Effect of Shape on the Fall Velocity of Gravel Particles

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As man has been forced to cope with the processes of sediment transportation and deposition — to protect himself and utilize the processes to his advantage — he has slowly developed the science of sediment engineering. Although this science has made tremendous strides, particularly in the past few decades, there yet remain many areas which require further study and research.

One of the areas in which considerable progress has been made is in relation to the characteristics of the sediment particles and the surrounding fluid, the susceptibility of the particles to erosion, transportation, and deposition. Considerable evidence has been gathered which shows that the terminal fall velocity is of paramount importance as a characteristic of the sediment particles. This fall velocity, in turn, has been related to the properties of the sediment and the surrounding fluid.

PREVIOUS INVESTIGATIONS

Many scientists have made contributions toward relating the properties of the sediment and fluid to the fall velocity of the particles. The fall velocity in turn, has been related to the susceptibility of the particles to movement. These include investigations by Richards in 1908 [1], Zegrzda in 1934 [2], Wadell in 1935 [3], Heywood in 1938 [4], Krumbein in 1942 [5, 6], Serr in 1948 [7], Corey in 1949 [8], and McNown and Malaika in 1950 [9].

Zegrzda attempted to explain the variations in Richards data by introducing shape as a factor. Wadell defined *sphericity* as the ratio of the surface area of a sphere having the same volume as the particle to the actual surface area of the particle. For simplicity and for practical considerations, Wadell suggested using the ratio of the diameter of a circle having the area of the maximum projected area of the particle to the area of the smallest circle circumscribing this maximum projected area. By this criterion, however, spheres and disks have the same sphericity. Heywood proposed a volume constant to express shape, viz., the ratio of the volume of the particle to the cube of the diameter of a sphere for which the cross-sectional area is equal to the projected area of the particle normal to the direction of motion.

Krumbein proposed an expression for sphericity

$$\psi = \sqrt[3]{\left(\frac{b}{a}\right)^2 \frac{c}{b}}$$

in which a, b, and c are the maximum, intermediate, and minimum mutually prependicular axes, respectively. Serr suggested using the ratio of the sieve diameter to the sedimentation diameter of the particle.

Corey investigated several shape factors and concluded that, from the viewpoint of simplicity and effective correlation, the following ratio was most significant as an expression of shape

$$sf = \frac{c}{\sqrt{ab}}$$

McNown and Malaika also concluded that $\frac{c}{\sqrt{ab}}$ was of major significance in expressing the shape of a particle; they also added a second shape factor a/b which expresses the relative length of the particle.

ANALYSIS OF THE PROBLEM

Within the Stokes range of flow ($\mathbf{R} < 1.0$), it is possible to develop mathematical solutions which relate the fall velocity to the properties of the particle and the fluid. If water is the fluid, however, gravel particles and most sand particles have a Reynolds number beyond the Stokes range. Therefore, if $\mathbf{R} > 1.0$ it is necessary to resort to empirical solutions. The general function relating the fall velocity to the pertinent variables is

$$\varphi_1 \left(V, W, d_n, sf, \rho, \mu\right) = 0 \qquad (1)$$

in which V is the fall velocity, W is the weight of the particle, d_n is the nominal diameter of the particle, sf is a dimensionless factor describing shape, ρ is the density of the fluid, and μ is the dynamic viscosity of the fluid. Equation (1) can be expressed in dimensionless form as

$$\varphi_2 \left(\frac{W}{d_n^2 \rho V^2}, \frac{V d_n \rho}{\mu}, sf\right) = 0 \qquad (2)$$

or

$$\varphi_3 (C_D, \mathbf{R}, sf) = 0 \qquad (3)$$

in which C_D is the drag coefficient and $\mathbf{R}$ is the Reynolds number.

Although Eq. (3) is a well established relationship, the shape factor sf is not generally expressed in terms of measurable quantities but rather by names, like sphere, disk, and ellipsoid. Although shape factors have been proposed by many individuals, it is doubtful whether any single factor will ever be a completely adequate expression of shape. The shape factor $\frac{c}{\sqrt{ab}}$ [8, 9] is relatively simple to determine, as is the ratio a/b as already mentioned.

From the foregoing discussion, it is reasonable to assume that as a first attempt the mutually perpendicular axes should be used to express shape. Refinements might be made by devising some means of expressing the texture of the particle surface.

According to McNown and Malaika [9], the stability of particle orientation while falling in a fluid depends upon the magnitude of the inertial effects relative to the viscous effects. For Reynolds numbers within the Stokes range, any orientation is stable so that a particle will retain its initial orientation as it falls. As the Reynolds number is increased beyond the Stokes range a single stable position generally becomes evident. This position is one with the maximum projected area normal to the direction of fall. At high Reynolds numbers the forces acting on the particles become unbalanced, the particle becomes unstable, and an oscillating motion develops; although the fall orientation is variable, the shortest axis tends to be parallel to the average direction of fall.

In the following pages, a description is given of recent investigations which have been conducted in an attempt to determine simple and yet significant factors expressing the shape of sand and gravel particles.

