Some Aspects of the Monthly Atmospheric Circulation Affecting Monthly Precipitation over the Colorado River Basin

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James L. Rasmussen

This research was conducted under contract No. Nonr 1610(06) between the Office of Naval Research and Colorado State University

Technical Paper No. 46
Department of Atmospheric Science
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Fort Collins, Colorado

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ABSTRACT

The fields of temperature, height, geostrophic wind, geostrophic vorticity, and the advection fields of vorticity, and temperature of the 700 to 500 mb atmospheric layer are studied in relationship to the area integrated monthly precipitation over the Colorado River Basin. The horizontal shear of the wind field in the layer and the position of the long wave ridge with respect to the basin are also shown to be related to the monthly precipitation.

A multiple linear regression technique is used to determine those parameters which are most important in describing the monthly basin precipitation.

An analysis of the long waves on monthly charts is shown. The fields of vorticity and temperature advection were used to evaluate the relationship between the fields of temperature and motion. With the restriction that the waves must be stationary the computed vertical motions over the Colorado River Basin agree quite well with those required by the restriction.

CHAPTER I

INTRODUCTION

The major problem facing the growth of the southwestern United States is the shortage of water resources. The situation requires that maximum utilization be made of the water available and that an intensive study be made of the water balance of the major western river basins. The Colorado River Basin, because of its location and the volume of water it yields, is the most important watershed in the southwestern United States.

Object

Large fluctuations occurred during the eleven years 1950-1960 in annual and monthly precipitation over the upper Colorado River Basin. The economic stability of the Southwest is governed to a large degree by these fluctuations in precipitation over the Basin. Since the atmosphere is the transfer medium of water from the source, the oceans, to the sink, the river basins, differences should be evident in the circulation of the atmosphere with respect to wet and dry months in the area of the Basin. This paper covers some aspects of relationship between monthly circulation parameters of the 700 to 500 mb atmospheric layer and the monthly precipitation regime of the Colorado River Basin over the period 1950-1960.

Review of Recent Related Studies

Research pertaining to average monthly circulation and its relationship to monthly precipitation has been quite limited. Jerome Namias, William Klein and Philip Clapp of the Extended Forecast Section, U. S. Weather Bureau, have been the researchers primarily concerned with the problem. Namias (1953) reviews the work done by this section of the Weather Bureau in attempting to forecast the monthly precipitation. Their work has largely been one of synoptically relating patterns of 700 mb flow to precipitation over the whole country with no specific emphasis on the Mountain West. Klein (1948) developed an empirical model of an atmospheric wave at 700 mb and showed the spatial distribution of precipitation classes light, moderate and heavy over this wave. His study was based on data from the Tennessee Valley Authority Stations. As pointed out by Namias (1953) the model is not applicable to the area west of the Great Divide and east of the Coastal Range. This area includes the Colorado River Basin. Namias and Klein have published numerous reports (Namias, 1948, 1956, 1960) (Klein, 1948, 1951) using this model and synoptic experience in relating precipitation to 700 mb monthly circulation patterns. These studies pertain almost entirely to the eastern United States and similar orographic areas.

Klein (1952) demonstrated some characteristics of long waves on monthly mean charts and concluded that the monthly mean map may be considered a dynamic entity in itself to which physical principles can be applied. Waves of the type Rossby (1939, 1942) investigated theoretically are quite similar to those found on monthly maps. His model (Rossby, 1942) might be modified or extended to apply to monthly patterns.

CHAPTER II

PRECIPITATION REGIME OF THE COLORADO RIVER BASIN 1950-1960

Complex Nature of the Distribution

The distribution of precipitation over an area the size of the Colorado River Basin, more than 100,000 square miles, is very complex. Precipitation is discontinuous in both time and space, occurring during short time periods and in varying amounts over the area during these periods. The distribution is further complicated in the Colorado River Basin by the orographic features of the watershed. All of these factors must be considered when evaluating the precipitation falling over the watershed.

Area Weighted Basin Precipitation

A method has been developed (Marlatt and Riehl, 1963) to obtain a measure of the total precipitation over the Colorado River Basin by using an area weighted summation. They computed the daily, monthly and annual basin precipitation for the period 1931-1960 using data from 13 locations with suitable length of record scattered over the basin. Each precipitation gauge was weighted according to the percentage of the total area of the basin it represents. The representative area for each gauge was chosen to follow topographic boundaries, thus helping to minimize the orographic problem.

River Runoff as a Measure of Basin Precipitation

The river discharge at the base of the basin is a measure of a portion of the basin precipitation. However, due to the seasonal fluctuations in runoff caused by water storage in snow pack during winter and subsequent melt in the spring, the runoff would be a representative value only over a time period of a year. Furthermore, the depletions due to domestic, industrial and agricultural uses, the loss due to evapotranspiration and leakage from the basin, and the storage in ground water must be known.

With values for all these factors the basin precipitation may be reconstructed on an annual basis. Marlatt and Riehl (1963) show a good correlation between their annual basin precipitation and the reconstructed annual runoff of the Colorado River Basin as evaluated by Yevdjevich (1961).

Basin Integrated Precipitation 1950-1960

The precipitation data used in this study will be that obtained by the area weighted summation procedure outlined above. Table I shows the values of monthly basin precipitation for each month 1950-1960, and the eleven-year mean monthly basin precipitation for each month. The extreme range of values between months is quite considerable. The range of values between the same months of different years is of more interest, running about 1.20 to 3.92 inches depending on the month. This leads one to ask the question whether or not there are different atmospheric conditions associated with the different precipitation amounts.

Marlatt and Riehl (1963) found that average monthly precipitation in the Colorado River Basin is practically uniform throughout the year with little seasonal variation, and that 50 percent of the annual precipitation occurs in 16 percent of the days with precipitation. This gives approximately 35 days in which 50 percent of the total annual precipitation falls. Furthermore, they have found that, as the total daily precipitation over the basin increases, the area over which the precipitation occurs increases regardless of season. For these reasons it may be expected that the same type of atmospheric disturbance produces precipitation throughout the year in the Colorado River Basin.

Precipitation Noise

Many of the daily basin precipitation amounts are very small with apparently little or no influence on the water balance of the basin. Marlatt and Riehl (1963) investigated this feature of the precipitation regime and concluded that daily basin precipitation amounts of less than .10 inches are insignificant with respect to the total water balance of the Colorado River Basin and can be considered "noise". Figures 1 and 2 show annual precipitation plotted against the number of days during the year having precipitation less than .10 inches and greater than or equal to .10 inches respectively. Figure 1 shows little correlation (correlation

Table I. Monthly Basin Precipitation 1950 through 1960
and 11 Year Mean Monthly Precipitation (Inches)

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1950	2.53	1.19	1.33	1.10	0.82	0.53	1.70	0.49	1.77	0.40	1.44	1.48
1951	1.98	1.15	1.08	1.52	1.24	0.92	1.11	2.26	0.62	2.18	1.49	4.66
1952	2.57	1.11	2.43	1.31	1.16	1.18	1.47	1.74	1.05	0.00	0.90	1.38
1953	1.66	0.74	1.73	1.60	1.57	1.13	1.78	1.70	0.12	1.65	1.12	0.74
1954	1.35	0.55	2.27	0.54	1.16	1.03	1.71	1.05	2.12	1.30	0.83	0.97
1955	1.57	1.73	0.97	0.90	1.36	0.84	1.19	2.60	0.59	0.71	1.79	1.88
1956	2.84	1.26	0.45	1.24	0.92	0.50	1.31	0.68	0.19	1.32	0.80	1.45
1957	3.64	1.16	1.79	2.53	3.46	1.70	1.91	2.54	0.35	2.27	1.93	1.49
1958	0.75	1.66	1.97	1.15	0.57	0.63	0.61	1.13	1.40	0.51	1.34	0.97
1959	0.73	1.90	0.88	1.18	0.99	1.00	0.93	1.81	2.59	1.75	0.43	1.10
1960	1.04	1.87	1.64	1.14	0.79	0.60	0.91	0.92	1.07	1.97	1.32	0.80
Mean	1.88	1.30	1.50	1.29	1.28	0.91	1.33	1.54	1.08	1.28	1.22	1.54

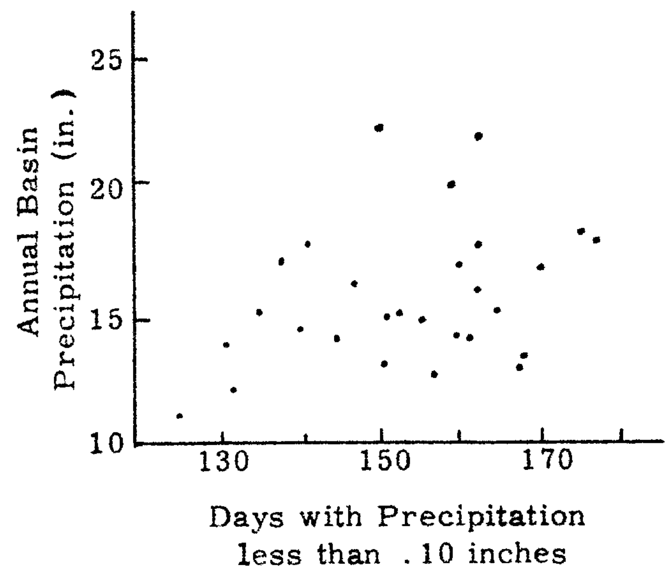

Fig. 1 Days with precipitation less than .10 inches in a year vs. basin precipitation for that year over the period 1930 through 1960. (From Marlatt and Riehl, 1963).

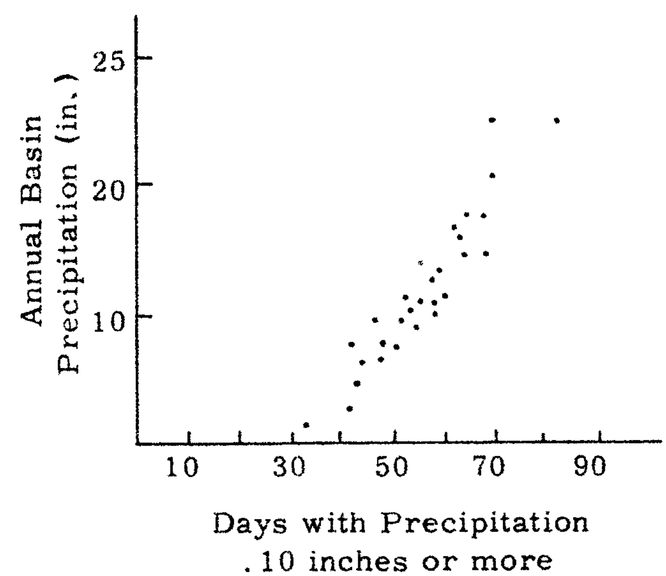

Fig. 2 Days with precipitation .10 inches or more in a year vs. basin precipitation for that year over the period 1930 through 1960. (From Marlatt and Riehl, 1963).

coefficient .42) whereas Figure 2 has good correlation (correlation coefficient .92) indicating that events producing daily basin precipitation of .10 inches or more contribute significantly to the water balance of the basin.

Daily maps of the 500 mb pressure level in the atmosphere were studied to observe what differences, if any, in the atmospheric circulation were evident during days with precipitation less than .10 inches and during days with more than .10 inches. In general it was evident that the "noise precipitation" was caused by very weak disturbances which at times were in fact hard to distinguish on weather charts. On the other hand, major precipitation events in the Colorado River Basin are almost always associated with strong cyclonic disturbances, often having closed cyclonic circulation at 500 mb, traversing the Colorado River Basin from southwest to northeast.

Because the significant precipitation occurs with a definite disturbance in the circulation of the atmosphere, the precipitation data used in this study will be that of the total precipitation minus the "noise" precipitation. Table II shows the monthly values of this parameter for the period 1950 through 1960 and the average monthly values for the eleven years.

Figure 3 shows the frequency distribution of precipitation[1)] for class intervals of .25 inches for all the months.

For the part of the study relating features of the monthly atmospheric circulation of the 700 to 500 mb layer to basin precipitation it was decided to classify the months in three broad classes according to precipitation. The heavy, moderate, and light precipitation classes were arrived at by dividing the accumulative precipitation into three equal groups.

Figure 4 shows the accumulative percentage distribution for class intervals of .25 inches. The light precipitation group consists of all

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[1)] The term "precipitation" refers to the total minus "noise" precipitation. This terminology will be used throughout the rest of the paper unless otherwise stated.

Table II. Monthly and 11 Year Mean Monthly Basin Precipitation.
Days with Less than 0.10 Inch Precipitation Neglected.

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1950	2.14	0.90	0.66	0.62	0.33	0.24	1.13	0.10	1.22	0.15	1.08	1.15
1951	1.20	0.49	0.47	0.94	0.75	0.41	0.59	1.67	0.42	1.71	1.26	4.01
1952	1.96	0.54	1.76	1.01	0.79	0.96	1.01	0.92	0.55	0.00	0.49	0.85
1953	1.30	0.26	1.39	1.27	0.99	0.86	1.30	1.11	0.00	1.38	0.78	0.19
1954	0.86	0.30	1.68	0.13	0.81	0.47	1.02	0.61	1.77	0.98	0.70	0.58
1955	1.09	1.25	0.41	0.31	0.90	0.31	0.79	1.89	0.34	0.28	1.33	1.23
1956	2.26	0.78	0.11	0.57	0.11	0.00	0.86	0.21	0.00	0.36	0.34	1.20
1957	3.13	0.78	1.16	1.83	3.15	1.41	1.27	1.80	0.10	1.72	1.47	1.03
1958	0.24	1.23	1.41	0.82	0.11	0.36	0.17	0.49	0.98	0.20	0.87	0.56
1959	0.10	1.08	0.24	0.96	0.42	0.48	0.25	1.45	2.36	1.31	0.18	0.98
1960	0.55	1.38	0.87	0.76	0.30	0.31	0.28	0.60	0.53	1.61	0.88	0.23
Mean	1.35	0.82	0.92	0.84	0.79	0.53	0.79	0.99	0.75	0.93	0.85	1.09

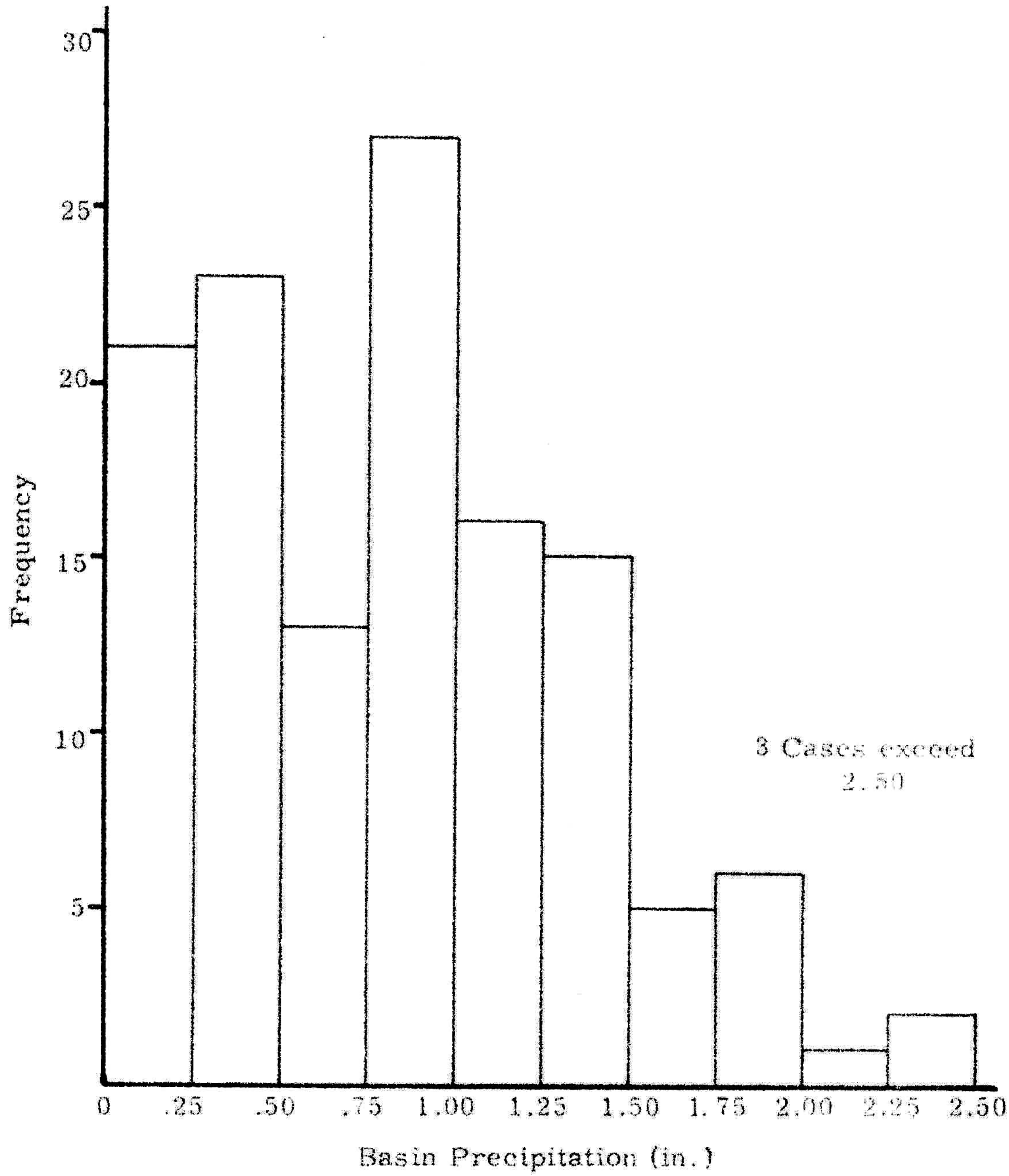

Fig. 3 Frequency distribution of basin precipitation for all 132 months (class intervals of .25 inches).

