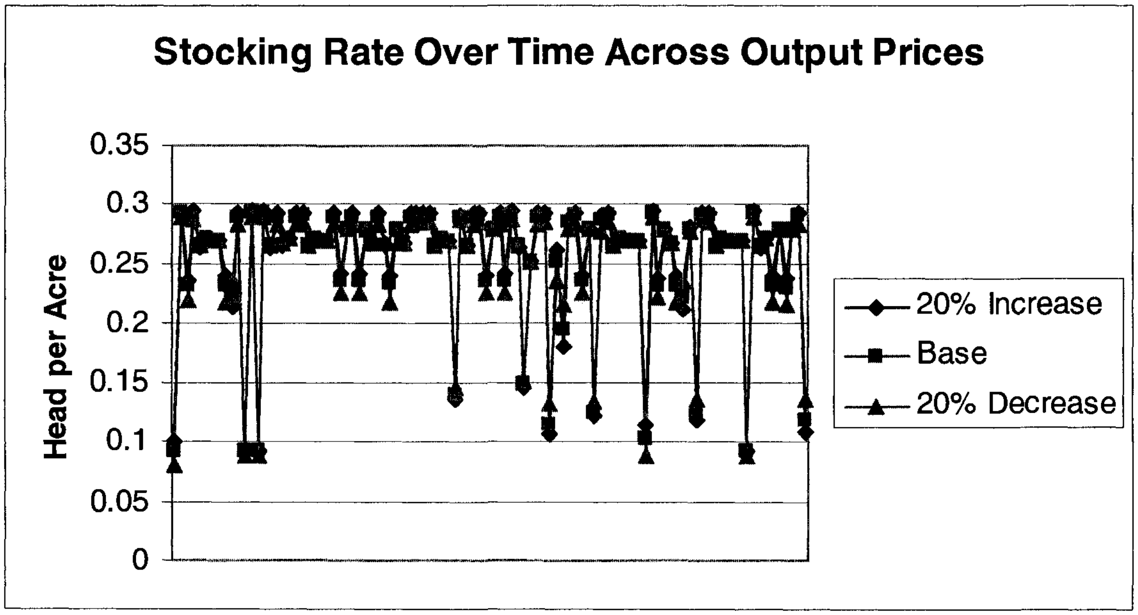

Figure A3.14 Time paths of Stocking Rate across Output Prices

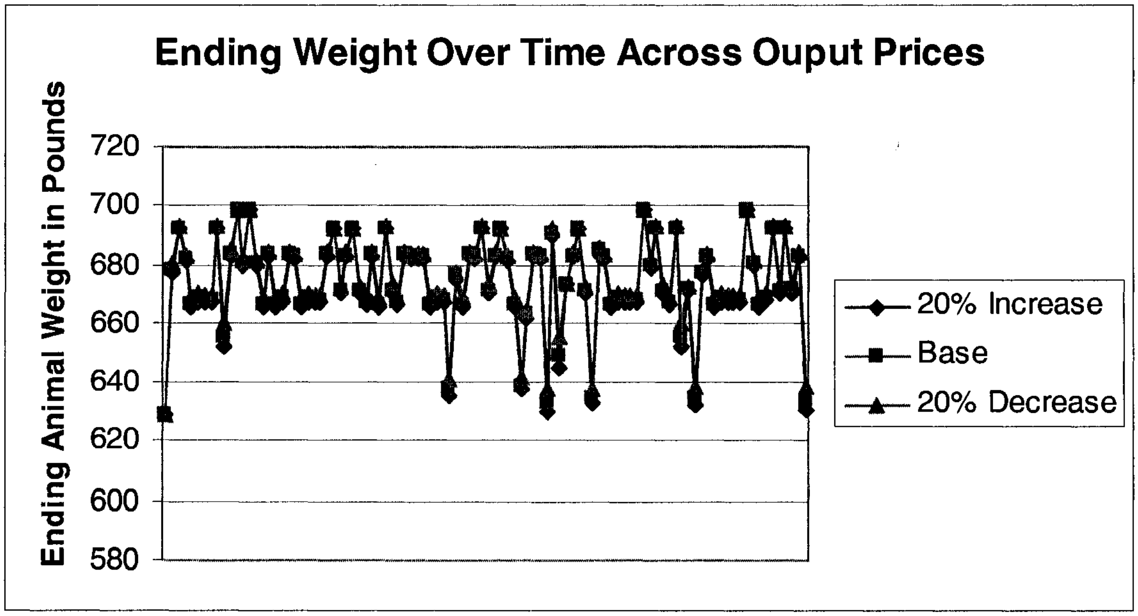

Figure A3.15 Time paths of Ending Weights across Output Prices

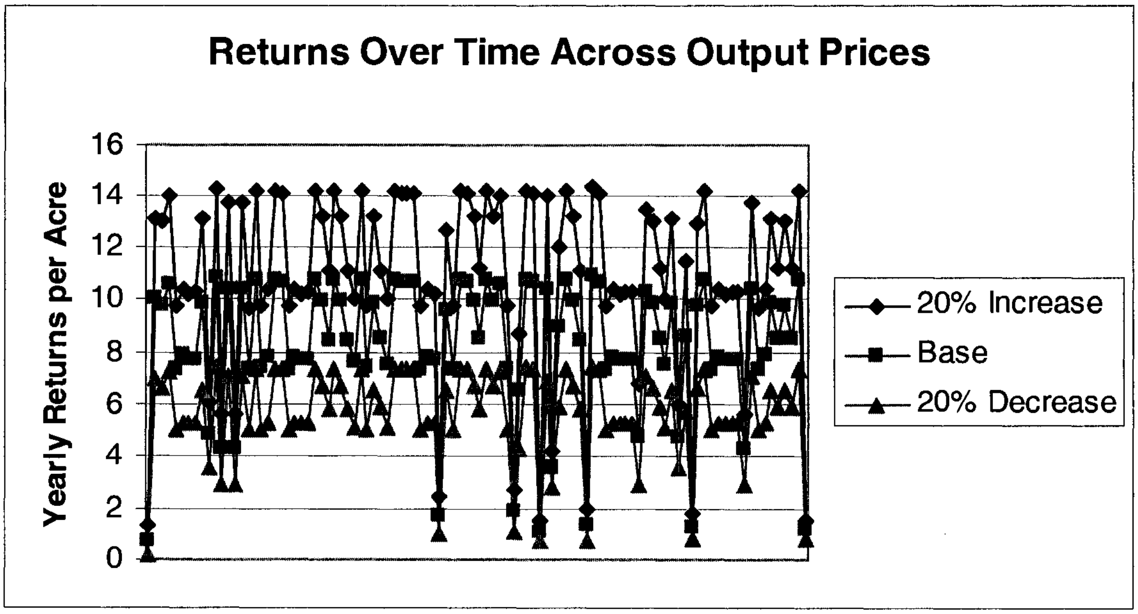

Figure A3.16 Time paths of Yearly Returns across Output Prices