Chapter 37

Suppositional Reasoning*

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The purpose of this paper is to show how to structure some typical examples of suppositional reasoning. Such reasoning is common, but unfortunately it is typically neglected by writers of logic textbooks, as Fisher points out in [Fisher, p. viii]. Fisher gives a method of diagramming suppositional reasoning and gives credit to Thomas for similar work in [Thomas]. I shall begin by laying out their methods of structuring suppositional reasoning.

1. Conditionalization
To illustrate suppositional reasoning involving a "conditionalization step" Thomas uses the following example:

Example T1. From [Thomas, p. 216].

> Suppose (1) George has two suitcases. Then (2) George has some luggage. So if George has two suitcases, then George has some luggage.

Thomas' picture of the reasoning uses boxes of the sort mentioned in [Jaśkowski, pp. 234-5]. (Throughout the discussion the pages indicated are those in the most recent reference.)

```
+--------+
| 1 -> 2 | -> If 1 then 2. (By conditionalization)
+--------+
```

Fischer' picture of the same reasoning looks like this:

```
(Suppose) u1 -> u2
              └──> If 1 then 2 (By conditionalization)
```

(The 'u' in the diagram is to indicate that 1 is unasserted.)

My main complaint about both Thomas' and Fisher's diagrams is that they do not call attention to the role of the arguer and the audience when portraying the reasoning. To accomplish this we modify Jaśkowski's S operator in [Jaśkowski, p. 233]. 'Sw:...' is short of 'We suppose that ....' Following Thomas and Fisher, read 'x -> y' as 'y follows from x.' And read 'x/y' as 'x; so y.'

```
Sw: 1
    ↓
    2
----------
 If 1 then 2
```

Adapting Frege's terminology, 'Sw' is a force operator. The content, ..., of 'Sw: ...' is a 'mere complex of ideas' [Frege, p. 2]. It is tempting to follow Frege in building a force operator into the conclusion, making the conclusion a judgment with assertoric force. But to keep the focus of the paper on suppositional reasoning, this temptation will be resisted.

Note that Example T1 is artificial. It seems to me that no one would persuade someone to believe the conclusion in this way. Granted, someone might not recognize that if George has two suitcases then George has luggage. To get such a person to recognize this you would help him recognize what a suitcase is. But the following example contains real suppositional reasoning.

Example E1. From [Euclid, p. 41], Euclid's proof that if AB is a finite straight line then an equilateral triangle can be constructed on it.

Let (1) AB be the given finite straight line. ... [Then by constructing circles with centers at A and B that intersect B and A, respectively, we find that the circles intersect at C.] Therefore [given (2) the postulates and definitions we have accepted, and ought to accept] (3) the triangle ABC is equilateral; and it has been constructed on the given finite straight line AB.

This is a picture of Euclid's reasoning:

```
Sw: 1               2
    |_______________|
            ↓
            3
    _________________
    If 1 then 3
```

Not everyone would agree. Russell in the opening line of [Russell, p. 3] says that 'pure mathematics is the class of all propositions of the form "p implies q".' And Gentzen in [Gentzen, p. 74] states his goal as that of reflecting 'as accurately as possible the actual reasoning used in mathematical proofs.' To reach this goal he sets up a system of natural deduction which starts from assumptions to which logical deductions are applied' (p. 75). Apparently both Russell and Gentzen would say that Euclid's conclusion is that if 1 and 2 then 3. But it seems clear to me that Euclid is not asking us to suppose that 2; he is asking us to accept 2. Thus, the stronger conclusion follows.

More than one supposition is involved in the following use of conditionalization.

Example A1. From [Aristotle, p. 43], Aristotle's proof of Cesare.

Let (1) M be predicated of no N, but let (2) M be predicated of every O. Since, then the negative is convertible, (3) N will belong to no M; but M was assumed to belong to every O: consequently (4) N will belong to no O.