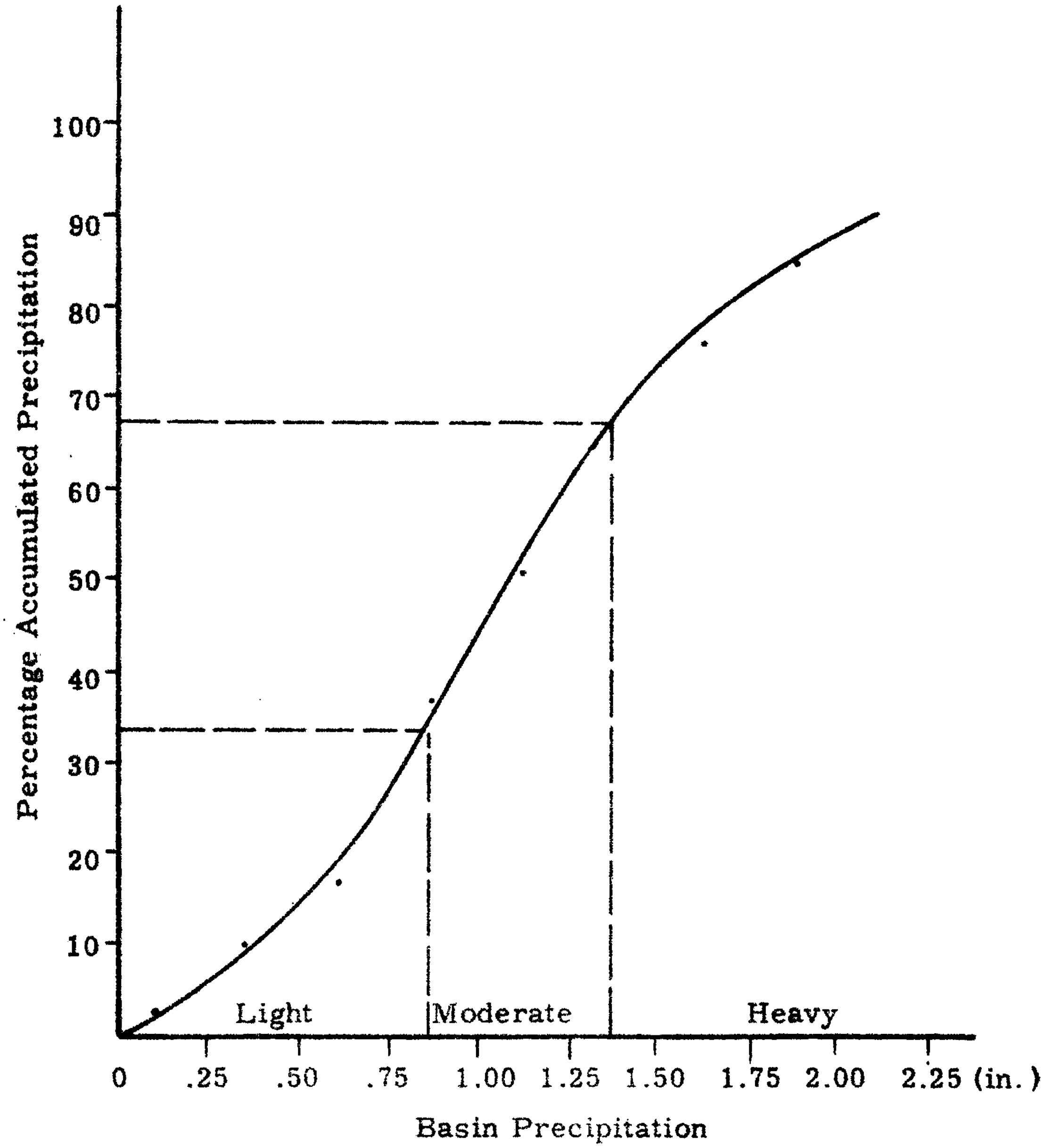

Fig. 4 Basin precipitation (class intervals .25 inches) vs. percent accumulated precipitation. Shown also are the light, moderate, and heavy precipitation classes.

samples less than .85 inches precipitation of which there are 67 cases. The moderate group consists of samples between .85 and 1.37 inches with 41 cases. The high group is made up of the months with precipitation greater than 1.37 inches and consists of 24 cases. The corresponding accumulative percentage distributions and precipitation classes for the winter and summer months were obtained in the same manner. The even distribution in number of months in each class further illustrates the lack of seasonal variation in the precipitation over the basin. Table III contains the precipitation classes, their range and number of cases in each for the total sample and winter and summer seasons respectively.

Table III. Precipitation Classes (Light, Moderate and Heavy), the Class Limits, and Number of Cases in each Class for the Total Sample and for Winter and Summer Seasons.

	PRECIPITATION CLASS					
	Light		Moderate		Heavy	
	Class Limits	Cases	Class Limits	Cases	Class Limits	Cases
Total Sample	0-0.85	67	0.85-1.37	41	1.37	24
Winter	0-0.88	33	0.88-1.35	21	1.35	12
Summer	0-0.76	34	0.76-1.28	18	1.28	14

CHAPTER III

GENERAL CIRCULATION DATA AND DERIVED FIELDS

Data Used in the Experiment

The geopotential height of the 700 mb[2)] and 500 mb[3)] pressure surfaces was obtained at each point of a grid (see Figure 5). The coverage of the 700 mb data was complete for each of the 132 maps, however, the southwest corner of the 500 mb maps was often impossible to obtain because of the lack of data in that area. The area covered by the grid is bounded by 90 degrees west longitude on the east and 140 degrees west longitude on the west and by 70 degrees latitude on the north and 25 degrees north latitude on the south. The grid points are located at even and odd intersections of latitude and longitude lines divisible by five. The resulting grid is of a diamond shape as can be seen from Figure 5.

Average Height of the 500-700 mb Layer

To get a representative measure of the height of the 500-700 mb layer, the heights of the two pressure levels were averaged at each grid point. The resulting height will be termed "layer height".

Temperature Field Computed from Thickness of 700-500 mb Layer

To obtain the thickness of the 700-500 mb layer, the height of the 700 mb surface was subtracted from the height of the 500 mb surface at each grid point for each month. These thickness values are related to

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[2)] The 700 mb height data were obtained from the Long Range Forecasting Section, U.S. Weather Bureau.

[3)] The 500 mb height data were obtained from the monthly charts published in the "Climatological Data, National Summary" issued by the U.S. Weather Bureau.

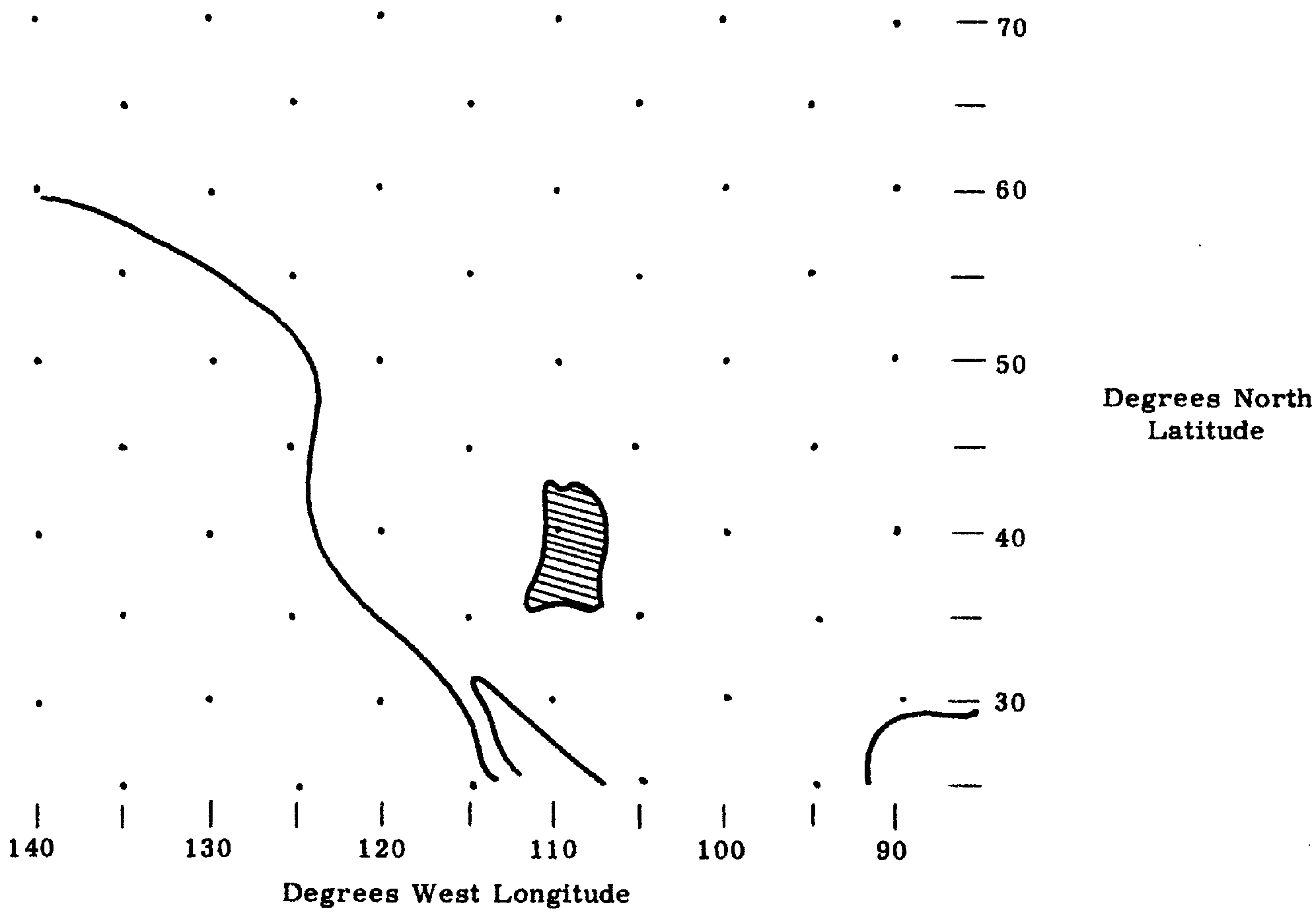

Fig. 5 Grid over which data was tabulated. Hatched area is the Upper Colorado River Basin.

the mean temperature of the layer through the well-known hydrostatic equation:

$$T = \frac{g}{R} \frac{z_1 - z_0}{\ell n\,(p_0/p_1)}$$

where g is acceleration of gravity, R the gas constant for air, z height of the pressure surface, and p pressure. The subscripts 1 and 0 refer to the upper and lower levels respectively.

Geostrophic Wind Field

Assuming the motion to be steady and that the pressure gradient, acting from high to low pressure at right angles to the contours, is balanced by the Coriolis effect, or "deviating force", acting across the direction of motion and towards the right in the northern hemisphere, the resulting motion is termed "geostrophic motion" (Figure 6). This balance may be expressed:

$$\mathbb{V}_g = -\frac{g}{f}\nabla z \times \mathbb{k}$$

in vector notation, where $\mathbb{V}_g$ is the geostrophic wind vector, g the acceleration of gravity, f the Coriolis effect equal to $2\,\Omega \sin \phi$ where Ω is the rotation of the earth and ϕ is latitude, z is the height of the pressure surface, and $\mathbb{k}$ the unit vector normal to the earth's surface. In approximating the actual wind by the geostrophic wind in the monthly circulation we assume that the non-geostrophic components of the instantaneous winds cancel out in the averaging process over a month. The wind was evaluated in component form by approximating the differentials by finite differences:

$$U_g = -\frac{g}{f}\frac{\Delta z}{\Delta y}, \quad V_g = \frac{g}{f}\frac{\Delta z}{\Delta x}$$

where y is distance northward, x is distance eastward, U_g the component of the geostrophic wind positive northward and V_g the component eastward. The wind was evaluated over the grid as shown in Figure 7 where

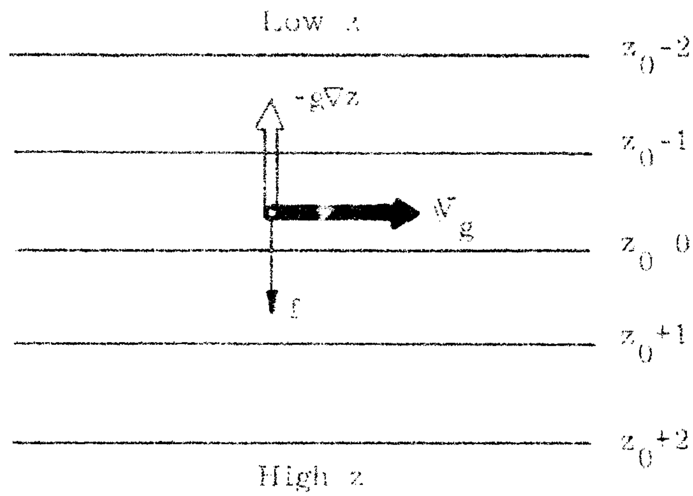

Fig. 6 Illustration of the balance between the pressure force and the Coriolis effect on a moving air parcel and the resulting geostrophic wind.

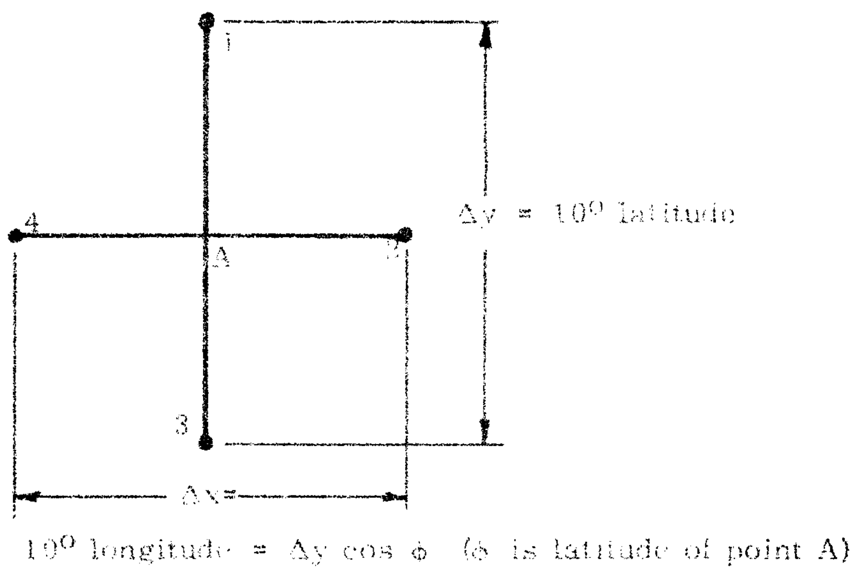

Fig. 7 Grid over which the geostrophic wind at point "A" was computed.

$$U_g = - \frac{g}{2\,\Omega \sin \phi_A} \frac{z_1 - z_3}{\Delta y}, \quad V_g = \frac{g}{2\,\Omega \sin \phi} \frac{z_2 - z_4}{\Delta y \cos \phi_A} .$$

Geostrophic Vorticity

The horizontal curl of the wind field or vertical component of relative vorticity, relative to a point on the earth, was evaluated over the grid. This can be expressed in vector and component form as follows:

$$\zeta_g = \nabla \times \mathbf{V}_g, \quad \zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}$$

where ζ_g is the vertical component of relative vorticity field; like the wind-field, it was derived by estimating the space differentials by finite differences.

