# SEDIMENT TRANSPORT THROUGH PIPES 

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## FOREWORD

The work reported herein was conducted in the Hydraulics Laboratory of Colorado $A$ and $M$ College under contract with Armco Drainage and Metal Products Inc., Middletown, Ohio. It is a continuation of an earlier program sponsored by the same corporation together with Research Corporation of Santa Monica, California. Mr. George E. Shafer, Chief Engineer, Armco Drainage and Metal Products, represented the sponsor, whose assistance was essential to the operation of this research project. Mr. R. J. Garde was the principal investigator and his studies resulted in a thesis leading to the degree of Master of Science in Irrigation Engineering.

Since the thesis fully reports and discusses the findings of this research, it serves the additional purpose of a final report to the sponsor of the project and others interested.

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## NOMENCLATURE

| Symbols | Definition | Unit |
| :---: | :---: | :---: |
| a | Distance measured from bed of channel where $C_{a}$ is measured | L |
| A | Area | $\mathrm{L}^{2}$ |
| $A_{1}, A_{2}, A_{3}$ | Constants |  |
| C | Local concentration in per cent by weight |  |
| $\mathrm{C}_{1}$ | Relative absolute volume of sediment |  |
| $\mathrm{C}_{\text {a }}$ | Local concentration in per cent by weight at distance a from the bed of channel |  |
| $\mathrm{C}_{\mathrm{T}}$ | Total sediment load concentration in per cent by weight |  |
| $\mathrm{C}_{\mathrm{x}}$ | Drag coefficient for a given sediment size |  |
| d | Median diameter of sediment in mm | L |
| $\mathrm{d}_{50}$ | Median diameter of sediment sample | L |
| D | Diameter of the pipe | L |
| $\mathrm{D}_{1}$ | Depth of flow | L |
| E | Energy required per foot of pipe per pound of sediment transported per second |  |
| f | Darcy-Weisbach resistance coefficient with or without sediment present |  |
| $\mathrm{f}_{1}$ | That component of $f$ for sediment-1aden flow, which is contributed by clear water flow |  |
| $\mathrm{f}_{2}$ | That component of $f$ for sediment-1aden flow, which is contributed by total sediment load carried; $f=f_{1}+f_{2}$ |  |
| g | Gravitational acceleration | $\mathrm{L} / \mathrm{T}^{2}$ |
| G | Total sediment discharge | L ${ }^{3} / \mathrm{T}$ |

## NOMENCLATURE --Continued

| Symbo1s | Definition | Unit |
| :---: | :---: | :---: |
| J | Hydraulic gradient along the pipe length with or without sediment |  |
| $\mathrm{J}_{\mathrm{e}}$ | Hydraulic gradient along the pipe length with clear water |  |
| k | Karman constant |  |
| k | Nikuradse equivalent uniform sand diameter | L |
| $\mathrm{k}_{1}, \mathrm{k}_{2}, \mathrm{k}_{3}$ | Constants |  |
| $\ell$ | Mixing length in theory of turbulence | L |
| L | Length of pipe | L |
| n | Constant |  |
| Q | Discharge of clear water or water sediment mixture | $L^{3} / T$ |
| r | Weight ratio of solids to air |  |
| Re | Reynolds number |  |
| s | Slope of energy gradient, slope of water surface for open channel flow, or slope of pipe |  |
| $\mathrm{S}_{\mathrm{f}}$ | Shape factor for sediment |  |
| t | Temperature in degrees centigrade |  |
| u | Mean velocity at a given point in the x -direction | L/T |
| $u^{\prime}, v^{\prime}, w^{\prime}$ | Velocity fluctuations in $x, y$ and $z$ directions respectively | L/T |
| v | Mean velocity over the total area of pipe | L/T |
| $\mathrm{v}_{\mathrm{m}}$ | Limit deposit velocity | L/T |
| $\omega$ | Settling velocity of the sediment particle | $\underline{L} \nleftarrow T$ |

## NOMENCLATURB --Continued

| Symbols | Definition | Unit |
| :---: | :---: | :---: |
| W | Rate of sediment transport | M/T |
| y | Depth of flow | L |
| $\mathrm{y}_{\mathrm{s}}$ | Depth of sand deposit | L |
| $z$ and $z_{1}$ | Constants |  |
| $\epsilon_{m}$ | Momentum exchange coefficient | $L^{2} / \mathrm{T}$ |
| $\epsilon_{s}$ | Sediment exchange coefficient | $\mathrm{L}^{2} / \mathrm{T}$ |
| $\gamma_{s}$ | Unit weight of sediment | $F / L^{3}$ |
| $\gamma_{m}$ | Unit weight of mixture | $F / L^{3}$ |
| $\gamma_{w}$ | Unit weight of water | F/L ${ }^{3}$ |
| $\rho$ | Mass density of mixture | $\frac{\mathrm{FT}^{2} / \mathrm{L}}{\mathrm{L}^{3}}$ |
| $\rho_{a}$ | Mass density of air | $\frac{\mathrm{FT}^{2} / \mathrm{L}}{\mathrm{L}^{3}}$ |
| $P_{s}$ | Mass density of sediment | $\frac{\mathrm{FT}^{2} / \mathrm{L}}{\mathrm{L}^{3}}$ |
| $\uparrow$ | Intensity of shear | $\mathrm{F} / \mathrm{L}^{2}$ |
| $\mu$ | Coefficient of dynamic viscosity | FT/L ${ }^{2}$ |

## Chapter I

## INTRODUCTION

During the past three decades, there has been significant progress in sedimentation engineering. This branch of science deals with the problems arising from the fact that flowing water has a variable capacity to carry sediment with it. Scouring or silting of canals, silting of reservoirs, degradation below dams, and scouring at hydraulic structures are some of the examples of sediment problems. The problems of sediment transport can be classified broadly as follows: (A) those involving alluvial channels, that is, channels formed of the same material as that being transported; and (b) those involving rigid boundaries such as pipes.

Considering the transportation of sediment within fixed boundaries, it has long been recognized that the process of transporting two phases (solids and liquid or gaseous phase) through closed pipes has a wide field of application, because of economic considerations. Attempts have been made to employ this method of transportation in many cases. But, before going into further detail about this matter, it is necessary to define "sediment" as used in the present discussion.

Definition of Sediment

The definition of sediment used herein has been accepted by the Subcommittee on Sediment Terminology of the American Geophysical Union as follows:

Fragmental material transported by, suspended in, or deposited by water or air or accumulated in beds by other natural agents; any detrital accumulation, such as loess.

Ordinarily this does not include ice or organic material floating on the surface. In this report it is used to describe noncohesive material such as fine sand, sand, and gravel being carried by water.

## Present Fields

of Application
Transportation of sediment through closed pipes, is used in various fields as described in the following paragraphs:

1. Dredging. -- One of the oldest and most important uses of sediment transport through pipes is the dredging process to remove sand, silt and other material from such places as rivers, canals, basins, and harbors. Improvement of the Delaware River has required, over the last 30 years, the dredging of about 980 million cubic feet (mcf) of earth. Today it continues to require maintenance dredging at a rate of about 25 mcf per year. (10). On the Loup River in Nebraska, dredging has been done for a number of years. In France also, dredging is carried out on a large scale. For example, the dredging of the Great Western Pass of the Gironde river mouth has removed 33 mcf of material in three years.
2. Construction of dams.-- Another operation utilizing sediment transport through pipes is the construction of hydraulic fill dams. In the construction of Fort Peck dam on Missouri river, about 3300 mcf of earth was placed by this method. This process was also used in the construction of Kingsley dam on the North Platte river.
3. Storm sewers.-- Knowledge of sediment transport by flowing water is also useful to the designers of storm and combined sewers. Usually a sewer is designed as an open channel with a free surface. Pressure sewers are those which flow full. Sewerage engineers define a self-cleansing velocity as that which will carry the detritus load in the flow without depositing on the bed of the sewer. Except in a few cases, an attempt is made to secure self-cleansing velocity for all flows.
4. Culverts.-- For economic reasons, pipe culverts are being used increasingly on highways to take care of surface drainage and cross drainage. The storm flow which the culvert must carry may contain an appreciable amount of clay, gravel and sometimes small rocks, and it is for the culvert designer to choose size, shape, and type of culvert in such a way that all this material can be efficiently carried, without deposition.
5. Coal and ore transport.-- Because of the many advantages, hydraulic transport is being employed increasingly for the transportation of mine and quarry products such as coal and ores. Below are listed some of the outstanding advantages (10):
a. Ease of installation.
b. Ease of overcoming both natural and man-made obstacles, such as hills, waterways and buildings.
c. Only a small number of persons is required for erection, operation, and maintenance of equipment.
d. All of the moving mechanical parts are grouped at the pumping station.
e. Low cost of transportation.

The Pittsburgh Consolidation Coal Company, recently spent about $\$ 550,000$ (24) to construct a test line at Catiz, Ohio, to assure the efficacy of this system of hauling coal. It is planned that about $18,000,000$ tons of pulverized coal will be delivered in 15 years to the Cleveland Electric Company. The cost of the entire project is estimated to be between 8 and 10 million dollars.
6. Chemical engineering.-- During the past two decades, chemical engineers have contributed to the knowledge of this process and they have used it in catalytic reactions and similar operations. It is because of the ease with which solids can be added to and removed from the reaction zone, by pneumatic transport, that this process finds many applications in chemical engineering.

In spite of all these applications of the sediment transport through pipes, there is a definite lack of sufficient information about this process to approach the related design problems in a rational way. Thus for example, a storm sewer designer has very little basis, at present, on which to determine the self-cleansing velocity, and must rely to a great extent on his intuition. Hence, it would be a decided advantage to the designer to have sufficient information to be able to arrive at a logical design of such factors as size and shape of sewer, hydraulic slope, and self-cleansing velocity. The same situation exists with respect to other applications. Therefore, designers are badly in need of such information as power and water required for the transport of sediment through pipes which may or may not flow full.

## Previous Studies

Some work has been done in this field by the dredging engineers, the hydraulic engineers working in laboratories, the chemical engineers and the mining engineers. Hydraulic engineers have studied the sediment transport through open channels and smooth pipes and have contributed to our knowledge of the mechanism of sediment transport. Dredging engineers have directed their attention primarily to the study of head loss or energy requirements as a function of sediment load carried. Chemical engineers have studied the problem essentially from the same approach, but by using gas as a fluid phase. Very little work has been done using pipes with different types of boundaries, such as helical corrugated or standard corrugated.

## Problem

Under the limitations set for the present studies, which are listed at the end of this chapter, the problem can be stated as a study of the sediment transport phenomenon through pipes with special reference to

1. Effect of boundary form, such as smooth, helical corrugated, and standard corrugated.
2. Effect of sediment characteristics, such as mean diameter and size distribution of particles.
3. Effect of nature of sediment transport, such as either in full suspension or partly in suspension and partly carried along the bottom, on the Darcy-Weisbach resistant coefficient $f$; the energy required to carry a given
amount of sediment; and the energy required per foot of pipe per pound of sediment transported and water used.

## Limitations

Following limitations were imposed on the present studies:

1. The experimental data taken for this thesis were limited to twelve-inch diameter pipes; (a) geometrically smooth pipe, (b) standard corrugated pipe, and (c) helical corrugated pipe. Using the data of other experimenters, pipe diameter was also varied.
2. Only one size of sediment of mean diameter 0.60 mm was used; but data taken by others being available, effect of sediment characteristics was also studied.
3. Only the full pipe flow condition was studied.
4. Water was used as a liquid phase.

The problem of sediment transport through pipes has received attention from investigators in various fields during the last fifty years or so. Hydraulic engineers, dredging engineers, and personnel working in chemical, mining, and pulp industries are some of those who have contributed to our present day knowledge of the problem. But due to the large number of variables which govern the phenomenon, very little work has been done to throw light on the role of these variables. For the purpose of this study, the literature reviewed was divided into five categories as listed below:

1. Studies relating to internal mechanism of sediment transport through pipes and open channe1s.
2. Studies of the energy requirements, and the resistance coefficient.
3. Studies of effect of boundary.
4. Pneumatic transport.
5. Misce11aneous.

Thus, the literature reviewed in the following pages is neither in chronological nor alphabetical order; instead, it has been arranged for convenience of writing.

Studies Relating to Internal
Mechanism of Sediment Transport
Ismail (19) published a paper in 1952 on the experiments conducted to study the sediment transport through a closed, horizontal,
rectangular channel 10.50 in. wide and 3 in. deep. The total length of the conduit was 40 ft and the length of test section was 28.50 ft . Sand with sediment diameters of 0.10 mm and 0.16 mm was used. For each sand content, experiments were performed with four different velocities. Measurements of the velocity distribution were made by a standard Prandt1 pitot tube 0.25 in. in diameter. Quantities measured were head loss, discharge, velocity profiles, and sediment concentration profiles and temperature.

The effect of sediment on $f, \epsilon_{m}, \epsilon_{s}$, and $K$ was studied, where $f$ is Darcy-Weisbach resistance coefficient, $\in m$ and $\epsilon_{s}$ are the transfer coefficients for momentum and sediment respectively, and $K$ is Von Karman "universal constant".

Ismail's conclusions were:
a. The Karman "universal constant" $X$ decreases as the sediment load in suspension increases. The minimum value obtained for $K$ was 0.20 when total concentration $C_{T}$ was 43 gm per liter.
b. The change in $k$ does not follow the change in concentration from point to point over one cross section, but it varies from section to section maintaining a constant value over each section.
c. The value of $\epsilon_{m}$ is affected by the presence of sediment only through the changes in $K$.
d. The Darcy-Weisbach resistance coefficient $f$ is not affected by the presence of sediment up to a point where sediment load is great enough to form dunes; then $f$ becomes greater than that for clear water.

Laursen and Lin made an analysis of the same data and concluded that $K$ was not adequately defined to permit reliable conclusions, that sediment has little or no effect on flow, and that the proportionality factor in $\epsilon_{s} \propto \epsilon_{m}$ is equal to or less than one.

Vanoni (31) 1946 , published results of the experiments conducted by him on transportation of suspended sediment by water in a flume 33.25 in. wide and 60 ft long, the slope of which could be adjusted. The experimental sediment distribution in the vertical was compared with the theoretical distribution. The formula

$$
\frac{C}{C_{a}}=\left[\begin{array}{cc}
\frac{D_{1}-y}{y} & \frac{a}{D_{1}-a} \tag{1}
\end{array}\right] \mathrm{Z}_{1}
$$

where $C$ is the concentration at distance $y, C_{a}$ is the concentration at distance $a, D_{1}$ is the depth of $f$ low and $Z_{1}$ is a constant, was obtained on the assumption that $\epsilon_{s}$ and $\epsilon_{m}$ are equal. His conclusions are summarized as follows:
a. The distribution of relative concentration of suspended load is given by the above equation, but the value of $Z_{1}$ given by the theory does not agree with the value of $Z_{1}$ that fits experimental data.
b. The above disagreement is attributed to the action of random turbulent fluctuations in suspended sediment and "slip" between the fluid and the sediment as the sediment is accelerated. This makes $\epsilon_{S}$ to differ from $\epsilon_{m}$.
c. For fine material, the coefficient of sediment transfer tends to exceed the coefficient of momentum transfer; for coarser material the opposite tendency is found.
d. Suspended load decreases the value of $K$, which characterises the effectiveness of turbulence in transferring momentum. Reduction of $K$ means the mixing is less effective and would indicate that the sediment tends to suppress or damp out the turbulence.

Chamberlain (6) 1955 , studied the internal mechanism of sediment transport with 0.20 mm sediment using 12 in. diameter smooth, helical corrugated, and standard corrugated pipes. Concentration profiles were taken along the vertical and horizontal diameters. The horizontal concentration profiles revealed that the concentration was constant along the horizontal diameters for smooth and the standard corrugated pipes. For the helical corrugated pipe there was a large deviation of concentration near the wall from the mean concentration across the section. Secondary circulation maintained a more uniform sediment concentration over the area normal to the flow in the helical corrugated pipe. Values of $K$ were computed for the smooth and standard corrugated pipes and they varied from 0.21 to 0.34 for the smooth pipe and from 0.34 to 1.33 for the standard corrugated pipe.

Dane1 (9) 1939, has given an explanation for the observed fact, that for certain material the head loss for a limited range of velocities is smaller than that for clear water. This may be explained as being due to the damping effect of density gradation on turbulence, just as the density variations with height cause the calmness of atmosphere at the sunset. He also mentioned the importance of the size of the material in this phenomenon. If the particles are coarse, their continued falling through the fluid creates the turbulence and there is
little chance that density variation can reduce turbulence to the point where the head loss is less than that for water at the same velocity.

## Studies of Energy Requirements and

 the Resistance CoefficientPerhaps the first reference regarding sediment transport through pipes is that of Blatch (5) 1906. One inch diameter brass and galvanized iron pipes of approximately 27 ft length were used, while the test section was only 12 ft long. The sediment used was of two sizes -0.20 mm and 0.59 mm median diameter, both of specific gravity 2.64. The range of velocities attained was from 0.75 fps to about $16.00 \mathrm{fps}$. One of the important concepts introduced was that of "economic velocity" which is the velocity at which, for a given per cent of sand in water, the head loss per unit length is a minimum. It was found that for 1 in. pipe, with velocity between 3.50 fps and 4.00 fps, there is a transition zone which depends upon size and uniformity of sediment. For fine and uniform sediment, the transition zone is narrow.

Gregory (15) 1927, published the results of experiments in pumping clay slurry. Friction losses which occurred while pumping slurry through 4 in. cast iron pipe were studied. The total length of pipe was 370 ft while the test section was about 250 ft , consisting of 200 ft of straight portion plus additional length of 30 ft and two ells. The discharge was measured in a tank and the velocity was computed. Other quantities measured were head loss, rate of sediment transport, and temperature. The size of slurry ranged from colloidal to microscopic and it did not settle quickly nor cake in the pipe.

Plotting the hydraulic gradient $J$ against the velocity $V$, it was found that when the material in flow was between 18.60 and 35.30 per cent by weight, the plot substantially agreed with that for clear water. When the velocity was decreased to a certain point, a region of critical velocity was reached, whereas for still smaller velocities the head loss was practically constant.

Wilson (33) 1942, approached the problem of sediment transport by assuming that the total head loss is equal to the head loss for clear water flow at the same velocity, plus the head required for transporting sediment. He arrived at the equation

$$
\begin{equation*}
J=\frac{f V^{2}}{2 g D}+\frac{\gamma_{s}-\gamma_{w}}{\gamma_{w}}\left(1+A_{1}\right) \frac{C T \omega}{v}, \tag{2}
\end{equation*}
$$

where $\omega$ is the settling velocity and $A_{1}$ is a constant. He also recognized the importance of settling velocity of sediment as the significant parameter to describe the sediment. For constant values of $C_{T}, \omega, f$, and $D$, an equation was developed to determine the condition for deposition.

Wilson (34) 1945, analyzed the data taken by Blatch, Howard, and others. A definite contribution was made by introducing a new dimensionless parameter to describe the energy required for sediment transport, namely,

$$
\mathrm{E}=\frac{\mathrm{J}}{\frac{\mathrm{C}_{\mathrm{T}}}{100}}=\begin{align*}
& \text { Energy required per foot of pipe per }  \tag{3}\\
& \text { pound of sediment transported. }
\end{align*}
$$

His analysis of data led to the following general conclusions:
a. Since $E$ is proportional to $J / C_{T}$, the most efficient transportation of sediment through pipes, as far as energy required is concerned, is when $C_{T}$ is a maximum, for a given amount of sediment to be transported.
b. A limit to the above statement is the point beyond which one cannot decrease the discharge for given solids of a particular size, without clogging the pipe.
c. Analysis of Howard's data (18) for pipe with rifiing showed that with a given quantity of mixture flowing, more energy per unit mass of the solids transported is required to transport a small solids load than a large solids load.

