

THESIS

AN ANALYSIS OF FACTORS AFFECTING STUDENT SUCCESS IN MATH
160 CALCULUS FOR PHYSICAL SCIENTISTS I

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WE HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER OUR SUPERVISION BY DANIEL LEE REINHOLZ ENTITLED “AN ANALYSIS OF FACTORS AFFECTING STUDENT SUCCESS IN MATH 160 CALCULUS FOR PHYSICAL SCIENTISTS I” BE ACCEPTED AS FULFILLING IN PART REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE.

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ABSTRACT OF THESIS

AN ANALYSIS OF FACTORS AFFECTING STUDENT SUCCESS IN MATH 160 CALCULUS FOR PHYSICAL SCIENTISTS I

The average success rate in MATH 160 Calculus for Physical Scientists I at Colorado State University has been near 60% for at least the past three years. Weak pre-calculus skills are often cited as one of the primary reasons students do not succeed in calculus. To investigate this conjecture we included the ALEKS Preparation for Calculus instructional software as a required component of MATH 160. Despite a perceived decrease in the number of algebra-related questions asked by students, we found no improvement in success rates. We also performed an analysis of other factors in relation to success, such as ACT scores and whether or not students had prior calculus experience. As a result of our investigations we conjecture that difficulty with conceptual thinking is a more significant factor than lack of mostly mechanical pre-calculus skills contributing to non-success in MATH 160.

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Chapter 1

Introduction

The calculus reform movement began in 1987 as a response to low success rates in university-level calculus across the United States of America. According to Kasten and others [7], during the 1986-87 academic year only 140,000 of 300,000 students across the United States initially enrolled in calculus successfully completed their courses, for a nationwide success rate of about 47%. Such low success rates are a major impediment to students pursuing mathematically-intensive fields. Apropos to this, the calculus reform movement is based on the idea that calculus should ‘be a pump and not a filter.’ Rather than preventing students from pursuing careers in mathematically-intensive fields, calculus should impel them to.

Broadly speaking, the calculus reform movement advocated a greater emphasis on a conceptual calculus, particularly the trinity of algebraic, graphical, and numerical perspectives to calculus [8]. Both an emphasis on concepts and a focus on this trinity of perspectives are parts of the approach in MATH 160 Calculus for Physical Scientists I, so it is fair to say that, at least in some sense, it is a reform curriculum.

Despite drawing pedagogical approaches from the calculus reform movement, the success rates in MATH 160 are comparable with the success rates that initially

prompted the calculus reform movement in 1987. In Dr. Kenneth Klopfenstein's Fall 2007 section of MATH 160 the overall success rate of students ($N = 77$) was 62.65%, where students who received a grade of A, B, or C are considered successful (so students receiving a grade of D, F, or W are considered unsuccessful). For students with prior calculus experience ($N = 49$) the success rate was 83.7%, and for students with no prior calculus experience ($N = 28$) the success rate was only 39.3%. Notably, these figures only include students who remained in the course long enough to take the first midterm, which we feel indicates a serious attempt at completing the course. These figures are typical and representative of students in other sections and semesters (see table 1.1).

Table 1.1: *Success Rates in MATH 160, Fall 2005 - Fall 2008*

Semester	Initial Population	Successful Population	Success Rate
Fall 2005	330	216	65.45%
Spring 2006	188	131	69.68%
Fall 2006	319	196	61.44%
Spring 2007	186	86	46.24%
Fall 2007	431	270	62.65%
Spring 2008	253	155	61.26%
Fall 2008	439	214	48.75%
Total	2146	1268	59.09%

At Colorado State University MATH 160 is a required course for engineering, physics, and mathematics majors, as well as a few additional majors. In most of these majors MATH 160 is the beginning of a sequence (MATH 160, MATH 161, MATH 261, MATH 340) that takes students from calculus through ordinary differential equations. Failure in MATH 160 results in the delay of the completion of this sequence, which can be a major setback to students who need to be proficient with this level of mathematics in order to move on to upper-level courses.

Alternatively, failure in MATH 160 can deter students from pursuing mathematics-intensive majors altogether. Moreover, with particular regards to students with no prior calculus experience, having such a low success rate is an inefficient use of both student and university resources.

We conjecture the following major causes for low success rates in calculus:

1. *Weak mechanical pre-calculus skills.* When we refer to mechanical pre-calculus skills, we mean the ability to perform algebraic and symbolic manipulations. Accordingly, a student with weak pre-calculus skills is prone to making algebraic errors while calculating limits and derivatives. The inability to perform such manipulations is a huge impediment to solving problems, such as optimization. Moreover, we believe that when a student is preoccupied with symbolic manipulations it detracts from his or her ability to focus on mastering the ideas and concepts of calculus.
2. *Perceived lack of importance of concepts.* Most students' prior mathematics experience has consisted largely of solving problems algorithmically. Upon being presented with a problem, the student sees his or her task as identifying the *type* of problem, choosing the appropriate algorithm, and applying it to solve the problem. Apropos to this background, students tend to resist focusing on the concepts of calculus, instead studying calculus from a procedural approach, which is largely ineffective.
3. *Difficulty learning and studying concepts.* Compounding the issue of perceived lack of importance of concepts is the actual difficulty in studying and learning concepts. Perhaps most notably is the difficulty students have with the function concept [2], [4]. MATH 160 requires students to think about

mathematics in ways fundamentally different from how they have previously. Students have great difficulty in reading and interpreting mathematical writing, as well as in expressing their ideas with precision. These issues are compounded by the fact that students enter the course with *incorrect* definitions and misconceptions of many concepts in calculus (asymptotes and tangent lines for instance). In general, correcting these misconceptions can be more difficult than teaching new concepts to students [6].

4. *Unrealistic expectations of the effort required to succeed.* The majority of the student population in MATH 160 is incoming freshmen, especially during Fall semesters. Many of these students had to use very little effort to succeed or even excel in their high school mathematics courses, and correspondingly feel that there will be very little effort required to succeed in university-level calculus. It seems to be a recurrent pattern that students only realize the amount of effort required after too much time has passed for them to remedy the situation.
5. *Poor study skills.* Even the students who recognize the amount of effort required to succeed in MATH 160 are often not equipped with the study skills required to make efficient and effective use of their time. This issue is compounded by their lack of attention to concepts, which tends to promote memorization rather than deep understanding as a learning strategy.

We performed a general analysis of factors related to success in MATH 160 in order to attempt to quantify our conjectures above. Although we feel that it is important to address all of the above issues, we also feel that they are largely independent, so our approach was to attempt to address each of the problems

individually and systematically. At this time we decided to forego the investigation of student study skills and expectations of MATH 160 in order to focus on the other issues we believe are related to student success.

We decided to focus primarily on pre-calculus skills in their relation to success rate because we were offered a unique opportunity to do so. The ALEKS Corporation offered us a free trial of their ALEKS (Assessment and LEarning in Knowledge Spaces) Preparation for Calculus software for usage during the Spring 2008 and Fall 2008 semesters. ALEKS essentially consists of two components: an adaptive assessment based on knowledge space theory, and a learning mode [1]. The set of topics used for assessment and instruction within the program is customizable; we used what would typically be considered as a standard set of pre-calculus topics, focusing primarily on algebra and trigonometry.

Knowledge space theory provides a conceptual framework for describing an individual's understanding or knowledge state in a given domain. Underlying the theory is the notion of a knowledge structure. The knowledge structure of a field consists of a delineation of the basic concepts or ideas, as well as the precedence relationships between different concepts [5]. If there is a precedence relationship between two concepts it means that understanding of one concept is prerequisite to the understanding of the other, more advanced topic.

