Dissertation

THE MEASUREMENT AND TRANSMISSION OF VOLATILITY IN FINANCIAL MARKETS: EVIDENCE FROM METAL FUTURES MARKETS

Submitted by
Ahmed Ali Abdel Alim Khalifa
Department of Economics

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WE HEREBY RECOMMEND THAT THE DISSERTATION PREPARED UNDER OUR SUPERVISION BY AHMED ALI ABDEL ALIM KHALIFA ENTITLED "THE MEASUREMENT AND TRANSMISSION OF VOLATILITY IN FINANCIAL MARKETS: EVIDENCE FROM METAL FUTURES MARKETS" BE ACCEPTED AS FULLFILLING IN PART REQUIRMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY.

Committee on Graduate Work

Charles F. Revier

Charles Revier

Hong Miao

D Basu

Deepanker Basu

Nancy Jianakoplos

Advisor Nancy Jianakoplos

R. Sanjay

Co-Advisor Sanjay Ramchander

Department Chair Steven Shulman

ABSTRACT OF DISSERTATION

THE MEASUREMENT AND TRANSMISSION OF VOLATILITY IN FINANCIAL MARKETS: EVIDENCE FROM METAL FUTURES MARKETS

The measurement and forecasting of asset-price volatility plays a critical role in the study of financial markets. This dissertation verifies the importance of using the integrated volatility using Fourier transformation (IVFT) measure to estimate integrated volatility efficiently. Consequently, studies of volatility that ignore intraday returns series and the IVFT measure are likely to yield misleading conclusions. The IVFT measure and the information provided by high-frequency returns are valuable to a broad range of issues in financial markets. The dissertation provides strong evidence based on the multi-chain Markov switching (MCMS) model of the interdependence, but no comovements between, the three metal markets, which is critical information for portfolio management, derivative pricing and economic policy making.

The dissertation makes a comprehensive comparison of three volatility measures: daily absolute returns, cumulative intraday squared returns, and integrated volatility via Fourier transformation (IVFT). The comparisons are made using intraday futures price data for the time period 1999 through 2008 for three metal markets: gold, silver and copper, at four frequency intervals: 1 minute, 2 minutes, 5 minutes and 15 minutes. The forecasted volatility from a GARCH model is used as a baseline to evaluate the performance of the three measures of volatility. The principal findings of the study are: (A) using heteroscedastic root mean square error and loss function criteria, the IVFT measure better fits the GARCH predictions of volatility than either the daily absolute returns or the cumulative intraday squared returns measures. In addition to this, the goodness of fit of the IVFT measure to the GARCH forecast of volatility improves as the time frequency increases from 15 minutes to 1 minute. (B) Using a multi-chain Markov switching model, the study shows a spillover and interdependence between gold futures, silver futures and copper futures, but there is no comovement between the three metal futures markets during the study period. The distinguishing feature of this dissertation is providing evidence of an accurate measure of volatility using the Fourier transformation which is crucial for accurate forecasting of volatility. For risk and portfolio management, the dissertation provides useful results, including the fact that one of the three metal markets is sufficient as a hedge against inflation or reducing risk.

Ahmed Ali Abdel Alim Khalifa
Department of Economics
Colorado State University
Fort Collins, CO 80523
Fall 2009

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Chapter 1

Introduction

Volatility estimation and forecasting have been the subject of extensive investigation in the financial economics literature (Poon and Granger, 2003; Andersen, Diebold, et al., 2001; Andersen, et al., 2005; and Figlewski, 1997). Volatility can be defined as a measure of the intensity of unpredictable changes in asset returns, and it is often calculated as the sample standard deviation of the asset returns. However, given the time varying nature of observed asset-return volatility, modeling volatility as a constant standard deviation is not valid.

According to the efficient-market hypothesis, past price movements give no information about the sign of the random component of asset returns in any period t. The mathematical formulation of the random-walk model places some restrictions on the efficient-markets hypothesis, including the prediction that the expected value of asset returns will be zero and the variance will be constant over time given the log of asset prices follows a (geometric, or proportional) random walk. Neither of these describes the actual behavior of security prices. The Black-Scholes model for deriving option prices is an extension of efficient-market pricing in continuous time. According to this option-pricing formula (Black-Scholes model), the expected value of asset returns in this model is $(u * T)$, where u is the nonrandom mean and T is the length of the time period until the maturity date of the option, and the volatility of this model is estimated by $(\sigma * \sqrt{T})$, where σ is a constant standard deviation. An important feature of this asset-pricing process is that with a constant standard deviation σ, the volatility of total return over the whole period depends on the square root of the length of the period (Figlewski, 1997).

There are several approaches to estimating the time-dependent volatility with discrete time series. One approach, introduced in the literature by Ding, Granger, and Engle (1993), suggests measuring volatility directly from realized absolute returns. This measure has been used extensively to estimate asset return volatility.

The second approach is *integrated volatility* which is derived from the quadratic variation process of the logarithm of asset prices. The *integrated volatility* approach is derived from the continuous-time model of Black and Scholes. Given the difficulty of obtaining the numerical estimates of the integrals, Andersen, Bollerslev, et al., (2001) introduced a new approach of time-dependent volatility, estimated from the historical data, called *realized volatility*, which uses the summation of intraday squared returns as a measure of integrated volatility. Andersen and Bollerslev (1998a) documented that the realized volatility measure provides better performance relative to realized daily absolute returns.

The third measure, introduced by Malliavin and Mancino (2002) is *integrated volatility using Fourier transformation (IVFT)* as a measure of *integrated volatility*. This IVFT measure was found to be relatively more efficient when compared with previous proxies for volatility, as documented in Reno and Barucci (2002) in the foreign exchange market using DM/$ exchange rate data (1989-1993) and Nielsen and Frederiksen (2008) in the stock market.

Forecasting volatility is the starting point for predicting the future prices of financial assets and for making investment decisions. The task of any volatility model is to describe the historical (stylized) pattern of volatility and to use this model to forecast future volatility, a key element in investment decisions, security valuation, risk management, and monetary policy. Volatility forecasting is important for at least three reasons. First, many investors interpret volatility as a measure of uncertainty or risk. Volatility, therefore, becomes a key input to many investment decisions and portfolio allocation choices. Consequently, a good forecast of the volatility of asset prices over an investment holding period is essential for assessing investment risk.

Second, forecasting of volatility is important for pricing derivative securities. The main determinants of options pricing are volatility of the underlying asset from the present time until the option expires, stock price, strike price, time to option expiration, and interest rate. Moreover, traders can buy financial derivatives (options and futures options) that are written on volatility itself, in which case the definition and measurement of volatility will be clearly specified in the derivative contracts. In these new contracts, volatility now becomes the underlying asset. Hence, volatility forecasts and a second prediction of the volatility of volatility over the defined period will be needed in order to price such derivative contracts.

Third, policy makers, including central banks around the world, benefit from obtaining an accurate forecast of volatility. They use volatility as an indicator of the stability of the economy, because financial market volatility can have wide repercussions on the economy as a whole. Policy makers often rely on market estimates of volatility as an indicator of the vulnerability of financial markets and the economy. For example, in the U.S., the Federal Reserve explicitly takes into account the volatility of stocks, bonds, currencies and commodities in establishing its monetary policy (Poon and Granger, 2003).

Gauging the usefulness of volatility forecasts requires a more refined articulation of the relevant volatility concepts, as well as the construction of a volatility measure that captures this notion in an empirically sensible fashion. Consequently, identifying the best proxy for or measure of volatility is a key to an accurate forecast of the volatility of a specific asset.

In addition to the importance of volatility measures, information flows from one market to another are a key element in risk management and portfolio diversification. Consequently, understanding the behavior of volatility transmission is an essential element of such fundamental theories as the efficient-market hypothesis, which posits that financial asset prices provide rational assessments of fundamental values and future payoffs. Consequently, volatility and price changes should reflect the arrival of relevant new information across financial markets. In particular, Ross (1976) suggests that under the condition of no arbitrage, volatility is directly related to the rate of information flow. An obvious extension to this argument is that interdependencies between markets can be viewed in the context of volatility linkages and information flows.

The purpose of this dissertation is to extend our current understanding of volatility estimation and transmission in the context of metal futures markets. Specifically, we consider intraday futures data for gold, silver and copper for the period 1999-2008. A study of the metal markets is important because precious metal markets such as gold and silver serve as monetary media and media of international exchange. These metal markets along with copper have a wide variety of uses in industry and commerce, such as manufacturing, computers, electronics, medicine, jewelry, oil refining, etc. Furthermore, the commodities markets in general, and the metal markets in particular, are often used as hedges against inflation and market uncertainty.

This study makes three contributions to the literature. First, the study provides a comprehensive analysis of the volatility behavior of metal futures prices using three different measures of volatility: daily absolute returns (realized absolute returns), cumulative intraday squared returns, and integrated volatility using the Fourier transformation (IVFT). Choosing the futures markets is appropriate given that these markets are highly liquid and are important media of price discovery, which is a general process used in determining the spot prices. Importantly, conclusions from past studies should be revised to the extent that they use relatively inefficient measures of volatility which underestimate the volatility of metal futures markets. From a methodological standpoint, the main novelty of the dissertation is that it provides evidence in favor of a new measure of volatility, i.e., the integrated volatility estimated via Fourier transformation (IVFT). The improved forecasting accuracy achieved using the IVFT measure suggests that financial managers should adopt this measure to better predict the price of financial derivatives when using high frequency data.

Second, the study comprehensively documents the behavior of volatility transmission across metal futures markets, where characterizations of volatility in these individual markets along with transmission or spillover mechanisms across these markets provide important insights into hedging and risk management. Specifically, a multi-chain Markov switching (MCMS) model is employed to measure the nature and degree of market integration across the various metal futures markets

The MCMS model, introduced by Otranto (2005) and later applied by Gallo and Otranto (2007), is distinguished by inserting asymmetries that make the transition probability of each market dependent on the lagged states of the other markets. Gallo and Otranto used MAE (mean absolute error) and RMSE (root mean square error) to measure the forecasting performance and they provide evidence that the MCMS model gives a better forecasting performance relative to other existing models such as VAR. Most notably, the MCMS model enables us to differentiate among inter-market linkages such as spillovers, interdependencies and comovements. Volatility spillover is defined as a situation in which a switch in regime of a dominating market leads to a change in regime in the dominated market (with lag). In contrast, interdependence of volatility is seen as a situation in which a switch in regime of one of the markets leads to a change in the regime of the other markets, and vice versa. Volatility comovements, on the other hand, are contemporaneous changes in regimes across markets. In addition to this, the results from MCMS include a detailed analysis of Granger causality

Finally, the study makes use of intraday data over an extension time period (1999 through 2008) to examine these issues. Using intraday data enables us to take full account of the process governing price variability. One of the stylized facts of the volatility of financial assets is volatility clustering, documented by Poon and Granger (2003) using daily, weekly and monthly returns. The number of periods or the long memory of the volatility differs from one interval to another; the intraday data and MCMS model enable us accurately to discover the number of days after which the high volatility state decays.

The following research questions are addressed in this study:

For Measurement and Modeling

1- Does integrated volatility via Fourier transformation (IVFT) capture stylized facts of volatility?

2- Does intraday data from the metal futures markets provide evidence to support IVFT as a superior measure of volatility in the financial markets?

3- Does increasing the frequency of intraday data from 15-minute intervals to one-minute intervals improve the fit of the volatility measures to the forecasts of volatility from a GARCH (1, 1) model?

For Volatility Transmission

4- What types of inter-market linkages are present in the metal futures markets?

Several studies measure the integration between the precious metal markets using intraday, daily, weekly and monthly data (Cai, et.al., 2001; Wahab, et. al., 1994; Escribano and Granger, 1998; Chan and Mountain, 1988 and Ciner, 2001). However, in the metal markets literature, neither the IVFT measure nor the MCMS model has been used. Consequently, the research questions are important from both theoretical and practical perspectives. For example, the potential for a diversification strategy in the metal markets will be reduced if greater integration leads to increased sensitivities and cross-market influences. Furthermore, a proper test for volatility transmission requires the correct determination of volatility itself. The dissertation's approach provides detailed markets characterizations and distinguishes between volatility spillovers, interdependences and comovements of volatility transmission across metal futures markets.

The remainder of this dissertation is organized as follows. Chapter 2 provides a preliminary data analysis and a comparison between volatility proxies in the forecasting performance using intraday data of metal futures markets and various statistical criteria, including heteroscedastic RMSE and logarithmic loss function. Chapter 3 focuses on testing the volatility transmission models in the metal futures markets for estimating the spillover effect, interdependence, and comovements across the precious metal futures markets and estimating the types of linkages across metal futures markets using MCMS model and IVFT measure. Chapter 4 summarizes the empirical evidence with an emphasis on the conclusions and further research extensions.

Chapter 2

The Measurement and Modeling of Volatility in Metal Futures Markets

This chapter addresses three important questions: First, does the integrated volatility via Fourier transformation (IVFT) estimate capture the stylized facts of volatility? Second, among the various volatility measures, which measure provides the best forecasting performance of future volatility? Finally, a third and related question is: Does the forecasting performance improve by increasing the time frequency used to estimate integrated volatility?

Measuring volatility is essential for the characterization of market dynamics, asset valuation including derivatives pricing, and in portfolio and risk management. With the availability of high-frequency or tick-by-tick data, intra-day volatility measures have been examined in a number of research studies (see, for example, Nielsen and Frederiksen, 2008; Andersen and Bollerslev, 1998a; among others). These studies provide new and important insights related to the distributional properties and dynamic dependencies in financial markets. They show how such volatility measures may be used in the formulation of highly informative and directly testable distributional implications for discretely observed asset returns. Furthermore, several studies (Andersen and Sorensen, 1996; Danielsson, 1994; and Ghysels, et.al., 1996) apply filtering and smoothing techniques to volatility time series to obtain a time series of the underlying daily volatilities, and the evaluation and comparison of volatility models, as well as reduced form volatility forecasting (see Andersen and Bollerslev, 1998b).

2.1. Literature Review

Volatility modeling has been the subject of voluminous literature over the past two decades. Empirical studies of intra-day volatility distinguish among three competing approaches for estimating volatility.

The first approach, introduced to the literature by Ding, Granger, and Engle (1993), suggests measuring volatility directly from realized absolute returns. This measure was used by Cumby, et.al. (1993), Figlewski (1997), and West and Cho (1995) as an alternative to constant standard deviation σ or $(\sigma * \sqrt{T})$ to calculate asset return volatility. This approach can be used both for intraday and daily data; however, both intra-day and daily frequencies will yield the same measure of volatility.

The second approach introduced by Anderson and Bollerslev (1998a) to measure integrated volatility is *realized volatility* and it is estimated by summing the intraday squared returns. This measure is valid for high frequency data (intraday data), which may be every second, every minute, every five minutes, etc. The empirical studies showed that this approach is unbiased and more efficient than the daily realized absolute returns measure. However, characteristics of financial market data used in these studies suggest that returns measured at an interval shorter than five minutes are plagued by spurious serial correlation caused by various market microstructure effects such as nonsynchronous trading, discrete price observations, intraday periodic volatility pattern, and bid-ask bounce. Poon and Granger (2003) and Cai, et.al (2001) provide a detailed characterization of the intraday return volatility of gold futures contracts traded on the COMEX division of the New York Mercantile Exchange (NYMEX) using cumulative intraday returns as a proxy of volatility. Anderson and Bollerslev (1998a) by appropriately filtering out the intraday patterns, they find that the high-frequency returns reveal long-memory volatility dependence in the gold market.

The third approach in the literature suggested by Malliavin and Mancino (2002) and subsequently applied by Reno and Barucci (2002) is integrated volatility using Fourier Transformation (IVFT). This proxy is relatively more efficient when compared with previous proxies for volatility, as shown by Reno and Barucci (2002) in the foreign exchange market using DM/$ exchange rate data for 1989-1993 and Nielsen and Frederiksen (2008) in the stock market using simulated data

2.2. Estimation of Volatility

There are many approaches to estimating volatility in financial markets. In this study, we focus on the following three approaches:

2.2.1. Realized Daily Absolute Returns

According to this approach, the time dependent volatility is estimated using the realized daily absolute returns. This measure can be used for data of any time frequency. It is estimated by the following formula:

$$\sigma_t = |\sum_{n=2}^{N} R_{t,n}| = |P_t - P_{t-1}|, \qquad (2.1)$$

where σ_t is the conditional standard deviation for period t and $|R_t|$ is absolute returns. Asset return is defined as the difference between the logarithms of two consecutive asset prices. P_t is the log of the current price of the asset at period t, where t might be a day, a week or a month. In this study, t will refer to "days" when the realized absolute returns measure is used, P_{t-1} is the log of the price on the previous day, and N is the number of observations in one day. In the case of a 15-minute interval, N will be approximately 14 observations per day, in the case of a 5-minute interval, N will be approximately 56 observations per day; in the case of a 2-minute interval, N will be approximately 140 observation per day; and in the case of a 1-minute intervals, N will be approximately 280 observations per day.