EQUIPMENT AND PROCEDURE

The purpose of this study was to determine the relationship between shape factor, Reynolds number, and drag coefficient for gravel particles. Therefore, because of the very irregular shapes of such particles, the entire study was based on the selection of random samples. To accomplish this, samples were taken from various sources and sieved into fractions. Each of the sieve fractions were then quartered to obtain a sample of about 10 particles to be used for testing. Once the individual particles had been obtained, each particle was subjected to a series of measurements. The lengths of the three mutually perpendicular axes *a*, *b*, and *c* were determined, the maximum projected area was measured in some cases, the volume or submerged weight was determined, and finally the fall velocity in water was measured.

In deciding what methods should be used for each of the measurements, the principal considerations were that the equipment should be simple, inexpensive, as precise as necessary, and flexible, and that the system should permit rapid measurements on a mass-production basis.

To accomplish these ends, a microscope was devised for the purpose of measuring the projected area and the *a* and *b* axes of the smaller particles. A grid system with a spacing of 25 microns was placed in the field of the microscope; the nearest 5 microns could be estimated. Initially, a dial type of micrometer, reading to one-thousandth of an inch, was used for measuring the *c* dimension. However, the use of this micrometer was slow and occasionally a particle was crushed by its weight. Therefore, another system was devised, using the microscope. It was first focused on the top of the particle and then on the base on which the particle was resting. By reading the micrometer on the microscope at each of these settings, it was possible to determine the *c* dimension. To simplify the process, it was assumed that while resting on the table a particle would assume a position with its center of gravity as low as possible and with the maximum projected area normal to the line of view of the microscope.

The three axes, *a*, *b*, and *c*, of the larger particles were measured to the nearest 0.01 cm with an ordinary vernier micrometer. The maximum projected area of the larger particles was determined by placing each in a pair of tweezers immediately below a sheet of translucent cross-ruled paper. A light was arranged below so that a shadow was cast on the paper and the projected area could be determined by counting the squares. The arrangement of the particle in the tweezers was adjusted so that the maximum projected area was cast on the screen.

The particles were weighed on an analytical balance. To obtain the specific weights, the particles were suspended with a fine wire and also weighed submerged in still water. Approximately 200 particles were used.

To measure the fall velocity, a plastic cylinder 10 in. in diameter and 10 ft. high was filled with water or with oil. As shown in Fig. 1, the cylinder was lighted so that the particle could be seen clearly

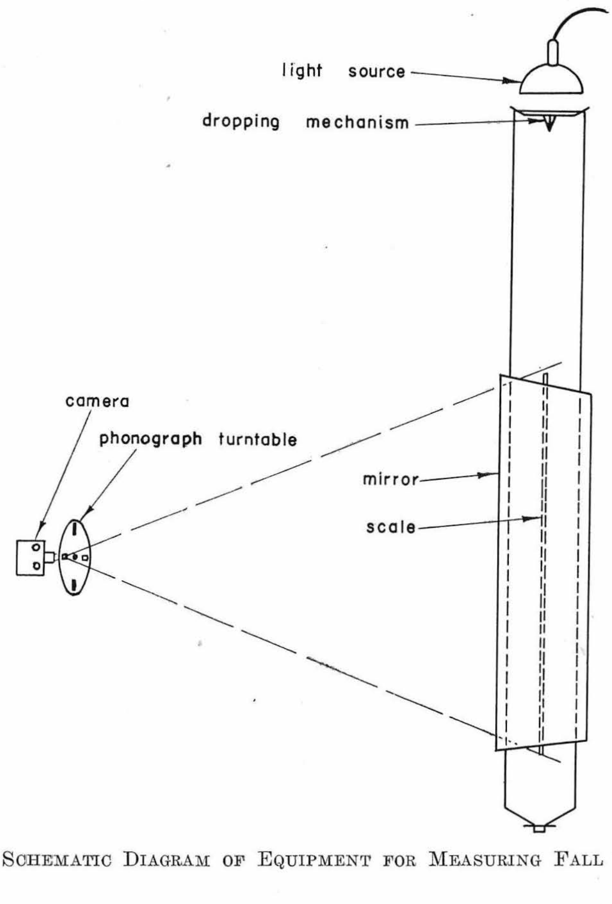

FIG. 1. SCHEMATIC DIAGRAM OF EQUIPMENT FOR MEASURING FALL VELOCITY.

as it fell. The curvature of the cylinder magnified the size of the particle, thereby increasing its visibility. Because the larger particles fell at rates which were too fast to time with a stop watch, a photographic procedure was adopted. A disk with four holes near the perimeter was mounted vertically on the turntable of a phonograph. A camera was placed behind the disk in line with the holes and the shutter was opened at the time the particle was dropped.

The holes in the revolving disk permitted an image to shine on the film at exact intervals of time as the particle fell. A graduated reference scale was placed beside the tube and photographed with the falling particle. The negatives were placed in a photographic enlarger and measurements made directly from the image.