Once a knowledge structure has been established, the task becomes determining an individual's specific knowledge state within the knowledge structure. An individual's knowledge state is a description of everything he or she understands within the knowledge structure. This task of determining an individual's knowledge state is performed by the ALEKS initial assessment. Once the assessment has determined a student's knowledge state, ALEKS determines which concepts

the student is most ready to learn, based on the precedence relationships among concepts in the knowledge structure. Relying on this framework, ALEKS then provides individually tailored instruction through the learning mode.

Chapter 2

The ACT and Algebra Background

We began our study by investigating the relationship between a student's performance on the ACT, a conceptual algebra pretest, and success in calculus. For the purpose of our study we have defined a successful student as one who completes MATH 160 with a letter grade of C or higher. A nonsuccessful student is one who completes Exam 1 but completes the course with a grade of D, F, or W. Students who did not complete Exam 1 were not included in our study.

2.1 Method

All participants in the study were students enrolled in Dr. Kenneth Klopfenstein's sections of MATH 160 at Colorado State University, during the Fall 2007 and Fall 2008 semesters. From the university's database we collected ACT Math and ACT Reading scores for all participants. For students who did not have ACT scores, we used SAT scores if available, using the conversion chart used by the University of California [9]. We converted SAT Math scores to ACT Math scores and SAT Verbal scores to ACT Reading scores. The conversion was made by converting the SAT score to a UC Score, and then rounding down to the nearest UC score that corresponded to an ACT score.

At the beginning of the Fall 2007 semester we administered an eight question algebra pretest (see appendix A), in order to gain background information about students' conceptual understanding of algebra topics. The test was developed by Dr. Kenneth Klopfenstein, and given Dr. Klopfenstein's extensive experience as a mathematics professor, we feel fairly confident in the content validity of the instrument. Since it is a content-based instrument, we do not feel that inter-item reliability is relevant.

The assessment rubric for the algebra pretest developed by Daniel Reinholz is given in appendix B. Each item is graded on a scale from 1-4, representing a student's range of conceptual understanding of each topic. A score of 1 represents little to no understanding of the problem, whereas a 2-3 represents some understanding of the problem, but at a mechanical level, and a 4 represents a strong conceptual understanding of the given aspect of algebra.

At the beginning of the Fall 2008 semester students completed a mathematical background survey (see appendix E), a different measure of conceptual pre-calculus skills. We developed this alternative instrument due to some difficulties in assessing student performance on the original pretest. The mathematical background survey was developed by Daniel Reinholz, with suggestions from Dr. Ken Klopfenstein. Each question was graded on a binary scale, with a 1 representing mastery of a given concept and a 0 representing a lack of mastery. Like the original algebra pretest, this survey should have high content validity. Unfortunately, we were unable to use the survey in the study due to a very low response rate of about 10%. Nevertheless, we have included the survey to provide a reference for future studies.

In order to begin with simple statistics we ran our set of research questions *backwards*, using success as the dependent variable, and the potential predictors

as independent variables. We performed independent samples t -tests between students who succeeded and did not succeed in MATH 160 in terms of ACT Math, ACT Reading, and Algebra Pretest scores. Next, we ran the questions *forwards* and used multiple regression to try to predict a student's total MATH 160 score from his or her Algebra Pretest, ACT Math, and ACT Reading scores. Complementarily, we used discriminant analysis to see if the above three predictors could discriminate between those who did and did not succeed in MATH 160.

2.2 Results

To investigate whether there is a difference between students who succeeded and did not succeed in MATH 160 during Fall 2007 in terms of Algebra Pretest scores, ACT math scores, and ACT reading scores, we ran 3 independent samples t -tests. Table 2.1 shows that students who succeeded in MATH 160 were significantly different from students who did not succeed in terms of Algebra Pretest scores ($p = 0.004$). The mean score for students who succeeded was 2.5 points higher than for students who did not succeed, where the test was out of a possible 32. The effect size was $d = 0.83$, which according to Cohen [3] is larger than typical. Students who succeeded and did not succeed MATH 160 did not differ significantly on ACT Math scores ($p = 0.781$) or ACT Reading scores ($p = 0.144$). We repeated these analyses for Fall 2008 students and once again there was no statistically significant difference with respect to ACT Reading or ACT Math scores.

We used multiple regression to investigate how well one can predict a total score in MATH 160 with a combination of three variables: ACT Math scores, ACT Reading scores, and Algebra Pretest scores. The means, standard deviations, and intercorrelations can be found in table 2.2. The combination of variables to

Table 2.1: *Comparison of Students who Succeeded and did not Succeed in MATH 160 (N = 63), Fall 2007*

Variable	<i>M</i>	<i>SD</i>	<i>t</i>	<i>df</i>	<i>p</i>
Algebra Pretest Score			-2.99	64	0.004
Unsuccessful in MATH 160	15.440	2.770			
Successful in MATH 160	17.958	3.130			
ACT Math Score			-0.28	61	0.781
Unsuccessful in MATH 160	25.920	3.068			
Successful in MATH 160	26.160	2.629			
ACT Reading Score			-1.48	61	0.144
Unsuccessful in MATH 160	23.000	4.673			
Successful in MATH 160	24.820	3.762			

predict a student's total score in MATH 160 was statistically significant, $F(3, 59) = 3.925$, $p < 0.05$. The Beta coefficients can be found in table 2.3. Only ACT Reading significantly predicted a student's total score when all three variables were considered. The adjusted R^2 value was 0.124, which means that 12% of the variance of total score could be explained by this model. This corresponds to $R \approx 0.352$, which according to Cohen [3] is a typical effect size.

Table 2.2: *Means, Standard Deviations, and Intercorrelations for Total Score in MATH 160 and Predictors (N = 63), Fall 2007*

Variable	M	SD	ACT Math	ACT Reading	Algebra
Total Course Score	487.112	123.719	0.199	0.348*	0.160
Predictor Variable					
1. ACT Math	26.110	2.700	–	0.171	0.173
2. ACT Reading	24.440	3.987		–	-0.066
3. Algebra Pretest	15.619	6.285			–

* $p < 0.05$

We used discriminant analysis to try and determine what combination of Algebra Pretest scores, ACT Math scores, and ACT Reading scores best distinguishes

Table 2.3: *Simultaneous Multiple Regression Analysis Summary for Predicting Total Score in MATH 160 (N = 63), Fall 2007*

Variable	B	SEB	Beta
ACT Math	5.162	5.626	0.113
ACT Reading	10.529	3.762	0.339*
Algebra Pretest	3.200	2.387	0.163
Constant	44.964	158.929	

* $p < 0.05$

students who succeed in MATH 160 from those who do not. It was verified that the predictor variables were not heavily skewed, and the population covariances were relatively equal between groups. Wilk's lambda was not significant, $\lambda = .99$, $\chi^2 = 2.161$, $p = .501$, which indicates that this combination of predictors was not able to significantly discriminate the two groups. The standardized function coefficients are featured in table 2.4, which suggest that the ACT Reading Scores are the primary factor for discriminating whether or not a student will succeed in Math 160. The classification showed that only 60.3% of cases were properly predicted by this model.

Table 2.4: *Discriminant Analysis Summary for Distinguishing Successful and Non-successful Students, (N = 64), Fall 2007*

Variable	Standardized Function Coefficients	Correlations Between Variables and Discriminant Function
ACT Math	-0.100	0.186
ACT Reading	0.999	0.985
Algebra Pretest	0.172	0.100

2.3 Discussion

The results of t -tests for Fall 2007 and Fall 2008 populations indicated there was no statistically significant difference between the students who succeeded and

did not succeed in terms of ACT Math and ACT Reading scores. However, the t -tests did indicate a statistically significant relationship between a student's Algebra Pretest scores and success. In contrast, when multiple regression was performed, only ACT Reading scores were found to be a significant predictor of success in MATH 160.