2.2.2. Cumulative Intraday Squared Returns

The second approach is the cumulative intraday squared returns measure, estimated as:

$$\sigma_t = (\sum_{n=2}^{N} R_{t,n}^2)^{1/2} = (\sum_{n=2}^{N} (P_{t,n} - P_{t,n-1})^2)^{1/2}, \qquad (2.2)$$

where σ_t is the volatility measure, $\sum_{n=2}^{N} R_{t,n}^2$ is cumulative intraday squared returns, $P_{t,n}$ is the log of the current price, and $P_{t,n-1}$ is the log of the price in the lagged period in the same day. Consequently, this measure is valid only for high frequency data (intraday data) which may be every second, every minute, every 5 minutes, etc., and N is the number of observations in one day.

2.2.3. Integrated Volatility via Fourier Transformation (IVFT)

According to the IVFT estimation approach suggested by Malliavin and Mancino (2002), suppose S(t), $0 \leq t \leq T$) is a time series of the asset prices. Let $p(t) = \log(S(t))$ which is the series of the logarithms of prices. Without loss of generality, the series p(t) can be described by the following stochastic process:

$$dp(t) = \sigma(t)dW(t) \tag{2.3}$$

where σ(t) is the instantaneous volatility at time t, a time dependent random function, and W(t) is a standard Brownian motion. If we normalize the time window [0, T], in which the time series is recorded to be [0,2π], then the Fourier coefficients of σ^2 can be computed by means of the Fourier coefficients of dp (see Malliavin and Mancino, 2002). It is then possible to reconstruct $\sigma^2(t)\ \forall\ t \in [0,2\pi]$, where σ_t^2 is the conditional variance estimated by the Fourier transformation and volatility is estimated by taking the square root of σ_t^2. That is, the measure of volatility is σ_t based on the classical results of the Fourier theory. The Fourier coefficients of dp are:

$$\begin{aligned} a_0(dp) &= \frac{1}{2\pi}\int_0^{2\pi} dp(t), \\ a_k(dp) &= \frac{1}{\pi}\int_0^{2\pi} \cos(kt)dp(t), \\ b_k(dp) &= \frac{1}{\pi}\int_0^{2\pi} \sin(kt)dp(t). \end{aligned} \tag{2.4}$$

where k is the number of sample oscillating functions, which is determined according to the sample theory, and it equals ((N/2) +1), where n is the number of observations during a day if we are estimating daily volatility. Consequently, N is the number of observations in a week if we are estimating weekly volatility, etc. Following Reno and Barucci (2002), we obtain the following Fourier coefficients expression of σ^2:

$$a_0(\sigma^2) = \lim_{n\to\infty} \frac{\pi}{n+1-n_0} \sum_{s=n_0}^{n} \frac{1}{2}[a_s^2(dp) + b_s^2(dp)],$$

$$a_k(\sigma^2) = \lim_{n\to\infty} \frac{2\pi}{n+1-n_0} \sum_{s=n_0}^{n} a_s(dp) a_{s+k}(dp), \qquad (2.5)$$

$$b_k(\sigma^2) = \lim_{n\to\infty} \frac{2\pi}{n+1-n_0} \sum_{s=n_0}^{n} a_s\,(dp) b_{s+k}(dp).$$

Note that there are two symbols for the Fourier coefficients in the set of equations (2.5) to match the convolution technique in computing Fourier coefficients. The definition of s is same as k and it has the same length. By the classical Fourier-Fejer inversion formula, we can reconstruct σ_t as follows:

$$\sigma_t = (\lim_{n\to\infty} \sum_{k=0}^{n} \left(1-\frac{k}{n}\right) [a_k(\sigma^2)\cos(kt) + b_k(\sigma^2)\sin(kt)]\,)^{1/2} \qquad (2.6)$$

This is one approach to estimating instantaneous volatility given a series of prices.

The second approach is as follows. Given a time series of N observations $(t_i, p(t_i)), i = 1, \ldots, N$, data is compacted in the interval$[0,2\pi]$. This interval is a normalization for the time window (day in our case), which means, the returns are integrated each day. Inside this time window, there is a partition determined by the number of observations during that day. Integrals in equation (2.4) are computed through integration by parts:

$$a_k(dp) = \frac{1}{\pi}\int_0^{2\pi} cos(kt)dp(t) = \frac{p(2\pi) - p(0)}{\pi} - \frac{k}{\pi}\int_0^{2\pi} \sin(kt)\, p(t)dt. \quad (2.7)$$

Further, by setting $p(t) = p(t_i)$ in the interval $[t_i, t_{i+1}]$, the integral in equation (2.7) in the interval $[t_i, t_{i+1}]$ becomes:

$$\begin{aligned} \frac{k}{\pi}\int_{t_i}^{t_{i+1}} \sin(kt)\, p(t)dt &= p(t_i)\int_{t_i}^{t_{i+1}} \sin(kt)\, dt \\ &= \frac{p(2\pi) - p(0)}{\pi} - p(t_i)\frac{1}{\pi}[\cos(kt_i) - \cos(kt_{i+1})]. \end{aligned} \quad (2.8)$$

Reno and Barucci (2002) show that adding a linear trend to obtain $p(2\pi) = p(0)$ does not affect the volatility estimate. However, our study includes this part in the estimation. Equation (2.7) can then be computed as:

$$a_k(dp) = \frac{p(2\pi) - p(0)}{\pi} - \sum_{i=1}^{N-1} p(t_i)\frac{1}{\pi}[\cos(kt_{i+1}) - \cos(kt_i)]. \quad (2.9)$$

Similarly we have the following:

$$b_k(dp) = \frac{p(2\pi) - p(0)}{\pi} - \sum_{i=1}^{N-1} p(t_i)\frac{1}{\pi}[\sin(kt_{i+1}) - \sin(kt_i)]. \quad (2.10)$$

Reno and Barucci (2002) propose an estimator of the integrated volatility:

$$\int_0^{2\pi} \sigma^2(s)ds = 2\pi a_0(\sigma^2), \quad (2.11)$$

$$\sigma_t = \left(2\pi a_0(\sigma^2)\right)^{1/2}. \quad (2.12)$$

Equation (2.5) gives the expression of $a_0(\sigma^2)$ which is a limit of the summation of $a_k(dp)$ and $b_k(dp)$. Equations (2.9) and (2.10) compute the Fourier coefficients $a_k(dp)$ and $b_k(dp)$. The integrated volatility can now be estimated without integration. For the purpose of this study and according to sampling theory, k is determined by the formula $(\frac{N}{2}+1)$ to avoid the aliasing effects.

The properties of the three estimation methods of volatility—realized daily absolute returns, cumulative intraday squared returns, and the Fourier estimator—have been examined briefly in the literature in the context of foreign exchange markets and simulated data. As mentioned previously, Anderson and Bollerslev (1998a) show that the cumulative intraday squared returns measure is more efficient relative to realized daily absolute returns using foreign exchange market data. Reno and Barucci (2002) compare the Fourier method to realized volatility in a Monte Carlo study to generate the latent instantaneous volatility process, and their simulations show that the Fourier method compares favorably to realized volatility. However, Reno and Barucci (2002) contrast a 5-minute realized volatility estimator to a Fourier estimator using all observations (which are measured every 14 seconds on the average for one day), resulting in the conclusion that IVFT is a superior measure in comparison to cumulative squared intraday returns measure of foreign exchange rate (DM/$) for the period 1989-1993) and simulated data using the parameters estimated by Anderson and Bollerslev (1998 a, b). Nielsen and Frederiksen (2008) undertake a comprehensive comparison across the three different measures of integrated volatility using the Monte Carlo simulation techniques and parameters estimated by Anderson and Bollerslev (1998b). They find the Fourier method to be superior compared to the other two estimators (cumulative intraday squared returns and wavelet transformation) in the presence of market microstructure noise. More strikingly, even after using the bias correction methods designed specifically to handle market microstructure effects, the Fourier method was shown to have a superior forecasting performance while having only a slightly higher bias.

2.3. Sources of Data

Our primary data set is closing prices of the metal futures market which consists of four time intervals: 1 minute, 2 minutes, 5 minutes and 15 minutes intervals of gold futures, silver futures and copper futures for the period January 1999 to December 2008.

The data are obtained from the *Futures Industry Institute*. All three futures contracts are traded on the NYMEX (New York Mercantile Exchange) and priced in U.S. dollars. For gold, the trading unit is 100 troy ounces, and the trading hours are from 8:20 AM EST until 1:30 PM EST. Trading in standardized contracts is conducted for the current calendar month, the next two calendar months, any February, April, August, and October falling within a 23-month period, and any June or December falling within a 60-month period beginning with the current month. At the expiration date, the seller must deliver 100 troy ounces (±5%) of refined gold, assaying not less than .995 fineness, cast either in one bar or in three one-kilogram bars, and bearing a serial number and identifying stamp of a refiner approved and listed by the Exchange (New York Mercantile Exchange, 2008a).

For silver, the trading unit is 5000 troy ounces, the trading hours are from 8:25 AM EST until 1:25 PM EST. Standardized contracts are traded for delivery during the current calendar month, the next two calendar months, any January, March, May, and September falling within a 23-month period; and any July and December falling within a 60-month period beginning with the current month. At the expiration date, the seller must deliver 5,000 troy ounces (±6%) of refined silver, assaying not less than .999 fineness, in cast bars weighing 1,000 or 1,100 troy ounces each and bearing a serial number and identifying stamp of a refiner approved and listed by the Exchange (New York Mercantile Exchange, 2008b).

For copper, the trading unit is 25000 pounds. The trading hours are conducted from 8:10 AM until 1:00 PM. The formal contract for trading is conducted for delivery during the current calendar month and the next 23 consecutive calendar months. At the expiration date, the seller must deliver grade 1 electrolytic copper conforming to the specification B115 as to chemical and physical requirements, as adopted by the American Society for Testing and Materials, and of a brand, approved and listed by the Exchange (New York Mercantile Exchange, 2008c).

The study estimates volatility for intraday data to obtain a time dependent daily integrated volatility measure for the period 1999-2008. The raw data specify the time, to the nearest second, and the exact price of the futures transaction. The intraday time series present the data in four frequencies. For each frequency, the closing prices for the nearby futures contracts are employed to calculate the 1-minute, 2-minute, 5-minute and 15-minute prices. The study uses the logarithm of futures metal prices (P). Based on the trading hours for each contract, there are approximately 280 one-minute time intervals, 140 two-minute time intervals, 56 five-minute time intervals, and 14 fifteen-minute time intervals during each trading day. Finally, there are approximately 2513 trading days for gold futures, 2511 for silver futures and 2507 for copper futures over the sample period.

The average number of observations for each market is a total of 702,800, 351400, 140,560 and 35,140 observations for the 1 minute, 2 minutes, 5 minutes and 15 minutes, respectively.

2.4. Stylized Asset Returns Facts for the Metal Futures Market

Previous studies that examine the stochastic characteristics of financial time series document the following stylized facts: (1) Daily returns, measured by the following formula, $R_t = P_t - P_{t-1}$ where P_t is the log of the current closing price at day t and P_{t-1} is the log of the price of the previous day, exhibit *very little autocorrelation.* (2) Volatility displays positive correlation with its own past. This is most evident at short horizons such as daily or weekly frequencies, which means that volatility has a long memory. (3) The unconditional distribution of daily returns has *fat tails* (i.e. leptokurtic distribution). The previous stylized facts are examined for gold, silver and copper futures markets using the IVFT measure and explained as follows.

The first stylized fact is that daily frequency returns in the metal futures markets have very little autocorrelation. This means that returns are almost impossible to predict from their own past, as shown in Figures (2.1, 2.2 and 2.3). The correlation of the daily frequency returns of gold futures, silver futures and copper futures with returns lagged from 1 to 20 days have correlation roughly equal to zero.

The second stylized fact is the long memory of volatility (or volatility clustering). This means that a low volatility period will be followed by low volatility and a high volatility period will be followed by high volatility. When we estimate daily volatility using the IVFT measure for 1-minute frequency, Figures (2.4, 2.5 and 2.6) illustrate the positive correlation of volatility with its own past. There is evidence of a long memory of volatility in the metal markets. Numerically, the autocorrelation of volatility in gold futures is approximately 0.5 during 20 lags, but it decreases sharply after 10 lags for the silver futures market and after 14 lags for copper futures, where each lag is equivalent to one day. Consequently, the data show that volatility persists, a feature documented in financial markets in general (Poon and Granger, 2003), and also supported by our study using evidence from the metal market.

This fact is useful for financial practitioners because a shock in the volatility series seems to have a very long 'memory' and impact on future volatility over a long horizon, and it is transmitted from one market to another. The integrated GARCH (IGARCH) model of Engle and Bollerslev (1986) captures this effect, but a shock in this model impacts future volatility over an infinite horizon. This fact is supported by volatility clustering which refers to the observation, as noted by Mandelbrot (1963) and documented by Cont (2005) that large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes. A quantitative manifestation of this fact is that, while returns themselves are uncorrelated, volatility of returns measured by realized absolute returns $|R_t|$, cumulative returns and/or their squares, and IVFT display a positive, significant and slowly decaying autocorrelation function. In other words, $\mathrm{Corr}(|R_t|, |R_{t+\tau}|) > 0$ for τ (1 minute, 2 minutes, 5 minutes, etc.) for intraday observations or equaling one day if daily data are used.

The third stylized fact is that the distribution of daily returns has fatter tails than the normal distribution. This fact is illustrated in Figure 2.7 and numerically in Table (2.1) as well. Using the Jarque-Bera test, Table (2.1), the data used in the study rejects the null hypothesis of a normal distribution at a significance level less than 1%, as shown in the probability in the table. Another criterion supporting this fact is skewness (a measure of symmetry), which is 0.19 for gold, -0.99 for silver, and -0.34 for copper, where it is zero for a standard normal distribution. Yet another criterion is that kurtosis, a measure of the peak of the distribution relative to a normal distribution, is 9.1 for gold, 11.8 for silver and 8.24 for copper. It is zero for a standard normal distribution, which means that the data for the three metal markets reflect a sharp peak at the mean. The histograms as illustrated in Figure (2.7) for all three metal contracts show fat tails and sharp peaks at the mean of the returns for the three metal markets in comparison to the standard normal distribution. For example, the histogram of the gold futures returns has a fat tail at 0.09 and -0.058, and peaks at the mean of the returns. In the case of silver futures returns, the histogram has a fat tail at 0.08% and at -0.098%, and a sharp peak at the mean of returns of silver in comparison with the standard normal distribution. Finally, with regard to returns for copper futures, the histograms indicate a fat tail at 0.07% and left tail -0.098%, and a sharp peak at the mean of returns in comparison to the standard normal distribution. Consequently, there is strong evidence to suggest that the returns from gold, silver and copper futures are not approximated well by a normal distribution. Instead, the returns in these markets are characterized by a non-normal distribution with fat tails and a sharp peak at the mean of the returns.

An observation illustrated in Table (2.1). is that, the standard deviation of daily returns completely dominates the mean of returns at short horizons such as daily, i.e. the standard deviation is greater than the mean of returns. The standard deviation of gold is 0.011644 which dominates the mean of returns of gold (0.000447). For silver, the standard deviation is 0.018866 which dominates the mean of silver futures returns

(0.00033). For copper futures, the standard deviation is 0.0182 which dominates the mean of copper futures' returns (0.0003). Hence, evidence from metal markets supports the fourth fact of financial assets, and it is clear that the returns for financial assets are risky. Similarly, the median of gold futures is 0.00036, the median of silver futures is 0.00107, and the median of copper futures is 0.000417, which differ from zero and are dominated by their standard deviations. This observation is consistent with the real world data and consistent with Figlewski's (1997) viewpoint that the means of returns of financial assets differ from zero. This fact is critical for the risk management, and it is an indicator that these financial assets have a high risk.