The evaluation procedure is as follows (note Figure 7):

$$\zeta_g = \frac{V_{g2} - V_{g4}}{\Delta y \cos \phi_A} - \frac{U_{g1} - U_{g3}}{\Delta y}$$

When measuring the vorticity in absolute frame the local vorticity of the surface of the earth at a point (Coriolis effect) must be added to the relative vorticity, thus the absolute vorticity takes the form

$$\zeta_{ga} = \zeta_g + f$$

where f is the Coriolis parameter.

The concept of vorticity offers an approach to the study of the circulation of the atmosphere. If we assume conservation of absolute vorticity by a system of air particles, the parameter becomes a powerful tool in the study of large-scale flow patterns as demonstrated by Rossby (1939) and others. For our purposes the vorticity of the motion and the advection of vorticity have application in the computation of vertical motion and in the discussion of wave types on the monthly charts. Both of these fields, relative and absolute vorticity, were computed for all months at the 500 and 700 mb levels and for the layer-mean wind.

Advection Fields of Vorticity and Heat

The local variation of the temperature and vorticity at a grid point are expressed by the partial derivatives $\partial T/\partial t$ and $\partial\zeta/\partial t$ where t represents time. The variations which are due to moving particles of air are termed advective and are expressed by $\mathbf{V} \cdot \nabla T$ and $\mathbf{V} \cdot \nabla\zeta$.

The quantities $\mathbf{V} \cdot \nabla T$, $\mathbf{V} \cdot \nabla\zeta_g$ and $\mathbf{V} \cdot \nabla\zeta_{ga}$ were computed for each grid point for the mean level of the 700 to 500 mb layer. The parameters

$$U_g \frac{\partial T}{\partial x} + V_g \frac{\partial T}{\partial y}, \quad U_g \frac{\partial \zeta_g}{\partial x} + V_g \frac{\partial \zeta_g}{\partial y}, \quad U_g \frac{\partial \zeta_{ga}}{\partial x} + V_g \frac{\partial \zeta_{ga}}{\partial y}$$

were evaluated by the same method as for vorticity and wind.

No account was taken in these computations of advection due to vertical motions. The wind fields used are those on the pressure surface, so the resultant advection fields are those of horizontal advection along the pressure surface.

Note should be taken of the fact that because such computations were performed by the finite difference method, the grid decreased in size with each successive computation. The area of computation was chosen quite large to handle this shrinkage. Figure 8 shows the resultant areas covered by the computations for the various fields.

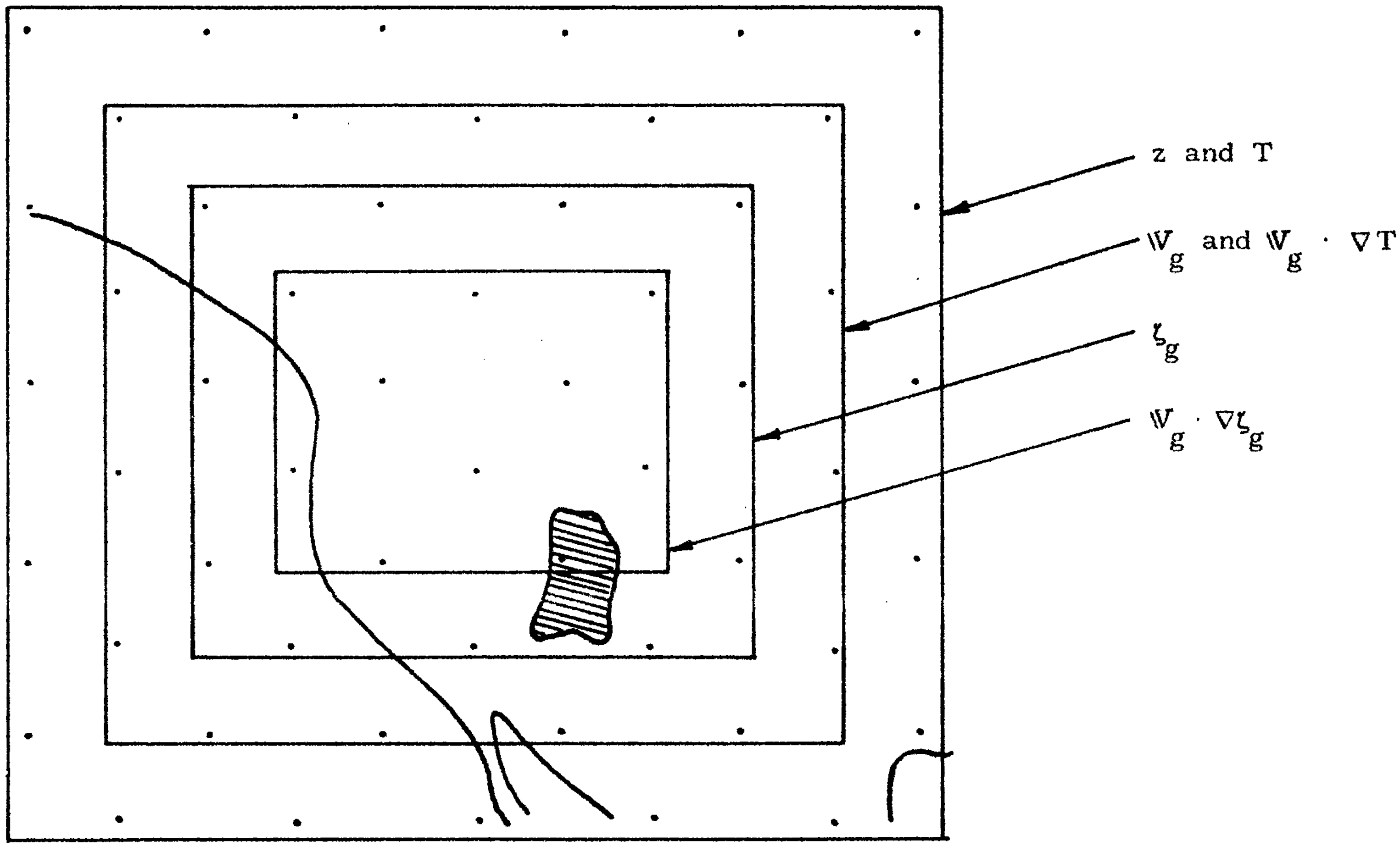

Fig. 8 Grid and location of the Colorado River Basin showing the coverage of each computed field.

CHAPTER IV

ANALYSIS OF GENERAL CIRCULATION PARAMETERS WITH RESPECT TO PRECIPITATION

The Monthly Atmospheric Circulation as Related to the Precipitation Distribution Within a Month

In the discussion in Chapter II of this paper mention was made of the fact that significant precipitation producing storms are reflected in the circulation on daily charts of the 500 mb pressure surface. A number of events of this type during a month should significantly alter the average monthly circulation so that anomalies having "wet" characteristics would be evident in the monthly circulation. This perhaps is an over-simplification because of differences between individual storms and because of the different contributions to the monthly precipitation by different storms.

The track of any storm has an important bearing on the amount of precipitation realized over the area it influences. Riehl and Gray (1962) describe the distribution of precipitation around a storm moving across the Mountain West. They found that the northeast quadrant of the storm had the maximum precipitation, in accord with experience elsewhere. If this quadrant does not track over the basin, the precipitation realized will be less than if it would have passed directly over the area. The effect on the monthly chart at 500 or 700 mb would, for practical reasons, be the same for both tracks because of the smoothing of the features in the averaging process.

Another effect that could influence the monthly circulation is the problem of "natural periods". The time increment of a month is chosen for convenience but it does not necessarily define periods of heavy or light precipitation. The atmospheric disturbances generally follow a regular pattern passing a location at fairly even intervals for a period of time. Then, with a shift in hemispheric circulation, this "natural period" is broken and another weather pattern, perhaps one with no storm passages, evolves. These breaks, of course, do not necessarily occur on or near the end of a month and beginning of another. Situations where this happens could produce a monthly circulation which perhaps is not representative with respect to the basin precipitation that occurred. Examples of this can be seen in Figures 9 and 10. The first (Figure 9) shows two months when the shift in frequency

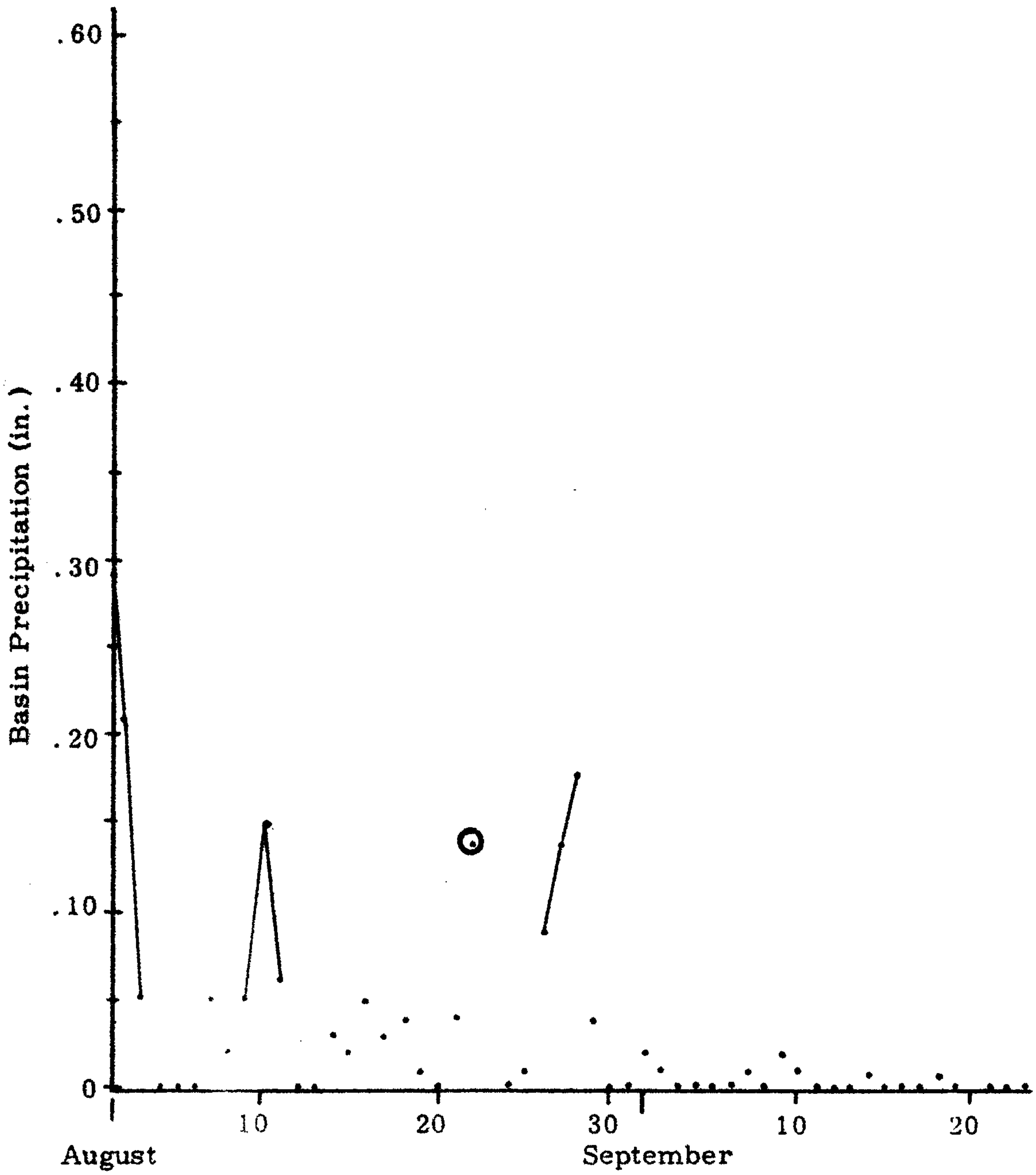

Fig. 9 Plot of daily basin precipitation for the period August-September 1953. The circled points and lines connecting points mark events or storms.

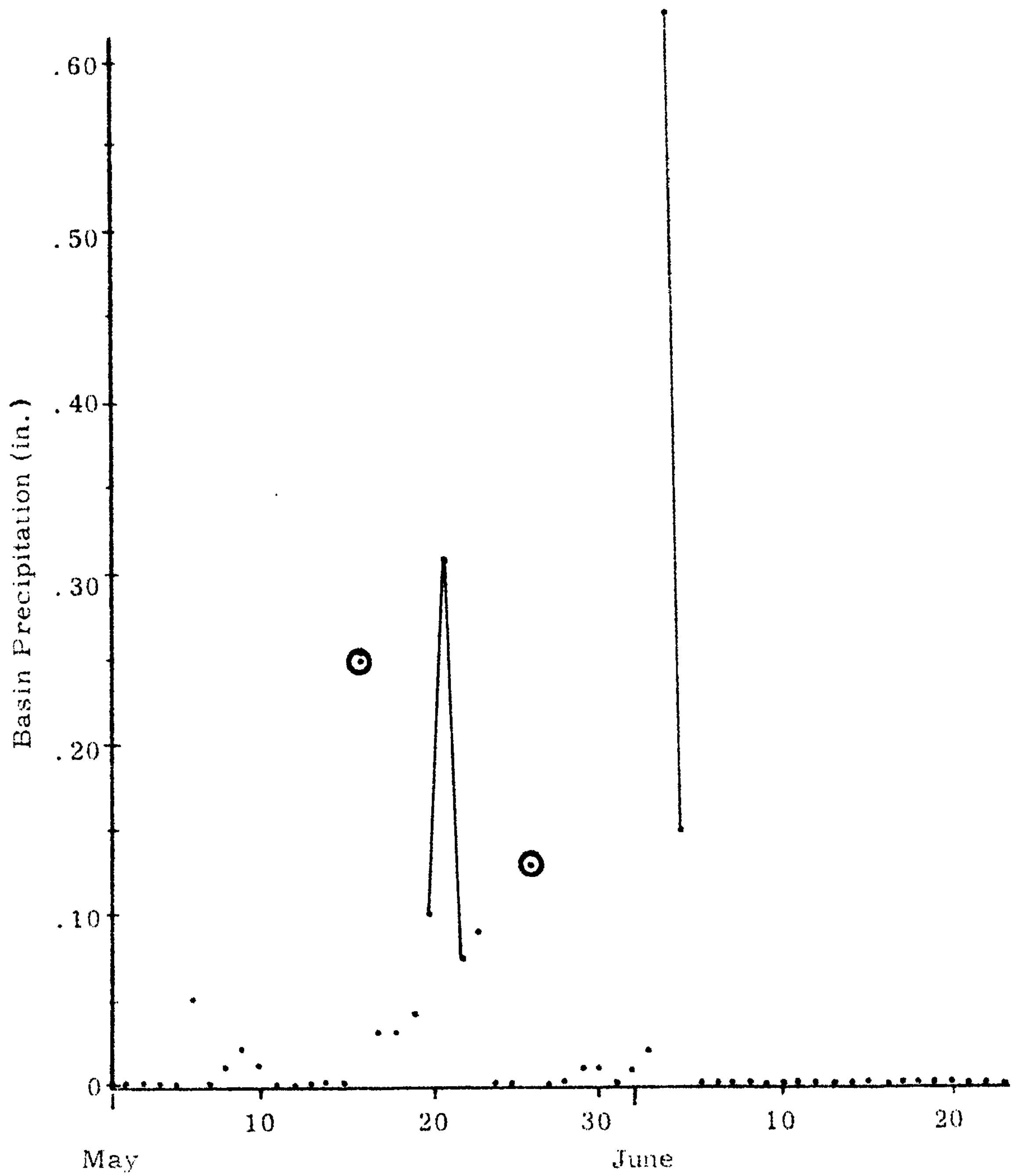

Fig. 10 Plot of daily basin precipitation for the period May-June 1952. The circled points and lines connecting points mark events or storms.

of events occurs at the end of the month. The second (Figure 10) shows a situation that is quite different. The frequency of events or large precipitation occurrences did not change at the end of a month but continued into the following month. The large storm during the first days of June 1952 gave a fairly large monthly precipitation, but the precipitation regime for the month was dry. The daily precipitation data, however, do not exhibit any obvious period that would approximate the natural periods better than do the calendar months.

These drawbacks will have to be kept in mind while analyzing the circulation parameters.