Howard (17) 1939, studied the transportation of sediment through 2 in. and 4 in. pipes. The length of pipes under test was about 14 ft . The sediments tested are given below:

| Commercial Name | 50 per cent size mm |  | Classification |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| Pea Gravel | 2.50 | 0.40 |  |
| Pearl River Sand | 0.024 |  | Medium grave1 |
| Laboratory Loess | 0.001 | Silt |  |
| Buck Shot |  | Clay |  |

The quantities measured were pressure loss, sediment concentration, and velocity. Important conclusions drawn from these tests are as follows:
a. There are three distinct ways in which sediment is transported through pipes: - by rolling when the velocity is small; by jerking when the velocity is medium; and by motion of all the particles over the entire cross-section of pipe when the velocity is large.
b. For pipes carrying sand, $f$ decreases with an increase
in velocity.
c. Values of $f$ will increase with an increase in solids concentration for any given velocity.
d. Bconomic velocity for transporting solids depends upon the character of sediment to be carried, and each class of sediment will probably have a different economic velocity for the same size of pipe.
e. A very fine sediment should be transported with much less head loss than large sediment.
f. Extension of results from a small pipe line to a pipe line of greater diameter must be qualitative and not governed by any 1aw of corresponding velocities.
$0^{\prime}$ Brien and Folsom (22) 1937, published a paper on transporm tation of sand in pipe lines, in which they discussed the results of experiments carried out using 2 in. and 3 in. wrought iron pipes, with three different sizes of sands ranging between 0.0065 in. and 0.05 in. in diameter. The analysis of data revealed that the Darcy-Weisbach equation

$$
J=\frac{f V^{2}}{2 g D}
$$

is adequate to study flow of homogeneous as we11 as non-homogeneous mixtures.

Durepaire (13) 1939, in the discussion of Howard's paper (17) described the results of tests carried out at Nantes Harbour. The inside diameter of pipe was 2.05 in. and Loire river sand with maximum grain size 0.30 mm was used. Results can be summarized as follows:
a. With all the sediment load in suspension and within the range of concentration encountered in the experiments (up to 40 per cent by volume), the head loss expressed in feet of mixture was the same as that for clear water, irrespective of concentration, except at the state when deposition is impending.
b. Critifal velocity (i.e., velocity at which deposition begins for a given concentration) and economic velocity (i.e., velocity at which head loss is minimum for a given concentration) occur approximately at the same velocity.
c. No jerking motion of the sediment was observed.
d. It was found in the partial deposition phase that for a constant concentration of sand, head loss is greater when total discharge decreases. At the same time the height of deposited sediment increases. The deposit of sediment in a given run was of rather uniform depth because there was no jerking motion.

Durand (10) 1953, published the results of experiments carried on in pipes of diameters varying from 1.50 in. to 28 in., while sediment diameter varied from 20 microns to 100 mm . The study revealed the necessity for classifying the sediment mixtures according to grain size into homogeneous and heterogeneous classes. The complete classification suggested was:
a. Homogeneous mixtures: clays, fine ash, and very finely powdered coal (up to 20 to 30 microns).
b. Intermediary mixtures: silts (25 to 50 microns)
c. Heterogeneous mixtures:
i) Heterogeneous mixtures transported by suspension - fine sand, powdered coal, and slurry (from 50 microns to 0.2 mm .)
ii) Transition category: coarse sand and fine grained coal (from 0.2 mm to 2 mm )
iii) Heterogeneous mixtures transported by saltation: gravels, pebbles, and lumps of coal (above 2 mm. )

The importance of the parameter $V^{2} / g D$ was also shown and it was possible to establish a sediment transport function for non-deposit regime in terms of $V^{2} / g D, J-J_{e} / J_{e} C_{1}$ and other variables describing fluid sediment, $C_{1}$ being the relative absolute volume of sediment.

Craven (8) in 1952 studied sediment transport through pipes in the range of deposition. The pipe diameter used was 5.55 in . and the sediment was uniform quartz sand of $0.25 \mathrm{~mm}, 0.58 \mathrm{~mm}$ and 1.62 mm diameter. It was found that for given values of parameters $V / \omega$ and $Q / D \sqrt[5]{\Delta \gamma / \rho}$ when $C_{T}$ is very small, isolated dunes appear in the pipe or are superimposed on the inert bed. As $C_{T}$ is increased, the dune spacing decreases until dunes join together. Further increase in $\mathrm{C}_{\mathrm{T}}$ causes dunes to lengthen and to flatten until the bed is perfectly plane.

Although bed-configuration for each of the sands evolved through the same pattern, the value of $\mathrm{C}_{\mathrm{T}}$ at which a given change in the form occurred, was different for each grain size. It was also noticed that, the larger the value of $V / \omega$, the greater is the tendency for the sediment to travel in a series of dunes. If actual shear values are far greater than critical in the open channel flow, the majority
of bed load equations take the form

$$
\begin{equation*}
\mathrm{J} \propto \frac{\Delta \gamma}{\gamma}\left(\mathrm{C}_{\mathrm{T}}\right)^{\mathrm{n}} \tag{4}
\end{equation*}
$$

where $n$ varies from $\frac{1}{2}$ to $\frac{2}{3}$. It was found that a similar relation holds good for pipe flow, the equation taking the form

$$
\begin{equation*}
J \approx 0.606 \frac{\Delta Y}{\gamma_{\mathrm{w}}}\left(\mathrm{C}_{\mathrm{T}}\right)^{2 / 3} \tag{5}
\end{equation*}
$$

Ambrose (1) 1952, studied free surface flow in pipes with a sediment-water mixture. With the help of dimensional analysis two functions were evolved:
a. Transport function

$$
\frac{Q / D^{2}}{\left(\frac{\gamma_{s}}{\gamma_{W}}-1\right)^{2 / 5} Q_{S}^{1 / 5} g^{2 / 5}},
$$

which was dependent on Reynolds number, specific gravity of sediment, $y / D, y_{S} / D, d / D$ and $K / D$, where $y$ and $y_{S}$ are the depths of flow and sand bed respectively. $K$ is Nikuradse equivalent sand grain size and $Q_{s}$ is absolute volume rate of transport.
b. The discharge function

$$
\frac{\mathrm{Q}}{\mathrm{~g}^{1 / 2} \mathrm{~s}^{1 / 2} \mathrm{D}^{5 / 2}}
$$

depending on the same parameters described above.
It was found that the transport function increased with an increase in $y / D$, but reached a maximum value of 2.90. Hence no deposition will occur in the pipes if the transport function has a value
greater than 2.90. It was also found that other variables have no influence on the transport function and therefore it depends solely on geometry of flow.

In regard to the discharge function, it was found that for impending deposition $d / D$ has no effect, while for an inert bed it was necessary to include a factor in the function, which represents the effect of $d / D$, in order to obtain good correlation. This was interpreted to mean that the parameter $d / D$ is an indication of relative roughness of bed.

## Studies of Effect of Boundary

Chamberlain (6) 1955, reported the studies on 12 in . diameter smooth, helical-corrugated and standard corrugated pipes using 0.20 mm diameter sediment, in the range of suspension. Effect of boundary form on the Darcy-Weisbach resistance coefficient $f$, horse power required, discharge of water-sediment mixture, amount of sediment transported, and incipient deposition velocity was studied. Important conclusions arrived at are summarized below:
a. The Darcy-Weisbach resistance coefficient $f$ was unaffected by the presence of sediment, in the regime studied.
b. The mean velocity at which deposition started became less dependent on the magnitude of total load as velocities were increased. This velocity was much higher in the smooth pipe than in helical corrugated or standard corrugated pipe for fixed total load.
c. The horsepower input required to maintain a certain discharge of sand-water mixture was not materially greater than that necessary to pump the same discharge of water, as long as sediment was in suspension. Horsepower was computed in terms of discharge of mixture, with unit weight and head loss expressed in terms of clear water.
d. Taking the minimum point for a constant $\mathrm{C}_{\mathrm{T}}$-curve on the $J: V$ diagram as an operating point, the helical-corrugated pipe required less horsepower for a fixed total sediment load, than did standard corrugated pipe and usually less than smooth pipe.
e. Helical corrugated pipe and standard corrugated pipe delivered more sediment for a given quantity of water sediment discharge than did smooth pipe.
U. S. Corps of Engineers (26 and 27) 1952, published the results of experiments carried out at the Bonnevilie Hydraulics Laboratory on the head loss in corrugated pipes with clear water flow. The resistance coefficient $f$ was computed from $J$ found in two ways: 1) with piezometer taps in crests of corrugated pipe, 2) with piezometer taps in troughs. The values of $f$ showed that the variation in the value of $f$ calculated as above, is within the experimental error.
U. S. Corps of Engineers (28 and 29) also described the results of experiments on corrugated pipes with part of the bottom paved to a smooth surface. These tests were on 5 ft and 7 ft diameter pipes. It was found that in general the effect of paving was a reduction in the value of $f$ and an increase in the carrying capacity, compared to the case of pipe without paving. The degree of these changes was a function of per cent of bottom paved.

Howard (18) 1941, reported the studies made on effect of rifling on transport of sand in 4 in. pipe. Results were also compared with those from 2 in. pipe to find the similarity in transportation characteristics if any. Object of the tests was to develop an optimum design of rifling for pipes which carry sediment. The materials tested were:

| Commercial name | 50 per cent size mm | Classification |
| :--- | :---: | :--- |
| Pea gravel | 2.50 | Medium gravel |
| Pearl river sand | 0.40 | Medium sand |
| Laboratory loess | 0.024 | Silt |
| Buck shot | 0.001 | Clay |

Using sand in 4 in. pipe, it was found that the length of rifling should be one third the length of the over all pipe section. The next series of runs were designed to develop an efficient rifling. Results thus obtained were then compared with 2 in. diameter pipe using geometrically similar rifling. From these investigations the following conclusions were drawn:
a. Rifling will increase the efficiency of the line when coarse sand and grave1 are transported.
b. Rifling will reduce the efficiency of the line when silt or clay is transported.

Morris (21) 1955, in studying flow through rough conduits recognized three basic types of flow:
a. Flow over individual elements, where the apparent friction factor (resistance coefficient) results from the form drag on roughness elements in addition to friction drag (surface drag) on the wall surface between elements.
b. Wake interference flow, in which the elements are sufficiently close together so that the zone of separation, vortex generation and dissipation associated with each element are not completely developed before the next element is encountered.
c. Quasi-smooth or skimming flow where the elements are so close together that the flow essentially skims the crests of elements. Criteria were given to determine which type of flow is prevailing in a particular case and equations were developed for the resistance coefficient $f$. It was also shown that these equations can be extended to surfaces of variable roughness by using the average values of the roughness dimensions. Data from various sources were reanalysed employing this concept and it showed the soundness of the approach.

## Pneumatic Transport

Wood and Bailey (35) 1939, presented the research done on transportation of sand and linseed sediments by air in horizontal pipe of 2.9 in. diameter and 25 ft long. The data were taken in the range where sediment moved in saltation and it was found that head loss is a linear function of $C_{T}$.

Vogt and White (32) 1948, published a paper on "Friction in the flow of suspensions - granular solids in gases through pipes." Friction losses were studied in $0.50-i n$. pipe carrying sand sizes 0.0088 in. 0.0138 in., 0.0018 in. and 0.0287 in.; steel shots 0.0165 in.; clover seeds 0.046 in.; and wheat 0.158 in. in diameter. Both horizontal and vertical pipes were used. Using $\left(J-J_{e} V_{\checkmark} J_{e}\right.$ as a
significant parameter it was found that

$$
\begin{equation*}
\frac{J-J_{e}}{J e}=A_{1} \quad\left(\frac{D}{d}\right\rangle^{2}\left[\frac{\rho_{a}}{\rho_{s}} \frac{r}{\operatorname{Re}}{ }^{-A_{2}},\right. \tag{6}
\end{equation*}
$$

where $A_{1}$ and $A_{2}$ are constants and $r$ is the ratio of solids to air.

Farbar (14) 1949 , in studying the flow characteristics of the solids-gas mixtures in horizontal and vertical pipes, used a $17-\mathrm{mm}$ tube with material ranging from 8 microns to 220 microns in diameter. It was found that at large concentrations there was a tendency for the pressure drop to be independent of concentration. He a1so found that the flow characteristics of solids-gas mixtures, in which size distribution covers wide ranges, differ considerably from mixtures of narrow size range.

Hariu and Mo1stad (16) in 1949 described the experiments in vertical glass tubes of 0.267 in. and 0.532 in. diameters. The object of investigation was to study the effect of gas velocity and concentration on pressure drop. The sediment sizes used were Ottawa sand 0.00165 ft and 0.00117 ft ; sea sand 0.00090 ft and 0.00070 ft ; microspheroid cracking catalyst 0.00036 ft ; and ground cracking catalyst 0.00036 ft . The pressure drop was divided into that due to gas alone when no sediment is present, that due to friction loss from contact between sediment and pipe, and that due to solids-static loss. The solids-static loss was described as the pressure drop in the gas due to supporting dispersed solids in the given length of vertical tube.

Maltby (20) 1905, carried out tests on the discharge pipe lines for 8 dredges. This work was done on the Mississippi River downstream from the junction of the Ohio River and the Mississippi River at Cairo, Illinois. The diameter of the pipes varied from 32 in. to 34.50 in. while the material transported was sand. Velocities in the pipes were measured by a pitot tube and they ranged from 14 fps to 24 fps. There is no record either of the size of sand or of the concentrations. No definite conclusions were drawn in the article. The coefficient of resistance $f$ was found to be between 0.013 and 0.015 .

Babbit and Caldwe11 (2) in 1939 published the studies made on laminar flow of sludges in 1 in., 2 in. and 3 in. pipes. The concluded that sludges, such as mixtures of clay and water used in deep we 11 boring, sewage sludges, sludges from water softening plants, and similar other aqueous suspensions of fine particles, act as free plastics. They gave formulae for the lower critical velocity and the upper critical velocity which were functions of coefficient of rigidity, yield stress, diameter of the pipe, and density of fluid. Thus, instead of using a coefficient of viscosity, he found it necessary to use the yield stress and the coefficient of rigidity. Among the important factors affecting yield value and coefficient of rigidity were: concentration of suspended matter, size and character of the particles of suspended matter, nature of continuous phase, temperature, thyxotropy, slippage, agitation, and gas content of the sludge.

The same authors (3) reported in 1940 the studies they made on the flow of sludge through pipes in the turbulent flow region. The
velocities attained ranged from 0.50 fps to 35.00 fps , and pipe diameters ranged from 0.50 in. to 3.00 in. Observations for the head loss were made with each of eight sludges tested in four test pipes with various velocities.

A plot of $f$ against $R_{e}$ showed that the agreement between this plot and similar plot with clear water as fluid was quite good. Friction factors obtained from the sludges were slightly higher than those obtained from clear water. For velocities less than the critical velocity, the hydraulic gradient deviated from that for clear water with the same diameter of pipe so that the hydraulic gradient was higher when compared to clear water flow at the same velocity.

Binder and Busher (4) 1946, published an article on "A Steady flow of plastics through pipes', in which it was stated that in the laminar flow range - with a Reynolds number less than 2100 - the denominator in $R_{e}$ be taken as apparent viscosity. As Babbit and Caldwell have pointed out, each yield value and coefficient of rigidity is independent of size and roughness of pipe. Accepting this conclusion, the authors suggested a method to determine yield value and coefficient of rigidity for a particular plastic by carrying out tests with model pipes.

## Chapter III

THEORETICAL CONSIDERATIONS

The transport of sediments by any fluid depends upon the turbulence which is created by a combination of the boundary conditions and the flow conditions. Therefore, to have a clear understanding of the mechanism of sediment transport - either through pipes or in open channels - one must study turbulence as a means of sediment transport.

For convenience of discussion the present chapter will be divided into: definitions of different kinds of fluids, mechanism of sediment transport, the current approach to sediment transport through pipes, and dimensional analysis on which the chapter on Presentation and Analysis of Data will be based.

Types of Fluids
A fluid is defined as a substance that deforms when subjected to a shear stress, no matter how small that shear stress may be.

Fig. 1 shows diagramatically the characteristics of the ideal fluid, the Newtonian fluid, and the non-Newtonian fluid. For a Newtonian fluid there exists a linear relationship between shear stress $\widehat{\sim}$ and the rate of angular deformation. While a non-Newtonian fluid is characterized by a non-linear relationship between shear stress and angular deformation. Similar to Newtonian fluids, ideal plastics also have a linear relationship between $\uparrow$ and rate of deformation but plastics sustain some stress before the flow starts. This stress is known as "yield-stress".

In the discussion that follows a mixture of sediment and water is assumed to satisfy the following requirements:
a. Sediment-water mixture is homogeneous.
b. Continuity equation

$$
\begin{equation*}
\frac{\partial \mathbf{u}}{\partial \mathbf{x}}+\frac{\partial \mathbf{v}}{\partial \mathbf{y}}+\frac{\partial \mathbf{w}}{\partial \mathbf{z}}=\mathbf{0} \tag{7}
\end{equation*}
$$

is applicable where $u, \nabla$, and $w$ are the three components of velocity in the $x, y$, anđ $z$ directions respectively.
c. Sediment-water mixture, in the limit case, is a homogeneous fluid of Newtonian type. For large-size sediment or for large concentrations, this assumption may not be valid.

Mechanism of Sediment Transport

Since the random velocities which make up turbulent flow also act as a means for carrying sediment particles into the interior of the fluid, the theory of turbulence provides a means of studying the mechanism of sediment transport and also its distribution in the vertical.

For large Reynolds numbers the individual particles in a fluid no longer have a velocity which is parallel to the direction of flow. Instead they have components in all three directions. If $u$ is the mean velocity in the $x$-direction (i.e. direction of flow) and $u^{\prime}, v^{\prime}$, and $w^{\prime}$ are velocity fluctuations in $x, y$ and $z$ directions respectively, then $i t$ will be seen that the mean-values of $u^{\prime}, v^{\prime}$ and $w^{\prime}$ over a long time interval or at any instant over a large area
are zero. This is stated symbolically as

$$
\begin{equation*}
\frac{1}{T} \int_{0}^{T} u^{\prime} d T=0=\frac{1}{A} \int_{0}^{A} u^{\prime} d A \tag{8}
\end{equation*}
$$

where $A$ and $T$ are area and time respectively.
Similar expressions can be written for $v^{\prime}$ and $w^{\prime}$. Since the flow is considered to be two-dimensional, the $w^{*}$ component is not considered.

Taking a unit horizontal area, it can be shown that the tangential stress resulting from exchange of momentum due to velocity fluctuations is given by

$$
\begin{equation*}
T=P u^{\prime} v^{\prime} \tag{9}
\end{equation*}
$$

If $\ell$ is defined as the distance from layer bb to oo (see Fig. 2) such that an element of fluid from layer bb, due to the vertical velocity fluctuation, can reach the region oo and mix with the fluid there, then

$$
\begin{equation*}
u^{\prime} \propto \ell \frac{d u}{d y}, \tag{10}
\end{equation*}
$$

where $d u / d y$ is the velocity gradient in the vertical direction. By further assuming that $v^{\prime}$ also is proportional to $\ell d u / d y$, then

$$
\begin{equation*}
\uparrow=\rho \ell^{2}\left(\frac{d u}{d y}\right)^{2}=\rho\left[\ell^{2} \frac{d u}{d y}\right] \frac{d u}{d y}=\rho \epsilon_{m} \frac{d u}{d y} . \tag{11}
\end{equation*}
$$

The term $\rho \ell^{2} d u / d y$, is called the "eddy viscosity", while $\ell^{2} d u / d y$ is called the "momentum transfer coefficient".

In the case where sediment is in suspension, the sediment transfer may be compared with the momentum transfer an equation analogous to (11) can be written,

$$
\begin{equation*}
g_{1}=\rho \epsilon_{\mathrm{s}} \frac{\mathrm{dC}}{\mathrm{dy}}, \tag{12}
\end{equation*}
$$

where $C$ denotes concentration at any given depth, dC/dy represents the concentration gradient, $\epsilon_{s}$ is the sediment transfer coefficient, and $g_{1}$ represents the amount of sediment moved upward through a unit area per second. If $\omega$ is the settling velocity of individual particles, then the amount of sediment falling downward through a unit area per second will be $\omega C$ where $C$ is the concentration at that depth. Under equilibrium conditions the sum of these two terms must be zero. Hence

$$
\begin{equation*}
\omega c+\epsilon_{s} \frac{d C}{d y}=0, \tag{13}
\end{equation*}
$$

which is the fundamental equation for the study of sediment transport in suspension.