2.5. Methodology for Testing the Study Questions

Following Anderson, Diebold, et al. (2001), Reno and Barucci (2002) and Baillie and Bollerslev (1992), we evaluate the three volatility measures based on how closely the measures fit forecasts of volatility from a GARCH model. The univariate autoregressive conditional heteroscedastic (ARCH) model was introduced by Engle (1982) and generalized by Bollerslev (1986). We use volatility estimated from a GARCH (1, 1) model as a benchmark for comparing the three integrated volatility measures because "true" volatility is unobservable and GARCH (1, 1) is the most commonly used model for forecasting volatility in financial time series econometrics[1]. Volatility estimated from GARCH (1, 1) is consistent with the Chicago Board of Options Exchange (CBOE) Volatility index (VIX) and other volatility measures.

[1] Since true volatility is unobservable, Nielsen and Frederiksen (2008) estimated integrated volatility using two consecutive day's of intraday data as a benchmark for true volatility to make comparisons among volatility measures. We replicated the Nielsen and Frederiksen measure, but found that it overestimated integrated volatility and was not consistent with the CBOE Volatility index or other measures of volatility including GARCH forecasts.

Suppose the mean equation is specified by an AR (p) model as:

$$R_t = c + \sum_{i=1}^{p} a_i R_{t-i} + \varepsilon_t. \tag{2.13}$$

Here R_t is the return at day t, c and a_i are parameters to be estimated, and the error term,ε_t is factorized as:

$$\varepsilon_t = z_t h_t^{1/2}. \tag{2.14}$$

where z_t is an i.i.d. sequence with mean zero and variance one, then the GARCH (p, q) model can be written as

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i h_{t-j}. \tag{2.15}$$

We run GARCH (1, 1) models on the daily return series. Symbols, p and q, are the number of lags of the error term and GARCH volatility; ω is the (kopa) coefficient of GARCH equation. Then we use the following equation to forecast the one day forward volatility:

$$F_{1,t} = \sigma_1^2 + (\alpha_1 + \beta_1)(h_{t-1} - \sigma_1^2). \tag{2.16}$$

where $\sigma_1 = \frac{\omega}{1-(\alpha_1+\beta_1)}$ and h_{t-1} is the conditional variance from GARCH model of the previous day and $F_{1,t}$ is the one-day-ahead forecast. In summary, there are two steps to compute the one-day-ahead forecast. The first step is computing the daily returns for the three metal markets and estimating the mean equation (2.13) and GARCH equation (2.15) using maximum likelihood to get the GARCH coefficients. The second step is estimating the one-day-ahead forecast of the volatility ($F_{1,t}$).

Before proceeding to a discussion of the methodology for evaluating forecasting performance, the reader should note that equation (2.16) is a special case of the following equation (2.17), the general formula introduced by Anderson, et.al (1999). The more general formula takes the form:

$$F_{m,t} = mb\sigma^2_{(m)} + (\alpha_m + \beta_m)[1 - (\alpha_m + \beta_m)^{mh}]\frac{\sigma_{(m),t} - \sigma^2_m}{1 - (\alpha_m + \beta_m)} \quad (2.17)$$

where m is the number of days of returns on which the forecast is based, which equals one in our case, and h is the number of days ahead that the forecast is being made, which is also equal to one in our case.

We use two measures to evaluate the forecasting performance of the volatility measures: the heteroscedastic root mean squared error (HRMSE) and the logarithmic loss function (LL):

$$HRMSE = \mathbb{E}\left[\left(1 - \frac{F_{m,t}}{\sigma^2_t}\right)^2\right]^{1/2}, \quad (2.18)$$

$$LL = \mathbb{E}\left[log\left(\frac{F_{m,t}}{\sigma^2_t}\right)\right]. \quad (2.19)$$

The criteria for determining the better volatility proxy are the relative values of the HRMSE and the LL. Smaller values of either HRMSE or LL indicate better forecasting performance. Note that, it is possible to find LL negative if the fraction of $\left(\frac{F_{m,t}}{\sigma^2_t}\right)$ is less than one.

2.6. Estimation Results

Both Figures 2.8, 2.9 and 2.10 and Tables 2.3, 2.4 and 2.5 illustrate that the IVFT measure is a relatively more accurate measure of volatility when compared to both cumulative intraday squared returns and daily absolute returns. Specifically, Table (2.2), panel (A), shows the estimated annual volatility, computed by the following formula

$$\sigma_t^{Annual} = \sqrt{252} \times \text{Average Daily Volatility} \qquad (2.20)$$

The estimated annual volatility in gold futures is 17.2 % using IVFT, 13.9% using cumulative intraday squared returns measure, and 12.9 % using the daily realized absolute returns. The results seem to indicate that the IVFT measure reflects more information about volatility in the metal futures markets. It is seen that the closest measure to the GARCH daily returns 17.6 % is IVFT. Furthermore, the magnitude of the IVFT measure decreases as the intraday time frequency decreases from 1 minute to 15 minutes. For example, the annual volatility estimated by cumulative intraday squared returns measure in gold futures decreased from 13.9 % using a 1-minute interval to 11.75% if a 15-minute interval is used, where annual volatility estimated by IVFT measure decreased from 17.2% using 1-minute intervals to 12.1% if 15-minute intervals are used. This result has important implications for financial practitioners, especially those who are trading options, since they can predict options prices better if they use the IVFT measure to estimate the volatility of the financial markets using high frequency data.

The corresponding results for silver are shown in Table (2.2), panel (B). These results are qualitatively similar to those of gold futures. For instance, the annual volatility of silver futures is 35.5 % using IVFT, 25.4% using cumulative intraday squared returns measure, and 19.9% using the daily absolute returns. The GARCH estimate indicates an annual volatility of 27.4 %. Similar to the results from gold futures, the results from silver also indicate that the IVFT measure captures more variation in returns relative to the other competing measures of volatility. Note that the highest value of volatility is the one estimated by the IVFT measure, which means that the IVFT measure reflects more information about the asset prices relative to other measures.

For copper, Table 2.2, panel (C), shows that both the cumulative intraday squared returns and daily absolute returns are closer to the GARCH benchmark relative to the IVFT measure. However, the values of the IVFT measure are bigger than the benchmark and the other two measures, which illustrates that the measures based on high-frequency returns such as IVFT contain extremely valuable information for the measurement of integrated volatility at the daily level.

2.7. Forecasting Performance of Volatility Measures

This section evaluates the goodness of fit of the three alternative volatility measures to one-day-ahead forecasts of volatility generated from a GARCH (1, 1) model using data from the gold, silver and copper futures markets for the period 1999 to 2008. Using the technique described previously (see equation 2-16), the GARCH (1, 1) coefficients and day-ahead forecasts are generated using metal futures markets data at differing intraday frequencies (1 minute, 2 minutes, 5 minutes and 15 minutes).

The results presented in Table (2.3) are in agreement with Nielsen and Frederiksen (2008), Andersen and Bollerslev (1998a), and Reno and Barucci (2002), namely, that increasing sample frequency from 15 minutes to 1 minute improves the fit to GARCH model forecasts of volatility. We find that the fit to GARCH (1, 1) forecasts of volatility associated with the Fourier estimators of one, two, five and fifteen minutes for gold, silver and copper futures markets are better than daily absolute returns and cumulative intraday squared returns measures if the same intraday intervals are used.

In general, the IVFT volatility measure provides the best fit to GARCH model forecasts of volatility for all three metal contracts and across different frequency intervals. The goodness of fit is judged by the HRMSE and LL measures. These results are presented in Table (2.3) – Panels A, B, and C, IVFT measure improves as frequency increases. In the gold futures market, for example, (see panel A) the HRMSE using 1 minute intervals is 481.4 for daily absolute returns, 3.04 for realized volatility measure (cumulative intraday squared returns), and it is 0.96 for IVFT during 1999-2008. The results imply that IVFT has the lowest forecasting error after adjusting for heteroscedasticity. To ensure that these results are robust, the data period is partitioned into two sub periods, the first is from 1999-2004 and the second period is 2004-2008. Similar results are documented for the sub periods 1999-2004 and 2005-2008.

The second criterion used to judge the goodness of fit of the three volatility measures is the logarithmic loss function metric (LL). Again, using gold as an example for the overall period using 1-minute intervals, the logarithmic loss function is 1.53 if realized daily absolute returns are used, 0.55 for cumulative intraday squared returns and 0.149 for IVFT. The previous analysis shows that the IVFT measure of volatility has a lower error relative to the other measures. For the 2-minute intervals, Table (2.3) panel (B) provides evidence that HRMSE for gold futures during the period 1999-2008 is 3.423, if the cumulative intraday squared returns is used and is equal to 1.36 if IVFT is used. The logarithmic loss function is 0.44 if IVFT is used. On the other hand the logarithmic loss function is 0.67 when the cumulative intraday squared returns measure is used. This indicates that the IVFT measure provides better performance relative to the other measures and the same trend exists using the sub periods (1999-2004 and 2005-2008) as it is shown in the Table (2.3) panel (B).

With respect to the impact of increasing the frequency on the performance of the IVFT measure, Table (2.3), panels (A, B, C and D), shows that the goodness of fit of the IVFT measure as frequency increases improves. The performance of the IVFT increases by increasing the time frequency from 15 minute intervals to 1 minute. For example, the HRMSE improved from 4.67 to 0.96 if time frequency is increased from 15 minutes to 1 minute intervals (note that decreasing HRMSE means better performance and vice versa), and the logarithmic loss function improved from 0.937 to 0.1495 if time frequency increased from 15 minutes intervals to 1 minute intervals (note that decreasing logarithmic loss function means better performance and vice versa). For silver futures, the same conclusion holds true if we use the sub periods 1999-2004 and 2004-2008, except for the 1-minute intervals for the sub period 1999-2004, when the realized volatility measure performs better than IVFT. Therefore, all cases show that the IVFT measure is a superior measure relative to other measures for the four intervals.

For the silver futures market, Table (2.4), panel (A), shows that HRMSE using 1 minute intervals is 289.33 for daily absolute returns, 39.7841 for cumulative intraday squared returns, and 0.8327 for the IVFT measure during the period 1999-2008. Consequently, for the time period 1999-2008 of the metal futures markets, the data shows that the IVFT measure has minimum error after adjusting for heteroscedasticity. Similarly, the same conclusion holds true for the sub period 1999-2004 and the sub period 2005-2008 which indicate that IVFT has the minimum error to fit with the GARCH (1, 1) forecasts of volatility relative to the other measures. A similar conclusion is reached if we use the logarithmic loss function for silver futures or copper futures for the four time frequencies, confirming that the IVFT measure better fits the GARCH (1, 1) forecast relative to the other volatility measures. On the other hand, both HRMSE and LL show that daily absolute returns measure is a very noisy measure of volatility. Table 2.3 (Gold Futures) shows that the HRMSE statistics are 195.26, 804.09 and 481.4 for the periods 1999-2004, 2005-2008 and 1999-2008, respectively, indicating that the daily absolute returns measure has the highest error relative to the other measures.

More evidence showing the deficiencies of the daily absolute returns measure is provided in Table 2.4. With respect to the volatility of silver futures, the HRMSE statistics for the daily absolute returns measure are 69.3, 406.31 and 289.33 for the periods 1999-2004, 2005-2008 and 1999-2008, respectively, which are the largest errors of the three volatility measures. Likewise with respect to the volatility of copper futures, Table 2.5 shows that the HRMSE statistics of the daily absolute returns measure are 169.99, 1768 and 1211 for the periods 1999-2004, 2005-2008 and 1999-2008 respectively. Again these are the largest errors among the three volatility measures.

When the logarithmic loss function is used to assess how well the volatility measures fit the GARCH forecasts, the daily absolute return measure also performs the worst of the three volatility measures. Table 2.3 shows that with respect to the volatility of gold futures the LL statistics of the daily absolute returns measure are 1.55, 1.46 and 1.53 for the periods 1999-2004, 2005-2008 and 1999-2008 respectively, the largest among the three volatility measures. For silver futures volatility Table 2.4 indicates that the LL statistics for the daily absolute returns measure are 1.33, 1.43 and 1.37 for the periods 1999-2004, 2005-2008 and 1999-2008 respectively, which are also the largest for the three volatility measures. For copper futures, Table 2.5 shows that the LL statistics for the daily absolute returns measure are 1.34, 1.39 and 1.38 for the periods 1999-2004, 2005-2008 and 1999-2008 respectively. Once again the daily absolute returns measure has the highest error relative to the other measures.

In summary, this chapter undertakes a comprehensive comparison among the three volatility measures (realized daily absolute returns, cumulative intraday squared returns and the IVFT measure) using intraday data from metal futures markets (gold, silver and copper). The IVFT measure estimate is based on integration of the time series, so that it naturally exploits the time structure of high frequency data by including all the observations in the volatility computation. Using historical tick-by-tick data from metal futures prices, the study illustrates the fact that this IVFT measure performs better and the fit to the forecasting performance of the GARCH model is superior relative to both the daily absolute returns and the cumulative intraday squared returns. When the study employed the IVFT measure, the fit to GARCH forecasts turned out to be more accurate than those associated with the sum of squared intraday returns and daily absolute returns. The study used the IVFT measure to evaluate the fit to the forecasting performance of the GARCH (1, 1) model when it is discretized at intraday frequencies.

Using the HRMSE and LL criteria, the results reported in recent literature are confirmed using metal futures markets. With respect to the impact of time frequency on volatility measures, the study provides evidence that the goodness of fit of the IVFT measure to the GARCH model forecasts of volatility improves as the time frequency increases from 15 minutes to 1 minute.

Table 2.1 Summary Statistics and Stylized Facts of Daily Returns (1999-2008)

Metal	Gold Futures	Silver Futures	Copper Futures
Mean	0.04%	0.03%	0.03%
Median	0.04%	0.11%	0.04%
std-Dev	1.16%	1.89%	1.82%
Skew	0.19452	-0.999243	-0.3411
Kurt	9.108317	11.76386	8.239152
Jarque-Bera	3921.116	8450.257	2914.688
Probability	0	0	0
# of observations	2512	2510	2506

Table 2.2 Annual Volatility of Gold Futures (Panel A), Silver Futures (Panel B) and Copper Futures (Panel C) Markets

Panel A: Gold Futures				
V-Measure	Time Frequency			
	1 minute	2 minutes	5 minutes	15 minutes
GARCH Daily	17.6	17.6	17.6	17.6
Daily Absolute Returns	12.9	12.9	12.9	12.9
Realized volatility	13.9	13.3	12.7	11.75
Fourier	17.2	14.9	13.9	12.1
Panel B: Silver Futures				
V-Measure	Time Frequency			
	1 minute	2 minutes	5 minutes	15 minutes
GARCH Daily	27.4	27.4	27.4	27.4
Daily Absolute Returns	19.9	19.9	19.9	19.9
Realized volatility	25.4	23.6	21.7	19.23
Fourier	35.5	28.9	24.3	20.25
Panel C: Copper Futures				
V-Measure	Time Frequency			
	1 minute	2 minutes	5 minutes	15 minutes
GARCH Daily	27	27	27	27
Daily Absolute Returns	20.3	20.3	20.3	20.3
Realized volatility	20.8	19.7	18.8	17.5
Fourier	37.8	29.7	25.6	20.47

Table 2.3 Heteroscedastic RMSE and Logarithmic Loss Function Using 1-Day-Ahead Forecast for Gold Futures

	Heteroscedastic RMSE			Logarithmic Loss Function		
Measures	Daily Absolute Returns	Realized Volatility	Fourier Method	Daily Absolute Returns	Realized Volatility	Fourier Method
Panel (A) 1 minute						
1999-2004	195.26	1.0851	0.5994	1.55	0.4348	-0.0654
2005-2008	802.09	4.7632	1.4452	1.46	0.6833	0.443
1999-2008	481.4	3.0427	0.9554	1.53	0.5451	0.1495
Panel (B) 2 minutes						
1999-2004	195.2	1.5187	0.9498	1.55	0.59	0.3259
2005-2008	802	5.2521	1.8581	1.46	0.7555	0.5833
1999-2008	481.4	3.4227	1.3637	1.53	0.667	0.4398
Panel (C) 5 minutes						
1999-2004	195.2	2.3097	1.4987	1.55	0.7519	0.5603
2005-2008	802	5.7659	1.7834	1.46	0.8305	0.57
1999-2008	481.4	3.9674	1.7041	1.53	0.7939	0.5746
Panel (D) 15 minutes						
1999-2004	195.2	3.8824	3.5428	1.55	0.9789	0.9261
2005-2008	802	10.9251	6.1939	1.46	0.9796	0.9281
1999-2008	481.4	7.3583	4.6696	1.53	0.9896	0.9373