The General Relationship Between the Circulation Parameters and Basin Precipitation

Mean monthly charts. The eleven year mean monthly contour charts of the average 700-500 mb pressure level show a persistent long wave ridge over the western United States. The position of this ridge on the mean monthly charts varies throughout the year; it is centered along the coast in winter (Figure 11) and over the Colorado River Basin in summer (Figure 12). Monthly charts display a variety of positions of this ridge ranging from well off the West Coast to east of the mountains.

Monthly contour patterns. The contour patterns on the monthly maps may be classified into two general types: (1) a wave pattern with large amplitude and (2) a quite flat pattern with little amplitude. The pattern types can best be seen from sample months (Figures 13-16). Figures 13 and 14 show two patterns for wet months and Figures 15 and 16 for dry months. The samples are taken from the winter season in order to obtain a comparable spacing of contours on each map. The summer maps have a similar variety of patterns but the contours are spaced more widely. Note should be taken of the position of the long wave ridge (dash-dot line) with respect to the Colorado River Basin for each case. As can be seen in Figure 13, the location of the ridge is east of the basin giving a southwest to northeast orientation of the contours over the basin. Figure 15 shows the location of the ridge line west to southwest of the basin. The same feature, but much less pronounced, is evident for the other pattern types (Figures 14, 16).

Positions of the ridge with respect to the Colorado River Basin. Table IV shows the percentage of cases of each precipitation class (low, moderate,

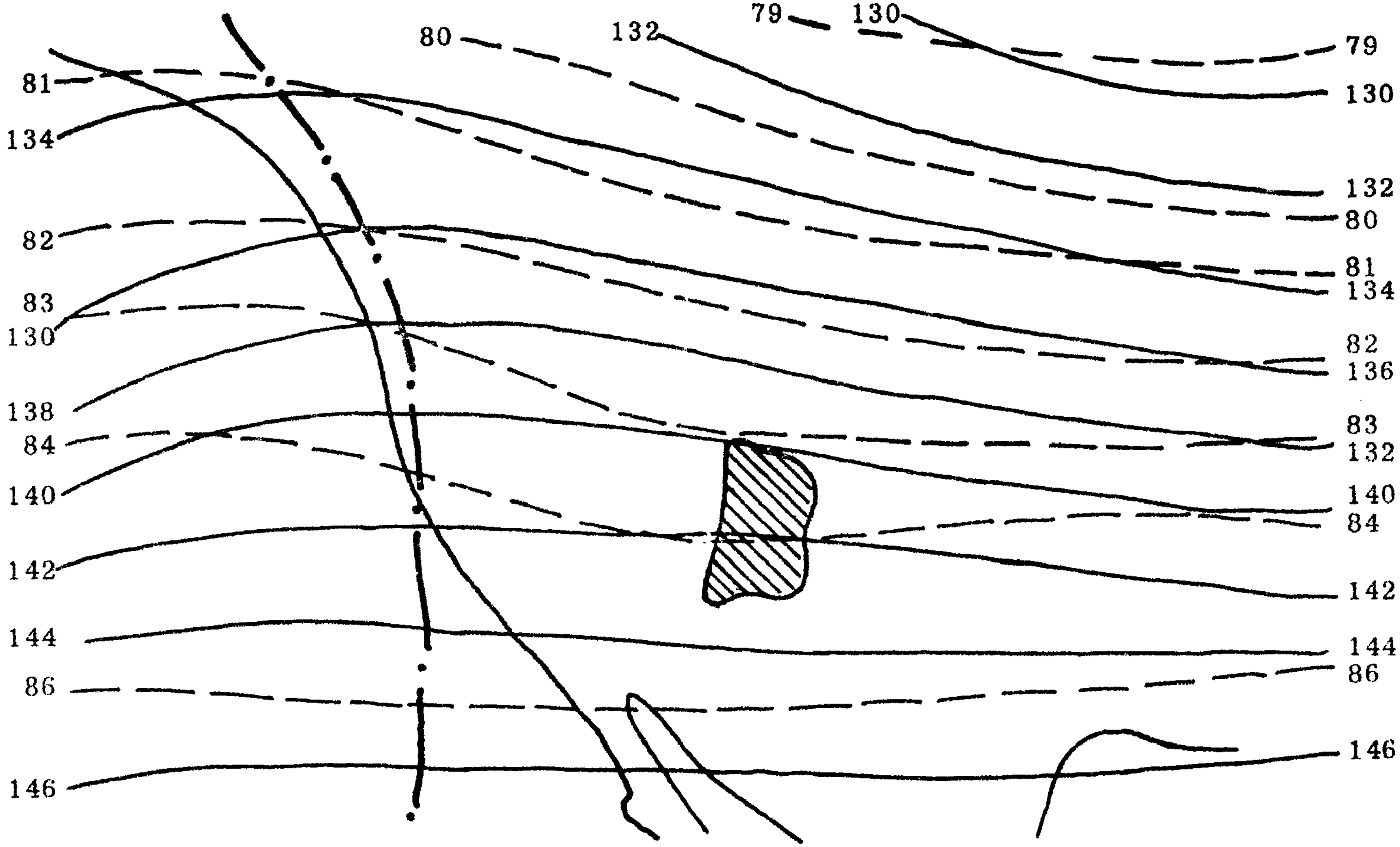

Fig. 11 Mean monthly average 700 to 500 mb map for January. Dash-dot line is the long wave ridge line, solid lines are contours (hundreds of feet), dashed lines are thickness lines (hundreds of feet).

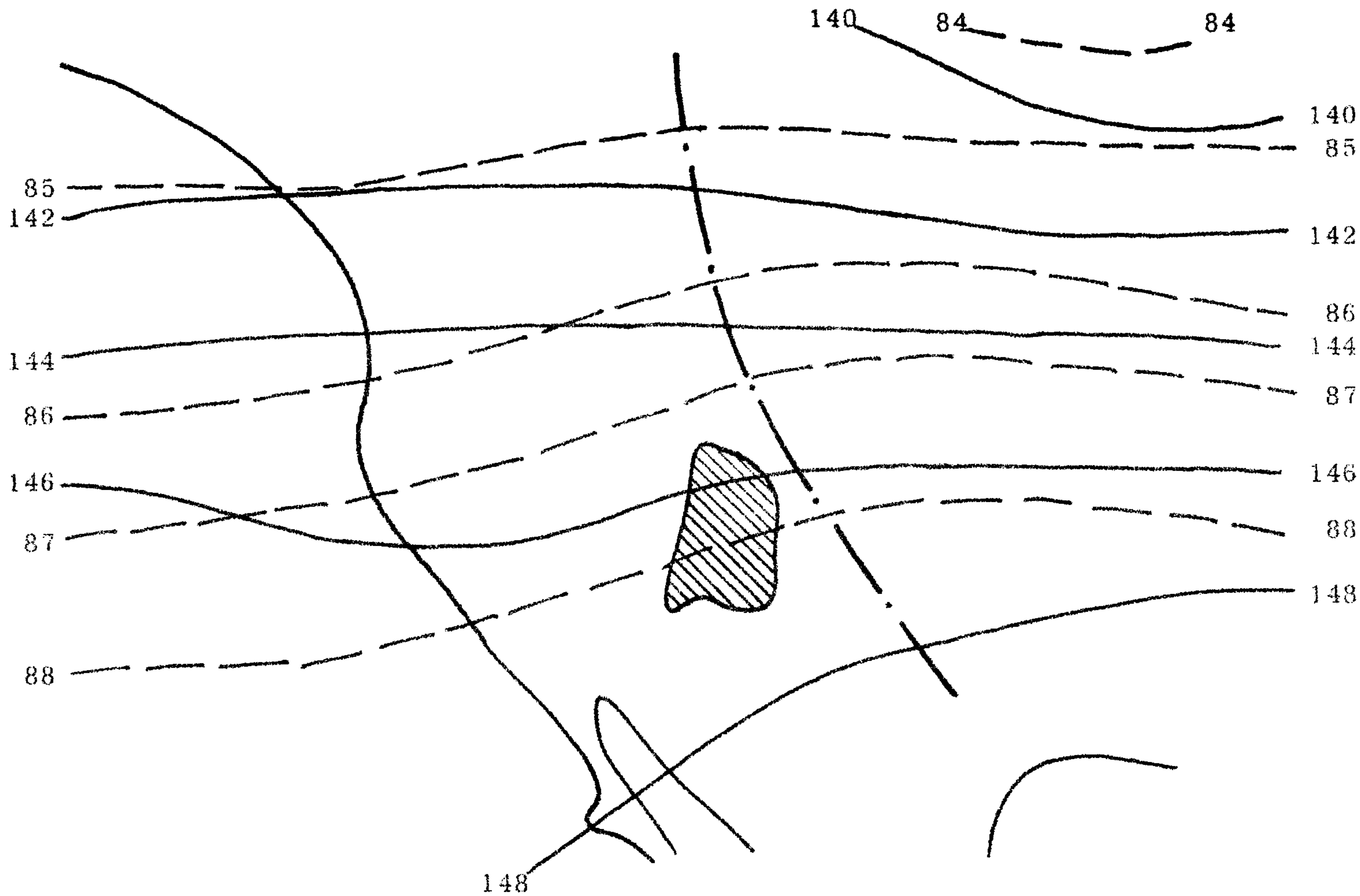

Fig. 12 Mean monthly average 700 to 500 mb map for June. Dash-dot line is the long wave ridge line, solid lines are contours (hundreds of feet), dashed lines are thickness lines (hundreds of feet).

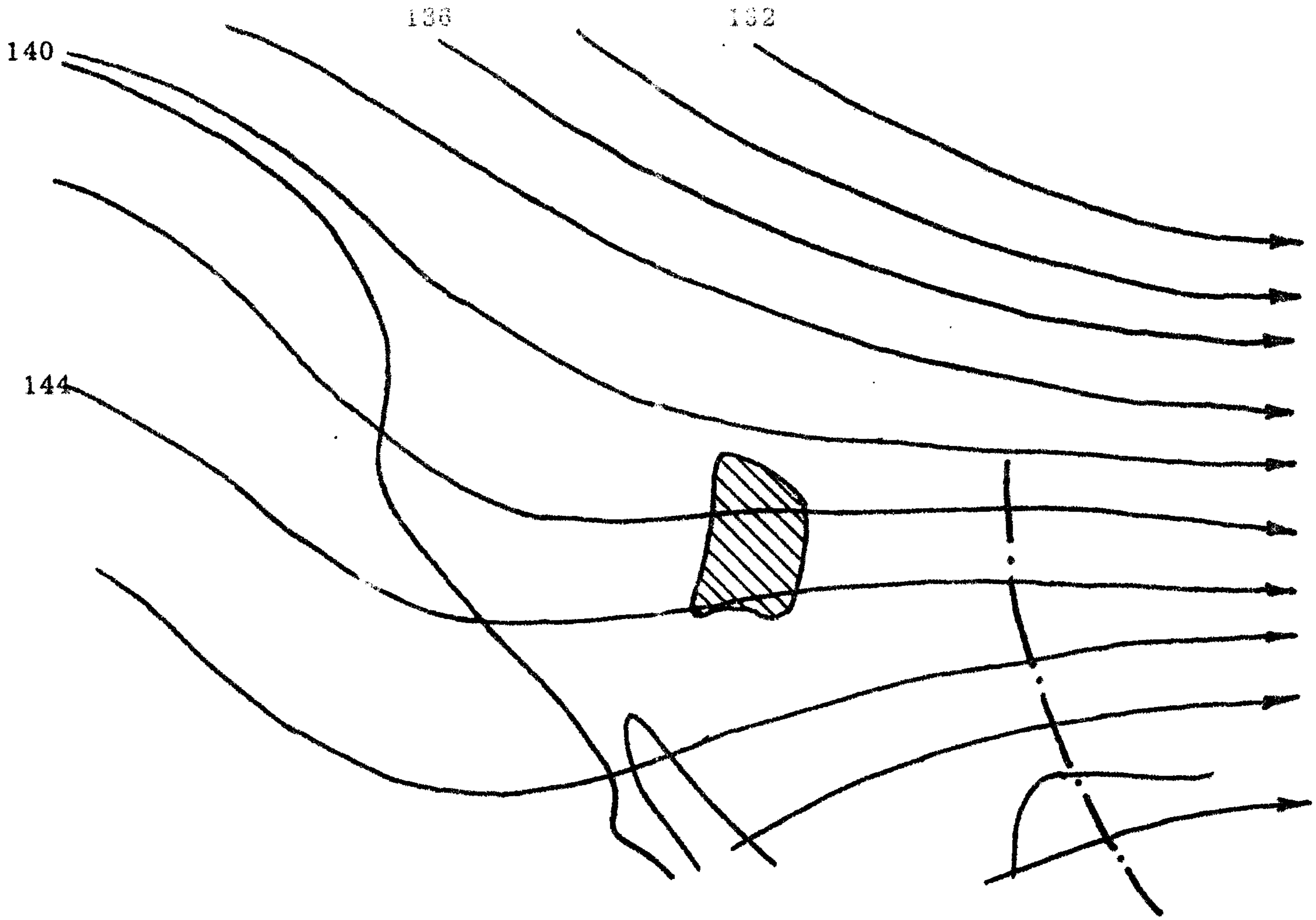

Fig. 13 Flow pattern (contours in hundreds of feet) of the 700-500 mb layer showing an example of the large-amplitude wave pattern for a heavy precipitation month. Map is for January 1957. Dash-dot line is the long wave ridge line.

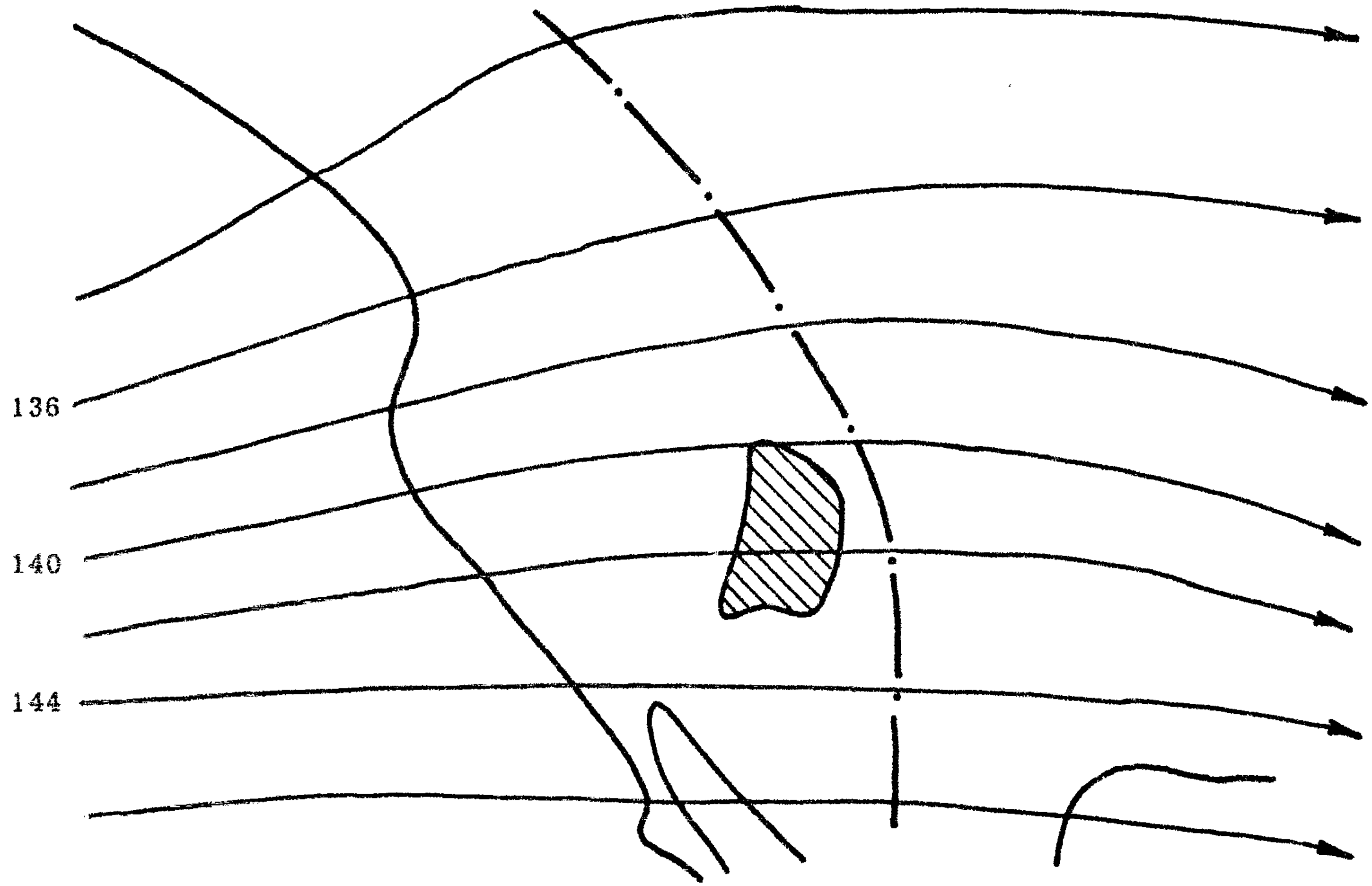

Fig. 14 Flow pattern (contours in hundreds of feet) of the 700 to 500 mb layer showing an example of the small amplitude wave pattern for a heavy precipitation month. Map is for January 1956. Dash-dot line is the long wave ridge line.