The oil was Texaco white mineral oil A, a light colorless fluid having an S.A.E. rating of approximately five. The velocity of particle fall in the oil was slow enough that the photographic method was not necessary and the particles could be timed with a stop watch over a distance of 100 cm. To obtain Reynolds numbers between those from oil and those from water, a mixture of kerosene and oil was used.

Discussion of Results

In an analysis of the drag coefficient as a function of the Reynolds number, with a shape factor as a third variable, there are two individual terms which must be studied — the area in the drag coefficient and the length in the Reynolds number, as well as their combination in the shape parameters. The area associated with the drag coefficient for spheres and disks is that projected normal to the di-

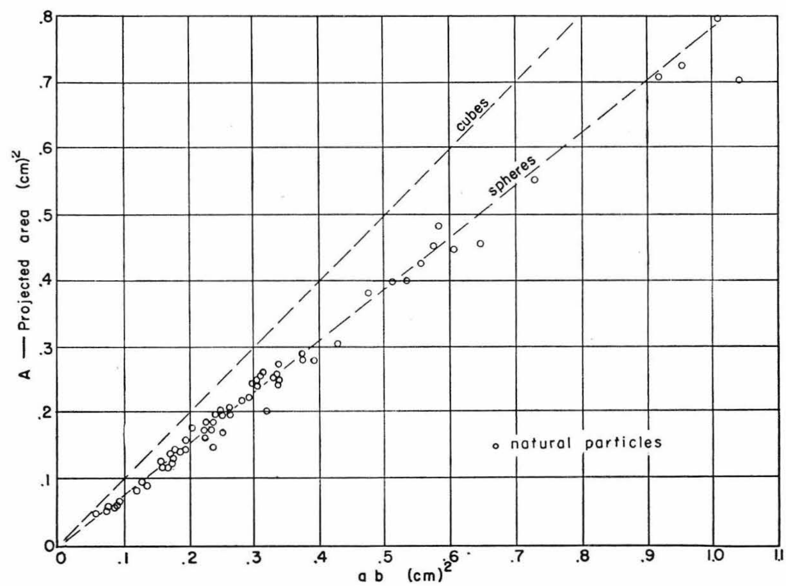

FIG. 2. RELATIONSHIP BETWEEN MAXIMUM PROJECTED AREA AND ab.

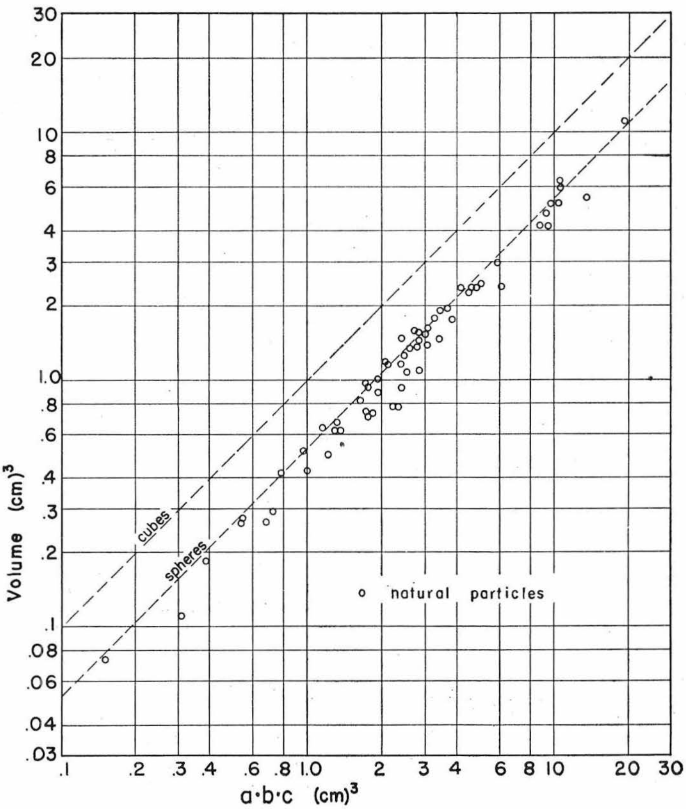

FIG. 3. RELATIONSHIP BETWEEN VOLUME AND *abc*.

rection of the flow. Therefore, it is logical to assume that the projected area of the particle normal to its direction of fall would be the area to use in its drag coefficient. Because most particles falling outside the Stokes range align their short axes parallel to the direction of flow, the projected area would logically be the maximum. This area was initially assumed to be the one most useful in the drag coefficient. Later, however, it was decided that the direct measurement of the projected area was an additional step which perhaps was unnecessary if it were possible to obtain a correlation between the maximum projected area and the product of the long and intermediate axes.