Table 2.4 Heteroscedastic RMSE and Logarithmic Loss Function

Using 1-Day -Ahead Forecast for Silver Futures

	Heteroscedastic RMSE			Logarithmic Loss Function		
Measures	Daily Absolute Returns	Realized Volatility	Fourier Method	Daily Absolute Returns	Realized Volatility	Fourier Method
Panel (A) 1 minute						
1999-2004	69.3	0.5691	0.669	1.3262	-0.2428	-1.0401
2005-2008	406.31	52.7703	1.0708	1.428	0.5595	0.3047
1999-2008	289.33	39.7841	0.8327	1.3743	0.0863	-0.4922
Panel (B) 2 minutes						
1999-2004	69.3	0.732	0.6835	1.3262	-0.0261	-0.5371
2005-2008	406.31	52.7216	1.4576	1.428	0.655	0.4887
1999-2008	289.33	39.7644	1.0453	1.3743	0.2543	-0.1175
Panel (C) 5 minutes						
1999-2004	69.3	1.2049	0.8677	1.3262	0.2584	0.0366
2005-2008	406.31	52.7888	1.8335	1.428	0.7682	0.559
1999-2008	289.33	39.8104	1.3647	1.3743	0.4701	0.2538
Panel (D) 15 minutes						
1999-2004	69.3	2.503	2.2176	1.3262	0.6395	0.5577
2005-2008	406.31	53.2035	4.9872	1.428	0.9729	0.8787
1999-2008	289.33	40.0668	3.7177	1.3743	0.7803	0.6939

Table 2.5 Heteroscedastic RMSE and Logarithmic Loss Function Using 1-Day- Ahead Forecast for Copper Futures

	Heteroscedastic RMSE			Logarithmic Loss Function		
Measures	Daily Absolute Returns	Realized Volatility	Fourier Method	Daily Absolute Returns	Realized Volatility	Fourier Method
Panel (A) 1 minutes						
1999-2004	169.99	0.8365	0.4389	1.34	0.2928	-0.3093
2005-2008	1768	8.8272	1.2373	1.3931	0.8711	-0.4677
1999-2008	1211	5.8537	0.8213	1.383	0.5424	-0.3552
Panel (B) 2 minutes						
1999-2004	169.99	1.0698	0.6549	1.34	0.4396	0.1049
2005-2008	1768	8.8701	1.4805	1.3931	0.9242	-0.0885
1999-2008	1211	5.9056	1.05	1.383	0.6516	0.0449
Panel (C) 5 minutes						
1999-2004	169.99	1.5662	0.8273	1.34	0.6047	0.3393
2005-2008	1768	9.0687	1.2099	1.3931	0.9806	0.0881
1999-2008	1211	6.079	1.014	1.383	0.7731	0.2562
Panel (D) 15 minutes						
1999-2004	169.99	2.9672	1.698	1.34	0.8391	0.6334
2005-2008	1768	11.8465	2.0027	1.3931	1.0924	0.6085
1999-2008	1211	7.6557	1.8483	1.383	0.9584	0.641

Figure 2.1 Daily Returns Autocorrelation for Gold Futures

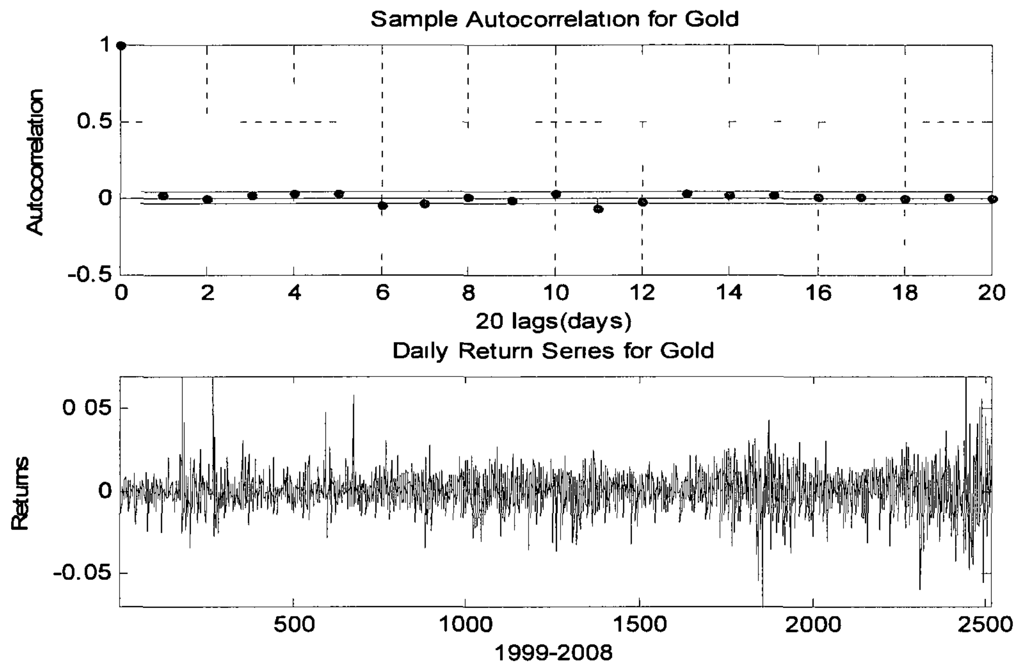


Figure 2.2 Daily Returns Autocorrelation for Silver Futures

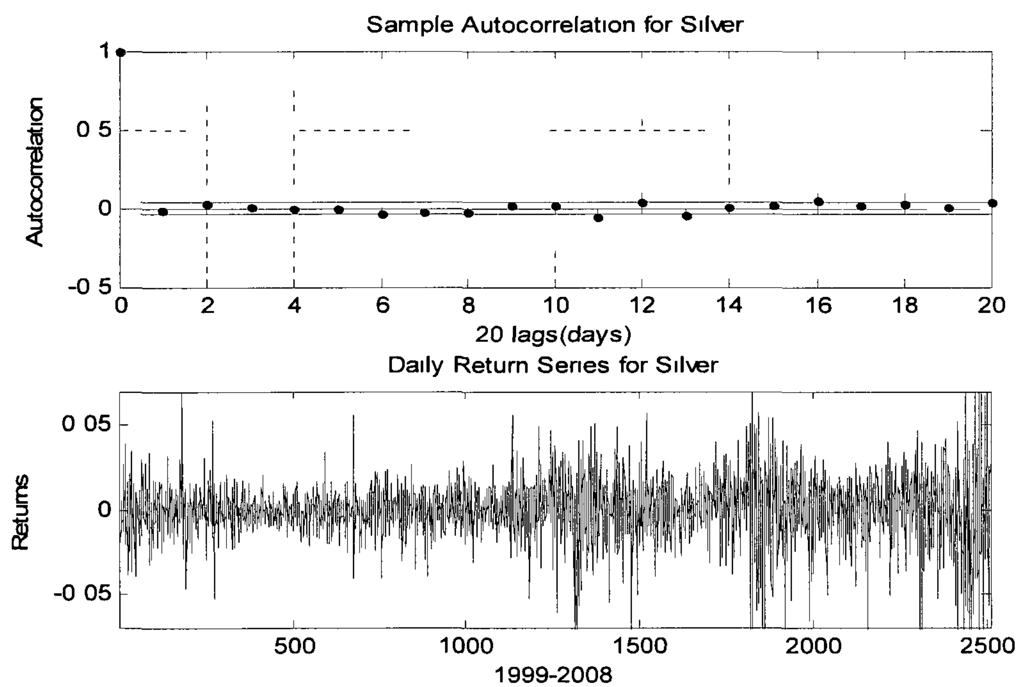


Figure 2.3 Daily Returns Autocorrelation for Copper Futures

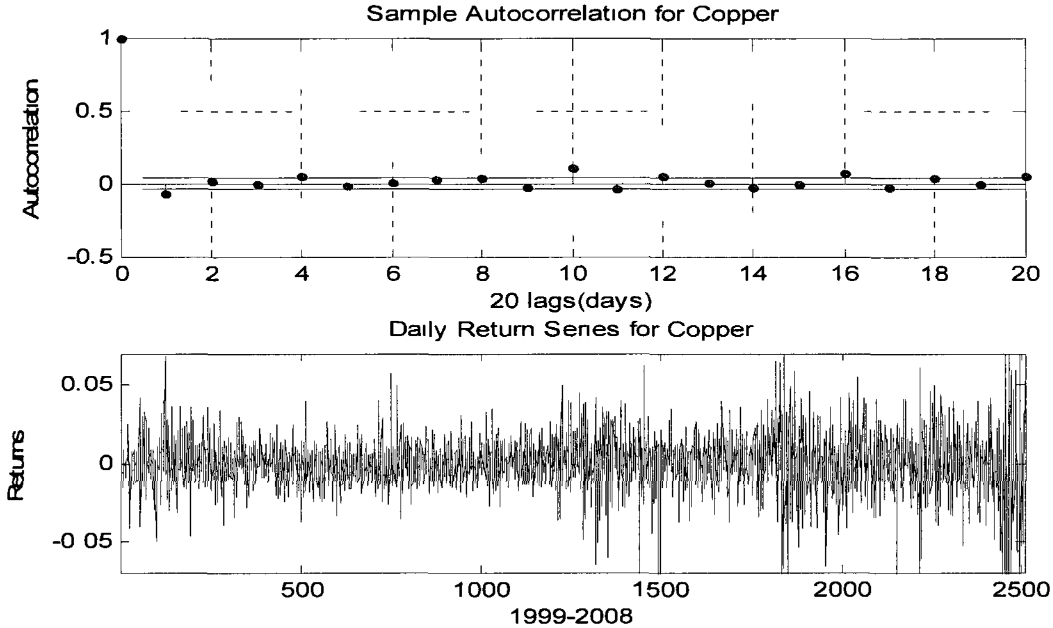


Figure 2.4 Daily Volatility Autocorrelation for Gold Futures

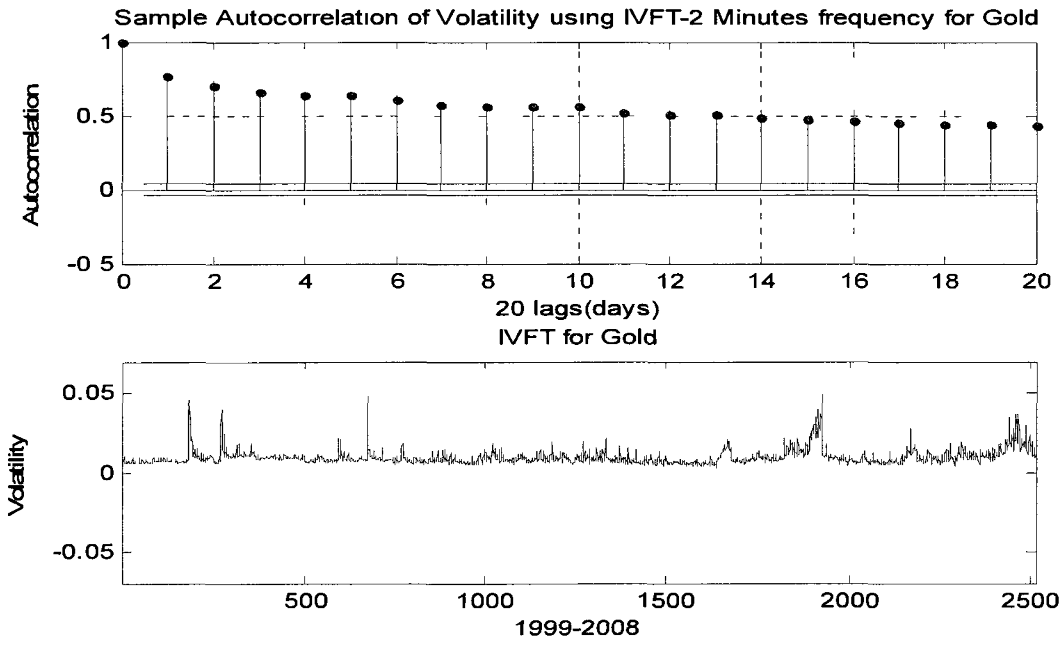


Figure 2.5 Daily Volatility Autocorrelation for Silver Futures

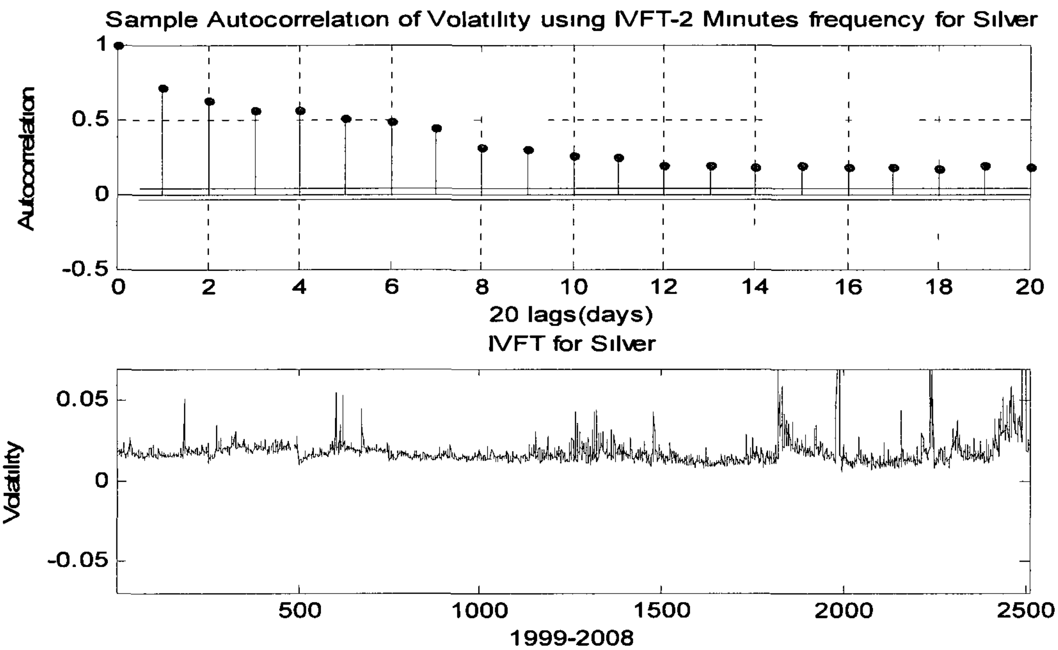


Figure 2.6 Daily Volatility Autocorrelation for Copper Futures

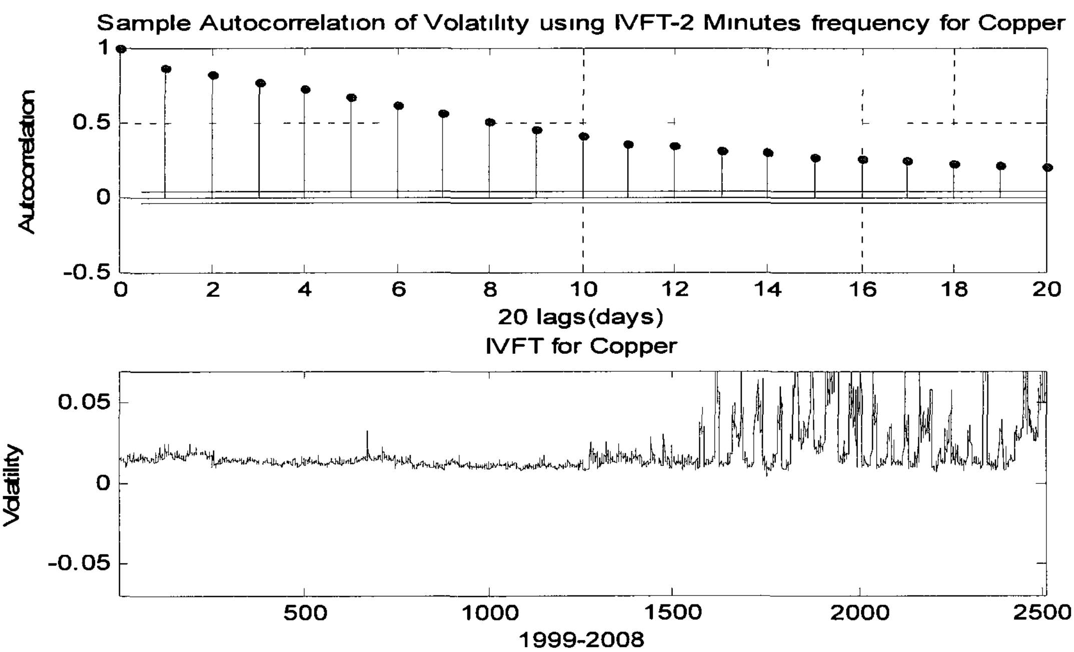


Figure 2.7 Daily Returns Distribution of the Metal Futures Markets (Gold-Silver-Copper)