In the early stages of the development of the theory of suspension of sediment, it was assumed that $\epsilon_{\mathrm{s}}$ is equal to $\epsilon_{\mathrm{m}}$. However, Vanoni (31) found that $\varepsilon_{\mathrm{s}}$ is not equal to $\varepsilon_{\mathrm{m}}$, but depends instead on the size of sediment in suspension. Fine sediment tends to make $\epsilon_{\mathrm{s}}$ greater than $\epsilon_{\mathrm{m}}$; coarse sediment has an opposite tendency. By solving Eq 13 for $C$ the equation for open channel flow,

$$
\begin{equation*}
\frac{c}{C_{a}}=\left[\frac{D_{1}-y}{y} \cdot \frac{a}{D_{1}-a}\right] \tag{14}
\end{equation*}
$$

can be obtained, where $C$ and $C_{a}$ are concentrations at levels $y$ and $a$ above the bottom respectively, $D_{1}$ is the depth of $f 10 w$, and $z$ is given by the equation

$$
\begin{equation*}
z=\frac{\omega}{K \sqrt{g D S}} \tag{15}
\end{equation*}
$$

where $s$ is the energy gradient or slope and $X$ is Von Karman's 'Universal constant". This constant is claimed to have a value 0.40, and it has been used as an index of the turbulence. Vanoni (31) found that for fine sediment of 0.10 mm size in suspension, values of $X$ were smaller or were reduced because of the presence of sediment.

## Current Approach to Sediment <br> Transport Through Pipes

A number of experimentors have found that the head loss (in feet of fluid flowing) for a uniform mixture of fine sediment and water flowing through pipe will be the same as that of a fluid of the same average density. Danel (9), Durepaire (13), and Durand (10) have also expressed the same view. A mathematical analysis is given for this by Chamberlain (6).

Another approach to the theory of sediment transport is that very fine sediment in suspension tends to damp the turbulence and therefore the coefficient of resistance decreases. Tison (6), while transporting chemical plant residue of 0.05 mm in size through 1 in . and 2 in . pipes, found that sediment reduces the resistance coefficient $f$ considerably. Durand (10) and Danel (9) have expressed similar views. In open channel flow a similar phenomenon is observed.

Makarechian has shown by experiments conducted recently with a turbulence tank at Colorado $A$ and $M$ College that, when fine sediment (wash load) is present in the water, more coarse bed material, such as sand, is held in suspension than that can be held in suspension in clear water. His explanation for this behavior is that, it is the combined effect of the increase in viscosity and the increase in density of fluid, which results in a decrease of fall velocity of the largersized sediment and more sediment is held in suspension.

Effect of Boundary Form on Mechanism of Sediment Transport

It has been found that type of the boundary form has a definite influence on the mechanism of sediment transport in pipes (6). In smooth and standard corrugated pipes, the horizontal concentration profiles are nearly uniform, i.e., there is very little variation in concentration at any point on a horizontal diameter from the mean concentration along this diameter. In helical corrugated pipe, there is deviation in concentration near walls from the mean concentration over the horizontal diameter.

The vertical concentration profiles are also affected by the boundary form. Smooth pipe has a vertical concentration profile similar to that in open channel flow. In general, there is a constant concentration (for fine sediment) in the vertical section of helical corrugated pipe. The vertical concentration profile for a standard corrugated pipe has a shape lying between the vertical concentration profiles for smooth and helical corrugated pipes.

It is not infrequently found that two extremes exist for solving problems in fluid mechanics. One solution may be completely empirical and the other may be completely deductive. The empirical approach depends principally upon drawing conclusions from experimental results, neglecting (if necessary) the physical phenomena occurring during the process. The purely deductive analysis may depend upon assumptions which are not valid in practice. It is seldom that these are reconciled without a compromise.

The approach based on dimensional analysis makes use of all the pertinent variables in order to arrive at dimensionless parameters which influence the phenomenon. This procedure reduces the number of variables and provides a means of systematizing research and analysis. Experience has shown that the functional relationships thus determined are not only adequate for numerical analysis and design requirements, but also provide in many cases the basis for a physical interpretation of the phenomenon under consideration.

For the foregoing reasons, dimensional analysis of the problem of sediment transport through pipes is given as follows.

The variables entering the problem can be classified into four categories:
a. Variables describing the sediment:
$P_{S}$ Density of sediment
d Mean diameter of sediment
$\sigma_{8}$ Geometric standard deviation
$S_{f}$ Shape factor
b. Variables describing the fluid:
$P_{w}$ Density of fluid
$\mu$ Coefficient of viscosity
c. Variables describing the flow:

V Mean velocity of flow
J Hydraulic gradient
$\mathrm{C}_{\mathrm{T}}$ Average sediment concentration
d. Variables describing the boundary

D Diameter of pipe
s Slope of pipe
$\emptyset$ Variable characterising form
Of all these variables only $J$ and $C_{T}$ are dependent variabies while the others are independent. Their functional relationship can be written as

$$
\begin{align*}
& \psi_{\mathrm{i}}\left(\rho_{\mathrm{s}}, \mathrm{~d}, \sigma_{\mathrm{g}}, \mathrm{~S}_{\mathrm{f}} ; \rho_{\mathrm{w}}, \mu ; \mathrm{v}, \mathrm{C}_{\mathrm{T}},\right.  \tag{16}\\
& \mathrm{J} ; \mathrm{D}, \mathrm{~s}, \varnothing)=0 .
\end{align*}
$$

During the present investigations, the slope of the pipe was held at horizontal, and $\sigma_{g}$ and $S_{f}$ were not considered as signifycant variables for the dimensional analysis given below. Therefore, Eq 16 can be written as

$$
\begin{equation*}
\psi_{2}\left(P_{\mathrm{s}}, \mathrm{~d}, P_{\mathrm{w}}, \mu, \mathrm{v}, \mathrm{C}_{\mathrm{T}}, \mathrm{~J}, \mathrm{D}, \varnothing\right)=0 \tag{17}
\end{equation*}
$$

and, with the proper assumption on $\psi_{2}, \mathrm{~J}$ can be expressed as

$$
\begin{equation*}
\mathrm{J}=\psi_{3}\left(\rho_{\mathrm{s}}, \mathrm{~d}, \rho_{\mathrm{w}}, \mu, \mathrm{~V}, \mathrm{C}_{\mathrm{T}}, \mathrm{D}, \varnothing\right) \tag{18}
\end{equation*}
$$

Selecting $P_{w}, V$, and $D$ as repeating variables and applying the $\pi$-theorem gives

$$
\begin{equation*}
\mathrm{J}=\psi_{4}\left(\frac{\rho_{\mathrm{S}}}{\rho_{\mathrm{w}}}, \frac{\mathrm{~d}}{\mathrm{D}}, \frac{\mathrm{VD} \mu}{\rho_{\mathrm{w}}}, \frac{\mathrm{v}^{2}}{\mathrm{gD}}, \mathrm{C}_{\mathrm{T}}, \frac{\emptyset}{\mathrm{D}}\right) \tag{19}
\end{equation*}
$$

Since for all the data used, sediment with specific gravity of about 2.65 was used and the continuous medium was water, $\rho_{s} / \rho_{w}$ was constant. Hence

$$
\begin{equation*}
\mathrm{f}=\frac{\mathrm{J}}{\mathrm{v}^{2} / 2 \mathrm{gD}}=\psi_{5}\left(\frac{\mathrm{~d}}{\mathrm{D}}, \quad \operatorname{Re}, \quad \mathrm{C}_{\mathrm{T}}, \frac{\varnothing}{\mathrm{D}}\right) \tag{20}
\end{equation*}
$$

This relationship will be used to study the variation of resistance coefficient with Reynolds number, concentration, nature of boundary and relative size of sediment.

In the phenomenon of surface resistance in pipe flow, it has been found that a dimensionless parameter $\operatorname{Re} \sqrt{\mathrm{f}}$ is very significant. More will be said about it in Chapter $V$, but suffice it to say, the relative thickness of the laminar sublayer in the pipe (i.e. the ratio of the thickness of laminar sublayer to pipe radius) is inversely proportional to $\operatorname{Re} \sqrt{\bar{f}}$. Therefore it is logical to assume that $\operatorname{Re} \sqrt{\sqrt{f}}$ may also play a significant role in sediment transport through pipes. The functional relationship for $\mathrm{Re}-/ \overline{\mathrm{f}}$ may be written as

$$
\begin{equation*}
\operatorname{Re}-\sqrt{\mathrm{f}}=\gamma_{6}\left(\frac{\mathrm{~d}}{\mathrm{D}}, \quad \mathrm{C}_{\mathrm{T}}, \frac{\varnothing}{\mathrm{D}}\right) . \tag{21}
\end{equation*}
$$

If $E$ is defined as (34) energy required for transporting a unit weight of sediment per second per foot of pipe, then a functional relationship between different variables will be of the form:

$$
\begin{equation*}
\mathrm{E}=\psi_{7}\left(W, \frac{\emptyset}{D}, \operatorname{Re}, \frac{d}{D}\right) \tag{22}
\end{equation*}
$$

where $W$ is the rate of sediment transport in pounds per second.
This is of course a dimensional equation since it contains a dimensional parameter $W$, the sediment discharge in weight per second. Thus dimensional analysis has shown that, in studying the problem of sediment transport through pipes, the significant dimensionless parameters will be: $J, \quad P_{S} / P_{w}, d / D, C_{T}, V^{2} / g D, \operatorname{Re}$, $\sqrt{\mathrm{f}}, \operatorname{Re} \sqrt{\mathrm{f}}$ together with the parameter describing the boundary form. These dimensionless parameters together with their combinations should be adequate to solve problems, such as influence of total sediment load on the resistance coefficient, for which a solution is sought in this thesis.

## Chapter IV

## EXPERIMENTAL BQUIPMENT AND PROCEDURE


#### Abstract

The study of three boundary forms of 12 -in. diameter pipe -smooth, helical corrugated, and standard corrugated -- with 0.60 mm sediment transport through them, was carried out at Fort Collins in the Hydraulics Laboratory of Colorado $A$ and $M$ College. A description of experimental equipment follows:


## General Lay-out

Fig. 3 shows the general lay-out of the complete recirculation system utilized. It consisted of a centrifugal pump of capacity approximately 10 cfs , which was driven by a $35-\mathrm{HP}, 870-\mathrm{RPM}$ motor. The fluid was discharged into a smooth vertical 12 -in. diameter pipe, 6 ft 3 in. long, at the end of which a 10 -in. diameter sharp-edged orifice was situated. With a right angle bend, the fluid was passed through 29 ft of $12-i n$. diameter helical-corrugated pipe, then through a 7 -ft vertical smooth pipe and then it entered the $76-\mathrm{ft}$ test section. At the end of the test section a plastic section 9 in. long was fixed, then a 3ft smooth pipe, a right angle bend of helical corrugated pipe above 8 $f t$ long, and finally a return helical corrugated pipe 50 ft long. Each of these was 12 in . in diameter. The end of the return pipe was connected to the suction of the pump. The system was filled by city water supply through a special valve。

Positions of two air valves, a valve for taking the sediment out, a 12-in. valve controling the discharge, the standpipe with a cone and quick-acting valve for adding the sediment are shown in the same figure.

The valve controlling discharge was situated 6 ft 6 in . upstream from the pump while the standpipe was 2 ft 6 in downstream of the valve. The standpipe was 6 in. diameter and $4 \mathrm{ft} h i g h$ and it was welded to the main pipe. At the other end of the 6-in. pipe a 6-in. quick-acting valve was fixed and on the top of it a cone 2 ft 6 in . diameter at top and 6 in. diameter at the bottom with 2 ft 6 in. height was joined.

## Quantities Measured

The purpose of this investigation was to study the overall phenomenon of sediment transport through pipes in the range where the whole quantity of sediment was transported in suspension and also when it was carried partly in suspension and partly along the bottom. In this connection the following quantities were measured me discharge of the mixture of sediment and water flowing through the pipe, the head loss at definite intervals along the test section (in order to determine the hydraulic gradient) the total sediment concentration, and the water temperature. Along with these data, sediment samples were also collected frequently to study sediment size distribution. The plastic section was effectively used to observe closely the movement of sediment along the bottom of the pipe.

Measurement of Discharge
The discharge was measured with the help of a calibrated 10 in. diameter sharp-edged orifice. It was located in the vertical section of the smooth pipe immediately after the pump at a distance of 6 ft 3 in. from the latter. The taps to measure differential head were connected through $3 / 8$-in. plastic tubing to sand traps in the form of bottles with rubber stoppers. There were three openings in the stoppers. One was connected to the tap near the orifice, another was connected to the manometer through $3 / 8$-in. plastic tubing, and the third was used for removing air bubbles from the system. The manometer was 5 ft high. Due to fluctuations, it was thought that accuracy of measurement of differential head was about $\pm 0.003$; therefore, numerous readings were taken to arrive at a mean reading.

Measurement of Head-10ss
The total length of the pipe used for testing was 76 ft . In order that effect of the bend immediately upstream of the test section should be avoided, the first tap for measuring the head loss was situated at a distance 12 diameters (in this case 12 feet) downstream of the bend. Hence the test section along which measurements were made was 60 ft.

The piezometer taps were located along the test section at 10-ft intervals. The openings were $3 / 32$ in. in diameter and were connected to 2 -in. long, $\frac{1}{4}$-in. diameter brass tubing. The brass tubing was connected to a sand trap made of 2 -in. diameter and $4.5-\mathrm{in}$. long bottles of approximately $4-0 z$ capacity. The rubber stopper sealing the
bottle had two openings. One was connected to the piezometer tap while the other was connected to the manometer through plastic tubing. The air bubbles collected near the piezometer tap were bled by lifting the stopper a little.

The bank of manometers used to measure head loss was made from a 4 ft long, $6-\mathrm{mm}$ (inside diameter) glass tubes 14 in number. The tops of these glass tubes were connected to a common copper tube which was completely closed except with one opening which could be opened or closed with a clamp. The bottoms of the glass tubes were connected to the piezometer taps with the help of plastic tubing. A scale was fixed at the center of the manometer bank and a wooden $T$ with a horizontal wire on the top, which could slide along the scale, was used for making the reading. The readings fluctuated considerably and therefore a clamping system was used which could clamp all the tubes at the same time. Throughout all the experiment only 7 tubes out of 14 were used.

Some remarks about the position of the piezometer taps on the corrugated pipe seem appropriate. Experiments conducted at Bonneville Hydraulic Laboratory (26) on the corrugated pipes were carried out using piezometer taps at crests and troughs of corrugations. It was found that the hydraulic gradient measured from either set of locations of piezometer taps was consistent.

Chamberlain (6) studied the variation in differential head between crests and troughs, with and without sediment running through the pipe. He found that the differential head between crests and troughs $\delta \pi$ is directly proportional to the Reynolds number of the
flow. For a given Reynolds number, as the concentration increased $\delta \pi$, in general, decreased. A plot of $J_{1} / J$ against Re, where $J_{1}$ and $J$ are the hydraulic gradients determined from taps located on troughs and crests respectively, showed that the hydraulic gradient was not affected significantly. Therefore, it was concluded that the hydraulic gradient as recorded by the corrugation crests gave a more accurate measure of energy dissipation.

For this reason, the piezometer taps were located on the crests (looking from inside) for standard corrugated and also for helicalcorrugated pipes. They were situated along the horizontal diameter of the pipe.

Measurement of Total Sediment Load
The samples were taken at the downstream edge of the 10 -in. orifice plate which was used to measure the discharge of mixture. The sampling device consisted of a $\frac{1}{4}-i n$. (inside diameter) L-shaped brass tube, as shown in Fig. 4. The sharp-edged opening of the short arm projected upstream parallel to the direction of flow. Two such tubes, which could traverse at right angles to each other, were used. The position of the tip was read on the scale fixed outside, as shown in Fig. 4.

The importance of sampling at the same velocity as that of the flow in the pipe need not be stressed. If the sample is collected at a velocity greater than the velocity of flow in pipe, then due to inertia of the sediment particles, the particles tend to converge towards the sampling tube to a lesser degree than the water resulting in
a sample too low in sediment concentration. On the other hand, if the sample is collected at a velocity less than that of the flow in the pipe, the sample will be too high in sediment concentration.

The sampling time was calculated for a given discharge and then samples were taken. The rate of sampling was controlled by changing the elevation of the end of the sampling tube and also by pinching the plastic tube to a certain extent.

The samples were collected in one-1iter glass cones. These cones were calibrated in the following manner. About 80 samples of one liter volume each with different concentrations were collected in cones and the sediment was allowed to settle for 3 or 4 minutes. Then the cone was tapped until most of the sediment settled at the bottom and was compacted as far as possible. The height of deposition in the cone was measured on a scale. The samples were transferred to clean bottles or cans and they were oven dried at about $110^{\circ} \mathrm{C}$ temperature. The samples were then weighed and sediment concentration in per cent by weight was found. The resulting calibration curve, expressed as cone reading or scale reading against sediment concentration in per cent weight of solids per weight of mixture was a straight line as shown in Fig. 5.

Measurement of Temperature
The temperature was measured by using a centigrade thermometer. Whenever the temperature was to be measured, the air valve (Fig. 3) was opened until the water-sediment mixture started flowing and then the thermometer was inserted to measure the temperature. The temperature was recorded every half an hour.

Description of Boundary-forms
Fig. 6 shows the details of the three boundary forms studied during the investigations reported herein. All the three boundaries studied viz., smooth, standard corrugated, and helical corrugated had the nominal diameter of 12 in.

## Description of Sediment

The sediment used for the experiments was from Greeley, about 40 miles east of Fort Collins. It is known locally as "Broughton Greeley sand". The size distribution of the sediment is shown in Fig. 7. Fifty per cent size or mean diameter of the sediment was 0.60 mm while the geometric standard deviation was 1.920 . These two characteristics of sediment, viz. mean diameter and geometric standard deviation, are used to describe the sediment in the present investigations.

Analysis of the sediment samples taken during the experiments showed that median size of samples was not always constant, viz. 0.60 mm ; the standard deviation $\sigma \mathrm{g}$ was also not constant. There were several reasons for this variation, which are discussed below:
a. It was found that the median size of the sample depended on the location where the sample was collected. Due to centrifugal action in the pump the sediment was separated into finer and coarser particles and there was a consistent tendency for larger size sediment to flow near the north edge of orifice, i.e. corresponding to outer rim of pump impeller. When this was noticed, the samples in the subsequent runs were collected at the same place.
b. If there was not enough total sediment load in the system to form a permanent bed, then under certain flow conditions the sediment moved in the form of waves. Therefore, in the vertical section the sediment moved in the form of clouds. The median size of sample collected at any fixed position across the orifice was, in such a case, a function of time at which it was sampled.
c. If the total sediment load was sufficient to form a permanent bed on the bottom of the pipe under all flow conditions, then most of the coarse sediment was deposited and the median size of the sample collected was relatively finer than when the total load was moving in full suspension.
d. Due to the circulation of sediment in the system, grinding of the sediment particles resulted. The finer particles resulting from such grinding were periodically removed by changing the water in the circulation system.

To study any consistent tendency in the variation of median diameter and geometric standard deviation of the samples collected, these two variables were plotted against Re. Figs. 8 and 9 show such variations in $d / d_{50}$ and $\sigma g$ as a function of $R e$, for smooth and helical corrugated pipes. In this case $d_{50}$ is the median size of sediment sample collected. Such data were not taken for standard corrugated pipe. Fig. 8 shows that for a given boundary form, as Re increases, the ratio $d / d_{50}$ decreases. If for all values of $\operatorname{Re}$ the sediment is in complete suspension, then both standard deviation and $\mathrm{d} / \mathrm{d}_{50}$ should be constant, the latter having the value unity. But, when there is a permanent deposition present in the pipe, a complicated
sorting action may be taking place inside the pipe. In some cases, for large values of $\operatorname{Re}, d / d_{50}$ is less than one; but the general tendency is to have a value unity. Fig. 9 shows the variation in geometric standard deviation as a function of Re. It shows that for smooth and helical corrugated pipe, there is in general an increase in standard deviation with an increase in Re.