A plot of these parameters was made as shown in Fig. 2. It can be seen that the correlation is reasonably good. For purposes of

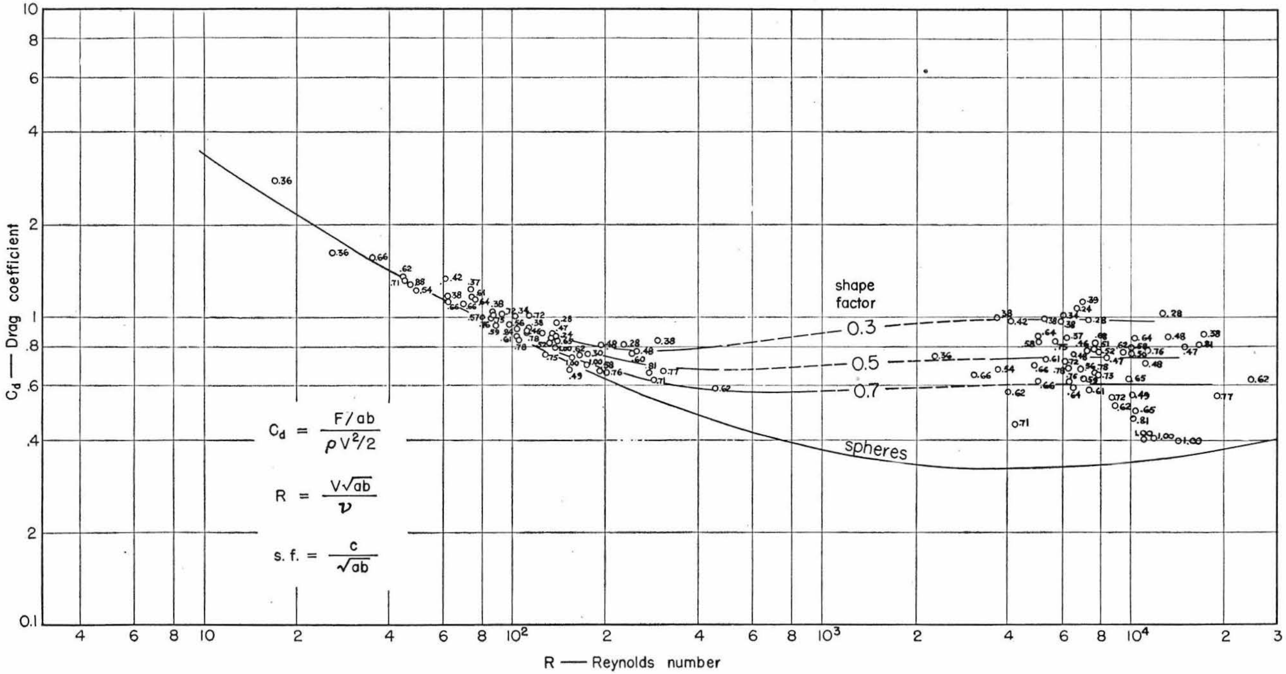

FIG. 4. VARIATION OF $\frac{F/(ab)}{(\rho V^2)/2}$ WITH $\frac{V\sqrt{ab}}{\nu}$ FOR RIVER GRAVEL.

comparison, the lines for spheres and cubes (with one face perpendicular to the direction of motion) are included in the plot. Obviously, the particles do not approach the cube. Because the projected area of the cube when oriented to present the maximum projected area is almost coincidental with that of the sphere, it was not plotted. The data are scattered reasonably well about the projected area for spheres which means the projected area is approximately 0.75 ab. The standard deviation of the variations from the mean was found to be 7.7 percent. In view of this rather close agreement, the product ab was considered a satisfactory substitute for the maximum projected area of the particle. This substitution simplified to a considerable degree the problem of obtaining data for each particle.

The process of weighing each particle was time consuming. Furthermore, it was difficult to obtain an accurate determination of the weight of the smaller particles, even with a micro-balance. Therefore, considerable advantage would be gained if the product abc could be correlated with the volume of the particle. Figure 3 is a plot of this relationship and shows that the scatter is considerably greater than in the plot of area vs. ab. In fact the standard deviation from the mean is 15.8 percent. Because of this rather large deviation, the correlation was not considered sufficiently good for use in the analysis of sediment samples.

Figure 4 is a plot of the drag coefficient (containing the product ab in place of the area) plotted against Reynolds number (containing $\sqrt{ab}$ as the length term) with the shape factor $\frac{c}{\sqrt{ab}}$ as the third variable. From this plot it can be seen that the smaller values of the shape factor give the higher values of the drag coefficient. There is considerable scatter in the data, however, and in some cases as much as a 50 percent variation in the drag coefficient for two particles having the same shape factor and Reynolds number. In spite of the scatter, however, the data show very clearly that the larger values of the shape factor plot nearest the line for spheres and the smaller values of the shape factor plot progressively farther from this line. It is interesting to note that all the data approach the line for spheres as the Reynolds number drops below approximately 100. Evidently, for low values of the Reynolds number, the product ab is adequate to describe the shape of the particle. In other words, the thickness of the particle seems to have only a minor effect on the