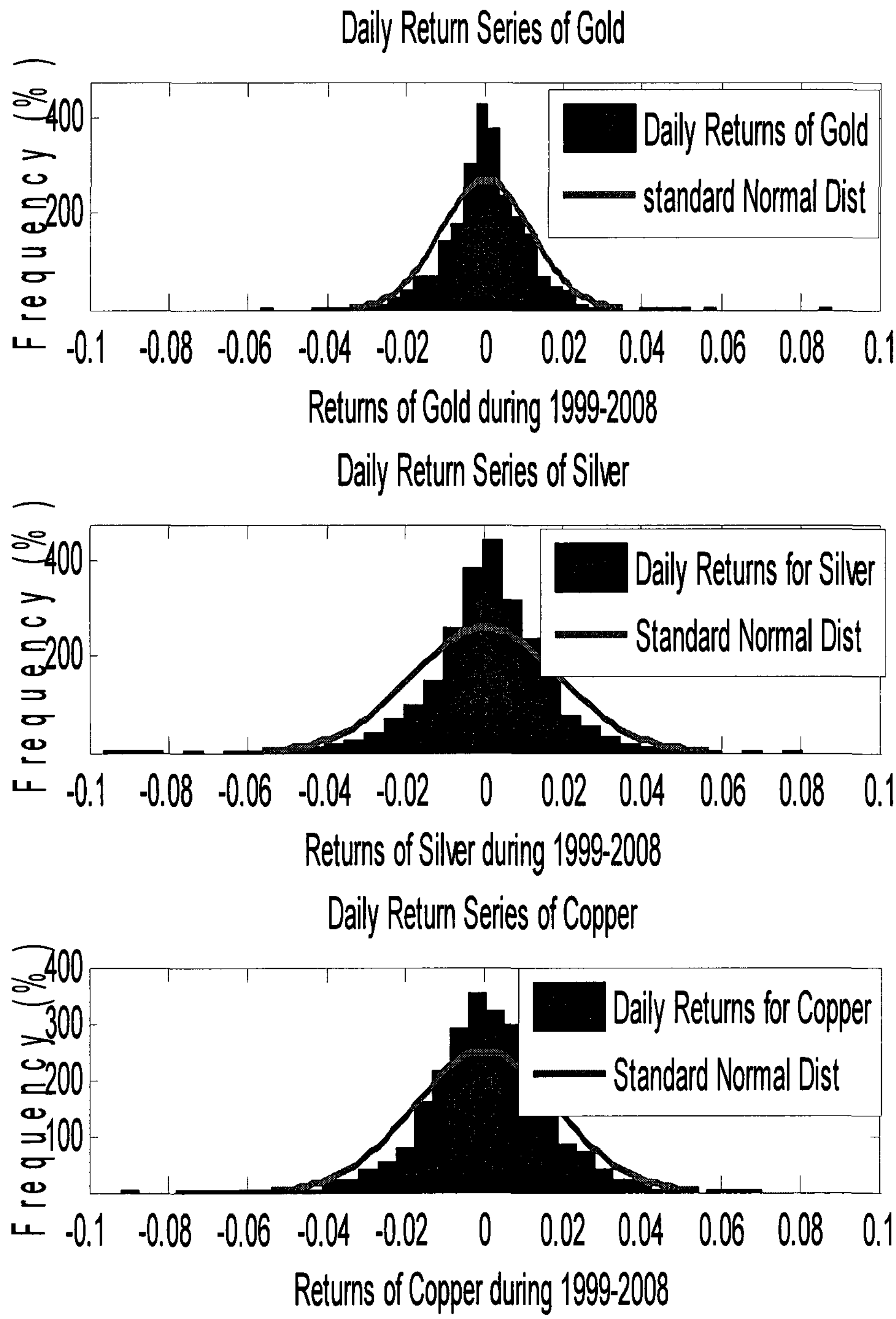


Figure 2.8 Integrated Daily Volatility (Three Measures) vs. GARCH (Gold Futures)

The dotted line is the GARCH benchmark forecast of daily volatility. The continuous lines are the volatility measures. (Daily Absolute Returns, Realized Volatility and IVFT).

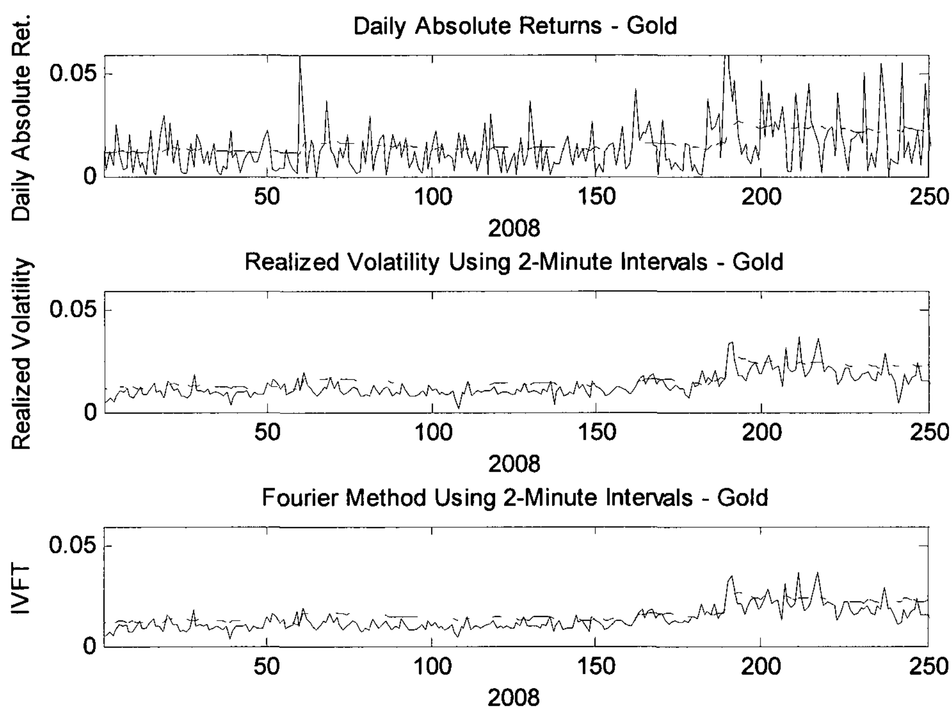


Figure 2.9 Integrated Daily Volatility (Three Measures) vs. GARCH (Silver Futures)

The dotted line is the GARCH benchmark forecast of daily volatility. The continuous lines are the volatility measures. (Daily Absolute Returns, Realized Volatility and IVFT).

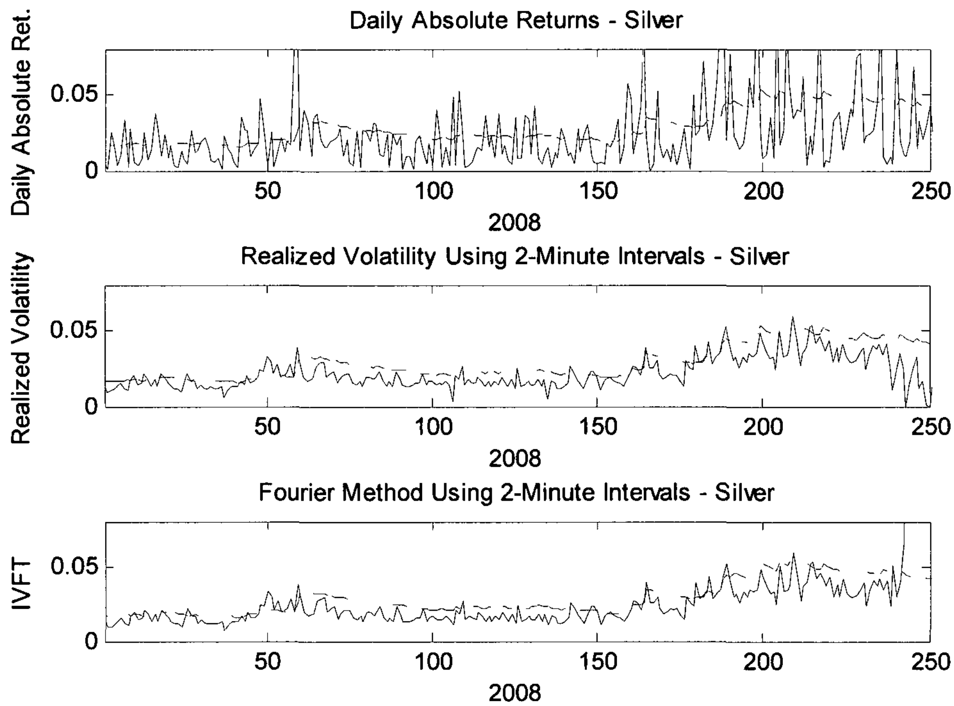
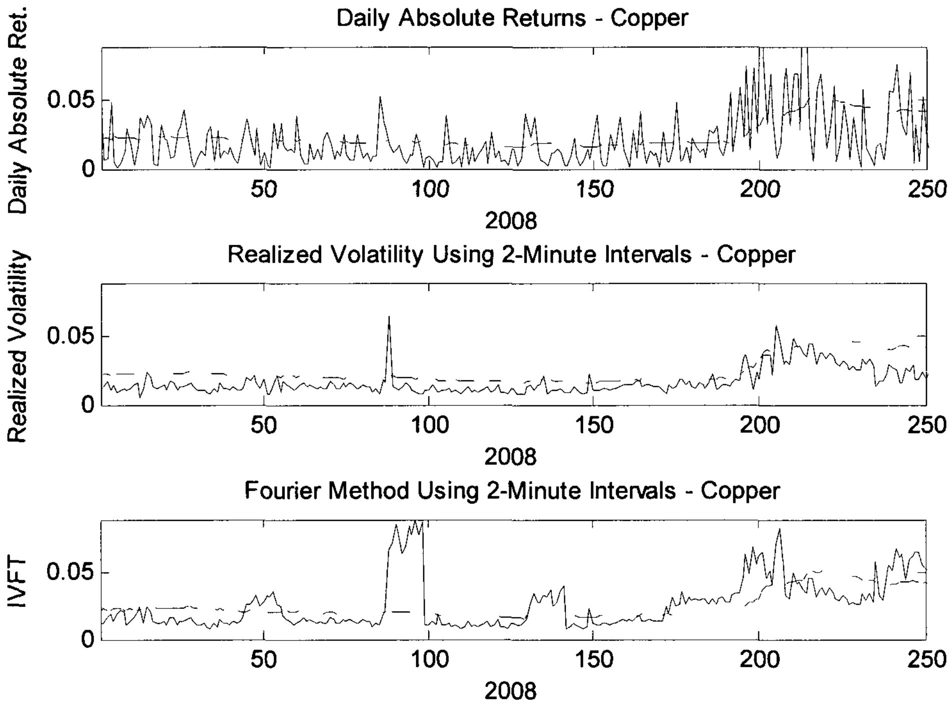


Figure 2.10 Integrated Daily Volatility (Three Measures) vs. GARCH (Copper Futures)

The dotted line is the GARCH benchmark forecast of daily volatility. The continuous lines are the volatility measures. (Daily Absolute Returns, Realized Volatility and IVFT).



Chapter 3

The Transmission of Volatility in Metal Futures Markets

In the previous chapter, we compared the three main approaches of estimating integrated volatility. We found that the integrated volatility via Fourier transformation approach gives better estimates of integrated volatility and it also fits GARCH forecasts of volatility better than the other two approaches. In this chapter, we will investigate the dynamic relationship among the return volatilities of gold, silver and copper futures. The characteristics of volatility in these individual markets along with transmission or spillover mechanisms across markets will carry important implications for hedging, risk management issues such as diversification, the forecasting of returns, and derivative pricing.

3.1. Literature Review

The empirical studies examining the dynamic relationship among the metal markets can be broadly classified along two lines. The first strand of literature focuses on the integration of the metal markets using cointegration and VAR-based models. Chan and Mountain (1988) use daily data for the period 1980 until 1983 to test the causal relationship among the spot prices of gold, silver and interest rates. Using Akaike's final prediction error and Schwarz's Bayesian information criterion test statistics, they find that the changes in silver prices exert a causal influence on the spot prices of gold and that treasury-bill rates exert instantaneous causal influence on the spot prices of silver, while the spot prices of gold and silver have no influence on treasury-bill rates.

Escribano and Granger (1998) use monthly data from September 1971 to March 1990 to study the long-run relationship between gold and silver prices. They find clear and strong evidence of a simultaneous relationship between the returns of gold and silver (see also Honga et.al., 2007). Ciner (2001) examines the cointegration between the two markets (gold and silver), using a data set that starts at the first trading day of 1992 and runs through the last trading day of 1998. He concludes that the stable relationship between gold and silver prices disappeared during the 1990s.

In contrast, Liu and Chou (2003) use daily prices from January 1983 through July 1995 for gold and silver futures contracts traded on COMEX, and the corresponding cash prices for the two metal markets. Using fractional cointegration analysis, they show that gold-silver and silver-gold parities are slow-adjusting, long-memory processes with a time-varying risk premium, and support the importance of information in relatively long-run spread trading in the precious metal markets. Results from the error correction model indicate that riskless profit can be generated by correctly forecasting the spread between the futures and spot prices of gold and silver.

Adrangi et.al. (2000) conduct a detailed study of gold and silver futures markets using 15-minute intraday data between late December 1993 and late December 1995 (11,979 observations). They analyze the price discovery process among the strategically linked gold and silver futures contracts and examine the role of the intermarket spread in their price dynamics. They use a multivariate VAR model that allows for intermarket volatility spillover and asymmetric-spread effects on the variance and covariance of the two contracts. Their analysis suggests that the silver contract bears the majority of the burden of convergence to the gold-silver spread. This is noteworthy considering that the silver contract was by far the more volatile of the two contracts over the period studied. However, their study does not distinguish between dependence, spillover effect, interdependence or long-run relationship between the spot prices of the two metal markets.

The second strand of literature examines the persistence of volatility originating from exogenous shocks on different metal markets. These studies use ARCH and GARCH type models to examine how shocks persist over time and across markets, and how exogenous factors affect volatility across markets. Akgiray, et al. (1991) use daily spot prices of the two metal markets for the time period 1975 through 1986. For the purpose of investigating the stochastic properties of the time series, they classify the whole period into three equal sub periods. They find that the price series exhibit time dependency and that GARCH effects persist even after splitting the data into various sub-periods. In addition, they find that the power exponential distribution accurately portrays the thick-tailed conditional variance which remains after the GARCH effect was removed. Notably the authors conclude that constant variance pricing models are inappropriate for the precious metal markets.

Batten and Lucey (2007) and Tully and Lucey (2005) examine the conditional and unconditional daily mean-return variance estimated from spot prices for gold and silver contracts during the period 1982 to 2002. In particular, they focus on whether there exists a detectable daily seasonality pattern in the first and second moments. They use COMEX cash and futures data, and find that under both parametric and nonparametric analysis the evidence of daily seasonality is weak for the mean and strong for the variance. They show that a negative Monday effect appears in both gold and silver across cash and futures markets. They note that when the mean and variance are analyzed simultaneously in a GARCH framework, a leveraged GARCH model provides the best fit for the data. They do not find any evidence of ARCH-in-mean effect; that is, there is no long memory in the two metal markets.

Xu and Fung (2005) use a bivariate asymmetric GARCH model to examine patterns of cross-market information flows for gold, platinum and silver futures contracts traded in both the U.S. and Japanese markets. Daily futures closing prices are used in their estimation and the study period runs from November 1994 until March 2001. Their results indicate that price transmission is strong across the two markets, but information flows appear to lead from the U.S. market to the Japanese market in terms of returns; i.e., there are strong volatility spillover feedback effects across both markets and their impacts appear to be comparable and similar. They also find that intraday pricing information transmission across the two precious metal futures markets is rapid, as offshore trading information can be absorbed in the domestic market within a trading day.

In conclusion, the existing literature does not fully account for the complexity involved in the estimation and transmission of volatility. For instance, previous intraday studies such as Cai, et.al. (2001) and Adrangi et.al. (2000) focus on realized volatility. Other studies using daily spot prices, such as, Xu and Fung (2005), Liu and Chou (2003) and Akgiray, et. al. (1991), or monthly data, such as Escribano and Granger (1998), ignore intraday information. Consequently, there is possibility of noise and biased estimation of the inter-market relations. As mentioned in Chapter 2, the volatility measures used in intraday studies, daily studies or monthly studies have been shown to be less efficient in forecasting and hence in estimating volatility transmission across metal markets. Furthermore, in testing volatility transmission across markets, prior studies fail to account for the presence of different regimes, to allow for the dominant market's having an asymmetric impact on other markets, and to distinguish spillover effect, interdependence and comovement in the long run in the spot and future prices of the metal markets.