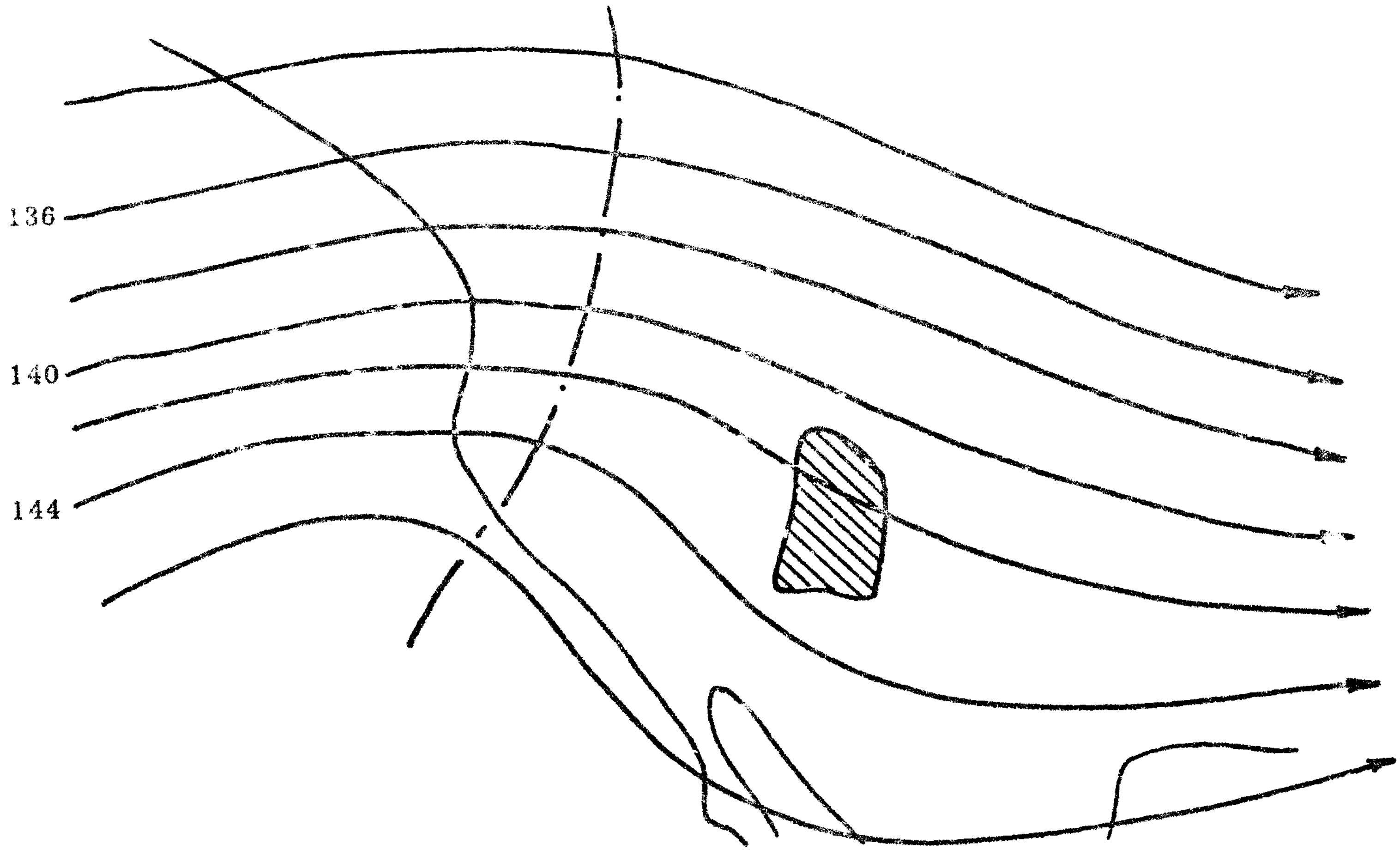

Fig. 15 Flow pattern (contours in hundreds of feet) of the 700 to 500 mb layer showing an example of the high amplitude wave pattern for a light precipitation month. Map is for December 1953. Dash-dot line is the long wave ridge line.

and high) having the ridge at the average 700-500 mb level to the east, over, and to the west of the basin respectively. Of note is the heavy concentration of cases showing the ridge to the west of the basin in dry months and to the east of the basin in wet months. Klein (1948) presents a similar picture in his model 700 mb surface for geographic areas such as the eastern United States.

Table IV. Percentage of the Cases of each Precipitation Class with the Location of the (Long Wave) Ridge East, Over and to the West of the Colorado River Basin.

Precipitation Class	Location		
	West	Over	East
Light	62	19	19
Moderate	66	27	7
Heavy	25	8	67

Height deviation of the layer height. Namias (1953, 1956) observed that positive and negative deviations from mean monthly 700 mb heights were negatively correlated to some extent with monthly precipitation. This feature of the circulation was tested over the Colorado River Basin with results tabulated in Table V. Here the percentage of cases of each precipitation class having positive, zero, and negative deviations are shown. The dry months have predominantly higher heights and the wet months predominantly lower heights than the mean value over the basin.

Horizontal shear normal to the flow. In observing the flow patterns for the various months it was noticed that the horizontal wind shear on the pressure surface increased toward the river basin on wet months and away from the basin on dry months. Shear of the geostrophic wind in the pressure surface and along a line normal to the flow pattern can be approximated by obtaining the height differences at points equally spaced along the line. Because there is a seasonal variation in the contour gradient and because some shear is present in the mean monthly circulation, the parameter was

Table V. Percentage of the Cases of each Precipitation Class having Positive, Zero and Negative Deviations of the Monthly Layer Height from Mean Monthly Layer Height over the Basin.

Precipitation Class	Deviations		
	Positive	Zero	Negative
Light	55	8	37
Moderate	46	10	44
Heavy	29	0	71

normalized by subtracting the mean monthly height values from the height values at each point along the normal line. These deviations then were plotted against distance along the line normal to the flow. Two examples of these plots are shown in Figure 17. The plot on the left in Figure 17 represents that of a typical heavy precipitation month and the plot on the right that of a typical light precipitation month.

The difference between the height deviations measured at the end points of a line segment normal to the flow indicates the deviation from normal of the geostrophic wind over that line segment. The difference between geostrophic wind deviations over adjoining line segments indicate the nature of the deviation from normal of the shear along the line normal to the flow.

The nature of the shear along the line normal to the flow, whether increasing or decreasing toward the basin, may be obtained directly from the plot of the height deviations as shown in Figure 17. The curve on the left (Figure 17) indicates that the shear has greater than normal increase toward the basin. This may be seen by inspecting the differences in the height deviations over the line segments between points 2-3, 3-4, and 4-5 respectively. The differences are positive, indicating less than normal geostrophic wind over the line segment 2-3, near 0 over the line segment 3-4 indicating normal geostrophic wind, and negative over the line segment 4-5 indicating greater than normal geostrophic wind. The shear then is

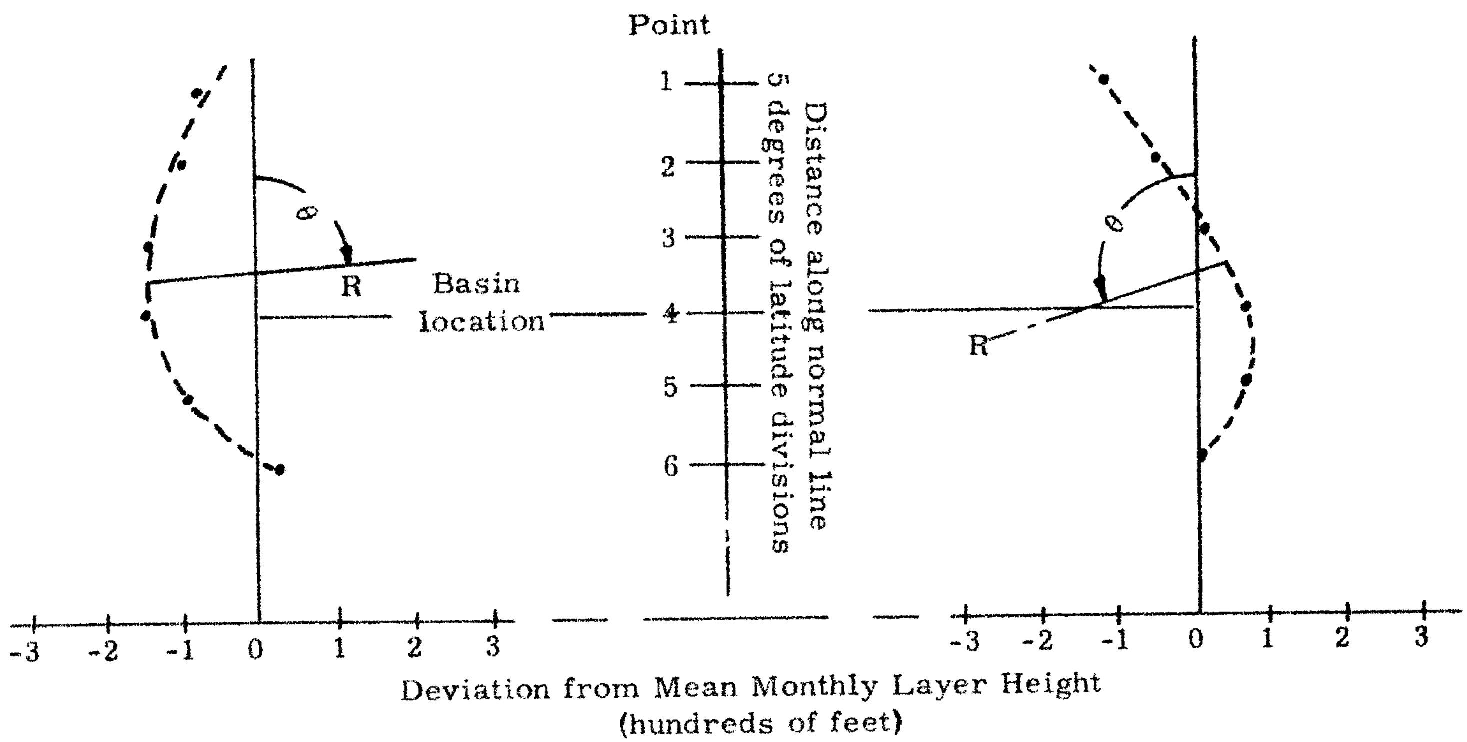

Fig. 17 Diagram of the shear plot. The plot on the left is that of a typical heavy precipitation month and the plot on the right that of a typical light precipitation month. The radius R of the curve and the angle from the normal line to the radius (θ) are also shown.

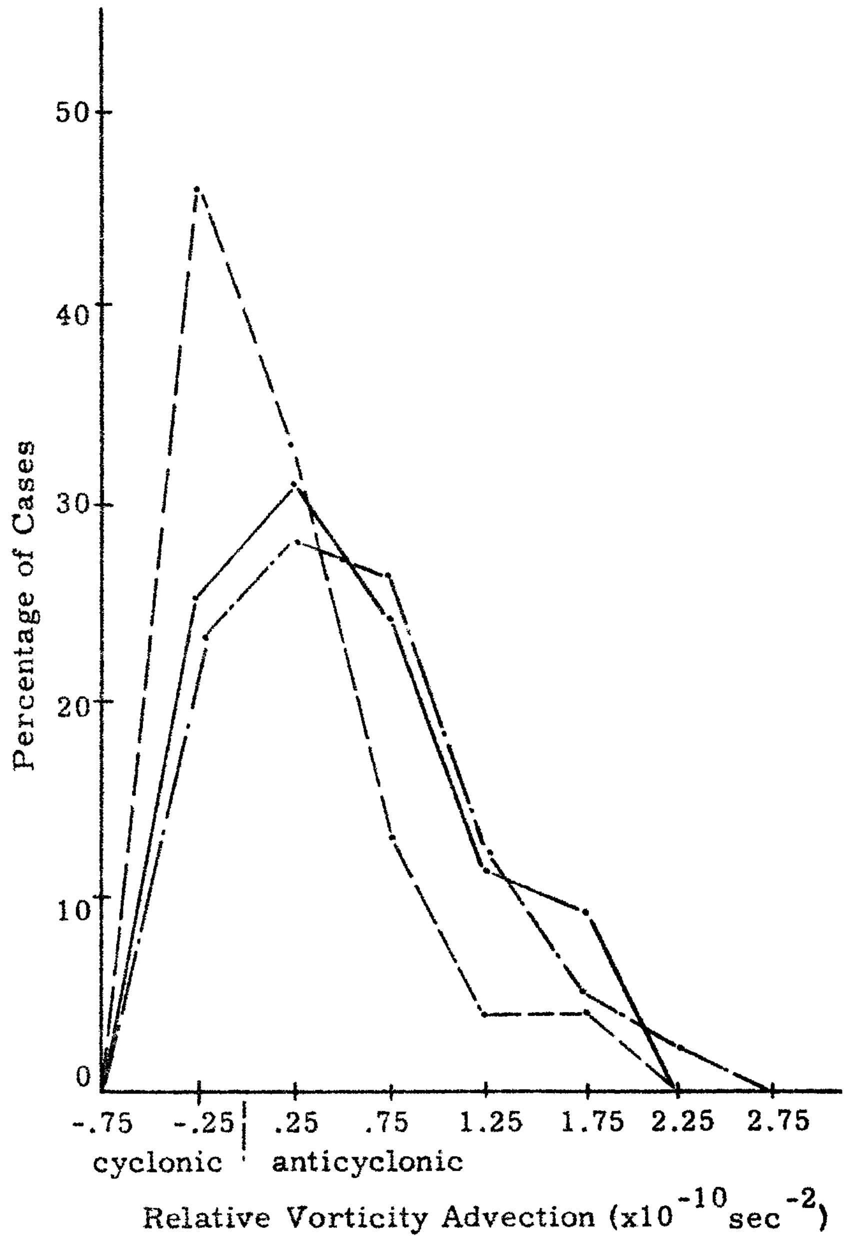

Fig. 18 Percentage distribution of relative vorticity advection for heavy (dashed line), moderate (solid line), and light (dot-dash line) precipitation classes.

in the 700-500 mb layer. Figure 19 demonstrates the opposite seasonal trends of these two parameters.

Temperature of the 700-500 mb layer. The temperature distribution at 500 mb of the individual storms producing precipitation in the Colorado River Basin usually is marked by a cold core centered in a storm at 500 mb. The accumulative effect of several of these storm passages on the monthly map should result in a lower than normal thickness between 500 and 700 mb pressure surfaces in the heavy precipitation class and the opposite for the light precipitation class. The thickness between the 500 and 700 mb levels has a seasonal trend, being large in summer and small in winter, so that to obtain a representative number, the mean monthly values over the basin were subtracted from the monthly values. These thickness or temperature deviations were grouped into 6 classes with class intervals of 50 feet. The relative frequency for each class of the light, moderate and heavy precipitation months is shown in Figure 20. The heavy precipitation months show a heavy concentration of cases with negative deviations or colder than normal for these months. The light precipitation months are quite evenly distributed between the positive and negative deviations with a slight preference for the positive deviations.

Temperature advection. All but 16 percent of the months show cold temperature advection over the basin. The distribution of values has no seasonal characteristics, and no significant relationship with precipitation class is evident.

Vertical velocity. The rate of precipitation is determined to a large degree by the adiabatic cooling resulting from upward motion in the atmosphere. Furthermore, the development of circulatory systems in the atmosphere is a result of divergence and hence vertical motions. Because of these importantconsequences of vertical motion it would be valuable to obtain a measure of this parameter over the river basin for the 132 months.