The structure:

```
Sw: 1                       Sw: 2
    ↓
    3
    |___________________________|
                ↓
    _____________________________
    If 1 and 2 then 4
```

2. Reductios

Thomas illustrates *reductio* reasoning with this example:

Example T2. From [Thomas, pp. 221-2].

Suppose ... that (1) the meaning of a word in a language were the same as the object or objects which that word names. Then it would follow that (2) words like "is," "the," and "nothing" that do not name anything have no meaning. But this is absurd. Though these words do not name anything they nevertheless have a meaning. Therefore, it is not the case that the meaning of a word in a language is the same thing as the object or objects which that word names.

Thomas and Fisher structure this reasoning by using conditionalization. This is Thomas's picture:

```
| 1 -> 2 | -> If 1 then 2; + 2 is false -> 1 is false.
```

And Fisher's picture is similar. But conditionalization is not used. This is the structure of the reasoning:

```
Sw: 1
    ↓
    2 + not 2
-------------
    Not 1
```

It is natural to ask why both Thomas and Fisher view *reductios* as making use of conditionalization. I think they are confusing the goals of formal logicians with those of informal logicians. Of course the elegance of a formal system is determined by the number of basic rules. And formalized *reductio* rules can be dispensed with if conditionalization and *modus tollendo tollens* are present. (See [Kalish et al., pp.. 44-45].) But as informal logicians we are not driven by a need to keep basic patterns of reasoning to a minimum. Our goal is to be able to present reasoning as accurately as possible.

The following example also is a *reductio* that also does not use a conditionalization step.

Example P. From [Plato, p. 869].

SOCRATES: Suppose someone to ask, 'Is it possible for a man who has once come to know something and still preserves a memory of it, not to know just that thing he remembers at the moment when he remembers it?' This is, perhaps, rather a long-winded way of putting the question. I mean, can a man who has become acquainted with something and remembers it, not know it?

THEAETETUS: Of course not, Socrates, the supposition is monstrous.

The reasoning can be structured as follows, where 'TSw: ...' is short for 'We can try to suppose that ...' and 'A' is short for 'Someone knew something, has a memory of what was known, and does not know what was known when what was known is remembered.

```
TSw: A Not (Sw: A)
__________________
A is necessarily false
```

'Not (Sw: A)' does not mean what '(Sw: not A)' means. (For more discussion of the negation of forces see [Prior, pp. 70-71] and see the discussion of "illocutionary denegation" in [Searle and Vandervecken].)

Though I prefer the above method of structuring Example P, one might picture it as:

```
TSw: 1
     ↓
   absurd
  ________
   Not 1
```

(This is close to Gentzen's Negation Introduction rule in [Gentzen, p. 77]. But Gentzen does not distinguish between supposing, where success can be obtained, and trying to suppose, where success may not be obtained.)

The following reasoning is similar to that of Example P.

Example K. From [Kripke, pp. 125-6].

Cats are in fact animals! Then is this truth a necessary or a contingent one? It seems to me that it is necessary. Consider the counterfactual situation in which in place of these creatures -- these animals -- we have in fact little demons which when they approached us brought bad luck indeed. Should we describe this as a situation in which cats were demons? It seems to me that these demons would not be cats. They would be demons in a cat-like form. We could have discovered that the actual cats that we have are demons. Once we have discovered, however, that they are not, it is part of their very nature that, when we describe a counterfactual world in which there were such demons around, we must say that the demons would not be cats. It would be a world containing demons masquerading as cats.

This is the reasoning -- Let us try to suppose that (1) cats are not animals. Then we will have to imagine them as being something, demons, perhaps (or robots controlled by Martians, using Putnam's example, or ....). But then these somethings that we are imagining are not cats. So, we cannot succeed in imagining cats as not being animals. So, we should assert that cats are necessarily animals. In symbols the reasoning is

```
TSw: 1      Not (S: 1)
_______________________
     Not 1
```

If we were to pipe the reasoning in Examples P and K through conditionalization, ignore the distinction between the two types of suppositional force modifiers, and leave the audience out of the picture (in other words if we were to follow Thomas and Fisher's method of structuring the reasoning) this would be the result:

```
[1] -> If 1 then 1; + 1 is false -> 1 is false.
```

Example A2. From [Aristotle, p. 44], Aristotle's proof of Baroco.