In view of the above facts, it was rather difficult to decide which size of sediment should be taken as representative. Since insufficient data were available to analyse the problem fully, it was decided to take the original size of 0.60 mm as the size of sediment.

## Procedure

During the entire experimental stage the procedure, in general, remained the same. When a particular pipe was installed in place, a few runs were made with clear water flowing through the pipe with the discharge being varied for each run. The main object of these runs was to establish the clear water relation between velocity and hydraulic gradient for that particular boundary form.

Following the preliminary runs, a certain quantity of sediment was added to the system and either three or four runs were made atdifferent discharges. More sediment was added and additional runs were made. This was continued until the pump began to plug. Experiments were made in the range where the entire load of sediment was carried in suspension and also where the entire sediment load was partly carried in suspension and partly along the bottom. After the foregoing runs were completed for one pipe, the pipe was changed and the procedure repeated for other boundaries.

Due to the difficulty in removing air from the system, in the case of standard corrugated pipe, it was not possible to take data in both the ranges described above; but in the cases of smooth pipe and helical corrugated pipe, the data were taken in both ranges.

For a particular run with sediment in the system, the following procedure was followed:

The system was full at the end of the previous run. The pump was started and, by using the city water supply, a small quantity of water was added to the system to fill the water lost due to leakage. At the same time the air valve was kept open to remove all the air from the system. When all the air was removed and the system was completely filled with water, both the city water supply valve and air valve were closed. The discharge valve was then adjusted so that a particular discharge was flowing through the pipe. It was found that usually fortyfive minutes to one hour was required for the system to reach a state of equilibrium. After this, the actual data were taken.

Taking the data included the measurement of discharge, recording the piezometer readings and temperature at regular intervals, taking sediment concentration samples across the 10 -in. orifice, and close observation of the movement of sediment in the plastic section. In addition to this, when part of the sediment was moving along the bottom, the portion of the circumference covered by the material moving on the bottom was also measured in the plastic section. The quantities discharge, temperature, and hydraulic gradient were averaged over the period required to complete the run. During the time required for completing one run temperature varied by a maximum amount of $1 \frac{1}{2}$ degrees
while there was no appreciable variation in discharge. Head loss between $10-\mathrm{ft}$ intervals was fairly constant in all runs except when there was movement of sand dunes on the bottom.

When part of the total sediment load was moving along the bottom of the pipe, it was found that for a particular discharge and particular total sediment load, the head loss between any two piezometer taps was a function of time. This matter is discussed in detail in Chapter $V$ but suffice it to say, this variation in head loss was direct1y related to movement of sand-dunes in the downstream direction. Therefore, for a few runs piezometer readings were taken as a function of time.

There were 6 stations along each of two diameters of the $10^{-}$ in. orifice where the sediment concentration data were taken. At each such station usually 3 or 4 samples were collected by siphoning. To record the time of sampling a stop-watch reading to 0.05 second was used. Thus a total of 36 or 48 samples were taken to find the mean sediment concentration.

Samples were collected in $1000-\mathrm{ml}$ cones. They were tapped 8 to 10 times and allowed to settle for 3 to 4 minutes. The sediment content of each cone was scaled, and then the cones were cleaned for using again. In certain runs the sediment samples were saved for sieveanalysis. In Appendix $C$, the data regarding the sieve-analysis, sampling station and run number are tabulated.

## Chapter V

PRESENTATION AND ANALYSIS OF DATA

The object of the present investigation was to throw light on the effect of: (1) boundary form, (2) sediment characteristics, (3) type of sediment transport, and (4) total sediment load, on the Darcy-Weisbach resistance coefficient, the energy required under different given conditions and the necessary water-discharge. In order to present the foregoing information in a significant manner it was found necessary to use various parameters. Their definitions are given below.

Parameters Used in Presentation of Data

Average velocity $\underline{v}$ in fps:-- The average velocity $V$ is defined as the discharge passing through the pipe in cfs divided by the area of pipe in sq ft . No consideration is given to the area which, in some cases, is blocked by permanent deposition of sediment. Darcy-Weisbach resistance coefficient $f$ :-- This resistance coefficient is defined by the following equation:

$$
\begin{equation*}
f=\frac{J}{v^{2} / 2 g D} \tag{23}
\end{equation*}
$$

As will be seen, this coefficient was originally defined for clear water flow and hence it represented the effect of boundary roughness on energy loss. $0^{\prime}$ Brien and Folsom (22) indicated that this definition can be applied to the case of suspended sediment transport through pipes.

In the present investigation this definition was extended further and used in the case where part of the sediment is transported along the bottom of the pipe. A further discussion of this point will be reserved for discussion of $f: \operatorname{Re}: C_{T}$ plots.

Reyno1ds number Re:-- As usual, Reynolds number for pipe is defined by the equation

$$
\begin{equation*}
\operatorname{Re}=\frac{V \cdot D \cdot \rho_{w}}{\mu}=\frac{V \cdot D}{\nu} \tag{24}
\end{equation*}
$$

For all the experiments, the kinematic viscosity of clear water at the temperature during the experiment was substituted for the kinematic viscosity of the water-sediment mixture. Also, in the range where part of the sediment load was carried along the bottom of the pipe the diameter of pipe was used in place of the actual depth of flow.

Parameter E :-- This parameter was used to express the energy requirement. It was first suggested by Wilson (34). It can be described as the energy required per foot of pipe per pound of sediment transported per second. Since the energy required per foot of pipe is equal to $Q \gamma_{m} J$, and solids transported per second is equal to $Q \gamma_{m} C_{T} / 100$, the parameter E will be,

$$
\begin{equation*}
\mathrm{E}=\frac{\mathrm{Q} \gamma_{\mathrm{m}} \mathrm{~J}}{\frac{\mathrm{Q} \gamma_{\mathrm{m}}^{\mathrm{C}}}{100}}=\frac{\mathrm{J}}{\mathrm{C}_{\mathrm{T}} \sqrt{100}} \tag{25}
\end{equation*}
$$

Analysis of Dimensional plots
Discussion of $J: V$ plots:- The $J: V$ plots are of prime importance in the study of flow of fluid through pipes because they are the source of information regarding both the resistance coefficient and
the energy expended in maintaining the flow. For convenience, the present discussion is subdivided into (1) clear water flow, and (2) watersediment mixture flow.

The data for clear water flow are plotted in Fig. 10, which shows the variation of $J$ with $V$ on $\log -10 g$ paper. Data presented by Chamberlain (6) for the same pipes for clear water flow are also plotted. It is evident from this figure that the slope of $\mathrm{J}: \mathrm{V}$ lines for helical-corrugated and standard-corrugated pipes is two, while that for smooth pipe is less than two.

The theory of turbulent flow has shown that the shear stress $\uparrow$ in fully-developed turbulence can be given by the expression

$$
\begin{equation*}
\uparrow=u \frac{d v}{d y}+\rho \ell^{2}\left(\frac{d V}{d y}\right)^{2} \tag{26}
\end{equation*}
$$

where $d V / d y$ is the gradient of mean velocity and $\ell$ is the mixing length. In such a case, the first term on the right-hand side, which represents the component of shear due to molecular viscosity is very sma11 compared to the second term, which represents the component due to turbulence or eddy viscosity. Hence for turbulent flow

$$
\begin{equation*}
T=\rho l^{2}\left(\frac{d v}{d y}\right)^{2} \tag{27}
\end{equation*}
$$

If one is dealing with the bulk flow, this equation leads to what is known in hydraulics as the quadratic resistance law implying that for a hydrodynamically rough boundary the energy loss is proportional to the square of the mean velocity.

The above comments together with Fig. 10 show that in the helical-corrugated pipe and the standard-corrugated pipe, the boundary is hydrodynamically rough. On the other hand, in smooth pipe there is sti11 a laminar sub-1ayer covering the protrusions so that the boundary is hydrodynamically smooth and the viscous effects are sufficiently pronounced to cause the slope of the line to be less than two.

Data for the flow of the water-sediment mixture are plotted in Fig. 10 which shows that when sediment is introduced in the flow, the points deviate from the line for clear water on the $J: V$ plot. This is true for all three boundary forms. Because of insufficient data for standard corrugated pipe, this is not as obvious as in the case of the other two pipes. Nevertheless in each case there is a definite tendency to deviate from the clear water line. Most of the points lie to the left side of the line for clear-water indicating that, for a given velocity, there is more head loss with sediment-laden water than with clear water. This point is discussed further later in this chapter.

Fig. 11 represents the variation of the hydraulic gradient $J$ with the velocity of flow $V$ for helical corrugated pipe, where the total concentration $\mathrm{C}_{\mathrm{T}}$ is the third variable. In general all the boundaries have similar $J: V$ plots. The sediment used was of relatively large size. Therefore, when the velocities were low, even with small total load in the circulation system, part of the total sediment load moved along the bottom in the smooth pipe. The portion of the $\mathrm{J}: \mathrm{V}$ plot in the range of suspension, therefore, is missing. In the case of helical-corrugated pipe, data in the range of suspension and deposition are also plotted.

As the velocity is decreased, the lines of constant concentration show a decreasing hydraulic gradient. At a certain point, where the hydraulic gradient is a minimum, the constant concentration line goes back up.

At a given concentration, if the rate of flow is decreased, a point is reached at which the flow is no longer able to maintain all the solids in suspension and some of them are deposited in the pipe. Thus, due to this deposition, the cross-section of the pipe is decreased and the velocity is increased. The resultant increase in the velocity permits the establishment of a new regime, which owing to high velocity is able to maintain solids in motion. If the total quantity of flow is decreased further, with a resultant decrease in average velocity, solids are deposited, further obstructing the flow and increasing the actual velocity. This leads to a new state of equilibrium. Over some range of discharge this equilibrium is maintained without much change in hydraulic gradient. If the quantity of discharge is further decreased, there is actually an increase in the hydraulic gradient.

There is special importance to the $\mathrm{J}: \mathrm{V}$ plot drawn on paper with cartesian co-ordinates (See Fig. 11). A line $A B$ can be drawn on the plot which delimits the zone of the regimes with and without the deposit in the pipe. It is also evident that it is in close agreement with the minimum head loss. Velocity corresponding to the passage from one regime to the other has been named the "economic velocity" by B1atch (5) or is commonly called the "limit deposit velocity". This curve is of great practical use from an economic point of view because
if a certain per cent of sand has to be carried, one can find the "economic velocity" at which the sand can be transported at minimum head loss.

Considering the curve of limit-deposit velocity for helicalcorrugated pipe, it will be seen that such a curve is nearly a straight 1ine with a slope of 2.30 for the sediment used in the present investigations. This shows that, for helical-corrugated pipe, if $\mathrm{V}_{\mathrm{m}}$ is the limit deposit velocity, then,

$$
\begin{equation*}
v_{m}=\psi\left(c_{T}\right) \tag{28}
\end{equation*}
$$

for the particular pipe and sediment used.
Water requirement relative to sediment discharge:-- To study the water requirements for transporting a given discharge of sediment, the water-sediment mixture discharge $Q$ was plotted against the sediment discharge $G$ on $\log -10 g$ paper for all the three boundaries and for two sediment sizes viz. 0.60 mm and 0.20 mm . The result is the composite Fig. 12. Deductions from this plot can be stated as follows:

1. There is practically no effect of size of sediment, within the range 0.2 mm to 0.60 mm , on discharge of water-sediment mixture required to transport a given discharge of sediment through helicalcorrugated pipe. This may be due to the combined effect of the intensity of turbulence and the spiral flow within the pipe being sufficiently great to transport a large part of sediment in suspension regardless of the size of the sediment.
2. In standard-corrugated pipe 0.60 mm sediment required a greater discharge of mixture, for a given discharge of sediment, than did 0.20 mm sediment.
3. Surprisingly enough, in the case of the smooth pipe a smallersize sediment required a little greater discharge of water-sediment mixture, than did the large-size sediment for a given discharge of sediment. This difference is so small, however, it cannot be considered conclusive.

Considering the economy of water for a given size of sediment, say 0.60 mm for different boundaries, it can be seen clearly from Fig. 12 that for a given sediment discharge standard-corrugated pipe requires about double the discharge of mixture of that required for helicalcorrugated pipe. Smooth pipe requires approximately one and one half times the discharge required for helical-corrugated pipe.

Hence as far as economic use of water is concerned, helicalcorrugated pipe is more economical than either smooth or standard corrugated pipe.

Discussion of $E: W$ plots:-- As mentioned in the beginning of this chapter, the parameter $E$, which represents the energy required per foot of the pipe for transporting a unit weight of sediment per second, is very useful in interpreting the data when it is plotted against $W$ on log-log paper. $W$ is the rate of sediment transport in pounds per second. Such $E: W$ plots were prepared for all the three boundaries and for two sizes of sediment 0.60 mm and 0.20 mm . They are shown in Figs. 13 and 14.

Considering the effect of sediment characteristics for any given boundary, it will be noticed that in the case of each of the three boundary forms - viz. smooth, helical corrugated, and standard corrugated - the curve for 0.6 mm size sediment lies above the curve for 0.20 mm
size sediment -- thus indicating that with an increase in the size of sediment transported, for a given $W$, $E$ also increases. This is in accordance with the current theory of turbulence and it can be explained by arguing that for large-size sediment the small and large eddies of the flow do not have sufficient energy to supply for transporting solids and therefore additional energy must be supplied.

This will show the advantage of transporting a given amount of solids in finer state than in coarser state, if a choice is left to the design engineer. Such cases arise in transportation of coal, ores etc. Of course along with this advantage, there are some practical problems which may not make it feasible to transport solids in a finer state -- such as the problem of grinding or crushing to reduce the size and the problem of separating the solids from the fluid once they have been transported.

It is also interesting to study the effect of boundary form on energy requirement for a given size sediment, say 0.60 mm , and for given amount of solids to be transported, say 10 pounds per second. For transporting $W$ equal to 10 pounds per second, helical corrugated pipe requires an energy input 0.92 , standard corrugated pipe requires 2.65 , and smooth pipe requires 1.00 energy per pound of sediment per foot of pipe, in fps units. Hence for a given amount of sediment to be transported, helical corrugated pipe in general will require the least amount of energy per foot of pipe per pound of sediment transported, as compared with smooth and standard corrugated pipes. Standard corrugated pipe requires the most energy.

Very interesting information is obtained (34) by a further analysis of the parameter $E$. Since $B \propto J / C_{T}, J$ must be a minimum in order to have $E$ a minimum, for the most economical transportation of sediment at a given concentration. This point will correspond to the "1imit deposit velocity" or the "economical velocity".

It is obvious also that the general tendency of all $\mathrm{E}: \mathrm{W}$ curves is for $E$ to decrease as $W$ increases. Therefore, it is more economical to transport a given volume of sediment at a large rate than to transport it at a small rate, as far as energy consumed per pound of sediment transported is concerned.

Since for a given amount of sediment, $E$ will be minimum when $C_{T}$ is maximum, one must study the case where $\mathrm{C}_{\mathrm{T}}$ is maximum. For given $W$, assuming $\gamma_{m}$ is fairly constant, if $Q$ is increased then the corresponding $C_{T}$ will decrease because

$$
\mathrm{w} \propto \mathrm{Q} \gamma_{\mathrm{m}} \mathrm{c}_{\mathrm{T}}
$$

and

$$
\begin{equation*}
c_{T} \propto \frac{W}{Q \gamma_{m}} \tag{29}
\end{equation*}
$$

Therefore, if $Q$ is decreased, the concentration for a fixed $W$ will increase. Furthermore, as the discharge is decreased more and more, E will reduce with a corresponding reduction in discharge. But there wi11 certainly be a practical limit beyond which the discharge cannot be decreased in order to take advantage of economy in energy consumption per pound of sediment transported. With a decrease in discharge of
water-sediment mixture, the section of the pipe will be reduced due to deposition and after a certain reduction in discharge, it may be impossible to maintain a steady flow and the pipe may become clogged. It can also be shown that after reaching a minimum point, $E$ will increase. For a given diameter of the pipe and a given size of sediment, this minimum value of $E$ will depend on the value of $W$. Refer to Appendix $D$.

Discussion of
Durand's Bquation
In 1953 Durand (10) gave an equation of the form

$$
\begin{equation*}
\frac{\mathrm{J}-\mathrm{Je}}{\mathrm{Je} \mathrm{C}_{1}}=\mathrm{K}_{1}\left(\frac{\sqrt{\mathrm{gD}}}{\mathrm{~V}}\right)^{3}\left(\frac{1}{\sqrt{\mathrm{C}_{\mathrm{x}}}}\right)^{1.50} \tag{30}
\end{equation*}
$$

where $K_{1}$ is a constant, $C_{1}$ is the relative absolute volume of the sediment, and $C_{x}$ is the drag coefficient defined by the equation

$$
\begin{equation*}
c_{x}=\frac{4}{3} \frac{g d}{w^{2}} \frac{\rho_{s}-\rho_{w}}{f_{w}}, \tag{31}
\end{equation*}
$$

where $w^{-}$is the settling velocity of particles. This equation incorporates the following variables in the sediment transport problem: size of pipe, size of sediment, specific gravity of sediment and fluid, discharge, sediment concentration and hydraulic gradients with and without sediment. Experimental data on which the above equation was based, had a large range e.g.

D varied from 40 to 580 mm
d varied from 0.2 to 25 mm
Concentration varied from 50 to 600 gm per liter.

At the same time, however, there were two important limitations imposed on Durand's equation

1. Sediment used was rather uniform in size.
2. This relationship was only for non-deposit regime.

To study the applicability of this equation under various conditions, data were collected from different sources and plotted according to Durand's equation. The result is Fig. 15. These are for

1. Generally graded material,
2. A smooth boundary only, and
3. Non-deposit as we 11 as deposit regimes.

Fig. 15 shows that there is considerable scatter aroundkthe line representing Durand's equation, in some cases as much as 200 per cent or more. This shows the inadequacy of Durand's equation when applied to data taken under different conditions. Therefore, there is a need for a better relationship between different variables which will be applicable under widely varying conditions. Such a relationship should naturally make use of some of the significant parameters used often in fluid mechanics and, which have a physical significance.

Resistance
Coefficient Approach
The resistance coefficient approach is discussed from the view point of both clear water flow and sediment-laden flow.

Clear water flow.-- As discussed earlier in this chapter, within the range of velocities in which data were taken, the boundaries were hydrodynamically rough and turbulent flow existed throughout the
cross-section of both the helical-corrugated pipe and the standard corrugated pipe. On the other hand, the slope of $V: J$ line on $\log -10 g$ paper for smooth pipe being less than two, it was concluded that the smooth pipe acted as a hydrodynamically smooth boundary.

In Chapter III it was shown that from dimensional analysis, the relationship between the Darcy-Weisbach resistance coefficient and the other dimensionless parameters is given by Eq 20 ,

$$
\begin{equation*}
f=\psi\left(\operatorname{Re}, \quad C_{T}, \frac{\mathrm{~d}}{\mathrm{D}}, \quad \frac{\emptyset}{\mathrm{D}}\right) \tag{20}
\end{equation*}
$$

For clear water flow $C_{T}$ and $d / D$ will not come into the picture and therefore one must begin with the equation

$$
\begin{equation*}
f=\psi_{1}\left(\operatorname{Re}, \frac{\emptyset}{D}\right) \tag{32}
\end{equation*}
$$

The relationship between $f$ and $\operatorname{Re}$ for different boundaries is shown in Figs 16 and 17. It will be seen that, for the case of clear water in helical corrugated pipe and standard corrugated pipe, $f$ has a constant value which is unaffected by Reynold"s number. The resistance coefficient $f$ for the helical corrugated pipe is 0.04 and that for standard corrugated pipe 0.12.

In the case of clear water in the smooth pipe, however, $f$ decreases as Re increases. Superimposed on the data is the line representing the Karman-Prandt1 equation for turbulent flow in smooth pipes

$$
\begin{equation*}
\frac{1}{\sqrt{f}}=2 \log _{10} \operatorname{Re} \sqrt{f}-0.80 \tag{33}
\end{equation*}
$$

It will be seen that the clear-water data fit the curve reasonably we 11 and hence the values of $f$ can be estimated for smooth pipe with clear water by using this equation.