With the computed fields available to us, two methods of evaluating vertical motion were possible. The methods are generally termed the vorticity method and the adiabatic method. These methods may be expressed:

$$w = \frac{\frac{\partial \zeta}{\partial t} + \nabla \cdot \zeta \, \mathbb{V}_g}{\frac{\partial \zeta}{\partial z}}$$

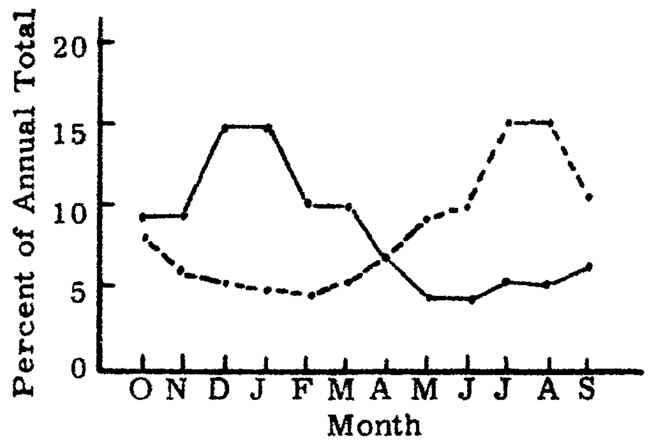

Fig. 19 Annual course of total precipitable water (dashed curve) and of geostrophic absolute vorticity advection (solid curve) of the Colorado River Basin for the layer 700 to 500 mb. For both quantities the ordinate refers to percent of mean annual total contributed by each month. (From Marlatt and Riehl, 1963).

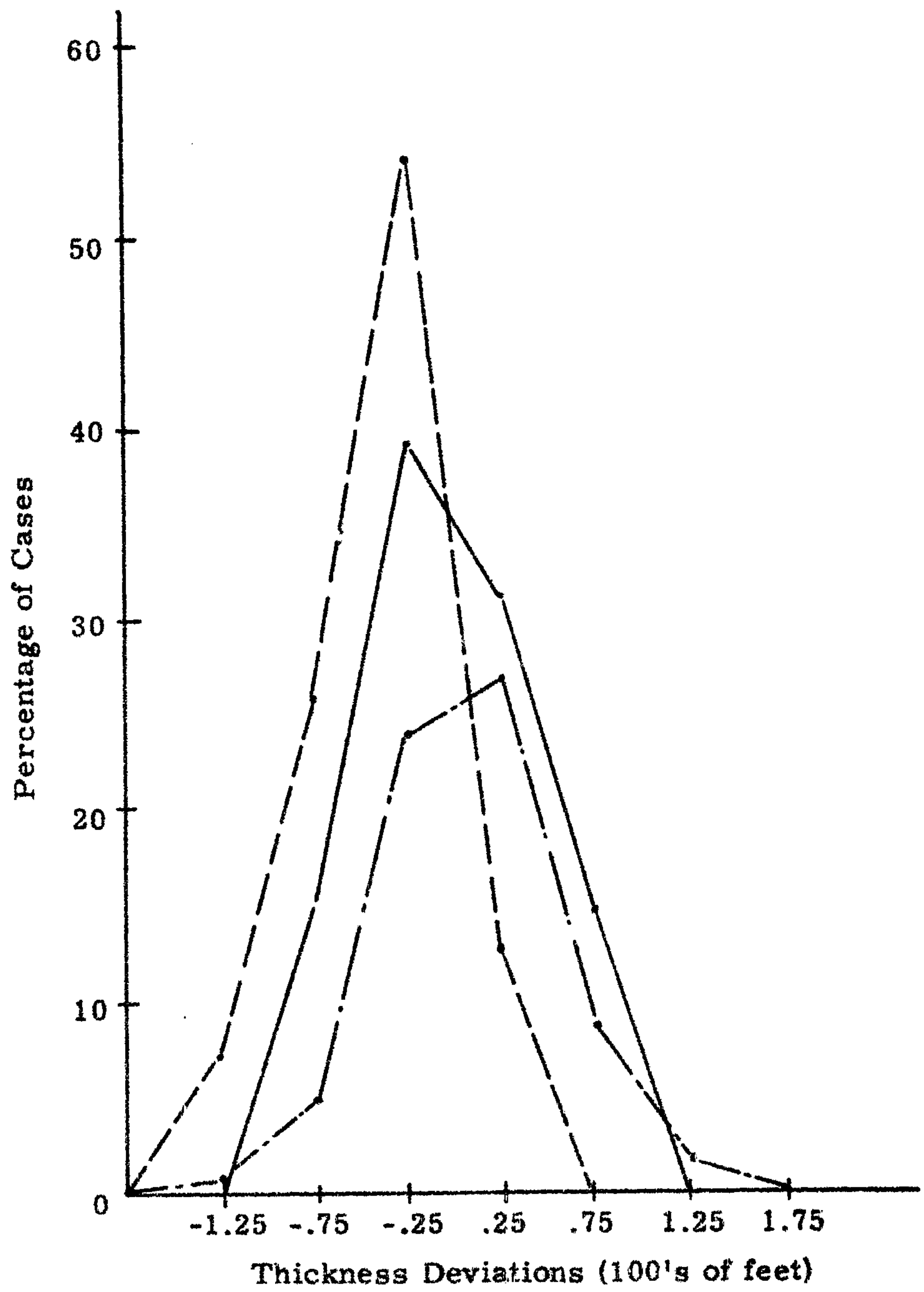

Fig. 20 The percentage distribution of thickness deviations from mean monthly thickness for the light (dot-dash line), moderate (solid line), and heavy (dashed line) precipitation classes.

for the vorticity method and

$$w = \frac{\frac{\partial T}{\partial t} - \mathbf{V}_g \cdot \nabla T}{\gamma_d - \gamma}$$

for the adiabatic method where warm advection is negative. Here w is vertical velocity, positive upward, γ_d the dry adiabatic lapse rate, the rate of adabatic cooling per unit distance of vertical displacement (-9.8°C/km), and γ is the lapse rate of the 700-500 mb layer.

The vorticity method presents some difficult problems. The term $\partial\zeta/\partial z$ is often near zero and the product $\zeta\mathbf{V}$ must be evaluated by interpolation of the velocity field over the basin. These features tend to make the computation unstable.

The adiabatic method, on the other hand, lends itself well for computational purposes. Assuming (1) that the lapse rate γ of the 700-500 mb layer to be that of the U.S. Standard Atmosphere (6.5°C/km), and (2) that the term $\partial T/\partial t$ is small compared with the advective term $\mathbf{V}_g \cdot \nabla T$, we may evaluate the vertical velocity over the basin.

Because the average monthly lapse rate of a layer in the free atmosphere is being considered,the first assumption is a good approximation. The U.S. Standard Atmosphere is based on the average atmospheric conditions at 40° north latitude, approximately the latitude at which the Colorado River Basin is located. The monthly lapse rates for the 700-500 mb layer over Grand Junction, Colorado, were evaluated for six different months distributed throughout the year. The average value for these six months was 7°C/km with a range of 6.1 to 8.3°C/km. The value 6.5°C/km used in this study, therefore, is reasonable. The term $\partial T/\partial t$ was evaluated for the period June-July 1953 by evaluating the temperature change between these two months. The value of the term $\partial T/\partial t$ for this one case was .09°C/day which is an order of magnitude less than the 2°C/day due to advection.

Figure 21 shows the frequency distribution of vertical motion values for the 132 month sample. Based on experience the range of values seems to be reasonable with the greatest concentration of cases between 0 and -.4 km/day.

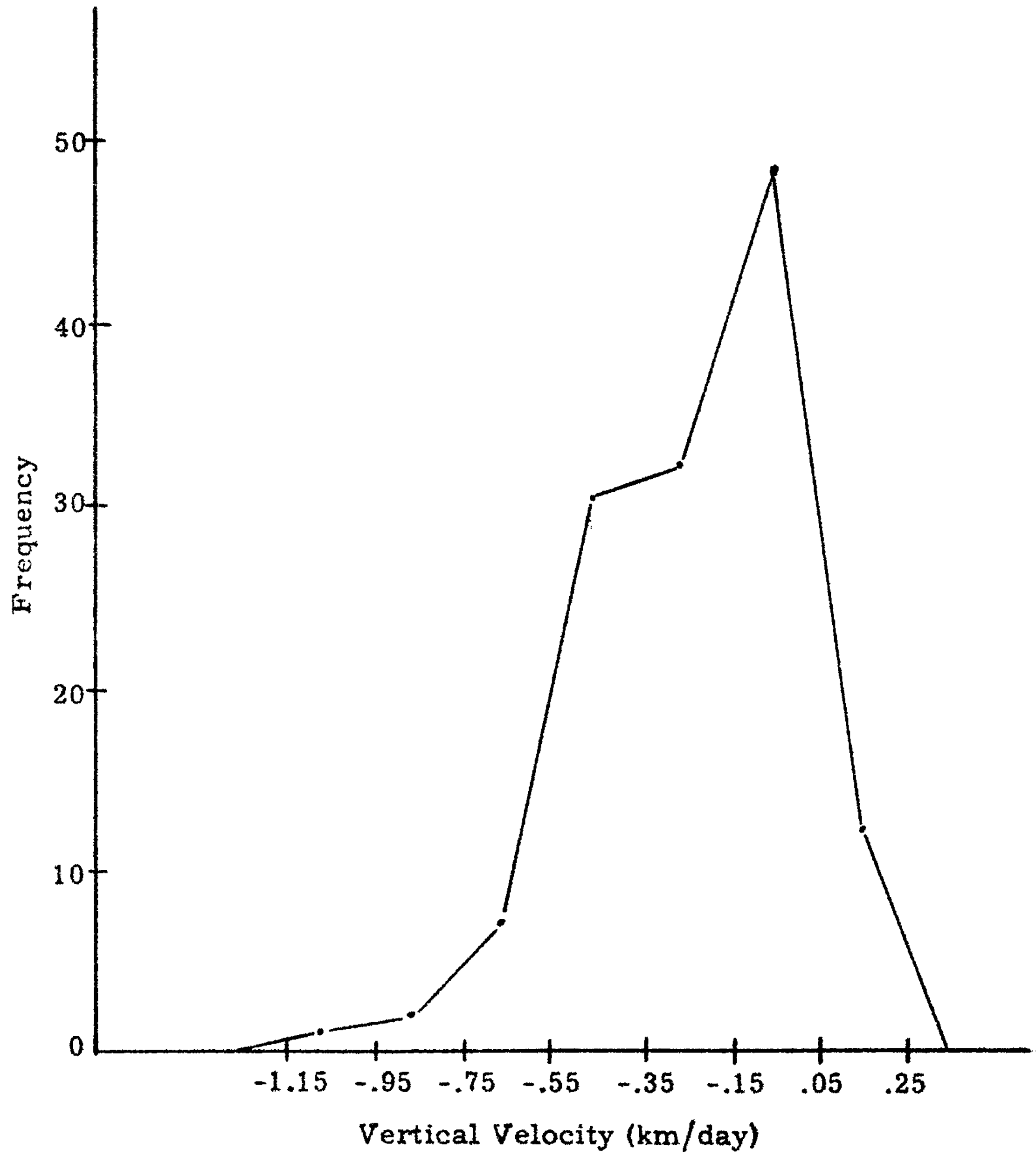

Fig. 21 Frequency distribution of monthly vertical motions over the Colorado River Basin.

As one might expect under the persistent ridge, the motion is downward or near zero in all but 12 cases. The positive values are of such small magnitude, 0 to .18 km/day, that they could easily be of opposite sign if the actual instead of geostrophic wind were used in calculation advection, the actual lapse rate known and the term evaluated. There is no significant change in distribution of vertical motion between precipitation classes.

Statistical Evaluation of the General Circulation Parameters With Respect to Basin Precipitation

Many of the parameters discussed in the last section of this paper have relationships which would indicate partial correlation with the basin precipitation. It is of interest to inspect these relationships further to obtain an idea of the relative importance of each to basin precipitation. Because of the large number of months and the number of parameters for each month, a statistical approach to the problem was chosen.

Multiple linear regression analysis. The simplest way to relate several independent variables to a dependent variable, basin precipitation in this case, would be to obtain a multiple linear regression function of the form:

$$Y = a + b_1 x_1 + b_2 x_2 + b_3 x_3 \ldots b_n x_n ,$$

where a is a constant and $b_1, b_2, \ldots b_n$ are linear regression coefficients. A good treatment of the multiple linear regression analysis can be found in the book "Methods of Correlation and Regression Analysis" by Ezekiel and Fox (1959). A separate treatment has not been included in this paper. A standard computer program was used to evaluate the regression equation. The particular program used in this study is based on stepwise multiple linear regression technique in which the independent variable is selected first. Then the rest of the variables are selected and entered into the regression equation separately. The stepwise procedure selects variables for the equation in an order according to their importance in reducing the variance in a single step. The coefficients and constant are determined by the least squares technique. The regression technique demands that certain restrictions be met to obtain the best possible results. These restrictions are:

1. The regression function must have normally distributed deviations and

2. The independent variables must not be correlated.

Analysis such as the stepwise multiple regression technique often is used to form a prediction equation; however, the purpose here is to determine the importance of each general circulation parameter and exhibit a regression equation explaining a portion of the total variance in the basin precipitation.

The variables, their means, standard deviations, and partial correlation coefficients. The twelve independent variables used in the regression analysis are listed below.

Variable	Symbol	Description
1.	C	Correlation coefficient between the advection fields of relative vorticity and temperature.
2.	$\zeta_{500} - \zeta_{700}$	Vorticity over basin at 500 mb minus that at 700 mb (10^{-5} sec^{-1}).
3.	$1/R$	Radius of curvature of shear plot discussed in earlier part (cm^{-1}).
4.	θ	Angle from the ordinate of the radius of the shear plot (degrees).
5.	$z - \bar{z}$	Deviation from normal of the layer height over the basin (100's of feet).
6.	α	Angle from north of the line normal to the flow over the basin (degrees).
7.	S_r	Distance on map normal to the ridge line to the basin (cm).
8.	$\mathbf{V}_g \cdot \nabla \zeta_g$	Relative vorticity advection over the basin (10^{-10} sec^{-2}).
9.	$\mathbf{V}_g \cdot \nabla \zeta_{ga}$	Absolute vorticity advection over the basin (10^{-10} sec^{-2}).

Table IX. Order in which the Variables were Selected and Standard Error of Estimate after each Selection.

Order in which Variable was Selected	Variable	Standard Error of Estimate
1	5	.5060823
2	2	.5060166
3	7	.5058927
4	3	.5058855
5	12	.5058870
6	6	.5058810
7	9	.5059026
8	11	.5059240
9	8	.5059472
10	1	.5059710
11	10	.5059949
12	4	.5060198

It is interesting that after the sixth step the standard error increases with each step. The coefficient of determination, R^2, has a value of .256 so the portions of total variance explained is 25.6 percent. The regression equation will not be shown here because of its length; however, all the pertinent information for it may be seen in Table VIII. The multiple correlation coefficient has the value $R = .506$.

Because the standard error of estimate reached a minimum after the six variables, $\zeta_{500} - \zeta_{700}$, $1/R$, $z - \bar{z}$, α, S_r, and $T - \bar{T}$ were entered, it was decided to try the same regression analysis, but limiting the independent variables to these six. Table X shows the six independent variables, their coefficients and "Student's" t values.

Table X. Regression Coefficient, Standard Error of Estimate for the six Independent Variables from the 12 Variables originally used and the Regression Constant for the 6-Variable Case.

Variable	Coefficient	Standard Error of Estimate	Student's t Values
2	.39411	.14437	2.72976
3	.74475	.60062	1.23996
5	-.31166	.09217	-3.38111
6	.00510	.00454	1.12213
7	.02774	.02240	1.23827
12	.00217	.00182	1.18971
Constant a = .77645			

The resulting regression equation

$$\text{Basic Precipitation} = .776 + .394\ (\zeta_{500} - \zeta_{700}) + .744\ (1/R) - .312\ (z - \bar{z}) + .005\ (\alpha) + .028\ (S_r) + .002\ (T - \bar{T})$$

with coefficient of determination $R^2 = .254$. The regression explains 25.4 percent of the total variance or only 0.2 percent less than with the 12 independent variables. The order of importance of the variables, of course, has not changed, but it is evident that these six variables are the important ones as far as linear relationships with basin precipitation are concerned. The multiple correlation coefficient is $R = .504$ showing no significant change from the 12-variable case.

One could expand from this point and investigate each of the independent variables to see if non-linear relationships exist and incorporate them by transforming them into linear functions. Perhaps more of the variance could be explained through such a procedure but this is out of the scope of this paper.