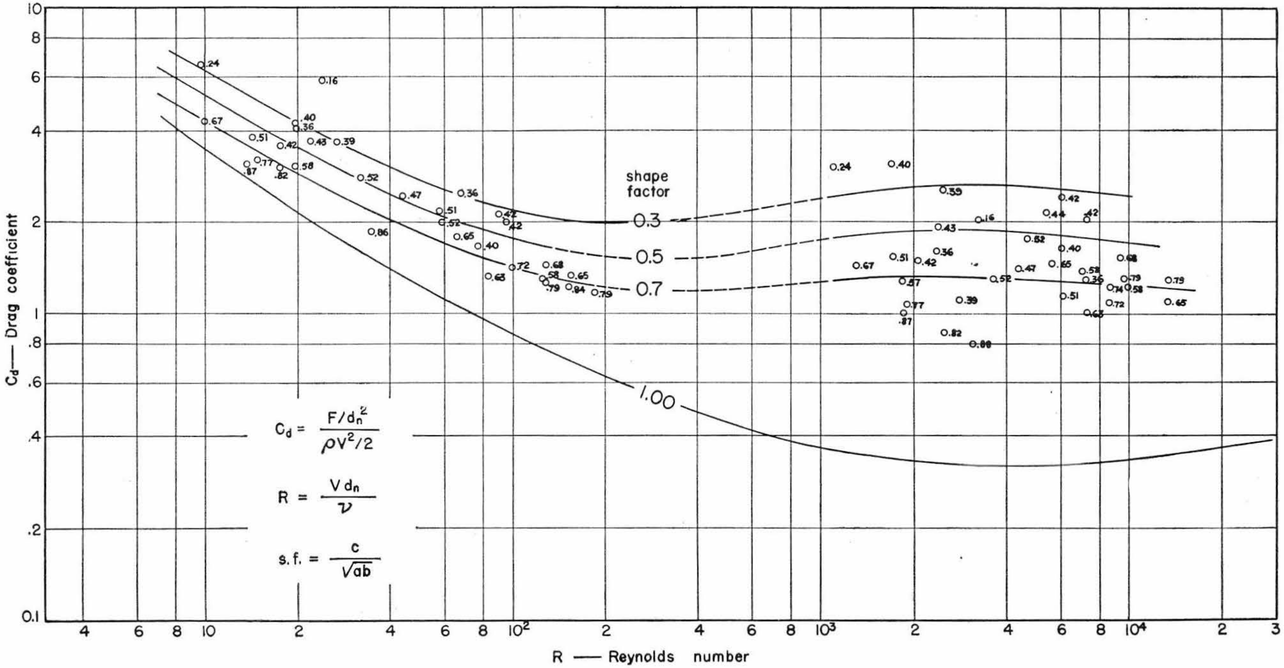

FIG. 5. VARIATION OF $\frac{F/d_n^2}{(\rho V^2)/2}$ WITH $\frac{V d_n}{\nu}$ FOR RIVER GRAVEL.

resulting drag. This observation is in accordance with the findings of other experimenters.

As the Reynolds number is increased, however, the effect of particle thickness becomes significant because of the importance of the point of separation and the character of the zone of separation in the wake behind the particle. There is considerably more scatter of the data in this region of higher Reynolds number, due, at least in part, to the fact that the unstable nature of the orientation of the particles results in oscillations which cause continual changes in both the horizontal and the vertical accelerations.

Because Malaika [10] and Corey [8] had suggested the use of nominal diameter in the drag coefficient to replace the projected area, a plot was made utilizing the nominal diameter (the diameter of a sphere having the same volume). This plot had the effect of spreading the data considerably, thereby permitting an easier correlation. Because the nominal diameter is zero for a disk which does not have thickness, the limit, as the shape factor goes to zero, is a drag coefficient of infinity. This problem, however, is not serious because the shape factors are seldom less than about 0.3. Figure 5 is a plot for gravel taken from the Poudre River near Fort Collins. The drag coefficient includes d_n^2 and the Reynolds number includes d_n as the length term. This system of plotting spread the Reynolds numbers more uniformly, and the drag coefficients were distributed over a greater distance from the curve for spheres. Careful examination of the data reveals that the lines of constant shape factor are more easily defined if the nominal diameter is used than if the two axes a and b are used. Because the drag coefficient now includes only the nominal diameter and no indication of shape, the lines of constant shape factor do not rapidly approach the line for spheres at low Reynolds number.