Sediment-1aden flow:-- The basic equation to be used for the analysis of the resistance coefficient with sediment-laden flow will be
$f=\psi\left(\operatorname{Re}, \quad C_{T}, \frac{d}{D}, \frac{\varnothing}{D}\right)$.

For a systematic detailed discussion, it seems necessary to subdivide the discussion into two parts, the suspended load regime and the deposition regime.

The data for the suspended load regime are plotted in Figs. 16 and 17 which show the variation in the resistance coefficient $f$ with the Reynolds number $R e$, using $C_{T}$ as the third variable. For the size of sediment used, it is evident that the presence of sediment increased the resistance coefficient $f$. On the other hand, Chamberlain (6) using the same pipes but with 0.20 mm diameter sediment, found that the values of $f$ were not affected appreciably by concentration up to the point of incipient deposition. This means that the increase in the size of sediment necessitates additional energy to increase the eddy viscosity so that the larger sediment will be held in suspension. Therefore, one may logically expect that an increase in concentration wi11 cause a corresponding increase in the resistance coefficient for a given Reynolds number. Validity of this statement is proved by drawing lines of constant-concentration in the case of smooth and helio cal corrugated pipes as shown in Figs. 16 and 17 . With such constant
$\mathrm{C}_{\mathrm{T}}-1$ ines one may naturally expect either a relationship among the three dimensionless parameters $\operatorname{Re}, C_{T}$, and $f$, for a given value of $d / D$ or a composite relationship among all the four parameters. This matter is further discussed in the latter part of this chapter.

From the same plot, it will be seen that for constant $\mathrm{C}_{\mathrm{T}}$, $f$ decreases with an increase in Re. This may be explained by saying that with an increase in $R e$, the turbulence-creating eddies supply more and more energy to hold the sediment in suspension and therefore for the same concentration $f$ should decrease with an increase in $C_{T}$. The data for the deposition regime also are plotted in Fig. 17. When the sediment load was in full suspension, head loss between any $10-\mathrm{ft}$ interval was reasonably constant with respect to position in the test section and also with respect to time. When part of the sediment was carried along the bottom, however, this was not true. In a given $10-\mathrm{ft}$ interval the head loss varied with respect to time, while at any given instant the head loss between successive 10-ft intervals was not constant. Fig. 18 shows such variation with respect to time. In this plot, the excess or deficiency in head loss in $10-\mathrm{ft}$ intervals over a time averaged head loss is plotted as a function of distance with time of observation as the third variable. If a particular $10-f t$ interval is taken into consideration, for a certain time, the head loss in this interval is more than the mean head loss in any $10-f t$ interval; then after some time it is less than the mean head loss which implies that there is a cyclic variation.

Observations in the transparent plastic section during this time showed that the material along the bed was moving in the form of sand dunes and that it was possible to locate the position of crests by reading the manometer. Usually two or three dunes were present in the test section at a given time, the number depending on the velocity.

For this reason the hydraulic gradient $J$, averaged over a long time ( $1 \frac{1}{2}$ to 2 hours) was taken to compute the values of $f$.

The resistance coefficient computed in this way gave a composite effect of various energy losses occurring in the pipe. These energy losses can be described as due to:

1. Boundary roughness of the pipe
2. Turbulence
3. Sediment load carried in suspension
4. Moving sand dunes
5. Increase in velocity as a result of decrease in cross section caused by deposition.

These effects are again interdependent; thus part of the boundary roughness becomes ineffective due to deposition. Therefore, the problem becomes very complicated making it difficult to explain the effect of each of these factors on the value of $f$. However, some general remarks can be made with a reasonable degree of certainty.

In the case of helical corrugated pipe it was possible to draw a few constant deposition lines on $f: R e: C_{T}-p l o t$ as shown in fig. 17. As the Reynolds number is increased along a given line of constant deposition, $f$ increases initially, then reaches a maximum value,
and with a further increase in $R e, f$ decreases. This can be explained by an analogy in open channel flow. Starting with an inactive alluvial bed, as the Reynolds number is increased, the individual particles start moving and then ripples are formed which move downstream. With further increase in Re , the ripples grow into dunes until their height and spacing are optimum for creating the maximum resistance to flow. After this stage is reached, with further increase in the Reynolds number, the dune spacing increases at a higher rate than the dune height and, in spite of the fact that dune height increases slight1y, the resistance coefficient actually decreases. In such cases the dunes may be considered somewhat like isolated obstacles in the f1ow. For even larger values of $R e$, dunes disappear and the bed surface as a whole becomes mobile and although plane, becomes hydrodynamically rough. These changes in bed roughness have an important effect on the sediment transport rate. When dunes are formed on the bed giving maximum resistance to flow, then, due to the turbulence created by these dunes, more sediment is thrown in suspension than when no dunes are present. Also the sediment in suspension is coarser.

On the $\mathrm{f}: \mathrm{Re}: \mathrm{C}_{\mathrm{T}}$ plot for helical-corrugated pipe, there is an important difference from the plots for smooth and standard corrugated pipe. There is a distinct discontinuity between suspended load regime and deposition regime for helical corrugated pipe. This is due to the fact that as deposition starts in helical-corrugated pipe, the helical action is reduced. This is not the case in smooth pipe and therefore such a discontinuity is not observed.

## Significance of

 the Parameter Re $\sqrt{\mathrm{f}}$In Chapter III, some comments were made about the significance of the parameter $\operatorname{Re} \sqrt{\mathrm{f}}$. It was said that the relative thickness of the laminar sub-1ayer was inversely proportional to $\operatorname{Re} \sqrt{\mathrm{f}}$ in the case of clear water flow.

Recently, use of this parameter has been made also in the analysis of resistance in alluvial channels based on the concept of the laminar sub-layer. Therefore it was thought that $\operatorname{Re}-/ \overline{\mathrm{f}}$ may also play a significant role in sediment-laden flow in pipes. Plots were made of $\operatorname{Re} \sqrt{\mathrm{f}}$ against $\mathrm{C}_{\mathrm{T}}$ for the data available from present studies and also data available from other sources. Fig. 19 shows such a plot for all boundaries with 0.60 mm sediment.

It is highly significant that in each of these plots the slope of the line for all the available data was one third and furthermore there was a tendency for the data to fall on this line irrespective of the regime (suspended load or deposition) in which data were collected.

The relationship may be viewed also from a different standpoint e.g. it may be written in the form,

$$
\operatorname{Re} \sqrt{\mathrm{f}} \propto \mathrm{C}_{\mathrm{T}}^{1 / 3}
$$

or

$$
\begin{equation*}
C_{T} \propto(\operatorname{Re} \sqrt{f})^{3} \tag{34}
\end{equation*}
$$

so it follows that the sediment load transported is proportional to the cube of discharge when $f$ is constant. When $f$ is subject to small variation, this proportionallity may still be used in application.

In carrying out a further step in the analysis of the data, it was assumed that the relative position of the lines on the $\operatorname{Re} \sqrt{\mathrm{f}}: \mathrm{C}_{\mathrm{T}}$ plots was a function of the parameter $d / D$ and a plot was made with $d / D$ as the ordinate and $P$, the intercept on $\operatorname{Re} \sqrt{f}: C_{T}$ plots, as the abscissa, see Fig. 20. The third variable on this plot was $d$, which gave series of converging lines for constant values of $d$. The equation of each line on this $d / D: P$ plot was obtained and the slopes and the intercepts were then plotted as functions of $d$. The slope relationship is represented by the equation

$$
\begin{equation*}
S=0.89 d^{1 / 3} \tag{35}
\end{equation*}
$$

and values of the intercept are given in the plot shown in Fig. 21.
The resulting equation obtained was of the form

$$
\begin{equation*}
\operatorname{Re} \sqrt{\mathrm{f}}=\left[\frac{I}{\mathrm{~d} / \mathrm{D}}\right]^{\mathrm{S}_{1}}{ }^{C_{\mathrm{T}}} \tag{36}
\end{equation*}
$$

where $s_{1}$ is equal to $1 / S$ and $I$ is the intercept obtained from Fig. 21.

It is particularly interesting to note that three parameters have been evolved by the foregoing process. The first parameter Req/ $\bar{f}$ describes the flow and may also be considered as $V_{\star} D / \nu$, which is the shear-velocity Reynolds number, or as the relative thickness of the laminar sub-layer. The second parameter $C_{T}$ is the concentration of the total load. The third parameter, $\left[1 / d / D_{]}^{\top} s_{1}\right.$, relates the sediment size to the size of pipe and therefore may be considered as describing the relative geometry of the pipe and the sediment.

To ascertain the applicability of this equation to varying conditions as well as to study the accuracy, all the available data were plotted (see Fig. 22) according to this equation.

These data used are from different sources and are tabulated in summary form in the Appendix. It is evident that the observed values of the variables have a wide range, viz.

D varied from 0.083 ft to 2.9 ft .
d varied from 0.170 mm to 2.60 mm .
$C_{T}$ varied from 1 to 60 per cent by weight.
Furthermore, in some cases, as much as one third or more of the periphery was covered by deposited material. Fig. 23 shows variation of $\operatorname{Re}-\sqrt{f}$ with (I/d/D) ${ }^{S_{1}} C_{T}{ }^{1 / 3}$ for standard corrugated and helical corrugated pipe.

Taking into consideration the facts that these data were taken by different personne1, that there is no way to judge the accuracy of some of the data found in the literature, and that in some cases assumptions had to be made with regard to such factors as temperature, the equation shows a remarkable correlation. However, there are certain limitations which must be considered.

Limitations:-- It is realized that when the slopes and the intercepts of the lines in the $d / D: P$ plot are plotted against the sediment size $d$, they are represented by relationships which are dimensiona1. To overcome this difficulty, it was thought that the standard deviation of the size distribution of the sediment might provide a dimensionless term. However, the limited data available on the standard
deviation did not help in this regard. It is possible also that the length term d might be a part of some sort of Richardson number or another parameter involving the difference in specific weight between the sediment and water. This was not investigated, however.

A second limitation in this treatment and the final relationship established is of minor importance as far as application is concerned, but since there could be occasion for its use, it is mentioned below. Any relationship between $d / D, C_{T}, \operatorname{Re}$ and $f$ should be a continuous function in the range

```
clear water m}\mathrm{ suspended load }->\mathrm{ - deposition
    regime regime regime
```

That is, if a relationship

$$
\begin{equation*}
\psi_{1}\left(f, \operatorname{Re}, \quad C_{T}, d / D\right)=0, \tag{37}
\end{equation*}
$$

is established, then it should also be valid when $C_{T}$ is zero or

$$
\begin{equation*}
\psi_{2}(f, \quad R e)=0 \tag{38}
\end{equation*}
$$

The relationship given by Eq 36 is not valid when $\mathrm{C}_{\mathrm{T}}$ is zero. The reason for this is that when sediment is not present $I, d / D$, and $s_{1}$ are not defined and $C_{T}$ is zero.

Bventual aim in establishing a relationship between these parameters should be to obtain an expression of the form

$$
\begin{equation*}
f=\psi_{3}\left(\operatorname{Re}, \quad C_{T}, \quad \frac{d}{D}\right)=f_{1}+f_{2}, \tag{39}
\end{equation*}
$$

where $f_{1}=$ that part of $f$ contributed by clear water
$f_{2}=$ that part of $f$ contributed by inclusion of sediment. Therefore, when sediment is not present, $f$ is equal to $f_{1}+0$.

To overcome this discrepancy the following analysis is suggested. It was shown that for the smooth pipe (commercial), in the case of clear water flow, the resistance coefficient could be obtained by using Karman-Prandtl equation for turbulent flow, e.g.

$$
\begin{align*}
& \frac{1}{\sqrt{f}}=2 \log _{10} \operatorname{Re}-\sqrt{\mathrm{f}}-0.8  \tag{33}\\
& \operatorname{Re} \sqrt{\mathrm{f}}=10\left[\frac{1}{-\sqrt{\mathrm{f}}}+0.8\right] \cdot 1^{1 / 2}=10^{M} . \tag{40}
\end{align*}
$$

Therefore, for any amount of sediment present $f$ must not be less than that given by the equation

$$
\begin{equation*}
\operatorname{Re} \sqrt{\mathrm{f}}=10^{\mathrm{M}} \tag{41}
\end{equation*}
$$

where $M$ is equal to $\frac{1}{2}(1 / \sqrt{f}+0.8)$. But for a concentration $C_{T}$

$$
\begin{equation*}
\operatorname{Re} \sqrt{ } / \bar{f}=\left[\frac{I}{d / D}\right]^{S_{1}} C_{T}^{1 / 3}=\mathrm{N} \mathrm{C}_{T}{ }^{1 / 3}, \tag{42}
\end{equation*}
$$

where

$$
\mathrm{N}=\left[\frac{\mathrm{I}}{\mathrm{~d} / \mathrm{D}}\right]^{\mathrm{s}_{1}} .
$$

Equating the two values of $\operatorname{Re} \sqrt{f}$ gives

$$
\begin{equation*}
10^{\mathrm{M}}=\mathrm{N} \mathrm{C}_{\mathrm{T}}{ }^{1 / 3} \tag{43}
\end{equation*}
$$

Therefore, in using the approach developed in the present research, especially for low concentrations, after finding the value of $f$, it should be seen if $C_{T}$ is less than that given by the equation

$$
\begin{equation*}
10^{\mathrm{M}}=\mathrm{N} \mathrm{C}_{\mathrm{T}}{ }^{1 / 3} . \tag{43}
\end{equation*}
$$

If $C_{T}$ is less than this computed value, it should be concluded that the relationship

$$
\begin{equation*}
\operatorname{Re} \sqrt{\mathrm{f}}=\left[\frac{I}{d / D}\right]^{S_{1}} C_{T}^{1 / 3} \tag{36}
\end{equation*}
$$

is not valid at such a low concentration. In such a case, the $f$ given by the Karman-Prandtl equation should be used.

A third limitation to the use of this equation is that it was developed for a constant value of $\gamma_{S} / \gamma_{w}$ equal to 2.65.

In spite of these limitations, it is believed that this relationship can be used with confidence in solving the practical problems of sediment transport through pipes. The results obtained may be within 10 to 15 per cent accuracy.

It now remains to show the application of this relationship to actual problems.

## Illustrative Examples

In practice, the designer will be interested in the following variables: diameter of sediment to be transported $d$, pipe diameter $D$, sediment discharge $W$, discharge of mixture $Q$ and horse power consumed or resistance coefficient. If the designer knows any
four of these five variables, the fifth can be estimated by direct use of the relationships above developed. But if only three out of five variables are known, trial and error method must be used along with the above relationships in order to get an estimation of the two unknowns, compatible with the given conditions. Two procedures are given below to show the method of approach.

## Problem 1

Given: size of sediment to be transported 1.00 mm , pipe diameter 12 in., kinematic viscosity $1.12 \times 10^{-5} \mathrm{ft}^{2} / \mathrm{sec}$, water-sediment mixture discharge is 8 cfs , and the sediment is to be transported at 10 per cent concentration by weight. Find the Darcy-Weisbach resistance coefficient.

## Solution

$$
\begin{aligned}
& \text { Knowing } d=1.00 \mathrm{~mm} \text { and } D=12 \mathrm{in} ., \\
& \frac{d}{D}=0.0033
\end{aligned}
$$

Also, for $d=1.00 \mathrm{~mm}$ using Fig. $21, I=65$ and using Eq 35

$$
s_{1}=\frac{1}{S}=1.11
$$

Since $Q=8.0 \mathrm{cfs}$ and $A=D^{2} / \alpha=0.785 \mathrm{sq} \mathrm{ft}$

$$
\begin{aligned}
V & =10.19 \mathrm{fps} \\
\operatorname{Re} & =\frac{10.19 \times 1}{1.12 \times 10^{-5}}=9.09 \times 10^{5}
\end{aligned}
$$

$$
\text { since } \begin{aligned}
& N=\left|\frac{I}{d / D}\right|^{S_{1}}=5.35 \times 10^{4} \\
& N_{T^{1}}{ }^{3}=\left(5.35 \times 10^{4}\right)(10.0)^{13} \\
&=1.15 \times 10^{5} \\
&=\operatorname{Re}-/ \overline{\mathrm{f}}
\end{aligned}
$$

Substituting $\operatorname{Re}=9.09 \times 10^{5}$ in above equation

$$
\begin{aligned}
\sqrt{f} & =\frac{1.15 \times 10^{5}}{9.09 \times 10^{3}}=0.1265 \\
f & =0.016
\end{aligned}
$$

Knowing $f=0.016$ the hydraulic gradient can be obtained as follows; since $J=f V^{2} / 2 g D$, substituting values of $f, V$, and $D$

$$
J=0.0258
$$

Simple computations will show that

$$
\gamma_{\mathrm{m}}=71.75 \mathrm{lbs} / \mathrm{cft}
$$

$$
\text { Hp per foot of pipe }=Q_{i m} J / 550
$$

$$
=8.0 \times 71.75 \times 0.0258 / 550
$$

$$
=0.028
$$

Problem 2
Other types of problems arise where the designer knows $d$, $D$, and $Q$ or $W$ and he needs the estimation of $f$. In such a case there are usually some conditions imposed, such as maximum capacity of
pump or minimum amount of sediment to be transported. A procedure is given below to solve the problem if the designer knows $d, D$ and $W$ and it is desired to find $f$ and $Q$.

Knowing $d$ and $D, d / D, I$ and $s_{1}$ can be found.
Therefore

$$
\left[\frac{I}{d / D}\right]^{S_{1}}=\mathrm{N} .
$$

Knowing the details about the pumping equipment, the designer knows the maximum discharge that can be pumped. With this knowledge, a reasonable discharge can be assumed and knowing $W, C_{T}$ can be computed. Knowing $N$ and $C_{T}$, the term $\mathrm{N} \mathrm{C}_{\mathrm{T}}{ }^{13}$ is computed. Therefore,

$$
\operatorname{Re} \sqrt{\mathrm{f}}=\mathrm{N} \mathrm{C}_{\mathrm{T}}{ }^{1 / 3} .
$$

Since discharge is assumed, the corresponding $\operatorname{Re}$ can be computed and then $f$ can be found. From this information, the horse power required to transport $W$ pounds of sand per second with $Q$ cfs discharge through $L$ ft can be found. This should be compared with horsepower available from pumping. It is necessary to repeat the same procedure for different discharges and compare the results before the final design can be achieved.

## Chapter VI

## SUMMARY AND CONCLUSIONS

Summary
Analysis of the data taken for smooth, helical corrugated and standard corrugated pipes 12 -inches in diameter using $0.60-\mathrm{mm}$ median diameter sediment and also the analysis of data collected from other sources has thrown light on the effect of boundary form, characteristics of sediment as we 11 as total sediment load transported, on DarcyWeisbach resistance coefficient, energy requirements and necessary water discharge.

1. In the range of Reynolds number in which experiments were carried, for clear water flow, the standard corrugated and helical corrugated boundaries behaved as hydrodynamically rough while in the smooth boundary the laminar sub-layer still covered the surface irregularities and hence it was hydrodynamically smooth.
2. For 0.60 mm size sediment, presence of sediment always made the hydraulic gradient greater than that for clear water at the same velocity.
3. For helical corrugated pipe, the limit deposit velocity was a function of sediment concentration.
4. Helical corrugated pipe required nearly the same discharge of water-sediment mixture to transport a given discharge of sediment of 0.60 mm median size as sediment of 0.20 mm median size. For the same sediment discharge, standard corrugated pipe required more water discharge
for 0.60 mm size sediment than for 0.20 mm size sediment. In the case of smooth pipe no conclusive results were obtained.
5. For each of the three boundaries -- smooth, helicalcorrugated and standard-corrugated -- it was found that, for a given sediment discharge, increasing the size of the sediment resulted in greater energy requirements per foot of pipe per pound of sediment transported.
6. For a given size of sediment and a given sediment discharge, helical corrugated pipe required the least amount of energy per foot per pound of sediment transported; standard corrugated required the most energy.
7. Durand's equation did not fit the data collected from various sources, to an acceptable degree of satisfaction.
8. For reasons given in $1, f$ for clear water was constant with respect to $R e$ for helical-corrugated and for standard corrugated pipes. For smooth pipe $f$ decreased with increase in $R e$ and the Karman-Prandtl equation for turbulent flow in smooth pipes was found adequate to estimate $f$.
9. In each of the three boundaries, $f$ increased with increase in $C_{T}$, for a given Reynolds number. In both the suspended load regime and deposition regime, it was found that the equation

$$
\operatorname{Re} \sqrt{f}=\left[\frac{I}{d / D}\right]^{S_{1}} C_{T}^{1 / 3}
$$

was adequate to give the variation in $f$ due to variation in $d, D$ and $\mathrm{C}_{\mathrm{T}}$ for smooth pipes.
10. Sediment movement on the bottom of the pipe is quite comparable to the sediment movement on the bed of an alluvial channe 1.