Investigation into this discrepancy uncovered that the populations used by the different statistics were significantly different. In the multiple regression the scores of seven individuals who did not succeed in Math 160 (out of a total of $N = 20$) were not used, because these individuals were missing ACT scores. As a result, nearly half of the population that did not succeed in MATH 160 was not included in the multiple regression (in contrast, only 5 out of 55 scores of students who were successful were not used due to lacking ACT scores). Thus, we can conclude the discrepancy in the statistical results is due to a difference in populations used. For this reason it would be prudent to repeat this study in a subsequent semester, and hope to find a population with less missing data. Unfortunately this is difficult, because international and transfer students are often lacking ACT scores. Nevertheless, accounting for the discrepancy in populations, the statistics seem to indicate that both Algebra Pretest scores and ACT Reading scores are related to success in Math 160.

The results do somewhat surprisingly indicate that there is no strong relationship between ACT Math scores and success in MATH 160 for the given populations. One interpretation is that ACT Math scores are not strongly related to success in MATH 160 because ACT Math scores do a poor job of distinguishing members of our given population, as most students in the population are those who did well in high-school mathematics. In contrast, ACT Reading scores are a better

predictor of success because they provide information as to which students have strong verbal skills in addition to strong mathematical skills. Thus, ACT Reading scores would provide information as to which students are the most well-rounded, at least with respect to the students in our population.

An alternative interpretation is that although ACT Math skills are important, reading comprehension and conceptual algebra skills are more important in the context of MATH 160. This is suggested by the fact that conceptual Algebra Pretest and ACT Reading scores are the best predictors of success, while ACT Math (representing mechanical pre-calculus skills) is a poor predictor of success.

Chapter 3

ALEKS

During the Spring 2008 and Fall 2008 semesters we included ALEKS as a required component of MATH 160. We conjectured that if weak mechanical pre-calculus skills were a significant obstacle to student success in calculus then the inclusion of ALEKS should have a positive impact on success rates. Once again for the purpose of our study we have defined a successful student as one who completes MATH 160 with a letter grade of C or higher. A nonsuccessful student is one who completes Exam 1 but completes the course with a grade of D, F, or W. Students who did not complete Exam 1 were not included in our study.

3.1 Method

In order to assess the effects of including ALEKS as a required component of the course we considered the student populations in Dr. Klopfenstein's sections from Fall 2007, Spring 2008, and Fall 2008. We restricted the populations to Dr. Klopfenstein's sections in order to minimize any differences due to instructor variation. Due to the small population sizes (about 70-90) we used every student in these sections for our samples. Overall we feel that a population in a given Fall (or Spring) semester will be representative of the population in another Fall

(or Spring) semester. Nevertheless, when comparing populations between Fall and Spring semesters one must be cognizant of the different dynamics between Fall and Spring semesters; Spring student populations not only consist of students who delayed taking MATH 160, but also many repeat students.

For all students considered in the study we have ACT Math and ACT Reading scores, as provided by the university's database. We followed the same procedure as in the previous investigation for converting SAT scores to ACT scores. We administered a mathematical background survey to students during both Fall 2007 and Fall 2008, which indicates whether or not they have prior calculus experience.

In order to motivate students to use ALEKS, their ALEKS score was included as about 10% of their final grade. During the Spring 2008 semester the cutoff date was March 13 (the date of the Exam 2), at which time a student's ALEKS score was determined from the ALEKS system and incorporated into their grade. If a student completed 80% of the topics on ALEKS he or she received full credit, and otherwise received a grade corresponding to the percentage of 80% of topics completed. During the Fall 2008 semester the cutoff date for ALEKS was September 15 (the Monday before Exam 1, which was administered on a Thursday). The cutoff date was set much earlier in Fall 2008 to try and impel students to spend more time in ALEKS earlier in the semester. For students in Spring 2008 and Fall 2008 we have both pre-test and post-test scores (the scores given at the cutoff date) provided by ALEKS, as well as the time spent in the system.

At the end the Spring 2008 semester we administered a survey to all students who were still enrolled in MATH 160 in order to assess student perceptions of ALEKS (see appendix C and appendix D for student responses). The student perceptions of ALEKS survey was developed by Dr. Ken Klopfenstein and Daniel

Reinholz. It consists of nine questions regarding ALEKS and its role in MATH 160, and a single question related to ALEKS and the precalculus PACe program at Colorado State University. All questions are given on a 5 point Likert scale, from strongly disagree to strongly agree, with space for written comments following each item.

In order to assess the relationship between ALEKS usage and performance and outcome in the course, we performed simultaneous multiple regression, using student populations from Spring 2008 and Fall 2008. Additionally, we compared student outcomes between Fall 2007 (without ALEKS) and Fall 2008 (with ALEKS) using an ANCOVA. We used ACT scores and prior calculus experience as covariates, in an attempt to compensate for possible differences in the student populations between these two semesters.

3.2 Results

The results of the student perceptions of ALEKS survey from Dr. Klopfenstein's Spring 2008 section are compiled in appendix D. Of the 64 students in the class, 39 completed the survey, for a response rate of 61%. Of these students 37 responded with their names on the survey, and two answered anonymously.

The overall impressions of students were that the review provided by ALEKS was helpful, but also that requiring ALEKS as a part of the grade was a helpful motivating factor. Students, however, did not feel that ALEKS was more helpful than a traditional classroom setting. Nevertheless, multiple students cited the convenience of being able to work at their own pace. The last question, whether or not ALEKS was unnecessary after taking the PACe program yielded interesting results. Most students did not feel that the two technologies were redundant, and

the verbal comments generally led towards the students feeling ALEKS was more helpful. This could be largely in part to ALEKS' adaptive assessment, which does not force students to work through topics they are already adept with.

Additionally, the impressions of instructors involved with MATH 160 during Spring 2008 and previous semesters were that the number of mechanical, algebra-related questions asked in class decreased after the inclusion of ALEKS. This seems to indicate that the inclusion of ALEKS did have a positive effect on the mechanical pre-calculus skills of students.

In order to assess the relationship between ALEKS usage and performance and outcome in the course, we performed simultaneous multiple regression. The means, standard deviations, and intercorrelations for these variables can be found in table 3.1. The combination of a student's initial ALEKS assessment scores, their most recent assessment score up to March 13 (when the ALEKS grade deadline was), and time spent in the ALEKS system to predict total score in MATH 160 was statistically significant $F(3, 58) = 4.159, p < 0.01$. The Beta coefficients can be found in table 3.2. Note that only the initial ALEKS score significantly predicted total score in the course when all three variables were considered. The adjusted R^2 value was 0.134, which indicates that 13.4% of the variance can be explained by this model. This corresponds to $R \approx 0.355$, which according to Cohen [3] is a typical effect size.