Figure 6 is a plot of the drag coefficient versus the Reynolds number, using the nominal diameter, for a sample taken from a rock-crusher plant. These shape factors correlate rather well but the lines of constant shape factor result in higher drag coefficients than for the river gravel. Figure 7 is a plot of several natural, water-borne gravels as compared with the material from the crusher plant. In this figure, it is particularly noticeable that the drag coefficients for a shape factor of 0.7 agree rather well for the rounded natural particles, whereas the particles from the crusher plant are considerably

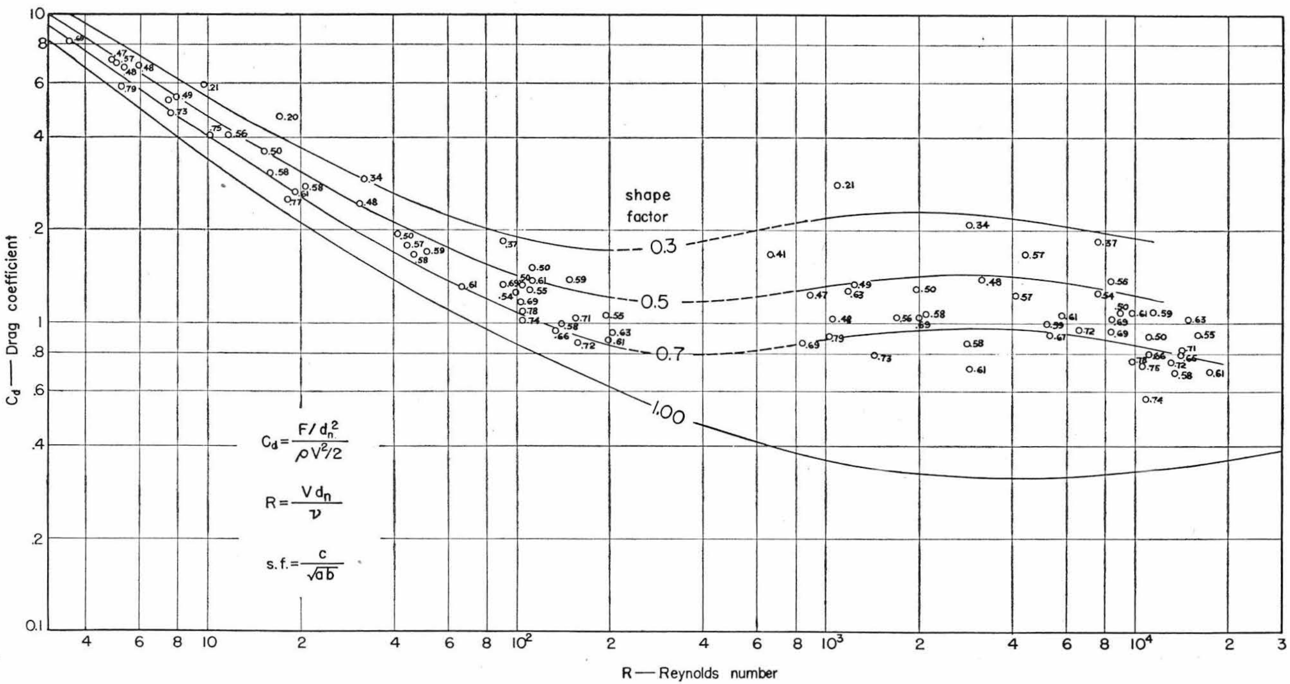

FIG. 6. VARIATION OF $\frac{F/d_n^{\,2}}{(\rho V^2)/2}$ WITH $\frac{Vd_n}{\nu}$ FOR CRUSHER GRAVEL.

higher, in fact as much as 30 percent higher, than the rounded gravel.

In an effort to reduce the scatter of these data to a minimum, an attempt was made to bring in the second length factor a/b. A plot was made of the drag coefficient vs. the shape factor with a/b as the third variable. The Reynolds number was held constant. For small values of the drag coefficient there was an indication that the larger values of a/b tended to give a larger drag coefficient for particles having the same shape factor $\frac{c}{\sqrt{ab}}$. Although there was a slight tendency for a correlation in this plot, the scatter was so great that the length factor a/b was not considered sufficiently important to be used, especially in view of the degree of refinement involved in this study.

Because of the possibility of a particle falling with different orientations, several runs were made to determine what manner of fluctuations in fall velocity occurred if one particle was dropped a large number of times. This study showed only a small variation for Reynolds numbers less than approximately 100. For larger Reynolds numbers, the fluctuation rapidly increased. Associated with this difference is the fact that $\mathbf{R} = 100$ is approximately the point at which the orientation of the particles becomes unstable and oscillations develop. It was found that the stability of a particle depended upon both the shape factor and the Reynolds number. If the particles were unsymmetrical but had large shape factors, there was no stable position for Reynolds numbers above 100. These particles would change their orientation continuously while falling. Particles having small values of the shape factor but an approximately symmetrical shape, were stable up to relatively high Reynolds numbers, 10,000 or more.

Since one of the primary causes for unsteadiness of motion is lack of symmetry in the particle, it is probable that no single shape factor could evaluate both the relative flatness of the particle and the degree of symmetry.