Conclusions

1. In the helica1-corrugated boundary, the water-sediment discharge necessary to transport a given discharge of sediment is not affected by the size of sediment from 0.2 to 0.6 mm diameter sediment.
2. Considering the aspect of energy requirement for transporting a given quantity of sediment, of a given size, helical corrugated pipe is the most economical.
3. The resistance coefficient $f$ is adequate to study the flow of water-sediment mixture through all the three boundary forms studied in the regime of suspended load and in the regime of deposition.
4. The resistance coefficient $f$ is affected by the sediment concentration and the characteristics of sediment and boundary form.
5. There is a need to investigate further the parameter re1ating the sediment size to the size of the pipe, with some ingenuity it may be possible to form a parameter different from $I / d / D S_{1}$ which will be more significant.
6. Ambrose, H. H. The transportation of sand in pipes: II. Free surface flow. Hydraulics Conference, Iowa City. Proceedings 5:77-88, 1952. (Iowa University, Studies in Engineering, Bulletin 34, 1953).
7. Babbit, H. B. and Caldwe11, D. H. Laminar flow of sludges in pipes. Urbana, Illinois. 1939. 58p. (I11inois University. Engineering Experiment Station. Bulletin Series No. 319)
8. Babbit, H. B. and Caldwe 11, D. H. Turbulent flow of sludges in pipes. Urbana, Illinois. 1940. 44p. (Illinois University. Bngineering Bxperiment Station. Bulletin Series No. 323).
9. Binder, R. C. and Busher, J. B. A study of flow of plastics through pipes. Journal of Appiied Mechanics, $13: A-101-A-105$, June 1946.
10. Blatch, N. S. Discussion: Water filtration at Washington, D. C. American Society of Civil Bngineers. Transactions, 57:400-408, 1906.
11. Chamberlain, A. R. Bffect of boundary form on fine sand transport in twelve-inch pipes. Fort Collins, Colo., 1955. 166, 47 p. (Colorado Agricultural and Mechanical College. Department of Civil Bngineering. CBR No. 55ARC 6.)
12. Cramp, Wiliiam. Pneumatic transport plants. Society of Chemicai Industries. Journal, 44:207T-213T, 1925.
13. Craven, J. P. The transpoxtation of sand in pipes: I. Full pipe flow. In Hydraulic Conference, Iowa City. proceedings, 5:07-77, 1952. (Iowa University. Studies in Engineering, Bulletin 34, 1953.)
14. Dane1, P. F. Discussion: Transportation of sand and gravei in a four inch pipe. American Society of Civil Bngineers. Transactions, 104:1372-74, 1939.
15. Durand, R. Basic relationships of the transportation of solids in pipes - experimental research. Minnesota International Hydraulics Convention. Proceedings, 1953:89-103.
16. Durand, R. Transport hydraulique de graviers et galets en conduite. La Houille Blanche, n.s.6:609-19, Octobre 1951.
17. Dougherty, R. W. A survey on the hydraulic transportation of coal. U. S. Bureau of Mines. Report of investigations, 4799:1-22, July 1951.
18. Durepaire, M. P. Discussion: Transportation of sand and gravel in a four-inch pipe. American Society of Civil Bngineers. Transactions, 104:1369-72, 1939.
19. Farbar, Leonard. Flow characteristics of solids - gas mixtures in a horizontal and vertical conduit. Industrial and Engineering Chemistry, 41:1184-1191, June 1949.
20. Gregory, W. B. Pumping clay slurry through a four inch pipe. Mechanical Bngineering, 49:609-616, June 1927.
21. Hariu, O. H. and Molstad, M. C. Pressure drop in vertical tubes in transport of solids by gases. Industrial and Engineering Chemistry, 41:1148-1160, June 1949.
22. Howard, G. W. Transportation of sand and gravel in a four inch pipe. American Society of Civil Engineers. Transactions, 104: 1334-48, 1939.
23. Howard, G. W. Effect of rifling on four inch pipe transporting solids. American Society of Civil Engineers. Transactions, 106:135-157, 1941.
24. Ismail, H. M. Turbulent transfer mechanism and suspended sediment in closed channels. American Society of Civil Engineers, Transactions, 117:409-434, 1952.
25. Maltby, F. B. Hydraulic dredging on the Mississippi River. American Society of Civil Engineers. Transactions, $54 \mathrm{Pt} . \mathrm{C}$ : 391-478, 1904.
26. Morris, H. N. Flow in rough conduits. American Society of Civil Bngineers. Transactions, 120:373-410, 1955.
27. O'Brien, M. P. and Folsom, R. G. The transportation of sand in pipe lines. California, University. Publications in engineering, 3:343-384, 1937.
28. $O^{\prime}$ Brien, M. P. Review of the theory of turbulent flow and its relation to sediment transportation. American Geophysical Union. Transactions, 14:487~491, April 1933.

## BIBLIOGRAPHY --Continued

24. Ohio to have first pipe-line for coal. Civil Engineering, Sept 1955. p.84. Vol. 25.
25. Soleil, G. and P. Ballade, Le transport hydraulique des materiaux dans les travaux publics. (In Societe Hydrotechnique de France. Transport hydraulique et decantation des materiaux solides. Grenoble, France, La Huille Blanche, 1953. p. 9-26) Compte rendu des Deuxiemes Journees de $1^{\prime}$ Hydraulique.
26. U. S. Engineer Department. Bonneville Hydraulic Laboratory. Friction losses in corrugated metal pipe. Portland, Oregon, Corps of Engineers, Oregon District, January 1952. 10 p., 16 p1., processed. (First preliminary report. CWI 828).
27. U. S. Engineer Department. Bonneville Hydraulic Laboratory. Friction losses in corrugated metal pipe. Portland, Oregon, Corps of Engineers, Oregon District, October 1952. 4p., 17 p1., processed. (Supplement to First preliminary report. CWI 828).
28. U. S. Engineer Department. Bonneville Hydraulic Laboratory. Friction losses in corrugated metal pipe. Portland, Oregon, Corps of Engineers, Oregon District, November 1953. 4 p., 6 p1., processed. (Memorandum report 2-2. CWI 828).
29. U. S. Engineer Department. Bonneville Hydraulic Laboratory. Friction losses in corrugated metal pipe. Portland, Oregon, Corps of Engineers, Oregon District, January 1954. 4 p., 6 pl., processed. (Memorandum report 2-3. CWI 828).
30. U. S. Engineer Department. Bonneville Hydraulic Laboratory. Friction losses in corrugated metal pipe. Portland, Oregon, Corps of Engineers, Oregon District. December 1954. 4 p., 8 p1., processed. (Memorandum report 3-1. CWI 828).
31. Vanoni, Vito A. Transportation of suspended sediment by water. American Society of Civil Engineers. Transactions, 111:67-133, 1946.
32. Vogt, B. G. and White, R. R. Friction in flow of suspensions. Industrial and Engineering Chemistry, 40:1731-1738, Sept 1948.
33. Wilson, W. E. Mechanics of flow, with non-colloidal inert solids. American Society of Civil Engineers. Transactions, 107:15761586, 1942.

## BIBLIOGRAPHY --Continued

34. Wilson, W. E. Transportation of suspended solids in pipe lines. American Institute of Mining and Metallurgical Engineers. Technical publication No. 1785:1-10, March 1945.
35. Wood, W. A. and Bailey, A. J. The horizontal carriage of granular material by an injector driven air stream. Institution of Mechanical Bngineers. Proceedings, 1420:149-173, 1939.


Fig. 1 Schematic representation of fluids.


Fig. 2 Relation between velocity fluctuation and velocity gradient.
cair valve



Fig. 5 Calibration curve for sediment sampling cone.


$$
\begin{array}{ll}
R_{1}=0.7622 & L=0.76 \\
R_{2}=0.6875 & \phi=53^{\circ} 44^{\prime} \\
M=0.500 & T=0.0745
\end{array}
$$


note: all length dimensions are in inches section views are normal to corrugation

Fig. Boundary forms under investigation.



Fig. 8 Variation of median sediment size with Reynolds number.



Fig. 9 variation of size distribution of sediment with Re.


Fig. 10 Variation of $J$ with Vand $G r$ for all boundaries.


Fig. 11 Variation of Jwith $V$ and $C_{T}$ for helical corrugated pipe




Fig. 12 Variation of $Q$ with $G$.



Fig. 14 Variation of $E$ with $W$ for all boundaries.


Fig. 15 Plot of Durands equation for smooth boundary.



Fig. 16 Variation of $f$ with $R_{e}$ and $G_{T}$ for smooth and standard corrugated pipes.



Fig. 18 Variation of head loss with position and time Run no. 41 H .


Fig. 19 Variation of Revf with concentration for all boundaries.




Fig. 22 Variation of Rel f with $\left[\frac{I}{d / D}\right]^{5} C_{T}^{3 / 3}$ for smooth boundary
$\operatorname{Re} \sqrt{f}=\frac{V D}{\gamma}$

APPENDIX

## TABLE OF CONTENTS

## Item

A Summary of laboratory data
B Summary of computed quantities
C Summary of sieve analysis data
D Derivation to find the velocity for a given $W$, at which $E$ is a minimum

APPENDIX A
SUMMARY OF LABORATORY DATA

I. hBLICAL CORRUGATBD PIPB

| $1_{H}$ | 2.06 | 2.62 | 15.80 | - | 8.352 | 8.291 | 8.245 | 8.195 | 8.147 | 8.094 | 8.038 | (Clear water runs) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 H | 2.60 | 3.31 | 17.00 | - | 8.077 | 7.981 | 7.896 | 7.798 | 7.694 | 7.589 | - | " " " |
| $3^{H}$ | 2.17 | 2.76 | 16.70 | - | 7.945 | 7.883 | 7.829 | 7.768 | 7.710 | 7.648 | 7.597 | " " " |
| 4H | 2.81 | 3.58 | 16.20 | - | 8.434 | 8.336 | 8.243 | 8.147 | 8.055 | 7.951 | - | + 1 |
| 5 H | 3.27 | 4.17 | 16.70 | - | 8.759 | 8.627 | 8.503 | 8.379 | 8.254 | 8.125 | - | " " " |
| ${ }^{6} \mathrm{H}$ | 4.50 | 5.73 | 17.00 | - | - | 8.774 | 8.558 | 8.309 | 8.107 | - | - | Clear water runs) |
| 7H | 5.89 | 7.50 | 14.65 | - | 8.057 | 7.688 | 7.334 | 6.981 | 6.632 | - | - | " " " |
| 8H | 6.25 | 7.97 | 15.95 | - | 8.250 | 7.832 | 7.438 | 7.042 | 6.641 | - | - | " " |
| $9^{4} \mathrm{H}$ | 6.56 | 8.35 | 16.30 | - | 9.089 | 8.654 | 8.287 | 7.829 | 7.401 | - | - | " " " |
| 10 H | 1.25 | 1.59 | 16.55 | - | 7.965 | 7.942 | 7.916 | 7.890 | 7.867 | - | 7.815 | " " " |
| 11H | 2.00 | 2.55 | 14.06 | 0.15 | 8.794 | - | 8.706 | 8.662 | 8.619 | 8.572 | 8.528 | No deposition |
| $12_{\mathrm{H}}$ | 2.83 | 3.61 | 13.35 | 0.31 | 9.189 | 9.108 | 9.027 | 8.930 | 8.824 | - | 8.635 | " " |
| 13 H | 5.06 | 6.44 | 13.46 | 0.41 | 8.416 | 8.171 | 7.806 | 7.591 | 7.293 | 6.976 | 6.705 | " " |
| 14H | 6.55 | 8.35 | 17.80 | 0.53 | 8.822 | 8.461 | 7.911 | 7.610 | 7.142 | 6.648 | 6.255 | " " |
| 15 H | 2.26 | 2.88 | 14.52 | 0.35 | 8.467 | 8.392 | 8.303 | 8.237 | 8.185 | 8.102 | 8.031 | " |
| 16H | 4.13 | 5.27 | 13.10 | 4.10 | 9.278 | 9.037 | 8.765 | 8.562 | 8.349 | 8.093 | - | No deposition |
| 17H | 5.06 | 6.44 | 14.00 | 4.28 | 8.758 | 8.433 | 8.076 | 7.795 | 7.470 | 7.124 | - | " |
| 18 H | 5.80 | 7.39 | 13.75 | 4.13 | 9.024 | 8.664 | 8.270 | 7.899 | 7.519 | 7.113 | - | " |
| 19 H | 6.59 | 8.40 | 15.70 | 3.80 | 9.044 | 8.614 | 8.031 | 7.693 | 7.195 | 6.685 | 6.253 | " " 0 , |
| 20 H | 2.62 | 3.34 | 13.75 | 0.50 | 8.462 | 8.373 | 8.279 | 8.196 | 8.106 | 8.011 | 7.914 | Maximum 6 in. wide, minimum 0 in.* |
| 21H | 6.59 | 8.40 | 15.50 | 5.00 | 9.344 | 8.896 | 8.455 | - | 7.505 | 6.973 | 6.513 | No deposition |
| $22_{\text {H }}$ | 5.82 | 7.41 | 14.52 | 3.96 | 8.942 | 8.580 | 8.094 | 7.758 | 7.383 | 6.934 | - | " " |
| 23 H | 3.95 | 5.04 | 15.35 | 2.53 | 8.902 | 8.668 | 8.410 | 8.178 | 7.960 | 7.693 | 7.455 | Maximum deposit 4 in. wide, min. 0 . in. |
| 24H | 3.00 | 3.82 | 12.47 | 1.00 | 8.899 | 8.761 | 8.613 | 8.469 | 8.320 | 8.150 | 7.988 | Maximum deposit 6 in. wide, min. 0.1 n . |
| 25H | 2.50 | 3.19 | 12.60 | 0.65 | 8.873 | 8.798 | 8.707 | 8.617 | 8.549 | - | - | Average 8 in. wide deposition all times |

* Deposition was measured along the circumference of pipe.

APPENDIX A --Contimued

| Run | Q | V | T | $\mathrm{C}_{\mathrm{T}}$ | Piezometer Readings |  |  |  |  |  |  | Bed Condition |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | cfs | fps | ${ }^{\circ} \mathrm{C}$ | \% | 1 | 2 | 3 | 4 | 5 | 6 | 7 | (As observed in 1 Ft plastic sec.) |
| ${ }^{26} \mathrm{H}$ | 3.78 | 4.81 | 13.40 | 2.35 | 9.139 | 8.933 | 8.709 | 8.500 | 8.254 | - | - | Average 3 in. wide deposition |
| ${ }^{27}{ }_{H}$ | 4.96 | 6.32 | 13.50 | 5.25 | 8.794 | 8.428 | 8.005 | 7.645 | 7.315 | 6.999 | 6.575 | No deposition |
| $28^{2} \mathrm{H}$ | 6.55 | 8.35 | 12.28 | 5.54 | 9.176 | 8.745 | 8.221 | 7.833 | 7.423 | 6.969 | 6.553 | No deposition |
| 29H | 2.98 | 3.80 | 13.00 | 1.63 | 8.584 | 8.397 | 8.163 | 7.987 | 7.750 | 7.562 | 7.365 | Average 6 in. wide deposition |
| 30H | 3.87 | 4.93 | 13.52 | 3.70 | 8.704 | 8.445 | 8.213 | 7.832 | 7.491 | - | - | No deposition |
| $31_{H}$ | 5.15 | 6.56 | 13.10 | 6.35 | 9.356 | 8.987 | 8.564 | - | 7.888 | 7.471 | 7.090 | No deposition |
| 32H | 6.55 | 8.35 | 13.92 | 7.10 | 9.256 | 8.775 | 8.262 | - | 7.294 | 6.759 | 6.319 | No deposition |
| 33 H | 3.19 | 4.05 | 13.90 | 2.52 | 9.261 | 8.964 | 8.689 | 8.448 | 8.173 | 7.868 | 7.665 | Average 9 in. wide deposition |
| 34 H | 2.03 | 2.58 | 14.03 | 0.58 | 8.829 | 8.768 | 8.676 | 8.559 | 8.422 | 8.299 | 8.200 | Average 12 in wide deposition |
| $35_{H}$ | 5.09 | 6.49 | 12.80 | 9.23 | 9.337 | 8.922 | 8.487 | 8.039 | 7.598 | 6.980 | 6.472 | Average 4 in. wide deposition. |
| 36H | 6.15 | 7.84 | 13.53 | 9.73 | 9.095 | 8.591 | 7.950 | - | 6.908 | 6.377 | - | No deposition |
| 37H | 2.59 | 3.30 | 13.10 | 1.29 | 9.020 | 8.808 | 8.577 | 8.415 | 8.233 | 8.012 | 7.827 | Average 10 in. wide deposition |
| 38H | 3.00 | 3.82 | 13.00 | 2.30 | 9.087 | 8.775 | 8.452 | 8.214 | 7.924 | 7.585 | 7.376 | Average 10 in. wide deposition |
| ${ }^{39} \mathrm{H}$ | 5.00 | 6.37 | 13.18 | 10.09 | 9.241 | 8.703 | 8.271 | 7.752 | 7.185 | 6.506 | - | Average 8 in. wide deposition |
| 40 H | 6.08 | 7.75 | 13.10 | 12.23 |  | 8.963 | 8.599 | 8.008 | 7.342 | 6.502 | - | No deposition |
| 41H | 2.58 | 3.28 | 14.50 | 1.32 | 8.883 | 8.653 | 8.456 | 8.290 | 8.097 | 7.872 | 7.671 | Average 10 in. wide deposition |
| 42H | 5.03 | 6.40 | 13.15 | 9.72 | 9.304 | 8.812 | 8.063 | 7.463 | 6.817 | - | - | Average 7 in. wide deposition |
| 43H | 6.28 | 7.83 | 13.30 | 14.08 | - | - | 9.262 | 8.599 | 7.872 | 7.057 | 6.319 | Max. 4 in. wide, min. 0 in. |
| 44H | 3.05 | 3.88 | 14.06 | 3.40 | 9.292 | 8.864 | 8.457 | 8.165 | 7.805 | 7.413 | - | Average 13 in. wide deposition |
| 45H | 5.02 | 6.40 | 12.30 | 12.31 | - | - | 9.398 | 8.835 | 8.085 | 7.315 | 6.605 | Average 8 in. wide deposition |
| 46H | 6.14 | 7.82 | 14.60 | 18.40 | 11.554 | 10.721 | 9.743 | 9.062 | 8.218 | 7.253 | 6.529 | Average 6 in. wide deposition |
| 47H | 3.20 | 4.08 | 14.30 | 5.10 | 9.311 | 8.783 | 8.265 | 7.849 | 7.397 | 6.861 | - | Average 13 in. wide deposition |
| 48 H | 3.72 | 4.74 | 13.70 | 8.02 | 9.424 | 8.767 | 8.073 | 7.489 | 6.837 | - | - | Average 11 in. wide deposition |
| 49H | 5.21 | 6.64 | 12.60 | 14.00 | - | - | - | 9.385 | 8.491 | 7.561 | 6.803 | Average 10 in. wide deposition |
| 50H | 5.91 | 7.55 | 15.10 | 18.57 | - | - | - | 9.138 | 8.344 | 7.406 | 6.576 | Average 8 in. wide deposition |