Multiple regression was also performed using the same three predictor variables to predict a student's score on the Final Exam. The combination of a student's initial ALEKS assessment scores, their most recent assessment score up to March 13, and time spent in the ALEKS system to predict Final Exam score was statistically significant $F(3, 53) = 2.974, p = 0.04$. In this situation no individual

Table 3.1: Means, Standard Deviations, and Intercorrelations for Total Score in MATH 160 and Predictors ($N = 62$), Spring 2008

Variable	M	SD	Pre-Test	March 13	Time
Total Course Score	533.414	133.8829	0.271*	0.333**	0.130
Predictor Variable					
1. Initial ALEKS Score	57.71	20.275	–	0.363**	-0.487**
2. March 13 ALEKS Score	80.21	15.851		–	0.231*
3. Hours Spent in ALEKS	9.16	7.249			–

* $p < 0.05$, ** $p < 0.01$

Table 3.2: Simultaneous Multiple Regression Analysis Summary for Predicting Total Score in MATH 160 ($N = 62$), Spring 2008

Variable	B	SEB	Beta
Initial ALEKS Score	2.297	1.087	0.348*
March 13 ALEKS Score	1.224	1.248	0.145
Hours Spent in ALEKS	4.912	2.910	0.266
Constant	257.677	85.543	

* $p < 0.05$

predictor variable significantly predicted Final Exam score when all three variables were considered. The adjusted R^2 value was 0.096, which indicates that 9.6% of the variance can be explained by this model. This corresponds to $R \approx 0.31$, which according to Cohen [3] is a slightly smaller than typical effect size.

Using information provided by the ALEKS system we had strong evidence to suggest that a large number of international students performed the initial assessment simultaneously and collaboratively, meaning that the inclusion of their scores would skew the study. Thus, we conducted the same analyses as above considering only domestic students. Using the same three predictors as above to predict total score in MATH 160 was statistically significant $F(3, 34) = 5.572, p = 0.003$. The adjusted R^2 value was 0.270, which indicates that 27% of the variance can be explained by this model. This corresponds to $R \approx 0.52$, which according to

Table 3.3: Means, Standard Deviations, and Intercorrelations for Final Exam Score and Predictors ($N = 57$), Spring 2008

Variable	M	SD	Pre-Test	March 13	Time
Final Exam Score	117.18	31.059	0.197	0.291*	0.163
Predictor Variable					
1. Initial ALEKS Score	58.88	19.819	–	0.307*	-0.497**
2. March 13 ALEKS Score	81.00	14.104		–	0.214
3. Hours Spent in ALEKS	9.05	7.140			–

* $p < 0.05$, ** $p < 0.001$

Table 3.4: Simultaneous Multiple Regression Analysis Summary for Predicting Final Exam Score ($N = 57$), Spring 2008

Variable	B	SEB	Beta
Initial ALEKS Score	0.457	0.263	0.292
March 13 ALEKS Score	0.312	0.328	0.142
Hours Spent in ALEKS	1.209	0.711	0.278
Constant	54.032	23.883	

* $p < 0.05$

Cohen [3] is a larger than typical effect size. ALEKS initial scores were the only significant predictor of total scores in this model.

Multiple regression was also performed using the same three predictor variables to predict a student's score on the Final Exam (for domestic students). The combination of all three predictors to predict Final Exam score was statistically significant $F(3, 32) = 3.117, p = 0.04$. Only ALEKS initial assessment scores significantly predicted Final Exam scores when all three variables were considered. The adjusted R^2 value was 0.154, which indicates that 15.4% of the variance can be explained by this model. This corresponds to $R \approx 0.392$, which according to Cohen [3] is a typical effect size.

To explore the usefulness of including ALEKS as a required component of the MATH 160 course we used ANCOVA to compare the success of students in

Table 3.5: Means, Standard Deviations, and Intercorrelations for Total Score in MATH 160 and Predictors ($N = 38$), Spring 2008 Domestic Students

Variable	M	SD	Pre-Test	March 13	Time
Total Course Score	557.129	124.574	0.442**	0.493**	0.078
Predictor Variable					
1. Initial ALEKS Score	60.08	18.065	–	0.418*	-0.369*
2. March 13 ALEKS Score	78.66	13.273		–	0.177
3. Hours Spent in ALEKS	7.48	5.34			–

* $p < 0.05$, ** $p < 0.01$

Table 3.6: Simultaneous Multiple Regression Analysis Summary for Predicting Total Score in MATH 160 ($N = 38$), Spring 2008 Domestic Students

Variable	B	SEB	Beta
Initial ALEKS Score	2.565	1.227	0.372*
March 13 ALEKS Score	2.904	1.577	0.309
Hours Spent in ALEKS	3.743	3.831	0.160
Constant	146.573	107.634	

* $p < 0.05$

Fall 2007 (without ALEKS) and Fall 2008 (with ALEKS), with prior calculus experience, ACT Math, and ACT Reading scores as covariates. The assumptions of homogeneity of regression slopes and homogeneity of variances were checked and met. After controlling for the covariates the results indicate that there is a statistically significant difference in Final Exam scores for individuals who did and did not use ALEKS, $F(4, 127) = 5.693$, $p < 0.001$, partial $\eta^2 = 0.152$.

However, the statistically significant difference was in the opposite of the direction we expected, because we actually found that the students who did not use ALEKS (Fall 2007) scored higher (see table 3.9). Note that this difference in scores is after a scaling factor has already been applied to the Final Exam scores. The scaling factor used in Fall 2007 was a total of 180 (rather than 200), and in Fall 2008 was 155. These scaling factors were determined by considering the means of

Table 3.7: Means, Standard Deviations, and Intercorrelations for Final Exam Score and Predictors ($N = 32$), Spring 2008 Domestic Students

Variable	M	SD	Pre-Test	March 13	Time
Final Exam Score	120.06	32.275	0.441*	0.270	-0.127
Predictor Variable					
1. Initial ALEKS Score	61.53	17.062	–	0.232	-0.455**
2. March 13 ALEKS Score	80.14	9.008		–	0.082
3. Hours Spent in ALEKS	7.46	5.338			–

* $p < 0.05$, ** $p < 0.001$

Table 3.8: Simultaneous Multiple Regression Analysis Summary for Predicting Final Exam Score ($N = 32$), Spring 2008 Domestic Students

Variable	B	SEB	Beta
Initial ALEKS Score	0.806	0.347	0.426*
March 13 ALEKS Score	0.598	0.587	0.167
Hours Spent in ALEKS	0.323	1.082	0.053
Constant	20.107	46.374	

* $p < 0.05$

the top 5% of scores and top 10% of scores on the exam, in order to estimate a realistic maximum possible score.

Table 3.9: Adjusted and Unadjusted Final Exam Score Means and Variability Using Prior Calculus Experience, ACT Math Scores, and ACT Reading Scores as Covariates

	N	Unadjusted		Adjusted	
		M	SD	M	SE
No ALEKS	60	0.696	0.18	0.722	0.026
ALEKS	72	0.642	0.22	0.621	0.023

3.3 Discussion

In the analysis of the relationship between ALEKS scores and success in MATH 160 during the Spring 2008 semester we will focus on the results for domestic students only, because as suggested before, we have considerable evidence

to suggest that the results of the initial ALEKS assessment were skewed for many international students. Moreover, we believe it is most important to focus on the relationship between ALEKS scores and Final Exam scores rather than total scores, because the ALEKS score is actually included as part of a student's total score. Making these considerations we found that a student's initial ALEKS score was a significant predictor of Final Exam score, and notably a better predictor than a student's ALEKS score from later in the semester.

These results suggest that including the instructional aspect of ALEKS as a component of MATH 160 did not have a significant impact on success, as a student's score before working with the ALEKS system was more predictive than his or her score after spending time in the system. Further analyses strongly support this conclusion, as we found no significant improvement in success during the semesters in which ALEKS was used compared to semesters where it was not used. Contrarily, we found a decrease in the success rate between Fall 2007 and Fall 2008 semesters, where ALEKS was used in Fall 2008 but not Fall 2007. It is likely that the decrease in success rate can be mostly attributed to differences in populations between semesters and differences in the exams given. Nevertheless, it is also possible that spending time in the ALEKS system detracted from the amount of time students spent working elsewhere in the course, resulting in a decrease in success rates. Either way, we do not feel there is any compelling reason to continue to include ALEKS as a required component of the MATH 160 course in the future.