SUMMARY AND CONCLUSIONS

This study involves the investigation of the effect of shape upon the fall velocity of natural gravel particles and crushed rock. It shows that the projected area is rather closely approximated by ab, the product of the long and intermediate axes. It also shows that

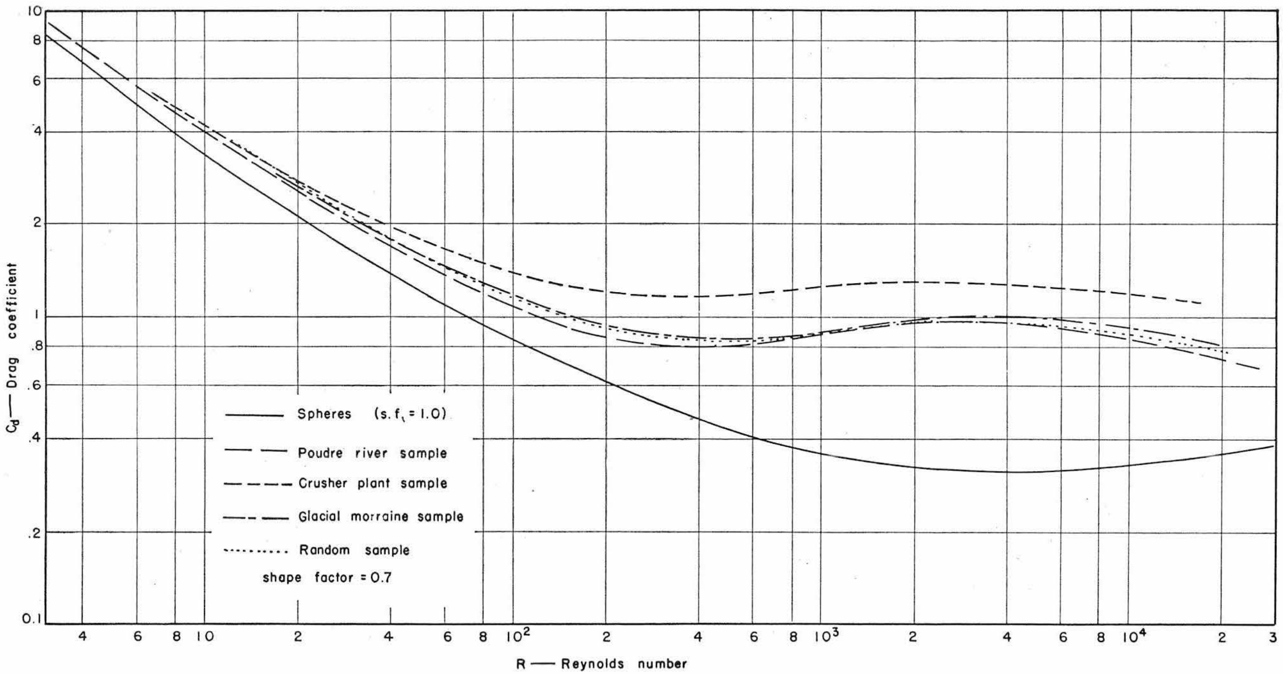

FIG. 7. COMPARISON OF C_D VERSUS Re FOR GRAVEL FROM VARIOUS SOURCES.

the volume cannot satisfactorily be represented by the product of all three mutually-perpendicular axes. The shape factor $\frac{c}{\sqrt{ab}}$ appears to be satisfactory as a single parameter to express shape at least for the degree of refinement which now exists on this subject. The additional refinement of adding the length factor a/b, which expresses the relative length of the particle, did not reduce the scatter sufficiently to make its use advisable at the present.

Although moderately good correlation is achieved by using the product ab for the area in the drag coefficient and $\sqrt{ab}$ as the length term in the Reynolds number, better correlation was obtained by using d_n^2 in the drag coefficient and d_n in the Reynolds number.

By means of curves such as those of Fig. 5, it is possible to improve to a marked extent the laboratory evaluation of the fall velocity of particles from simple data involving the physical properties of the particles. There is still considerable scatter in the data and much to be desired in regard to the expression of the shape factor. However, the correlation which is obtained can be regarded as a considerable improvement over the information previously available.

ACKNOWLEDGMENTS

The data included in this paper represent various parts of four master's theses in Irrigation Engineering at Colorado A & M College. Messrs. Arthur T. Corey and Edmund F. Shulz studied the influence of shape on fall velocity for sand-size particles. Robert H. Wilde studied the same problem for gravel-size particles and Eugene Serr investigated the correlation of sieve and sedimentation diameters. The thesis by Mr. Serr was supervised by N. A. Christensen, formerly dean of engineering, the other three by the writer. The writer wishes to acknowledge particularly the assistance of Mr. Wilde, Research Fellow, in the collection of the data and the preparation of this paper.

Recently the Corps of Engineers, through Don C. Bondurant, has supplied funds for assembling and analyzing the data from these theses and other investigations.

DISCUSSION

Mr. McNown expressed his interest in the results presented by the author, particularly because his own work with Malaika and Pramanik had been restricted to regular shapes. From his earlier work on the effect of shape McNown emphasized two points. The first was the fact that the motion of particles which are approximately symmetrical with respect to three mutually perpendicular planes is essentially the same as that for an ellipsoid of the same proportions, if the Reynolds number is less than unity. The second point dealt with stability of orientation. The limiting Reynolds number below which one position is stable and above which no stable position exists is definitely dependent upon particle shape. For thin and sharp-edged particles values as low as 100, the value cited by Albertson, were found, but for well-rounded particles the limiting values approached 1000.