## APPRNDIX A --Continued

| $\overline{\text { Run }}$ | Q | V | T | $\mathrm{C}_{\text {T }}$ | Piezometer Readings |  |  |  |  |  |  | Bed Condition(As observed in 1Ft plastic sec.) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | cfs | fps | ${ }^{\circ} \mathrm{C}$ | \% | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |
| $51_{\text {H }}$ | 3.90 | 4.97 | 16.25 | 9.73 | - | - | - | 8.811 | 8.121 | 7.437 | 6.820 | Average 11.5 in. wide deposition |
| 52H | 5.15 | 6.56 | 16.00 | 17.09 | - | - | - | 9.055 | 8.203 | 7.280 | 6.567 | Average 10 in . wide deposition |
| $5^{5} \mathrm{H}$ | 6.18 | 7.88 | 13.60 | 19.90 | - | - | - | 9.360 | 8.485 | 7.419 | 6.575 | Average 8 in. wide deposition |

II. SMOOTH PIPB

| $1_{\text {s }}$ | 2.03 | 2.58 | 12.45 | - | 8.397 | 8.356 | 8.343 | 8.324 | 8.303 | 8.292 | 8.270 | (Clear water) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 s | 2.99 | 3.81 | 13.30 | - | 8.462 | 8.408 | 8.373 | 8.344 | 8.299 | 8.270 | 8.228 | " " |
| 3 s | 4.08 | 5.20 | 14.50 | - | 8.332 | 8.276 | 8.223 | 8.151 | 8.104 | 8.021 | - | " " |
| 4 s | 4.72 | 6.02 | 15.15 | - | 8.466 | 8.363 | 8.283 | 8.215 | 8.118 | 8.061 | 7.963 | " " |
| 5 s | 5.15 | 6.56 | 12.55 | - | 8.722 | 8.549 | 8.460 | 8.370 | 8.259 | 8.163 | 8.036 | " " |
| $6{ }_{5}$ | 6.03 | 7.68 | 13.90 | - | - | 8.169 | 8.057 | 7.932 | 7.802 | 7.667 | 7.516 | " |
| 7 s | 7.20 | 9.17 | 14.50 | - | - | 8.308 | 8.160 | 8.018 | 7.836 | 7.663 | 7.468 | " ${ }^{\prime}$ - |
| 8 S | 3.00 | 3.82 | 13.80 | 0.40 | 8.869 | 8.789 | 8.732 | 8.675 | 8.621 | 8.583 | 8.509 | Sediment moving in form of waves |
| 9 s | 4.03 | 5.14 | 13.02 | 0.63 | 8.891 | 8.747 | 8.717 | 8.623 | 8.500 | 8.389 | 8.259 | " ${ }^{\prime \prime}$ |
| 10 s | 5.18 | 6.60 | 15.17 | 0.95 | 8.804 | 8.627 | 8.547 | 8.419 | 8.243 | 8.106 | 7.880 | Occasional deposit in form of waves |
| $11{ }_{s}$ | 6.02 | 7.67 | 14.00 | 1.45 | 8.805 | 8.635 | 8.520 | 8.343 | 8.162 | 7.980 |  | Very little deposit in form of wave |
| 12 s | 7.32 | 9.33 | 13.07 | 1.89 | 8.323 | 8.119 | 7.935 | 7.708 | 7.467 | 7.302 |  | No deposition |
| 13 s | 3.55 | 4.53 | 12.94 | 0.61 | 8.968 | 8.885 | 8.806 | 8.699 | 8.628 | 8.562 | 8.485 | Max. 11.50 in wide, min. 0 in. |
| 14 s | 4.96 | 6.32 | 13.85 | 1.37 | 8.951 | 8.856 | 8.728 | 8.520 | 8.309 | 8.154 | 8.031 | Average 9 in. wide deposition |
| 15 s | 5.91 | 7.54 | 12.45 | 1.78 | 8.922 | 8.781 | 8.643 | 8.387 | 8.128 | 7.954 | 7.656 | Average 8 in . wide deposition |
| 16 s | 7.28 | 9.27 | 13.40 | 2.74 | 8.537 | 8.332 | 8.136 | 7.912 | 7.621 | 7.331 | - | Average 4 in. wide deposition |
| 17s | 3.06 | 3.90 | 13.62 | 1.00 | 8.955 | 8.855 | 8.751 | 8.624 | 8.499 | 8.416 | 8.290 | Max. 12.50 in . wide, min. 0 in. |
| 18s | 4.59 | 5.84 | 12.80 | 1.52 | 9.111 | 8.997 | 8.828 | 8.626 | 8.416 | 8.296 | 8.079 | Average 10 in . wide deposition |
| 19 s | 5.72 | 7.29 | 14.45 | 2.09 | 9.042 | 8.886 | 8.688 | 8.462 | 8.125 | 7.916 | 7.570 | Average 9 in. wide deposition |
| 20 s | 7.28 | 9.29 | 13.72 | 3.48 | 8.559 | 8.323 | 8.115 | 7.809 | 7.381 | 7.103 | - | Average 7 in . wide deposition |

APPENDIX A --Continued

| Run | Q | V | T | CT | Piezometer Readings |  |  |  |  |  |  | Bed Condition(As observed in 1Ft. plastic sec.) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | cfs | fps | ${ }^{\circ} \mathrm{C}$ | \% | 1 | 2 | 3 | 4 | 5 | 6 | 7 |  |
| 21 | 2.99 | 3.71 | 14.10 | 1.24 | 9.130 | 9.031 | 8.886 | 8.741 | 8.667 | 8.559 | 8.351 | Max. 15 in. wide, min. 0 in. |
| 22 s | 4.41 | 5.62 | 12.75 | 1.73 | 9.425 | 9.232 | 9.032 | 8.800 | 8.619 | 8.490 | 8.246 | Avg. 11 in. wide deposition |
| 23s | 5.90 | 7.51 | 13.20 | 2.87 | 8.754 | 8.576 | 8.240 | 7.880 | 7.539 | 7.298 | 6.939 | Avg. 10 in. wide deposition |
| 24 s | 7.20 | 9.18 | 13.10 | 4.69 | 8.824 | 8.608 | 8.388 | 7.832 | 7.358 | 7.064 | 6.582 | Avg. 8 in. wide deposition |
| 25 s | 7.18 | 9.15 | 13.88 | 5.25 | 9.123 | 8.863 | 8.291 | 7.803 | 7.377 | 7.094 | - | Avg. 9 in. wide deposition |
| 26 s | 5.80 | 7.39 | 13.43 | 3.39 | 6.886 | 8.631 | 8.162 | 7.808 | 8.495 | 7.241 | 6.846 | Avg. 11 in. wide deposition |
| 27 s | 4.80 | 6.12 | 14.40 | 2.20 | 8.790 | 8.537 | 8.213 | 7.903 | 7.705 | 7.495 | 7.217 | Avg. 12 in. wide deposition |
| 28 s | 3.49 | 4.45 | 12.67 | 1.53 | 8.709 | 8.557 | 8.344 | 8.204 | 8.001 | 7.851 | 7.667 | Avg. 12 in. wide deposition |
| 29 s | 7.15 | 9.10 | 14.88 | 6.35 | 9.293 | 8.931 | 8.235 | 7.680 | 7.169 | 6.853 | 6.290 | Avg. 10.50 in. wide deposition |
| 30 s | 5.20 | 6.63 | 13.20 | 3.19 | 8.929 | 8.515 | 8.049 | 7.746 | 7.448 | 7.213 | 6.862 | Avg. 12.50 in. wide deposition |
| 31 s | 3.60 | 4.59 | 13.37 | 1.73 | 8.901 | 8.673 | 8.412 | 8.184 | 8.134 | 7.960 | 7.759 | Avg. 12 in. wide deposition |
| 32 s | 7.10 | 9.04 | 12.94 | 7.15 | - | 9.043 | 8.291 | 7.771 | 7.239 | 6.866 | 6.315 | Avg. 11.50 in. wide deposition |
| 33 s | 5.66 | 7.21 | 13.26 | 4.51 | - | 3.744 | 8.272 | 7.899 | 7.567 | 7.271 | 6.848 | Avg. 13 in. wide deposition |
| 34s | 3.65 | 4.65 | 13.70 | 2.12 | 8.874 | 8.579 | 8.355 | 8.167 | 7.980 | 7.832 | 7.612 | Avg. 13.50 in. wide deposition |
| 35 s | 7.01 | 8.94 | 12.93 | 7.57 | - | 8.881 | 8.193 | 7.682 | 7.217 | 6.016 | 6.245 | Avg. 12 in. wide deposition |
| 36 s | 5.54 | 7.05 | 13.00 | 5.60 | - | 9.138 | 8.623 | 8.236 | 7.903 | 7.576 | 7.145 | Avg. 13 in. wide deposition |
| 37 s | 3.48 | 4.43 | 13.10 | 2.25 | 9.292 | 9.000 | 8.763 | 8.559 | 8.375 | 8.209 | 8.037 | Avg. 14.50 in. wide deposition. |
| 38 s | 6.94 | 8.84 | 12.95 | 8.45 | - | 9.207 | 8.489 | 8.007 | 7.481 | 7.008 | 6.447 | Avg. 12.50 in. wide deposition |
| 39 s | 5.31 | 6.77 | 12.12 | 5.63 | - | 9.215 | 8.724 | 8.344 | 7.961 | 7.570 | 7.138 | Avg. 13.50 in. wide deposition |
| 40 s | 3.61 | 4.60 | 12.85 | 2.17 | 9.413 | 9.067 | 8.849 | 8.597 | 8.398 | 8.231 | 7.993 | Avg. 15.50 in. wide deposition |

## APPENDIX A --Continued


III. STANDARD CORRUGATED PIPE

| $1_{c}$ | 5.00 | 6.37 | 19.50 | - | - | 11.073 | 10.103 | 9.393 | 8.418 | 7.413 | 6.283 | (Clear water) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 c | 5.21 | 6.65 | 17.75 | - | - | - | 9.216 | 8.366 | 7.529 | 6.573 | - | " " |
| 3 c | 5.70 | 7.26 | 19.00 | - | - | - | 9.345 | 8.186 | 7.406 | 6.243 | - | " " |
| 4 c | 5.70 | 7.26 | 19.20 | - | - | - | 9.145 | 8.373 | 7.438 | 6.415 | - | " " |
| 5 c | 5.00 | 6.37 | 20.35 | 0.03 | - | 9.250 | 8.510 | 7.882 | 7.172 | 6.410 | - | No deposition |
| 6 c | 3.20 | 4.05 | 20.25 | 0.09 | 9.289 | 8.881 | 8.560 | 8.275 | 7.941 | 7.624 | 7.256 | No deposition |
| 7 c | 4.35 | 5.55 | 23.10 | 0.23 | - | 9.225 | 8.729 | 8.265 | 7.727 | 7.179 | 6.605 | " " |
| 8 c | 4.66 | 5.95 | 22.80 | 0.26 | - | - | 9.236 | 8.674 | 7.994 | 7.226 | 6.572 | " " |
| ${ }^{9} \mathrm{c}$ | 4.99 | 6.36 | 25.25 | 0.90 | - | - | 9.403 | 8.818 | 8.110 | 7.384 | 6.563 | " " |
| 10 c | 3.80 | 4.86 | 21.25 | 0.26 | - | 9.128 | 8.638 | 8.261 | 7.819 | 7.370 | 6.857 | " " |
| 11c | 4.40 | 5.61 | 23.70 | 0.29 | - | 9.351 | 8.707 | 8.273 | 7.707 | 7.150 | 6.513 | " " |
| 12 c | 3.28 | 4.17 | 23.25 | 0.57 | 9.256 | 8.880 | 8.475 | 8.176 | 7.832 | 7.485 | 7.085 | " " |
| 13 c | 3.95 | 5.03 | 24.30 | 0.63 | - | 9.007 | 8.541 | 8.176 | 7.743 | 7.304 | 6.819 | " " |
| 14 c | 3.50 | 4.46 | 17.80 | 0.61 | 9.286 | 8.884 | 8.493 | 8.179 | 7.824 | 7.454 | 7.043 | " " |
| 15 c | 2.44 | 3.09 | 21.25 | 0.17 | 7.995 | 7.780 | 7.562 | 7.374 | 7.184 | 6.997 | 6.789 | " " |
| 16 c | 3.04 | 3.82 | 22.70 | 0.28 | 9.019 | 8.725 | 8.425 | 8.177 | 7.906 | 7.634 | 7.318 | " |
| 17 c | 2.95 | 3.76 | 23.80 | 0.26 | 9.185 | 8.858 | 8.549 | 8.305 | 7.945 | 7.742 | 7.425 | " " |
| 18 c | 4.78 | 6.10 | 22.00 | 1.21 | - | 9.561 | 8.865 | 8.319 | 7.687 | 7.045 | 6.270 | " " |
| 19 c | 4.60 | 5.85 | 24.95 | 1.26 | - | 9.083 | 8.471 | 8.010 | 7.420 | 6.863 | - | " " |
| 20 c | 5.20 | 6.64 | 25.80 | 6.31 | - | - | - | 9.287 | 8.484 | 7.711 | 6.801 | " |

SUMMARY OF COMPUTED QUANTITIBS
I. HELICAL CORRUGATED PIPE

| $\begin{aligned} & \text { Run } \\ & \text { No. } \end{aligned}$ | $\begin{gathered} \mathrm{Q} \\ \mathrm{cfs} \end{gathered}$ | $\begin{aligned} & \hline \mathbf{C}_{\mathbf{T}} \\ & \% \end{aligned}$ | $\begin{gathered} \mathrm{GX1O}^{2} \\ \mathrm{cfs} \end{gathered}$ | $\begin{gathered} \mathrm{W} \\ \mathrm{lbs} / \mathrm{sec} \end{gathered}$ | $\mathrm{HP} \times 10^{2}$ | B | $\mathrm{J} \times 10^{2}$ | Rex $10^{-5}$ | $\mathrm{fx10}{ }^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{1} \mathrm{H}$ | 2.06 | - | - | - | 0.12 | - | 0.50 | 2.18 | 4.78 |
| 2 H | 2.60 | - | - | - | 0.28 | - | 0.95 | 2.84 | 5.59 |
| $3 H$ | 2.17 | - | - | - | 0.15 | - | 0.59 | 2.35 | 5.00 |
| 4H | 2.81 | - | - | - | 0.31 | - | 0.96 | 3.02 | 4.83 |
| 5 H | 3.27 | - | - | - | 0.47 | - | 1.27 | 3.56 | 4.70 |
| 6H | 4.50 | - | - | - | 1.13 | - | 2.20 | 4.93 | 4.35 |
| 7H | 5.89 | - | - | - | 2.38 | - | 3.56 | 6.08 | 4.08 |
| 8 H | 6.25 | - | - | - | 2.85 | - | 4.02 | 6.66 | 4.08 |
| ${ }^{9} \mathrm{H}$ | 6.56 | - | - | - | 3.15 | - | 4.22 | 7.05 | 3.90 |
| ${ }^{10} \mathrm{H}$ | 1.25 | - | - | - | 0.04 | - | 0.25 | 1.35 | 6.40 |
| $11_{H}$ | 2.00 | 0.15 | 0.12 | 0.19 | 0.10 | 2.88 | 0.44 | 2.11 | 4.03 |
| 12 H | 2.83 | 0.31 | 0.33 | 0.54 | 0.31 | 3.16 | 0.97 | 2.82 | 4.77 |
| 13H | 5.06 | 0.41 | 0.79 | 1.29 | 1.64 | 6.95 | 2.85 | 5.05 | 4.43 |
| 14H | 6.55 | 0.53 | 1.31 | 2.16 | 3.25 | 8.25 | 4.37 | 7.33 | 4.04 |
| 15 in | 2.26 | 0.35 | 0.30 | 0.49 | 0.18 | 2.02 | 0.70 | 2.32 | 4.94 |
| 16H | 4.13 | 4.10 | 6.48 | 10.70 | 1.14 | 0.57 | 2.37 | 4.09 | 5.50 |
| 17 H | 5.06 | 4.28 | 8.40 | 13.87 | 1.93 | 0.74 | 3.27 | 5.11 | 5.08 |
| 18 H | 5.80 | 4.13 | 9.28 | 15.31 | 2.59 | 0.90 | 3.83 | 5.85 | 4.53 |
| 19H | 6.59 | 3.80 | 9.68 | 15.99 | 3.64 | 1.21 | 4.74 | 6.97 | 4.33 |
| 20 H | 2.62 | 0.50 | 0.50 | 0.82 | 0.27 | 1.83 | 0.92 | 2.64 | 5.29 |
| 21H | 6.59 | 5.00 | 12.83 | 21.20 | 3.69 | 0.93 | 4.77 | 6.95 | 4.36 |
| $22_{\mathrm{H}}$ | 5.82 | 3.96 | 8.90 | 14.69 | 2.78 | 1.01 | 4.10 | 5.97 | 4.80 |
| 23 H | 3.95 | 2.53 | 3.83 | 6.32 | 1.10 | 0.95 | 2.41 | 4.14 | 6.15 |
| 24H | 3.00 | 1.00 | 1.14 | 1.88 | 0.51 | 1.49 | 1.50 | 2.91 | 6.61 |
| 25 H | 2.50 | 0.66 | 0.63 | 1.03 | 0.23 | 1.23 | 0.81 | 2.44 | 5.13 |
| ${ }^{26} \mathrm{H}$ | 3.78 | 2.35 | 3.40 | 5.61 | 0.94 | 0.91 | 2.17 | 3.77 | 6.05 |
| 27H | 4.96 | 5.25 | 10.19 | 16.80 | 2.12 | 0.68 | 3.70 | 4.95 | 5.95 |
| 28H | 6.55 | 5.54 | 14.18 | 23.40 | 3.36 | 0.76 | 4.37 | 6.32 | 4.04 |
| 29 H | 2.98 | 1.63 | 1.85 | 3.07 | 0.69 | 1.24 | 2.03 | 2.94 | 9.07 |
| 30 H | 3.87 | 3.70 | 5.54 | 9.13 | 1.37 | 0.80 | 3.05 | 3.86 | 8.07 |
| 31 H | 5.15 | 6.35 | 12.87 | 21.22 | 2.30 | 0.57 | 3.78 | 5.09 | 5.67 |
| 32H | 6.55 | 7.10 | 18.33 | 30.30 | 3.81 | 0.66 | 4.90 | 6.63 | 4.53 |
| 33H | 3.19 | 2.52 | 3.06 | 5.05 | 0.98 | 1.05 | 2.66 | 2.91 | 10.40 |
| 34H | 2.03 | 0.58 | 0.45 | 0.74 | 0.36 | 2.66 | 1.55 | 2.05 | 11.10 |
| 35 H | 5.09 | 9.23 | 18.83 | 31.10 | 2.83 | 0.45 | 4.39 | 5.00 | 6.72 |
| 36 H | 6.15 | 9.73 | 24.00 | 39.60 | 4.05 | 0.53 | 5.44 | 6.15 | 5.70 |
| $33^{\text {H }}$ | 2.59 | 1.29 | 1.27 | 2.10 | 0.58 | 1.51 | 1.96 | 4.25 | 11.60 |
| 38 H | 3.00 | 2.30 | 2.64 | 4.35 | 1.015 | 1.25 | 2.94 | 4.94 | 13.30 |
| ${ }^{39} \mathrm{H}$ | 5.00 | 10.09 | 20.25 | 33.40 | 3.16 | 0.49 | 5.23 | 8.19 | 8.30 |
| $40^{H}$ | 6.08 | 12.23 | 30.40 | 50.10 | 5.36 | 0.54 | 7.17 | 9.98 | 7.70 |