Alison Ahlgren at The University of Illinois at Urbana-Champaign (UIUC) has also spent considerable time investigating the usage of ALEKS. Dr. Ahlgren's investigations showed that ALEKS was a strong predictor of success in their introductory calculus courses. Such findings informed the current placement policy

at UIUC, in which all students complete the ALEKS initial assessment at the beginning of the semester to determine their placement in mathematics courses [10]. Since implementing this new placement policy UIUC has seen their D, F, W rate drop from around 38% to around 18% (A. Ahlgren, personal communication, October 30, 2008).

The findings at UIUC contrast with our findings with ALEKS. As a predictor the ALEKS initial score alone only explained about 20% of the variance in our Final Exam scores, meaning that it is not a particularly strong predictor. Thus, we suspect that although using ALEKS for placement may have some positive impact on our success rates, that we are unlikely to see such a drastic improvement as at UIUC. We are not sure of the reason for the discrepancy in apparent findings with ALEKS, but conjecture there may be considerable differences in the syllabi of MATH 160 and the calculus course offered at UIUC.

Chapter 4

Predicting Success

The results of our previous investigation indicated that while the inclusion of ALEKS as a component of MATH 160 did not have a positive impact on student success, ALEKS might still be useful as a predictor of student success. Thus, we decided to further investigate the predictive value of ALEKS.

4.1 Method

In order to investigate the predictive value of ALEKS assessments we considered the entire population (all sections) of MATH 160 during the Fall 2008 semester. We performed multiple regression using ALEKS initial assessment scores and Exam 1 scores as predictor variables, and both Final Exam scores and total course scores as dependent variables. We also used logistic regression to determine how well we could classify the students that would and would not succeed using these variables.

4.2 Results

When we performed simultaneous multiple regression we found that the combination of ALEKS initial assessment scores and Exam 1 scores was a statistically

significant predictor of total score, $F(2, 386) = 277.982, p < 0.001$. Both ALEKS initial scores and Exam 1 scores were significant predictors with both variables considered. The adjusted R^2 was 0.588, indicating that 58.8% of the variance in total course scores can be explained by this model. This corresponds to $R \approx 0.767$, which according to Cohen [3] is a much larger than typical effect size.

Since both ALEKS and Exam 1 scores are contained as a part of a student's total course score, we performed the same analysis instead using the Final Exam score as the dependent variable. The combination of ALEKS initial assessment scores and Exam 1 scores was a statistically significant predictor of Final Exam score, $F(2, 328) = 168.441, p < 0.001$. Both ALEKS initial scores and Exam 1 scores were significant predictors with both variables considered. The adjusted R^2 was 0.504, indicating that 50.4% of the variance in Final Exam scores can be explained by this model. This corresponds to $R \approx 0.71$, which according to Cohen [3] is a much larger than typical effect size. It is worth noting that by itself the Exam 1 score explains 49% of the variance, so in this situation the addition of the ALEKS initial assessment score does very little to increase the predictive value of the model.

Table 4.1: Means, Standard Deviations, and Intercorrelations for Final Exam Score and Predictors ($N = 331$), Fall 2008

Variable	M	SD	ALEKS	Exam 1
Final Exam Score	93.19	36.3869	0.389**	0.700**
Predictor Variable				
1. Initial ALEKS Score	58.63	16.686	–	0.388**
2. Exam 1 Score	49.56	15.982		–

** $p < 0.001$

Given that both initial ALEKS scores and Exam 1 scores were found to be significant predictors of Final Exam score and total score in MATH 160 we de-

Table 4.2: *Simultaneous Multiple Regression Analysis Summary for Predicting Final Exam Score (N = 331), Fall 2008*

Variable	B	SEB	Beta
Initial ALEKS Score	0.301	0.092	0.138**
Exam 1 Score	1.472	0.096	0.647**
Constant	2.540	5.801	

** $p < 0.001$

cided to investigate their ability to predict success in MATH 160, using a logistic regression. The results of the model were that the Cox & Snell $R^2 = 0.437$ and the Nagelkerke $R^2 = 0.583$. When both variables were considered only Exam 1 scores were found to be statistically significant ($p < 0.001$).

As a result we decided to run a logistic regression using only ALEKS as a predictor. We found that ALEKS was a statistically significant predictor of success in MATH 160 ($p < 0.001$), and that the Cox & Snell $R^2 = 0.111$ and the Nagelkerke $R^2 = 0.148$. Nevertheless, the model only properly classified 64.2% of students who did not succeed and 64.5% of students who did succeed.

Corresponding to these results we ran a logistic regression using only Exam 1 score as a predictor. Exam 1 score was found to be a statistically significant predictor of success in MATH 160 ($p < 0.001$). The model properly classified 80.9% of students who did not succeed and 79.9% of students who did succeed. The regression equation given by the model was

$$z = -5.887 + 0.129 \cdot (\text{Exam 1 Score}),$$

which we converted into a predictor of the probability of success using the transformation

$$P = \frac{1}{1 + e^{-z}}.$$

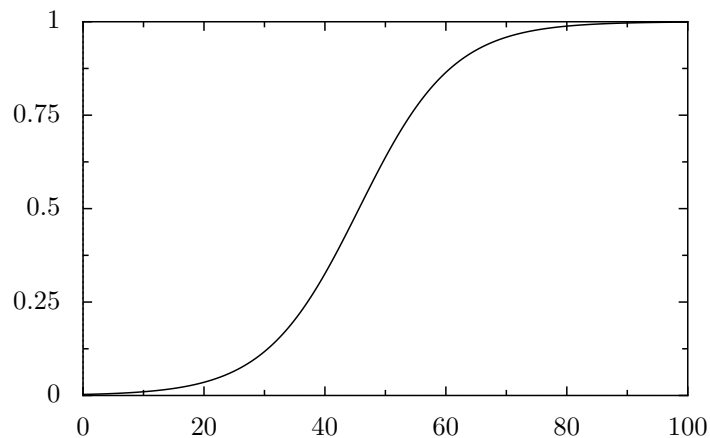


Figure 4.1: Probability of Success Given Exam 1 Score.

See figure 4.1 for a graph of the model predicting student success in MATH 160.

In order to put the above model into perspective, it is helpful to consider the score distributions in Fall 2008 for Exam 1 ($N = 437$). The mean score was 45.12, with a standard deviation of 17.406. The average score of the top 5% of the population was 79.3, and of the top 10% of population was 75.2. Correspondingly, scores were scaled to be out of 80 points rather than 100 in the calculation of final grades (the scores used in the predictive model were unscaled). A summary of the cumulative score distributions for unscaled Exam 1 scores are given in table 4.3.

Table 4.3: *Cumulative Score Distribution for Exam 1, Fall 2008 ($N = 437$)*

Exam 1 Score	Number of Students Below Cutoff	Percentage of Population
20	34	8.0%
30	95	21.9%
35	141	32.3%
40	183	42.1%
45	224	51.3%

Finally, we also performed logistic regression using prior calculus experience in addition to Exam 1 scores as a predictor, but this made no improvement upon

the model that used only Exam 1 scores. This analysis was performed using only Dr. Klopfenstein's section, as we did not collect data as to whether students had prior calculus experience for the other sections of MATH 160.