...if (1) M belongs to every N, but (2) [M does] not [belong] to some O, it is necessary that (3) N does not belong to some O; for if (not 3) N belongs to every O, and M is predicated also of every N, it must be the case that (not 2) M belongs to every O; but we assumed that M does not belong to some O.
The structure:

```
Sw: 1        Sw: 2     Sw: not 3
               ↓
            Not 2
------------------------------------
          If 1 and 2 then 3
```

A simpler proof (Antilogism) of the same result is obtained as follows:

```
Sw: 1       Sw: not 3
     ↓
     Not 2
-------------------------
If 1 and 2 then 3
```

3. Counterexamples
The following example illustrates the simplest sort of suppositional reasoning.

Example T. From [Thomson].

...let me ask you to imagine this. You wake up in the morning and find yourself back to back in bed with an unconscious violinist. A famous unconscious violinist. He has been found to have a fatal kidney ailment, and the Society of Music Lovers has canvassed all the available medical records and found that you alone have the right blood type to help. They have therefore kidnapped you, and last night the violinist's circulatory system was plugged into yours, so that your kidneys can be used to extract poisons from his blood as well as your own. ... [To unplug him would be to kill him. Should you remain plugged?] I imagine you would regard this [a yes answer] as outrageous.

The suppositional reasoning can be structured as follows:

Sw: A has a right to life and B has a right to let A die.

It may be right to let someone die, even though this person has a right to life.

4. Proof by cases
Example E2. From [Euclid, pp. 255-6].

Let (1) ABC be a triangle having the angle ABC equal to the angle ACB. I say that the side (2) AB is also equal to the side AC. For, if (not 2) AB is unequal to AC, one of them is greater. Let (3) AB be greater, ...[then, given 4, the postulates and definitions] an absurdity follows. If (5) AC is greater an absurdity then, given 4, an absurdity follows. So, if 1 then 2.] Structure:

```
Sw: 1          Sw: not 2
                    ↓
                 3 or 5
             Sw: 3 + 4     Sw: 5 + 4
                   ↓             ↓
                absurd        absurd
             ____________________________
                    If 1 then 2
```

5. Naming suppositions
Example EP. From [Epstein, p. 550].

... even though the number of fractions [1/2, 1/4, 1/8, ...] is infinite, their sum is not infinite. This is why. Suppose s = 1/2 + 1/4 + 1/8 + ... Now multiply everything by two, so you get 2s = 1 + 1/2 + 1/4 + 1/8 + ... Now subtract equations. (1 + 1/2 + 1/4 + 1/8 + ...) - (1/2 + 1/4 + 1/8 + ...) = 1. So s = 1. So 1 = 1 + 1/2 + 1/4 + 1/8 + ...

The reasoning should be structured as follows:

```
Sw: s is the name of      Multiplying equals by equals
    1/2 + 1/4 + ...              yields equals, etc.
       |______________________________|
                    ↓
            1 = 1/2 + 1/4 + ...
```

Though this reasoning is good, not all reasoning of this general form is. Suppose the content of the supposition is '2s is the name of 1/2 + 1/4 + ... Using the above pattern of reasoning we could conclude that 2 = 1. We can avoid this unwanted conclusion by requiring that names be simple and neutral. When using names in suppositions we also must be assured that there is something that is named.

6. Concluding remarks
The structuring of reasoning endorsed above emphasizes that reasoning is an activity involving a listener and an audience. And we have argued that it is a mistake to view all suppositional reasoning as involving conditionalization. But much more work needs to be done. (Suppositions also figure in practical inference. 'Suppose' is used in reasoning that is not suppositional. Suppositional force operators interact with other force operators than those mentioned above.) Certainly Thomas and Fisher should be given high praise for leading the way.

Notes

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