## APPENDIX B --Continued

| $\begin{aligned} & \text { Run } \\ & \text { No. } \end{aligned}$ | $\begin{gathered} \mathrm{Q} \\ \mathrm{cfs} \end{gathered}$ | $\begin{aligned} & \mathrm{C}_{\mathrm{T}} \\ & \% \end{aligned}$ | $\begin{gathered} {\mathrm{Gx} 10^{2}}^{\text {cfs }} \end{gathered}$ | $\mathrm{lbs} / \mathrm{sec}$ | HPx $10^{2}$ | E | $\mathrm{Jx} 10^{2}$ | Rex $10^{-5}$ | $\mathrm{fx} 10^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 41H | 2.58 | 1.32 | 1.29 | 2.13 | 0.60 | 1.53 | 2.02 | 3.98 | 12.70 |
| $42^{\text {H }}$ | 5.03 | 9.72 | 19.62 | 32.40 | 3.08 | 0.49 | 5.08 | 8.23 | 8.00 |
| $43_{\mathrm{H}}$ | 6.28 | 14.08 | 36.48 | 60.25 | 5.86 | 0.49 | 7.50 | 10.00 | 7.90 |
| 44H | 3.05 | 3.40 | 4.00 | 6.60 | 1.33 | 1.08 | 3.76 | 4.88 | 16.10 |
| 45 H | 5.02 | 12.31 | 25.30 | 41.75 | 4.44 | 0.54 | 7.19 | 8.45 | 11.30 |
| $4_{6} \mathrm{H}$ | 6.14 | 18.40 | 47.95 | 79.10 | 6.67 | 0.41 | 8.50 | 9.70 | 8.90 |
| 47 H | 3.20 | 5.10 | 6.37 | 10.52 | 1.96 | 0.93 | 4.90 | 3.27 | 18.90 |
| $48_{\mathrm{H}}$ | 3.72 | 8.02 | 11.86 | 19.58 | 2.88 | 0.77 | 6.47 | 3.74 | 18.50 |
| ${ }^{49} \mathrm{H}$ | 5.21 | 14.00 | 30.18 | 49.70 | 5.92 | 0.60 | 9.12 | 5.08 | 13.40 |
| $50_{\mathrm{H}}$ | 5.91 | 18.57 | 47.00 | 77.50 | 6.56 | 0.41 | 8.63 | 6.17 | 9.75 |
| $51^{1}$ | 3.90 | 9.73 | 15.26 | 25.18 | 3,13 | 0.64 | 6.63 | 4.20 | 17.30 |
| $5_{2} \mathrm{H}$ | 5.15 | 17.09 | 37.10 | 59.52 | 5.49 | 0.43 | 8.25 | 5.49 | 12.40 |
| $53_{H}$ | 6.18 | 19.90 | 52.84 | 87.20 | 7.49 | 0.41 | 9.28 | 6.20 | 9.65 |

II. SMOOTH PIPB

| $\begin{aligned} & \text { Run } \\ & \text { No. } \\ & \hline \end{aligned}$ | $\begin{gathered} \mathrm{Q} \\ \mathrm{cfs} \end{gathered}$ | $\begin{aligned} & \mathrm{C}_{\mathrm{T}} \\ & \% \\ & \hline \end{aligned}$ | $\begin{gathered} \mathrm{Gx10}^{2} \\ \mathrm{cf} \mathrm{~s} \end{gathered}$ | 1bs/sec | $\mathrm{HP} \times 10^{2}$ | B | $\mathrm{J} \times 10^{2}$ | Rex $10^{-5}$ | $\mathrm{fx} 10^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1_{s}$ | 2.03 | - | - | - | 0.04 | - | 0.18 | 1.96 | 1.75 |
| 2 s | 2.99 | - | - | - | 0.10 | - | 0.30 | 2.97 | 1.33 |
| 3 s | 4.08 | - | - | - | 0.31 | - | 0.66 | 4.18 | 1.58 |
| 4 s | 4.72 | - | - | - | 0.44 | - | 0.82 | 4.94 | 1.46 |
| 5 s | 5.15 | - | - | - | 0.58 | - | 0.99 | 5.00 | 1.48 |
| 6 s | 6.03 | - | - | - | 0.86 | - | 1.25 | 6.08 | 1.36 |
| 7 s | 7.20 | - | - | - | 1.33 | - | 1.63 | 7.38 | 1.25 |
| 8 s | 3.00 | 0.40 | 0.45 | 0.75 | 0.19 | 1.41 | 0.56 | 3.02 | 2.47 |
| 9 s | 4.03 | 0.63 | 0.97 | 1.61 | 0.46 | 1.53 | 0.97 | 3.97 | 3.79 |
| $10_{s}$ | 5.18 | 0.95 | 1.86 | 3.08 | 0.84 | 1.49 | 1.42 | 5.41 | 2.10 |
| $11_{s}$ | 6.02 | 1.45 | 3.31 | 5.48 | 1.14 | 1.13 | 1.65 | 6.09 | 1.80 |
| 12 s | 7.32 | 1.89 | 5.27 | 8.72 | 1.72 | 1.07 | 2.04 | 7.21 | 1.51 |
| 13 s | 3.55 | 0.61 | 0.82 | 1.35 | 0.32 | 1.32 | 0.80 | 3.50 | 2.50 |
| 14 s | 4.96 | 1.37 | 2.58 | 4.27 | 1.04 | 1.16 | 1.60 | 5.00 | 2.58 |
| 15 s | 5.91 | 1.78 | 4.03 | 5.66 | 1.47 | 1.20 | 2.16 | 5.73 | 2.44 |
| $16_{s}$ | 7.28 | 2.74 | 7.64 | 1.26 | 1.96 | 0.84 | 2.33 | 7.25 | 1.75 |
| 17 s | 3.06 | 1.00 | 1.17 | 1.93 | 0.39 | 1.11 | 1.12 | 3.07 | 4.55 |
| 18 s | 4.59 | 1.52 | 2.66 | 4.40 | 0.96 | 1.20 | 1.84 | 4.48 | 3.47 |
| 19 s | 5.72 | 2.09 | 4.57 | 7.56 | 1.58 | 1.13 | 2.40 | 5.85 | 2.91 |
| $20_{s}$ | 7.28 | 3.48 | 9.75 | 13.00 | 2.11 | 0.71 | 2.51 | 7.34 | 1.88 |
| 21 s | 2.99 | 1.24 | 1.41 | 2.32 | 0.44 | 1.04 | 1.29 | 2.95 | 6.04 |
| 22 s | 4.41 | 1.73 | 2.91 | 4.81 | 0.98 | 1.10 | 1.93 | 4.28 | 3.94 |
| 23 s | 5.90 | 2.87 | 6.50 | 10.75 | 2.10 | 1.06 | 3.08 | 5.84 | 3.58 |
| 24 s | 7.20 | 4.69 | 12.98 | 21.48 | 3.39 | 0.85 | 4.08 | 7.11 | 3.12 |
| 25 s | 7.18 | 5.25 | 14.73 | 24.39 | 4.21 | 0.91 | 4.93 | 7.27 | 3.78 |

## APPENDIX B --Continued

| $\begin{aligned} & \text { Run } \\ & \text { No. } \end{aligned}$ | $\begin{gathered} Q \\ \text { cfs. } \\ \hline \end{gathered}$ | $\begin{aligned} & \mathrm{C}_{\mathrm{T}} \\ & \% \\ & \hline \end{aligned}$ | $\begin{gathered} \mathrm{Gx}^{10^{2}} \\ \mathrm{cfs} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{W} \\ \mathrm{lbs} / \mathrm{sec} \\ \hline \end{gathered}$ | HP $\times 10^{2}$ | B | $\mathrm{J} \times 10^{2}$ | $\operatorname{Rex} 10^{-5}$ | $\mathrm{f} \times 10^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $26_{s}$ | 5.80 | 3.39 | 7.55 | 12.58 | 2.28 | 0.99 | 3.40 | 5.78 | 4.01 |
| $27{ }_{s}$ | 4.80 | 2.20 | 4.04 | 6.66 | 1.42 | 1.16 | 2.58 | 4.91 | 4.43 |
| 28 S | 3.49 | 1.53 | 1.99 | 3.29 | 0.70 | 1.13 | 1.74 | 3.40 | 5.65 |
| 29 s | 7.15 | 6.35 | 17.88 | 29.55 | 4.22 | 0.76 | 5.00 | 7.40 | 3.89 |
| $30_{s}$ | 5.20 | 3.19 | 6.40 | 10.59 | 1.92 | 0.92 | 3.00 | 5.15 | 4.39 |
| 31 s | 3.60 | 1.73 | 2.38 | 3.94 | 0.75 | 1.04 | 1.82 | 3.58 | 5.55 |
| 32 s | 7.10 | 7.15 | 20.00 | 33.10 | 4.14 | 0.66 | 4.90 | 6.96 | 3.85 |
| $33_{s}$ | 5.66 | 4.51 | 9.90 | 16.39 | 2.44 | 0.80 | 3.70 | 5.62 | 4.58 |
| 34 s | 3.65 | 2.12 | 2.96 | 4.90 | 0.81 | 0.90 | 1.93 | 3.67 | 5.75 |
| 35 s | 7.01 | 7.57 | 21.00 | 34.75 | 4.25 | 0.64 | 5.10 | 6.89 | 4.10 |
| 36 s | 5.54 | 5.60 | 12.12 | 20.06 | 2.54 | 0.67 | 3.90 | 5.45 | 5.05 |
| 37 s | 3.48 | 2.25 | 2.99 | 4.95 | 0.74 | 0.81 | 1.84 | 2.43 | 6.04 |
| 38 s | 6.94 | 8.45 | 23.30 | 38.54 | 4.33 | 0.59 | 5.22 | 6.80 | 4.30 |
| $3^{39}$ | 5.31 | 5.63 | 11.69 | 19.34 | 2.56 | 0.71 | 4.10 | 5.10 | 5.74 |
| 40 s | 3.61 | 2.17 | 2.99 | 4.95 | 0.88 | 0.97 | 2.12 | 3.54 | 6.46 |

III. STANDARD CORRUGATED PIPB

| $\begin{aligned} & \text { Run } \\ & \text { No. } \end{aligned}$ | $\begin{gathered} Q \\ \mathrm{cfs} \end{gathered}$ | $\begin{aligned} & \mathrm{C}_{\mathrm{T}} \\ & \% \\ & \hline \end{aligned}$ | $\begin{gathered} \mathrm{G} \mathrm{\times 10}^{2} \\ \mathrm{cfs} \end{gathered}$ | $\begin{gathered} \mathrm{W} \\ \mathrm{lbs} / \mathrm{sec} \end{gathered}$ | $\mathrm{HP} \times 10^{2}$ | E | Jx10 ${ }^{2}$ | Rex $10^{-5}$ | $\mathrm{f} \mathrm{\times 10}{ }^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{1}{ }_{c}$ | 5.00 | - | - | - | 5.25 | - | 9.24 | 5.90 | 14.60 |
| 2 c | 5.21 | - | - | - | 5.04 | - | 8.50 | 5.83 | 12.40 |
| 3 c | 5.70 | - | - | - | 6.94 | - | 10.70 | 6.55 | 13.10 |
| 4 c | 5.70 | - | - | - | 5.51 | - | 8.50 | 6.60 | 10.40 |
| 5 c | 4.99 | 0.03 | 0.06 | 0.10 | 4.06 | 22.40 | 7.16 | 5.95 | 11.40 |
| $6^{6}$ | 3.20 | 0.09 | 0.11 | 0.18 | 1.19 | 37.00 | 3.33 | 3.78 | 5.85 |
| $7{ }_{c}$ | 4.35 | 0.23 | 0.38 | 0.63 | 2.57 | 21.90 | 5.10 | 5.55 | 10.70 |
| 8 c | 4.66 | 0.26 | 0.46 | 0.75 | 3.55 | 25.80 | 6.70 | 5.92 | 12.20 |
| ${ }^{9} \mathrm{c}$ | 4.99 | 0.90 | 1.70 | 2.80 | 3.99 | 7.77 | 7.00 | 6.66 | 11.20 |
| $10{ }_{c}$ | 3.81 | 0.26 | 0.37 | 0.62 | 1.91 | 17.00 | 4.40 | 4.65 | 12.00 |
| $11^{1}$ | 4.40 | 0.29 | 0.48 | 0.80 | 2.80 | 19.25 | 5.60 | 5.69 | 11.40 |
| 12 c | 3.28 | 0.57 | 0.70 | 1.16 | 1.32 | 6.18 | 3.52 | 4.17 | 13.10 |
| 13 c | 3.95 | 0.63 | 0.95 | 1.57 | 2.00 | 7.01 | 4.46 | 5.19 | 11.40 |
| 14 c | 3.50 | 0.61 | 0.81 | 1.34 | 1.49 | 6.15 | 3.74 | 3.91 | 15.20 |
| 15 c | 2.44 | 0.17 | 0.16 | 0.26 | 0.55 | 11.80 | 2.00 | 2.96 | 13.40 |
| 16 c | 3.02 | 0.28 | 0.32 | 0.53 | 0.97 | 10.10 | 2.83 | 3.82 | 12.80 |
| 17 c | 2.95 | 0.26 | 0.30 | 0.48 | 0.98 | 11.30 | 2.93 | 3.82 | 13.30 |
| 18 c | 4.78 | 1.21 | 2.20 | 3.62 | 3.52 | 5.29 | 6.45 | 5.95 | 11.20 |
| $19{ }^{19}$ | 4.60 | 1.26 | 2.21 | 3.64 | 2.84 | 4.26 | 5.42 | 6.09 | 10.20 |
| $20^{20}$ | 5.20 | 6.31 | 12.50 | 20.60 | 4.94 | 1.31 | 8.32 | 7.03 | 12.20 |

APPENDIX C
SUMMARY OF SIEVE ANALYSIS DATA

| $\begin{aligned} & \text { Run } \\ & \text { No. } \end{aligned}$ | Sample <br> No. | Station of Sampling | Mean Sieve Diameter mm | Geometric Standard Deviation |
| :---: | :---: | :---: | :---: | :---: |
| $11_{H}$ | 49 | 0.55 NS | 0.210 | 1.670 |
| $13^{H}$ | 34 | 0.25 NS | 0.680 | 3.400 |
| ${ }^{16} \mathrm{H}$ | 23 | 0.25 NS | 0.535 | 2.500 |
| ${ }^{19} \mathrm{H}$ | 15 | 0.55 BW | 0.580 | 3.000 |
| $20_{\mathrm{H}}$ | 17 | 0.45 EW | 0.223 | 2.180 |
| $21_{H}$ | 14 | 0.55 EW | 0.600 | 2.730 |
| $21_{H}$ | 19 | 0.55 NS | 0.582 | 2.650 |
| 24 H | 5 | 0.25 BW | 0.326 | 2.700 |
| 25 H | 3 | 0.25 EW | 0.173 | 1.730 |
| $2^{26} \mathrm{H}$ | 10 | 0.40 NS | 0.384 | 2.860 |
| ${ }^{29} \mathrm{H}$ | 32 | 0.55 EW | 0.241 | 2.160 |
| ${ }^{30} \mathrm{H}$ | 24 | 0.55 EW | 0.360 | 2.340 |
| 33 H | 57 | - | 0.300 | 2.210 |
| 42 H | 40 | 0.55 EW | 0.485 | 2.270 |
| $46_{6}$ | A | 0.55 EW | 0.680 | 2.840 |
| $5_{1}{ }_{\text {H }}$ | B | 0.55 EW | 0.705 | 2.650 |
| $52_{\mathrm{H}}$ | C | 0.55 EW | 0.590 | 2.820 |
| $53_{\mathrm{H}}$ | D | 0.55 EW | 0.640 | 3.660 |
| 13 s | 15 | 0.35 NS | 0.145 | 2.140 |
| 14 s | 14 | 0.35 NS | 0.455 | 3.250 |
| 15 s | 34 | 0.35 NS | 0.590 | 4.130 |
| $16_{s}$ | 3 | 0.35 NS | 0.510 | 2.850 |
| $21_{s}$ | 49 | 0.35 NS | 0.265 | 3.230 |
| $22_{s}$ | 17 | 0.35 NS | 0.385 | 2.700 |
| 23 s | 19 | 0.35 NS | 0.692 | 4.150 |
| 24 s | 23 | 0.35 NS | 0.690 | 2.380 |
| 29 s | 11 | 0.35 NS | 0.720 | 4.190 |
| $30_{s}$ | 40 | 0.35 NS | 0.650 | 3.100 |
| $31{ }_{\text {s }}$ | 10 | 0.35 NS | 0.280 | 2.800 |
| 38 s | 45 | 0.35 NS | 0.795 | 1.870 |

## APPENDIX D

Derivation to find the velocity at which $E$ is a minimum
for a given $W$
It can be shown that there exists a velocity at which $E$ is a minimum for a given $W$. Since

$$
\begin{align*}
& W=Q \gamma_{\mathrm{m}} \mathrm{C}_{\mathrm{T}}  \tag{44}\\
& \mathrm{C}_{\mathrm{T}} \propto \frac{1}{\mathrm{Q} /_{\mathrm{m}}} \tag{45}
\end{align*}
$$

for a given $W$.
Since,

$$
J=\frac{f V^{2}}{2 g D}
$$

$$
\begin{equation*}
J \propto f V^{2} \tag{46}
\end{equation*}
$$

for a given $D$.
Equation 36 can be written as

$$
\begin{equation*}
\operatorname{Re} \sqrt{\mathrm{f}}=\mathrm{NC}_{\mathrm{T}}{ }^{1 / 3} \tag{42}
\end{equation*}
$$

Where $N$ depends on the values of $d$ and $D$. For given values of d and $D, N$ will be constant. Therefore

$$
\begin{equation*}
\mathrm{f} \times \frac{\mathrm{C}_{\mathrm{T}}^{2 / 3}}{\mathrm{R}_{\mathrm{e}}^{2}} \tag{47}
\end{equation*}
$$

Since $R_{e}$ is proportional to $V$ for a given temperature and diameter of pipe, Eq 47 can be written as

## APPENDIX D --Continued

$$
\begin{equation*}
\mathrm{f} \propto{\frac{\mathrm{C}_{\mathrm{T}}}{\mathrm{~V}^{2}}}^{2 / 3} \tag{48}
\end{equation*}
$$

Combining Eqs 45, 46 and 48, the relationship

$$
\begin{equation*}
f \propto \frac{1}{v^{8 / 3}} \gamma_{m}^{2 / 3} \tag{49}
\end{equation*}
$$

is obtained.
From Equation 3

$$
\begin{equation*}
\mathrm{B} \propto \frac{\mathrm{~J}}{\mathrm{C}_{\mathrm{T}}} . \tag{50}
\end{equation*}
$$

Substituting the relationships for $J$ and $C_{T}$ in Bq 50 wila give

$$
\begin{equation*}
\mathrm{B} \propto \mathrm{v}^{1 / 3} \gamma_{\mathrm{m}}^{1 / 3} \tag{51}
\end{equation*}
$$

But $\gamma_{m}$ can be expressed as

$$
\begin{equation*}
\gamma_{m}=\left(A_{1}+\frac{A_{3}}{v}\right), \tag{52}
\end{equation*}
$$

where $\quad A_{1}=\gamma_{w}$, and

$$
A_{2}=\frac{4 W}{D^{2}}\left(1-\frac{\gamma_{W}}{\gamma_{s}}\right)
$$

Therefore,

$$
\begin{equation*}
B=X_{1}\left(A_{1} V^{1 / 3}+A_{2} V^{-2 / 3}\right), \tag{53}
\end{equation*}
$$

where $K_{1}$ is a constant of proportionality. It can easily be shown that, if a general case is taken in which $d$ and $D$ both can assume different values, $\mathrm{K}_{1}$ will depend on $\mathrm{N}, \mathrm{W}, \mathrm{D}$, and temperature.

Differentiating Eq 53 with respect to $V$, equation

$$
\begin{equation*}
\frac{\mathrm{dE}}{\mathrm{dV}}=\mathrm{K}_{1} \frac{1}{3} \mathrm{~V}^{-2 / 3}\left(\mathrm{~A}_{1}-2 \mathrm{~A}_{2} \mathrm{~V}^{-1}\right) \tag{54}
\end{equation*}
$$

can be obtained. Therefore, for $B$ to be a minimum the necessary condition is that

$$
\mathrm{v}=\frac{2 \mathrm{~A}_{2}}{\mathrm{~A}_{1}}
$$

that is

$$
\begin{equation*}
v=\frac{8 w}{D^{2}} \frac{1}{\gamma_{w}}\left(1-\frac{\gamma_{w}}{\gamma_{s}}\right) \tag{55}
\end{equation*}
$$