4.3 Discussion

Using the entire MATH 160 population from Fall 2008 showed that both ALEKS and Exam 1 scores were significant predictors of Final Exam scores. Notably, however, is that when Exam 1 information was already available, adding the ALEKS initial assessment provided very little new predictive value, increasing the explained variance by less than 2%. It is clear from these analyses that although the ALEKS initial assessment does have some predictive value, Exam 1 scores are much more useful in predicting a student's outcome in the course. Moreover, when we attempted to use ALEKS scores to predict success, only 64% of cases were properly classified. This is in contrast to the 80% of cases which were properly classified using Exam 1 scores.

Chapter 5

Recommendations and Future Directions

Despite instructor impressions that the inclusion of ALEKS reduced the number of mechanical pre-calculus questions asked in class, the inclusion of ALEKS as a required component of MATH 160 failed to elicit any improvement in student success. In fact, we saw a decrease in success rate between Dr. Klopfenstein's sections of MATH 160 after the inclusion of ALEKS. Whether or not we can attribute this decrease to the inclusion of ALEKS is questionable, but irrespective we have found no compelling evidence to support the usage of ALEKS in MATH 160 in future semesters.

Our analyses also showed that ACT Math scores were not a significant predictor of success. We believe that a student's ACT Math scores are largely indicative of his or her mechanical pre-calculus skills. This finding combined with the fact that the inclusion of ALEKS did not improve success in MATH 160 seems to suggest that mechanical pre-calculus skills are not as important to success in MATH 160 as originally believed. While mechanical pre-calculus skills are undoubtedly important to success in MATH 160, it seems that we would be more likely to improve success rates by focusing on interventions related to other important factors, such as developing students' conceptual thinking and study skills.

Although we found that the inclusion of ALEKS as a required component of MATH 160 did not help in improving success rates, we were still interested in the usage of ALEKS as a predictor of success in MATH 160, as it might still be useful for placement purposes. However, when we found that ALEKS initial assessment scores properly classified students who succeeded or did not succeed in only 64% of cases we realized that ALEKS was not especially useful as a predictor for success in MATH 160 either.

We did find that a student's Exam 1 score is a very strong predictor of student success. Typically, however, an Exam 1 score is not a useful predictor because it already requires a student to have committed 4 weeks to the course, making it unacceptable for placement purposes. Nevertheless, we believe this predictor can be used to make an informed decision as to how to implement an intervention to help students who are struggling in the course. We believe the focus of the intervention should be improving the conceptual mathematical thinking skills of students. This belief is informed by both the lack of improvement afforded by ALEKS, and the fact that both ACT Reading scores and conceptual Algebra Pretest scores had some predictive value of student success in MATH 160. Such an intervention is further supported by the fact that a student's Exam 1 score is such a strong predictor of success, with the material covered on Exam 1 being the most conceptual part of MATH 160.

Our recommendation is the design and implementation of a new, experimental course, with working title "Concepts for Calculus." Ideally this course would be implemented as a 10 or 11 week course, offered during the last 2/3 of the semester, so that Exam 1 scores from MATH 160 students could be used to place or advise students into this alternate course. The idea is that MATH 160 students would

complete Exam 1 as usual, and those who were identified as having a low chance of success in MATH 160 would be strongly advised to drop MATH 160 and enroll in Concepts for Calculus for the remainder of the semester.

We feel this design is particularly advantageous because most students who perform poorly on Exam 1 will not benefit greatly from continuing in the course, because they are not prepared to understand the new concepts introduced. Moreover, many students who perform poorly on Exam 1 become discouraged and stop attending class. Nevertheless, many of these students remain enrolled because of financial aid or other reasons. As a result, students who do not succeed in MATH 160 during one semester may not be much better prepared to succeed during a subsequent semester. However, by moving such students to an alternate course Concepts for Calculus we can utilize, rather than waste, the remaining time during the semester in which students began MATH 160, and hopefully prepare them to succeed during the next semester. This gives students a chance to experience success, rather than failure, in university-level mathematics.

The thrust of the Concepts for Calculus course would be conceptual mathematical thinking skills, with an emphasis on the function concept and the usefulness of multiple perspectives, both of which are critical in MATH 160. In this experimental course students would develop the type of thinking required by calculus in the more familiar context of pre-calculus topics. This would provide students with scaffolding for the development of mathematical thinking, which they would be able to apply and further develop in the less-familiar context of calculus.

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Appendix A

Conceptual Algebra Pretest

For all questions, explain fully why you believe your answer is correct.

1. (a) Without using a calculator, list the following numbers in order from smallest to largest

$$2/5, 5/8, \frac{\sqrt{2}}{2}, 1/2, 7/9$$

- (b) Locate and label the numbers listed above on the number line.

2. In your opinion, is $(AB)^2$ equal to A^2B^2 ?
3. In your opinion, is \sqrt{AB} equal to $\sqrt{A}\sqrt{B}$.
4. Is $(x + y)^2$ (A) never, (B) sometimes, or (C) always equal to $x^2 + y^2$?
5. Is the equation $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$ (A) never, (B) sometimes, or (C) always true?
6. In your opinion, are the rational functions $W = \frac{2B^2 + B + 4}{B^2 + 2}$ and $W = \frac{B + 6}{3}$ equal?
7. In your opinion, are the rational functions $y = \frac{x^2 - 1}{x + 1}$ and $y = \frac{x^2 - 2x + 1}{x - 1}$ equal?

Appendix **B**

Conceptual Algebra Pretest Rubric

For all questions, explain fully why you believe your answer is correct.

1. (a) Without using a calculator, list the following numbers in order from smallest to largest

$$2/5, 5/8, \frac{\sqrt{2}}{2}, 1/2, 7/9$$

(1/4): One of the simple fractions, or multiple numbers out of order.

(2/4): $\sqrt{2}/2$ somewhere else, but other numbers okay.

(3/4): $\sqrt{2}/2$ transposed with either $5/8$ or $7/9$, but numbers otherwise in the proper order.

(4/4): All numbers in the proper order ($2/5, 1/2, 5/8, \sqrt{2}/2, 7/9$)

- (b) Locate and label the numbers listed above on the number line.

(1/4): One or more numbers outside of the range 0 to 1, or numbers inconsistent with (a) (if (a) blank, then one or more numbers outside of 0 and 1).

(2/4): All numbers other than $\sqrt{2}/2$ are between 0 and 1, and consistent with (a) (if (a) blank, then $\sqrt{2}/2$ outside of 0 and 1, other numbers inside 0 and 1).

(3/4): Numbers between 0 and 1, and consistent with (a) (if (a) blank, then numbers between 0 and 1 but out of order).

(4/4): All numbers in the proper order.

2. In your opinion, is $(AB)^2$ equal to A^2B^2 ?

(1/4): No. -or- Yes, because if we plug in a specific value the equation holds true.

- (2/4): Yes, because of the rules of algebra (no explanation why or an improperly cited rule).
- (3/4): Yes, because for any value we plug in the equation is true. Alternatively, an incomplete attempt at displaying commutativity.
- (4/4): Yes, because $(AB)^2 = ABAB = AABB = A^2B^2$ (the commutative property of multiplication).