CHAPTER V

SOME CHARACTERISTICS OF THE LONG WAVES ON MONTHLY MAPS

The Relationship Between the Temperature and Motion Fields in Simple Rossby Waves

Rossby (1942) investigated the interrelations between the horizontal fields of motion and temperature in simple atmospheric waves. His theoretical waves have the following properties:

1. They travel without changing shape.
2. They occur only in the horizontal plane.
3. The motion is rotational.

In the development of his model, Rossby assumes that the waves are sinusoidal, superimposed on a steady broad west wind. The following restrictions were imposed:

1. The motion is horizontal.
2. The motion of air particles is isothermal.
3. The temperature distribution is steady with respect to the moving wave.
4. The meridional temperature gradient everywhere is the same.

With these assumptions Rossby demonstrates that the relationship between the amplitudes of the isotherms (A_T) and streamlines (A_S), the steady west wind, U, and the speed of propogation of the wave, c, takes the form

$$\frac{A_T}{A_S} = \frac{U}{U - c}$$

which may also be written

$$c = U \left(1 - \frac{A_S}{A_T}\right) .$$

Figures 22 and 23 show two interesting schematic examples of these waves. The first (Figure 22) shows a wave where A_S/A_T is greater than 1, and the resultant wave speed c is negative or toward the west. The second (Figure 23) is the type where A_S/A_T is less than 1, producing a slow eastward moving wave. An attempt will be made at modifying this treatment by Rossby so that we may apply the theory to our monthly charts.

Application of Rossby Waves to Monthly Maps

The assumptions and restrictions imposed by Rossby in developing his wave model are to some extent met by monthly maps. The waves on monthly maps are not necessarily sinusoidal but they are of a much simpler nature than the waves on daily charts. The fields of temperature and motion on monthly maps closely resemble Rossby's wave model.

The Relationships Between the Temperature and Motion Fields of the Monthly Circulation 1950-1960

A stumbling block in the analysis of monthly charts, especially of the mean 700-500 mb layer is determining the amplitude of the waves and isotherms. It would be quite impossible to estimate their amplitudes to any degree of accuracy by visual inspection. This is especially true of the isotherms over a grid the size used in this study.

A measure of the relationship between the two fields may be obtained from the correlation between the vorticity advection and the temperature advection fields. Using the convention that anticyclonic relative vorticity advection and cold thermal advection are positive, then a wave similar to Rossby's type 1 would have a positive correlation between the advection fields (Figure 24), and his type 2 would have a negative correlation coefficient between the advection fields (Figure 25). The simple correlation coefficients between the two fields were obtained from between 8 and 10 data values depending on how many were available for each map. The distribution of coefficients over the total sample is shown in Figure 26. Because the correlation is developed with only 8 to 10 pairs of data for each map, the values of magnitude .45 or larger are the only ones significant to any degree statistically. The distribution is interesting in that 64 cases have positive correlations greater than .45; only 10 samples have negative correlations between -.45 and -1.00. It is evident that the dominant monthly circulation over the Western U.S. has waves of a nature where A_S/A_T is greater than 1.

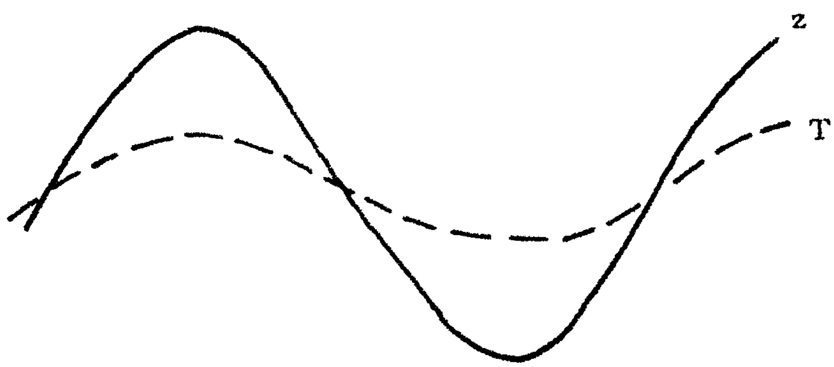

Fig. 22 Rossby wave of type 1 where $A_s/A_t > 1$.

Fig. 23 Rossby wave of type 2 where $A_s/A_t < 1$.

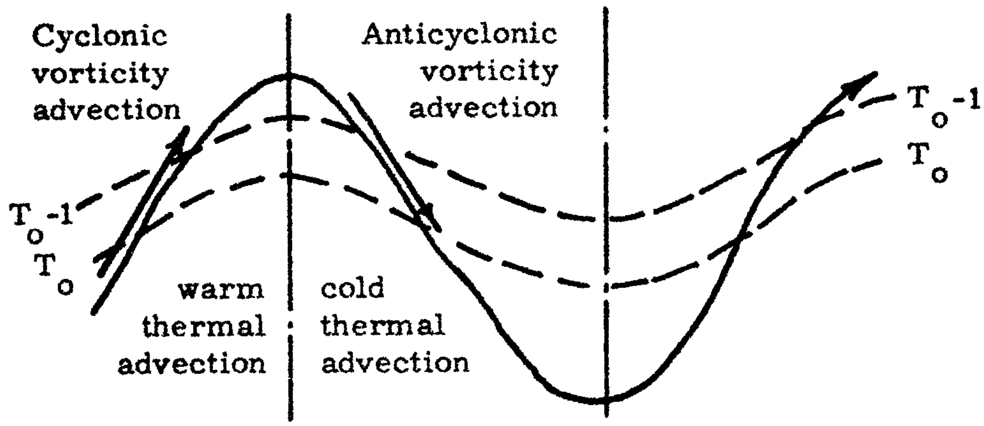

Fig. 24 Fields of vorticity and temperature advection around a wave where $A_s/A_t > 1$ (positive correlation between the advection fields).

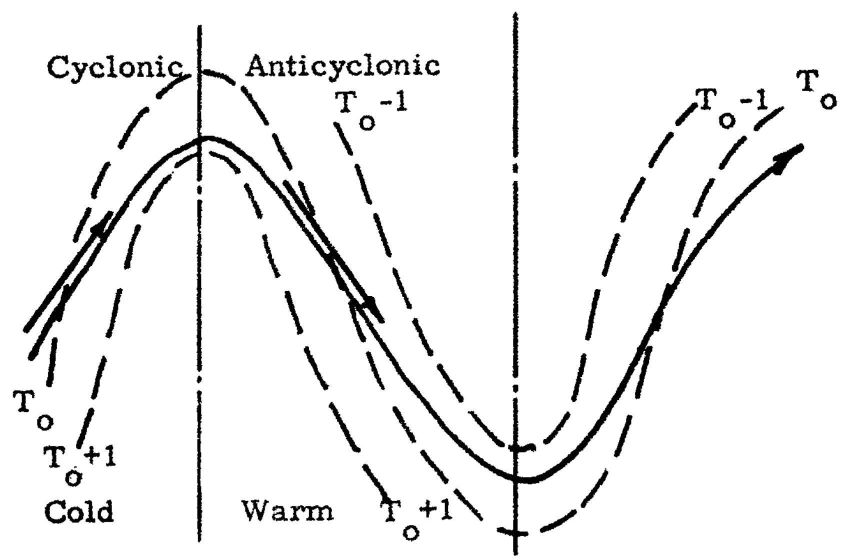

Fig. 25 Fields of relative vorticity advection and temperature advection around a wave where $A_s/A_t < 1$ (negative correlation between the advection fields).

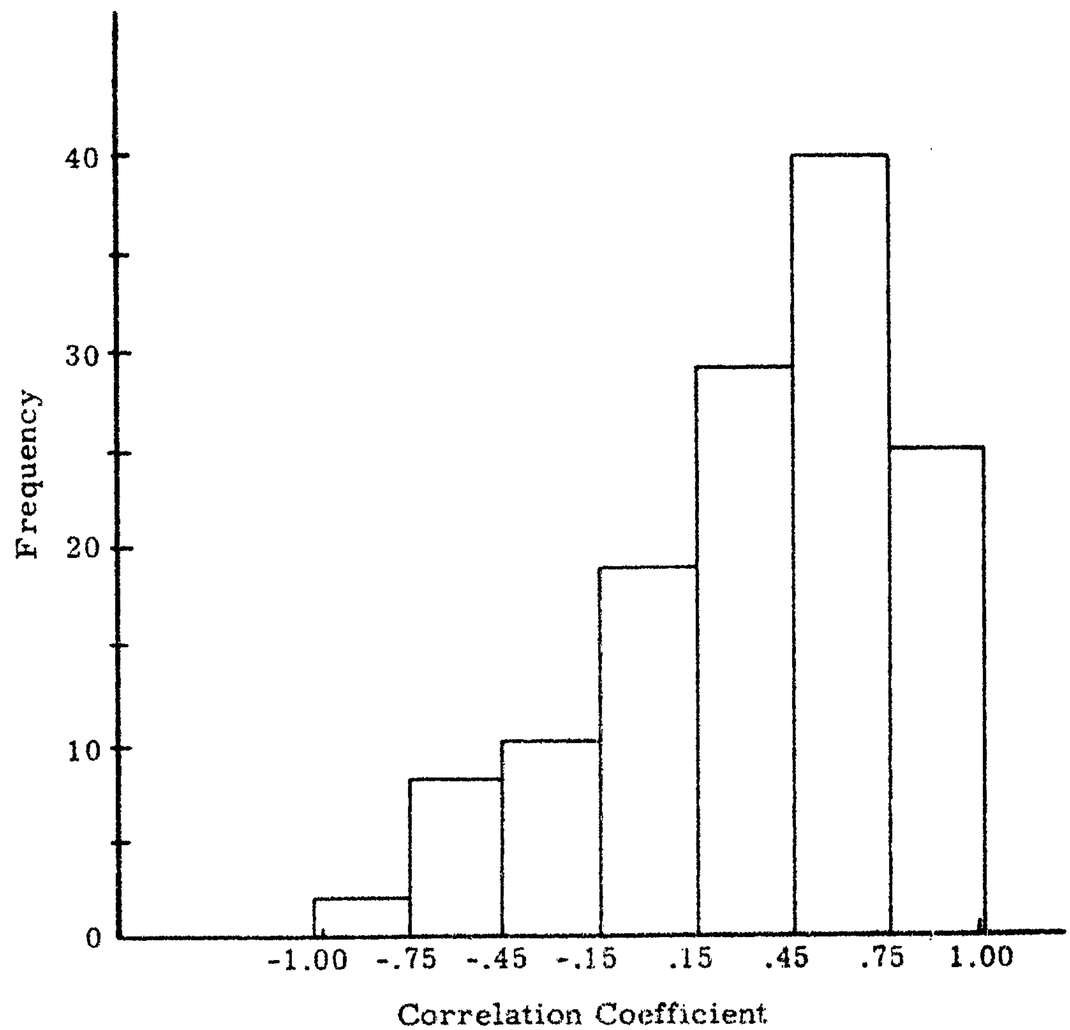

Fig. 26 Frequency diagram showing the distribution of correlation coefficients between the fields of relative vorticity advection and temperature advection.

If we now inspect this type of wave to see what distribution of vertical motion is necessary around a ridge to maintain a stationary pattern, we observe that east of the ridge, where cold temperature advection is taking place, sinking motion with adiabatic warming must compensate for the advective cooling. The opposite effect must occur west of the ridge where warm advection is occurring (Figure 24).

The interesting result is that of the 46 cases where the basin was located over or east of the ridge and where a significant correlation of +.45 or more existed between the advection field, 43 of the cases had cold advection over the basin.

Of the cases where a positive correlation existed, and where the basin was located west of the ridge, only 2 showed warm advection. However, as pointed out by Riehl and Gray (1962), the strong positive vertical motions in a synoptic scale storm crossing the Mountain West occur in about 1/10 of the area of the storm. This idea may be extended to the monthly maps with a result that downward motions are the rule over the whole map.

The opposite distribution of vertical motions should be featured around a stationary wave with negative correlation between the advection fields. In this case (Figure 25) ascent and adiabatic cooling must counter the warm advection east of the ridge; descent with adiabatic warming must counter the cold advection west of the ridge. Because we have a sample of only 10 months with significant correlations, a conclusion is difficult to draw. However, of the six cases where the basin was located east of the ridge, three of the cases had warm and three had neutral advection. Of the cases where the basin was located over or west of the ridge, two had cold and two had neutral advection. Since warm advection indicates ascent and cold advection descent from the foregoing model, these results seem to have merit.

The reasoning that we can approach the study of the monthly atmospheric circulation of a mid-tropospheric layer through the use of the advection fields of relative vorticity and temperature seems to be valid within limits.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Certain parameters of the monthly general circulation seem to have a relationship to Colorado River Basin precipitation. The results of the multiple linear regression analysis indicate that the important parameters are:

1. Deviation from normal of the 700-500 mb average height.
2. Difference in vorticity between the 700 mb and 500 mb levels.
3. Horizontal shear of the wind field.
4. Direction of flow at 700-500 mb layer over the basin.
5. Location of the long wave ridge line with respect to the basin.

These parameters have a multiple correlation coefficient of .50 with respect to monthly basin precipitation. Because the derived regression equation explains only 25 percent of the variance in the monthly precipitation, it is not suitable as a tool for estimating the monthly precipitation. There are a multitude of reasons for this failure in estimating precipitation more closely, some of which are:

1. The month is not a natural period.
2. Inaccuracies in measuring circulation and basin precipitation.
3. Tracks the individual storms take relative to the basin.

The waves on monthly maps resemble simple "Rossby Waves." By imposing the condition that the waves on monthly maps are stationary agreement between the computed vertical motions and those required by the restrictions is generally found.

The monthly flow pattern of the 700-500 mb layer is usually of the type where the amplitude of the stream lines is greater than the amplitude of the isotherms. This is contrary, perhaps, to what one would expect from observing daily charts of the same layer.

Possibilities for Future Study

Two new approaches should be undertaken in the study of the monthly precipitation. The first should concern itself with a more thorough evaluation of daily circulation and its effect in the monthly circulation. The second approach should be to investigate further the applicability of wave analysis to the monthly circulation. Specifically this would involve evaluating the vertical motions over the whole field of motion and correlating this with the flow pattern.

Finally a more accurate measurement of area precipitation is needed especially for mountainuous regions. There is little hope of studying the water balance of river basins such as the Colorado without basin precipitation data that is accurate. The precipitation data used in this study is the best available, but it neglects the contribution from high altitudes.

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APPENDIX

TABLE XI. -- LIST OF VALUES FOR EACH VARIABLE FOR EACH MONTH. MONTHS RANKED ACCORDING TO PRECIPITATION.