We apply the multi-chain Markov switching (MCMS) model to study the integrated volatilities of the returns of gold, silver and copper futures estimated by IVFT. The MCMS model, introduced by Otranto (2005) and later applied by Gallo and Otranto (2007), inserts asymmetries to make the transition probabilities of each market dependent on its own state and those of other markets. Gallo and Otranto (2007) document that the MCMS model has better forecasting performance relative to other existing models.

Most notably, the MCMS model can distinguish inter-market linkages such as spillovers, interdependencies and comovements. Volatility spillover is defined as a situation in which a switch in the regime of a dominating market leads to a change in the regime of the dominated market with a lag. In contrast, interdependence of volatility is seen as a situation in which a switch in the regime of one of the markets leads to changes in regime of other markets. Volatility comovement, on the other hand, is a contemporaneous change in regimes across markets.

3.2. Methodology

3.2.1. Multi-Chain Markov Switching Model

Suppose we have the integrated volatility series of n markets in a time interval $[0, T]$, $y_{i,j,t}\ for\ (1 \leq i \leq n),\ (1 \leq j \leq q)\ and (0 \leq t \leq T)$, where i, j and t refer to the market, state and time, respectively[1]. We define an n-dimension vector $Y_t \equiv (y_{1,j,t}, y_{2,j,t}, \dots, y_{n,j,t})'$ representing the integrated volatility estimates at time t and assume that Y_t follows a VAR (p) process, where p refers to the number of lags in the model with state dependent parameters:

$$y_t = \mu(s_t) + \sum_{i=1}^{p} \Phi_i(s_t) y_{t-i} + \epsilon_t$$
$$\epsilon_t \sim N\big(0, \Sigma(s_t)\big). \tag{3-1}$$

We assume the structure of the covariance matrix as follows:

$$\sum(S_{1\,t}, S_{2\,t}) = \begin{bmatrix} \sigma_1^2(S_{1\,t}, .) & \rho(S_{1\,t}, S_{2\,t})\sigma_1(S_{1\,t}, .)\sigma_2(., S_{2\,t}) \\ \rho(S_{1\,t}, S_{2\,t})\sigma_1(S_{1\,t}, .)\sigma_2(., S_{2\,t}) & \sigma_2^2(., S_{2\,t}) \end{bmatrix} \tag{3-2}$$

Here the parameters of the conditional mean, $\mu(s_t)$, and $\Phi_m(s_t), 1 \leq m \leq p$ as well as the variance-covariance matrix of the error terms, ϵ_t all depend on the state vector $s_t = (s_{1,t}, s_{2,t}, \dots, s_{q,t})'$ with $s_{j,t}, 1 \leq j \leq q$ presenting the state associated with variable $y_{i,j,t}$ and where each state can have q regimes. In the variance-covariance matrix, the

[1] This chapter is using y_t as a symbol of volatility at day t for metal futures market, which is same as σ_t mentioned in the previous chapter for two reason First y_t is used to avoid confusion with the unconditional standard deviation in the variance covariance matrix result and known in the econometrics literature as σ The second reason is, the estimation symbols are consistent with the VAR(P) symbols that are traditionally used in the econometrics time series literature

variances of each variable (related to fourth moments of returns, which we assume to exist) depend only on the variable's own state. $\rho(S_{1,t}, S_{2,t})$ refers to the correlation coefficients between the two markets at state j. These correlation coefficients vary between negative to positive one. This specification implies that volatility is transmitted from one market to another, also causing some changes in the covariance structure, whereas the changes or movements in volatility depend solely on the own state. $\sigma_1^2(s_{1,t},.)$ is the variance at the state of market 1 and the state of the other market. The same interpretation can be extended to each symbol.

The main difference between the multi-chain Markov switching model and the classical Markov process is that the variables $y_{i,j,t}, 1 \leq j \leq q$ depend on separate but potentially related state variables. That is, the state of one variable $y_{j,t}$, can be made to depend on the lagged states of all variables under consideration$y_{j,t}, 0 < j \leq n$. The transition probability matrix therefore captures the volatility transmission mechanism among variables because the change in the state of one variable can be transmitted to all the others. We can specify the transition probability matrix by assuming one variable as dominant and making the switching dynamics asymmetric, which will be useful in investigating volatility spillover phenomena in metal futures markets. The MCMS model can also be modified to examine the independence of variables.

To illustrate how the MCMS model can be applied to investigate volatility transmission among assets, we consider a two-asset, two-state and two-lag model (i.e., $n = p = q = 2$). The state vector s_t can take one of four possible values: $(0,0)', (0,1)', (1,0)', or\ (1,1)'$, at anytime$0 \leq t \leq T$. These possible values can be considered as a high volatility state (state 1) or a low volatility state (state 0). Consequently, $s_t = (1,0)'$ means that asset one is at a high volatility state, whereas asset two is at the low volatility state. The transition probability matrix $P = \{\Pr[s_t|s_{t-1}]\}$ is a 4×4 matrix. We further suppose that conditional on $(s_{1,t-1}, s_{2,t-1})$, the states $s_{1,t}$, and $s_{2,t}$ are independent, that is:

$$\Pr[s_{1,t}, s_{2,t}|s_{1,t-1}, s_{2,t-1}] = \Pr[s_{1,t}|s_{1,t-1}, s_{2,t-1}] \times \Pr[s_{2,t}|s_{1,t-1}, s_{2,t-1}]. \quad (3\text{-}3)$$

We can parameterize the right hand side of equation (3.3) with logistic functions where the function explicitly depends on past states[2]:

$$\begin{aligned}
\Pr[s_{1,t} = h|s_{1,t-1} = h, s_{2,t-1}] &= \frac{\exp[\alpha_1(h,.) + \beta_1(h,1)s_{2,t-1}]}{1 + \exp[\alpha_1(h,.) + \beta_1(h,1)s_{2,t-1}]} \\
\Pr[s_{2,t} = h|s_{1,t-1}, s_{2,t-1} = h] &= \frac{\exp[\alpha_2(.,h) + \beta_2(1,h)s_{1,t-1}]}{1 + \exp[\alpha_2(.,h) + \beta_2(1,h)s_{1,t-1}]} \quad (3\text{-}4)
\end{aligned}$$

For $h = 0{,}1$. Here, $\alpha_1(h,.)$, $\alpha_2(.,h)$, $\beta_1(h,1)$, and $\beta_2(1,h)$ are parameters to be estimated. The transition probability matrix makes the probability of staying at the same state for asset *i* conditional on the previous states of both assets. Since each asset has only two states, the probabilities of switching to another state can be estimated by the following equation.

$$\Pr[s_{j,t} = k|s_{j,t-1} = h, s_{i,t-1}] = 1 - \Pr[s_{j,t} = h|s_{j,t-1}, s_{i,t-1}]. \quad (3\text{-}5)$$

for $h, k = 0{,}1, h \neq k$, and $i, j = 1{,}2, i \neq j$.

And the transition matrix will be as follows:

[2] See Gallo and Otranto (2008) for additional details.

$$\begin{bmatrix} P(00|00) & \cdots & P(11|00) \\ \vdots & \ddots & \vdots \\ P(00|11) & \cdots & P(11|11) \end{bmatrix} \qquad (3\text{-}6)$$

Now we have a system of equations (3.1, 3.2 and 3.6) that can be estimated simultaneously in order to investigate the volatility dependence structure. Various hypothesis tests can be performed on the estimated model. The main advantage of the MCMS model is that it can divide the volatility dependence relationships into four different types: independence, interdependence, spillover and comovement. The statistical significance of the state parameters in (3.4) $(\beta_1(h,1)s_{2,t-1}, \beta_2(1,h)s_{1,t-1})$, supports the case of interdependence. On the other hand, if the coefficients are insignificant, this would imply that the two assets are independent. If the coefficients $\beta_j(h,k)\ for\ k,h = 0,1$ are not significantly different from zero, then the MCMS model indicates a case of spillover from variable i to variable j. The case of comovement corresponds to the case in which the state of the first market and the second market are contemporaneously the same for every time period t. As Gallo and Otranto (2007) show, this condition leads to four restrictions on the parameters $\alpha_1(h,.)$, $\alpha_2(.,h)$, $\beta_1(h,1)s_{2,t-1}$, and $\beta_2(1,h)s_{1,t-1}$ to be jointly tested as described below in hypothesis 10. The case of comovement corresponds to the case in which the state of the first market and the second market is the same for each t; this situation can justify the adoption of the classical Markov switching model and the four constraints to be verified are:

$$\Pr[s_{1,t}=0,s_{2,t}=1|s_{1,t-1}=0,s_{2,t-1}=0]=\Pr[s_{1,t}=1,s_{2,t}|s_{1,t-1}=0,s_{2,t-1}=0]$$
$$\Pr[s_{1,t}=0,s_{2,t}=1|s_{1,t-1}=0,s_{2,t-1}=0]=\Pr[s_{1,t}=1,s_{2,t}|s_{1,t-1}=0,s_{2,t-1}=0]$$
$$\Pr[s_{1,t}=0,s_{2,t}=1|s_{1,t-1}=1,s_{2,t-1}=0]=\Pr[s_{1,t}=1,s_{2,t}|s_{1,t-1}=1,s_{2,t-1}=0] \quad (3\text{-}7)$$
$$\Pr[s_{1,t}=0,s_{2,t}=1|s_{1,t-1}=1,s_{2,t-1}=1]=\Pr[s_{1,t}=1,s_{2,t}|s_{1,t-1}=1,s_{2,t-1}=1]$$

3.2.2.Study Hypotheses

After estimating the model, we test the following hypotheses to evaluate the nature of the dependency between two markets at a time.

State Dependence in Volatility (equation 3-1)

1- No dependence of the intercept of y_1 on the state of y_2:

$$H_0: \mu_1(0{,}0) = \mu_1(0{,}1) \;\; and \; \mu_1(1{,}0) = \mu_1(1{,}1)$$

2- No dependence of the intercept of $y_2$on the state of y_1:

$$H_0: \mu_2(0{,}0) = \mu_2(1{,}0) \;\; and \; \mu_2(0{,}1) = \mu_2(1{,}1)$$

Dynamic Dependence in Volatility (equation 3-1)

3- y_2 does not linearly Granger cause y_1 (Note that there is no impact of the states in the VAR, hence, the hypothesis testing is the standard VAR):

$$H_0: \emptyset_{12}^1 = \emptyset_{12}^2 = 0$$

4- y_1 does not linearly Granger cause y_2 (Note that there is no impact of the states in the VAR, hence, the hypothesis testing is the standard VAR):

$$H_0: \emptyset_{21}^1 = \emptyset_{21}^2 = 0$$

State Dependence in Correlations (variance-covariance matrix) (equation 3-2)

5- No dependence in the correlation on the state of y_2:

$$H_0: \rho(0,0) = \rho(0,1) \; and \; \rho(1,0) = \rho(1,1)$$

6- No dependence in the correlation on the state of y_1:

$$H_0: \rho(0,0) = \rho(1,1) \; and \; \rho(0,1) = \rho(1,1)$$

Characterization of Market Dependence that distinguishes various volatility transmission mechanisms (set of equations 3-4)

7- No spillover effect from y_2 to y_1:

$$H_0: \beta_1(0,1) = \beta_1(1,1) = 0$$

8- No spillover effect from y_1 to y_2:

$$H_0: \beta_2(1,0) = \beta_2(1,1) = 0$$

9- No interdependence (no reciprocal spillover):

$$H_0: \beta_1(0,1) = \beta_1(1,1) = \beta_2(1,0) = \beta_2(1,1) = 0$$

10- Comovement between y_1 and y_2:

$$H_0: \begin{cases} \alpha_1(0,.) = \alpha_2(.,0) \\ \alpha_1(0,.) + \beta_1(0,1) + \alpha_2(.,1) = 0 \\ \alpha_1(1,.) + \beta_2(1,0) + \alpha_2(.,0) = 0 \; and \\ \alpha_1(1,.) + \beta_1(1,1) = \alpha_2(.,1) + \beta_2(1,1) \end{cases}$$

Global Causality (equation 3-1 and 3-4)

11- y_2 does not cause y_1

$$H_0: \emptyset_{12}^1 = \emptyset_{12}^2 = 0 \;\; and \; \beta_1(0,1) = \beta_1(1,1) = 0$$

12- y_1 does not cause y_2

$$H_0: \emptyset_{21}^1 = \emptyset_{21}^2 = 0 \;\; and \; \beta_2(1,0) = \beta_2(1,1) = 0$$

All of these hypotheses are tested using the classical Wald statistics.

3.3. Empirical Results

In order to estimate the volatility transmission for two assets at a time, we run three pair-wise MCMS models between the volatility series of (a) gold y_g and silver y_s, (b) gold y_g and copper y_c, and (c) silver y_s and copper y_c.

The volatility transmission effects are examined by using the daily integrated volatility estimated by the IVFT method with 1 minute interval. The estimated parameters are reported in Table 3.1 to Table 3.4.

3.3.1. *Interpreting the Estimated Coefficients*

Further explanation of the following symbols may help in interpreting the estimated coefficients. In the tables and the discussion that follow, the symbols g, *s*, and *c* are used to denote the gold, silver, and copper futures markets, respectively. $S_{t-1} = 0$ is the lagged low volatility regime and $S_{t-1} = 1$ is the lagged high volatility regime (Note that S stands for state or regime). In the symbol $\emptyset_{ij}^{i}$, the superscript is the number of lags in the VAR(p); for example, $\emptyset_{sg}^{lag1}$, the subscript lag1 is the first lag (t-1), s in the symbol stands for the silver futures market and subscript g for the gold futures market. For example, $\emptyset_{sg}^{lag1}$ measures the impact of the gold futures market at (t-1) on the current volatility of the silver futures market. In the case of $\mu_s(0,0)$, $\sigma_s(0,.)$, the subscript refers to the silver futures market; the first number in the parenthesis is the lagged regime for the first market and the second number in the parentheses is the lagged regime for the second market.

According to estimated MCMS model (equation 3-1 through 3-6) and the symbols explained above, the coefficients can be classified into four groups. The first group of coefficients is obtained by estimating equation (3-1), including the mean of volatility of each market, conditional on the lagged states of the two markets.

The first group of the coefficients for the silver-gold futures pair is summarized in Table 3.1, Panel A. The $\mu_s(0,0)$ shows that the average volatility of silver futures conditional on both gold and silver experiencing low volatility regimes during the most recent period is 16%, and it is significantly different from zero at the 1% level. $\mu_s(0,1)$, the average volatility of the silver futures markets conditional on silver futures at the lagged low regime and gold futures at the lagged high regime, is 16.8 %. This is significantly different from zero at the 1% level. The same interpretation can be extended to $\mu_s(1,0)$ and $\mu_s(1,1)$, where the first element is the lagged state in the silver futures market and the second element is the lagged state of the second market (gold futures).

With respect to the gold futures equation, the $\mu_g(1,1)$ coefficient is 34%, which means that the average volatility of gold conditional on both gold and silver experiencing high volatility regimes during the most recent period is 34%, which is significantly different from zero at the 1% significance level. The coefficient $\mu_g(0,1)$shows that the average volatility of gold futures at the lagged low regime of silver futures and lagged high regime of gold futures is 15 %, which is statistically different from zero at 5% significance level.

The conclusion from Table 3.1, panel A, is that silver futures have the highest volatility when silver futures and gold futures are at lagged high regime; whereas gold has the highest volatility when both gold and silver are at lagged high regime. Moreover, silver and gold futures, each has the lowest volatility when both markets are at lagged low volatility

The second group of coefficients is shown in Table 3.1, panel B. This table summarizes the impact of lagged volatility of silver futures and gold futures on current volatility in the silver futures and gold futures markets. In $\emptyset_{sg}^{lag1}$, the superscript *lag1* indicates the first lagged value, the subscript *s* reflects the silver market, and the subscript *g* indicates the gold futures market. Consequently, these coefficients are independent of the states and carry the same interpretation as the traditional VAR model. For example, $\emptyset_{ss}^{lag1}$ shows the impact of the volatility of silver futures at t-1 on the volatility of silver futures at time t, which is 0.53, and it is statistically significant at 1% significance level. $\emptyset_{sg}^{lag1}$ is 0.008, measuring the impact of volatility of gold futures at t-1 on the current volatility of silver. However, this coefficient is insignificant at 5% significance level. The same interpretation can be extended to the coefficients of the traditional VAR. The coefficient $\emptyset_{gg}^{lag2}$ from the gold futures equation, 0.3, reflects the impact of the volatility of gold futures at t-2 on the current volatility of gold futures, which is statistically significant at 1% significance level. The same interpretation can be extended to the other coefficients in this group. To determine linear Granger causality between the two markets, we use $\emptyset_{sg}^{lag1}, \emptyset_{sg}^{lag2}, \emptyset_{gs}^{lag1}, \emptyset_{gg}^{lag2}$. For example, the coefficient $\emptyset_{gs}^{lag2}$ equals 0.012 with a standard error of 0.005, and it is significantly different from zero at the 1% level. This means that silver futures linearly Granger cause gold futures.