3. In your opinion, is \sqrt{AB} equal to $\sqrt{A}\sqrt{B}$.

- (1/4): No. -or- Yes, because if we plug in a specific value the equation holds true.
- (2/4): Yes, because of the rules of algebra (no explanation why).
- (3/4): Yes, because both sides of the equation are nonnegative, the truth of the above statement is equivalent to the truth of the square of the statement. Thus, $AB = (\sqrt{A}\sqrt{B})^2 = \sqrt{A}\sqrt{B}\sqrt{A}\sqrt{B} = \sqrt{A^2}\sqrt{B^2} = AB$. Alternatively, $\sqrt{AB} = (AB)^{1/2} = A^{1/2}B^{1/2} = \sqrt{A}\sqrt{B}$.
- (4/4): Sometimes. If A and B are negative, then the right hand side of the equation is not defined, and thus, not equal to the left hand side which is defined. If A and B are positive, then both expressions are equal.

4. Is $(x + y)^2$ (A) never, (B) sometimes, or (C) always equal to $x^2 + y^2$?

- (1/4): (C) Always equal, because this is what it means to square $(x + y)$.
- (2/4): (A) Never equal. $(x + y)^2 = x^2 + y^2 + 2xy$. Alternatively, sometimes equal, for somewhat flawed reasoning.
- (3/4): (B) Sometimes equal. They are equal if $x = y = 0$ and not equal if $x = y = 1$ (other correct examples will suffice).
- (4/4): (B) Sometimes equal. $(x + y)^2 = x^2 + y^2 + 2xy$ so if $2xy = 0$, then both expressions are equal.

5. Is the equation $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$ (A) never, (B) sometimes, or (C) always true?

- (1/4): (C) Always true, because this is how we add square roots.
- (2/4): (A) Never true. The rules of algebra do not allow us to split square roots like above.
- (3/4): (B) Sometimes true. They are equal if $x = y = 0$ and not equal if $x = y = 1$ (other correct examples will suffice).

(4/4): (B) Sometimes true. If $x + y$ is positive but either x or y is negative, then the left side will be defined and the right undefined, so not equal. If both sides of the equation are defined, since they are non-negative, the truth of the above statement is equivalent to the truth of the square of the above statement. $x + y = (\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{x}\sqrt{y}$ is a true statement iff $2\sqrt{x}\sqrt{y} = 0$.

6. In your opinion, are the rational functions $W = \frac{2B^2 + B + 4}{B^2 + 2}$ and $W = \frac{B + 6}{3}$ equal?

(1/4): Always equal. If we let $B = 0$ they are equal.

(2/4): Never equal. It is not possible to simplify one equation to the other.

(3/4): Sometimes equal. They are equal for $B = 0$, and not for $B = 2$.

(4/4): Sometimes equal. They intersect at the points $B = -1, 0, 1$, but are otherwise unequal.

7. In your opinion, are the rational functions $y = \frac{x^2 - 1}{x + 1}$ and $y = \frac{x^2 - 2x + 1}{x - 1}$ equal?

(1/4): Never equal.

(2/4): Always equal. By factoring/simplifying both sides of the equation are the same.

(3/4): Sometimes equal. The left hand quotient is not defined at $x = -1$ (or the left hand not defined at $x = 1$), but they are equal for $x = 0$.

(4/4): Sometimes equal. We can factor/simplify the quotients to be equal, but implicit in doing so is the assumption that $x \neq \pm 1$, the points where one of the two quotients is not defined. Thus, they are equal for all $x \neq \pm 1$.

Additional Grading Notes:

- If an answer is given without explanation, the answer receives a score of 1/4 (except for question 1, where explanation is not required).
- Plugging in a single example case and concluding that something holds in all cases warrants 1/4. If a student plugs in a value to see that functions are not equal, and then concludes the functions are not equal because they are different at a single point, this warrants 3/4.
- Scratched out work will be ignored - not counted in favor of or against the student.

Appendix C

ALEKS Survey

Survey on Student Perceptions of ALEKS in MATH 160, Spring 2008

Please respond honestly to the following survey questions. We will use the information students provide on this survey to help us decide whether and how to incorporate ALEKS into MATH 160 in the future. Your signature is not required. However, if you complete this survey thoughtfully and sign your name, we will replace one of your lowest homework scores with a 10. Thank you for your help. Prof. Ken Klopfenstein, MATH 160 Course Coordinator

NAME: (printed) _____ SIGNATURE: _____

1. I improved my skill with algebra by working on ALEKS.

Strongly disagree	Disagree	No opinion	Agree	Strongly Agree
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 Comments:

2. I understood clearly how ALEKS worked and what I needed to do to earn the score I wanted.

Strongly disagree	Disagree	No opinion	Agree	Strongly Agree
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 Comments:

3. The algebra I learned from using ALEKS helped me in MATH160.

Strongly disagree	Disagree	No opinion	Agree	Strongly Agree
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 Comments:

4. I did the work on ALEKS only because it counted toward my grade.

Strongly disagree	Disagree	No opinion	Agree	Strongly Agree
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 Comments:

5. My work on ALEKS gave me more confidence in my skill with algebra.

Strongly disagree	Disagree	No opinion	Agree	Strongly Agree
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 Comments:

6. The algebra I saw on ALEKS was almost all review.

Strongly disagree	Disagree	No opinion	Agree	Strongly Agree
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 Comments:

7. ALEKS was more helpful for me than traditional classroom lectures and homework.

Strongly disagree	Disagree	No opinion	Agree	Strongly Agree
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 Comments:

8. I would recommend ALEKS to a friend taking MATH 160.

Strongly disagree	Disagree	No opinion	Agree	Strongly Agree
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 Comments:

9. The ALEKS program is easy to use.

Strongly disagree	Disagree	No opinion	Agree	Strongly Agree
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 Comments:

10. After taking the precalculus courses in the PACe Program, I didn't need ALEKS.

Not applicable	Strongly disagree	Disagree	No opinion	Agree	Strongly Agree
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 Comments:

Appendix D

ALEKS Survey Responses (Spring 2008)

Table D.1: *Student Responses to ALEKS Survey (N = 39)*

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Mean	3.97	3.95	3.74	3.74	3.82	3.45	2.64	3.46	3.49	2.97
Median	4	4	4	4	4	4	2	4	4	2
Mode	4	4	4	4	4	4	2	4	4	2
Std. Dev	.707	.999	.85	.938	.823	1.201	1.063	1.072	.997	1.224
Min	2	2	1	2	2	1	1	1	1	2
Max	5	5	5	5	5	5	5	5	5	5

1 = Strongly Disagree, 5 = Strongly Agree

Survey Items:

1. I improved my skill with algebra by working with ALEKS.
2. I understood clearly how ALEKS worked and what I needed to do to earn the score I wanted.
3. The algebra I learned from ALEKS helped me in MATH 160.
4. I did the work on ALEKS only because it counted toward my grade.
5. My work on ALEKS gave me more confidence in my skill with algebra.
6. The algebra I saw on ALEKS was almost all review.
7. ALEKS was more helpful for me than traditional classroom lectures and homework.

8. I would recommend ALEKS to a friend taking MATH 160.
9. The ALEKS program is easy to use.
10. After taking the pre-calculus courses in the PACe program, I didn't need ALEKS.

Student Written Comments:

1. I improved my skill with algebra by working with ALEKS.
 - There was a great amount of practicing.
 - ALEKS is a helpful tool - it is like a review - refreshes your mind.
 - I needed to refresh my algebra skills, and I did by using ALEKS.
 - There were a lot of algebra problems for me.
 - It helped me with some new skills.
 - Very repetitive; not necessarily a good thing.
 - It's been years since I've taken an algebra course and this program really helped.
 - It was a good refresher, but you can't replace social/visual learning.
 - It was review.
 - I agree, but I would say barely, very slight improvement.
 - The only thing I didn't like is I had to start a new assessment every time.
 - I hated the program. Trying to get it to work or do what I wanted was harder than the actual work.
 - I realized after doing the first ALEKS assignment I needed some work. By the first test I had improved my algebra skills.
2. I understood clearly how ALEKS worked and what I needed to do to earn the score I wanted.
 - Make it clear that ALEKS will reset after a period of time.
 - It was easy to work on ALEKS and understand it.
 - It was very easy to understand.
 - It's easy to understand.