Mr. John Dawson commented on his experiments at Oklahoma A & M College which were conducted in order to determine the effects of surges on drilling rates and the capacity of drilling mud to transport particles.

In answer to a request by Mr. Mitchell, South Dakota School of Mines and Technology, Mr. Dawson stated that marbles had been used to calibrate his equipment.

Mr. Posey expressed his interest in the fact that Mr. Albertson had found the nominal diameter useful in the attainment of consistent results.

Mr. Bondurant said that the fall-velocity method now being used by the Corps of Engineers does not give accurate results for sizes greater than $\frac{1}{4}$ mm. Microscopic measurements have indicated that the average values for the two methods differed so much that the fall-velocity method was no longer thought to be reliable.

Mr. Howe asked if the proximity of the particles to the boundary, which he had observed in the photographs, might not have affected significantly the fall velocities. Mr. Albertson replied that the particles sometimes struck the wall and that there was surely some effect, although he could not say how much. Mr. McNown commented that the effect of the boundary was surely very much less for the comparatively large Reynolds numbers employed in these experiments than it would have been for motion in the Stokes range.

Mr. Baines recommended the use of an independent parameter which does not contain the fall velocity in place of the Reynolds number. A combination of C_D and $\mathbf{R}$ in which V is eliminated [11] results in the parameter $F/(\rho\nu^2)$, or $\pi/(8\,C_D\mathbf{R}^2)$, which is dependent only on the properties of the pebble and the fluid. Apparently the clearest way of plotting the data is with C_D as ordinate and

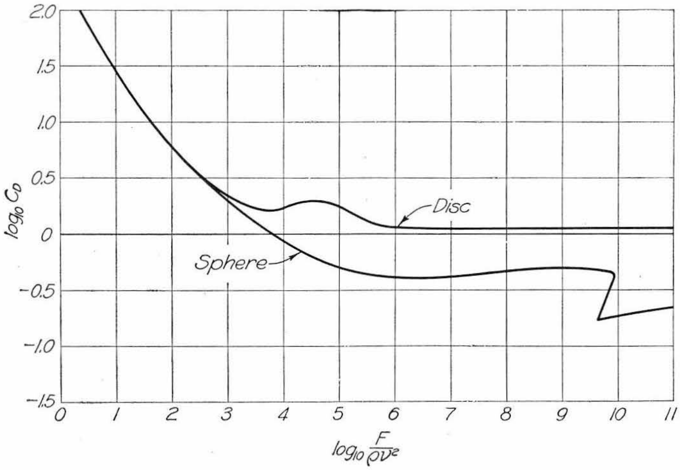

FIG. 8. VARIATION OF C_D WITH $\frac{F}{\rho \nu^2}$ FOR SPHERES AND DISCS.

$F/(\rho\nu^2)$ as abscissa. Figure 8 is a plot of C_D against this parameter for the sphere and disk. The curves thereon are similar in shape to those given in the paper so that they should be as easily interpreted. The advantages of using such a plot are that prior to performing experiments the range to be studied can be predicted, and that the fall velocity of a particle can be very quickly obtained if the weight of the particle, its shape factor, and the fluid properties are known. Otherwise, using the plots given in the paper, the method of successive approximation is necessary.

Mr. Albertson, in closing the discussion, said the orientation of a particle depends upon the forces acting on it. As an extreme example, the flow pattern and pressure distribution around a disk falling outside the Stokes range are unsymmetrical except in the two cases where the disk is oriented either normal to the flow or parallel to the flow. The disk in each of these orientations is subject to a torque if the flow pattern is changed in the slightest. In each case, however, such a torque tends to orient the disk normal to the flow — which is evidently the most stable position. A natural, unsymmetrical particle, such as most gravel particles, is subject to unbalanced forces almost without exception regardless of orientation. Hence most natural particles oscillate and move from side to side as they fall. The average orientation, however, remains that with the maximum cross section normal to the direction of flow.

By using the nominal diameter, the disk becomes a special, limiting case because it has zero thickness. Thus the nominal diameter and the drag become infinite and the fall velocity goes to zero.

The suggestion made by Mr. Baines is excellent. In the limited space permitted for this paper, however, it was necessary to consider only the question of whether a simple shape factor could be found which would improve the correlation between the fall velocity of the particle, the weight of the particle, the size of the particle, and the properties of the fluid.

From the practical viewpoint, a plot of the drag coefficient versus the Reynolds number is useful only if the weight of the particle is the single unknown to be determined. If the fall velocity is desired, a direct solution is obtained by the method pointed out by Mr. Baines. If a direct solution for the nominal diameter is required, it is necessary to employ a parameter not containing d_n, such as $\frac{\Delta\gamma\nu}{V^2\rho}$, in which $\Delta\gamma$ is the difference between the specific weights of the particle and the fluid. Finally, for compact usage these parameters can be shown conveniently on a single plot in a manner similar to that used by Rouse [11].

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