In summary, from the Table 3.1, panel B, the silver futures equation shows that the first lag of silver futures has a greater impact on the volatility of silver futures compared to the second lag, and it is statistically significant. On the other hand, the second lag of gold futures has a greater magnitude, but it is not statistically significant on silver futures compared to the impact of the first lag of volatility of gold futures on silver futures. With respect to the gold futures equation, the first lag of gold has a greater impact on the current volatility of gold futures compared to the second lag. However, the second lag of silver futures' volatility has a greater impact on the current volatility of gold futures, and it is statistically significant compared to the impact of the first lag of silver futures on the current volatility of gold futures. This result has significant implications for traders and financial practitioners in the options and futures options sector of financial markets. If they are trading in one of the metal markets by determining the volatility and prices of gold future using the current volatility of silver futures, they may be able to carry out their forecasting two days in advance

The third group of coefficients is the variance-covariance matrix shown in Table 3.1, panel C. $\sigma_s(0,.)$ is 0.105. This symbol represents the square root of the variance of volatility of silver futures at the lagged low regime regardless of the lagged regime of the gold market, and it is statistically significant. $\sigma_g(.,1)$ 0.25, is the square root of variance of gold futures at the lagged high regime of gold regardless of the lagged regime of silver, and it is statistically significant at 1% significance level. The interpretation of the remaining coefficients is similar. With respect to the correlation coefficient of this group, $\rho(0,.)$, the correlation coefficient of silver futures volatility to gold futures volatility at lagged low regimes for both of the two markets, is 0.4 and it is statistically significant at 1% significance level. It is 0.37 when the lagged state of silver futures is at high regime and the lagged state of gold is at the low regime, and it is statistically significant at the 5% significance level.

In summary, Table 3.1, panel C, includes silver futures and gold futures. Each has the highest standard deviation when both markets are at lagged high volatility. With respect to the switching coefficients-correlation, the interesting result is that silver futures and gold futures have the highest correlation when both markets are at lagged low volatility and both markets have no correlation when both markets are at lagged high volatility. Consequently, financial practitioners and portfolio managers have to construct their portfolios differently according to differing volatility regimes.

The fourth group of coefficients consists of the probability parameters and transition probability matrix (equations 3-2 and 3-3), summarized in Table 3.1, panels D and E. For example, $\alpha_s(0,.)$ is 2.9. This is the intercept of the logistic function (equation 3-4) for silver futures at lagged low regime of silver futures regardless of the lagged regime of gold futures, and it is statistically significant at the 1% significance level. $\beta_s(0,1), -1.0026$, measures the impact of the lagged high regime of gold on the probability of silver futures. We recall that the first element in the parenthesis reflects the low regime of the first market (silver futures) at low regime, but it is insignificant. Joint hypothesis testing of the parameters in panel (D) determines the characterization of interdependence and comovement between silver and gold futures as specified in hypotheses 7 through 10 and summarized in Table 3.4. With respect to the transition probability matrix, each element can be interpreted according to the interaction between the two states of the two markets. For example, the first element of the matrix (Table 3.1, panel D) is 87%. This means that the probability that silver will remain at low regime

$p(0,0)$ when gold is at lagged low regime $(0,0)$ is 87%. The probability that silver futures will remain at a high regime $(1,1)$ when gold futures remain at a high regime $(1,1)$ is 24%. The probability that silver futures will remain at a low regime $(0,1)$ when gold futures remain at a high regime $(0,1)$ is 50%.

In summary, Table 3.1, panels D and E, includes two important features of the probability parameters of silver futures and gold futures: first, the lagged state of each market has an insignificant impact on others; second, the transition probability is at the highest value when both of the two markets are in low volatility at current state, $\rho(0,0)$ and they change gradually from 0.87 to 0.37 to 0.4 to 0.24 with changing the lagged state from $(0,0)$ to $(0,1)$ to $(1,0)$ to$(1,1)$. The important implication of this result is that the lagged low volatility state for silver futures, high current volatility state of gold futures and lagged volatility state of gold futures are crucial in forecasting next-period volatility in both silver and gold futures markets.

Table 3.1 Estimated Parameters of the MCMS Model for Silver Futures-Gold Futures

Panel A

Switching Coefficients-Constant Term

Market	Silver Futures Equation				Gold Futures Equation			
	$\mu_s(0,0)$	$\mu_s(0,1)$	$\mu_s(1,0)$	$\mu_s(1,1)$	$\mu_g(0,0)$	$\mu_g(0,1)$	$\mu_g(1,0)$	$\mu_g(1,1)$
Coeff.	0.1677*	0.1680*	0.3045	1.1020*	0.0717*	0.0717	0.1539*	0.3417*
Standard Error	0.0153	0.0415	0.4614	0.4336	0.0140	0.0479	0.0470	0.0561

Panel B

Autoregressive Terms

Market	Silver Futures Equation				Gold Futures Equation			
	$\emptyset_{ss}^{lag1}$	$\emptyset_{sg}^{lag1}$	$\emptyset_{ss}^{lag2}$	$\emptyset_{sg}^{lag2}$	$\emptyset_{gs}^{lag1}$	$\emptyset_{gg}^{lag1}$	$\emptyset_{gs}^{lag2}$	$\emptyset_{gg}^{lag2}$
Coeff.	0.5352*	0.0088	0.3013*	0.0120	-0.0045	0.5380*	0.0115*	0.3038*
Standard Error	0.0188	0.0391	0.0092	0.0305	0.0031	0.0453	0.0048	0.0289

Panel C

	Switching Coefficients –Standard Deviation				Switching Coefficients-Correlation Terms			
Market	Silver Futures		Gold Futures					
	$\sigma_s(0,.)$	$\sigma_s(1,.)$	$\sigma_g(0,.)$	$\sigma_g(.,1)$	$\rho(0,0)$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
Coeff.	0.1058*	1.5670*	0.0531*	0.2497*	0.4180*	0.0982	0.372**	0.0000
Standard Error	0.0055	0.3568	0.0039	0.0345	0.0370	0.1598	0.220	0.0200

Panel D

Probability Parameters

Market	Silver Futures Equation			Gold Futures Equation		
	$\alpha_s(0,.)$	$\beta_s(0,1)$	$\alpha_s(1,.)$	$\beta_g(1,1)$	$\alpha_g(.,0)$	$\alpha_g(.,1)$
Coeff.	2.990*	-1.0026**	-0.2920	0.0000	2.3860*	0.2830
Standard Error	0.3019	0.5780	0.3731	0.0250	0.2930	0.3108

Panel E(Transition Probability Matrix (P 4x4))

State at t | | State at t-1	(0,0)	(0,1)	(1,0)	(1,1)
(0,0)	0.8720	0.3779	0.4061	0.2459
(0,1)	0.0801	0.5017	0.1664	0.3265
(1,0)	0.0438	0.0517	0.3032	0.1837
(1,1)	0.0040	0.0687	0.1243	0.2439

* indicates that the coefficient is significantly different from zero at the 1% level.

** indicates that the coefficient is significantly different from zero at the 5% level.

For the copper futures and gold futures equations, the interpretation of Table 3.2 is simply a repetition of the explanation of Table 3.1. We will therefore focus on the important differences and the implications of these finding.

First, the conclusion from Table 3.2, panel A, is that the copper futures and gold futures markets have the highest volatility when both gold futures and copper futures are at a lagged high regime. Moreover, copper futures and gold futures each have the lowest volatility when both markets are at lagged low volatility, or when only the copper futures market is at lagged low volatility. The interesting result derived from this panel is that the average volatility of copper futures is higher than the average volatility of gold futures, and the average volatility of copper futures increases if there is a switch from low regime to high regime in copper futures.

Second, in Table 3.2, panel B, the copper futures equation shows that the first lag of copper futures has a greater impact on the volatility of gold futures compared to the second lag of the same metal, and it is statistically significant. The first lag of gold futures has a greater impact, and it is statistically significant on copper futures compared to the impact of the second lag of volatility of gold futures on copper futures. With respect to the gold futures equation, the first lag of gold has a greater impact on the volatility of gold futures compared to the second lag. However, neither the first lag nor the second lag of copper futures' volatility has a significant impact on the current volatility of gold futures. In conclusion, Table 3.2 and Table 3.1, panel B, have a significant implication for traders and financial practitioners in the options and futures options sectors of financial markets. It is better for them predict gold futures behavior using the volatility of silver futures, but not copper futures.

Third, Table 3.2, panel C, demonstrates that the copper futures market has the highest standard deviation when the copper futures market is at lagged high regime or low regime compared to the gold futures market. Furthermore, copper and gold futures each have the highest standard deviation when both markets are at lagged high volatility. With respect to the switching coefficients-correlation, the interesting result is that copper futures and gold futures have the highest correlation when both markets are at lagged low volatility, and there is no correlation when both markets at lagged high volatility. Hence, financial practitioners and portfolio managers need to construct their portfolios differently according to volatility regimes.

Fourth, Table 3.2, panels D and E, demonstrate two important features of the probability parameters of copper futures and gold futures. First, the lagged high volatility of gold has a significant impact on copper futures but not vice versa; second, the transition probability is at the highest value when both of the two markets are in low volatility at current State, (0,0) and they change gradually from 0.86 to 0.42 to 0.17 to 0.11 with changing the lagged state from (0,0) to (0,1) to (1,0) to (1,1)Also, one of the interesting results from panel (D) is, at the current (1,0) and lagged (1,0), the probability is 0.55 and at the current (0,1) and lagged (0,1), the probability is 0.45. The important implication of this result is that the correlation between the two metal markets is increasing in two cases; the first case is at the low current and lagged low regimes of both of the two metal markets; Second, when the two metal markets are at different regimes (current and lagged states). The important implication of this result is that there is a strong correlation between copper futures and gold futures at different states.

Therefore, there is a possibility for better forecasting of the volatility of the two metal markets for next periods using the information of the states of one of the two markets.

Table 3.2 Estimated Parameters of the MCMS Model for Copper Futures-Gold Futures

Panel A								
Switching Coefficients-Constant Term								
Market	Copper Futures Equation				Gold Futures Equation			
	$\mu_c(0,0)$	$\mu_c(0,1)$	$\mu_c(1,0)$	$\mu_c(1,1)$	$\mu_g(0,0)$	$\mu_g(0,1)$	$\mu_g(1,0)$	$\mu_g(1,1)$
Coeff.	0.1223*	0.1223*	0.382*	0.8169*	0.0668*	0.0668*	0.1446*	0.22*
Standard Error	0.0146	0.0202	0.139	0.1600	0.0123	0.0250	0.0318	0.08

Panel B								
Autoregressive Terms								
Market	Copper Futures Equation				Gold Futures Equation			
	$\emptyset_{cc}^{lag1}$	$\emptyset_{cg}^{lag1}$	$\emptyset_{cc}^{lag2}$	$\emptyset_{cg}^{lag2}$	$\emptyset_{gc}^{lag1}$	$\emptyset_{gg}^{lag1}$	$\emptyset_{gc}^{lag2}$	$\emptyset_{gg}^{lag2}$
Coeff.	0.5817*	0.045**	0.2474*	0.0143	-0.0044	0.5701*	0.0016	0.3022*
Standard Error	0.0275	0.0196	0.0210	0.0153	0.0076	0.0399	0.0028	0.0274

Panel C								
	Switching coefficients –Standard Deviation				Switching Coefficients- Correlation Terms			
Market	Copper Futures		Gold Futures					
	$\sigma_c(0,.)$	$\sigma_c(1,.)$	$\sigma_g(0,.)$	$\sigma_c(.,1)$	$\rho(0,0)$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
Coeff.	0.0692*	0.9181*	0.049*	0.2460*	0.1997*	0.2699*	0.0000	0.0000
Standard Error	0.0019	0.0740	0.0034	0.0317	0.0371	0.0805	0.0061	0.0204

Panel D						
Probability Parameters						
Market	Copper Futures Equation			Gold Futures Equation		
	$\alpha_c(0,.)$	$\beta_c(0,1)$	$\alpha_c(1,.)$	$\beta_g(1,1)$	$\alpha_g(.,0)$	$\alpha_g(.,1)$
Coeff.	2.9980*	-1.0784*	1.1713*	0.0000	2.2541*	0.0667
Standard Error	0.1598	0.3978	0.1539	0.0174	0.2221	0.2196

Panel E				
Transition Probability Matrix (P 4x4)				
State at t|| State at t-1	(0,0)	(0,1)	(1,0)	(1,1)
(0,0)	0.8620	0.4215	0.1727	0.1102
(0,1)	0.0905	0.4506	0.0639	0.1264
(1,0)	0.0430	0.0618	0.5573	0.3556
(1,1)	0.0045	0.0661	0.2061	0.4078

* indicates that the coefficient is significantly different from zero at the 1% level.
** indicates that the coefficient is significantly different from zero at the 5% level.

For copper futures and silver futures, the interpretation of Table 3.3 is similar to that of Tables 3.1 and 3.2. Accordingly, we will focus on the importance differences between these metal markets and the implications of these differences.

First, Table 3.3, panel A, shows that the silver futures market has the highest volatility when both silver futures and copper futures are at lagged high regime. Moreover, copper and silver futures each have the lowest volatility when both markets are at lagged low volatility. The interesting result demonstrated by this panel is that the average volatility of silver is higher than the average volatility of copper futures, and the magnitude of this result increases if there is a switch from low regime to high regime in the copper futures.

Second, Table 3.3, panel B, shows that the first lag of copper futures has a greater impact on the volatility of silver futures compared to the second lag of the same metal, and they are statistically significant. The second lag of silver futures is insignificant on copper futures compared to the first lag of silver futures on copper futures. With respect to the silver futures equation, the first lag of silver has a greater impact on the current volatility of silver futures compared to the second lag. However, neither of the two lags of copper futures' volatility has a significant impact on the current volatility of silver futures. Neither of the volatilities of the two markets can explain the dynamic of the other's volatility.

Third, Table 3.3, panel C, indicates that the silver futures market has the highest standard deviation when the silver futures market is at lagged high regime or lagged low regime compared to the copper futures market. Moreover, copper and silver futures each have the highest standard deviation when both markets are at lagged high volatility. With respect to the switching coefficients-correlation, the interesting result is that copper futures and silver futures have the highest correlation when both markets are at lagged low volatility, and both markets have an insignificant correlation when both markets at lagged high volatility. Consequently, financial practitioners and portfolio managers must construct their portfolios differently according to volatility regimes.

Fourth, Table 3.3, panels D and E, demonstrate three important features of the probability parameters of silver futures and copper futures. (1) The lagged state of silver has a significant impact on copper futures, but not vice versa; (2) The transition probability is at the highest value when both of the two markets are in low volatility at current State, (0,0) and it changes gradually 0.90 to 0.28 to 0.22 to 0.11 with changing the lagged state from (0,0) to (0,1) to (1,0) to (1,1). Also, one of the interesting results from panel (D) is, at the current (1,0) and lagged (1,0), the probability is 0.63 and at the current (0,1) and lagged (0,1), the probability is 0.34. The important implication of this result is that both the lagged high volatility state and current low volatility state for copper futures and silver futures are crucial in forecasting the volatility of the next periods for the two markets.