- The first assignment on it helped me to know how to work on it. For example, how to draw.
 - It was easy to work with.
 - I was just a little confused with all of the assessments we had to do and which score counted.
 - Straight forward.
 - I was so confused the first month that I was working on ALEKS. It took me forever to figure out the pie, doing certain things to get credit, and so on.
 - Yes, easy to understand.
 - At first I didn't know I had to take the test again.
 - I did understand how the ALEKS program worked. It was very easy and straight forward.
3. The algebra I learned from ALEKS helped me in MATH 160.
- More practicing helped me understand more algebra.
 - Yes it is like a review, it refreshed my mind.
 - I needed a lot of basic skills for MATH 160.
 - It helped me in other courses not only MATH 160.
 - I learned new skills that helped me with algebra.
 - Not really cause the tests are more conceptual than actual math problems.
 - Most of the algebra I used in MATH 160 I learned in high school previous to this class.
 - I really didn't feel that ALEKS helped me in MATH 160.
 - The algebra I learned really made learning calculus a lot easier.
4. I did the work on ALEKS only because it counted toward my grade.
- I don't think that if it wasn't counted toward the grade that I would do it.
 - I'm glad it counted or I wouldn't have made time to do it.
 - Also because I knew it would help me with my algebra.
 - I worked on ALEKS because I wanted to refresh my algebra skills after a long break.

- I did it because I felt it helped.
- A requirement does place emphasis on doing the application.
- It was an easy 75 points.
- While it did help me a little, I felt it was pointless.
- Yes, I remembered it the night before the first test, and ended up spending too much time on it and not studying.
- At first I did, but it helped a lot.
- I did the ALEKS work so I could improve my proficiency as I continue on in calculus.

5. My work on ALEKS gave me more confidence in my skill with algebra.

- Working with ALEKS helps you to remember how to do things.
- Sometimes I know how to solve a problem using algebra but I am not sure. ALEKS helped me be more confident.
- I learned new things, but no more confidence.
- Some of the algebra in ALEKS my friend in calc 2 couldn't help me with.
- Didn't really affect my confidence.
- I now feel confident in my algebra skills.

6. The algebra I saw on ALEKS was almost all review.

- I didn't see the different denominator addition with different numerals before.
- It was the review I needed.
- There were some problems that I haven't seen or solved before.
- I saw some things that were new for me.
- I had not seen algebra in 9 years (high school calc).
- At some point I had done everything, but it was a long time ago.
- It wasn't all review, but there wasn't enough review of algebra we use in calc like trig identities and foiling/expanding.
- It was review from high school and PACE.
- Except for the higher up algebra questions, it was about 90

- Yes, almost all of it. I just had to remember from high school the algebra, except for logs.
- It was mainly a review of concepts I had previously learned but I think that we all could use a review.

7. ALEKS was more helpful for me than traditional classroom lectures and homework.

- Convenient!
- I like the classroom discussion I think a bit better.
- Yes, mostly because I could do it as many times as needed, forcing me to understand.
- The program helps but I still believe classroom experience is irreplaceable.
- Traditional class would have been more effective.
- I like classroom lectures more than online homework.
- It sucked at helping with my questions that I had. I don't even like the online math classes that CSU offers.
- I learn best from examples and then relating that to homework, so no, ALEKS was less helpful.
- They are both equally good, but we should have one or the other during calculus.

8. I would recommend ALEKS to a friend taking MATH 160.

- Great resource!
- Yes, because one need to learn a lot lot of algebra for MATH 160.
- It helps a lot.
- Depends from person to person.
- Only if it was required by the course for a grade.
- I would recommend it to them only if they had to take it for class.

9. The ALEKS program is easy to use.

- Sometimes good sometimes bad.
- Really easy.

- ALEKS as a program was very basic and easy to use.
 - It was kinda hard getting the program to work on my computer.
 - No, I had a lot of issues with the website.
 - ALEKS was easy to use, and when I was confused it was easy to find help in the program.
 - Only because it barely works.
 - Very good instructions on how to use it were given.
10. After taking the pre-calculus courses in the PACe program, I didn't need ALEKS.
- ALEKS is ongoing help and review. PACE is just there for you when you are enrolled in the class.
 - Only complaint is that some sections were overly tedious.
 - I still needed ALEKS' help.
 - I had one semester between precalculus and m160, and that me forget some skills.
 - The precalculus course didn't have the algebra that MATH 160 needed, as ALEKS did. so ALEKS is important for this class.
 - ALEKS is a new thing which is more helpful than the PACe program.
 - PACe not that helpful!
 - I took all of the PACe program courses and they helped a lot more than ALEKS. ALEKS was more of a review.
 - I only had to take the trig PACe courses - ALEKS helped me a lot.
 - I dislike the general idea of online learning.
 - PACe was hell. ALEKS was easier to use and review.
 - I felt that PACe and ALEKS pretty much repeated each other, but ALEKS did offer more.
 - I only took 125 and 126 so that covered the trig functions and stuff. The rest I just had to remember from awhile ago.
 - ALEKS helps a lot!

Appendix	E
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Mathematical Background Survey

Please answer all of the following questions to the best of your ability, using the space provided below. Explain fully why you believe your answers are correct (answers without explanation will be counted as incorrect). You are welcome to use a calculator, but we do not think it will be helpful. We will use feedback from these surveys in order to improve the MATH 160 course in the future, so your time and effort is greatly appreciated.

Name: _____ Section: _____

1. Have you previously taken a calculus course (in high school, another college, MATH 160 a previous semester, etc)?

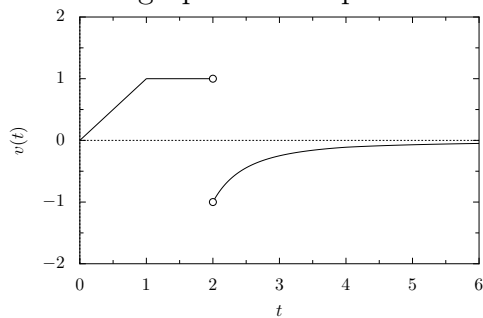
2. At the beginning of a storm, a raindrop counter starts with a value of 0. Each time a raindrop hits the roof of a specific building, the counter increases by one. The raindrop counter tells how many drops of rain have hit the roof after a given amount of time. Does the raindrop counter describe a function? Explain why or why not.

3. Are the functions $f(x) = \frac{x^2 - 1}{x + 1}$ and $g(x) = \frac{x^2 - 2x + 1}{x - 1}$ equal? Explain why or why not.
4. Let $B(T) = (T + 1)^2 / (T - 1)^2$. What is the largest subset of the real numbers that can be chosen for this function's domain? What is the corresponding range?
5. What do we mean when we write $2^{1/n}$ (what is the meaning of the $(1/n)^{th}$ power of 2)?
6. Explain how one might find $\sqrt[3]{2}$, without using a calculator (recall $\sqrt[3]{2}$ means $2^{1/3}$).

7. Use the unit circle to explain why $\cos(\pi/3) = \cos(5\pi/3)$.

8. Give an example of a pair of functions f and g so that $(f \circ g)(x) \neq (g \circ f)(x)$.

9. Does the graph below represent a function? Explain why or why not.



10. Does the graph below represent an invertible function? Explain why or why not.