BASIN PRECIPITATION (inches)	MONTH - YEAR	C	$\zeta_{500} - \zeta_{700}$ (10^{-5} sec^{-1})	1/R (cm^{-1})	θ (DEGREES)	$z - \bar{z}$ (100s of feet)	α (degrees)	S_r (cm)	$V \cdot \nabla \zeta_r$ (10^{-10} sec^{-2})	$V \cdot \nabla \zeta_a$ (10^{-10} sec^{-2})	$V \cdot \nabla T$ (.9 °C/day)	ζ_r (10^{-5} sec^{-1})	$T - \bar{T}$ (feet)	ω (vertical velocity; km/day)
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0.00	10-52	-.55	-0.562	.20	0085	01.9	-20	-2.5	00.00	-.50	-3	-1.323	0086	0.03
0.00	9-53	0.16	-0.782	.14	0102	00.5	010	-2.5	00.50	0.00	02	-1.161	0044	-.06
0.00	9-56	0.60	-0.808	.17	0095	00.7	010	-1.9	00.30	0.12	-6	-1.283	0044	0.18
0.00	6-56	0.42	-0.688	.20	0101	00.8	023	03.3	-0.10	0.50	02	-0.905	0050	-.06
0.10	9-57	0.05	-0.396	.00	0080	00.0	-03	-1.4	00.50	0.10	-2	-0.686	-018	0.05
0.10	8-50	-.57	-0.445	.00	0090	00.2	022	-0.2	00.01	0.25	-1	-0.792	-039	0.03
0.10	1-59	0.73	00.589	.25	0086	00.8	-17	-3.5	01.55	0.62	-3	-0.551	0030	0.09
0.11	3-56	0.75	00.573	.25	0090	01.3	-20	-2.3	01.30	0.50	24	-0.510	0027	-.72
0.11	5-55	-.40	-0.265	.25	0085	01.5	019	-0.5	-0.10	0.10	01	-0.430	0038	-.03
0.11	5-56	0.06	-0.169	.10	-089	00.6	020	02.1	-0.10	-.27	02	-0.251	0055	-.06
0.13	4-54	0.51	-0.479	.10	0128	01.0	007	00.0	00.00	0.05	12	-0.888	0072	-.36
0.15	10-50	0.72	-0.461	.05	0125	00.0	015	01.5	00.50	0.90	07	-1.060	0078	-.21
0.17	7-58	0.24	-0.055	.20	-091	-0.7	009	-2.0	00.00	-.13	05	-0.605	-021	-.15
0.18	11-59	0.42	-0.204	.20	0103	00.5	-33	-4.9	01.50	0.00	16	-1.072	0047	-.48
[illegible]	12-53	0.71	00.929	.17	0091	-0.5	-27	-4.3	01.80	0.50	20	-0.614	-083	-.60
0.20	10-58	0.22	00.501	.17	0075	00.7	-21	-2.1	00.60	-.10	05	-0.562	0005	-.15
0.21	6-56	0.22	-0.326	.04	-105	00.0	021	00.0	-0.30	0.03	00	-0.677	-025	0.00
0.23	12-60	0.07	00.195	.25	0075	01.2	-33	-3.8	01.50	-.05	08	-0.612	0031	-.24
0.24	6-50	0.77	00.121	.17	-093	-0.3	025	02.9	-0.25	-.24	01	-0.166	-034	-.03
0.24	1-58	0.28	00.538	.05	0060	01.2	-38	-2.9	01.20	0.25	09	-0.698	0045	-.27
0.24	3-59	0.25	00.882	.04	0074	01.0	-30	-3.1	01.40	0.25	15	-0.264	-022	-.45
0.25	7-59	-.51	-1.026	.17	0081	00.5	003	-1.9	00.50	0.15	00	-1.279	0016	0.00
0.26	2-55	0.46	00.921	.12	0087	00.3	-32	-4.0	01.50	0.20	14	00.223	-013	-.42
0.28	10-55	0.55	-0.512	.17	0097	00.5	-09	00.0	00.00	-.20	10	-1.375	0028	-.30
0.28	7-60	0.00	-0.970	.04	0081	00.9	008	-0.7	-0.30	-.15	01	-1.116	0006	-.03
0.30	2-54	0.14	[illegible]	.25	0068	02.4	-28	-2.8	-1.30	0.30	[illegible]	-1.375	0115	-.39

TABLE XI. — CONTINUED.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0.30	5-60	0.49	-0.274	.12	0097	-0.1	005	00.0	00.25	0.20	08	-0.430	0014	-.24
0.31	6-60	-.33	-0.660	.20	0098	00.6	002	00.0	00.00	-.20	12	-0.651	0015	-.36
0.31	6-55	-.07	-0.255	.14	-097	00.0	007	00.0	00.20	0.20	-2	-0.286	-017	0.06
0.31	4-55	0.55	00.724	.33	-083	-1.6	020	03.4	-0.40	-.05	25	00.178	-066	-.75
0.33	5-50	0.51	00.664	.08	-081	-0.7	-16	-3.7	00.76	0.35	12	00.549	-075	-.36
0.34	9-55	0.18	-0.223	.04	-089	-0.3	030	01.5	-0.50	-.45	03	-0.537	0015	-.09
0.34	11-56	-.07	01.126	.14	-098	00.5	-37	-4.3	01.55	0.25	12	-0.391	-036	-.36
0.36	6-58	0.19	-0.568	.00	0090	00.0	019	00.0	-0.50	-.20	03	-0.500	-001	-.09
0.41	6-51	0.28	-0.249	.04	-090	-0.9	-11	-3.9	-0.50	-.60	02	00.050	-033	-.06
0.41	3-55	0.33	00.889	.20	0113	-0.4	-10	-8.8	01.00	0.20	35	-0.293	-041	-1.05
0.42	9-51	0.08	-0.481	.07	-081	-0.3	000	-4.3	00.25	0.10	-1	-0.630	0035	0.03
0.42	5-59	0.59	00.029	.17	-076	-0.9	028	01.7	-0.30	-.10	07	00.252	-028	-.21
0.47	6-54	0.45	-0.274	.05	-081	-0.9	023	03.0	-0.10	0.50	11	-0.454	0038	-.33
0.47	3-51	0.85	00.603	.05	-085	-0.1	-10	00.0	00.90	0.10	12	00.130	-034	-.36
0.48	6-59	0.50	-0.575	.12	0092	00.2	008	00.0	00.10	0.40	02	-0.677	0025	-.06
0.49	11-52	0.37	00.474	.20	-111	-1.1	-05	-4.7	00.50	0.20	04	00.036	-043	-.12
0.49	8-56	0.12	-0.844	.04	0091	00.8	009	00.0	00.00	0.27	02	-1.249	0046	-.06
0.49	2-51	0.95	00.534	.14	-100	-0.1	-03	06.0	00.50	0.10	14	-0.279	0046	-.42
0.53	9-60	-.68	-0.046	.17	0095	00.9	010	-0.3	-0.20	-.22	00	-0.687	-028	0.00
0.54	2-52	0.50	00.797	.25	-100	-0.4	-10	-1.7	00.80	0.24	16	-0.226	-056	-.48
0.55	1-60	0.57	00.441	.05	-111	-0.2	-02	-3.4	00.35	0.10	10	-0.208	-009	-.30
0.55	9-52	-.57	-0.203	.09	-111	00.4	035	00.0	-0.10	-.10	02	-0.954	0017	-.06
0.55	12-58	-.36	00.184	.33	0095	01.8	-27	-3.6	00.70	-.10	01	-1.121	0083	-.03
0.57	4-46	0.38	00.291	.05	-107	-0.2	001	-1.5	00.00	-.15	01	-0.027	-018	-.03
0.58	12-54	0.63	00.595	.14	-105	00.5	-03	00.0	01.00	0.40	03	-0.888	-021	-.09
0.59	7-51	-.14	-0.995	.06	-103	-0.3	023	00.0	-0.10	-.10	02	-0.963	0076	-.06
0.60	8-60	-.83	-0.480	.04	-087	-0.5	014	01.1	00.10	0.10	04	-0.573	0009	-.12
0.61	8-54	0.53	-0.063	.14	-095	-0.2	-26	01.6	-0.20	0.35	00	-0.899	-015	0.00
0.62	4-50	0.05	-0.487	.20	0096	01.1	-05	00.3	00.50	0.00	05	-0.194	0067	-.15
0.66	3-50	0.91	00.275	.20	0097	00.8	-02	-2.7	01.40	0.40	23	-0.296	0045	-.69
0.70	11-54	0.41	00.416	.17	0075	01.5	-22	-1.1	00.75	0.00	03	-1.141	0065	-.09
0.75	5-51	0.51	00.00	.04	0084	00.5	002	-4.8	-0.10	0.10	03	00.005	0017	-.09
0.76	4-60	0.75	00.139	.00	0095	00.5	-03	-1.6	00.20	0.00	19	-0.201	-008	-.57

TABLE XI. -- CONTINUED.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0.78	11-53	0.86	00.260	.06	0086	00.6	-06	-1.1	00.60	0.30	06	-1.106	0053	-.18
0.78	2-57	0.83	-0.122	.16	0102	01.0	003	-0.6	00.30	0.00	05	-1.073	0120	-.15
0.78	2-56	0.45	01.277	.20	-097	-1.9	-14	-7.4	00.80	0.25	10	00.242	-126	-.30
0.79	5-52	-.13	-0.545	.00	0090	00.8	-09	-2.5	00.20	-.05	06	-0.255	0042	-.18
0.79	7-55	0.18	-0.266	.09	-086	-0.9	038	03.9	-0.30	0.40	-1	-0.767	-015	0.03
0.81	5-54	-.50	-0.71	.08	0098	00.2	000	-2.5	00.30	-.02	-3	-0.344	0056	0.09
0.81	4-58	0.83	00.475	.25	-102	-0.9	-10	-4.4	00.50	0.12	14	00.292	-060	-.42
0.85	12-52	0.94	00.610	.17	-108	-0.6	-06	-1.5	00.70	0.60	08	-0.777	-043	-.24
0.86	6-53	-.31	-0.610	.00	0104	00.0	028	01.0	-0.40	0.30	02	-0.670	0038	-.06
0.86	7-56	0.66	-0.629	.04	0076	00.7	009	00.0	00.25	0.00	-3	-0.960	-007	0.09
0.86	10-56	0.60	-0.246	.00	0096	-0.8	020	01.9	00.15	0.50	12	-0.579	-020	-.36
0.86	1-54	0.71	00.265	.25	0090	00.6	-05	-0.5	00.20	0.00	-3	-0.909	0056	0.09
0.87	11-58	0.77	00.110	.20	0097	-0.1	-10	-4.7	01.00	0.30	17	-0.761	0029	-.51
0.87	3-60	0.66	-0.073	.25	0091	01.8	-18	-2.6	00.80	0.20	10	-0.676	0082	-.30
0.88	11-60	0.59	00.495	.00	0106	-0.4	006	00.0	00.40	0.20	14	-0.617	0022	-.42
0.90	2-50	0.16	-0.253	.12	0094	01.0	-22	-1.7	00.90	0.40	07	-1.160	0074	-.21
0.90	5-55	0.64	00.127	.00	0090	00.0	012	05.5	-0.30	-.20	09	-0.037	-016	-.27
0.92	8-52	0.10	-0.465	.11	0094	00.3	024	-1.0	-0.20	0.10	03	-1.046	0027	-.09
0.94	4-51	-.32	-0.271	.08	-102	-0.3	-15	-5.4	00.55	0.10	02	-0.199	0025	-.06
0.96	4-59	-.34	00.164	.14	0092	00.5	-11	-4.5	00.62	0.10	01	-0.295	0011	-.03
0.96	6-52	-.28	-0.069	.11	0103	00.2	035	03.4	-0.20	0.10	06	-0.249	0003	-.18
0.98	12-59	0.83	00.610	.04	-121	00.6	-27	-2.9	00.60	0.10	15	-0.777	0041	-.45
0.98	10-54	0.36	00.317	.00	0094	00.1	-05	-0.3	00.00	-.10	07	-0.783	-024	-.21
0.98	9-58	-.01	00.145	.20	0090	-0.3	009	-5.8	-0.20	-.25	08	-0.772	-014	-.24
0.99	5-53	0.32	00.622	.25	-090	-1.3	003	06.5	-0.20	0.25	12	00.365	-087	-.36
1.01	4-52	-.14	00.300	.04	0062	01.3	013	01.9	00.30	0.15	-2	-0.169	0040	0.06
1.01	7-52	0.67	-0.424	.20	0095	00.5	014	00.0	00.05	0.05	04	-0.783	-007	-.12
1.02	7-54	-.76	-0.507	.33	0090	01.0	024	02.9	-0.05	0.45	00	-1.121	0016	0.00
1.03	12-57	0.55	00.242	.14	0104	00.4	-13	-1.0	02.20	1.10	04	-1.113	0035	-.12
1.08	11-50	0.75	-0.080	.14	0090	-0.3	-15	-1.2	01.20	0.25	12	-1.251	0053	-.36
1.08	2-59	0.68	00.303	.20	-097	-1.0	-10	-7.9	00.30	0.00	11	-0.093	-035	-.33
1.09	1-55	0.32	00.870	.25	0068	-0.5	-20	-5.4	00.50	-.10	15	-0.369	-054	-.45
1.11	8-53	0.29	-0.124	.05	-094	00.0	024	03.3	-0.50	0.40	00	-0.937	-022	0.00

TABLE XI. — CONTINUED.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1.13	7-50	-.67	00.102	.14	-100	-0.4	-18	-3.0	00.00	-.05	00	-0.345	-061	0.00
1.15	12-50	0.87	-0.369	.20	0090	01.2	-32	-1.6	01.50	0.50	13	-1.284	0103	-.39
1.16	3-57	0.83	00.508	.08	0076	00.5	-14	-0.8	00.70	0.15	08	-0.161	0012	-.24
1.20	12-56	0.73	00.735	.20	0110	00.9	-34	-2.9	01.30	0.25	20	-0.728	0001	-.60
1.20	1-51	0.89	00.829	.04	-090	-0.8	-10	-4.6	00.75	-.20	15	-0.222	-052	-.45
1.22	9-50	0.18	-0.003	.25	-103	-0.5	012	-6.6	-0.10	0.25	-2	00.055	-027	0.06
1.23	2-58	0.92	00.135	.10	-114	00.4	-16	-1.0	00.80	0.25	09	-0.965	0069	-.27
1.23	12-55	0.68	00.990	.25	-080	-0.3	004	00.3	00.30	0.00	14	-0.520	-043	-.42
1.25	2-55	0.91	01.178	.25	-092	-0.2	-15	-7.5	01.00	0.10	15	00.183	-088	-.45
1.26	11-51	0.70	00.403	.20	-087	-0.6	-10	-1.7	01.00	0.20	10	-0.406	-007	-.30
1.27	4-55	0.41	-0.405	.17	-090	-1.1	-00	-3.5	00.40	0.00	17	-0.406	-003	-.51
1.27	7-57	0.19	-0.259	.20	0100	00.6	034	05.7	-0.20	0.29	04	-0.340	-036	-.12
1.30	1-55	0.78	00.330	.25	0099	01.6	-17	-1.3	01.80	1.00	23	-1.299	0090	-.69
1.32	7-53	-.30	-0.692	.25	0089	00.2	008	00.5	00.10	0.10	02	-1.030	0033	-.06
1.31	10-59	0.36	00.319	.25	-094	-1.5	-26	-6.6	00.70	-.07	12	-0.448	-003	-.36
1.33	11-55	0.89	00.573	.25	-070	-1.8	-12	-3.2	00.75	0.00	23	-0.325	-065	-.69
1.32	10-55	0.35	00.399	.20	-110	00.1	017	02.6	-0.20	0.00	-3	-0.3[illegible]	-008	0.09
1.38	2-60	0.68	00.869	.32	-098	-1.9	-31	-6.4	01.10	-.15	12	00.539	-106	-.36
1.39	3-53	0.51	00.430	.14	0087	00.5	03	01.9	00.35	0.10	10	-0.273	0044	-.30
1.41	6-57	0.46	00.246	.04	0102	-0.5	-03	-3.4	00.10	-.05	10	-0.249	-044	-.30
1.41	3-58	0.01	00.174	.00	0047	-0.2	010	00.4	-0.20	0.00	04	00.047	-058	-.12
1.45	8-59	-.01	-0.237	.20	0094	-0.9	020	01.3	-0.20	0.10	03	-0.915	-036	-.09
1.47	11-57	0.21	00.773	.14	-100	-1.5	-23	-5.3	00.50	-.25	08	-0.013	-108	-.24
1.51	10-50	0.56	00.411	.14	-097	-1.0	-08	-4.7	00.20	0.00	00	-0.104	-039	0.00
1.67	8-51	0.92	-0.392	.00	-089	00.2	034	00.7	-0.10	0.15	-2	-0.568	0001	0.06
1.68	3-54	0.80	00.535	.03	-095	-0.2	001	00.0	00.50	-.10	10	-0.177	-014	-.30
1.71	10-51	0.67	00.206	.14	-081	-0.4	-02	00.0	00.23	-.10	06	-0.008	-029	-.18
1.72.	10-57	0.09	00.208	.33	-102	-1.6	019	02.5	-0.40	-.10	-2	-0.288	-068	0.06
1.76	3-52	0.65	00.484	.25	-102	-1.6	-06	03.0	00.30	-.20	09	00.451	-041	-.27
1.77	9-54	0.64	00.099	.12	-084	-0.3	028	00.5	-0.20	0.00	03	-0.595	0025	-.09
1.80	6-57	0.40	-0.555	.11	0100	00.1	036	02.8	-0.10	0.50	03	-0.746	-012	-.09
1.85	4-57	0.05	00.647	.25	-098	-0.5	-09	-2.7	00.80	0.00	02	00.878	-050	-.06

TABLE XI. — CONTINUED.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1.89	8-55	-.33	-0.719	.12	0084	01.2	045	02.0	-0.10	0.25	02	-0.998	0073	-.06
1.96	1-52	0.93	00.782	.05	-083	00.1	005	01.5	-0.30	0.10	18	-0.337	-009	-.54
2.14	1-50	0.55	00.364	.05	0125	-2.1	007	05.6	00.05	0.00	17	-0.115	-060	-.51
2.26	1-56	0.64	00.651	.20	-099	00.4	-01	01.6	00.30	0.30	13	-0.428	0036	-.39
2.36	9-59	0.77	00.404	.33	-085	-1.4	013	02.3	00.30	0.40	02	-0.050	-085	-.06
3.13	1-57	0.60	00.572	.25	-093	-0.7	007	04.0	-0.50	-.50	13	-0.023	-054	-.39
3.15	5-57	-.74	00.106	.17	-112	-0.9	023	05.2	-0.10	0.10	00	00.332	-008	0.00
4.01	12-51	0.83	00.837	.14	-070	-2.2	-13	-8.2	01.50	0.50	26	-0.067	-099	-.78