Table 3.3 Estimated Parameters of the MCMS Model for Copper Futures-Silver Futures

Panel A								
Switching Coefficients-Constant Term								
Market	Copper Futures Equation				Silver Futures Equation			
	$\mu_c(0,0)$	$\mu_c(0,1)$	$\mu_c(1,0)$	$\mu_c(1,1)$	$\mu_s(0,0)$	$\mu_s(0,1)$	$\mu_s(1,0)$	$\mu_s(1,1)$
Coeff.	0.1306*	0.1306	0.4421*	0.5906*	0.2108*	0.2108	0.4519	0.729**
Standard Error	0.0104	0.1567	0.0816	0.0798	0.0289	0.4688	0.3587	0.2485
Panel B								
Autoregressive Terms								
Market	Copper Futures Equation				Silver Futures Equation			
	$\emptyset_{cc}^{lag1}$	$\emptyset_{cs}^{lag1}$	$\emptyset_{cc}^{lag2}$	$\emptyset_{cs}^{lag2}$	$\emptyset_{sc}^{lag1}$	$\emptyset_{ss}^{lag1}$	$\emptyset_{sc}^{lag2}$	$\emptyset_{ss}^{lag2}$
Coeff.	0.5678*	0.0027	0.2276*	0.0210	-0.0062	0.4863*	0.0000	0.2887*
Standard Error	0.0301	0.0132	0.0258	0.0150	0.0532	0.0009	0.0301	0.0294
Panel C								
	Switching coefficients–Standard Deviation				Switching Coefficients-Correlation Terms			
Market	Copper Futures		Silver Futures					
	$\sigma_c(0,.)$	$\sigma_c(1,.)$	$\sigma_s(0,.)$	$\sigma_s(.,1)$	$\rho(0,0)$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
Coeff.	0.0661*	0.598*	0.0957*	0.9406*	0.1589*	0.0000	0.0128	0.0152
Standard Error	0.0027	0.0440	0.0041	0.1933	0.0640	0.0176	0.0712	0.0437

Panel D						
Probability Parameters						
Market	Copper Futures Equation			Silver Futures Equation		
	$\alpha_c(0,.)$	$\beta_c(0,1)$	$\alpha_c(1,.)$	$\beta_s(1,1)$	$\alpha_s(.,0)$	$\alpha_s(.,1)$
Coeff.	2.8647*	-1.8724*	1.0326*	0.0000	3.0148*	-0.0889
Standard Error	0.2219	0.4413	0.1780	0.0620	0.1781	0.4189

Panel E				
Transition Probability Matrix (P 4x4)				
State at t|| State at t-1	(0,0)	(0,1)	(1,0)	(1,1)
(0,0)	0.9018	0.2810	0.2273	0.1092
(0,1)	0.0442	0.3486	0.0353	0.1534
(1,0)	0.0514	0.1412	0.6383	0.3067
(1,1)	0.0025	0.1292	0.0991	0.4307

* indicates that the coefficient is significantly different from zero at the 1% level.
** indicates that the coefficient is significantly different from zero at the 5% level.

3.3.2. Summary of Hypothesis Testing

Table 3.4 summarizes the hypothesis testing results of the Wald test statistics for the twelve hypotheses above. The estimated models (equations 3-1 and 3-6) show some interdependence between pairs of series according to various categories detailed above and explained as follows.

The first bivariate model (gold futures/silver futures) data used in this study fail to reject the null hypothesis of no dependence of the mean volatility of each market on the other (hypotheses 1 and 2). With respect to dynamic dependence in volatility, the tests conclude that silver linearly Granger causes gold futures but gold does not linearly Granger cause silver futures (hypotheses 3 and 4). With respect to the characterization of market dependence, the test results indicate that there is a spillover and interdependence from the gold futures to the silver futures market, and the same trend holds from silver futures to gold futures at 1% significance level (hypotheses 7 through 9). On the other hand, we conclude that there is no comovement of volatility of gold and silver in the long run (hypothesis 10). With respect to global causality[3] from gold futures to silver futures, data used in this study reject both the null hypothesis that gold futures do not cause silver futures and that silver does not cause gold futures.

[3] Global causality includes Granger linear causality, the dynamic relationship between the volatility in each pair of equations. But global causality encompasses more than Granger linear causality by including the impact of lagged states of each market on the transition probability of the other market.

Table 3.4 Market Characterization Based on MCMS Models (Silver Futures-Gold Futures).

Hypothesis	(Silver-Gold)
State Dependence in the Volatility	
1. No dependence of the intercept of silver futures on the state of gold futures	
2. No dependence of the intercept of gold futures on the state of silver futures	
Dynamic Dependence in the Volatility	
3. Gold futures do not linearly Granger cause silver futures	
4. Silver futures do not linearly Granger cause gold futures	*
State Dependence in the Correlations	
5. No dependence in the correlation on the state of gold futures	
6. No dependence in the correlation on the state of silver futures	
Characterization of Market Dependence	
7. No spillover from gold futures to silver futures	**
8. No spillover from silver futures to gold futures	**
9. No interdependence	**
10. Comovement between silver futures and gold futures	**
Global Causality	
11. Gold futures do not cause silver futures	**
12. Silver futures do not cause gold futures	**
Plausible Market Characterization	
• Spillover from gold futures to silver futures	
• Spillover from silver futures to gold futures	
• Interdependence	X
• Comovement	

The '*' and '**' symbols represent rejection of the hypothesis at 5% and 1% significance level respectively, on the basis of corresponding Wald-type tests on estimated MCMS models. X represents the existence of spillover, interdependence, comovement or independence.

Similar to the interpretation of Table 3.4, the second bivariate model (copper futures–gold futures) as shown in Table 3.5, the data used in this study fail to reject the null hypothesis of no dependence of the mean of volatility of each market on the other (hypotheses 1 and 2). With respect to dynamic dependence in volatility, the study supports the finding that gold futures is causally prior, in the Granger sense, to copper futures; but copper futures does not have a causal influence on gold futures (hypotheses

3 and 4). With respect to state dependencies in correlation, the study confirms the finding that there is no dependence in the correlation on the state of gold futures but there is a state of dependence in the correlation on the state of copper futures at 1% significance level (hypotheses 5 and 6) (The reader should distinguish between the correlation coefficients stated in panel E of Table 3.2 and the impact of the states on the correlation coefficients in panel D of Table 3.2. With respect to characterization of market dependence, the tests reveal that there is a spillover and interdependence from the gold futures to the copper futures market and vice versa at 1% significance level (hypothesis 7:9). On the other hand, the evidence concludes that there is no comovement of volatility of gold futures and copper futures in the long run (hypothesis 10). Similarly, there is a global causality from gold futures to copper futures, and the trend holds from copper futures to gold futures (hypotheses 11 and 12).

Table 3.6 summarizes the third bivariate model (copper futures–silver futures). The data used in this study fail to reject the null hypothesis of no dependence of the mean of volatility of each market on the other (hypotheses 1 and 2). With respect to dynamic dependence in volatility, the test results support the finding that silver does linearly Granger cause copper futures at 5% significance level. And similarly, the same trend holds from copper futures to silver futures at 1% significance level (hypotheses 3 and 4). With respect to state dependence in the correlation, the test results indicate the finding that there is dependence in the correlation of copper futures on the state of silver futures at 1% significance level. Similarly, there is state dependence in the correlation of silver to the state of copper futures at 5% significance level (hypotheses 5 and 6).

Table 3.5 **Market Characterization Based on MCMS Models (Copper Futures–Gold Futures).**

Hypothesis	(Copper-Gold)
State Dependence in the Volatility	
1. No dependence of the intercept of copper futures on the state of gold futures	
2. No dependence of the intercept of gold futures on the state of copper futures	
Dynamic Dependence in the Volatility	
3. Gold futures do not linearly Granger cause copper futures 4. Copper futures do not linearly Granger cause gold futures	**
State Dependence in the Correlations	
5. No dependence in the correlation on the state of gold futures	
6. No dependence in the correlation on the state of copper futures	**
Characterization of Market Dependence	
7. No spillover from gold futures to copper futures	**
8. No spillover from copper futures to gold futures	**
9. No interdependence	**
10. Comovement between copper futures and gold futures	**
Global Causality	
11. Gold futures do not cause copper futures	**
12. Copper futures do not cause gold futures	**
Plausible Market Characterization	
• Spillover from gold futures	
• Spillover from copper futures	
• Interdependence	X
• Comovement	

The '*' and '**' symbols represent rejection of the hypothesis at 5% and 1% significance level respectively, on the basis of corresponding Wald-type tests on estimated MCMS models. X represents the existence of spillover, interdependence, comovement or independence.

With respect to the characterization of market dependence, the study finds that there is a spillover and interdependence from silver futures to copper futures, and similarly from copper futures to silver futures at 1% significance level (hypotheses 7 through 9). On the other hand, the study concludes that there is no comovement of the volatility of silver and copper in the long run (hypothesis 10). With respect to the global causality between silver futures to copper futures, the data used in this study reject the null hypothesis that silver futures do not cause copper futures, and similarly, the test fails to reject the null hypothesis that copper futures do not cause silver futures (hypotheses 11 and 12).

In summary, this chapter shows the interaction of gold futures volatility with silver futures volatility, gold futures volatility with copper futures volatility, and silver futures volatility with copper futures volatility for the period 1999 through 2008. Our tests results show that gold futures do not play the lead role in metal futures markets, and that comovement in the long run between gold futures and silver futures does not exist during the study period. It turns out that a plausible market characterization from the estimated models and the hypotheses testing performed is a spillover from gold futures to silver futures, and from silver to gold futures. This first direction, spillover from gold futures to silver futures, was found in the previous literature, but the dissertation tested both of the two directions for three bivariate MCMS models. There is evidence of interdependence between gold futures and silver futures, which offers an addition to the literature provided by the current study. There is a spillover from gold futures to copper futures, and similarly, there is a spillover from copper futures to gold futures, which is a contribution to the existing literature. Previous studies have ignored including copper futures market (which is used intensively in different industries) when they are studying the cointegration across metal markets and therefore they ignored the interaction between the metal markets, economic activities and the business cycle. This chapter included copper futures as a metal intensively involved in economic activities, concluding that there are interactions between metal markets and various industries, and probably business cycles. Consequently, the economic environment and economic policies are determinants for metal futures markets volatility. There is evidence of interdependence between silver futures and copper futures, which is an addition to the literature provided by the current study, reflecting the interaction between economic activities and metal futures markets. Escribano and Granger (1998) presented evidence that any previously existing comovement between gold and silver prices has disappeared. Our study supports this finding of no comovement between gold and silver futures prices using more recent data. Evidence of no comovement between gold futures and copper futures markets is provided by the estimated results of the current study. Evidence of no comovement between the silver and copper futures markets is supported by the estimated results of the current study.

The applications of this chapter are many. One implication is that including three metals in one portfolio does not provide diversification. Consequently, decreasing portfolio risk requires more assets from different sectors. Providing useful information for traders and financial practitioners is a second application. For example, knowing the volatility in one of the three metal markets will help in determining the volatility of the two other metal markets and consequently in predicting the price of financial derivatives (options and futures options) in those markets. A third implication of the results presented in this chapter is that forecasting the volatility of returns in one of the three metals markets will help in pricing the financial derivatives (options and futures options) of the other two markets.

Table 3.6 Market Characterization Based on MCMS Models (Copper Futures-Silver Futures).

Hypothesis	(Copper-silver)
State Dependence in the Volatility	
1. No dependence of the intercept of copper futures on the state of gold futures	
2. No dependence of the intercept of gold futures on the state of copper futures	
Dynamic Dependence in the Volatility	
3. Silver futures do not linearly Granger cause copper futures	*
4. Cooper futures do not linearly Granger cause silver futures	**
State Dependence in the Correlations	
5. No dependence in the correlation on the state of silver futures	**
6. No dependence in the correlation on the state of copper futures	*
Characterization of Market Dependence	
7. No spillover from silver futures to copper futures	**
8. No spillover from copper futures to silver futures	**
9. No interdependence	**
10. Comovement between copper futures and silver futures	**
Global Causality	
11. Silver futures do not cause copper futures	**
12. Copper futures do not cause silver futures	**
Plausible Market Characterization	
• Spillover from silver futures	
• Spillover from copper futures	
• Interdependence	X
• Comovement	

The '*' and '**' symbols represent rejection of the hypothesis at 5% and 1% significance level respectively, on the basis of corresponding Wald-type tests on estimated MCMS models. X represents the existence of spillover, interdependence, comovement or independence.

Chapter 4

Summary and Extensions

Recently, a large amount of literature has been devoted to estimating and forecasting volatility for financial time series. In this field, the importance of high frequency data has been stressed, in particular for evaluating how well alternative measures of volatility fit the forecasts of volatility generated by GARCH models. This study introduced a comprehensive comparison between volatility measures (daily absolute returns, cumulative intraday squared returns and IVFT) using intraday data from metal futures markets (gold, silver and copper). A principal innovation of this comprehensive comparison is the inclusion of the IVFT measure, which is based on the integration of time series data. This measure naturally exploits the time structure of high frequency data by including all the observations in the volatility computation. Using historical tick-by-tick data from metal futures prices, the study illustrates that this IVFT measure performs better. When the study employed the IVFT, the fit to GARCH forecasts were more accurate than those associated with the sum of squared intraday returns and daily absolute returns. This study examined how varying the intraday frequencies used to construct the IVFT measure affected the fit of IVFT measure to the GARCH forecasts. The results obtained in the recent literature are confirmed using metal futures markets. This study provides evidence that the fit of the IVFT measure to the GARCH forecasts of volatility improves, based on heteroscedastic RMSE and logarithmic loss function criteria, when the time frequency of volatility measures are increased from 15-minute to one-minute intervals.

The second part of the study shows the interaction of gold futures volatility with silver futures volatility, gold futures volatility with copper futures volatility, and silver futures volatility with copper futures volatility during the period 1999-2008. Our study confirms the previous finding that gold futures do not play the leading role in the metal futures markets and that comovement in the long run between gold and silver futures does not exist during the study period. It turns out that a plausible market characterization from the estimated models and the hypotheses testing performed is a spillover from gold futures to silver futures, and similarly, there is a spillover from silver to gold futures. This first direction, spillover from gold to silver futures, was found in the previous literature, but the dissertation tested both of the two directions for three bivariate MCMS models. There is evidence of interdependence between gold and silver futures which is an addition to the literature provided by the current study. There is a spillover from gold futures to copper futures, and similarly, there is a spillover from copper to gold futures, which is also a contribution to the existing literature. The previous studies ignored the interaction between economic activities. Our study included copper futures as a metal intensively involved in economic activities, concluding that there is an interaction between metal markets and different industries, and probably the business cycle. Consequently, the economic environment and economic policies are determinants of the volatility of the metal futures markets. There is evidence of interdependence between silver and copper futures, which is an addition to the literature provided by the current study reflecting the interaction between economic activities and metal futures markets.

Evidence of no comovement between gold and copper futures markets is provided by the estimated results of the current study. In addition the estimated results of the current study provide evidence of no comovement between silver and copper futures markets.

The implications of our study are many. One implication is that portfolios including the three metal markets do not provide adequate diversification. A second implication provides useful information for traders and financial practitioners. Knowing the volatility of one of the three metal markets will help in determining the volatility of the other two metal markets, and consequently in predicting the price of financial derivatives (options and futures options) in those markets. A third implication relates to the fact that forecasting the volatility of one of the three markets will help in pricing the financial derivatives (options and futures options) of the other two markets. The fourth implication is related to the fact that in case of high frequency data, financial practitioners can more accurately predict the options prices of the three metal markets by using IVFT relative to the other measures.

The measurement and forecasting of asset-price volatility plays a critical role in the study of financial markets. This dissertation verifies the importance of using the IVFT measure to estimate the integrated volatility accurately. Consequently, studies of volatility that ignore intraday returns series and the IVFT measure will yield misleading conclusions for the following reasons. First, the high-frequency returns and IVFT contain extremely valuable information for the measurement of integrated volatility at the daily level. Second, the intraday returns and IVFT reveal that there are significant long-memory features in the return dynamics. Third, the IVFT estimates the volatility accurately and better fit the GARCH(1,1) forecasts of volatility. These features are critical for portfolio management and derivative pricing and they are relevant for the analysis of volatility transmission contemporaneously across metal futures markets. In summary, the IVFT measure and the information provided by high-frequency returns are valuable to a broad range of issues in financial markets. Moreover, the dissertation provides strong evidence based on the MCMS model of the interdependence, but no comovements between, the three metal markets, which is critical for portfolio management, derivative pricing and economic policy making.

There are two possible extensions in the short run. Since there are interdependences between the three metal futures markets, knowing the main factors (economic policies, announcements, etc.) that affect volatility is important in determining the factors that shift the supply-and-demand curves of these markets. Consequently, the next research question is: what is the impact of economic policies and macroeconomic news on the volatility of metal futures markets? A related question is: what is the impact of macroeconomic news on volatility transmission between these markets? The second possible extension of the current study is related to the importance of the study. Since many financial practitioners and portfolio managers use gold and silver as hedges against inflation, a possible extension of the current study is to answer the following question: what is the impact of the dollar index (foreign exchange rate of the dollar against a basket of major foreign currencies) on the volatility of gold futures, and similarly, the impact of the volatility of gold futures on the dollar index?

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